15. With mean μ the exponential distribution has parameter $1/\mu$, and median M_0 which satisfies $P[X > M_0] = \frac{1}{2} = e^{-M_0/\mu} \rightarrow M_0 = -\mu \ln \frac{1}{2}$.
Next year, with mean 1.1μ, the exponential distribution has parameter $1/1.1\mu$, and median M_1 which satisfies $P[X' > M_1] = \frac{1}{2} = e^{-M_1/1.1\mu} \rightarrow M_1 = -1.1\mu \ln \frac{1}{2}$.
Thus, $M_1/M_0 = 1.1$. Answer: C

16. The amount paid by the insurer is a mixture of 3 components.
There is a 0.94 probability that the damage and claim is 0.
There is a 0.04 probability of partial damage, and a 0.02 probability of total loss on the car.
If there is total loss on the car, the insurer pays 14,000 (15,000 minus the deductible of 1,000). If there is partial damage, then the expected amount paid by the insurer after deductible is $1000\int_1^{15}(x-1)(0.5003e^{-x/2})\,dx$. This integral can be simplified by integration by parts: $u(x) = x - 1$, $dv(x) = 0.5003e^{-x/2}dx \rightarrow v(x) = -1.0006e^{-x/2}$.

$$\int_1^{15}(x-1)(0.5003e^{-x/2})\,dx = (x-1)(-1.0006e^{-x/2})\Big|_{x=1}^{x=15} - \int_1^{15} -1.0006e^{-x/2}dx$$
$$= -0.00775 + \int_1^{15}1.0006e^{-x/2}dx = -0.00775 + 1.21268 = 1.205.$$

Therefore, if there is partial damage, the expected amount paid by the insurer is $1000 \times 1.205 = 1205$. The overall expected amount paid by the insurer is $0.94 \times 0 + 0.04 \times 1205 + 0.02 \times 14,000 = 328.20$. Answer: B

17. A_1 - the number of accidents in the first year,
A_2 - the number of accidents in the second year.
We wish to find $P[A_2 = 0|A_1 = 0] = \frac{P[(A_2=0)\cap(A_1=0)]}{P[A_1=0]}$.
We find these probabilities by conditioning over the age of the driver:

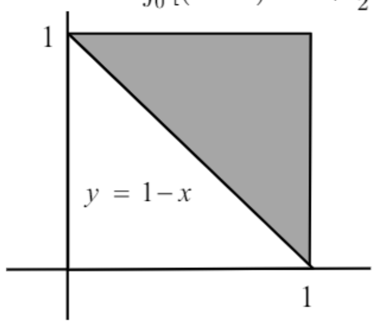
$$P[A_1 = 0] = P[A_1 = 0|\text{high risk}] \times P[\text{high risk}] + P[A_1 = 0|\text{low risk}] \times P[\text{low risk}]$$
$$= (0.98)^2 \times 0.25 + 0.99 \times 0.75 = 0.9826, \text{ and}$$
$$P[(A_1 = 0) \cap (A_2 = 0)] = P[(A_1 = 0) \cap (A_2 = 0)|\text{high risk}] \times P[\text{high risk}]$$
$$+ P[(A_1 = 0) \cap (A_2 = 0)|\text{low risk}] \times P[\text{low risk}]$$
$$= [(0.98)^2]^2 \times 0.25 + (.99)^2 \times 0.75 = .96567$$
$$\rightarrow P[A_2 = 0|A_1 = 0] = \frac{0.96567}{0.9826} = 0.9828.$$
Answer: B

18. The losses Y not paid by the insurance policy can be described in terms of total losses X as follows $Y = \begin{cases} X & \text{if } X \le 2 \\ 2 & \text{if } X > 2 \end{cases}$. The expected value of Y is

$$E[Y] = \int_{.6}^{2} x \times f(x)\,dx + \int_2^{\infty} 2 \times f(x)\,dx = \int_{.6}^{2} x \times \frac{2.5(0.6)^{2.5}}{x^{3.5}}\,dx + \int_2^{\infty} 2 \times \frac{2.5(0.6)^{2.5}}{x^{3.5}}\,dx$$
$$= \int_{.6}^{2} \frac{2.5(0.6)^{2.5}}{x^{2.5}}\,dx + \frac{5(0.6)^{2.5}}{(-2.5)x^{2.5}}\Bigg|_{x=2}^{x=\infty} = \frac{2.5(0.6)^{1.5}}{(-1.5)x^{1.5}}\Bigg|_{x=.6}^{x=2} + \left(-0 - \frac{5(0.6)^{2.5}}{(-2.5)2^{2.5}}\right)$$
$$= \frac{2.5(0.6)^{2.5}}{(-1.5)2^{1.5}} - \frac{2.5(0.6)^{2.5}}{(-1.5)(.6)^{1.5}} + 0.09859 = 0.934.$$
Answer: C

19. Given a function $h(x, y)$, to find the expectation of that function of the two random variables X and Y with joint density function $f(x, y)$, we calculate the integral $\int\int h(x, y) \times f(x, y)\, dy\, dx$, where the integral is taken over the two dimensional region of density for the joint distribution. Any region over which $h(x, y) = 0$ can be ignored. In this problem, $h(x, y)$ is the insurer's payment when the random losses are amounts x and y. What it means to say that the policy has a deductible of 1 is that the insurer pays losses in excess of 1. The insurer pays $X + Y - 1$ if $X + Y > 1$ and if $X + Y \leq 1$, the insurer pays 0. The amount paid by the insurer can also be described as $h(x, y) = max\{x + y - 1\,,\, 0\}$.
The bivariate distribution of X and Y has density only on the unit square, and $x + y > 1$ in the shaded upper triangular region of the unit square (above the line $x + y = 1$), so that $h(x, y) = 0$ on the lower triangular region. Therefore, the expectation can found by integrating over the upper triangular region. The upper triangular region corresponds to $\{(x, y) : 0 \leq x \leq 1\,,\, 1 - x \leq y \leq 1\}$
The expectation is

$$\int\int h(x, y) \times f(x, y)\, dy\, dx = \int_0^1 \int_{1-x}^1 (x + y - 1) \times 2x\, dy\, dx$$
$$= \int_0^1 [(x - 1) \times x + \tfrac{1}{2}(1 - (1 - x)^2)] \times 2x\, dx = \int_0^1 x^3\, dx = \tfrac{1}{4}$$



Answer: A

20. The insurance payment is $Y = \begin{cases} 0 & X \leq c \\ X - C & C < X < 1 \end{cases}$.
The insurance payment is less than .5 if the $X - C < 0.5$, or equivalently, if $X < C + 0.5$.
It must be true that $C \leq 0.5$, because if $C > 0.5$ then $C + 0.5 > 1$ and then $P[X < C + 0.5] = 1$ since $P[X < 1] = 1$. $P[X < C + 0.5] = \int_0^{c+0.5} 2x\, dx = (c + 0.5)^2$.
In order for this to be equal to 0.64 we must have $(c + 0.5)^2 = 0.64 \rightarrow c + 0.5 = 0.8$ (we ignore the negative square root since $X > 0$) $\rightarrow c = 0.3$. Answer: B

21. If the repair amount is X, then the insurance pays

$$Y = \begin{cases} 0 & X \leq 250 \\ X - 250 & X > 250 \end{cases}$$

Then $E[Y] = \int_{250}^{1500} (x - 250) \times f_X(x)\, dx = \int_{250}^{1500} (x - 250) \times \frac{1}{1500}\, dx = 520.83$
and $E[Y^2] = \int_{250}^{1500} (x - 250)^2 \times f_X(x)\, dx = \int_{250}^{1500} (x - 250)^2 \times \frac{1}{1500}\, dx = 434,028$.

Then $Var[Y] = 434,028 - (520.83)^2 = 162,764$, and the standard deviation of Y is $\sqrt{Var[Y]} = 403$. Answer: B

22. The probability of a non-zero claim occurring is $q = 1 - 0.3 = 0.7$.

B can take on the values $50,\ 200,\ 500,\ 1000$ or $10,000$, with probabilities

$P[B = 50] = P[X = 50|\text{non-zero claim occurs}] = \frac{P[X=50]}{P[X>0]} = \frac{0.1}{q} = \frac{0.1}{0.7} = \frac{1}{7}$, and in a similar way,

$P[B = 200] = \frac{1}{7}\ ,\ P[B = 500] = \frac{2}{7}\ ,\ P[B = 1000] = \frac{2}{7},\ P[B = 10,000] = \frac{1}{7}$. Then,

$Var[B] = E[B^2] - (E[B])^2 = 14,648,983 - (1892.86)^2 = 11,066,000$ (nearest 1000)

and $\sqrt{Var[B]} = 3327$. Answer: B

23. We denote by N the Poisson random variable representing the number of major snowstorms for the year. We are given that $E[N] = 1.5$. The probability function for this Poisson distribution is

$p(n) = P[N = n] = \frac{e^{-1.5}(1.5)^n}{n!}$.

The amount paid to the company is $c_0 = 0$ if $N = 0$ or 1, but is $c_n = 10,000n$ if $N = n \geq 2$.

N :	0	1	2	3	4	$\ldots$
Amt. paid to company, c_N	0	0	$10,000$	$20,000$	$30,000$	$\ldots$
Loss absorbed by company	0	$10,000$	$10,000$	$10,000$	$10,000$	$\ldots$
Total loss	0	$10,000$	$20,000$	$30,000$	$40,000$	$\ldots$

Note that $E[\text{Total loss}] = E[\text{Amt. paid to company}] + E[\text{Loss absorbed by company}]$

Total loss $= 10,000N \Rightarrow E[\text{Total loss}] = E[10,000N] = 10,000E[N] = 15,000$.

E[Loss absorbed by company]

$$= 0 \times p(0) + 10,000 \times [p(1) + p(2) + p(3) + p(4) + \cdots = 10,000[1 - p(0)]$$
$$= 10,000 \times \left[1 - \frac{e^{-1.5}(1.5)^0}{0!}\right] = 7,769.$$

Therefore, $E[\text{Amt. paid to company}] = 15,000 - 7,769 = 7,231$.

Note that the amount paid to the company is the total loss above a deductible of 10,000.

Answer: C

24. The expected insurance payment is $\int_2^{\infty}(x-2) \times f(x)\,dx = \int_2^{\infty}(x-2) \times xe^{-x}\,dx$.

With the change of variable $y = x - 2$, this integral becomes

$\int_0^{\infty} y(y+2)e^{-y-2}\,dy = e^{-2}[\int_0^{\infty} y^2 e^{-y}\,dy + 2\int_0^{\infty} ye^{-y}] = e^{-2}\left[\frac{2}{1^3} + 2 \times \frac{1}{1^2}\right] = 0.5413$.

Answer: C

25. The overall expected loss is $\int_0^1 x(2-2x)\,dx = \frac{1}{3}$.

With policy limit u, the expected insurance payment is $\int_0^u x \times f(x)\,dx + u \times [1-F(u)]$.

In this case, $F(u) = \int_0^u (2-2x)\,dx = 2u - u^2$.

Therefore, the expected insurance payment is

$\int_0^u x(2-2x)\,dx + u[1-2u+u^2] = u^2 - \frac{2u^3}{3} + u - 2u^2 + u^3 = \frac{u^3}{3} - u^2 + u$.

In order for this to be one-half of expected total loss we must have $\frac{u^3}{3} - u^2 + u = \frac{1}{6}$.

We do not solve the cubic equation. We substitute in the possible answers to see which is closest. We see that with $u = 0.21$ we get $\frac{(0.21)^3}{3} - (0.21)^2 + 0.21 = 0.169$, which is the closest to $\frac{1}{6}$ of all the possible values given in the answers. Answer: C

26. Let N be the number of consecutive days of rain starting on April 1. Then the amount paid by insurance is

N :	0	1	≥ 2
Amt paid by ins.:	0	1000	2000

We are told that N has a Poisson distribution with mean 0.6, so that $P[N=k] = \frac{e^{-0.6}(0.6)^k}{k!}$.

We note that $P[N \geq 2] = 1 - P[N = 0 \text{ or } 1] = 1 - e^{-0.6} - e^{-0.6} \times 0.6 = 0.1219$.

The first two moments of X are

$$E[X] = 0 \times e^{-0.6} + 1000e^{-0.6} \times 0.6 + 2000 \times 0.1219 = 573 \text{ and}$$

$$E[X^2] = 0 \times e^{-.6} + 1000^2 \times e^{-0.6} \times 0.6 + 2000^2 \times 0.1219 = 816,893.$$

Then $Var[X] = E[X^2] - (E[X])^2 = 488,461$, and the standard deviation is $\sqrt{488,461} = 699$.

Answer: B

27. The pdf of the exponential distribution with mean 300 is $f(x) = \frac{1}{300}e^{-x/300}$ $x > 0$, and the cdf is $F(x) = 1 - e^{-x/300}$. "Actual losses that exceed the deductible" refers to losses above 100, given that the loss is above 100. The 95-th percentile is c, where

$$0.95 = \frac{P[100<X\leq c]}{P[X>100]} = \frac{F(c)-F(100)}{1-F(100)} = \frac{(1-e^{-c/300})-(1-e^{-100/300})}{1-(1-e^{-100/300})} = \frac{e^{-1/3}-e^{-c/300}}{e^{-1/3}}.$$

Solving for c results in $c = 998.7$ Answer: E

28. Information is given for individual policies in the form

$q_i =$ probability of a non-zero claim from policy i

$\mu_i =$ claim amount from policy i, given that a claim occurs on policy i, $\sigma_i^2 = 0$

Let S denote the aggregate claims. Then the expected claim from policy i is $q_i \times \mu_i$ and the variance is $q_i \times (1-q_i) \times \mu_i^2$.

$E[S] = \sum q_i \times \mu_i = 1000 \times 0.01 \times 1 + 2000 \times 0.02 \times 1 + 500 \times 0.04 \times 2 = 90$, and

$$Var[S] = \sum q_i(1-q_i) \cdot \mu_i^2 = 1000 \times 0.01 \times 0.99 \times 1^2 + 2000 \times 0.02 \times 0.98 \times 1^2$$

$$+ 500 \times 0.04 \times 0.96 \times 2^2 = 125.9.$$

The 95-th percentile of S is Q, where $P[S \le Q] = 0.95$. Standardizing S results in $P\left[\frac{S-E[S]}{\sqrt{Var[S]}} \le \frac{Q-E[S]}{\sqrt{Var[S]}}\right] = 0.95$. Applying the normal approximation results in $\frac{Q-E[S]}{\sqrt{Var[S]}} = \frac{Q-90}{\sqrt{125.9}} = 1.645 \rightarrow Q = 108.5$ (the 95-th percentile of the standard normal distribution is 1.645). Answer: E

29. With the insurer's initial assumption, information is given for individual policies in the form
$q_i =$ probability of a non-zero claim from policy i
$\mu_i =$ claim amount from policy i, given that a claim occurs on policy i , $\sigma_i^2 = 0$
The insurer's revised assumption results in $\sigma_i^2 = \sigma^2$ for all i.
Under the initial assumption, the variance of a claim from policy i is $q_i(1-q_i) \times \mu_i^2$,
so that the variance of aggregate claims is
$\sum q_i(1-q_i) \times \mu_i^2 = 1000(.01)(.99)(1^2) + 2000(.02)(.98)(1^2) + 500(.04)(.96)(2^2) = 125.9$

Under the revised assumption, the variance of a claim from policy i is $q_i(1-q_i) \cdot \mu_i^2 + q_i \cdot \sigma^2$, so that the variance of aggregate claims is
$\sum[q_i(1-q_i) \cdot \mu_i^2 + q_i \cdot \sigma^2] = 1000[0.01 \times 0.99 \times 1^2 + 0.01\sigma^2]$

$+ 2000[0.02 \times 0.98 \times 1^2 + 0.02\sigma^2] + 500[0.04 \times 0.96 \times 2^2 + 0.04\sigma^2] = 125.9 + 70\sigma^2$
We are given that $125.9 + 70\sigma^2 = 1.67(125.9) \rightarrow \sigma^2 = 1.2$. Answer: B

30. The ceding insurer' initial premium is $(1+\theta) \times E[S]$. The expected claim on the reinsurer is $\alpha \times E[S]$, so the premium paid by the ceding insurer to the reinsurer is
$(1+\theta') \times \alpha \times E[S]$. The retained premium for the ceding insurer is
$(1+\theta) \times E[S] - (1+\theta') \times \alpha \times E[S] = [1+\theta-\alpha(1+\theta')] \times E[S]$, and the expected retained claim for the ceding insurer is $(1-\alpha) \times E[S]$. Thus, the effective relative security loading for the ceding insurer after reinsurance is θ'', where
$(1+\theta'') \times (1-\alpha) \times E[S] = [1+\theta-\alpha(1+\theta')] \times E[S]$, from which we can solve for α in terms of θ, θ' and θ'': $\alpha = \frac{\theta-\theta''}{\theta'-\theta''}$. Answer: B

31. The density function for an exponential distribution with mean $\frac{1}{\lambda}$ is $\lambda e^{-\lambda x}$, $x > 0$. In order for $f(x)$ to be a proper probability density function, we must have $c = .004$.
Alternatively, $\int_0^\infty f(x)\,dx = 1 \rightarrow \int_0^\infty ce^{-0.004x}\,dx = \frac{c}{0.004} = 1 \rightarrow c = 0.004$.
If R is the reimbursed amount and X is the actual expense, then
$R = \begin{cases} X & X \le 250 \\ 250 & X > 250 \end{cases}$. The median benefit is the amount k for which
$P[R \le k] = 0.5$. From the distribution of X, we see that
$P[X \le r] = \int_0^r f(x)\,dx = 1 - e^{-0.004r}$. Solving $P[X \le r] = 0.5$
results in $1 - e^{-0.004r} = 0.5 \rightarrow r = 173.29$.
Therefore, the median of R is 173.29. Answer: C

32. In 2000, with no policy limit and with a deductible of 100, the expected amount paid per loss is

$$E[Y] \text{ , where } \quad Y = \begin{cases} 0 & X < 100 \\ X - 100 & X \geq 100 \end{cases}$$

The density function of X is $f_X(x) = 0.001$ for $0 < x < 1000$ (0 otherwise) and the distribution function $F_X(x) = 0.001x$ for $0 < x < 1000$.

The expectation is

(i) $E[Y] = \int_{100}^{1000} (x - 100) \times 0.001 \, dx = 405$ or

(ii) $\int_{100}^{1000} [1 - F_X(x)] \, dx = \int_{100}^{1000} [1 - 0.001x] \, dx = 405$

The variance of amount paid is $E[Y^2] - (E[Y])^2$.
$E[Y^2] = \int_{100}^{1000} (x - 100)^2 \times 0.001 \, dx = 243,000$, so that the standard deviation of amount paid per loss in 2000 is $\sqrt{243,000 - (405)^2} = 281.0$.

In 2001, the loss W is uniform on $(0, 1050)$. The amount paid by the insurance is
$Z = \begin{cases} 0 & W < 100 \\ W - 100 & W \geq 100 \end{cases}$, and the variance of Z is

$$\begin{aligned} E[Z^2] - (E[Z])^2 &= \int_{100}^{1050} (w - 100)^2 \times \frac{1}{1050} \, dw - [\int_{100}^{1050} (w - 100) \times \frac{1}{1050} \, dx]^2 \\ &= 272,182 - (429.76)^2 = 87,487 \end{aligned}$$

and the standard deviation is 295.8. The percentage increase in standard deviation from 2000 to 2001 is $\frac{295.8}{281.0} - 1 = 0.053$. Answer: B

33. The probability of a given loss exceeding 500 is $e^{-500/1000} = e^{-1/2} = 0.60653$.
If there are n exposures, then the expected number of losses exceeding the deductible will be $ne^{-1/2} = 0.60653n$. We are told that this is 10, so that $n = 10e^{1/2}$.
If all loss amounts doubled, the loss distribution will be exponential with mean 2000, so that $F(x) = 1 - e^{-x/2000}$, and the expected number of losses exceeding 500 will be $n[1 - F_{new}(500)] = 10e^{1/2}e^{-1/4} = 10e^{1/4} = 12.84$. Answer: C

34. In year 2000, $E[\text{payment}] = \int_0^1 [1 - F(x)] \, dx = \int_0^1 e^{-x} dx = 1 - e^{-1} = 0.6321$ (million).
After 5% inflation, in year 2001,
$P[Y < y] = P[1.05X < y] = P[X < \frac{y}{1.05}] = 1 - e^{-y/1.05}$, exponential with mean 1.05 (million). In year 2001, $E[\text{payment}] = \int_0^1 [1 - F_Y(x)] \, dx = \int_0^1 e^{-x/1.05} dx = 1.05 - 1.05e^{-1/1.05} = 0.6449$.
The inflation rate on expected losses is $\frac{0.6449}{0.6321} - 1 = 0.0202$. Answer: B

35. This is a case in which the single claim amount distribution X (severity distribution) is given, and the distribution of the number of claims per year N (frequency distribution) is given.

We can regard the total claim amount as a mixture of 3 distributions:

$Z_1 = 0$ when there are $N = 0$ suits (prob. 0.96)

$Z_2 = X$ when there is $N = 1$ suit (prob. 0.03) and

$Z_3 = X_1 + X_2$ when there are $N = 2$ suits (prob. 0.01)

Then $E[Y] = 0.96 \times E[Z_1] + 0.03 \times E[Z_2] + 0.01 \times E[Z_3]$

$= 0.96 \times 0 + 0.03 \times 550,000 + 0.01 \times 550,000 \times 2 = 27,500.$

To find $E[Y^2]$ note that $E[Z_2^2] = E[X^2] = \int_{100,000}^{1,000,000} x^2 \times \frac{1}{900,000}\,dx = 3.7 \times 10^{11}$

and $E[Z_3^2] = E[(X_1 + X_2)^2] = E[X_1^2] + 2E[X_1X_2] + E[X_2^2]$,

where by independence $E[X_1X_2] = E[X_1] \cdot E[X_2] = 550,000^2$.

Then $E[Y^2] = 0.96 \times E[Z_1^2] + 0.03 \times E[Z_2^2] + 0.01 \times E[Z_3^2]$

$= 0.96 \times 0 + 0.03 \times 3.7 \times 10^{11}$

$+ 0.01 \times (3.7 \times 10^{11} + 2 \times 550,000^2 + 3.7 \times 10^{11}) = 2.455 \times 10^{10}$.

$Var[Y] = E[Y^2] - (E[Y])^2 = 2.38 \times 10^{10}$

The premium charged by the insurer is

$E[Y] + \sqrt{Var[Y]} = 27,500 + 154,250 = 181,750.$ Answer: D

36. Last year, for a loss of amount X, the amount paid by the insurer was

$$Y^* = \begin{cases} 0 \text{ if } X \le 100 \\ X - 100 \text{ if } X > 100 \end{cases}$$

Last year the pdf of the loss random variable X was

$f_X(x) = \frac{1}{1000}$ (uniform distribution on the interval $(0, 1000)$).

The expected payment by the insurer last year was $\int_{100}^{1000} (x - 100) \times \frac{1}{1000}\,dx = 405.$

This year, for a loss of amount X, the amount paid by the insurer is still

$$Y^* = \begin{cases} 0 \text{ if } X \le 100 \\ X - 100 \text{ if } X > 100 \end{cases}$$

but this year the pdf of the loss random variable X is

$f_X(x) = \frac{1}{1050}$ (uniform distribution on the interval $(0, 1050)$).

The expected payment by the insurer this year is $\int_{100}^{1050} (x - 100) \times \frac{1}{1050}\,dx = 429.76.$

The percentage increase is $100 \times (\frac{429.76}{405} - 1) = 6.11.$ Answer: C

37. The ceding insurer will cover all claims from classes 1 and 2, and will cover the first 2 units of claim from any policy in class 3. The ceding insurer purchases 2 units of reinsurance for each of the policies with benefit amount 4, for a total of $4500(2) = 9000$ units reinsured. The cost of the reinsurance is $R = 9000(.03) = 270$. The retained claim distribution S consists of 8000 (Class 1) policies with $q = 0.025$ and $E[B_i] = 1$ and 8000 policies ($3500 + 4500$, Classes 2 and 3 combined) with $q = 0.025$ and $E[B_i] = 2$. We are using the notation mentioned earlier, B_i is the conditional claim from policy i given that a claim occurs. Then, X_i is related to B_i through the relationships $E[X_i] = q_i E[B_i]$ and $Var[X_i] = q_i(1-q_i)(E[B_i])^2 + q_i \times Var[B_i]$. In this case $Var[B_i] = 0$ for all policies; this is generally assumed for term life insurances. Then,

$E[S] = \sum E[X_i] = 8000 \times 0.025 \times 1 + 8000 \times 0.025 \times 2 = 600$ and

$Var[S] = \sum Var[X_i] = 8000 \times 0.025 \times 0.975 \times 1^2 + 8000 \times 0.025 \times 0.975 \times 2^2 = 975$

Then, $Q = 600 + 2\sqrt{975} + 270 = 932.45$. Answer: E

38. The distribution of costs is exponential with a mean of 100. From the lack-of-memory property of the exponential distribution, the conditional distribution of costs given cost is greater than 20 is also exponential with mean 100. Reimbursement is 100 if health care cost is 120 and reimbursement is 115 if health care cost is 120+30 (since 50% of the additional 30 is reimbursed). The conditional probability that reimbursement is below 115 given that reimbursement is positive is the probability that an exponential random variable with mean 100 is less than 130 (conditional probability that total cost is less than 150 given that it is at least 20). This is $1 - e^{-130/100} = .727$. Answer: B

39. Suppose that the mean of the exponential distribution is λ. Then with deductible amount d, the expected payout is $E[(X-d)_+] = \int_d^\infty [1-F_X(x)]\,dx = \int_d^\infty e^{-x/\lambda}\,dx = \lambda e^{-d/\lambda}$.

Since the expected payout with deductible is 90% of the expected claim payment, it follows that $e^{-d/\lambda} = .9$. This is also $P(X > d)$ for the exponential distribution.

Then $E[(X-d)^2|X > d] = \int_d^\infty (x-d)^2 \, \frac{e^{-x/\lambda}}{\lambda}\,dx$. If we apply the change of variable $y = x - d$, we get $\int_0^\infty y^2 \, e^{-d/\lambda} \, \frac{e^{-y/\lambda}}{\lambda}\,dy = e^{-d/\lambda} \times 2\lambda^2 = 0.9 \times 2\lambda^2 = 1.8\lambda^2$.

This is because $\int_0^\infty y^2 \, \frac{e^{-y/\lambda}}{\lambda}\,dy = 2\lambda^2$ is the second moment of an exponential distribution and $e^{-d/\lambda}$ was already determined to be 0.9.

Then $Var[(X-d)_+] = E[(X-d)^2|X > d] - (\,E[(X-d)_+]]^2 = 1.8\lambda^2 - (0.9\lambda)^2 = 0.99\lambda^2$.

This is $0.01\lambda^2$ less (1%) than the variance of X. Answer: A

40. The number of accidents has a binomial distribution with $p = 0.25$ and $n = 3$, so the mean number of driving errors is $E[N] = 3 \times 0.25 = 0.75$ and the variance of the number of driving errors is $Var[N] = 3 \times 0.25 \times 0.75 = 0.5625$.
If the loss is X, then $E[X] = 0.8$ and $Var[X] = 0.8^2 = 0.64$.
The unreimbursed loss for a single accident is $Y = .3X$, with mean $0.3 \times 0.8 = 0.24$ and variance $0.3^2 \times 0.64 = 0.0576$. The variance of the total unreimbursed loss S is $Var[E[S|N]] + E[Var[S|N]]$. But $E[S|N] = 0.24N$ and $Var[S|N] = 0.0576N$ since there are N independent accidents.
Then $E[Var[S|N]] = E[0.0576N] = 0.0576 \times E[N] = 0.0576 \times 0.75 = 0.0432$,
and $Var[E[S|N]] = Var[0.24N] = 0.24^2 \times Var[N] = 0.24^2 \times 0.5625 = 0.0324$.
Finally, $Var[S] = 0.0432 + 0.324 = 0.0756$. Answer: B

41. We will be applying the normal approximation, since we are considering probabilities for a sum of many independent payouts. We must fund the mean and variance of an individual payout on a reported loss. The payout on a particular reported loss is either 0, with probability .25 (this is the prob. that the reported loss is below 5,000, in which case there is 0 payout), or is uniformly distributed between 0 and 15,000, with probability .75, if the reported loss is above the deductible of 5,000. The payout X is a mixture of 0 with probability .25, and Y, which is uniform from 0 to 15,000 with prob. 75. The moments of a mixed distribution are the weighted averages of the moments of the component distributions, using the `mixing probabilities. It follows that $E[X] = 0.25 \times 0 + 0.75 \times E[Y] = 0.75 \times 7500 = 5,625$ and $E[X^2] = 0.25 \times 0 + 0.75 \times E[Y^2] = 0.75 \times \int_0^{15,000} \frac{y^2}{15,000}\, dy = 56,250,000$.
Then, $Var[X] = 56,250,000 - (5,625)^2 = 24,609,375$.
With $T = \sum_{i=1}^{200} X_i$, where the X_i's are independent payouts, we have
$E[T] = 200 \times E[X] = 1,125,000$, and $Var[T] = 200 \times Var[X] = 4,921,875,000$.
Then $P[1,000,000 < T \leq 1,200,000]$

$$= P\left[\frac{1,000,000-1,125,000}{\sqrt{4,921,875,000}} < \frac{T-1,125,000}{\sqrt{4,921,875,000}} \leq \frac{1,200,000-1,125,000}{\sqrt{4,921,875,000}}\right]$$

$$= P[-1.78 < Z \leq 1.07] = \Phi(1.07) - \Phi(-1.78) = 0.8577 - 0.0375 = 0.8202.$$ Answer: D

42. Let Y be the insurance company payout when an accident occurs. We wish to find c for which $P[Y \leq c] = 0.95$. We can write the probability as
$.95 = P[Y \leq c] = P[\text{no accident}] + P[\text{accident} \cap (Y \leq c)] = 0.8 + 0.2 \times P[Y \leq c|\text{accident}]$.
Therefore, we wish to find c for which $P[Y \leq c|\text{accident}] = 0.75$.
The 75th percentile of the exponential loss random variable X with mean 3000 is a, where $F_X(a) = 1 - e^{-a/3000} = 0.75$, so that $a = 4159$. There is a 75 per cent chance that when an accident occurs, the payout will be less than 3659 (after application of the deductible). There is a 95 per cent chance that policy will pay out less than 3659. Answer: B

TABLE FOR THE

NORMAL DISTRIBUTION

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)

The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

PRACTICE EXAMS

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Note that some of the questions on these practice exams are somewhat more challenging than the typical exam questions.

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PRACTICE EXAM 1

1. If E and F are events for which $P[E \cup F] = 1$, then $P[E' \cup F'] =$

A) 0 B) $P[E'] + P[F'] - P[E'] \cdot P[F']$ C) $P[E'] + P[F']$

D) $P[E'] + P[F'] - 1$ E) 1

2. Sixty percent of new drivers have had driver education. During their first year, new drivers without driver education have a probability of 0.08 of having an accident, but new drivers with driver education have only a 0.05 probability of an accident. What is the probability a new driver has had driver education, given that the driver has had no accidents the first year?

A) $\frac{5}{6}$ B) $\frac{0.92 \times 0.4}{0.95 \times 0.6 + 0.92 \times 0.4}$ C) $\frac{0.95 \times 0.4}{0.95 \times 0.6 + 0.92 \times 0.4}$

D) $\frac{0.95 \times 0.4}{0.95 \times 0.4 + .92 \times 0.6}$ E) $\frac{0.95 \times 0.6}{0.95 \times 0.6 + 0.92 \times 0.4}$

3. A loss distribution random variable X has a pdf of $f(x) = ae^{-x} + be^{-2x}$ for $x > 0$. If the mean of X is 1, find the probability $P[X < 1]$.

A) 0.52 B) 0.63 C) 0.74 D) 0.85 E) 0.96

4. If $f_X(x) = xe^{-x^2/2}$ for $x > 0$, and $Y = ln\, X$, find the density function for Y.

A) $e^{2y - \frac{1}{2}e^{2y}}$ B) $(ln\, y)e^{-(ln\, y)^2/2}$ C) $e^{y - \frac{1}{2}e^{2y}}$ D) $ye^{-y^2/2}$ E) $e^{-\frac{1}{2}e^{2y}}$

5. An insurer estimates that Smith's time until death is uniformly distributed on the interval $[0, 5]$ and Jones' time until death is uniform on the interval $[0, 10]$. The insurer assumes that the two times of death are independent of one another. Find the probability that Smith is the first of the two to die.

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{3}{4}$

6. If X has a normal distribution with mean 1 and variance 4, then $P[X^2 - 2X \leq 8] = ?$

A) 0.13 B) 0.43 C) 0.75 D) 0.86 E) 0.93

7. The pdf of X is $f(x) = \begin{cases} 2x \text{ for } 0<x<1 \\ 0\text{, elsewhere} \end{cases}$. The mean of X is μ 0.
Find $\frac{E[|X-\mu|]}{Var[X]}$.

A) $\frac{20}{9}$ B) $\frac{26}{9}$ C) $\frac{32}{9}$ D) $\frac{19}{81}$ E) $\frac{22}{81}$

8. Two players put one dollar into a pot. They decide to throw a pair of dice alternately. The first one who throws a total of 5 on both dice wins the pot. How much should the player who starts add to the pot to make this a fair game?

A) $\frac{9}{17}$ B) $\frac{8}{17}$ C) $\frac{1}{8}$ D) $\frac{2}{9}$ E) $\frac{8}{9}$

9. An analysis of economic data shows that the annual income of a randomly chosen individual from country A has a mean of \$18,000 and a standard deviation of \$6000, and the annual income of a randomly chosen individual from country B has a mean of \$31,000 and a standard deviation of \$8000. 100 individuals are chosen at random from Country A and 100 from Country B. Find the approximate probability that the average annual income from the group chosen from Country B is at least \$15,000 larger than the average annual income from the group chosen from Country A (all amounts are in US\$).

A) 0.9972 B) 0.8413 C) 0.5000 D) 0.1587 E) 0.0228

10. Three individuals are running a one kilometer race. The completion time for each individual is a random variable. X_i is the completion time, in minutes, for person i.

X_1 : uniform distribution on the interval $[2.9\,,\,3.1]$
X_2 : uniform distribution on the interval $[2.7\,,\,3.1]$
X_3 : uniform distribution on the interval $[2.9\,,\,3.3]$

The three completion times are independent of one another.
Find the expected latest completion time (nearest 0.1).

A) 2.9 B) 3.0 C) 3.1 D) 3.2 E) 3.3

11. The amount of liability claim Y in a motor vehicle accident has a uniform distribution on the interval $(0, 1)$, and the amount of property damage in that accident has a uniform distribution on the interval $(0, \sqrt{y})$. Find the density function of X, the amount of property damage in an accident.

A) $2(1-x)$ B) $2x$ C) $2(1-x^{1/4})$ D) $\frac{1}{\sqrt{x}}-1$ E) $\frac{1}{2\sqrt{x}}$

12. A loss random variable X has a uniform distribution on the interval $[0, 1000]$.
Find the variance of the insurer payment per loss if there is a deductible of amount 100 and a policy limit (maximum insurance payment) of amount 400 (nearest 1000).

A) 20,000 B) 21,000 C) 22,000 D) 23,000 E) 24,000

13. Let $X_1, ..., X_n$ be independent Poisson random variables with expectations $\lambda_1, ..., \lambda_n$, respectively. $Z = \sum_{i=1}^{n} aX_i$, where a is a constant. Find the moment generating function of Z.

A) $exp\left(t\sum_{i=1}^{n} a\lambda_i + \frac{1}{2}t^2\sum_{i=1}^{n} a^2\lambda_i\right)$ B) $exp\left(\sum_{i=1}^{n} a\lambda_i(e^t - 1)\right)$

C) $exp\left(t\sum_{i=1}^{n} a\lambda_i + \frac{1}{2}t^2\sum_{i=1}^{n} a^2\lambda_i^2\right)$ D) $exp\left(\sum_{i=1}^{n} \lambda_i(e^{at} - 1)\right)$ E) $\left[\prod_{i=1}^{n} \lambda_i\right][e^{at} - 1]^n$

14. Let X be a random variable with mean 3 and variance 2, and let Y be a random variable such that for every x, the conditional distribution of Y given $X = x$ has a mean of x and a variance of x^2. What is the variance of the marginal distribution of Y?

A) 2 B) 4 C) 5 D) 11 E) 13

15. Let X and Y be discrete random variables with joint probabilities given by

	X	1	5
Y	2	$\theta_1 + \theta_2$	$\theta_1 + 2\theta_2$
	4	$\theta_1 + 2\theta_2$	$\theta_1 + \theta_2$

Let the parameters θ_1 and θ_2 satisfy the usual assumption associated with a joint probability distribution and the additional constraints $-0.25 \le \theta_1 \le 0.25$ and $0 \le \theta_2 \le 0.35$. If X and Y are independent, then $(\theta_1, \theta_2) =$

A) $(0, \frac{1}{6})$ B) $(\frac{1}{4}, 0)$ C) $(-\frac{1}{4}, \frac{1}{3})$ D) $(-\frac{1}{8}, \frac{1}{4})$ E) $(\frac{1}{16}, \frac{1}{8})$

16. A machine has two components. The machine will continue to operate as long as at least one of the two components is working. Measured from when a new machine begins continuous operation, the time (in years) until failure of component 1 is X and the time (in years) until failure of component 2 is Y. The density function of the joint distribution between X and Y is $f(x, y) = x + y$, $0 < x < 1$, $0 < y < 1$. Find the probability that a new machine is still in operation 6 months after it began operating.

A) $\frac{31}{32}$ B) $\frac{15}{16}$ C) $\frac{7}{8}$ D) $\frac{3}{4}$ E) $\frac{1}{2}$

17. For a Poisson random variable X with mean λ it is found that it is twice as likely for X to be less than 3 as it is for X to be greater than or equal to 3. Find λ (nearest .1).

A) 2.0 B) 2.2 C) 2.4 D) 2.6 E) 2.8

18. Let X and Y be discrete random variables with joint probability function

$$f(x,y) = \begin{cases} \frac{2^{x+1-y}}{9} \text{ for } x{=}1,2 \text{ and } y{=}1,2 \\ 0, \text{otherwise} \end{cases}.$$

Calculate $E[\frac{X}{Y}]$.

A) $\frac{8}{9}$ B) $\frac{5}{4}$ C) $\frac{4}{3}$ D) $\frac{25}{18}$ E) $\frac{5}{3}$

19. People passing by a city intersection are asked for the month in which they were born. It is assumed that the population is uniformly divided by birth month, so that any randomly passing person has an equally likely chance of being born in any particular month. Find the minimum number of people needed so that the probability that no two people have the same birth month is less than .5 .

A) 2 B) 3 C) 4 D) 5 E) 6

20. Under a group insurance policy, an insurer agrees to pay 100% of the medical bills incurred during the year by employees of a small company, up to a maximum total of one million dollars. The total amount of bills incurred, X, has probability density function

$$f(x) = \begin{cases} \frac{x(4-x)}{9}, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

where x is measured in millions.

Calculate the total amount, in millions of dollars, the insurer would expect to pay under this policy.

A) 0.120 B) 0.301 C) 0.935 D) 2.338 E) 3.495

21. Let X be a random variable with moment generating function $M(t) = (\frac{2+e^t}{3})^9$ for $-\infty < x < \infty$. Find the variance of X.

A) 2 B) 3 C) 8 D) 9 E) 11

22. A carnival gambling game involves spinning a wheel and then tossing a coin. The wheel lands on one of three colors, red, white or blue. There is a 1/2 chance that the wheel lands on red, and there is a 3/8 chance of white and a 1/8 chance of blue. A coin of the color indicated by the wheel is then tossed. Red coins have a 50% chance of tossing a head, white coins have a 3/4 chance of tossing a head, and blue coins have a 7/8 chance of tossing a head. If the game player tosses a head, she wins \$100, if she does not toss a head she wins 0. Find the cost to play the game so that the carnival wins an average of \$1 per play of the game.

A) Less than 62 B) At least 62 but less than 64 C) At least 64 but less than 66
D) At least 66 but less than 68 E) At least 68

23. Fred, Ned and Ted each have season tickets to the Toronto Rock (Lacrosse). Each one of them might, or might not attend any particular game. The probabilities describing their attendance for any particular game are

$P[\text{at least one of them attends the game}] = 0.95$,

$P[\text{at least two of them attend the game}] = 0.80$ and

$P[\text{all three of them attend the game}] = 0.50$

Their attendance pattern is also symmetric in the following way

$$P(F) = P(N) = P(T) \text{ and } P(F \cap N) = P(F \cap T) = P(N \cap T)$$

where F, N and T denote the events that Fred, Ned and Ted attended the game, respectively. For a particular game, find the probability that Fred and Ned attended.

A) .15 B) .30 C) .45 D) .60 E) .75

24. An insurer will pay the amount of a loss in excess of a deductible amount α. Suppose that the loss amount has a continuous uniform distribution between 0 and $C > \alpha$. When a loss occurs, let the expected payout on the policy be $f(\alpha)$. Find $f'(\alpha)$.

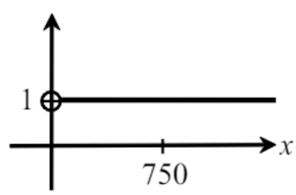
A) $\frac{\alpha}{C}$ B) $-\frac{\alpha}{C}$ C) $\frac{\alpha}{C}+1$ D) $\frac{\alpha}{C}-1$ E) $1-\frac{\alpha}{C}$

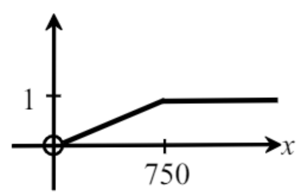
25. Coins K and L are weighted so the probabilities of heads are 0.3 and 0.1, respectively. Coin K is tossed 5 times and coin L is tossed 10 times. If all the tosses are independent, what is the probability that coin K will result in heads 3 times and coin L will result in heads 6 times?

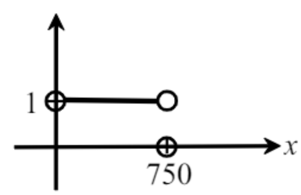
A) $\binom{5}{3}(0.3)^3(0.7)^2 + \binom{10}{6}(0.1)^3(0.9)^2$ B) $\binom{5}{3}(0.3)^3(0.7)^2\binom{10}{6}(0.1)^6(0.9)^4$

C) $\binom{15}{9}(0.4)^9(0.6)^6$ D) $\frac{\binom{5}{3}\binom{10}{6}}{\binom{15}{9}}$ E) 0.6×0.9

26. An insurance policy is written that reimburses the policyholder for all losses incurred up to a benefit limit of 750 . Let $f(x)$ be the benefit paid on a loss of x . Which of the following most closely resembles the graph of the derivative of f ?

(A)


(B)


(C)


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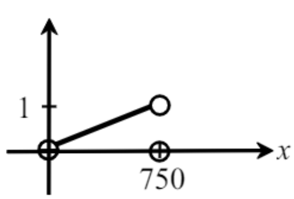
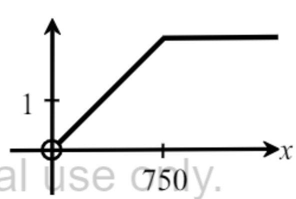


(E)


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27. The value, v, of an appliance is based on the number of years since purchase, t, as follows: $v(t) = e^{(7-0.2t)}$. If the appliance fails within seven years of purchase, a warranty pays the owner the value of the appliance. After seven years the warranty pays nothing. The time until failure of the appliance has an exponential distribution with a mean of 10. Calculate the expected payment from the warranty.

A) 98.70 B) 109.66 C) 270.43 D) 320.78 E) 352.16

28. A test for a disease correctly diagnoses a diseased person as having the disease with probability 0.85. The test incorrectly diagnoses someone without the disease as having the disease with a probability of 0.10. If 1% of the people in a population have the disease, what is the chance that a person from this population who tests positive for the disease actually has the disease?

A) 0.0085 B) 0.0791 C) 0.1075 D) 0.1500 E) 0.9000

29. Let X and Y be discrete random variables with joint probability function $f(x, y)$ given by the following table:

		x			
		2	3	4	5
	0	0.05	0.05	0.15	0.05
y	1	0.40	0	0	0
	2	0.05	0.15	0.10	0

For this joint distribution, $E[X] = 2.85$ and $E[Y] = 1$. Calculate $Cov[X, Y]$.

A) -0.20 B) -0.15 C) 0.95 D) 2.70 E) 2.85

30. One of the questions asked by an insurer on an application to purchase a life insurance policy is whether or not the applicant is a smoker. The insurer knows that the proportion of smokers in the general population is 0.30, and assumes that this represents the proportion of applicants who are smokers. The insurer has also obtained information regarding the honesty of applicants:

- 40% of applicants that are smokers say that they are non-smokers on their applications,
- none of the applicants who are non-smokers lie on their applications.

What proportion of applicants who say they are non-smokers are actually non-smokers?

A) 0 B) $\frac{6}{41}$ C) $\frac{12}{41}$ D) $\frac{35}{41}$ E) 1