22. Fans of ML and BJ but not TA:

$n(ML \cap BJ \cap TA') = n(ML \cap BJ) - n(ML \cap BJ \cap TA) = 600 - 400 = 200.$

Fans of ML and TA but not BJ:

$n(ML \cap BJ' \cap TA) = n(ML \cap TA) - n(ML \cap BJ \cap TA) = 500 - 400 = 100.$

Fans of TA and BJ but not ML:

$n(ML' \cap BJ \cap TA) = n(TA \cap BJ) - n(ML \cap BJ \cap TA) = 500 - 400 = 100.$

Total is 400. Answer: E

23. $P[X > a \,|\, |X| > a] = \frac{P[(X>a)\cap(|X|>a)]}{P[|X|>a]}$.

If $a < 0$, then $P[|X| > a] = 1$

and $P[X > a] = \int_a^0 \frac{1}{2}e^x\,dx + \int_0^\infty \frac{1}{2}e^{-x}\,dx = \frac{1}{2}(1 - e^a) + \frac{1}{2} = 1 - \frac{1}{2}e^a,$

so that $\frac{P[(X>a)\cap(|X|>a)]}{P[|X|>a]} = P[X > a] = 1 - \frac{1}{2}e^a.$

If $a \geq 0$, then $P[|X| > a] = \int_{-\infty}^{-a} \frac{1}{2}e^x\,dx + \int_a^\infty \frac{1}{2}e^{-x}\,dx = e^{-a}$

and $P[(X > a) \cap (|X| > a)] = \int_a^\infty \frac{1}{2}e^{-x}\,dx = \frac{1}{2}e^{-a},$

so that $\frac{P[(X>a)\cap(|X|>a)]}{P[|X|>a]} = \frac{1}{2}.$ Answer: D

24. The total running time of Smith and Green has a normal distribution with mean 5.1 hours and variance .25 hours^2. The average of those two running times, say X, is normal with mean 2.55 hours and variance 0.0625 hours^2. If Y is Jones' running time, we wish to find the probability $P[Y < X]$. Y is independent of X, and $X - Y$ is normal with mean $2.55 - 2.5 = 0.05$ and variance $0.0625 + 0.04 = 0.1025$. Then,

$P[Y < X] = P[X - Y > 0] = P\left[\frac{X-Y-0.05}{\sqrt{0.1025}} > \frac{-0.05}{\sqrt{0.1025}}\right] = P[Z > -0.156]$

$= P[W < 0.156] = 0.562$ (Z and W are standard normal random variables). Answer: B

25. $f(t) = \frac{1}{\theta}$, for $0 < t < \theta$, and $F(t) = P[T \leq t] = \frac{t}{\theta}$.

$P[Y > t] = P[(X_1 > t) \cap (X_2 > t) \cap (X_3 > t)] = [1 - F(t)]^3 = [1 - \frac{t}{\theta}]^3$ for $0 < t < \theta$.

$E[Y] = \int_0^\theta [1 - F_Y(t)]\,dt = \int_0^\theta P[Y > t]\,dt = \int_0^\theta [1 - \frac{t}{\theta}]^3\,dt = \frac{\theta}{4} = 0.25\theta$.

Alternatively, we can find $f_Y(t) = -\frac{d}{dt}P[Y > t] = 3 \times [1 - \frac{t}{\theta}]^2 \times \frac{1}{\theta}$.

We could then find $E[Y]$ from $\int_0^\theta t \times f_Y(t)\,dt$. Answer: B

26. $P[Y \geq 1] = 1\,,\quad P[Y \geq 2] = P[\text{all three tosses are } \geq 2] = (\frac{5}{6})^3$,
so $P[Y = 1] = P[Y \geq 1] - P[Y \geq 2] = 1 - (\frac{5}{6})^3$.
$P[Y \geq 3] = P[\text{all three tosses are } \geq 3] = (\frac{4}{6})^3$,
so $P[Y = 2] = P[Y \geq 2] - P[Y \geq 3] = (\frac{5}{6})^3 - (\frac{4}{6})^3$.
In a similar way, $P[Y = 3] = (\frac{4}{6})^3 - (\frac{3}{6})^3\,,\ P[Y = 4] = (\frac{3}{6})^3 - (\frac{2}{6})^3$,
$P[Y = 5] = (\frac{2}{6})^3 - (\frac{1}{6})^3$ and $P[Y = 6] = (\frac{1}{6})^3$.
Then $E[Y] = 1 - (\frac{5}{6})^3 + 2 \times [(\frac{5}{6})^3 - (\frac{4}{6})^3] + 3 \times [(\frac{4}{6})^3 - (\frac{3}{6})^3]$
$+ 4 \times [(\frac{3}{6})^3 - (\frac{2}{6})^3] + 5 \times [(\frac{2}{6})^3 - (\frac{1}{6})^3] + 6 \times (\frac{1}{6})^3$
$= 1 + (\frac{5}{6})^3 + (\frac{4}{6})^3 + (\frac{3}{6})^3 + (\frac{2}{6})^3 + (\frac{1}{6})^3 = \frac{49}{24} = 2.042.$ Answer: D

27. The coefficient of variation of Y is $\frac{\sigma_Y}{\mu_Y} = \frac{\sqrt{Var[Y]}}{E[Y]}$.
For the die toss random variable X we have $E[X] = \frac{1+2+3+4+5+6}{6} = \frac{7}{2}$,
and $E[X^2] = \frac{1^2+2^2+3^2+4^2+5^2+6^2}{6} = \frac{91}{6}$, so $Var[X] = \frac{35}{12}$.
Recall that the mean and variance of a Poisson random variable are the same, so that
$E[Y|X] = Var[Y|X] = X$.
We use conditioning rules for the variance of Y:
$Var[Y] = E[Var[Y|X]] + Var[E[Y|X]] = E[X] + Var[X] = \frac{7}{2} + \frac{35}{12} = \frac{77}{12}$.
Answer: E

28. Since the pgf is $P(t) = \sum_k p_k t^k$, we see that X and Y have the following distributions:

k	0	1	2	3
$P(X = k)$	0.5	0.2	0.3	0
$P(Y = k)$	0.4	0.3	0.2	0.1

$P[X + Y = 4] = P[X = 1, Y = 3] + P[X = 2, Y = 2] = 0.2 \times 0.1 + 0.3 \times 0.2 = 0.08$.
Answer: D

29. For a particular question that they got wrong, there is a $\frac{1}{4}$ probability that they got the same answer (answering randomly on one of the four incorrect answers). The number of questions of those 5 for which they got the same answer has a binomial distribution with $n = 5$ and $p = \frac{1}{4}$.
$P[\text{at least 3 answers the same on those 5 questions}]$
$= \binom{5}{3}(\frac{1}{4})^3(\frac{3}{4})^2 + \binom{5}{4}(\frac{1}{4})^4(\frac{3}{4}) + \binom{5}{5}(\frac{1}{4})^5 = \frac{106}{1024} = 0.1035.$ Answer: C

30. Suppose that the loss amount is denoted X. Then $f(x) = .001,\ 0 < x < 1000$.

The insurer pays 0 if $0 < X < 100$,

the insurer pays $X - 100$ if $100 \le X < 500$, and

the insurer pays $400 + \frac{1}{2}(X - 500)$ is $500 \le X < 1000$.

The expected insurance payment is

$\int_{100}^{500} (x - 100) \times 0.001\, dx + 400 \times P[X \ge 500] + \int_{500}^{1000} \frac{1}{2} \times (x - 500) \times 0.001\, dx$

$= 80 + 200 + 62.5 = 342.5.$ Answer: D

PRACTICE EXAM 10

1. The distribution of a discrete loss random variable X is uniform on the integers from 1 to 10. For losses less than or equal to 5 an insurance policy pays the full amount of the loss. For losses greater than 5, the insurer pays 5 plus half of the excess of the loss above 5. Calculate the variance of the insurance payment in the event of a loss.

   A) Less than 2 B) At least 2 but less than 4 C) At least 4 but less than 6
   D) At least 6 but less than 8 E) At least 8

2. The random variable X has a three-point discrete distribution with $P[X = k] = p_k$ for $k = 0, 1, 2$. The probability generating function of X is $P_X(t)$ and the moment generating function is $M_X(t)$. You are given the following: $P_X(0) = 0.40$ and $M_X(1) = 3.4322$. Calculate the value of $P[X = 1]$.

   A) 0.15 B) 0.20 C) 0.25 D) 0.30 E) 0.353.

3. Smith has developed a gambling strategy for a certain casino game that, Smith believes, will win 51% of the time. A participant either wins \$10 or loses \$10 on each play of the game. Assuming successive plays of the game are independent of one another, find the approximate probability that after 100 plays of the game, Smith will have won at least \$200 (use the normal approximation).

   A) Less than 0.05 B) At least 0.05 but less than 0.10 C) At least 0.10 but less than 0.15
   D) At least 0.15 but less than 0.20 E) At least 0.20

4. An urn contains 2 balls, X of which are blue and $2 - X$ of which are green. An assumption is made that X has the following distribution: $P[X = 0] = \frac{1}{6}$, $P[X = 1] = \frac{1}{3}$, $P[X = 2] = \frac{1}{2}$. A second urn contains 2 blue and 2 green balls. A ball is chosen at random from urn 1 and placed into urn 2. A ball is then chosen at random from urn 2. Calculate the probability that the ball chosen from urn 2 is blue.

   A) 0 B) $\frac{2}{15}$ C) $\frac{4}{15}$ D) $\frac{6}{15}$ E) $\frac{8}{15}$

5. X has the following distribution: $P[X = 1] = p$, $P[X = -1] = 1 - p$. $X_1, X_2, \ldots, X_n$ are independent random variables, each of which has the same distribution as X. The random variable Y is defined as follows: $Y = \sum_{k=1}^{n} X_k$. You are given $\frac{Var[Y]}{E[Y]} = 2.1$.

   Calculate p.

   A) 0.3 B) 0.4 C) 0.5 D) 0.6 E) 0.7

6. X_1 , X_2 and X_3 are independent exponential random variables with means of 1, 2 and 3, respectively. Y is defined to the $Y = max\{X_1, X_2, X_3\}$. Determine $E[Y]$.

A) Less than 3 B) At least 3 but less than 4 C) At least 4 but less than 5

D) At least 5 but less than 6 E) At least 6

7. A survey is done of people who own a cell phone, a home phone, or both. It is found that 75% of people who owned home phones also owned a cell phone and 40% of those who owned cell phones also owned home phones. Of the people in this survey that owned a cell phone or a home phone or both, what is the percentage that own both (nearest 1%)?

A) 31 B) 35 C) 39 D) 42 E) 45

8. Smith is planning a camping trip and wants to bring enough flashlight batteries so that the flashlight can operate for at least 48 hours. The flashlight uses one battery. An operating battery has a lifetime that is normally distributed with a mean of 8 hours and standard deviation of 2 hours. Assume that the battery lifetimes are independent of one another. What is the smallest number of batteries that Smith should have in order to have a 99% probability that the flashlight will operate for at least 48 hours?

A) 6 B) 7 C) 8 D) 9 E) 10

9. A loss random variable X has probability density function $\frac{3x^{1/2}}{2000}$ for $0 < x < 100$.
An insurance policy pays the excess of the loss above 50. Determine
$E[\text{Amount paid by insurance}|X > 50]$.

A) Less than 25 B) At least 25 but less than 27 C) At least 27 but less than 29

D) At least 29 but less than 31 E) At least 31

10. Which of the following statements are true?

I. If events A and B are independent, then events A and B' (complement) are independent.

II. If events A and B are independent, then events A' and B' are independent.

III. If events A and B are independent, then events $A \cup B$ and $A' \cup B'$ are independent.

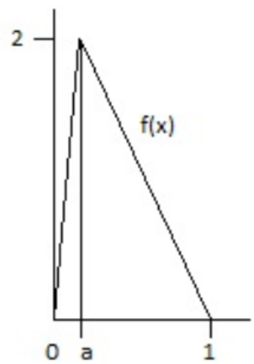
A) All but I B) All but II C) All but III D) All E) I only

11. X and Y are random variables. $W = X + Y$ and $Z = X - Y$. Which of the following statements is always true.

A) $Cov(W, Z) \geq 0$ B) $Cov(W, Z) > 0$ C) $Cov(W, Z) \leq 0$ D) $Cov(W, Z) < 0$

E) None of A, B, C or D

Questions 12 and 13 are based on the following graph of the pdf of the random variable X.



12. Assuming that $0 < a < 1$, find $E[X|X \leq a]$.

A) $\frac{a}{6}$ B) $\frac{a}{3}$ C) $\frac{a}{2}$ D) $\frac{2a}{3}$ E) $\frac{5a}{6}$

13. Assuming that $0 < a < 1$, what is the maximum of $Var[X]$?

A) $\frac{1}{18}$ B) $\frac{1}{9}$ C) $\frac{1}{6}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

14. For a certain type of bacterial culture, the amount of time it takes for the number of bacteria to double has a continuous uniform distribution between 8 and 12 hours. For another type of bacterial culture, the amount of time it takes for the number of bacteria to double has a continuous uniform distribution between 10 and 15 hours. Assuming that the two culture grow independently, what is the probability that the first culture will double in size before the second one doubles?

A) Less than 0.50 B) At least 0.50 but less than 0.55 C) At least 0.55 but less than 0.60
D) At least 0.60 but less than 0.65 E) At least 0.65

15. X and Y are continuous random variables with joint cumulative distribution function

$$F(x, y) = \frac{1}{250}(20xy - x^2y - xy^2) \text{ for } 0 \leq x \leq 5 \text{ and } 0 \leq y \leq 5$$

Calculate $E[Y|X = 0]$.

A) $\frac{5}{2}$ B) $\frac{20}{9}$ C) 2 D) $\frac{20}{11}$ E) $\frac{5}{3}$

16. A fair coin is tossed 6 times in a row. Assuming the successive tosses are independent of one another find the probability that in the sequence of 6 tosses there are at least 3 consecutive heads or at least 3 consecutive tails or both.

A) Less than 60% B) At least 60% but less than 62%
C) At least 62% but less than 64% D) At least 64% but less than 66%
E) At least 66%

17. A mass produced finished product coming off of an assembly line is subject to a quality test of randomly chosen finished products. The test will indicate that the product is defective for 98% of defective products. The test will indicated that the product is non-defective for 100% of non-defective products. It is estimated that the actual percentage of defective products is $\frac{1}{2}\%$. Suppose that a randomly chosen finished product is tested and found to be not defective. What is the probability that it is actually defective?
A) Less than 0.00011 B) At least 0.00011 but less than 0.00013
C) At least 0.00013 but less than 0.00015 D) At least 0.00015 but less than 0.00017
E) At least 0.00017

Questions 18 and 19 are based on the random variable X, which has a "zero-modified" Poisson distribution defined in the following way. Y has a Poisson distribution with mean $\lambda > 0$.
The probability function for X is defined as follows:
$P[X=0] = \frac{P[Y=0]}{2}$ and $P[X=k] = c \times P[Y=k]$ for $k = 1, 2, ...$

18. Which of the following is a correct expression for c?
A) $\frac{1}{2}$ B) $\frac{2-e^{-y}}{2-2e^{-y}}$ C) $\frac{2-e^{-y}}{2-e^{-y}}$ D) $\frac{1-2e^{-y}}{2-2e^{-y}}$ E) $\frac{1-e^{-y}}{1-2e^{-y}}$

19. Which of the following is a correct expression for $Var[X]$?
A) $c\lambda + c\lambda^2 + c^2\lambda^2$ B) $c\lambda - c\lambda^2 + c^2\lambda^2$ C) $c\lambda + c\lambda^2 - c^2\lambda^2$
D) $c\lambda - c\lambda^2 - c^2\lambda^2$ E) $c\lambda^2 - c\lambda - c^2\lambda^2$

20. Smith has developed a gambling strategy for a certain casino game that, Smith believes, will win 51% of the time. A participant either wins \$1 or loses \$1 on each play of the game. Assume successive plays of the game are independent of one another. Of the following numbers of plays of the game, which is the minimum number of plays Smith must make in order to have a probability of at least 0.99 of winning at least \$1,000,000 in total (use the normal approximation).
A) 5,100 B) 51,000 C) 510,000 D) 5,100,000 E) 51,000,000

21. The model for the number of members of a randomly chosen family in a certain town is $1 + X$, where X has a binomial distribution with $n = 3$, $p = 0.4$. The town's medical facility estimates the annual medical cost (in \$1,000s) for a family is as follows:

Number of Family Members	1	2	3	4
Annual Medical Cost	2	3	3.75	4.25

Calculate the average annual medical cost per family in the town.
A) Less than 3000 B) At least 3000 but less than 3100
C) At least 3100 but less than 3200 D) At least 3200 but less than 3300
E) At least 3300

22. A telemarketing company has an automated message sent to a randomly chosen 7-digit phone number in a particular area code. The randomly chosen number cannot have first digit 0 or 1 but the remaining 6 digits can be any of the digits from 0 to 9. The random numbers are chosen successively and independently of one another (so numbers can be repeated), with 1000 numbers (and calls) occurring per hour of operation. The calls are made for 12 hours per day. Your home phone, cell phone and office phone all have different numbers in the telemarketer's area code. Find the probability that you will not get a telemarketer call sometime during the coming (365 day) year on any of your three phones.

A) Less than 0.15 B) At least 0.15 but less than 0.18 C) At least 0.18 but less than 0.21
D) At least 0.21 but less than 0.24 E) At least 0.24

23. X has a normal distribution with mean μ and variance σ^2. Find the coefficient of variation of Y, where $Y = e^X$.

A) $e^{\mu^2}(e^{\sigma^2}-1)$ B) $e^{2\mu^2}(e^{\sigma^2}-1)$ C) $\sqrt{e^{\mu^2}(e^{\sigma^2}-1)}$ D) $\sqrt{e^{\sigma^2}-1}$ E) $e^{\sigma^2}-1$

24. 3000 fair dice are tossed independently of one another. N_1 denotes the number of "1"s that turned up. Calculate $P[N_1 > 550]$ using the normal approximation with integer correction.

A) Less than .010 B) At least .010 but less than .012 C) At least .012 but less than .014
D) At least .014 but less than .016 E) At least .016

25. There are 6 people on a subway car. Assume that each person's birth month is uniformly distributed on the 12 months of the year, and the birth months of different individuals on the subway car are independent of one another. Find the probability that at least two people in the subway car have the same birth month.

A) Less than 0.5 B) At least 0.5 but less than 0.6 C) At least 0.6 but less than 0.7
D) At least 0.7 but less than 0.8 E) At least 0.8

26. Y is the growth factor for the coming year in an investment fund. The model for Y is $Y = e^X$, where X has a uniform distribution on the interval $[-1,1]$. Find the 80th percentile of Y.

A) Less than 1.50 B) At least 1.50 but less than 1.75 C) At least 1.75 but less than 2.0
D) At least 2.0 but less than 2.25 E) At least 2.25

27. In a certain village the number of visits per year to the village physician an individual will make follows a Poisson distribution that depends on the individual's dietary habits. The distribution of the population of the village and the mean number of doctor visits are as follows:

	Proportion of population	Mean number of doctor visits
Strict vegan	0.15	1
Vegetarian (not vegan)	0.25	2
Non-vegetarian (or vegan)	0.60	3

Calculate the probability that a person with exactly 2 doctor visits in a year is a strict vegan.

A) Less than 0.095 B) At least 0.095 but less than 0.105 C) At least 0.105 but less than 0.115

D) At least 0.115 but less than 0.125 E) At least 0.125

28. According to the model of auto accident damage in a portfolio of insurance policies, an auto accident results in either minor damage or major damage. If the damage is minor the amount of damage has an exponential distribution with a mean of 1 and if the damage is major then the amount of damage has an exponential distribution with a mean of 2. 80% of auto accidents have minor damage. Find the median amount of damage for an insurance policy in this portfolio when an auto accident occurs.

A) Less than 0.75 B) At least 0.75 but less than 0.80 C) At least 0.80 but less than 0.85

D) At least 0.85 but less than 0.90 E) At least 0.90

29. Annual losses follow a uniform distribution on the interval $[0, 2000]$. A insurance premium of 1200 is charged to cover losses. If the loss is below the premium, the risk manager receives a bonus of 25% of the amount by which losses are below the premium. Find the expected bonus the risk manager will receive.

A) 70 B) 90 C) 110 D) 130 E) 150

30. X has the following probability function: $P[X = k] = \frac{k+1}{6}$, for $k = 0, 1, 2$.

Y has the following probability function given X

$P[Y = j|X = k] = \frac{1}{k+2}$ for $j = 0, 1, ..., k+1$.

Calculate $E[Y]$, the mean of the marginal distribution of Y.

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{5}{6}$ D) 1 E) $\frac{7}{6}$

PRACTICE EXAM 10 - SOLUTIONS

1. If the insurance payment is denoted Y, then $Y =$ pays the following $\begin{cases} X & \text{for } X \leq 5 \\ \frac{X+5}{2} & \text{for } 6 \leq X \leq 10 \end{cases}$.

Then, $E[Y] = \frac{1+2+3+4+5+5.5+6+6.5+7+7.5}{10} = 4.75$
and $E[Y^2] = \frac{1^2+2^2+3^2+4^2+5^2+5.5^2+6^2+6.5^2+7^2+7.5^2}{10} = 26.875$.
Then, $Var[Y] = 4.3125$. Answer: C

2. $P_X(t) = p_0 + p_1 t + p_2 t^2$ and $M_X(t) = p_0 + p_1 e^t + p_2 e^{2t}$.
We are given $P_X(0) = p_0 = .4$, and
$M_X(1) = p_0 + p_1 e^1 + p_2 e^2 = 0.4 + p_1 e^1 + (1 - p_0 - p_1)e^2$
$= .4 + p_1 e^1 + (.6 - p_1)e^2 = 0.4 + 2.718282 p_1 + 7.389056(0.6 - p_1)$
$= 4.833434 - 4.670774 p_1 = 3.4322$. Solving for p_1 results in $p_1 = 0.3$. Answer: D

3. Each particular game has random outcome $X = \begin{cases} -10 & \text{prob. } 0.49 \\ 10 & \text{prob. } 0.51 \end{cases}$.
$E[X] = 0.2$, $E[X^2] = 100$ and $Var[X] = 99.96$.
$W = \sum_{k=1}^{100} X_k$, where X_k is the outcome from the k-th play of the game, with the X_k's being independent of one another. $E[W] = 100E[X] = 20$, $Var[W] = 100Var[X] = 9,996$.
$P[W \geq 200] = P\left[\frac{W-20}{\sqrt{9996}} \geq \frac{200-20}{\sqrt{9996}}\right] = P[Z \geq 1.80] = 0.04$. Answer: A

4. The event A denotes the ball chosen from urn 2 is blue. $P[A]$ is
$P[A \cap \text{ball chosen from urn 1 is blue}] + P[A \cap \text{ball chosen from urn 1 is green}]$
$= P[A|\text{ball chosen from urn 1 is blue}] \times P[\text{ball chosen from urn 1 is blue}]$
$+ P[A|\text{ball chosen from urn 1 is green}] \times P[\text{ball chosen from urn 1 is green}]$
$= 0.6 \times P[\text{ball chosen from urn 1 is blue}] + 0.4 \times P[\text{ball chosen from urn 1 is green}]$.
The event B denotes the event that ball chosen from urn 1 is blue. To find $P[B]$ we condition over the number of blue balls in urn 1.
$P[B] = P[B \cap (X = 0)] + P[B \cap (X = 1)] + P[B \cap (X = 2)]$
$= P[B|X = 0] \times P[X = 0] + P[B|X = 1] \times P[X = 1] + P[B|X = 2] \times P[X = 2]$
If $X = 0$ the ball chosen from urn 1 must be green, so $P[B|X = 0] = 0$, and if $X = 2$ the ball chosen from urn 1 must be blue, so $P[B|X = 2] = 1$. If $X = 1$, there is a .5 chance that the ball chosen from urn 2 is blue, so by the symmetry of the situation, $P[B|X = 1] = 0.5$.
Then, $P[B] = 0 \times \frac{1}{6} + 0.5 \times \frac{1}{3} + 1 \times \frac{1}{2} = \frac{2}{3}$.
And then, $P[A] = 0.6 \times \frac{2}{3} + 0.4 \times \frac{1}{3} = \frac{8}{15}$. Answer: E

5. $E[X] = 2p - 1$, $Var[X] = 4p(1-p)$. $E[Y] = n(2p-1)$, $Var[Y] = 4np(1-p)$
$\frac{Var[Y]}{E[Y]} = \frac{4np(1-p)}{n(2p-1)} = \frac{4p(1-p)}{2p-1} = 2.1 \rightarrow 4p^2 + 0.2p - 2.1 = 0 \rightarrow p = 0.7$ or -0.75.
We reject the negative root. Alternatively, by trial and error, we see that only answer E satisfies the equation $\frac{4p(1-p)}{2p-1} = 2.1$. Answer: E

6. $P[Y \le t] = P[(X_1 \le t) \cap (X_2 \le t) \cap (X_3 \le t)]$
$= P[X_1 \le t] \times P[X_2 \le t] \times P[X_3 \le t]]$
$= (1 - e^{-t})(1 - e^{-t/2})(1 - e^{-t/3})$
$= 1 - e^{-t} - e^{-t/2} - e^{-t/3} + e^{-3t/2} + e^{-4t/3} + e^{-5t/6} + e^{-11t/6}$
$\rightarrow P[Y > t] = 1 - P[Y \le t] = e^{-t} + e^{-t/2} + e^{-t/3} - e^{-3t/2} - e^{-4t/3} - e^{-5t/6} - e^{-11t/6}$.
$E[Y] = \int_0^\infty P[Y > t\}\, dt = 1 + 2 + 3 - \frac{2}{3} - \frac{3}{4} - \frac{6}{5} - \frac{6}{11} = 3.93$. Answer: B

7. $P[C|H] = \frac{P[C \cap H]}{P[H]} = 0.75$, $P[H|C] = \frac{P[C \cap H]}{P[C]} = 0.40$
Then $P[C \cap H] = 0.75P[H] = 0.4P[C]$.
$P[C \cap H | C \cup H] = \frac{P[C \cap H]}{P[C \cup H]} = \frac{P[C \cap H]}{P[C] + P[H] - P[C \cap H]} = \frac{1}{\frac{P[C]}{P[C \cap H]} + \frac{P[H]}{P[C \cap H]} - 1}$
$= \frac{1}{\frac{1}{P[H|C]} + \frac{1}{P[C|H]} - 1} = \frac{1}{\frac{1}{.4} + \frac{1}{.75} - 1} = \frac{6}{17} = 0.353$. Answer: B

8. With k batteries, the expected lifetime of power, say L, is normal with mean $8k$ and variance $4k$.
$P[L \ge 48] = P[\frac{L - 8k}{\sqrt{4k}} \ge \frac{48 - 8k}{\sqrt{4k}}] = 0.99 \rightarrow \frac{48 - 8k}{\sqrt{4k}} \le -2.33$.
Solving $\frac{48 - 8k}{\sqrt{4k}} = -2.33$ results in $8k - 2.33\sqrt{4k} - 48 = 0$, with roots
$\sqrt{k} = 2.75$, -2.18 . Since the quadratic is actually the inequality
$8k - 2.33\sqrt{4k} - 48 \ge 0$, the solution is $\sqrt{k} \ge 2.75$ or $\sqrt{k} \le -2.18$.
The minimum value of k that would satisfy the condition must also satisfy
$k \ge 2.75^2 = 7.6$, so that the minimum number of batteries needed is 8 Answer: C

9. $E[\text{Amount paid by insurance}] = \int_{50}^{100} (x - 50) \times \frac{3x^{1/2}}{2000}\, dx = 17.0711$.
$P[X > 50] = \int_{50}^{100} \frac{3x^{1/2}}{2000}\, dx = 0.6464$.
$E[\text{Amount paid by insurance}|X > 50] = \frac{E[\text{Amount paid by insurance}]}{P[X > 50]} = 26.4$.
Answer: B

10. Independent of A and B means that $P[A \cap B] = P[A] \times P[B]$

I. $P[A \cap B'] = P[A] - P[A \cap B] = P[A] - P[A] \times P[B]$

$= P[A] \times (1 - P[B]) = P[A] \times P[B']$. True.

II. $P[A' \cap B'] = 1 - P[A \cup B] = 1 - P[A] - P[B] + P[A \cap B]$

$= P[A'] - P[B] + P[A] \times P[B] = P[A'] - P[B] \times P[A'] = P[A'] \times P[B']$. True.

III. With $X = A \cup B$ and $Y = A' \cup B'$, we have

$P[X \cap Y] = (A \cup B) \cap (A' \cup B') = (A' \cap B) \cup (A \cap B')$, which is a disjoint union, so that

$P[(A' \cap B) \cup (A \cap B'] = P[A' \cap B] + P[A \cap B'] = P[A'] \times P[B] + P[A] \times P[B']$

$(1 - P[A]) \times P[B] + (1 - P[B]) \times P[A]$.

Therefore, $P[X \cap Y] = P[A] + P[B] - 2 \times P[A] \times P[B]$.

$P[X] = P[A \cup B] = P[A] + P[B] - P[A] \times P[B]$ (because of independence of A and B)

and $P[Y] = P[A' \cup B'] = P[A'] + P[B'] - P[A'] \times P[B']$

$= 1 - P[A] + 1 - P[B] - (1 - P[A]) \times (1 - P[B]) = 1 - P[A] \times P[B]$

so that $P[X] \times P[Y] = P[A \cup B] \times P[A' \cup B']$

$= (P[A] + P[B] - P[A] \times P[B]) \times (1 - P[A] \times P[B])$

This is not the same as $P[X \cap Y] = P[A] + P[B] - 2 \times P[A] \times P[B]$.

Alternatively, consider two fair coins, one red and one green. A coin is chosen at random and tossed. With event A being that the coin chosen is red and B being that the toss is heads, we see that A and B are independent, each with probability $\frac{1}{2}$. Also, $P[A \cup B] = P[A' \cup B'] = \frac{3}{4}$, so that $P[A \cup B] \times P[A' \cup B'] = \frac{9}{16}$, but $(A \cup B) \cap (A' \cup B')$ is the event $(A' \cap B) \cup (A \cap B')$, which is "red and tail" $\cup$ "green and head", which has probability $\frac{1}{2}$, and is not equal to $\frac{9}{16}$.

Statement III is false. Answer: C

11. $Cov(W, Z) = E[W \times Z] - E[W] \times E[Z]$

$= E[(X + Y)(X - Y)] - E[X + Y] \times E[X - Y]$

$= E[X^2 - Y^2] - [(E[X])^2 - (E[Y])^2] = Var[X] - Var[Y]$.

This can be ≥ 0 or ≤ 0 depending on the relative sizes of $Var[X]$ and $Var[Y$.

Answer: E

12. The pdf of X is $f(x) = \begin{cases} \frac{2x}{a} & \text{for } 0<x\leq a \\ \frac{2(1-x)}{1-a} & \text{for } a\leq x<1 \end{cases}$.

$P[X \leq a] = \int_0^a \frac{2x}{a}\, dx = a$.

$f(x|X \leq a) = \frac{f(x)}{P[X\leq a]} = \frac{2x}{a^2}$ for $0 < x \leq a$.

$E[X|X \leq a] = \int_0^a x \times \frac{2x}{a^2}\, dx = \frac{2a}{3}$. Answer: D

13. $E[X] = \int_0^a x \cdot \frac{2x}{a}\, dx + \int_a^1 x \times \frac{2(1-x)}{1-a} dx = \frac{2a^2}{3} + \frac{1-3a^2+2a^3}{3(1-a)} = \frac{1+a}{3}$

$E[X^2] = \int_0^a x^2 \times \frac{2x}{a}\, dx + \int_a^1 x^2 \times \frac{2(1-x)}{1-a} dx = \frac{1+a+a^2}{6}$

$Var[X] = \frac{1+a+a^2}{6} - \left(\frac{1+a}{3}\right)^2 = \frac{1-a+a^2}{18}$.

The maximum variance occurs at $a = 0$ or 1, with $Var[X] = \frac{1}{18}$. Answer: A

14. $f_1(t) = \frac{1}{4}$, $8 < t < 12$ and $f_2(t) = \frac{1}{5}$, $10 < t < 15$.

$P[T_1 < T_2] = \int_8^{12}$ "density that $T_1 = t$ and $T_1 < T_2$" dt

$= \int_8^{10}$ "density that $T_1 = t$ and $T_1 < T_2$" dt

$+ \int_{10}^{12}$ "density that $T_1 = t$ and $T_1 < T_2$" dt

$= \int_8^{10} f_1(t) \times 1\, dt + \int_{10}^{12} f_1(t) \times P[T_2 > t]\, dt$

$= \int_8^{10} \frac{1}{4}\, dt + \int_{10}^{12} \frac{1}{4} \times \left(\frac{15-t}{5}\right) dt = \frac{1}{2} + \frac{8}{20} = 0.9$ Answer: E

15. The joint pdf is $f(x,y) = \frac{\partial^2}{\partial y\, \partial x} F(x,y) = \frac{20-2x-2y}{250}$ for $0 \le x \le 5$ and $0 \le y \le 5$.

The marginal pdf of X is $f_X(x) = \int_0^5 f(x,y)\, dy = \frac{75-10x}{250}$ for $0 \le x \le 5$.

The conditional pdf of Y given $X = x$ is $f_{Y|X}(y|X = x) = \frac{20-2x-2y}{75-10x}$ for $0 \le y \le 5$.

$E[Y|X = 0] = \int_0^5 y \times f_{Y|X}(y|X = 0)\, dy = \int_0^5 y \times \frac{20-2y}{75}\, dy = \frac{20}{9}$. Answer: B

16. There are $2^6 = 64$ different sequences of heads and tails for the tosses. The following indicates the sequences that have at least 3 heads, X indicates the toss can be H or T.

Toss:	1	2	3	4	5	6	Number of sequences
	H	H	H	X	X	X	8
	T	H	H	H	X	X	4
	X	T	H	H	H	X	4
	X	X	T	H	H	H	4

There are a total of 20 sequences with at least 3 heads in a row. By the symmetry of the situation there will also be 20 sequences with at least 3 tails in a row. 2 of the sequences have both 3 heads and 3 tails in a row, so that the total number of unique sequences with at least 3 heads or at least 3 tails in a row, or both is $20 + 20 - 2 = 38$ out of the 64 possible sequences. The probability is $\frac{38}{64} = 0.59375$. Answer: A

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17. We define the following events:

D - the product is defective , TD - a tested product is found to be defective

We are given $P[D] = 0.005$, $P[TD|D] = 0.98$ and $P[TD'D'] = 1.0$.

We wish to find $P[D'|TD']$.

This is $\frac{P[D' \cap TD']}{P[TD']}$.

From $P[D] = .005$ and $P[TD|D] = \frac{P[D \cap TD]}{P[D]} = 0.98$

we get $P[D \cap TD] = 0.98 \times 0.005 = 0.0049$.

From $P[D'] = .995$ and $P[TD'|D'] = \frac{P[D' \cap TD']}{P[D']} = 1.0$ we get $P[D' \cap TD'] = 0.995$.

Also, $.005 = P[D] = P[D \cap TD] + P[D \cap TD'] = 0.0049 + P[D \cap TD']$

so that $P[D \cap TD'] = 0.0001$.

Then $P[TD'] = P[D \cap TD'] + P[D' \cap TD'] = 0.9951$,

and $P[D|TD'] = \frac{P[D \cap TD']}{P[TD']} = \frac{0.0001}{0.9951} = 0.00010049$ Answer: A

18. $\sum_{k=0}^{\infty} P[X = k] = 1 \rightarrow \sum_{k=1}^{\infty} P[X = k] = 1 - P[X = 0] = 1 - \frac{P[Y=0]}{2} = 1 - \frac{e^{-y}}{2}$.

$\rightarrow \sum_{k=1}^{\infty} P[X = k] = c \times \sum_{k=1}^{\infty} P[Y = k] = c \times (1 - P[Y = 0]) = c \times (1 - e^{-y})$

$\rightarrow 1 - \frac{e^{-y}}{2} = c \times (1 - P[Y = 0]) = c \times (1 - e^{-y}) \rightarrow c = \frac{2 - e^{-y}}{2 - 2e^{-y}}$. Answer: B

19. $E[Y] = \sum_{k=0}^{\infty} k \times P[Y = k] = \sum_{k=1}^{\infty} k \times P[Y = k] = \lambda$, so

$E[Y^2] = \sum_{k=0}^{\infty} k^2 \times P[Y = k] = Var[Y] + (E[Y])^2 = \lambda + \lambda^2$.

$E[X] = \sum_{k=0}^{\infty} k \times P[X = k] = \sum_{k=1}^{\infty} k \times P[X = k] = c \times \sum_{k=1}^{\infty} k \times P[Y = k] = c\lambda$

$E[X^2] = \sum_{k=1}^{\infty} k^2 \times P[X = k] = c \times E[Y^2] = c(\lambda + \lambda^2)$.

$Var[X] = c(\lambda + \lambda^2) - (c\lambda)^2 = c\lambda + c\lambda^2 - c^2\lambda^2$. Answer: C

20. Smith's outcome on one play of the game is $X = \begin{cases} -1 & \text{prob. } 0.49 \\ 1 & \text{prob. } 0.51 \end{cases}$, with mean $E[X] = 0.02$

and variance $V[X] = 0.9996$. After n independent plays of the game, Smith's outcome is

$W = \sum_{k=1}^{n} X_i$, where each X_i has the distribution of X and the X_i's are independent of one

another. The mean and variance of W are $E[W] = 0.02n$ and $V[W] = 0.9996n$.

We wish to find n so that $P[W \geq 1,000,000] \geq 0.99$. Applying the normal approximation to W

we have $Z = \frac{W - .02n}{\sqrt{.9996n}}$ is approximately a standard normal random variable. Then,

$P[W \geq 1,000,000] = P\left[\frac{W - 0.02n}{\sqrt{0.9996n}} \geq \frac{1,000,000 - 0.02n}{\sqrt{0.9996n}}\right]$. In order for this to be at least 0.99 we

must have $\frac{1,000,000 - .02n}{\sqrt{.9996n}} \leq -2.33$. E is the only answer that satisfies the inequality. Answer: E

21. The probabilities for the various family sizes are

Family Size	1	2	3	4
Probability	$P[X=0]$ $=0.216$	$P[X=1]$ $=0.432$	$P[X=2]$ $=0.288$	$P[X=3]$ $=0.064$
Cost	2,000	3,000	3,750	4,250

Expected cost per family is

$2000 \times 0.216 + 3000 \times 0.432 + 3750 \times 0.288 + 4250 \times 0.064 = 3,080$. Answer: B

22. There are $10^7 = 10,000,000$ 7-digit numbers in the area code if digits 0 and 1 are allowed as first digits. Eliminating 0 and 1 as possible first digits results in 8,000,000 possible phone numbers. The probability that none of your three numbers is not chosen in a particular random choice is $\frac{7,999,997}{8,000,000}$. The probability that none of your numbers is chosen in the coming year ($12,000 \times 365$ successive random numbers) is $\left(\frac{7,999,997}{8,000,000}\right)^{12,000 \times 365} = 0.1935$. Answer: C

23. $E[Y] = M_X(1)$ (moment generating function of X), which is $e^{\mu + \frac{\sigma^2}{2}}$ and
$E[Y^2] = M_X(2) = e^{2\mu + 2\sigma^2}$.
Then $Var[Y] = e^{2\mu+2\sigma^2} - \left(e^{\mu+\frac{\sigma^2}{2}}\right)^2 = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} = e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$.
The coefficient of variation of Y is $\frac{\sqrt{Var[Y]}}{E[Y]} = \frac{e^{\mu+\frac{1}{2}\sigma^2}\sqrt{e^{\sigma^2}-1}}{e^{\mu+\frac{\sigma^2}{2}}} = \sqrt{e^{\sigma^2}-1}$. Answer: D

24. N_1 has a binomial distribution with mean $3000 \times \frac{1}{6} = 500$ and variance
$3000 \times \frac{1}{6} \times \frac{5}{6} = 416.7$. Applying the normal approximation to N_1 results in
$P[N_1 > 550] = P[N_1 \geq 551] = P[N_1 \geq 550.5] = P\left[\frac{X_1 - 500}{\sqrt{416.7}} \geq \frac{550.5-500}{\sqrt{416.7}}\right] = P[Z \geq 2.47]$
$= 0.0068$, Answer: A

25. The probability that no two people have the same birth month is
P[person 2 has different birth month than person 1
$\cap$ person 3 has different birth month than persons 1 or 2
$\cap \cdots \cap$ person 6 has different birth month than persons 1 to 5]
$= \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} \times \frac{7}{12} = .223$.
The probability that at least two people have the same birth month is $1 - 0.223 = 0.777$.

Answer: D

26. The 80th percentile of Y is the point c for which $P[Y \le c] = 0.8$.
This $P[e^X \le c] = P[X \le ln\, c] = .8$. Therefore $ln\, c$ is the 80th percentile of X, which is $-1 + .8 \times (1-(-1)) = .6$, and then $c = e^{.6} = 1.822$. Answer: C

27. X is the number of doctor visits in a year and we define the following:
SV - strict vegan , V - vegetarian (non-vegan) , NV - non-vegetarian or vegan
We are given $P[SV] = 0.15$, $P[V] = .25$, $P[NV] = 0.60$.
We are trying to calculate $P[SV|X=2] = \frac{P[SV \cap (X=2)]}{P[X=2]}$.
We calculate the numerator and denominator using conditioning rules.
$P[SV \cap (X=2)] = P[X=2|SV] \times P[SV] = \frac{1^2 \times e^{-1}}{2!} \times 0.15 = .075e^{-1} = 0.0276.$
$P[X=2] = P[SV \cap (X=2)] + P[V \cap (X=2)] + P[NV \cap (X=2)]$
$= P[X=2|SV] \times P[SV] + P[X=2|V] \times P[V] + P[X=2|NV] \times P[NV]$
$= \frac{1^2 \times e^{-1}}{2!} \times 0.15 + \frac{2^2 \times e^{-2}}{2!} \times 0.25 + \frac{3^2 \times e^{-3}}{2!} \times 0.6$
$= 0.0276 + 0.0677 + 0.1344 = 0.2297.$
Then $P[SV|X=2] = \frac{0.0276}{0.2297} = 0.120$. Answer: D

28. The amount of damage is X, and the median is m, where $P[X \le m] = 0.5$.
$P[X \le m] = P[X \le m|\text{minor damage}] \times P[\text{minor damage}$
$+ P[X \le m|\text{major damage}] \times P[\text{major damage}]$
$= (1-e^{-m}) \times 0.8 + (1-e^{-m/2}) \times 0.2 = 0.5$.
This results in the quadratic equation $8a^2 + 2a - 5 = 0$, where $a = e^{-m/2}$.
Solving for a results in $a = 0.67539$ or -0.92539. We ignore the negative root, since $a > 0$.
The median is $m = -2\, ln\, a = 0.785$. Answer: B

29. The pdf of the loss amount is $\frac{1}{2000}$. There will be bonus of amount $.25(1200-x)$ if the loss amount is $x \le 1200$. The expected bonus is $\int_0^{1200} 0.25 \times (1200-x)\, \frac{1}{2000}\, dx = 90$.
Answer: B

30. The joint probability function of X and Y is

$$P[X=k,Y=k]=P[Y=j|X=k]\times P[X=k].$$

This is

$P[X=0,Y=0]=P[Y=0|X=0]\times P[X=0]=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$,
$P[X=0,Y=1]=P[Y=1|X=0]\times P[X=0]=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$,
$P[X=1,Y=0]=P[Y=0|X=1]\times P[X=1]=\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$,
$P[X=1,Y=1]=P[Y=1|X=1]\times P[X=1]=\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$,
$P[X=1,Y=2]=P[Y=2|X=1]\times P[X=1]=\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$,
$P[X=2,Y=0]=P[Y=0|X=2]\times P[X=2]=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$,
$P[X=2,Y=1]=P[Y=1|X=2]\times P[X=2]=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$,
$P[X=2,Y=2]=P[Y=2|X=2]\times P[X=2]=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$,
$P[X=2,Y=3]=P[Y=3|X=2]\times P[X=2]=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$.

The marginal probability function of Y is

$P[Y=0]=\frac{1}{12}+\frac{1}{9}+\frac{1}{8}=\frac{23}{72}$, $P[Y=1]=\frac{1}{12}+\frac{1}{9}+\frac{1}{8}=\frac{23}{72}$,
$P[Y=2]=\frac{1}{9}+\frac{1}{8}=\frac{17}{72}$, $P[Y=3]=\frac{1}{8}$.
$E[Y]=1\times\frac{23}{72}+2\times\frac{17}{72}+3\times\frac{1}{8}=\frac{7}{6}$. Answer: E

PRACTICE EXAM 11

1. You are given the following information regarding the distribution of Y and the conditional distribution of X given Y:

$$P(Y = -1) = \tfrac{1}{6}\ ,\ P(Y = 0) = \tfrac{1}{3}\ ,\ P(Y = 1) = \tfrac{1}{2}\ ,$$
$$P(X = -1\,|\,Y = -1) = \tfrac{1}{3}\ ,\ P(X = 1\,|\,Y = -1) = \tfrac{2}{3}\,,$$
$$P(X = -1\,|\,Y = 0) = \tfrac{1}{2}\ ,\ P(X = 1\,|\,Y = 0) = \tfrac{1}{2}\,,$$
$$P(X = -1\,|\,Y = 1) = \tfrac{2}{3}\ ,\ P(X = 1\,|\,Y = 1) = \tfrac{1}{3}\ ,$$

Calculate $P(XY = 1)$.

A) $\frac{1}{18}$ B) $\frac{1}{9}$ C) $\frac{1}{6}$ D) $\frac{2}{9}$ E) $\frac{5}{18}$

2. The random variable X represents the loss amount given that a loss occurs. The insurer uses a model for X that is a continuous uniform distribution on the interval $[\,0\,,\,100,000\,]$. As part of the insurer's model, the insurer assumes that there is a 1% chance that a loss will occur. The insurance coverage pays 90% of the loss that is above 10,000. Calculate the expected payment by the insurer for a possible single occurrence of the loss (nearest 5).

A) 285 B) 305 C) 325 D) 345 E) 365

3. X_1, X_2 and X_3 are independent random variables each uniformly distributed on the interval $[0,1]$. Suppose that the three variables are arranged in increasing order $Y_1 \le Y_2 \le Y_3$ (Y_1 is the smallest of the three Xs, etc.). Calculate the variance of Y_2.

A) $\frac{1}{20}$ B) $\frac{1}{10}$ C) $\frac{1}{5}$ D) $\frac{1}{4}$ E) $\frac{1}{3}$

4. The random variable N represents the annual number of claims for a randomly selected insured. The random variable S is defined to be $S = X_1 + \cdots + X_N$, and the Xs are independent.
You are given:
$P(N = 0) = 0.8\,,\ P(N = 1) = 0.15\,,\ P(N = 2) = 0.05$ and
X has the following distribution: $P(X = 1) = 0.8\,,\ P(X = 2) = 0.15\,,\ P(X = 3) = 0.05$.
Calculate the probability $P(S < 4)$.

A) Less than 0.97 B) At least 0.97 but less than 0.98 C) At least 0.98 but less than 0.99
D) At least 0.99 but less than 1.0 E) 1.0

5. A fair die is tossed until the first 6 turns up. A record is kept of each toss until this happens.
The sum of all the numbers that have been tossed is calculated, including the first 6 and is denoted X.
Calculate $E(X)$. You are given that $\sum_{k=1}^{\infty}\left[k \times t^k\right] = \frac{t}{(1-t)^2}$ for $0 \le t < 1$.

A) 15 B) 18 C) 21 D) 24 E) 27

6. X_1, X_2 is a random sample from a discrete distribution with probability function
$P(X=1) = P(X=2) = \frac{1}{2}$.
$P_Y(t)$ is the probability generating function of $Y = \frac{X_1}{X_2}$. Calculate $P_Y(0)$.
A) -1 B) 0 C) 1 D) $\frac{3}{2}$ E) 2

7. X and Y are random variables which have a joint distribution with joint cdf $F(x,y) = y(x^2 + xy - y^2)$ for $0 \le y \le x \le 1$.
Calculate the conditional expectation of Y given $X = 0.5$.
A) $\frac{1}{18}$ B) $\frac{1}{9}$ C) $\frac{1}{6}$ D) $\frac{2}{9}$ E) $\frac{5}{18}$

8. You are given $P(A|B) = 0.5$, $P(B|A) = 0.4$, $P(A \cup B) = 0.7$.
Determine $P(A \cap B)$.
A) 0.05 B) 0.10 C) 0.15 D) 0.20 E) 0.25

9. A medical study has 1,200 participants. The study is looking at the relationship between
(i) gender: male vs female
(ii) time per day spent using social media: less than two hours vs two hours or more
(iii) weight: overweight vs not overweight
You are given the following information about the participants in the study:
520 are male, 420 spend less than two hours per day on social media, 480 are overweight,
100 are male and spend less than two hours per day on social media,
300 are male and overweight,
100 spend less than two hours per day on social media and are overweight
80 are male, spend less than two hours per day on social media and are overweight
How many participants are female, spend less than two hours per day on social media and are not overweight?
A) 280 B) 300 C) 320 D) 380 E) 400

10. You are given the following joint density function: $f(x,y) = 2e^{-(x+y)}$, $0 < x < y < \infty$.
Calculate $Cov(X+Y, X-Y)$.
A) -2 B) -1 C) 0 D) 1 E) 2