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PROBLEM SET 2

Conditional Probability and Independence

1. Let A, B, C and D be events such that $B = A'$, $C \cap D = \emptyset$, and
$P[A] = \frac{1}{4}$, $P[B] = \frac{3}{4}$, $P[C|A] = \frac{1}{2}$, $P[C|B] = \frac{3}{4}$, $P[D|A] = \frac{1}{4}$, $P[D|B] = \frac{1}{8}$
Calculate $P[C \cup D]$.
A) $\frac{5}{32}$ B) $\frac{1}{4}$ C) $\frac{27}{32}$ D) $\frac{3}{4}$ E) 1

2. You are given that $P[A] = .5$ and $P[A \cup B] = .7$.
Actuary 1 assumes that A and B are independent and calculates $P[B]$ based on that assumption.
Actuary 2 assumes that A and B mutually exclusive and calculates $P[B]$ based on that assumption. Find the absolute difference between the two calculations.
A) 0 B) 0.05 C) 0.10 D) 0.15 E) 0.20

3. (SOA) An actuary studying the insurance preferences of automobile owners makes the following conclusions:
   (i) An automobile owner is twice as likely to purchase collision coverage as disability coverage
   (ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.
   (iii) The probability that an automobile owner purchases both collision and disability coverages is 0.15.

   What is the probability that an automobile owner purchases neither collision nor disability coverage?
   A) 0.18 B) 0.33 C) 0.48 D) 0.67 E) 0.82

4. Two bowls each contain 5 black and 5 white balls. A ball is chosen at random from bowl 1 and put into bowl 2. A ball is then chosen at random from bowl 2 and put into bowl 1. Find the probability that bowl 1 still has 5 black and 5 white balls.
A) $\frac{2}{3}$ B) $\frac{3}{5}$ C) $\frac{6}{11}$ D) $\frac{1}{2}$ E) $\frac{6}{13}$

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5. (SOA) An insurance company examines its pool of auto insurance customers and gathers the following information:

(i) All customers insure at least one car.
(ii) 70% of the customers insure more than one car.
(iii) 20% of the customers insure a sports car.
(iv) Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

A) 0.13 B) 0.21 C) 0.24 D) 0.25 E) 0.30

6. (SOA) An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims. Calculate the probability that a claim submitted to the insurance company includes operating room charges.

A) 0.10 B) 0.20 C) 0.25 D) 0.40 E) 0.80

7. Let A, B and C be events such that $P[A|C] = 0.05$ and $P[B|C] = 0.05$. Which of the following statements must be true?

A) $P[A \cap B|C] = (.05)^2$ B) $P[A' \cap B'|C] \geq .90$ C) $P[A \cup B|C] \leq .05$
D) $P[A \cup B|C'] \geq 1 - (.05)^2$ E) $P[A \cup B|C'] \geq .10$

8. A system has two components placed in series so that the system fails if either of the two components fails. The second component is twice as likely to fail as the first. If the two components operate independently, and if the probability that the entire system fails is .28, find the probability that the first component fails.

A) $\frac{0.28}{3}$ B) 0.10 C) $\frac{0.56}{3}$ D) 0.20 E) $\sqrt{0.14}$

9. A ball is drawn at random from a box containing 10 balls numbered sequentially from 1 to 10. Let X be the number of the ball selected, let R be the event that X is an even number, let S be the event that $X \geq 6$, and let T be the event that $X \leq 4$. Which of the pairs (R, S), (R, T), and (S, T) are independent?

A) (R, S) only B) (R, T) only C) (S, T) only D) (R, S) and (R, T) only
E) (R, S), (R, T) and (S, T)

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10. (SOA) A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers. A randomly selected participant from the study died over the five-year period. Calculate the probability that the participant was a heavy smoker.

A) 0.20 B) 0.25 C) 0.35 D) 0.42 E) 0.57

11. If E_1, E_2 and E_3 are events such that $P[E_1|E_2] = P[E_2|E_3] = P[E_3|E_1] = p$, $P[E_1 \cap E_2] = P[E_1 \cap E_3] = P[E_2 \cap E_3] = r$, and $P[E_1 \cap E_2 \cap E_3] = s$, find the probability that at least one of the three events occurs.

A) $1 - \frac{r^3}{p^3}$ B) $\frac{3p}{r} - r + s$ C) $\frac{3r}{p} - 3r + s$ D) $\frac{3p}{r} - 6r + s$ E) $\frac{3r}{p} - r + s$

12. (SOA) A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease. Determine the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

A) 0.115 B) 0.173 C) 0.224 D) 0.327 E) 0.514

13. In a T-maze, a laboratory rat is given the choice of going to the left and getting food or going to the right and receiving a mild electric shock. Assume that before any conditioning (in trial number 1) rats are equally likely to go the left or to the right. After having received food on a particular trial, the probability of going to the left and right become 0.6 and 0.4, respectively on the following trial. However, after receiving a shock on a particular trial, the probabilities of going to the left and right on the next trial are 0.8 and 0.2, respectively. What is the probability that the animal will turn left on trial number 2?

A) 0.1 B) 0.3 C) 0.5 D) 0.7 E) 0.9

14. In the game show "Let's Make a Deal", a contestant is presented with 3 doors. There is a prize behind one of the doors, and the host of the show knows which one. When the contestant makes a choice of door, at least one of the other doors will not have a prize, and the host will open a door (one not chosen by the contestant) with no prize. The contestant is given the option to change his choice after the host shows the door without a prize. If the contestant switches doors, what is the probability that he gets the door with the prize?

A) 0 B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) $\frac{2}{3}$

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15. (SOA) A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

(i) 14% have high blood pressure.

(ii) 22% have low blood pressure.

(iii) 15% have an irregular heartbeat.

(iv) Of those with an irregular heartbeat, one-third have high blood pressure.

(v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.

What portion of the patients selected have a regular heartbeat and low blood pressure?

A) 2% B) 5% C) 8% D) 9% E) 20%

16. (SOA) An insurance company issues life insurance policies in three separate categories: standard, preferred, and ultra-preferred. Of the company's policyholders, 50% are standard, 40% are preferred, and 10% are ultra-preferred. Each standard policy-holder has probability 0.010 of dying in the next year, each preferred policyholder has probability 0.005 of dying in the next year, and each ultra-preferred policyholder has probability 0.001 of dying in the next year. A policyholder dies in the next year. What is the probability that the deceased policyholder was ultra-preferred?

A) 0.0001 B) 0.0010 C) 0.0071 D) 0.0141 E) 0.2817

17. (SOA) The probability that a randomly chosen male has a circulation problem is 0.25. Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem. What is the conditional probability that a male has a circulation problem, given that he is a smoker?

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{1}{2}$ E) $\frac{2}{3}$

18. (SOA) A study of automobile accidents produced the following data:

Model Year	Proportion of all vehicles	Probability of involvement in an accident
1997	0.16	0.05
1998	0.18	0.02
1999	0.20	0.03
Other	0.46	0.04

An automobile from one of the model years 1997, 1998, and 1999 was involved in an accident. Determine the probability that the model year of this automobile is 1997.

A) 0.22 B) 0.30 C) 0.33 D) 0.45 E) 0.50

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19. (SOA) An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident.

Calculate the probability that the driver was age 16-20.

A) 0.13 B) 0.16 C) 0.19 D) 0.23 E) 0.40

20. (SOA) Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year:

(i) 10% of the emergency room patients were critical;

(ii) 30% of the emergency room patients were serious;

(iii) the rest of the emergency room patients were stable;

(iv) 40% of the critical patients died

(v) 10% of the serious patients died; and

(vi) 1% of the stable patients died.

Given that a patient survived, what is the probability that the patient was categorized as serious upon arrival?

A) 0.06 B) 0.29 C) 0.30 D) 0.39 E) 0.64

21. Let A, B and C be mutually independent events such that $P[A] = .5$, $P[B] = .6$ and $P[C] = .1$. Calculate $P[A' \cup B' \cup C]$.

A) 0.69 B) 0.71 C) 0.73 D) 0.98 E) 1.00

22. (SOA) An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto and a homeowners policy will renew at least one of those policies next year. Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto and a homeowners policy. Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

A) 20 B) 29 C) 41 D) 53 E) 70

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23. (SOA) An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young Adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver?

A) 0.06 B) 0.16 C) 0.19 D) 0.22 E) 0.25

24. Urn 1 contains 5 red and 5 blue balls. Urn 2 contains 4 red and 6 blue balls, and Urn 3 contains 3 red balls. A ball is chosen at random from Urn 1 and placed in Urn 2. Then a ball is chosen at random from Urn 2 and placed in Urn 3. Finally, a ball is chosen at random from Urn 3.

Find the probabilities that all three balls chosen are red.

A) $\frac{5}{11}$ B) $\frac{5}{12}$ C) $\frac{5}{21}$ D) $\frac{5}{22}$ E) $\frac{5}{33}$

25. A survey of 1000 Canadian sports fans who indicated they were either hockey fans or lacrosse fans or both, had the following result.

(i) 800 indicated that they were hockey fans (ii) 600 indicated that they were lacrosse fans

Based on the sample, find the probability that a Canadian sports fan is not a hockey fan given that she/he is a lacrosse fan.

A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

26. (SOA) An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44 Calculate the number of blue balls in the second urn.

A) 4 B) 20 C) 24 D) 44 E) 64

27. (SOA) An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is $\frac{1}{3}$. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A?

A) 0.280 B) 0.311 C) 0.467 D) 0.484 E) 0.700

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PROBLEM SET 2 SOLUTIONS

1. Since C and D have empty intersection, $P[C \cup D] = P[C] + P[D]$.

   Also, since A and B are "exhaustive" events (since they are complementary events, their union is the entire sample space, with a combined probability of $P[A \cup B] = P[A] + P[B] = 1$).
   We use the rule $P[C] = P[C \cap A] + P[C \cap A']$, and the rule $P[C|A] = \frac{P[A \cap C]}{P[A]}$ to get
   $P[C] = P[C|A] \times P[A] + P[C|A'] \times P[A'] = \frac{1}{2} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} = \frac{11}{16}$ and
   $P[D] = P[D|A] \times P[A] + P[D|A'] \times P[A'] = \frac{1}{4} \times \frac{1}{4} + \frac{1}{8} \times \frac{3}{4} = \frac{5}{32}$.
   Then, $P[C \cup D] = P[C] + P[D] = \frac{27}{32}$. Answer: C

2. Actuary 1: Since A and B are independent, so are A' and B'.
   $P[A' \cap B'] = 1 - P[A \cup B] = 0.3$.
   But $0.3 = P[A' \cap B'] = P[A'] \times P[B'] = 0.5 \times P[B'] \rightarrow P[B'] = 0.6 \rightarrow P[B] = 0.4$.
   Actuary 2: $0.7 = P[A \cup B] = P[A] + P[B] = 0.5 + P[B] \rightarrow P[B] = 0.2$.
   Absolute difference is $|0.4 - 0.2| = 0.2$. Answer: E

3. We identify the following events:
   $D =$ an automobile owner purchases disability coverage, and
   $C =$ an automobile owner purchases collision coverage.

   We are given that
   (i) $P[C] = 2P[D]$,
   (ii) C and D are independent, and (iii) $P[C \cap D] = 0.1$.
   From (ii) it follows that $P[C \cap D] = P[C] \times P[D]$, and therefore,
   $0.15 = 2P[D] \times P[D] = 2(P[D])^2$, from which we get $P[D] = \sqrt{0.075} = 0.27386$. Then,
   $P[C] = 2P[D] = 0.54772$, $P[D'] = 1 - P[D] = 0.72614$, and $P[C'] = 1 - P[C] = 0.45228$.
   Since C and D are independent, so are C' and D', and therefore, the probability that an automobile owner purchases neither disability coverage nor collision coverage is $P[C' \cap D'] = P[C'] \times P[D'] = 0.328$.
   Answer: B

4. Let C be the event that bowl 1 has 5 black balls after the exchange.
   Let B_1 be the event that the ball chosen from bowl 1 is black, and let B_2 be the event that the ball chosen from bowl 2 is black.

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Event C is the disjoint union of $B_1 \cap B_2$ and $B_1' \cap B_2'$ (black-black or white-white picks), so that $P[C] = P[B_1 \cap B_2] + P[B_1' \cap B_2']$.

The black-black combination has probability $\frac{6}{11} \times \frac{1}{2}$, since there is a $\frac{5}{10}$ chance of picking black from bowl 1, and then (with 6 black in bowl 2, which now has 11 balls) $\frac{6}{11}$ is the probability of picking black from bowl 2. This is

$$P[B_1 \cap B_2] = P[B_2|B_1] \cdot P[B_1] = \frac{6}{11} \times \frac{1}{2}.$$

In a similar way, the white-white combination has probability $\frac{6}{11} \times \frac{1}{2}$.

Then $P[C] = \frac{6}{11} \times \frac{1}{2} + \frac{6}{11} \times \frac{1}{2} = \frac{6}{11}$. Answer: C

5. We identify the following events:

A - the policyholder insures exactly one car (so that A' is the event that the policyholder insures more than one car), and

S - the policyholder insures a sports car.

We are given $P[A'] = 0.7$ (from which it follows that $P[A] = 0.3$), and $P[S] = 0.2$ (and $P[S'] = 0.8$). We are also given the conditional probability $P[S|A'] = 0.15$; "of those customers who insure more than one car", means we are looking at a conditional event given A'.

We are asked to find $P[A \cap S']$.

We create the following probability table, with the numerals in parentheses indicating the order in which calculations are performed.

	A, 0.3	A', 0.7
S, 0.2	**(2)** $P[S \cap A]$ $= P[S] - P[S \cap A']$ $= 0.2 - 0.105 = 0.095$	**(1)** $P[S \cap A'] = P[S\|A'] \cdot P[A']$ $= 0.15 \times 0.7 = 0.105$
S', 0.8	**(3)** $P[A \cap S']$ $= P[A] - P[A \cap S]$ $= 0.3 - 0.095 = 0.205$	

We can solve this problem with a model population of 1000 individuals with auto insurance. $\#A = 300$ (since 70% insure more than one car), and $\#S = 200$. From $P[S|A'] = 0.15$ we get $\#S \cap A' = 0.15 \times \#A' = 0.15 \times 700 = 105$. Then $\#S \cap A = \#S - \#S \cap A' = 200 - 105 = 95$, and $\#S' \cap A = \#A - \#S \cap A = 300 - 95 = 205$ is the number that insure exactly one car and the car is not a sports car.

Therefore $P[S' \cap A] = 0.205$. Answer: B

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6. We define the following events.

E - the claim includes emergency room charges ,

O - the claim includes operating room charges.

We are given $P[E \cup O] = 0.85$, $P[E'] = 0.25$ and E and O are independent.

We are asked to find $P[O]$.

We use the probability rule $P[E \cup O] = P[E] + P[O] - P[E \cap O]$.

Since E and O are independent, we have $P[E \cap O] = P[E] \times P[O] = 0.75 \times P[O]$

(since $P[E] = 1 - P[E'] = 1 - 0.25 = 0.75$).

Therefore, $0.85 = P[E \cup O] = 0.75 + P[O] - 0.75 \times P[O]$.

Solving for $P[O]$ results in $P[O] = 0.40$. Answer: D

7. $P[A' \cap B'|C] = P[(A \cup B)'|C] = 1 - P[A \cup B|C] \geq 0.9$,

since $P[A \cup B|C] \leq P[A|C] + P[B|C] = 0.1$. Answer: B

8. $0.28 = P[C_1 \cup C_2] = P[C_1] + P[C_2] - P[C_1 \cap C_2] = P[C_1] + 2P[C_1] - 2(P[C_1])^2$

Solving the quadratic equation results in $P[C_1] = 0.1$ (or 1.4, but we disregard this solution since $P[C_1]$ must be ≤ 1). Alternatively, each of the five answers can be substituted into the expression above for $P[C_1]$ to see which one satisfies the equation. Answer: B

9. $P[R] = 0.5$, $P[S] = 0.5$, $P[T] = 0.4$.

$P[R \cap S] = P[6, 8, 10] = 0.3 \neq 0.5 \times .5 = P[R] \times P[S] \rightarrow R, S$ are not independent

$P[R \cap T] = P[2, 4] = 0.2 = 0.5 \times 0.4 = P[R] \times P[T] \rightarrow R, T$ are independent

$P[S \cap T] = P[\emptyset] = 0 \neq 0.5 \times 0.4 = P[S] \times P[T] \rightarrow S, T$ are not independent. Answer: B

10. We identify the following events

N - non-smoker,

L - light smoker,

H - heavy smoker,

D - dies during the 5-year study.

We are given $P[N] = 0.50$, $P[L] = 0.30$, $P[H] = 0.20$.

We are also told that $P[D|L] = 2 \times P[D|N] = \frac{1}{2} \times P[D|H]$

(the probability that a light smoker dies during the 5-year study period is $P[D|L]$;

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it is the conditional probability of dying during the period given that the individual is a light smoker). We wish to find the conditional probability $P[H|D]$.We will find this probability from the basic definition of conditional probability, $P[H|D] = \frac{P[H \cap D]}{P[D]}$. These probabilities can be found from the following probability table. The numerals indicate the order in which the calculations are made.
We are not given specific values for $P[D|L]$, $P[D|N]$, or $P[D|H]$, so will let $P[D|N] = k$, and then $P[D|L] = 2k$ and $P[D|H] = 4k$.

	N, 0.5	L, 0.3	H, 0.2
D	**(1)** $P[D \cap N]$ $= P[D|N] \times P[N]$ $= k \times 0.5 = 0.5k$	**(2)** $P[D \cap L]$ $= P[D|L] \times P[L]$ $= 2k \times 0.3 = .6k$	**(3)** $P[D \cap H]$ $= P[D|H] \times P[H]$ $= 4k \times 0.2 = 0.8k$

(4) $P[D] = P[D \cap N] + P[D \cap L] + P[D \cap H] = 0.5k + 0.6k + 0.8k = 1.9k$.

(5) $P[H|D] = \frac{P[H \cap D]}{P[D]} = \frac{0.8k}{1.9k} = 0.42$. Answer: D

11. $P[E_1|E_2] = \frac{P[E_1 \cap E_2]}{P[E_2]} = p \to P[E_2] = \frac{r}{p}$, and similarly $P[E_3] = P[E_1] = \frac{r}{p}$.
Then, $P[E_1 \cup E_2 \cup E_3]$
$= P[E_1] + P[E_2] + P[E_3] - (P[E_1 \cap E_2] + P[E_1 \cap E_3] + P[E_2 \cap E_3])$
$+ P[E_1 \cap E_2 \cap E_3] = 3 \times \frac{r}{p} - 3r + s$. Answer: C

12. In this group of 937 man, we regard proportions of people with certain conditions to be probabilities. We are given the population of 937 men. We identify the following conditions:
DH - died from causes related to heart disease, and
PH - had a parent with heart disease.
We are given $\#PH = 312$, so if follows that $\#PH' = 937 - 312 = 625$.
We are also given $\#DH = 210$ and $\#DH \cap PH = 102$.
It follows that $\#DH \cap (PH') = \#DH - \#DH \cap PH = 210 - 102 = 108$.
Then the probability of dying due to heart disease given that neither parent suffered from heart disease is the proportion $\frac{\#DH \cap (PH')}{\#PH'} = \frac{108}{625}$. The solution in terms of conditional probability rules is as follows. From the given information, we have
$P[DH] = \frac{210}{937}$ (proportion who died from causes related to heart disease)
$P[PH] = \frac{312}{937}$ (proportion who have parent with heart disease)
$P[DH|PH] = \frac{102}{312}$ (prop. who died from heart disease given that a parent has heart disease).

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We are asked to find $P[DH|PH']$ (PH' is the complement of event PH, so that PH' is the event that neither parent had heart disease). Using event algebra, we have

$P[DH|PH] = \frac{P[DH \cap PH]}{P[PH]} \Rightarrow P[DH \cap PH] = P[DH|PH] \times P[PH] = \frac{102}{312} \times \frac{312}{937} = \frac{102}{937}$.

We now use the rule $P[A] = P[A \cap B] + P[A \cap \overline{B}]$.

Then $P[DH] = P[DH \cap PH] + P[DH \cap PH'] \rightarrow \frac{210}{937} = \frac{102}{937} + P[DH \cap PH']$
$\Rightarrow P[DH \cap PH'] = \frac{108}{937}$.

Finally, $P[DH|PH'] = \frac{P[DH \cap PH']}{P[PH']} = \frac{108/937}{1-P[PH]} = \frac{108/937}{1-\frac{312}{937}} = \frac{108}{625} = 0.1728$.

These calculations can be summarized in the following table.

	DH , 210 given	DH' , 727 $= 937 - 210$
PH , 312 given	$DH \cap PH = 102$ given	$DH' \cap PH = 210$ $= 312 - 102$
PH' , 625 $= 937 - 312$	$DH \cap PH' = 108$ $= 210 - 102$	$DH' \cap PH' = 517$ $= 727 - 210$ or $= 625 - 108$

$P[DH|PH'] = \frac{P[DH \cap PH']}{P[PH']} = \frac{\#[DH \cap PH']}{\#[PH']} = \frac{108}{625} = 0.1728$.

In this example, probability of an event is regarded as the proportion of a group that experiences that event.
Answer: B

13. $L1 =$ turn left on trial 1, $R1 =$ turn right on trial 1, $L2 =$ turn left on trial 2.

We are given that $P[L1] = P[R1] = 0.5$.

$P[L2] = P[L2 \cap L1] + P[L2 \cap R1]$ since $L1, R1$ form a partitio.

$P[L2|L1] = 0.6$ (if the rat turns left on trial 1 then it gets food and has a 0.6 chance of turning left on trial 2). Then $P[L2 \cap L1] = P[L2|L1] \times P[L1] = 0.6 \times 0.5 = 0.3$.

In a similar way, $P[L2 \cap R1] = P[L2|R1] \times P[R1] = 0.8 \times 0.5 = 0.4$.

Then, $P[L2] = 0.3 + 0.4 = 0.7$.

In a model population of 10 rats, $\#L1 = \#R1 = 5$, and $\#L2 \cap L1 = 0.6 \times 5 = 3$ and $\#L2 \cap R1 = 0.8 \times 5 = 4$. Then the number turning left on trial 2 will be

$\#L2 = \#L2 \cap L1 + \#L2 \cap R1 = 3 + 4 = 7$,

so the probability of a rat turning left on trial 2 is $7/10 = 0.7$. Answer: D

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14. We define the events $A =$ prize door is chosen after contestant switches doors , $B =$ prize door is initial one chosen by contestant. Then $P[B] = \frac{1}{3}$, since each door is equally likely to hold the prize initially. To find $P[A]$ we use the Law of Total Probability.

$$P[A] = P[A|B] \times P[B] + P[A|B'] \times P[B'] = 0 \times \frac{1}{3} + 1 \times \frac{2}{3} = \frac{2}{3}.$$

If the prize door is initially chosen, then after switching, the door chosen is not the prize door, so that $P[A|B] = 0$. If the prize door is not initially chosen, then since the host shows the other non-prize door, after switching the contestant definitely has the prize door, so that $P[A|B'] = 1$.
Answer: E

15. This question can be put into the context of probability event algebra. First we identify events:

$H =$ high blood pressure,
$L =$ low blood pressure,
$N =$ normal blood pressure,
$R =$ regular heartbeat ,
$I = R' =$ irregular heartbeat

We are told that 14% of patients have high blood pressure, which can be represented as $P[H] = 0.14$, and similarly $P[L] = .22$, and therefore $P[N] = 1 - P[H] - P[L] = 0.64$.
We are given $P[I] = 0.15$, so that $P[R] = 1 - P[I] = 0.85$.
We are told that "of those with an irregular heartbeat, one-third have high blood pressure". This is the conditional probability that given I (irregular heartbeat) the probability of H (high blood pressure) is $P[H|I] = \frac{1}{3}$. Similarly, we are given $P[I|N] = \frac{1}{8}$.

We are asked to find the portion of patients who have both a regular heartbeat and low blood pressure; this is $P[R \cap L]$. Since every patient is exactly one of H, L or N, we have

$P[R \cap L] + P[R \cap H] + P[R \cap N] = P[R] = 0.85$, so that
$P[R \cap L] = 0.85 - P[R \cap H] - P[R \cap N]$.

From the conditional probabilities we have

$$\frac{1}{3} = P[H|I] = \frac{P[H \cap I]}{P[I]} = \frac{P[H \cap I]}{0.15} \rightarrow P[H \cap I] = 0.05, \text{ and}$$
$$\frac{1}{8} = P[I|N] = \frac{P[I \cap N]}{P[N]} = \frac{P[I \cap N]}{0.64} \rightarrow P[I \cap N] = 0.08.$$

Then, since all patients are exactly one of I and R, we have
$P[H \cap I] + P[H \cap R] = P[H] = 0.14 \rightarrow P[H \cap R] = 0.14 - 0.05 = 0.09$, and
$P[I \cap N] + P[R \cap N] = P[N] = 0.64 \rightarrow P[R \cap N] = 0.64 - 0.08 = 0.56$.

Finally, $P[R \cap L] = .85 - P[R \cap H] - P[R \cap N] = 0.85 - 0.09 - 0.56 = 0.20$.

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H , 0.14 given $\Downarrow$ N , 0.64 $= 1 - 0.14 - 0.22$ $\Downarrow$ L , 0.22 given

I , .15 given

$P(H|I) = \frac{1}{3}$ given

$P(H \cap I)$
$= P(H|I) \times P(I)$
$= \frac{1}{3} \times 0.15) = 0.05$

$P(I|N) = \frac{1}{8}$ given

$P(N \cap I)$
$= P(I|N) \times P(N)$
$= \frac{1}{8} \times 0.64) = 0.08$ $\Rightarrow$

$P(L \cap I)$
$= P(I) - P(H \cap I) - P(N \cap I)$
$= 0.15 - 0.05 - 0.08 = 0.02$
$\Downarrow$

R , 0.85
$= 1 - 0.15$

$P(R \cap L)$
$= P(L) - P(L \cap I)$
$= 0.22 - 0.02 = 0.2$

Note that the entries $P(R \cap H)$ and $P(R \cap N)$ can also be calculated from this table.
The model population solution is as follows. Suppose that the model population has 2400 individuals. Then we have the following

$\#H = 0.14 \times 2400 = 336, \quad \#L = 528 , \ \#N = 1536, \quad \#I = 360 , \ \#R = 2040.$

Since one-third of those with an irregular heartbeat have high blood pressure, we get $\#I \cap H = 120$, and since one-eighth of those with normal blood pressure have an irregular heartbeat we get $\#N \cap I = 192$. We wish to find $\#R \cap L$.

From $\#I = \#I \cap H + \#I \cap L + \#I \cap N$, we get

$$360 = 120 + \#I \cap L + 192,$$

so that $\#I \cap L = 48$. Then from $\#L = \#I \cap L + \#R \cap L$ we get $528 = 48 + \#R \cap L$, so that $\#R \cap L = 480$.

Finally, the probability of having a regular heartbeat and low blood pressure is the proportion of the population with those properties, which is $\frac{480}{2400} = 0.2$. Answer: E

16. This is a typical exercise involving conditional probability.
We first label the events, and then identify the probabilities.

S - standard policy

P - preferred policy

U- ultra-preferred policy

D- death occurs in the next year.

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We are given

$P[S]=0.50\,,\ P[P]=0.40\,, P[U]=0.10\,,\ P[D|S]=0.01\,,\ P[D|P]=0.005\,,\ P[D|U]=0.001.$

We are asked to find $P[U|D]$.

The model population solution is as follows. Suppose there is a model population of 10,000 insured lives. Then $\#S=5000$, $\#P=4000$ and $\#U=1000$.

From $P[D|S]=0.01$ we get $\#D\cap S=.01\times 5000=50$, and we also get
$\#D\cap P=0.005\times 4000=20$ and $\#D\cap U=0.001\times 1000=1$.
Then $\#D=50+20+1=71$, and $P[U|D]$ is the proportion who are ultra-preferred as a proportion of all who died. This is $\frac{1}{71}=0.0141$.

The conditional probability approach to solving the problem is as follows.
The basic formulation for conditional probability is $P[U|D]=\frac{P[U\cap D]}{P[D]}$.
We use the following relationships:

$$P[A\cap B]=P[A|B]\cdot P[B],$$

and

$$P[A]=P[A\cap C_1]+P[A\cap C_2]+\cdots+P[A\cap C_n],$$

for a partition $C_1,C_2...,C_n$.

In this problem, events S , P and U form a partition of all policyholders.
Using the relationships we get

$$P[U\cap D]=P[D|U]\times P[U]=0.001\times 0.1=0.0001, \text{ and}$$

$$\begin{aligned}P[D]&=P[D\cap S]+P[D\cap P]+P[D\cap U]\\&=P[D|S]\times P[S]+P[D|P]\times P[P]+P[D|U]\times P[U]\\&=0.01\times 0.5+0.005\times 0.4+0.001\times 0.1=0.0071.\end{aligned}$$

Then, $P[U|D]=\frac{P[U\cap D]}{P[D]}=\frac{P[D|U]\times P[U]}{P[D|S]\times P[S]+P[D|P]\times P[P]+P[D|U]\times P[U]}$
$=\frac{0.001\times 0.1}{0.01\times 0.5+0.005\times 0.4+0.001\times 0.1}=\frac{0.0001}{0.0071}=0.0141.$

Notice that the numerator is one of the factors of the denominator. This will always be the case when we are "reversing" conditional probabilities such as has been done here; we are to find $P[U|D]$ from being given information about $P[D|U]$, $P[D|S]$, $P[D|P]$, etc.

From the calculations already made it is easy to find the probability that the deceased policyholder was preferred;

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$$P[P|D] = \frac{P[P\cap D]}{P[D]} = \frac{P[D|P]\times P[P]}{P[D|S]\times P[S]+P[D|P]\times P[P]+P[D|U]\times P[U]}$$
$$= \frac{0.005\times 0.4}{0.01\times 0.5+0.005\times 0.4+0.001\times 0.1} = \frac{0.0020}{0.0071} = 0.2817.$$

And $P[S|D]$ is $\frac{0.01\times 0.5}{.01\times 0.5+0.005\times 0.4+0.00\times 0.1} = \frac{0.0050}{0.0071} = 0.7042.$

The calculations can be summarized in the following table.

	S, 0.5 given	P, 0.4 given	U, 0.1 given
D	$P(D\|S) = 0.01$ given	$P(D\|P) = 0.005$ given	$P(D\|U) = 0.001$ given
	$P(D\cap S)$ $= P(D\|S)\times P(S)$ $= 0.01\times 0.5 = 0.005$	$P(D\cap P)$ $= P(D\|P)\times P(P)$ $= 0.005\times 0.4 = 0.002$	$P(D\cap U)$ $= P(D\|U)\times P(U)$ $= 0.001\times 0.1 = 0.0001$

$P(D) = P[D\cap S] + P[D\cap P] + P[D\cap U] = 0.005 + 0.002 + 0.0001 = 0.0071.$

$P[U|D] = \frac{P[U\cap D]}{P[D]} = \frac{0.0001}{0.0071} = 0.0141.$ Answer: D

17. We identify the following events:

C - a randomly chosen male has a circulation problem ,

S - a randomly chosen male is a smoker.

We are given the following probabilities:
$P[C] = 0.25$, $P[S|C] = 2P[S|C']$.

From the rule $P[A\cap B] = P[A|B]P[B]$, we get

$P[S\cap C] = P[S|C]\times P[C] = 0.25\times P[S|C]$, and
$P[S\cap C'] = P[S|C']\times P[C'] = P[S|C']\times(1-P[C]) = 0.75\times\frac{1}{2}\times P[S|C]$,
so that $P[S] = P[S\cap C] + P[S\cap C'] = 0.25\times P[S|C] + 0.75\times\frac{1}{2}\times P[S|C] = 0.625\times P[S|C]$.
We are asked to find $P[C|S]$. This is $P[C|S] = \frac{P[C\cap S]}{P[S]} = \frac{0.25\times P[S|C]}{0.625\times P[S|C]} = 0.4.$

Note that the way in which information was provided allowed us to formulate various probabilities in terms of $P[S|C]$ (but we do not have enough to find $P[S|C]$). Answer: C

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18. We identify events as follows:

97: the model year is 1997 , 98: the model year is 1998 , 99: the model year is 1999

OO : other, the model year is not 1997, 1998 or 1999

A : the car is involved in an accident

We are given $P[97] = 0.16$, $P[98] = 0.18$, $P[99] = 0.20$, $P[00] = 0.46$,

$P[A|97] = 0.05$, $P[A|98] = 0.02$, $P[A|99] = 0.03$, $P[A|\text{other}] = 0.04$.

The model population solution is as follows. Suppose there are 10,000 automobiles in the study.

Then $\#97 = 1600$, $\#98 = 1800$, $\#99 = 2000$, $\#OO = 4600$.

From $P[A|97].05$ we get $\#A \cap 97 = .05 \times 1600 = 80$, and in a similar way we get

$\#A \cap 98.02 \times 1800 = 36$, $\#A \cap 99 = .03 \times 2000 = 60$

and $\#A \cap OO.04 \times 4600 = 184$.

We are given that an automobile from one of 97, 98 or 99 was involved in an accident, and we wish to find the probability that it was a 97 model. This is the conditional probability $P[97|A \cap (97 \cup 98 \cup 99)]$. This will be the proportion

$\frac{\#A\cap 97}{\#A\cap 97+\#A\cap 98+\#A\cap 99} = \frac{80}{80+36+60} = \frac{80}{176} = 0.4545.$

The conditional probability apporach to solve the problem is as follows.

We use the conditional probability rule $P[C|D] = \frac{P[C\cap D]}{P[D]}$, so that

$P[97|A \cap (97 \cup 98 \cup 99)] = \frac{P[97\cap[A\cap(97\cup 98\cup 99)]]}{P[A\cap(97\cup 98\cup 99)]}$.

From set algebra, we have $97 \cap [A \cap (97 \cup 98 \cup 99)] = 97 \cap A$, and

$A \cap (97 \cup 98 \cup 99) = (A \cap 97) \cup (A \cap 98) \cup (A \cap 99)$.

Since the events 97, 98 and 99 are disjoint, we get

$$\begin{aligned} P[A \cap (97 \cup 98 \cup 99)] &= P[(A \cap 97) \cup (A \cap 98) \cup (A \cap 99)] \\ &= P[A \cap 97] + P[A \cap 98] + P[A \cap 99]. \end{aligned}$$

From conditional probability rules we have

$P[A \cap 97] = P[A|97] \times P[97] = 0.05 \times 0.16 = 0.008$, and similarly

$P[A \cap 98] = 0.02 \times 0.18 = 0.0036$, and $P[A \cap 99] = 0.03 \times 0.20 = 0.006$.

Then, $P[A \cap (97 \cup 98 \cup 99)] = 0.008 + 0.0036 + 0.006 = 0.0176$.

Therefore, the probability we are trying to find is

$$\begin{aligned} P[97|A \cap (97 \cup 98 \cup 99)] &= \frac{P[97\cap[A\cap(97\cup 98\cup 99)]]}{P[A\cap(97\cup 98\cup 99)]} \\ &= \frac{P[97\cap A}{P[A\cap(97\cup 98\cup 99)]} = \frac{0.008}{0.0176} = 0.4545 \quad . \end{aligned}$$

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These calculations can be summarized in the following table.

	97 , 0.16 given	98 , 0.18 given	99 , 0.20 given	Other , 0.46 given
A	$P(A\|97)$ $=0.05$ given	$P(A\|98)$ $=0.02$ given	$P(A\|99)$ $=0.03$ given	$P(A\|\text{Other})$ $=0.04$ given
	$P(A\cap 97)$ $P[A\|97]\times P[97]$ $=0.05\times 0.16$ $=0.008$	$P(A\cap 98)$ $P[A\|98]\times P[98]$ $=0.02\times 0.18$ $=0.0036$	$P(A\cap 99)$ $P[A\|99]\times P[99]$ $=0.03\times 0.20$ $=0.006$	$P(A\cap \text{Other})$ $P[A\|\text{O}]\times P[\text{O}]$ $=0.04\times 0.46$ $=0.0184$

Then, $P[97|A\cap(97\cup 98\cup 99)]=\frac{0.008}{0.008+0.0036+0.006}=0.4545.$

Note that the denominator is the sum of the first three of the intersection probabilities, since the condition is that the auto was 97, 98 or 99. If the question had asked for the probability that the model year was 97 given that an accident occurred (without restricting to 97, 98, 99) then the probability would be $\frac{0.008}{0.008+0.0036+0.006+0.0184}$; we would include all model years in the denominator. If the question had asked for the probability that the model year was 97 given that an accident occurred and the automobile was from one of the model years 97 or 98, then the probability would be $\frac{0.008}{0.008+0.0036}$; we would include only the 97 and 98 model years. Answer: D

19. We identify the following events:

A - the driver has an accident,

T (teen) - age of driver is 16-20,

Y (young) - age of driver is 21-30,

M (middle age) - age of driver is 31-65,

S (senior) - age of driver is 66-99.

The final column in the table lists the probabilities of T, Y, M and S, and the middle column gives the conditional probability of A given driver age. The table can be interpreted as

Age	Probability of Accident	Portion of Insured Drivers
16-20	$P[A\|T]=0.06$	$P[T]=0.08$
21-30	$P[A\|Y]=0.03$	$P[Y]=0.15$
31-65	$P[A\|M]=0.02$	$P[M]=0.49$
66-99	$P[A\|S]=0.04$	$P[S]=0.28$

We are asked to find $P[T|A]$.

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We construct the following probability table, with numerals in parentheses indicating the order of the calculations.

	$T, 0.08$	$Y, 0.15$	$M, 0.49$	$S, 0.28$
A	**(1)** $P[A \cap T]$	**(2)** $P[A \cap Y]$	**(3)** $P[A \cap M]$	**(4)** $P[A \cap S]$
	$= P[A\|T] \times P[T]$	$= P[A\|Y] \times P[Y]$	$= P[A\|M] \times P[M]$	$= P[A\|S] \times P[S]$
	$= 0.06 \times 0.08$	$= 0.03 \times 0.15$	$= 0.02 \times 0.49$	$= 0.04 \times 0.28$
	$= 0.0048$	$= 0.0045$	$= 0.0098$	$= 0.0112$

(5) $P[A] = P[A \cap T] + P[A \cap Y] + P[A \cap M] + P[A \cap S] = 0.0303$

(6) $P[T|A] = \frac{P[A \cap T]}{P[A]} = \frac{0.0048}{0.0303} = 0.158.$ Answer: B

20. We label the following events:

C - critical , S - serious , T -stable , D - died , D' - survived.

The following information is given

$P(C) = 0.1$, $P(S) = 0.3$,

$P(T) = 0.6 = 1 - P(C) - P(S)$,

$P(D|C) = 0.4$, $P(D|S) = 0.1$, $P(D|T) = 0.01$.

We are asked to find $P(S|D')$. This can be done by using the following table of probabilities.

The rules being used here is $P(A \cap B) = P(A|B) \times P(B)$, and

$P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n)$ if $B_1, B_2, \ldots, B_n$

form a partition of the probability space. In this case, C, S, T form a partition since all patients are exactly one of these three conditions.

	C	S	T
D	$P(D \cap C)$	$P(D \cap S)$	$P(D \cap T)$
	$= P(D\|C) \times P(C)$	$= P(D\|S) \times P(S)$	$= P(D\|T) \times P(T)$
	$= 0.4 \times 0.1 = 0.04$	$= 0.1 \times 0.3 = 0.03$	$= 0.01 \times 0.6 = 0.006$

$\rightarrow P(D) = P(D \cap C) + P(D \cap S) + P(D \cap T) = 0.04 + 0.03 + 0.006 = 0.076$

	C	S	T
D'	$P(D' \cap C)$	$P(D' \cap S)$	$P(D' \cap T)$
	$= P(D'\|C) \times P(C)$	$= P(D'\|S) \times P(S)$	$= P(D'\|T) \times P(T)$
	$= 0.6 \times 0.1 = 0.06$	$= 0.9 \times 0.3 = 0.27$	$= 0.99 \times 0.6 = 0.594$

$\rightarrow P(D') = P(D' \cap C) + P(D' \cap S) + P(D' \cap T) = 0.06 + 0.27 + 0.594 = 0.924$

It was not necessary to do the calculations for D', since $P(D') = 1 - P(D) = 1 - 0.076 = 0.924$.

The probability in question is $P(S|D') = \frac{P(S \cap D')}{P(D')} = \frac{0.27}{0.924} = 0.292.$ Answer: B

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21. $P[A' \cup B' \cup C]$

$$= P[A'] + P[B'] + P[C] - (P[A' \cap B'] + P[A' \cap C] + P[B' \cap C]) + P[A' \cap B' \cap C]$$
$$= 0.5 + 0.4 + 0.1 - [0.5 \times 0.4 + 0.5 \times 0.1 + 0.4 \times 0.1] + 0.5 \times 0.4 \times 0.1 = 0.73 .$$

If events X and Y are independent, then so are X' and Y, X and Y', and X' and Y'. Alternatively using DeMorgan's Law, we have

$$P[A' \cup B' \cup C] = 1 - P[(A' \cup B' \cup C)'] = 1 - P[A'' \cap B'' \cap C'] = 1 - P[A \cap B \cap C']$$
$$= 1 - P[A] \cdot P[B] \cdot P[C'] = 1 - 0.5 \times 0.6 \times 0.9 = 0.73.$$

Answer: C

22. We define the following events

R - renew at least one policy next year
A - has an auto policy , H - has a homeowner policy

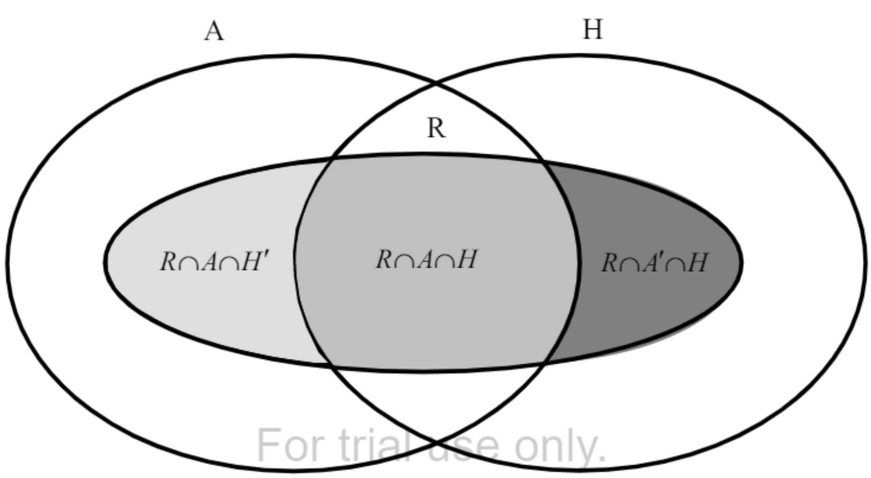
A policyholder with an auto policy only can be described by the event $A \cap H'$, and a policyholder with a homeowner policy only can be described by the event $A' \cap H$.

We are given $P[R|A \cap H'] = 0.4$, $P[R|A' \cap H] = 0.6$ and $P[R|A \cap H] = 0.8$.

We are also given $P[A] = 0.65$, $P[H] = 0.5$ and $P[A \cap H] = 0.15$.

We are asked to find $P[R]$.

We use the rule $P[R] = P[R \cap A \cap H] + P[R \cap A' \cap H] + P[R \cap A \cap H'] + P[R \cap A' \cap H']$.
Since renewal can only occur if there is at least one policy, it follows that $P[R \cap A' \cap H'] = 0$; in other words, if there is no auto policy (event A') and there is no homeowner policy (event H'), then there can be no renewal. An alternative way of saying the same thing is that R (the inside oval in the diagram) is a subset (subevent) of $A \cup H$.
(Note also that $P[A \cup H] = P[A] + P[H] - P[A \cap H] = 0.65 + 0.5 - 0.15 = 1$, so this also show that R must be a subevent of $A \cup H$, and it also shows that
$P[A' \cap H'] = 1 - P[A \cup H] = 1 - 1 = 0$ so that $A' \cap H' = \phi$).
This can be illustrated in the following diagram.



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We find $P[R \cap A \cap H]$, $P[R \cap A' \cap H]$ and $P[R \cap A \cap H']$ by using the rule

$P[C \cap D] = P[C|D] \times P[D]$:

$P[R \cap A \cap H] = P[R|A \cap H] \times P[A \cap H] = 0.8 \times 0.15 = 0.12$,

$P[R \cap A' \cap H] = P[R|A' \cap H] \times P[A' \cap H] = 0.6 \times P[A' \cap H]$,

$P[R \cap A \cap H'] = P[R|A \cap H'] \times P[A \cap H'] = 0.4 \times P[A \cap H']$.

In order to complete the calculations we must find $P[A' \cap H]$ and $P[A \cap H']$.

From the diagram above, or using the probability rule, we have

$$P[A] = P[A \cap H] + P[A \cap H'] \rightarrow 0.65 = 0.15 + P[A \cap H'] \rightarrow P[A \cap H'] = 0.5, \text{ and}$$
$$P[H] = P[A \cap H] + P[A' \cap H] \rightarrow 0.5 = 0.15 + P[A' \cap H] \rightarrow P[A' \cap H] = 0.35.$$

Then $P[R \cap A' \cap H] = 0.6 \times 0.35 = 0.21$ and $P[R \cap A \cap H'] = 0.4 \times 0.5 = 0.2$.

Finally, $P[R] = 0.12 + 0.21 + 0.2 = 0.53$. 53% of policyholders will renew. Answer: D

23. We are given $P(\text{teen}) = 0.08$, $P(\text{young adult}) = 0.16$, $P(\text{midlife}) = 0.45$ and $P(\text{senior}) = 0.31$. We are also given the conditional probabilities

$P(\text{at least one collision}|\text{teen}) = 0.15$, $P(\text{at least one collision}|\text{young adult}) = 0.08$,

$P(\text{at least one collision}|\text{midlife}) = 0.04$, $P(\text{at least one collision}|\text{senior}) = .05$.

We wish to find $P(\text{young adult}|\text{at least one collision})$.

Using the definition of conditional probability, we have

$P(\text{young adult}|\text{at least one collision}) = \frac{P(\text{young adult} \cap \text{at least one collision})}{P(\text{at least one collision})}$.

We use the rule $P(A \cap B) = P(A|B) \times P(B)$, to get

$P(\text{young adult} \cap \text{at least one collision}) = P(\text{at least one collision} \cap \text{young adult})$

$= P(\text{at least one collision}|\text{young adult}) \times P(\text{young adult}) = 0.08 \times 0.16 = 0.0128.$

We also have

$$\begin{aligned}P(\text{at least one collision}) &= P(\text{at least one collision} \cap \text{teen}) \\ &\quad + P(\text{at least one collision} \cap \text{young adult}) + P(\text{at least one collision} \cap \text{midlife}) \\ &\quad + P(\text{at least one collision} \cap \text{senior}) \\ &= P(\text{at least one collision}|\text{teen}) \times P(\text{teen}) \\ &\quad + P(\text{at least one collision}|\text{young adult}) \times P(\text{young adult}) \\ &\quad + P(\text{at least one collision}|\text{midlife}) \times P(\text{midlife}) \\ &\quad + P(\text{at least one collision}|\text{senior}) \times P(\text{senior}) \\ &= 0.15 \times 0.08 + 0.08 \times 0.16 + 0.04 \times 0.45 + 0.05 \times 0.31 = 0.0583 .\end{aligned}$$

Then $P(\text{young adult}|\text{at least one collision}) = \frac{0.0128}{0.0583} = 0.2196.$

These calculations can be summarized in the following table.

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