9. X has a binomial distribution with 100 trials and probability $\frac{1}{2}$ of success. The expected number of heads is $100(\frac{1}{2}) = 50$ and the variance of the number of heads is $100(\frac{1}{2})(\frac{1}{2}) = 25$.
Using the normal approximation with integer correction, we want to satisfy the relationship
$P(50 - k - 0.5 \le X \le 50 + k + 0.5) \ge 0.95$.

Applying the normal approximation, we have
$P(50 - k - .5 \le X \le 50 + k + .5) = P\left(\frac{-k-0.5}{\sqrt{25}} \le \frac{X-50}{\sqrt{25}} \le \frac{k+0.5}{\sqrt{25}}\right) = P(-c \le Z \le c)$,
where Z is standard normal. In order for this probability to be at least .95, it must be true that $\Phi(c) \ge 0.975$. This is true because we want to eliminate less than .025 probability from the left and right side of Z.

From the standard normal table, $\Phi(1.96) = 0.975$, and therefore, we must have $c \ge 1.96$.
Then, $\frac{k+0.5}{5} \ge 1.96 \rightarrow k \ge 9.3$. The smallest integer is $k = 10$.
Using the normal approximation, $P(40 \le X \le 60) \ge 0.95$ but $P(41 \le X \le 59) < 0.95$.
Answer: E

10. $$\begin{aligned} E[Y] &= \int_0^\theta \frac{x}{2} \times \frac{1}{\theta} e^{-x/\theta}\,dx + \int_\theta^\infty x \times \frac{1}{\theta} e^{-x/\theta}\,dx \\ &= \frac{1}{2}\left(-xe^{-x/\theta} - \theta e^{-x/\theta}\right)\Big|_{x=0}^{x=\theta} + \left(-xe^{-x/\theta} - \theta e^{-x/\theta}\right)\Big|_{x=\theta}^{x=\infty} \\ &= \frac{\theta}{2}(1 - 2e^{-1}) + 2\theta e^{-1} = \frac{\theta}{2}(1 + 2e^{-1}) \end{aligned}$$ Answer: B

11. The pdf of X is $f_X(x) = 1$ for $0 \le x \le 1$, and the conditional pdf of Y given $X = x$ is $f_{Y|X}(y|x) = \frac{1}{2-x}$ for $x \le Y \le 2$.

The joint density of X and Y is $f_{X,Y}(x,y) = f_{Y|X}(y|x) \times f_X(x) = \frac{1}{2-x}$ defined on the region $0 \le x \le 1$ and $x \le y \le 2$.
$E[Y] = \int_0^1 \int_x^2 y \times \frac{1}{2-x}\,dy\,dx = \int_0^1 \frac{4-x^2}{2(2-x)}\,dx = \int_0^1 \frac{2+x}{2}\,dx = \frac{5}{4}$.

An alternative, solution makes use of the rule $E[Y] = E[\,E[Y|X]\,]$.
Since the conditional distribution of $Y|X = x$ is uniform on the interval from x to 2, it follows that $E[Y|X] = \frac{X+2}{2}$. Then,
$E[Y] = E[\,E[Y|X]\,] = E\left[\frac{X+2}{2}\right] = \frac{1}{2}E[X] + 1$.

Since X is uniform on the interval from 0 to 1, $E[X] = \frac{1}{2}$.
Then, $E[Y] = \frac{1}{2} \times \frac{1}{2} + 1 = \frac{5}{4}$. Answer: C

12. $Y = \begin{cases} X & X \le 40 \\ 40 + \frac{1}{2}(x-40) & 40 < X \le 80 \\ 60 & X > 80 \end{cases}$. $Var[Y] = E[Y^2] - (E[Y])^2$.

$E[Y] = \int_0^{40} x \times 0.01\,dx + \int_{40}^{80}(20+.5x) \times 0.01\,dx + \int_{80}^{100} 60 \times 0.01\,dx = 8 + 20 + 12 = 40.$

$$E[Y^2] = \int_0^{40} x^2 \times 0.01\,dx + \int_{40}^{80}(20+.5x)^2 \times 0.01\,dx + \int_{80}^{100} 60^2 \times 0.01\,dx$$
$$= \frac{640}{3} + \frac{3040}{3} + 720 = \frac{5840}{3}.$$

$Var[Y] = \frac{5840}{3} - 40^2 = \frac{1040}{3}$. Answer: B

13. $A =$ have driver's license , $B =$ own a bike

$P(A \cup B) = .8 = P(A) + P(B) - P(A \cap B)$

$P(B|A) = \frac{1}{3} = \frac{P(A \cap B)}{P(A)}$, $P(A|B) = \frac{1}{2} = \frac{P(A \cap B)}{P(B)}$.

$\frac{P(A \cup B)}{P(A \cap B)} = \frac{0.8}{P(A \cap B)} = \frac{P(A)+P(B)-P(A \cap B)}{P(A \cap B)} = \frac{1}{1/3} + \frac{1}{1/2} - 1 = 4,$

and it follows that $P(A \cap B) = 0.2$.

Then $P(A) = 3 \times P(A \cap B) = 0.6$ and $P(B) = 2 \times P(A \cap B) = 0.4$.

We wish to find $P(A'|B') = \frac{P(A' \cap B')}{P(B')} = \frac{P[(A \cup B)']}{P(B')} = \frac{1-P(A \cup B)}{1-P(B)} = \frac{1-0.8}{1-0.4} = \frac{1}{3}$. Answer: A

14. A student can drop the course after the first test but before the second test. The fraction of the original group of students that drop the course after the first test but before the second test is

P[drop after 1st test but before 2nd test]

$= P$[drop after 1st test but before 2nd test $\cap$ pass 1st test]

$+ P$[drop after 1st test but before 2nd test $\cap$ fail 1st test]

$= P$[drop after 1st test but before 2nd test | pass 1st test] $\cdot P$[pass 1st test]

$+ P$[drop after 1st test but before 2nd test | fail 1st test] $\cdot P$[fail 1st test]

$= 0.1 \times 0.8 + 0.3 \times 0.2 = 0.14$

A student can drop the course after the second test but before the final exam.

P[drop after 2nd test but before final exam]

$= P$[drop after 2nd test but before final exam $\cap$ pass 1st test $\cap$ take 2nd test $\cap$ pass 2nd test]

$+ P$[drop after 2nd test but before final exam $\cap$ pass 1st test $\cap$ take 2nd test $\cap$ fail 2nd test]

P[drop after 2nd test but before final exam $\cap$ fail 1st test $\cap$ take 2nd test $\cap$ pass 2nd test]

$+ P$[drop after 2nd test but before final exam $\cap$ fail 1st test $\cap$ take 2nd test $\cap$ fail 2nd test]

We find these probabilities in the following way:

P[drop after 2nd test but before final exam $\cap$ fail 1st test $\cap$ take 2nd test $\cap$ fail 2nd test]

$= P$[drop after 2nd test but before final exam|fail 1st test $\cap$ take 2nd test $\cap$ fail 2nd test]

$\times P$[fail 2nd test|take 2nd test $\cap$ fail 1st test] $\times P$[take 2nd test|fail 1st test] $\times P$[fail 1st test]

$= 0.5 \times 0.2 \times 0.7 \times 0.2 = 0.014.$

Similarly,

P[drop after 2nd test but before final exam $\cap$ pass 1st test $\cap$ take 2nd test $\cap$ fail 2nd test]

$= .5 \times 0.1 \times 0.9 \times 0.8 = 0.036$.

P[drop after 2nd test but before final exam $\cap$ fail 1st test $\cap$ take 2nd test $\cap$ pass 2nd test] and

P[drop after 2nd test but before final exam $\cap$ pass 1st test $\cap$ take 2nd test $\cap$ pass 2nd test]

are both 0, since anyone who passes the 2nd test does not drop the course.

The probability of dropping the course is $0.14 + 0.014 + 0.036 = 0.19$. Answer: D

15. In order for there to be one blue ball in the urn after the n application, it must be true that a blue ball was chosen exactly once in the n applications of the procedure. The blue ball could have been chosen on the 1st, or 2nd, . . . , or n-th application.

P(blue ball chosen on 1st application and no blue ball chosen in next $n-1$ applications)

$= \frac{1}{3} \times (\frac{5}{6})^{n-1}$.

P(1st blue ball chosen on 2nd application and no blue ball chosen in next $n-2$ applications)

$= \frac{2}{3} \times \frac{1}{3} \cdot (\frac{5}{6})^{n-2}$.

$\vdots$

P(1st blue ball chosen on k-th application and no blue ball chosen in next $n-k$ applications)

$= (\frac{2}{3})^{k-1} \times \frac{1}{3} \times (\frac{5}{6})^{n-k}$.

$\vdots$

P(1st blue ball chosen on n-th application) $= (\frac{2}{3})^{n-1} \times \frac{1}{3}$.

The probability in question is the sum of these:

$$\sum_{k=1}^{n} (\tfrac{2}{3})^{k-1} \times \tfrac{1}{3} \times (\tfrac{5}{6})^{n-k} = \tfrac{1}{3} \times (\tfrac{2}{3})^{-1} \times (\tfrac{5}{6})^{n} \times \sum_{k=1}^{n} (\tfrac{2}{3})^{k} (\tfrac{5}{6})^{-k}$$

$$= \tfrac{1}{2} \times (\tfrac{5}{6})^{n} \times \sum_{k=1}^{n} (\tfrac{2/3}{5/6})^{k} = \tfrac{1}{2} \times (\tfrac{5}{6})^{n} \times \sum_{k=1}^{n} (0.8)^{k} = \tfrac{1}{2} \times (\tfrac{5}{6})^{n} \times 0.8 \times \sum_{k=1}^{n} (0.8)^{k-1}$$

$$= 0.4 \times (\tfrac{5}{6})^{n} \cdot \tfrac{1-(0.8)^{n}}{1-0.8} = 2 \times [(\tfrac{5}{6})^{n} - (\tfrac{2}{3})^{n}].$$

Answer: D

16. The region $(c < X < g) \cap (X < d)$ is $c < X < d$, so the probability is

$$P[b < X < e \mid c < X < d] = \frac{P[(b<X<e) \cap (c<X<d)]}{P[c<X<d]}.$$

The region $(b < X < e) \cap (c < X < d)$ is $c < X < d$, so

$$P[b < X < e \mid c < X < d] = \frac{P[c<X<d]}{P[c<X<d]} = 1.$$

Answer: E

17. Since $\sum_{x=0}^{\infty} P(X=x)=1$, it follows that $\sum_{x=1}^{\infty} P(X=x)=1-P(X=0)=1-e^{-1}$.

Then, $\sum_{x=1}^{\infty} P(Y=x)=c\times\sum_{x=1}^{\infty} P(X=x)=c(1-e^{-1})$.

But it is also true that $\sum_{x=1}^{\infty} P(Y=x)=1-P(Y=0)=1-\alpha$.

Therefore, $c(1-e^{-1})=1-\alpha$, so that $c=\frac{1-\alpha}{1-e^{-1}}$.

The mean of Y is

$$E[Y]=\sum_{x=0}^{\infty} x\times P(Y=x)=\sum_{x=1}^{\infty} x\times P(Y=x)=\sum_{x=1}^{\infty} x\times c\times P(X=x)$$
$$=c\times\sum_{x=o}^{\infty} x\times P(X=x)=c\times E[X]=c=\frac{1-\alpha}{1-e^{-1}}.$$

Answer: C

18. We define the following events:

C - shipment is from Crosscheck Lumber
S - shipment is from Sticks R Us
$2D$ - 2 sticks are defective

We wish to find $P(C|2D)$. This is $\frac{P(C\cap 2D)}{P(2D)}$.
The numerator can be formulated as $P(2D|C)\cdot P(C)$.

We are given that $P(C)=.5$. For a shipment from Crosscheck Lumber, the number of sticks that are defective in a batch of 10 sticks has a binomial distribution with $n=10$ and $p=.1$ (prob. of a particular stick being defective).

Therefore, $P(2D|C)=\binom{10}{2}(0.1)^2(0.9)^8=0.193710$.

The numerator is $P(C\cap 2D)=0.193710\times 0.5=0.096855$.

The denominator can be formulated as $P(2D)=P(C\cap 2D)+P(S\cap 2D)$ since the shipment must be either C or S. We find $P(S\cap 2D)$ in the same way as $P(C\cap 2D)$. $P(S\cap 2D)=P(2D|S)\cdot P(S)=\binom{10}{2}(0.2)^2(0.8)^8\times 0.5=0.150995$.

Then, $P(C|2D)=\frac{P(C\cap 2D)}{P(2D)}=\frac{P(C\cap 2D)}{P(C\cap 2D)+P(S\cap 2D)}=\frac{0.096855}{0.096855+0.150995}=0.39$. Answer: D

19. The donation is $\begin{cases} 0 & \text{Prob. } e^{-3} \\ K & \text{Prob. } 3e^{-3} \\ 2K & \text{Prob. } \frac{9e^{-3}}{2} \\ 3K & \text{Prob. } 1-(e^{-3}+3e^{-3}+\frac{9e^{-3}}{2}) \end{cases}$

The expected donation is $K \times 3e^{-3} + 2K \times \frac{9e^{-3}}{2} + 3K \times [\,1-(e^{-3}+3e^{-3}+\frac{9e^{-3}}{2})] = 2.328K$.

Setting this equal to 5000 results in $K = 2148$. Answer: C

20. Since X has a symmetric distribution about the point $X = 1$, it follows that $E[X] = 1$.

The second moment of X is

$$E[X^2] = \int_0^1 x^2 \times \frac{1-p}{2}\,dx + 1^2 \times p + \int_1^2 x^2 \times \frac{1-p}{2}\,dx$$
$$= \frac{1}{3} \times \frac{1-p}{2} + p + \frac{7}{3} \times \frac{1-p}{2} = \frac{4}{3} - \frac{p}{3}.$$

The variance of X is $Var[X] = E[X^2] - (E[X])^2 = \frac{4}{3} - \frac{p}{3} - 1 = \frac{1-p}{3}$. Answer: A

21. The cdf of X is $F_X(t) = \int_0^t f(x)\,dx = \begin{cases} \frac{t^2}{2} - \frac{t^3}{6} & \text{if } 0 < t \le 1 \\ \frac{1}{3} + \frac{t^3-1}{6} - \frac{t^2-1}{2} + t - 1 & \text{if } 1 < x < 2 \end{cases}$

The cdf of Y is $F_Y(y) = P(Y \le y) = P(X^2 \le y) = P(X \le \sqrt{y}) = F_X(\sqrt{2})$.

Since $1 < \sqrt{2} < 2$, we get

$F_Y(2) = F_X(\sqrt{2}) = \frac{1}{3} + \frac{(\sqrt{2})^3-1}{6} - \frac{(\sqrt{2})^2-1}{2} + \sqrt{2} - 1 = 0.55.$ Answer: C

22. X_1 is binomial with $np = 2$ and $np(1-p) = 1$.

It follows that $1 - p = \frac{1}{2}$, and $p = \frac{1}{2}$, and $n = 4$.

The probability function of X_1 is $P(X_1 = k) = \binom{4}{k}(\frac{1}{2})^k(\frac{1}{2})^{4-k} = \binom{4}{k}(\frac{1}{2})^4$.

The probability function of X_2 is $P(X_2 = j) = \frac{2^j e^{-2}}{j!}$.

$P(Y < 3) = P(Y = 0) + P(Y = 1) + P(Y = 2)$.

$P(Y = 0) = P(X_1 = 0 \cap X_2 = 0) = P(X_1 = 0) \times P(X_2 = 0) = \binom{4}{0}(\frac{1}{2})^4 \times \frac{2^0 e^{-2}}{0!} = \frac{1}{16}e^{-2}$.

$P(Y = 1) = P(X_1 = 0 \cap X_2 = 1) + P(X_1 = 1 \cap X_2 = 0)$

$= P(X_1 = 0) \times P(X_2 = 1) + P(X_1 = 1) \times P(X_2 = 0)$

$= \binom{4}{0}(\frac{1}{2})^4 \times \frac{2^1 e^{-2}}{1!} + \binom{4}{1}(\frac{1}{2})^4 \times \frac{2^0 e^{-2}}{0!} = \frac{2}{16}e^{-2} + \frac{4}{16}e^{-2} = \frac{6}{16}e^{-2}$.

$P(Y = 2) = P(X_1 = 0 \cap X_2 = 2) + P(X_1 = 1 \cap X_2 = 1) + P(X_1 = 2 \cap X_2 = 0)$

$= P(X_1 = 0) \times P(X_2 = 2) + P(X_1 = 1) \times P(X_2 = 1) + P(X_1 = 2) \times P(X_2 = 0)$

$= \binom{4}{0}(\frac{1}{2})^4 \times \frac{2^2 e^{-2}}{2!} + \binom{4}{1}(\frac{1}{2})^4 \times \frac{2^1 e^{-2}}{1!} + \binom{4}{2}(\frac{1}{2})^4 \times \frac{2^0 e^{-2}}{0!} = \frac{2}{16}e^{-2} + \frac{8}{16}e^{-2} + \frac{6}{16}e^{-2} = e^{-2}$

Then, $P(Y < 3) = \frac{1}{16}e^{-2} + \frac{6}{16}e^{-2} + e^{-2} = \frac{23}{16}e^{-2}$. Answer: D

23. In order to be a properly defined random variable, we must have
$P(X=0)+P(0<X<1)+P(X=1)=1$, so that
$a+\int_0^1 x\,dx+b=a+\frac{1}{2}+b=1$. Therefore, $a+b=\frac{1}{2}$.
$Var(X)=E(X^2)-[E(X)]^2$.
$E(X)=0\times a\,+\,\int_0^1 x\times x\,dx\,+\,1\times b=\frac{1}{3}+b$, and
$E(X^2)=0\times a^2\,+\,\int_0^1 x^2\times x\,dx\,+\,1^2\times b=\frac{1}{4}+b$.

Then, $Var(X)=\frac{1}{4}+b-(\frac{1}{3}+b)^2=\frac{5}{36}+\frac{b}{3}-b^2$.
$Var(X)$ will be maximized if $\frac{d}{db}\left[\frac{5}{36}+\frac{b}{3}-b^2\right]=\frac{1}{3}-2b=0$.
This occurs at $b=\frac{1}{6}$. Then $a=\frac{1}{2}-b=\frac{1}{3}$. Answer: D

24. $P(A|B)=\frac{P(A\cap B)}{P(A)}$ and $P(B|A)=\frac{P(A\cap B)}{P(B)}$.
Since $P(A\cap B)>0$ it follows that $P(A)=P(B)$, so II is true.

If $A=\{1,2,3\}$ and $B=\{3,4,5\}$ when tossing a fair die then the conditions are satisfied, but I is false since $P(A\cap B)=\frac{1}{6}\neq P(A)\times P(B)$, and III is false. Answer: B

25. The amount paid by the insurance is Y, where $Y=\begin{cases}0 & \text{if } X\leq 500\\ X-500 & \text{if } 500<X\leq 1000\\ \frac{X}{2} & \text{if } 1000<X<2000\end{cases}$.

$Var(Y)=E(Y^2)-[E(Y)]^2$.
$E(Y)=\int_{500}^{1000}(x-500)\times\frac{1}{2000}\,dx+\int_{1000}^{2000}\frac{x}{2}\times\frac{1}{2000}\,dx=\frac{125}{2}+375=\frac{875}{2}$.
$E(Y^2)=\int_{500}^{1000}(x-500)^2\times\frac{1}{2000}\,dx+\int_{1000}^{2000}(\frac{x}{2})^2\times\frac{1}{2000}\,dx=\frac{62{,}500}{3}+\frac{875{,}000}{3}=\frac{937{,}500}{3}$.
$Var(Y)=\frac{937{,}500}{3}-(\frac{875}{2})^2=121{,}093.75$.
Standard deviation of Y is $\sqrt{Var(Y)}=\sqrt{121{,}093.75}=348$. Answer: C

26. $P(X+Y\geq 2|X\leq 1)=\frac{P(X+Y\geq 2\cap X\leq 1)}{P(X\leq 1)}$.
$P(X\leq 1)=\int_0^1\int_0^2\frac{2x+y}{12}\,dy\,dx=\frac{1}{3}$.
$P(X+Y\geq 2\cap X\leq 1)=\int_0^1\int_{2-x}^2\frac{2x+y}{12}\,dy\,dx=\int_0^1\frac{3x^2+4x}{24}\,dx=\frac{1}{8}$.
$P(X+Y\geq 2|X\leq 1)=\frac{1/8}{1/3}=\frac{3}{8}$. Answer: C

27. Since $f(x)$ is a pdf, we know that $\int_0^2 f(x)\,dx = 2a + 2b = 1$.
$F(x) = \int_0^t f(t)\,dt = \frac{at^2}{2} + bt$, so $F(\frac{5}{4}) = \frac{25a}{32} + \frac{5b}{4} = \frac{1}{2}$.
Solving these two equations results in $a = \frac{4}{15}$, $b = \frac{7}{30}$.
The mean of X is $E(X) = \int_0^2 x(\frac{4x}{15} + \frac{7}{30})\,dx = \frac{53}{45}$
and the second moment of X is $E(X^2) = \int_0^2 x^2(\frac{4x}{15} + \frac{7}{30})\,dx = \frac{76}{45}$.
The variance of X is $E(X^2) - [E(X)]^2 = \frac{76}{45} - \left(\frac{53}{45}\right)^2 = \frac{611}{45^2} = .302$. Answer: D

28. Let Y denote the net profit of Gambler 1 for one spin.
Then Y is either -1 with probability $\frac{37}{38}$ or Y is 36 with probability $\frac{1}{38}$.
Then $E(Y) = -1 \times \frac{37}{38} + 36 \times \frac{1}{38} = -\frac{1}{38}$.
The net profit after n spins for Gambler 1 is $X_1 = Y_1 + Y_2 + \cdots + Y_n$,
and the expected profit is
$E(X_1) = E(Y_1) + E(Y_2) + \cdots + E(Y_n) = -\frac{1}{38} \times n = -\frac{n}{38}$.
Let Z denote the net profit of Gambler 2 for one spin.
Then Z is either -1 with probability $\frac{20}{38}$ or Z is 1 with probability $\frac{18}{38}$.
Then $E(Z) = -1 \times \frac{20}{38} + 1 \times \frac{18}{38} = -\frac{2}{38}$.
The net profit after n spins for Gambler 2 is $X_2 = Z_1 + Z_2 + \cdots + Z_n$,
and the expected profit is
$E(X_2) = E(Z_1) + E(Z_2) + \cdots + E(Z_n) = -\frac{2}{38} \times n = -\frac{2n}{38}$.

Then $E(X_2 - X_1) = E(X_2) - E(X_1) = -\frac{2n}{38} - (-\frac{n}{38}) = -\frac{n}{38}$. Answer: B

29. The expected insurance payment is $P(X \geq 1) = 0.8892$.
Therefore $P(X = 0) = 0.1108 = e^{-\lambda}$, and it follows that $\lambda = -ln(0.1108) = 2.200$.
The expected insurance payment on a policy with a policy limit of 2 is

$$\begin{aligned} &1 \times P(X = 1) + 2 \times P(X \geq 2) \\ &\quad = P(X = 1) + 2 \times [1 - P(X = 0 \text{ or } 1)] \\ &\quad = \frac{\lambda e^{-\lambda}}{1!} + 2 \times [1 - e^{-\lambda} - \frac{\lambda e^{-\lambda}}{1!}] \\ &\quad = 0.1108 \times 2.2 + 2 \times [1 - 0.1108 - 0.1108 \times 2.2] = 1.53. \end{aligned}$$

Answer: E

30. We use the following notation:

A - customer has an auto policy
F - customer has a fire insurance policy
L - customer has flood insurance coverage

We are given: $P(A) = 0.8$, $P(F) = 0.4$, $P(F|A) = 0.25$, $P(L|F) = 0.5$, $P(A|L) = 0.5$

We also know that $L \cap F = L$, so from $.5 = P(L|F) = \frac{P(L \cap F)}{P(F)} = \frac{P(L)}{P(F)} = \frac{P(L)}{0.4}$
we get $P(L) = 0.2$.

We wish to find $P(A' \cap L'|F) = \frac{P(A' \cap L' \cap F)}{P(F)}$.

$$\begin{aligned} P(A' \cap L' \cap F) &= P(F) - P[(A \cup L) \cap F] = P(F) - P[(A \cap F) \cup (L \cap F)] \\ &= P(F) - P[(A \cap F) \cup L] = P(F) - [P(A \cap F) + P(L) - P(A \cap F \cap L)] \\ &= P(F) - [P(A \cap F) + P(L) - P(A \cap L)] \end{aligned}$$

From the given information we have $0.25 = P(F|A) = \frac{P(F \cap A)}{P(A)} = \frac{P(F \cap A)}{.8}$,

so that $P(F \cap A) = 0.2$, and $0.5 = P(A|L) = \frac{P(A \cap L)}{P(L)} = \frac{P(A \cap L)}{0.2}$ so that $P(A \cap L) = 0.1$.

Then, $P(A' \cap L'|F) = \frac{P(A' \cap L' \cap F)}{P(F)} = \frac{P(F) - [P(A \cap F) + P(L) - P(A \cap L)]}{P(F)} = \frac{0.4 - [0.2 + 0.2 - .1]}{0.4} = 0.25.$

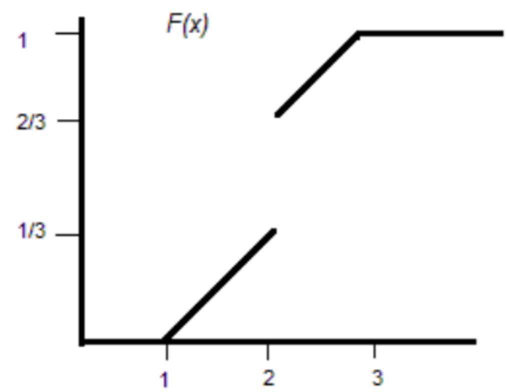
Answer: E

PRACTICE EXAM 9

1. The number of days that elapse between the beginning of a calendar year and the moment T that a high-risk driver has an accident is considered to be a continuous random variable with pdf ce^{-ct}. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year. What portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year?

   A) 0.15 B) 0.34 C) 0.43 D) 0.57 E) 0.66

2. You are given the following cumulative distribution function for the random variable X. What is $Var[X]$?



   A) $\frac{1}{9}$ B) $\frac{2}{9}$ C) $\frac{1}{3}$ D) $\frac{4}{9}$ E) $\frac{5}{9}$

The following is information for Questions 3 and 4.
Two similar bowls each contain 10 similarly shaped numbered balls. Bowl 1 contains 5 balls with the number 1 and 5 balls with the number 2 (equally likely to be chosen). Bowl 2 contains 3 balls with the number 1 and 7 balls with the number 2 (equally likely to be chosen). A bowl is chosen at random, and a ball is chosen from that bowl.

3. Find the probability that the ball chosen has number 1.

   A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$ E) 1

4. Find the probability that bowl 1 was chosen given that the ball chosen had number 1.

   A) $\frac{1}{8}$ B) $\frac{2}{8}$ C) $\frac{3}{8}$ D) $\frac{1}{2}$ E) $\frac{5}{8}$

5. A new factory requires two generators operating in order for the factory to keep working. The factory just purchased two new generators which will be operating independently of one another.

The manufacturer of the generators informs the factory owner that based on the manufacturer's experience, from the time two new generators begin operating, the average time until the first failure of the two generators is 11.5 months. The factory owner assumes the following model for the cumulative distribution function for the time until failure for an individual generator:

$F(t) = t^n$ for $0 < t < 1$, where t is measured in years.

Based on this information calculate n.

A) 35 B) 37 C) 39 D) 41 E) 43

6. A veterinarian does a 3-year mortality study on cats and diabetes. In the study, 60% of the cats are healthy, 30% have pre-diabetes and 10% have diabetes. In the three year period it is found that the pre-diabetic cats are twice as likely to die than the healthy cats and one-half as likely to die than the diabetic cats. A randomly chosen cat from the study group was found to have died during the three year period. What is the probability that the randomly chosen cat was diabetic?

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

7. When a loss occurs for a particular risk, the loss amount X is a random variable with the following pdf: $f(x) = \frac{10-x}{50}$, $0 < x < 10$.

An insurance policy for a single occurrence of a loss on the risk has a deductible of 2 and a maximum payment of 6. Find the expected insurance payment if a loss occurs on the risk.

A) Less than 1 B) At least 1 but less than 1.25 C) At least 1.25 but less than 1.50

D) At least 1.50 but less than 1.75 E) At least 1.75

8. A fair coin has the number one on one side and the number two on the other side. A fair die has the usual markings of one to six on the six sides. The coin and the die are tossed independently of one another. You are given the following three events:

A: The number on the coin and the number on the die are both even

B: The number on the coin and the number on the die are both odd

C: The product of the numbers on the coin and die is odd

How many of the following collections of events are independent?

I. A and B II. A and C III. B and C IV. A, B and C

A) None B) One C) Two D) Three E) Four

9. Discrete random variables X and Y have a joint distribution describe by the following joint probability table:

X / Y	0	1	2
0	0	0.1	0.2
1	0.1	0.2	0
2	0.2	0.2	0

Calculate the coefficient of correlation between X and Y.

A) At least -1 but less than -0.6 B) At least -0.6 but less than -0.2

C) At least -0.2 but less than 0.2 D) At least 0.2 but less than 0.6 E) At least 0.6

10. Smith has either chicken or steak for dinner every night. The choice of dinner meal is random every night with 75% of Smith's dinners being chicken. If Smith has steak for dinner, Smith will always have a beer with dinner, but when Smith has chicken for dinner, there is a 50% chance that Smith will have a beer with dinner . On a randomly chosen day, what is the probability that Smith has a beer with dinner?

A) $\frac{3}{8}$ B) $\frac{1}{2}$ C) $\frac{5}{8}$ D) $\frac{3}{4}$ E) $\frac{7}{8}$

11. Smith believes that there is parity in the 30-team NHL (National Hockey League), so he believes that his team has a $\frac{1}{30}$ chance in winning the league championship Stanley Cup in any given year. Smith also likes to play the Powerball lottery every Wednesday and Sunday each week. The Powerball lottery is played as follows: five integers are chosen from the integers from 1 to 69 (call them "blue integers") and one integer is chose from the integers from 1 to 26 (call that the "red integer"). A Powerball ticket consists of a choice of five blue integers and one red integer. If the ticket matches the five blue integers and one red integer chosen when the draw is made, then the ticket holder wins the grand prize. Suppose that Smith purchases one Powerball ticket every Wednesday and every Sunday. There is one Stanley Cup championship per year. Assume that there are exactly 52 weeks each year. On average, how many times would Smith's NHL team win the Stanley Cup before Smith win's the grand prize in the Powerball lottery?

A) Less than 50,000 times B) At least 50,000 times but less than 100,000 times

C) At least 100,000 times but less than 150,000 times

D) At least 150,000 but less than 200,000 times E) At least 200,000 times

12. In the case of an accident, an auto insurance policy pays X for damage to the automobile and Y for a liability claim. The model for the joint distribution of X and Y satisfies the following relationships:

(i) conditional distribution of Y given X: $f_{Y|X}(y|x) = \frac{1}{x}$ for $0 < y < x$

(ii) unconditional distribution of X: $f_X(x) = 3x^2$ for $0 < x < 1$

Suppose that the liability claim for a particular accident is $y = 0.40$. Determine the expected damage claim.

A) Less than 0.4 B) At least 0.4 but less than 0.55 C) At least 0.55 but less than 0.70

D) At least 0.70 but less than 0.85 E) At least 0.85

13. Bob repeatedly tosses a fair coin. Each time Bob tosses the coin ,Doug, independent from Bob, also tosses a fair coin. Each pair of tosses is referred to as a "trial". We define the following random variable:

X= the trial number in which the first "head" appears on either or both coins. Calculate $E[X]$.

A) Less than 0.5 B) At least 0.5 but less than 1.0 C) At least 1.0 but less than 1.5

D) At least 1.5 but less than 2.0 E) At least 2.0

14. A loss distribution is uniformly distributed on the interval from 0 to 100.

Two insurance policies are being considered to cover part of the loss.

Insurance policy 1 insures 80% of the loss.

Insurance policy 2 covers the loss up to a maximum insurance payment of $L < 100$.

Both policies have the same expected payment by the insurer.

Calculate L.

A) Less than 50 B) At least 50 but less than 60 C) At least 60 but less than 70

D) At least 70 but less than 80 E) At least 80

15. Let X and Y be independent continuous random variables with common density function

$$f(t) = \begin{cases} 1 \text{ for } 0<t<1 \\ 0, \text{otherwise} \end{cases}.$$

What is $P[X + Y^2 > 1]$?

A) 0 B) $\frac{1}{12}$ C) $\frac{1}{6}$ D) $\frac{1}{4}$ E) $\frac{1}{3}$

16. The lifetime of an individual electronic component has an exponential distribution with a mean lifetime of 100 hours from the time it begins operating. 1000 independent components begin operating at the same time. Use the normal approximation with integer correction to determine the probability that at least 100 components are still operating after 250 hours.

A) Less than 0.01 B) At least 0.01 but less than 0.02 C) At least 0.02 but less than 0.03

D) At least 0.03 but less than 0.04 E) At least 0.04

17. An insurer's fire insurance portfolio consists of three risk classifications: Low, Medium and High. 50% of the insurer's policy are Low risk, 35% are Medium risk and 15% are High risk.

The distributions of the number of annual claims from each risk type are as follows:

Low Risk: Binomial with $n = 2$, $p = 0.01$

Medium Risk: Binomial with $n = 3$, $p = 0.02$

High Risk: Binomial with $n = 4$, $p = 0.05$

Find the probability that a randomly chosen policy has no claims in a year.

A) Less than 0.95 B) At least 0.95 but less than 0.96 C) At least 0.96 but less than 0.97

D) At least 0.97 but less than 0.98 E) At least 0.98

18. An insurer offers two related insurance policies. Suppose that the loss random variable is X.
For policy A there is an ordinary deductible of amount d and the insurer will pay $X-d$ if the loss is in excess of d and the insurer pays nothing if the loss is below d. For policy B there is a franchise deductible, which means that if the loss is in excess of d, the insurer pays the full loss amount X but the insurer pays nothing if the loss is below d. Suppose the L_A and L_B denote the amounts paid by the insurer on policies A and B.
Which of the following is the correct expression for $E[L_B - L_A]$?
A) $d \times P[X \le d]$ B) $P[X \le d]$ C) $d \times P[X > d]$ D) $P[X > d]$ E) d

19. X and Y are independent exponential random variables, each with a mean of 1. Find the probability $P[Y > X+1]$.
A) e^{-1} B) $\frac{e^{-1}}{2}$ C) $\frac{e^{-1}}{3}$ D) $\frac{e^{-1}}{4}$ E) $\frac{e^{-1}}{5}$

20. An insurer is considering insuring a random loss X that is continuously uniformly distributed between 0 and 1. The insurer will apply a policy limit of amount L, which means that when a loss of amount X occurs the insurer pays $\begin{cases} X \text{ if } 0<X<L \\ L \text{ if } L \le X \le 1 \end{cases}$. The insurer wishes to choose the value of L so that the variance of the amount paid by the insurer is 0.0396. Find the value of L.
A) 0.3 B) 0.4 C) 0.5 D) 0.6 E) 0.7

21. Bob and Doug play a betting game. Bob tosses two fair dice and Doug independently tosses one fair die. If the total on the two dice that Bob tosses is greater than twice the number of the die that Doug tossed, then Bob wins. Calculate the probability that Bob wins.
A) $\frac{1}{8}$ B) $\frac{5}{24}$ C) $\frac{7}{24}$ D) $\frac{3}{8}$ E) $\frac{11}{24}$

22. A survey was done of 1000 sports fans in the Greater Toronto Area (GTA) who were fans of one or more of the following Toronto based teams:
Toronto Maple Leafs (ML), Toronto Blue Jays (BJ), Toronto Argonauts (TA)
Fans were asked to indicate which teams they were fans of and which they were not fans of.
You are given the following information on teams and combinations of teams preferred (those identified as being fans of a particular team can include being fans or not fans of other teams):
ML fans - 780 , BJ fans -750 , TA fans - 670 , ML $\cap$ BJ - 600 fans , ML $\cap$ TA - 500 fans ,
BJ $\cap$ TA - 500 fans , ML $\cap$ BJ $\cap$ TA - 400 fans.
How many of the 1000 fans are fans of exactly two teams?
A) 0 B) 100 C) 200 D) 300 E) 400

23. X is a continuous random variable with density function $f(x) = \frac{1}{2}e^{-|x|}$ for $-\infty < x < \infty$.

What is the correct expression for $P[X > a \,|\, |X| > a]$ for a real number a?

A) $\frac{1}{2}$ B) $\begin{cases} \frac{1}{2} \text{ for } a<0 \\ 1-\frac{1}{2}e^{-a} \text{ for } a\geq 0 \end{cases}$ C) $\begin{cases} 1-\frac{1}{2}e^{a} \text{ for } a<0 \\ \frac{1}{2}e^{-a} \text{ for } a\geq 0 \end{cases}$

D) $\begin{cases} 1-\frac{1}{2}e^{a} \text{ for } a<0 \\ \frac{1}{2} \text{ for } a\geq 0 \end{cases}$ E) E) $\begin{cases} \frac{1}{2}(1-e^{a}) \text{ for } a<0 \\ \frac{1}{2}e^{-a} \text{ for } a\geq 0 \end{cases}$

24. Smith, Jones and Green are marathon race runners, each of whom has a running time that is normally distributed, with the following distributions:

Smith: Average 2.4 hours, Variance: 0.09 hours^2

Jones: Average 2.5 hours, Variance: 0.04 hours^2

Green: Average 2.7 hours, Variance: 0.16 hours^2

Assuming their running times are independent, find the probability that in the next marathon race, Jones' running time is less than the average of Smith's and Green's running times.

A) Less than 0.5 B) At least 0.5 but less than 0.6 C) At least 0.6 but less than 0.7

D) At least 0.7 but less than 0.8 E) At least 0.8

25. X_1, X_2 and X_3 are independent random variables each of which has a continuous uniform distribution on the interval $[0, \theta]$, where $\theta > 0$. $Y = min\{X_1, X_2, X_3\}$. Calculate $E[Y]$.

A) $.2\theta$ B) $.25\theta$ C) $.3\theta$ D) $.33\theta$ E) $.4\theta$

26. X_1, X_2 and X_3 are the outcomes of three independent tosses of a fair six-sided die.

$Y = min\{X_1, X_2, X_3\}$. Calculate $E[Y]$.

A) Less than 1.0 B) At least 1.0 but less than 1.5 C) At least 1.5 but less than 2.0

D) At least 2.0 but less than 2.5 E) At least 2.5

27. X is the outcome of the toss of a fair die. The conditional distribution of $Y|X$ is Poisson with mean X.

Calculate the variance of Y.

A) Less than 2 B) At least 2 but less than 3 C) At least 3 but less than 4

D) At least 4 but less than 5 E) At least 5

28. X and Y are independent integer-valued random variable with the following probability generating functions: $P_X(t) = .5 + .2t + .3t^2$, $P_Y(t) = .4 + .3t + .2t^2 + .1t^3$

Calculate the probability $P[X + Y = 4]$.

A) 0.02 B) 0.04 C) 0.06 D) 0.08 E) .10

29. Smith and Jones study together for the upcoming actuarial exam. The exam has 30 questions and is a multiple choice exam with five possible answers for each question. Suppose that Smith and Jones both answered the same 25 questions correctly but answered the other 5 questions incorrectly. Assuming they answered randomly on the 5 questions for which they got incorrect answers, and assuming their performance on those 5 questions were independent, calculate the probability that they got the same answers for at least 3 of those 5 questions.

A) Less than 0.05 B) At least 0.05 but less than 0.10 C) At least 0.10 but less than 0.15

D) At least 0.15 but less than 0.20 E) At least 0.20

30. A loss random variable has a uniform distribution on the interval $[0, 1000]$.

An insurance policy has the following provisions:

(i) a deductible of 100,

(ii) if the size of the loss is between 100 and 500, the insurer pays the loss amount minus 100

(ii) if the size of the loss is greater than 500, the insurer pays 400 plus half the loss in excess of 500

Calculate the expected insurance payment when a loss occurs.

A) Less than 100 B) At least 100 but less than 200 C) At least 200 but less than 300

D) At least 300 but less than 400 E) At least 400

PRACTICE EXAM 9 - SOLUTIONS

1. We interpret proportion as probability. The statement "30% of high-risk drivers will be involved in an accident in the first 50 days of the year" is interpreted as
P[a high-risk driver is involved in an accident in the first 50 days of the year] $= 0.3$.
This can be written as $P[T \leq 50] = 0.3$, where T is the time, in days, until an accident occurs for a high-risk driver. Since the pdf of T is ce^{-ct}, this probability is $\int_0^{50} ce^{-ct}\,dt = 1 - e^{-50c} = 0.3$, and therefore, $e^{-50c} = 0.7$. The proportion of high-risk drivers that are expected to have an accident in the first 80 days of the year is also interpreted as a probability,
$P[T \leq 80] = \int_0^{80} ce^{-ct}\,dt = 1 - e^{-80c} = 1 - (e^{-50c})^{8/5} = 1 - (0.7)^{1.6} = 0.43$. Answer: C

2. From the graph of $F(x)$ we see
(i) X is continuous between $X = 1$ and $X = 2$, with constant pdf $f(x) = \frac{1}{3}$ for $1 < x \leq 2$,
(ii) X has a discrete point of probability at $X = 2$ with probability $P[X = 2] = \frac{1}{3}$, and
(iii) X is continuous between $X = 2$ and $X = 3$, with constant pdf $f(x) = \frac{1}{3}$ for $2 < x \leq 3$.
Note that a "jump" in the cdf corresponds to a discrete point of probability at the jump point and the vertical distance of the jump is the probability at that point.
$Var[X] = E[X^2] - (E[X])^2$.
$E[X] = \int x\, f(x)\, dx$ on the continuous region PLUS $\sum x\, P[X = x]$ at the discrete points of probability.

A similar formulation applies to $E[X^2]$.
$E[X] = \int_1^2 x \times \frac{1}{3}\, dx \,+\, 2 \times \frac{1}{3} \,+\, \int_2^3 x \times \frac{1}{3}\, dx = 2.$
$E[X^2] = \int_1^2 x^2 \times \frac{1}{3}\, dx \,+\, 2^2 \times \frac{1}{3} \,+\, \int_2^3 x^2 \times \frac{1}{3}\, dx = \frac{38}{9}.$
$Var[X] = \frac{38}{9} - 2^2 = \frac{2}{9}.$
Note that since the graph of the cdf is symmetric around $x = 2$, it follows that the mean of X must be 2. Answer: B

3. We identify the following events: $A =$ ball chosen has number 1, $A' =$ ball chosen has number 2, $C =$ Bowl 1 is chosen, and $C' =$ Bowl 2 was chosen. We are given that the probability of C is $P[C] = \frac{1}{2}$, since we are told that a bowl is chosen at random. In general, in this sort of situation, without any additional information about how the bowl is chosen, the phrase "a bowl is chosen at random" is interpreted to mean that each bowl has the same chance of being chosen. It then follows that $P[A|C] = \frac{5}{10}$ (given that bowl 1 was chosen, there is a $\frac{5}{10}$ probability that the ball chosen is a "1"), and $P[A|C'] = \frac{3}{10}$. Then the "overall" probability that the ball is a "1" is
$P[A] = P[A|C] \times P[C] + P[A|C'] \times P[C'] = \frac{5}{10} \times \frac{1}{2} + \frac{3}{10} \times \frac{1}{2} = \frac{2}{5}.$ Answer: B

4. We are asked to find the probability that bowl 1 was chosen given that the ball chosen was a "1".
This is The "reversed conditioning" probability $P[C|A]$. Using Bayes rule, we have

$$P[C|A] = \frac{P[C \cap A]}{P[A]} = \frac{P[A|C] \times P[C]}{P[A \cap C] + P[A \cap C']}$$

$$= \frac{P[A|C] \times P[C]}{P[A|C] \times P[C] + P[A|C'] \times P[C']} = \frac{\frac{5}{10} \times \frac{1}{2}}{\frac{5}{10} \times \frac{1}{2} + \frac{3}{10} \times \frac{1}{2}} = \frac{5}{8}.$$

Answer: E

5. The time until the first failure is $T = min\{T_1, T_2\}$ so
$P(T > t) = P(T_1 > t) \cdot P(T_2 > t) = (1 - t^n)^2 = 1 - 2t^n + t^{2n}$, $0 < t < 1$.
Then $E[T] = \int_0^1 [1 - F_T(t)]\, dt = \int_0^1 (1 - 2t^n + t^{2n})\, dt = 1 - \frac{2}{n+1} + \frac{1}{2n+1}$.
Since t is measure in years, we have $E[T] = 1 - \frac{2}{n+1} + \frac{1}{2n+1} = \frac{11.5}{12}$.
This becomes a quadratic equation in n (or use substitution of the various answers), whose solution is $n = 35$ (nearest integer).

Answer: A

6. We define the following events

H - healthy P - pre-diabetic

D - diabetic X - dies during the 3-year study

We are given $P[N] = 0.60$, $P[P] = 0.30$, $P[D] = 0.10$, and
$P[X|P] = \, = 2 \times P[X|H] = \frac{1}{2} \times P[X|D]$
Then $P[D|X] = \frac{P[D \cap X]}{P[X]} = \frac{P[X|D] \times P[D]}{P[X|H] \times P[H] + P[X|P] \times P[P] + P[X|D] \times P[D]}$

$$= \frac{P[X|D] \times .1}{\frac{1}{4} \times P[X|D] \times 0.6 + \frac{1}{2} \times P[X|D] \times 0.3 + P[X|D] \times 0.1} = \frac{0.1}{\frac{1}{4} \times 0.6 + \frac{1}{2} \times 0.3 + 0.1} = \frac{1}{4}.$$

Answer: C

7. A payment of $y = x - 2$ occurs if the loss amount $x > 2$, with the maximum payment of 6 occurring if the loss is $8 \leq x < 10$. The expected payment is
$\int_2^8 (x-2)(\frac{10-x}{50})\, dx \, + \, 6 \times P[X > 8] = 1.68$. Answer: D

8. $P(A) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, $P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$,
$P(C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (this occurs if the coin toss is a 1 and the die toss is 1,3 or 5)
$P(A \cap B) = 0 \rightarrow A, B$ are not independent
$P(A \cap C) = 0 \rightarrow A, C$ are not independent
$P(B \cap C) = \frac{1}{4} \rightarrow B, C$ are not independent
$P(A \cap B \cap C) = 0 \rightarrow A, B, C$ are not independent Answer: A

9. $\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y}$. $Cov(X,Y) = E[XY] - E[X] \times E[Y]$ and

$\sigma_X = \left(E(X - E[X])^2\right)^{1/2}$, $\sigma_Y = \left(E(Y - E[Y])^2\right)^{1/2}$

The marginal distributions of X and Y are

$P[X=0] = 0.3$, $P[X=1] = 0.5$, $P[X=2] = 0.2$

$P[Y=0] = 0.3$, $P[X=1] = 0.3$, $P[X=2] = 0.4$

Then, $E[X] = 1 \times 0.5 + 2 \times 0.2 = 0.9$, $E[Y] = 1 \times 0.3 + 2 \times 0.4 = 1.1$,

$Var[X] = E[(X - E[X])^2] = E[(X - 0.9)^2]$

$= (-0.9)^2 \times 0.3 + 0.1^2 \times 0.5 + 1.1^2 \times 0.2 = 0.49$, and $\sigma_X = 0.49^{1/2} = 0.70$.

Alternatively,

$Var[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2 = 1.3 - 0.81 = 0.49$.

In a similar way, we get $E[Y] = 1.1$ and $Var[Y] = .69$ and $\sigma_Y = .8307$.

$E[XY] = \sum_x \sum_y x \times y \times P(x,y) = 1 \times 0.2 + 2 \times 0.2 = 0.6$.

(the only X, Y pairs for which the probability is non-zero and neither of X or Y is 0 are the pairs $(X=1, Y=1)$ and $(X=1, Y=2)$).

Then, $\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y} = \frac{E[XY] - E[X] \cdot E[Y]}{\sigma_X \cdot \sigma_Y} = \frac{0.6 - 0.9 \times 1.1}{0.7 \times 0.8307} = -0.67$.

Answer: A

10. S - Smith has steak for dinner , C - Smith has chicken for dinner ,

B - Smith has a beer with dinner

$P(C) = 0.75$, $P(S) = 0.25$, $P(B|S) = 1$, $P(B|C) = 0.5$

$P(B) = P(B \cap C) + P(B \cap S) = P(B|C) \times P(C) + P(B|S) \times P(S)$

$= 0.5 \times 0.75 + 1 \times 0.25 = 0.625$. Answer: C

11. The number of possible tickets is $\binom{69}{5} \times 26 = 292,201,338$. On average, Smith will win the grand prize once in every 292,201,338 plays of the lottery. Smith plays the lottery 104 times per year, so it will take an average of $\frac{292,201,338}{104} = 2,809,628.25$ years until winning the lottery. Smith's team wins the Stanley Cup once every 30 years, on average, so Smith's team will have won the Stanley Cup $\frac{2,809,628.25}{30} = 93,654$ times in the time it takes Smith to win the grand prize in the Powerball lottery. Answer: B

12. The joint density of X and Y is $f_{X,Y}(x,y) = f_{Y|X}(y|x) \times f_X(x) = 3x$ for $0 < y < x < 1$.

The marginal density of Y is $f_Y(y) = \int_y^1 f(x,y)\,dx = \frac{3}{2}(1 - y^2)$ for $0 < y < 1$.

The conditional density of X given Y is $f(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{2x}{(1-y^2)}$ for $y < x < 1$.

The conditional density of X given $y = 0.4$ is $\frac{2x}{0.84}$ for $.4 < x < 1$.

Then $E[X|Y = .4] = \int_{0.4}^1 x \times \frac{2x}{0.84}\,dx = 0.743$. Answer: D

13. The probability of at least one head on a trial is .75, since the probability of no heads on a trial is the probability that both tosses are tails, which is $.5 \times .5 = .25$. X has the following distribution

X:	1	2	3	$\ldots$	n	$\ldots$
Prob.	0.75	0.25×0.75	$0.25^2 \times 0.75$	$\ldots$	$0.25^{n-1} \times 0.75$	$\ldots$

If we denote "tossing at least one head on a given trial" as success, then X has a geometric distribution which counts the trial number of the first success. In general, if the probability of success on a given trial is p, then $E[X] = \frac{1}{p}$, which in this case is $\frac{1}{0.75} = 1.33$. Answer: C

14. Expected payment under policy 2 is

$$\int_0^L y(.01)dy + L \times P[X > L] = 0.005L^2 + L \times \left(\frac{100-L}{100}\right) = L - 0.005L^2.$$

This is equal to the expected payment under policy 1, which is $0.8 \times E[X] = .8 \times 50 = 40$.
Solving $L - 0.005L^2 = 40$ results in $L = 55.28\,,\ 144.72$. Answer: B

15. $x + y^2 > 1$ on the two-dimensional region $0 < x < 1$ and $1 - x < y^2 < 1$,
or equivalently., $0 < x < 1$ and $(1-x)^{1/2} < y < 1$
The probability in question is
$\int_0^1 \int_{(1-x)^{1/2}}^1 f(x,y)\,dy\,dx = \int_0^1 \int_{(1-x)^{1/2}}^1 1\,dy\,dx = \int_0^1 [1 - (1-x)^{1/2}]\,dx$
$= 1 + \frac{2(1-x)^{3/2}}{3}\Big|_{x=0}^{x=1} = 1 - \frac{2}{3} = \frac{1}{3}$. Answer: E

16. $N =$ number of components operating after 250 hours. N has a binomial distribution with 1000 trials and probability $P(\text{operating after 250 hours}) = \int_{250}^{\infty} \frac{1}{100} e^{-x/100}\,dx = 0.0821$.
The mean of N is $E[N] = 1000 \times 0.0821 = 82.1$ and the variance is
$Var[N] = 1000 \times .0821 \times (1 - 0.0821) = 75.4$.
The normal approximation with integer correction to $P[N \geq 100]$ is
$P[N \geq 99.5] = P[N \geq 99.5] = P\left[\frac{N-82.1}{\sqrt{75.4}} \geq \frac{99.5-82.1}{\sqrt{75.4}}\right] = P[Z \geq 2.00] = 0.0228$.
Answer: C

17. $P[N=0] = P[N=0|\text{Low Risk}] \times P[\text{Low Risk}]$
$+ P[N=0|\text{Medium Risk}] \times P[\text{Medium Risk}] + P[N=0|\text{High Risk}] \times P[\text{High Risk}]$
$= (0.99)^2 \times 0.5 + (0.98)^3 \times 0.35 + (0.95)^4 \times 0.15 = 0.9416$. Answer: A

18. $E[L_A] = \int_d^\infty (x-d)\,f(x)\,dx$. $E[L_B] = \int_d^\infty x\,f(x)\,dx$.
$E[L_B - L_A] = \int_d^\infty d\,f(x)\,dx = d \times P(X > d)$
Similar solution for discrete X.
Alternatively, policy B pays d more than policy A for losses above d, so the expected amount paid for policy B is $d \times P(X > d)$ larger than paid for policy A. Answer: C

19. $P[Y > X+1] = \int_0^\infty \int_{x+1}^\infty e^{-x}\,e^{-y}\,dy\,dx = \int_0^\infty e^{-x}\,e^{-x-1}\,dx = \frac{e^{-1}}{2}$. Answer: B

20. The expected amount paid by the insurer is $E_1 = \int_0^L x\,dx \, + \, L \cdot (1-L) = L - \frac{L^2}{2}$.
The second moment of the amount paid by the insurer is
$E_2 = \int_0^L x^2\,dx \, + \, L^2 \times (1-L) = L^2 - \frac{2L^3}{3}$.
The variance of the amount paid by the insurer is
$L^2 - \frac{2L^3}{3} - (L - \frac{L^2}{2})^2 = \frac{L^3}{3} - \frac{L^4}{4}$.
We must solve $\frac{L^3}{3} - \frac{L^4}{4} = 0.0396$ for L. We cannot solve the equation algebraically, but by trial and error we see that $\frac{0.6^3}{3} - \frac{0.6^4}{4} = 0.0396$. Answer: D

21. The probabilities of Doug's tosses are all $\frac{1}{6}$ for tosses 1,2,3,4,5,6.
The probabilities of Bob's tosses are
2 with prob. $\frac{1}{36}$ (tosses two 1's)
3 with prob. $\frac{2}{36}$ (tosses 1 and 2 or 2 and 1)
4 with prob. $\frac{3}{36}$ (tosses 1 and 3 , etc.)
5 with prob. $\frac{4}{36}$, 6 with prob. $\frac{5}{36}$, 7 with prob. $\frac{6}{36}$
8 with prob. $\frac{5}{36}$, 9 with prob. $\frac{4}{36}$, 10 with prob. $\frac{3}{36}$,
11 with prob. $\frac{2}{36}$, 12 with prob. $\frac{1}{36}$

The combinations which result in Bob winning, with probabilities are
Doug tosses 1: prob. $\frac{1}{6}$, Bob tosses 3 or more: prob. $1 - \frac{1}{36} = \frac{35}{36}$;
Doug tosses 2: prob. $\frac{1}{6}$, Bob tosses 5 or more; prob. $1 - \frac{1}{36} - \frac{2}{36} - \frac{3}{36} = \frac{30}{36}$;
Doug tosses 3: prob. $\frac{1}{6}$, Bob tosses 7 or more; prob. $1 - \frac{1}{36} - \frac{2}{36} - \frac{3}{36} - \frac{4}{36} - \frac{5}{36} = \frac{21}{36}$;
Doug tosses 4: prob. $\frac{1}{6}$, Bob tosses 9 or more; prob. $\frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36}$;
Doug tosses 5: prob. $\frac{1}{6}$, Bob tosses 11 or more; prob. $\frac{2}{36} + \frac{1}{36} = \frac{3}{36}$;

The overall probability that Bob wins is $\frac{1}{6} \times (\frac{35}{36} + \frac{30}{36} + \frac{21}{36} + \frac{10}{36} + \frac{3}{36}) = \frac{11}{24}$.
Answer: E