10. If X and Y have a bivariate normal distribution for which

X has mean μ_X and standard deviation σ_X, and

Y has mean μ_Y and standard deviation σ_Y, and

the coefficient of correlation between X and Y is ρ, then

the general bivariate normal joint pdf is

$f(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \times exp[-\frac{1}{2(1-\rho^2)} \times [(\frac{x-\mu_X}{\sigma_X})^2 + (\frac{y-\mu_Y}{\sigma_Y})^2 - 2\rho(\frac{x-\mu_X}{\sigma_X})(\frac{y-\mu_Y}{\sigma_Y})]]$.

We are given that $f(x,y) = \frac{0.3125}{\pi} \times e^{-0.78125(x^2-0.6xy+0.25y^2)}$.

From the general form of the joint pdf, we see that $\frac{2\rho}{\sigma_X\sigma_Y} = 0.6$, so that $\rho = 0.6$.

The covariance between $X+Y$ and $X-Y$ is

$Cov(X+Y, X-Y) = Cov(X,X) + Cov(X,-Y) + Cov(Y,X) + Cov(Y,-Y)$

$= Var(X) - Cov(X,Y) + Cov(Y,X) - Var(Y) = Var(X) - Var(Y) = 1 - 4 = -3.$

The coefficient of correlation between $X+Y$ and $X-Y$ is $\frac{Cov(X+Y,X-Y)}{\sqrt{Var(X+Y)\times Var(X-Y)}}$.

$Var(X+Y) = Var(X) + Var(Y) + 2\rho\sqrt{Var(X)\times Var(Y)} = 1+4+2(0.6)\sqrt{(1)(4)} = 7.4$ and

$Var(X-Y) = Var(X) + Var(Y) - 2\rho\sqrt{Var(X)\times Var(Y)} = 1+4-2(0.6)\sqrt{(1)(4)} = 2.6.$

The coefficient of correlation between $X+Y$ and $X-Y$ is

$\frac{Cov(X+Y,X-Y)}{\sqrt{Var(X+Y)\cdot Var(X-Y)}} = \frac{-3}{\sqrt{(7.4)(2.6)}} = -0.684.$ Answer: A

11. Suppose that X is an exponential random variable with mean 1.

The pdf of X is $f_X(x) = e^{-x}$, $x > 0$. The insurance policy for a single risk with policy limit 2 will pay

$\begin{cases} x & \text{if } 0 < x \le 2 \\ 2 & \text{if } x > 2 \end{cases}$.

The expected amount paid for one policy is

$A = \int_0^2 x \times f_X(x)\,dx + 2 \times P(X>2) = \int_0^2 x \times e^{-x}\,dx + 2\times e^{-2}$

$= (-xe^{-x} - e^{-x})\Big|_{x=0}^{x=2} + 2e^{-2} = (-2e^{-2} - e^{-2}) - (0-1) + 2e^{-2} = 1 - e^{-2}.$

Note that, for a non-negative random variable $X \ge 0$, with a policy limit u, the expected insurance payment is $\int_0^u [1 - F_X(x)]\,dx$. In the case of the exponential distribution with mean 1, $F_X(x) = 1 - e^{-x}$, so the expected insurance payment with a policy limit of 2 is

$\int_0^2 [1 - (1-e^{-x})]\,dx = \int_0^2 e^{-x}\,dx = 1 - e^{-2}.$

Suppose that X_1 and X_2 are the independent exponential losses on the two risks. The combined loss is $Y = X_1 + X_2$, and the insurance on the combined losses will apply a limit of 4 to Y.

The sum of two independent exponential random variables, each with a mean of 1, is a gamma random variable with pdf $f_Y(y) = ye^{-y}$, $y > 0$. This can be verified a couple of ways.

(i) Convolution:

$f_Y(y) = \int_0^y f_{X_1}(x) \times f_{X_2}(y-x)\,dx = \int_0^y e^{-x} \times e^{-(y-x)}\,dx = \int_0^y e^{-y}\,dx = ye^{-y}$

(ii) Transformation of random variables:

Since X_1 and X_2 are independent, the joint distribution of X_1 and X_2 has pdf

$f(x_1, x_2) = f_{X_1}(x_1) \times f_{X_2}(x_2) = e^{-x_1} \times e^{-x_2}$

$U = X_1\,,\ Y = X_1 + X_2 \rightarrow X_1 = U\,,\ X_2 = Y - U$

$\rightarrow$ pdf of U, Y is $g(u,y) = f(u, y-u) \cdot \begin{vmatrix} \frac{\partial}{\partial u}\,u & \frac{\partial}{\partial u}\,y-u \\ \frac{\partial}{\partial y}\,u & \frac{\partial}{\partial y}\,y-u \end{vmatrix} = e^{-u} \times e^{-(y-u)} \cdot 1 = e^{-y}$,

and the joint distribution of U and Y is defined on the region $0 < u < y$

(this is true because $u = x_1 < x_1 + x_2 = y$).

The marginal density of Y is $f_Y(y) = \int_0^y g(u,y)\,du = \int_0^y e^{-y}\,dy = ye^{-y}$.

We impose a limit of 4 for the insurance policy on Y, the combination of the two exponential losses. The amount paid by the insurance is $\begin{cases} y & \text{if } 0 < y \le 4 \\ 4 & \text{if } y > 4 \end{cases}$

The expected insurance payment is $\int_0^4 y \times f_Y(y)\,dy + 4 \times P(Y > 4)$.

$\int_0^4 y \times f_Y(y)\,dy = \int_0^4 y \times ye^{-y}\,dy = \int_0^4 y^2 \times e^{-y}\,dy$.

Applying integration by parts, this becomes

$-y^2e^{-y}\Big|_{y=0}^{y=4} - \int_0^4(-e^{-y})(2y)\,dy = -16e^{-4} + 2\int_0^4 ye^{-y}\,dy$

$= -16e^{-4} + 2 \times [-ye^{-y} - e^{-y}\Big|_{y=0}^{y=4}] = -16e^{-4} + 2[-4e^{-4} - e^{-4} - (0-1)] = 2 - 26e^{-4}$.

$P(Y > 4) = \int_4^\infty f_Y(y)\,dy = \int_4^\infty ye^{-y}\,dy = (-ye^{-y} - e^{-y})\Big|_{y=4}^{y=\infty}$

$= (-0-0) - (-4e^{-4} - e^{-4}) = 5e^{-4}$.

Expected insurance payment of the combined policy is $2 - 26e^{-4} + 4(5e^{-4}) = 2 - 6e^{-4} = B$.

The ratio B/A is $\frac{2-6e^{-4}}{1-e^{-2}} = 2.186$. Answer: C

12. If the index closes below 20, then $Y = Min\{Max\{X,20\}, 50\} = 20$,
and if the index closes above 50, then $Y = Min\{\, Max\{X,20\}\,, 50\} = 50$.

If the index closes between 20 and 50, then
$Y = Min\{\, Max\{X,20\}\,, 50\} = Min\{X, 50\} = X$.

Therefore, $Y = \begin{cases} 20 & X \le 20 \\ X & 20 < X \le 50 \\ 50 & X > 50 \end{cases}$.

$E(Y) = \int_0^{20} 20 \times f_X(x)\,dx + \int_{20}^{50} x \times f_X(x)\,dx + \int_{50}^{100} 50 \times f_X(x)\,dx$.

X has pdf $f_X(x) = \frac{1}{100} = 0.01$, so

$E(Y) = \int_0^{20} 20 \times .01\,dx + \int_{20}^{50} x \times .01\,dx + \int_{50}^{100} 50 \times .01\,dx = 4 + 10.5 + 25 = 39.5$.

Answer: E

13. There are 6 possible rankings that a surveyed fan can choose:
BEG , BGE , EBG , EGB , GEB , GBE

We are given the following:

$P(BEG) + P(BGE) = 0.5$, $P(EBG) + P(GBE) = 0.3$, $P(BEG) + P(GEB) = 0.3$,
$P(BGE) + P(GBE) = 0.5$, $P(BEG) = 0.2$.

We wish to find $P(EGB|EGB \cup EBG) = \frac{P(EGB)}{P(EGB)+P(EBG)}$.

Since 80% ranked England either second or third, it follows that 20% ranked England first, so

$P(EGB \cup EBG) = P(EGB) + P(EBG) = 0.2$.

From the given information, we have $0.2 + P(BGE) = 0.5 \rightarrow P(BGE) = 0.3$.

Then, $0.3 + P(GBE) = 0.5 \rightarrow P(GBE) = 0.2$.

Then, $P(EBG) + 0.2 = 0.3 \rightarrow P(EBG) = 0.1$, and then $P(EGB) + 0.1 = 0.2$
$\rightarrow P(EGB) = 0.1$.

Finally, $\frac{P(EGB)}{P(EGB)+P(EBG)} = \frac{0.1}{0.1+0.1} = \frac{1}{2}$. Answer: C

14. We define the following events:

$B2$ - a surveyed individual ranked Brazil second ,

$G1$ - a surveyed individual ranked Germany first .

We wish to find $P(G3|B2) = \frac{P(B2 \cap G3)}{P(B2)}$. We are given $P(B2) = 0.3$,

and we are given the conditional probabilities $P(B2|G1) = \frac{2}{3}$ and $P(B2|G1') = \frac{1}{7}$.

From $P(B2|G1) = \frac{P(B2 \cap G1)}{P(G1)} = \frac{2}{3}$ we get $P(B2 \cap G1) = \frac{2}{3} \cdot P(G1)$, and from

$P(B2|G1') = \frac{1}{7}$ we get $P(B2 \cap G1') = \frac{1}{7} \times P(G1') = \frac{1}{7} \times [1 - P(G1)]$.

Therefore $0.3 = P(B2) = P(B2 \cap G1) + P(B2 \cap G1') = \frac{2}{3} \times P(G1) + \frac{1}{7} \times P(G1')$

$= \frac{2}{3} \times P(G1) + \frac{1}{7} \times [1 - P(G1)]$, from which we get $P(G1) = 0.3$.

Then, $P(B2 \cap G3) = P(B2 \cap G1') = \frac{1}{7} \times [1 - P(G1)] = 0.1$

and $P(G3|B2) = \frac{P(B2 \cap G3)}{P(B2)} = \frac{0.1}{0.3} = \frac{1}{3}$. Answer: B

15. In order to have no matching number on either ticket, the 6 randomly chosen numbers must come from the 37 other numbers, 13 , 14 , . . . , 49. The probability in question is the ratio of the number of random ticket draws that result in the event over the total possible number of random ticket draws.

$P(A) = \frac{\binom{37}{6}}{\binom{49}{6}} = \frac{\text{\# randomly chosen tickets that avoid 1,2,...,12}}{\text{total number of possible randomly chosen tickets}} = \frac{37!/(31!\,6!)}{49!/(43!\,6!)} = 0.166248$.

Answer: C

16. Y can be thought of as the conditional distribution of X given that X is not 4, 5 or 6.

The probability function of Y is

$P(Y = 1) = P(X = 1|X \neq 4, 5, 6) = \frac{P(X=1)}{P(X \neq 4,5,6)} = \frac{1/9}{4/9} = \frac{1}{4}$,

$P(Y = 2) = P(X = 2|X \neq 4, 5, 6) = \frac{P(X=2)}{P(X \neq 4,5,6)} = \frac{2/9}{4/9} = \frac{1}{2}$,

$P(Y = 3) = P(X = 3|X \neq 4, 5, 6) = \frac{P(X=3)}{P(X \neq 4,5,6)} = \frac{1/9}{4/9} = \frac{1}{4}$.

$E[Y] = (1)(\frac{1}{4}) + (2)(\frac{1}{2}) + (3)(\frac{1}{4}) = 2$,

$E[Y^2] = (1^2)(\frac{1}{4}) + (2^2)(\frac{1}{2}) + (3^2)(\frac{1}{4}) = \frac{9}{2}$.

$Var[Y] = E[Y^2] - (E[Y])^2 = \frac{9}{2} - 2^2 = \frac{1}{2}$. Answer: C

17. We wish to find $Var[Z] = E[Z^2] - (E[Z])^2 = E[Z^2] - (5.3)^2$.

Let us denote $P(X=0) = p_0$, $P(X=1) = p_1$, $P(X=2) = p_2$, etc.

Then $P(Y=0) = 0$, $P(Y=1) = p_0 + p_1$, $P(Y=2) = p_2$, etc.

Then $P(Z=0) = P(Z=1) = 0$, $P(Z=2) = p_0 + p_1 + p_2$,

$5.0 = E[X] = p_1 + 2p_2 + 3p_3 + \cdots$ and

$5.1 = E[Y] = (p_0 + p_1) + 2p_2 + 3p_3 + \cdots$ and

$5.3 = E[Z] = 2(p_0 + p_1 + p_2) + 3p_3 + \cdots$.

Therefore, $0.1 = E[Y] - E[X] = p_0$ and $.2 = E[Z] - E[Y] = p_0 + p_1 = .1 + p_1$,

so that $p_1 = 0.1$.

From $10 = Var[X] = E[X^2] - (E[X])^2 = E[X^2] - 25$, we have

$E[X^2] = 35 = p_1 + 4p_2 + 9p_3 + \cdots$

Then, $E[Z^2] = 4(p_0 + p_1 + p_2) + 9p_3 + \cdots = 4p_0 + 3p_1 + E[X^2]$

$= 4 \times 0.1 + 3 \times 0.1 + 35 = 35.7$, and

$Var[Z] = 35.7 - (5.3)^2 = 7.61$. Answer: D

18. Suppose that T is the time until failure of the machine. $P(a < T \le b) = e^{-a/3} - e^{-b/3}$.

The fraction of the purchase price refunded is a random variable X that can be described in the following way:

$$X = \begin{cases} 1 & 0 < T \le 1 & \text{prob. } 1 - e^{-1/3} \\ 3/4 & 1 < T \le 2 & \text{prob. } e^{-1/3} - e^{-2/3} \\ 1/2 & 2 < T \le 3 & \text{prob. } e^{-2/3} - e^{-4/3} \\ 1/4 & T > 3 & \text{prob. } e^{-4/3} \end{cases}$$

Then, $E[X] = 1 - e^{-1/3} + \frac{3}{4}(e^{-1/3} - e^{-2/3}) + \frac{1}{2}(e^{-2/3} - e^{-4/3}) + \frac{1}{4}e^{-4/3}$

$= 1 - \frac{1}{4}e^{-1/3} - \frac{1}{4}e^{-2/3} - \frac{1}{4}e^{-4/3} = 0.627$. Answer: D

19. The coefficient of variation of Y is $\frac{\sqrt{Var(Y)}}{E(Y)}$.

$$E(Y) = \int_{200}^{500} \tfrac{1}{2}(x-200)(0.001)\,dx + \int_{500}^{1000} [150 + \tfrac{1}{4}(x-500)](0.001)\,dx = \frac{45}{2} + \frac{425}{4} = \frac{515}{4}$$

$$E(Y^2) = \int_{200}^{500} [\tfrac{1}{2}(x-200)]^2(0.001)\,dx + \int_{500}^{1000} [150 + \tfrac{1}{4}(x-500)]^2(0.001)\,dx$$
$$= 2250 + \frac{139{,}375}{6} = \frac{152{,}875}{6}.$$

$$Var(Y) = \frac{152{,}875}{6} - \left(\frac{515}{4}\right)^2 = \frac{427{,}325}{48} = 8902.6.$$

The coefficient of variation is $\frac{\sqrt{8902.6}}{128.75} = 0.733$. Answer: D

20. P[At least 4 tosses are needed] $= 1 - P$[at most 3 tosses are needed] .

It is not possible to reach the total of 14 on 1 or 2 tosses.

There are $6 \times 6 \times 6 = 216$ possible sets of 3 consecutive tosses.

The following sets of 3 consecutive tosses result in a total of at least 14 on the faces that turn up.

(a) Three 6's (6 on each toss); 1 set.

(b) Two 6's and 2 to 5 on the other toss ; $4 \times 3 = 12$ sets
(6,6,2 , and 6,2,6 and 2,6,6 , and the same with 3 or 4 or 5 instead of 2).

(c) One 6 and either 5-5, or 4-5, or 4-4, or 3-5; $3 + 6 + \ + 3 + 6 = 18$ sets
(6,5,5 or 5,6,5 or 5,5,6, and 6,4,5 in six arrangements, and 6-3-5 in six arrangements).

(d) No 6's, and either three 5's, or two 5's and a 4; $1 + 3 = 4$ sets.

Total of $1 + 12 + 18 + 4 = 35$ sets out of 216 possible sets.

Probability is P[At least 4 tosses are needed] $= 1 - \frac{35}{216} = 0.838.$ Answer: E

21. The exponential distribution with mean θ has pdf $f(t) = \frac{1}{\theta}e^{-t/\theta}$ and cdf $F(x) = 1 - e^{-x/\theta}$.

For a non-negative loss random variable L with cdf $F(y)$, if a policy limit of u is imposed, the expected payment by the insurer when a loss occurs is $\int_0^u [1 - F(y)]\,dy$.

For the exponential loss random variable with mean 800 and with limit u, the expected amount paid by the insurer when a loss occurs is $\int_0^u e^{-x/\theta}\,dx = 800[1 - e^{-u/800}]$.

If the limit is $2u$, the expected payment by the insurer when a loss occurs is $800[1 - e^{-2u/800}]$.

We are given that $800[1 - e^{-2u/800}] = 1.2865(800[1 - e^{-u/800}])$.

After canceling 800 and factoring the difference of squares

$1 - e^{-2u/800} = (1 - e^{-u/800})(1 + e^{-u/800})$,

this equation becomes $1 + e^{-u/800} = 1.2865$, so that $u = 1000$. Answer: C

22. The mean and variance of the exponential loss with mean 1 are 1 and 1, and the mean and variance of the exponential distribution with mean 2 are 2 and 4.

(a) $S_a = X_1 + \cdots + X_{400}$. $E[S_a] = 400(1) = 400$, $Var[S_a] = 400(1) = 400$.
$P[S_a \le A] = P[\frac{S_a - 400}{\sqrt{400}} \le \frac{A - 400}{\sqrt{400}}] = 0.95 \rightarrow \frac{A-400}{\sqrt{400}} = 1.645 \rightarrow A = 432.9$.

(b) $S_b = Y_1 + \cdots + Y_{400}$. $E[S_b] = 400(2) = 800$, $Var[S_b] = 400(4) = 1600$.
$P[S_b \le B] = P[\frac{S_b - 800}{\sqrt{1600}} \le \frac{B - 800}{\sqrt{1600}}] = 0.95 \rightarrow \frac{B-800}{\sqrt{1600}} = 1.645 \rightarrow B = 865.8$.

(c) $S_c = X_1 + \cdots + X_{400} + Y_1 + \cdots + Y_{400}$.
$E[S_c] = 400 + 800 = 1200$, $Var[S_c] = 400 + 1600 = 2000$.
$P[S_c \le C] = P[\frac{S_c - 1200}{\sqrt{2000}} \le \frac{C - 1200}{\sqrt{2000}}] = 0.95 \rightarrow \frac{C-1200}{\sqrt{2000}} = 1.645 \rightarrow A = 1273.6$
$\frac{C}{A+B} = \frac{1273.6}{432.9+865.8} = 0.9807.$ Answer: E

23. Since $x < y < 2x$, it follows that $\frac{y}{2} < x < y$.

Also, since $x > 1$ it follows that $x > max\{\frac{y}{2}, 1\}$, and $y > 1$.

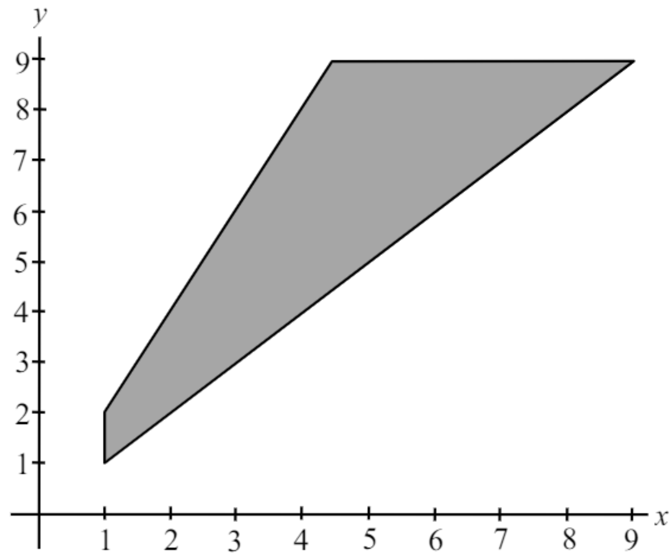
Therefore, if $1 < y \leq 2$, it follows that $x > 1$, and if $y > 2$ then $x > \frac{y}{2}$.

The joint density of X and Y is $f(x,y) = f(y|x) \times f_X(x) = \frac{1}{x} \times \frac{1}{x^2} = \frac{1}{x^3}$.

If $1 < y < 2$, then this joint pdf is defined for $1 < x < y$,

and if $y \geq 2$, then this joint pdf is defined for $\frac{y}{2} < x < y$.

The shaded region below is the region of joint density.



The pdf of the marginal distribution of Y is $f_Y(y) = \int f(x,y)\,dx$.

For $1 < y < 2$, we get $f_Y(y) = \int_1^y \frac{1}{x^3}\,dx = \frac{1}{2} - \frac{1}{2y^2}$.

For $y \geq 2$, we get $f_Y(y) = \int_{y/2}^y \frac{1}{x^3}\,dx = \frac{4}{2y^2} - \frac{1}{2y^2} = \frac{3}{2y^2}$. Answer: A

24. Suppose that the covariance between X and Y is C. Then $X - Y$ has a normal distribution with mean $1 - 1 = 0$ and variance

$Var[X - Y] = Var[X] + Var[Y] - 2\,Cov(X,Y) = 1 + 1 - C = 2 - C$.

Then, $P(X - Y > 1) = P(\frac{X-Y}{\sqrt{2-C}} > \frac{1}{\sqrt{2-C}}) = 0.2119$.

$Z = \frac{X-Y}{\sqrt{2-C}}$ has a standard normal distribution, and from the standard normal table,

we get $\frac{1}{\sqrt{2-C}} = 0.80$.

Then, $P(X > Y + 2) = P(X - Y > +2) = P(\frac{X-Y}{\sqrt{2-C}} > \frac{2}{\sqrt{2-C}}) = P(Z > 1.6) = 0.0548$.

Answer: B

25. N denotes the number of home runs hit in the game. $E[N] = 4$ and N has a Poisson distribution. The amount donated X (multiples of 100,000) can be summarized as follows:

Define Y to be $Y = N - X$.

N	0	1	2	3	4	5	...
X	0	0	0	1	2	3	...
Y	0	1	2	2	2	2	...

We know that $X + Y = N$ so that $E[X] + E[Y] = E[N] = 4$.

But we also can see that Y can only be 0, 1 or 2, and

$P(Y = 0) = P(N = 0) = e^{-4}$, $P(Y = 1) = P(N = 1) = 4e^{-4}$

and $P(Y = 2) = P(N \geq 2) = 1 - P(N = 0, 1) = 1 - 5e^{-4}$.

Therefore, $E[X] = 4 - E[Y] = 4 - (1)(4e^{-4}) - (2)[1 - 5e^{-4}] = 2.11$

and the expected amount paid by the Blue Jays is 211,000. Answer: D

26. There are 6 possible pairs of aces (Spade-Heart, Spade-Diamond, Spade-Club, Heart-Diamond, Heart-Club, Diamond-Club, and there are 13 possible ranks (ace, king,...),

for a total of 78 possible pairs in the first two cards. There are $\binom{52}{2} = \frac{52 \cdot 51}{2} = 1326$ possible two-card combinations that can be received in the first two cards. The probability of getting a pair in the first two cards is $\frac{78}{1326} = 0.0588$. Answer: E

27. $Var(aX + b) = a^2 Var(X) \rightarrow 4a^2 = 1 \rightarrow a = \frac{1}{2}$.

$E(aX + b) = aE(X) + b \rightarrow 2a + b = 5 \rightarrow b = 4 \rightarrow ab = 2$. Answer: B

28. In order to be a properly defined joint distribution, it must be true that $c + 2c + \frac{c}{2} + c = 1$.

Therefore, $c = \frac{2}{9}$.

Then $E(XY) = (1)(1)(\frac{2}{9}) + (1)(2)(\frac{1}{9}) + (2)(1)(\frac{4}{9}) + (2)(2)(\frac{2}{9}) = \frac{20}{9}$.

The marginal distribution of X has $P(X = 1) = \frac{2}{9} + \frac{1}{9} = \frac{1}{3}$, and $P(X = 2) = \frac{2}{3}$, so $E(X) = \frac{5}{3}$.

The marginal distribution of Y has $P(Y = 1) = \frac{2}{9} + \frac{4}{9} = \frac{2}{3}$, and $P(Y = 2) = \frac{1}{3}$, so $E(Y) = \frac{4}{3}$.

The covariance is $COV(X, Y) = E(XY) - E(X)E(Y) = \frac{20}{9} - (\frac{5}{3})(\frac{4}{3}) = 0$.

An alternative solution follows from the observation that X and Y are independent. Once we have determined the marginal distributions of X and Y, we can check to see if

$P(X = x, Y = y) = P(X = x) \cdot P(Y = y)$ for each (x, y) pair. For instance,

$P(X = 1, Y = 1) = c = \frac{2}{9}$ and $P(X = 1) \times P(Y = 1) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$. This turns out to be true for all (x, y) pairs. It follows that X and Y are independent, from which it follows that the covariance between X and Y is 0. Answer: C

29. If $X > \frac{2\theta}{3}$, then $2\theta > 3X > Y$, so
$P(Y < 3X) = \int_0^{2\theta/3}\int_0^{3x} \frac{1}{\theta} \times \frac{1}{2\theta}\, dy\, dx + \int_{2\theta/3}^{\theta} \frac{1}{\theta}\, dx = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$. Answer: D

30. For Policy 1, the maximum payment amount of 500 is reached if the loss is 600 or more because the deductible of 100 is applied first.

Expected insurance payment with Policy 1 is

$$\int_{100}^{600} (x - 100) \times \frac{1}{1000}\, dx + 500P(X > 600) = 125 + 500(\tfrac{4}{10}) = 325.$$

Expected insurance payment with Policy 2 is

$$\int_0^{500} x \times \frac{1}{1000}\, dx + 400P(X > 500) = 125 + 400(\tfrac{5}{10}) = 325.$$ Answer: C

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PRACTICE EXAM 8

1. X is a continuous random variable with density function $f(x) = \begin{cases} |x| & \text{for } -1 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$.

Find $E[\,|X|\,]$.

A) 0 B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) 1 E) $\frac{4}{3}$

2. As part of the underwriting process for insurance, each prospective policyholder is tested for diabetes. Let X represent the number of tests completed when the first person with diabetes pressure is found. The expected value of X is 8. Calculate the probability that the fourth person tested is the first one with diabetes.

A) 0.000 B) 0.050 C) 0.084 D) 0.166 E) 0.394

3. If X has a normal distribution with mean 1 and variance 4, then $P[X^2 - 4X \le 0] = ?$

A) Less than 0.15 B) At least 0.15 but less than 0.35 C) At least 0.35 but less than 0.55
D) At least 0.55 but less than 0.75 E) At least 0.75

4. Let X and Y be discrete random variables with joint probability function $f(x, y)$ given by the following table:

		x			
		2	3	4	5
	0	0.05	0.05	0.15	0.05
y	1	.40	0	0	0
	2	0.05	0.15	0.10	0

Calculate $Cov[X - Y, X + Y]$.

A) Less than -1 B) At least -1 but less than 0 C) At least 0 but less than 1
D) At least 1 but less than 2 E) At least 2

5. Let X and Y be continuous random variables with joint density function
$f(x, y) = \begin{cases} c(y-x) & \text{for } 0<x<y<1 \\ 0, & \text{otherwise} \end{cases}$.

What is the mean of the marginal distribution of X?

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$ E) $\frac{5}{8}$

6. According to NBA playoff statistics, if a team has won 3 games and lost 1 game out of the first 4 games during a "best of 7" playoff series, that team has an 80% chance of winning the series. Statistics also show that if a team has won 3 games and lost 1 game out of the first 4 games and then loses the 5th game, that team has a 65% chance of winning the series. Find the probability that a team that has won 3 games and lost 1 game out of the first 4 games will win the next game.

A) $\frac{2}{7}$ B) $\frac{3}{7}$ C) $\frac{4}{7}$ D) $\frac{5}{7}$ E) $\frac{6}{7}$

7. A statistician for the National Hockey League has created a model for the number of goals scored per 60-minute game by the Ottawa Senators and the Buffalo Sabres. According to the model, the number of goals scored per game by the Senators has a geometric distribution, $X_{OTT} = 0, 1, 2, ...$ with a mean of 3.5. The model also has a similar geometric distribution for the number of goals scored per 60-minute game by the Sabres, X_{BUF}, with a mean of 3.0. Assuming that X_{OTT} and X_{BUF} are independent, find the probability that Buffalo wins the game in 60 minutes by at least 2 goals.

A) 0.1 B) 0.2 C) 0.3 D) 0.4 E) 0.5

8. n fair six-sided dice are tossed independently of one another.
Find the probability that the sum is even.

A) $\frac{1}{2} - \frac{(n-1)(n-2)(n-3)}{6n^3}$ B) $\frac{1}{2} - \frac{(n-1)(n-2)}{6n^2}$ C) $\frac{1}{2}$

D) $\frac{1}{2} + \frac{(n-1)(n-2)(n-3)}{6n^3}$ E) $\frac{1}{2} + \frac{(n-1)(n-2)}{6n^2}$

9. A fair coin is tossed 100 times. The tosses are independent of one another. The number of heads tossed is X. It is desired to find the smallest integer value k which satisfies the probability relationship $P(50 - k \leq X \leq 50 + k) \geq 0.95$.
Find k by applying the normal approximation with integer correction to the distribution of X.

A) 6 B) 7 C) 8 D) 9 E) 10

10. The loss random variable X has an exponential distribution with a mean of $\theta > 0$.
An insurance policy pays Y, where $Y = \begin{cases} \frac{X}{2} & \text{if } X \leq \theta \\ X & \text{if } X > \theta \end{cases}$. Find $E[Y]$.

A) $\frac{\theta}{2}(1 + e^{-1})$ B) $\frac{\theta}{2}(1 + 2e^{-1})$ C) $\frac{\theta}{2}(1 - e^{-1})$ D) $\frac{\theta}{2}(1 - 2e^{-1})$ E) $\theta(1 - e^{-1})$

11. X has a continuous uniform distribution on the interval $[0, 1]$ and the conditional distribution of Y given $X = x$ is a continuous uniform distribution on the interval $[x, 2]$. Find $E[Y]$.

A) $\frac{3}{4}$ B) 1 C) $\frac{5}{4}$ D) $\frac{3}{2}$ E) $\frac{7}{4}$

12. A loss random variable X has a continuous uniform distribution on the interval $(0, 100)$.

An insurance policy on the loss pays the full amount of the loss if the loss is less than or equal to 40. If the loss is above 40 but less than or equal to 80, then the insurance pays 40 plus one-half of the loss in excess of 40. If the loss is above 80, the insurance pays 60. If Y denotes the amount paid by the insurance when a loss occurs, find the variance of Y.

A) $\frac{1020}{3}$ B) $\frac{1040}{3}$ C) $\frac{1060}{3}$ D) $\frac{1080}{3}$ E) $\frac{1110}{3}$

13. A survey of a large number of adult city dwellers identified two characteristics involving personal transportation: • have a driver's licence • own a bicycle .

The following information was determined.

- 80% of those surveyed had a driver's licence or owned a bicycle, or both
- $\frac{1}{3}$ of those who had a driver's license also owned a bike
- $\frac{1}{2}$ of those who owned a bike also had a driver's license.

Of those surveyed who didn't own a bike, find the fraction that didn't have a driver's license.

A) $\frac{1}{3}$ B) $\frac{4}{9}$ C) $\frac{5}{9}$ D) $\frac{2}{3}$ E) $\frac{7}{9}$

14. A particular large calculus class has two term tests and a final exam.

Students are not allowed to drop the course before the first term test.

Class records for past years show the following:

- 80% of students pass the first test
- 30% of students who fail the first term test drop the course before the second test
- 10% of students who pass the first term test drop the course before the second test
- 90% of students who pass the first term test and take the second test pass the second test
- 80% of students who fail the first term test and take the second test pass the second test
- 50% of students who fail the second term test drop the course before the final exam
- none of students who pass the second term test drop the course before the final exam.

Find the fraction of students who drop the course.

A) Less than $\frac{1}{20}$ B) At least $\frac{1}{20}$ but less than $\frac{1}{10}$ C) At least $\frac{1}{10}$ but less than $\frac{3}{20}$
D) At least $\frac{3}{20}$ but less than $\frac{1}{5}$ E) At least $\frac{1}{5}$

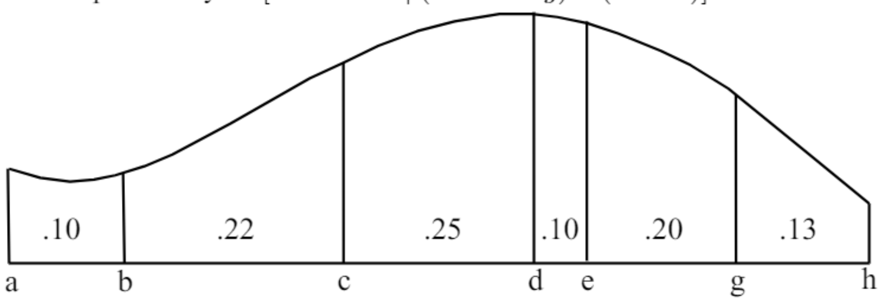
15. An urn has 6 identically shaped balls. 4 of the balls are white and 2 of the balls are blue.

A ball is chosen at random from the urn and replaced with a white ball. The procedure is done repeatedly.

Find the probability that after the n-th application of this procedure there is exactly one blue ball in the urn.

A) $(\frac{5}{6})^n + (\frac{2}{3})^n$ B) $(\frac{5}{6})^n - (\frac{2}{3})^n$ C) $2[(\frac{5}{6})^n + (\frac{2}{3})^n]$
D) $2[(\frac{5}{6})^n - (\frac{2}{3})^n]$ E) $2(\frac{5}{12})^n$

16. The graph below is the pdf of a continuous random variable X on the interval $[a, h]$.
The numerical values represent probabilities for the subintervals.
Find the conditional probability $P[b < X < e \mid (c < X < g) \cap (X < d)]$.



A) Less than 0.15 B) At least 0.15 but less than 0.35 C) At least 0.35 but less than 0.55
D) At least 0.55 but less than 0.75 E) At least 0.75

17. X has a Poisson distribution with a mean of 1, so the probability function for X is
$P(X = x) = \frac{e^{-1}}{x!}$ for $x = 0, 1, 2, \ldots$
Y is a new random variable on the non-negative integers. The probability function of Y is related to that of X as follows. A number α is given, with $0 < \alpha < 1$.
$P(Y = 0) = \alpha$, $P(Y = x) = c \cdot P(X = x)$ for $x = 1, 2, \ldots$
The number c is found so that Y satisfies the requirement for being a random variable $\sum_{x=0}^{\infty} P(Y = x) = 1$.
Find the mean of Y in terms of α and e.
A) $\frac{1-\alpha}{1-e^{-\alpha}}$ B) $\frac{1-\alpha}{e^{\alpha}-1}$ C) $\frac{1-\alpha}{1-e^{-1}}$ D) $\frac{1-\alpha}{e-1}$ E) $\frac{\alpha}{e-1}$

18. The Toronto Maple Leafs have two suppliers for hockey sticks, Crosscheck Lumber, and Sticks R Us. The Leafs get equal numbers of sticks from each supplier, and since the team logo is branded on every stick, after the sticks are delivered, it is not possible to tell what supplier provided any particular stick. The team estimates that on average, 10% of the sticks from Crosscheck lumber are defective and 20% of the sticks from Sticks R Us are defective. A Leaf player examines 10 sticks from a recent shipment from a supplier but doesn't know who the supplier was. The player finds 2 defective sticks out of the 10 sticks. Find the probability that the supplier of those sticks was Crosscheck Lumber.
A) Less than 0.11 B) At least 0.11 but less than 0.22 C) At least 0.22 but less than 0.33
D) At least 0.33 but less than 0.44 E) At least 0.44

19. The Winnipeg Rangers hockey team is considering a one-time charitable program of making a donation to the Winnipeg Children's Hospital. The donation will be related to how many goals they score in their next game. The team statistician has determined that the number of goals scored by the Rangers in a game has a Poisson distribution with a mean of 3. The Rangers are planning donate $K for each goal they score up to a maximum of 3 goals. Find the value of K that would make the Rangers' expected donations for a game to be $5000.

A) Less than 2000 B) At least 2000 but less than 2100 C) At least 2100 but less than 2200
D) At least 2200 but less than 2300 E) At least 2300

20. X has a distribution which is partly continuous and partly discrete.
X has a discrete point of probability at $X = 1$ with probability p, where $0 < p < 1$.
On the interval $(0, 1)$ X has a constant density of $\frac{1-p}{2}$,
and on the interval $(1, 2)$ X has a constant density of $\frac{1-p}{2}$.
Find the variance of X in terms of p.

A) $\frac{1-p}{3}$ B) $\frac{2-p}{3}$ C) $\frac{1-p}{2}$ D) $\frac{2-p}{2}$ E) $\frac{1+p}{2}$

21. X has the following pdf: $f(x) = \begin{cases} x - \frac{x^2}{2} & \text{if } 0 < x \le 1 \\ \frac{x^2}{2} - x + 1 & \text{if } 1 < x < 2 \end{cases}$, and 0 otherwise.

The random variable Y is defined as follows: $Y = X^2$. Find $F_Y(2)$.

A) 0.33 B) 0.48 C) 0.55 D) 0.67 E) 0.80

22. You are given the following:

- X_1 has a binomial distribution with a mean of 2 and a variance of 1
- X_2 has a Poisson distribution with a variance of 2
- X_1 and X_2 are independent
- $Y = X_1 + X_2$

What is $P(Y < 3)$?

A) $\frac{11}{16}e^{-2}$ B) $\frac{15}{16}e^{-2}$ C) $\frac{19}{16}e^{-2}$ D) $\frac{23}{16}e^{-2}$ E) $\frac{27}{16}e^{-2}$

23. X has pdf $f(x) = x$ for $0 < x < 1$.
Also, $P(X = 0) = a$ and $P(X = 1) = b$, and $P(X < 0) = P(X > 1) = 0$.

For what value of a is $Var(X)$ maximized?

A) $0 \le a < 0.1$ B) $0.1 \le a < 0.2$ C) $0.2 \le a < 0.3$ D) $0.3 \le a < 0.4$ E) $a \ge 0.4$

24. You are given the events $A \neq \emptyset$ and $B \neq \emptyset$ satisfy the relationships

(i) $P(A \cap B) > 0$ and

(ii) $P(A|B) = P(B|A)$ (conditional probabilities) .

How many of the following statements always must be true?

I. A and B are independent. II. $P(A) = P(B)$ III. $A = B$

A) None B) 1 C) 2 D) All 3 E) None of A,B,C or D is correct

25. A loss random variable is uniformly distributed on the interval $(0, 2000)$.

An insurance policy on this loss has an ordinary deductible of 500 for loss amounts up to 1000.

If the loss is above 1000, the insurance pays half of the loss amount.

Find the standard deviation of the amount paid by the insurance when a loss occurs.

A) Less than 250 B At least 250, but less than 300 C) At least 300, but less than 350

D) At least 350, but less than 400 E) At least 400

26. Random variables X and Y have a joint distribution with joint pdf

$f(x,y) = \frac{2x+y}{12}$ for $0 \leq x \leq 2$ and $0 \leq y \leq 2$

Find the conditional probability $P(X+Y \geq 2 | X \leq 1)$.

A) $\frac{1}{8}$ B) $\frac{1}{4}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$ E) $\frac{5}{8}$

27. The pdf of X is $f(x) = ax + b$ on the interval $[0, 2]$ and the pdf is 0 elsewhere.

You are given that the median of X is 1.25. Find the variance of X.

A) Less than 0.05 B) At least 0.05 but less than 0.15 C) At least 0.15 but less than 0.25

D) At least 0.25 but less than 0.35 E) At least 0.35

28. In the casino game of roulette, a wheel with 38 equally likely spots is spun, and a ball is dropped at random into one of the 38 spots. The 38 spots are numbers 1 to 36 along with 0 and 00. On a spin of the wheel, a gambler can bet that the ball will drop into a specified spot. If the ball does drop into that spot, the gamble gets back the amount that he bet plus 36 time the amount that he bet. If that spot does not turn up, the gambler loses the amount bet. A gambler can also bet that the outcome of the spin will be even. If the ball drops into an even number spot from 2 to 36, the gambler gets back his bet plus an amount equal to the amount that he bet (the bet is lost if the spot is 0 or 00). On every spin, Gambler 1 always bets 1 that the ball will drop in the spot with the number 1, and Gambler 2 always bets 1 that the ball will drop into an even numbered spot. X_1 denotes the net profit of Gambler 1 after n spins, and X_2 denotes the net profit of Gambler 2 after the n spins. Find $E(X_2 - X_1)$.

A) $-\frac{n}{19}$ B) $-\frac{n}{38}$ C) 0 D) $\frac{n}{38}$ E) $\frac{n}{19}$

29. A loss random variable X has a Poisson distribution with a mean of λ

An insurance policy on the loss has a policy limit of 1.

The expected insurance payment when a loss occurs is 0.8892 .

Find the expected insurance payment when a loss occurs for a policy on the same loss variable if the policy limit is 2.

A) Less than 0.35 B) At least 0.35 but less than 0.70 C) At least 0.70 but less than 1.05

D) At least 1.05 but less than 1.4 E) At least 1.4

30. An insurer has two lines of business: auto insurance and home fire insurance.

People with a home fire insurance policy can add flood insurance coverage, but only if the policy already has fire coverage. You are given the following information about the insurer's customers:

- 80% of all customers have an auto insurance policy
- 40% of all customers have a fire insurance policy
- 25% of customers with an auto insurance policy also have a fire insurance policy
- 50% of customers with a fire insurance policy also have flood insurance
- 50% of customers with flood insurance coverage also have auto insurance

Of the insurer's customers that have fire insurance, find the fraction that have neither auto insurance nor flood insurance coverage.

A) 0.05 B) 0.10 C) 0.15 D) 0.20 E) 0.25

PRACTICE EXAM 8 - SOLUTIONS

1. $E[\,|X|\,] = \int_{-1}^{1} |x|\, f(x)\, dx = \int_{-1}^{1} |x| \times |x|\, dx = \int_{-1}^{1} |x|^2\, dx = \int_{-1}^{1} x^2\, dx = \frac{2}{3}$.
Answer: C

2. This problem makes use of the geometric distribution. The experiment being performed is the diabetes test on an individual. We define "success" of the experiment to mean that the individual has high diabetes. We denote the probability of a success occurring in a particular trial by p. Since X is the number of persons tested until the first person with diabetes is found, it is a version of the geometric distribution, where Y is the trial number of the first success (the trial number of the first success is 1, or 2, or 3, ...).
The probability function if $P(Y = k) = (1-p)^{k-1}p\ ,\ k = 1, 2, 3, ...$
The mean of this form of the geometric distribution is $\frac{1}{p}$, so that $\frac{1}{p} = 8$ and therefore $p = \frac{1}{8}$.
The probability that the first success occurs on the 4th trial (first case of diabetes is the 4th individual) is $(1-p)^3 p$, since there will be 3 failures and then the first success. This probability is $(\frac{7}{8})^3 \times \frac{1}{8} = 0.08374$.
Answer: C

3. Since $X \sim N(1, 4)$, $Z = \frac{X-1}{2}$ has a standard normal distribution. The probability in question can be written as
$$P[X^2 - 4X \le 0] = P[X^2 - 4X + 4 \le 4] = P[(X-2)^2 \le 4] = P[-2 \le X - 2 \le 2]$$
$$= P[-1 \le X - 1 \le 3]$$
$$= P[-0.5 \le \tfrac{X-1}{2} \le 1.5] = P[-.5 \le Z \le 1.5] = \Phi(1.5) - [1 - \Phi(0.5)]$$
$= 0.9332 - 0.3085 = 0.6247$. (from the standard normal table). Answer: D

4. $Cov[X - Y, X + Y] = Cov[X, X] + Cov[X, Y] - Cov[Y, X] - Cov[Y, Y] = Var[X] - Var[Y]$
The marginal distribution of X has probability function
$P(X = 2) = 0.5\,,\ P(X = 3) = 0.2\,,\ P(X = 4) = 0.25\,,\ P(X = 5) = 0.05$.

$E[X] = 2 \times 0.5 + 3 \times 0.2 + 4 \times 0.25 + 5 \times .05 = 2.85$.

$E[X^2] = 4 \times 0.5 + 9 \times 0.2 + 16 \times 0.25 + 25 \times 0.05 = 9.05$.

$Var[X] = E[X^2] = (E[X])^2 = 9.05 - 2.85^2 = 0.9275$.
The marginal distribution of Y has probability function
$P(Y = 0) = 0.3\,,\ P(Y = 1) = 0.4\,,\ P(Y = 2) = 0.3$.
$E[Y] = 1 \times 0.4 + 2 \times 0.3 = 1.0$.
$E[Y^2] = 1 \times 0.4 + 4 \times 0.3 = 1.6$.
$Var[Y] = E[Y^2] = (E[Y])^2 = 1.6 - 1^2 = 0.6$.
$Cov[X - Y, X + Y] = Var[X] - Var[Y] = 0.9275 - 0.6 = 0.3275$. Answer: C

5. In order for this to be a properly defined joint pdf, we must have

$\int_0^1 \int_x^1 c(y-x)\,dy\,dx = 1$.

$\int_x^1 c(y-x)\,dy = c[\frac{1-x^2}{2} - x(1-x)] = \frac{c(1-x)^2}{2}$, and $\int_0^1 \frac{c(1-x)^2}{2}\,dx = \frac{c}{6}$.

Therefore, $c = 6$.

$f_X(x) = \int_x^1 6(y-x)\,dy = 3(1-x)^2$, $0 < x < 1$

$E[X] = \int_0^1 3x(1-x)^2\,dx = 3\int_0^1 [x - 2x^2 + x^3]\,dx = 3[\frac{1}{2} - 2 \times \frac{1}{3} + \frac{1}{4}] = 0.25$. Answer: B

6. We define the following events and probabilities:

$W =$ team wins the best-of-7 series,

$G =$ team loses game 5,

$T =$ team wins 3 of the first 4 games,

$q =$ probability team wins 5th game given that it has won 3 of the first 4 games.

Our objective is to find $q = P[G'|T]$.

We are given $P[W|T] = .8$ and $P[W|T \cap G] = 0.65$.

$P[W \cap G|T] = \frac{P[W \cap G \cap T]}{P[T]} = \frac{P[W \cap G \cap T]}{P[G \cap T]} \cdot \frac{P[G \cap T]}{P[T]}$

$= P[W|G \cap T] \times P[G|T] = 0.65 \times (1-q)$.

$.8 = P[W|T] = P[W \cap G|T] + P[W \cap G'|T] = 0.65 \times (1-q) + P[W \cap G'|T]$.

$P[W \cap G'|T] = \frac{P[W \cap G' \cap T]}{P[T]} = \frac{P[W \cap G' \cap T]}{P[G' \cap T]} \times \frac{P[G' \cap T]}{P[T]} = P[W|G' \cap T] \times P[G'|T] = q$

(this is true, since $P[W|G' \cap T] = 1$, because winning 3 out of the first 4 and then winning the 5th game results in winning the series) . Therefore, $0.8 = 0.65 \times (1-q) + q \rightarrow q = \frac{0.15}{0.35} = \frac{3}{7}$.

An alternative solution is as follows.

$P[W'|T] = 0.2$ and $P[W'|T \cap G] = 0.35$ (these are the complement of the given probabilities $P[W|T] = 0.8$ and $P[W|T \cap G] = 0.65$).

First note that $P[W' \cap T] = P[W' \cap G \cap T]$, because in order to win 3 of the first 4 games and lose the series, it must be true that the team loses the 5th (and all subsequent games). Therefore,

$0.2 = P[W'|T] = \frac{P[W' \cap T]}{P[T]} = \frac{P[W' \cap G \cap T]}{P[T]} = \frac{P[W' \cap G \cap T]}{P[G \cap T]} \times \frac{P[G \cap T]}{P[T]}$

$= P[W'|T \cap G] \times P[G|T] = 0.35 \times P[G|T]$. It follows that $P[G|T] = \frac{0.2}{0.35} = \frac{4}{7}$,

and then $q = P[G'|T] = 1 - \frac{4}{7} = \frac{3}{7}$. Answer: B

7. The geometric distribution $X = 0, 1, 2, \ldots$ has probability function $P[X = k] = (1-p)^k p$ and has mean $\frac{1-p}{p}$. For the Senators, we have $\frac{1-p_{OTT}}{p_{OTT}} = 3.5$, so that $p_{OTT} = \frac{1}{4.5}$.

For the Sabres, we have $\frac{1-p_{BUF}}{p_{BUF}} = 3\ 0$, so that $p_{BUF} = \frac{1}{4}$.

$P[X_{OTT} = k] = (1 - \frac{1}{4.5})^k \times \frac{1}{4.5} = (\frac{7}{9})^k \times \frac{2}{9}$ and $P[X_{BUF} = k] = (1 - \frac{1}{4})^k \times \frac{1}{4} = (\frac{3}{4})^k \times \frac{1}{4}$.

$P[X_{BUF} \geq n] = \sum_{k=n}^{\infty} (\frac{3}{4})^k \times \frac{1}{4} = (\frac{3}{4})^n$.

$P[X_{BUF} \geq X_{OTT} + 2]$

$= P[X_{BUF} \geq 2 | X_{OTT} = 0] \times P[X_{OTT} = 0] + P[X_{BUF} \geq 3 | X_{OTT} = 1] \times P[X_{OTT} = 1] + \cdots$

$= \sum_{n=0}^{\infty} P[X_{BUF} \geq n+2 | X_{OTT} = n] \times P[X_{OTT} = n]$

$= \sum_{n=0}^{\infty} P[X_{BUF} \geq n+2] \times P[X_{OTT} = n]$ (because of independence of X_{BUF} and X_{OTT})

$= \sum_{n=0}^{\infty} (\frac{3}{4})^{n+2} (\frac{7}{9})^n \times \frac{2}{9} = (\frac{3}{4})^2 \times \frac{2}{9} \times \sum_{n=0}^{\infty} (\frac{3}{4} \times \frac{7}{9})^n = \frac{1}{8} \times \frac{1}{1 - \frac{7}{12}} = 0.30.$ Answer: C

8. The probability of an even outcome when tossing a single $(n = 1)$ die is $\frac{1}{2}$.
The probabilities for the sum when tossing two dice are

Sum	2	3	4	5	6	7	8	9	10	11	12
Prob	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

The probability that the sum is even is $\frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{5}{36} + \frac{3}{36} + \frac{1}{36} = \frac{18}{36} = \frac{1}{2}$.

To see that the probability is always $\frac{1}{2}$, suppose that E_{n-1} is the event that sum of the first $n-1$ tosses is even. Then in order for the sum of the n dice to be even, we must have either E_{n-1} occurring and the n-th toss is even, or E'_{n-1} occurring (complement) and the n-th toss is odd.
Because of independence of the tosses, we get
$P(\text{sum of } n \text{ tosses is even}) = P(E_{n-1}) \times \frac{1}{2} + P(E'_{n-1}) \times \frac{1}{2} = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$.
Since $P(E_1) = \frac{1}{2}$, it follows that $P(E_k) = \frac{1}{2}$ for any k. Answer: C