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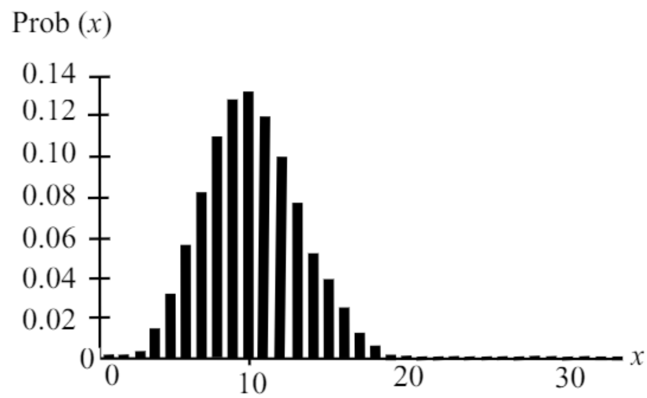
The mean in Example 6-5 is $\lambda = ln(10,000) = 9.21$. The histogram for this Poisson distribution is given below. As λ increases, the histogram becomes more bell-shaped.



Note that for the Poisson distribution with mean λ, we have the following relationship between successive probabilities: $P(X = n+1) = P(X = n) \times \frac{\lambda}{n+1}$.

For Exam P it is very important to be familiar with the binomial and Poisson distributions. The following distributions have all come up on the exam at one time or another as well, but not as often as the binomial and Poisson distributions.

Geometric distribution with parameter p $(0 \leq p \leq 1)$

Suppose that a single trial of an experiment results in either success with probability p, or failure with probability $1 - p = q$. The experiment is performed with successive independent trials until the first success occurs. **If X represents the number of failures until the first success**, then X is a discrete random variable that can be $0, 1, 2, 3, \ldots$. X is said to have a **geometric distribution with (probability of success) parameter p**. The probability that $X = 0$ is p, which is the probability that the first trial is a success. The probability that $X = 1$ is the probability that the first trial is a failure and the second trial is a success, which, because of independent trials, is $(1-p) \times p$. The probability that $X = 2$ is the probability that the first and second trials are both failures and the third trial is a success, which is $P[X = 2] = (1-p) \times (1-p) \times p = (1-p)^2 \times p$. Continuing in this way, we get that **the probability function for X is $P[X = x] = p(x) = (1-p)^x \times p$ for $x = 0, 1, 2, 3, \ldots$** .
The **mean and variance of X** are $\boldsymbol{E[X] = \frac{1-p}{p}} = \frac{q}{p}$, $\boldsymbol{Var[X] = \frac{1-p}{p^2}} = \frac{q}{p^2}$.
$M_X(t) = \frac{p}{1-(1-p)e^t}$ and $P_X(t) = \frac{p}{1-(1-p)t}$.
The geometric distribution has the lack of memory property, $P[X = n+k | X \geq n] = P[X = k]$.
An equivalent description of the geometric distribution states the mean $\beta = \frac{1-p}{p}$ as the parameter.

An alternative version of a geometric distribution is the random variable Y, which is the number of the trial on which the first success occurs. **Y is related to X just defined: $Y = X + 1$** and
$\boldsymbol{P[Y = y] = P[X = y-1] = (1-p)^{y-1} \times p}$ for $y = 1, 2, 3, \ldots$ **An example of this was seen with tossing a coin until a head turns up. Y is the toss number of the first head, X is the number of tails until the first head**: $E[Y] = E[X] + 1 = \frac{1}{p}$, $Var[Y] = Var[X] = \frac{1-p}{p^2}$.

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Example 6-6:

In tossing a fair die repeatedly (and independently on successive tosses), find the probability of getting the first "1" on the t-th toss. Find the expected number of tosses before the first "1" is tossed.

Solution:

The probability that the first "1" occurs on the first toss is $\frac{1}{6}$ (this is $P(Y=1)$ using the definition above). The probability that the first "1" occurs on the second toss is $(\frac{5}{6})(\frac{1}{6})$ (a toss other than "1" followed by a "1", this is $P(Y=2)$). The probability that the first "1" occurs on the t-th toss is $(\frac{5}{6})^{t-1}(\frac{1}{6}) = P(Y=t)$ ($t-1$ rolls other than "1" followed by a "1"). If we define tossing a "1" as a success, then the number of failures until the first success has a geometric distribution described as X above, with $p=\frac{1}{6}$:

$P[X=0]=\frac{1}{6}$, $P[X=1]=\frac{5}{6}\times\frac{1}{6}$, ..., $P[X=t]=(\frac{5}{6})^{t-1}\times\frac{1}{6}$. The mean of X is $E[X]=\frac{1}{p}-1=6-1=5$. We expect 5 "failures" until the first success.

If the question had asked us to find the expected toss number of the first "1", then this would be the version of the geometric distribution defined as Y above, with $p=\frac{1}{6}$. Then Y is the toss number at which the first "1" occurs. The mean of Y is $E[Y]=\frac{1}{p}=6$. This is reasonable, since there are six possible outcomes from the die toss, we would expect 6 tosses would be needed on average to get the first "1". □

Negative binomial distribution with parameters r and p ($r>0$ and $0<p\le 1$)

The probability function is

$$p(x)=\binom{r+x-1}{x}p^r(1-p)^x=\binom{r+x-1}{r-1}p^r(1-p)^x \text{ for } x=0,1,2,3,...,$$

The mean and variance and moment and probability generating functions are

$$E[X]=\frac{r(1-p)}{p}\ ,\ Var[X]=\frac{r(1-p)}{p^2}\ ,\ M_X(t)=\left[\frac{p}{1-(1-p)e^t}\right]^r\ ,\ P_X(t)=\left[\frac{p}{1-(1-p)t}\right]^r$$

If r is an integer, then the negative binomial random variable X can be interpreted as follows. Suppose that an experiment ends in either failure or success, and the probability of success for a particular trial of the experiment is p. Suppose further that the experiment is performed repeatedly (independent trials) until the r-th success occurs. If X is the number of failures until the r-th success occurs, then X has a negative binomial distribution with parameters r and p. The distribution is defined even if r is not an integer. Note that $r+x$ is the total number of trials until the r-th success. If r is not an integer then

$$\binom{r+x-1}{r-1}=\frac{(r+x-1)(r+x-2)\cdots(r+1)(r)}{x!}$$

The notation q is sometimes used to represent $1-p$.

The geometric distribution is a special case of the negative binomial with $r=1$.

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We can get some insight into the algebraic form of the negative binomial probability function by considering an example. Suppose that a fair die is tossed until the 3rd "1" turns up. We will define "success" to be a "1" turning up on a die toss, and "failure" will be the result that a "1" didn't turn up on the toss. Suppose that we wish to find the probability that there are (exactly) 2 failures before the 3rd success. In order for there to be 2 failures before the 3rd success, the 3rd success must be on the 5th toss, and of the previous 4 tosses there must be 2 failures and 2 successes. The number of non-"1"'s on the first 4 trials has a binomial distribution, with $n = 4$ and probability $\frac{5}{6}$ (we are assuming that the die is fair).

Therefore, the probability of 2 failures on the first 4 trials is $\binom{4}{2}(\frac{1}{6})^2(\frac{5}{6})^2$.

The probability of 2 failures in the first 4 trials followed by a success on the 5th trial is (because of independence)

$$\binom{4}{2}(\tfrac{5}{6})^2(\tfrac{1}{6})^2 \times \tfrac{1}{6} = \binom{4}{2}(\tfrac{5}{6})^2(\tfrac{1}{6})^3 .$$

This is the negative binomial probability function $P[X = 2]$ for $r = 3$, $p = \frac{1}{6}$.

More generally, in order to have x failures before the r-th success, we must have the r-th success occur on trial number $r + x$ (x failures and $r - 1$ successes in the first $x + r - 1$ trials, followed by a success on the r-th trial). Therefore, we must have x failures in the first $x + r - 1$ trials, followed by a success on trial number $x + r$. The number of failures in the first $x + r - 1$ trials has a binomial distribution, and the probability of x failures in the first $x + r - 1$ trials is $\binom{x+r-1}{x}(1-p)^x p^{r-1}$.

The probability of a success on the r-th trial is p, so the total probability of x failures before the r-th success is $\binom{x+r-1}{x}(1-p)^x p^{r-1} \times p = \binom{x+r-1}{x}(1-p)^x p^r$.

Keep in mind the binomial coefficient relationship $\binom{n}{k} = \binom{n}{n-k}$, which might be useful in some circumstances.

Example 6-7:

In tossing a fair die repeatedly (and independently on successive tosses), find the probability of getting the third "1" on the t-th toss.

Solution:

The negative binomial random variable X with parameters $r = 3$ and $p = \frac{1}{6}$ is the number of failures (failure means tossing 2,3,4,5 or 6) until the 3rd success. The probability that the 3rd success (3rd "1") occurs on the t-th toss is the same as the probability of $x = t - 3$ failures before the 3rd success. Thus, if X is the number of failures until the 3rd success, X has a negative binomial distribution with $r = 3$ and $p = \frac{1}{6}$. Then, the probability that $X = t - 3$ is

$$P[X = t - 3] = p(t-3) = \binom{r+x-1}{x} p^r (1-p)^x = \binom{3+t-3-1}{t-3}(\tfrac{1}{6})^3(\tfrac{5}{6})^{t-3}$$
$$= \binom{t-1}{t-3}(\tfrac{1}{6})^3(\tfrac{5}{6})^{t-3} = \tfrac{(t-1)!}{(t-3)!\cdot 2!}(\tfrac{1}{6})^3(\tfrac{5}{6})^{t-3} = \tfrac{(t-1)(t-2)}{2}(\tfrac{1}{6})^3(\tfrac{5}{6})^{t-3}. \quad \square$$

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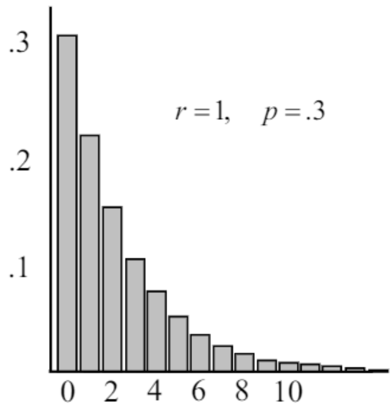
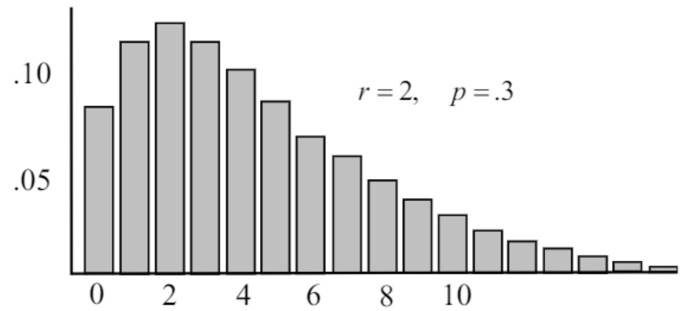
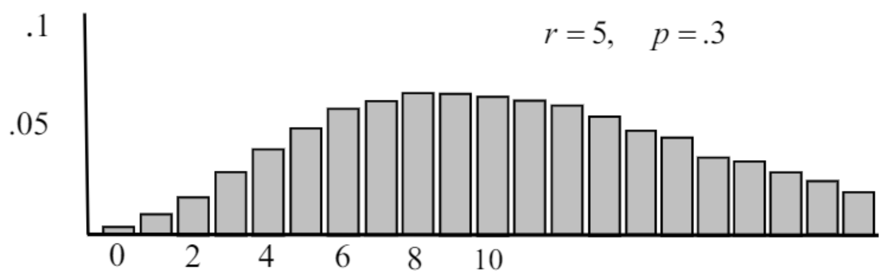
As mentioned above, the geometric distribution is a special case of the negative binomial with $r = 1$. The following graphs show the histogram for negative binomial distributions with $p = .3$ and (i) $r = 1$ (geometric) , (ii) $r = 2$, and (iii) $r = 5$.







Hypergeometric distribution with integer parameters M, K and n
($M > 0$, $0 \le K \le M$ and $1 \le n \le M$):

In a group of M objects, suppose that K are of Type I and $M - K$ are of Type II.

If a subset of n objects is randomly chosen without replacement from the group of M objects, let X denote the number that are of Type I in the subset of size n. X is said to have a hypergeometric distribution. X is a non-negative integer that satisfies $X \le n$, $X \le K$, $0 \le X$ and $n - (M - K) \le X$.

The probability function for X is $p(x) = \frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}}$.

x can't be larger than n or K, so $x \le min[n, K]$, and since there are $M - K$ Type II objects in total, x must be at least $n - (M - K)$.

The probability function is explained as follows. There are $\binom{M}{n}$ ways of choosing the subset of n objects from the entire group of M objects. The number of choices that result in x objects of Type I and $n - x$ objects of Type II is $\binom{K}{x}\binom{M-K}{n-x}$.

The mean and variance of X are $E[X] = \frac{nK}{M}$, $Var[X] = \frac{nK(M-K)(M-n)}{M^2 \cdot (M-1)}$.

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Example 6-8:

An urn contains 6 blue and 4 red balls. 6 balls are chosen at random and without replacement from the urn. If X is the number of red balls chosen, find the standard deviation of X.

Solution:

This is a hypergeometric distribution with $M = 10$, $K = 4$ and $n = 6$.

The probability function of X is $f(x) = \frac{\binom{4}{x}\binom{6}{6-x}}{\binom{10}{6}}$, for $x = 0, 1, 2, 3, 4$.

The variance is $Var[X] = \frac{nK(M-K)(M-n)}{M^2 \cdot (M-1)} = 0.64$. Standard deviation is $\sqrt{0.64} = 0.8$. □

The hypergeometric distribution has rarely come up on the exams that have been publicly released.

Multinomial distribution with parameters n, p_1, $p_2, \ldots, p_k$ (where n is a positive integer and $0 \leq p_i \leq 1$ for all $i = 1, 2, \ldots, k$ and $p_1 + p_2 + \cdots + p_k = 1$)

Suppose that an experiment has k possible outcomes, with probabilities p_1, $p_2, \ldots, p_k$ respectively. Each time the experiment is performed, it results in one of the possible outcomes.

If the experiment is performed n successive times (independently), let X_i denote the number of experiments that resulted in outcome i, so that $X_1 + X_2 + \cdots + X_k = n$.

The multinomial distribution probability function is

$$P[X_1 = x_1, X_2 = x_2, \cdots, X_k = x_k] = p(x_1, x_2, \ldots, x_k) = \frac{n!}{x_1! \times x_2! \times \cdots \times x_k!} \times p_1^{x_1} \times p_2^{x_2} \times \cdots \times p_k^{x_k}$$

For each i from $i = 1$ to $i = k$, X_i is a random variable with a mean and variance similar to the binomial mean and variance: $E[X_i] = np_i$, $Var[X_i] = np_i(1 - p_i)$.

Also, for i and j between 1 and n, X_i and X_j are related. A little later on in this study guide we will review joint distributions and relationships between random variables. One measure of the relationship between random variables is the covariance. For the multinomial distribution, the covariance between X_i and X_j is $Cov[X_i, X_j] = -np_ip_j$. Covariance will be covered in Section 8.

For example, the toss of a fair die results in one of $k = 6$ outcomes, with probabilities $p_i = \frac{1}{6}$ for $i = 1, 2, 3, 4, 5, 6$. If the die is tossed n times, then with $X_i = \#$ of tosses resulting in face "i" turning up, the distribution of $X_1, X_2, \ldots, X_6$ is a multinomial distribution. In $n = 10$ tosses of the die, the probability that there are exactly 2-"1"'s , 1-"2" , 0-"3"'s , 3-"4"'s , 1-"5" and 3-"6"'s is $\frac{10!}{2! \times 1! \times 0! \times 3! \times 1! \times 3!} \times (\frac{1}{6})^2 (\frac{1}{6})^1 (\frac{1}{6})^0 (\frac{1}{6})^3 (\frac{1}{6})^1 (\frac{1}{6})^3$.

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Recursive relationship for the binomial, Poisson and negative binomial:

The probability function for each of these three distributions satisfies the following recursive relationship $\frac{p_k}{p_{k-1}} = a + \frac{b}{k}$ for $k = 1, 2, 3, \dots$.

Poisson with parameter λ:

$$\frac{p_k}{p_{k-1}} = \frac{e^{-\lambda}\lambda^k/k!}{e^{-\lambda}\lambda^{k-1}/(k-1)!} = \frac{\lambda}{k} \rightarrow a = 0 \ , \ b = \lambda$$

Binomial with parameters n and p: $a = -\frac{p}{1-p}$, $b = \frac{(n+1)p}{1-p}$

Negative binomial with parameters r and p: $a = 1 - p$, $b = (r-1)(1-p)$

For instance, for a Poisson distribution with $\lambda = 2$, we have $a = 0$, $b = 2$, and

$$p_k = (a + \frac{b}{k}) \times p_{k-1} = \frac{2}{k} \cdot \frac{e^{-2} \cdot 2^{k-1}}{(k-1)!} = \frac{e^{-2} \cdot 2^k}{k!} \ .$$

In the released exams, the binomial, Poisson and geometric have come up regularly. The negative binomial arises occasionally, and the multinomial and hypergeometric have rarely occurred.

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SUMMARY OF DISCRETE DISTRIBUTIONS

Distribution	Parameters	Prob. Fn., $p(x)$	Mean, $E[X]$	Variance, $Var[X]$	$M_X(t)$, $P_X(t)$
Uniform	$N > 0$, integer	$\frac{1}{N}$, $x = 1, 2, ..., N$	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$	$\frac{e^t(e^{Nt}-1)}{N(e^t-1)}$, $\frac{t(t^N-1)}{N(t-1)}$
Binomial	$n > 0$ integer, $0 < p < 1$	$\binom{n}{x} p^x (1-p)^{n-x}$ $x = 0, 1, ..., n$	np	$np(1-p)$	$(1-p+pe^t)^n$, $(1-p+pt)^n$
Poisson	$\lambda > 0$	$\frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, ...$	λ	λ	$e^{\lambda(e^t-1)}$, $e^{\lambda(t-1)}$
Geometric	$0 < p < 1$	$(1-p)^x \times p$, $x = 0, 1, 2, ...$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^t}$, $\frac{p}{1-(1-p)t}$
Negative Binomial	$r > 0$, $0 < p < 1$	$\binom{r+x-1}{x} p^r (1-p)^x$, $x = 0, 1, 2, ...$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{p}{1-(1-p)e^t}\right]^r$, $\left[\frac{p}{1-(1-p)t}\right]^r$
Hypergeometric	$M > 0$, $0 \leq K \leq M$, $1 \leq n \leq M$, integers	$\frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}}$ $x \leq min[n, K]$	$\frac{nK}{M}$	$\frac{nK(M-K)(M-n)}{M^2(M-1)}$	
Multinomial	n, $p_1, p_2, ..., p_k$ $0 < p_i < 1$	$\frac{n!}{x_1! \cdot x_2! \cdots x_k!} \cdot p_1^{x_1} \cdot p_2^{x_2} \cdots p_k^{x_k}$ $x_1 + x_2 + \cdots + x_k = n$	$E[X_i] = np_i$	$Var[X_i] = np_i(1-p_i)$	

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PROBLEM SET 6

Frequently Used Discrete Distributions

1. X has a discrete uniform distribution on the integers $0, 1, 2, \ldots, n$ and Y has a discrete uniform distribution on the integers $1, 2, 3, \ldots, n$. Find $Var[X] - Var[Y]$.

A) $\frac{2n+1}{12}$ B) $\frac{1}{12}$ C) 0 D) $-\frac{1}{12}$ E) $-\frac{2n+1}{12}$

2. The probability that a particular machine breaks down in any day is .20 and is independent of the breakdowns on any other day. The machine can break down only once per day. Calculate the probability that the machine breaks down two or more times in ten days.

A) .1075 B) .0400 C) .2684 D) .6242 E) .9596

3. (SOA) A company prices its hurricane insurance using the following assumptions:

(i) In any calendar year, there can be at most one hurricane.

(ii) In any calendar year, the probability of a hurricane is 0.05 .

(iii) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

A) 0.06 B) 0.19 C) 0.38 D) 0.62 E) 0.92

4. (SOA) A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?

A) 0.096 B) 0.192 C) 0.235 D) 0.376 E) 0.469

5. (SOA) A hospital receives $1/5$ of its flu vaccine shipments from Company X and the remainder of its shipments from other companies. Each shipment contains a very large number of vaccine vials. For Company X's shipments, 10% of the vials are ineffective. For every other company, 2% of the vials are ineffective. The hospital tests 30 randomly selected vials from a shipment and finds that one vial is ineffective. What is the probability that this shipment came from Company X?

A) 0.10 B) 0.14 C) 0.37 D) 0.63 E) 0.86

6. (SOA) A company establishes a fund of 120 from which it wants to pay an amount, C, to any of its 20 employees who achieve a high performance level during the coming year. Each employee has a 2% chance of achieving a high performance level during the coming year, independent of any other employee. Determine the maximum value of C for which the probability is less than 1% that the fund will be inadequate to cover all payments for high performance.

A) 24 B) 30 C) 40 D) 60 E) 120

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7. Let X be a Poisson random variable with $E[X] = ln\,2$. Calculate $E[cos(\pi X)]$.

A) 0 B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1 E) $2\,ln\,2$

8. A manufacturing process has three quality control inspectors, each of whom inspect each manufactured product independently of one another. For an individual inspector, the probability of approving a product that is flawed is 0.02. A product is approved if an only if at least two inspectors approve it. Calculate the probability that a particular flawed product is approved.

A) Less than .0010 B) At least .0010 but less than .0011 C) At least .0011 but less than .0012 D) At least .0012 but less than .0013 E) At least .0013

9. (SOA) An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims. If the number of claims filed has a Poisson distribution, what is the variance of the number of claims filed?

A) $\frac{1}{\sqrt{3}}$ B) 1 C) $\sqrt{2}$ D) 2 E) 4

10. (SOA) An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. What is the expected benefit under this policy?

A) 2234 B) 2400 C) 2500 D) 2667 E) 2694

11. (SOA) A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists. Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 (ticket cost + 50 penalty) to the tourist. What is the expected revenue of the tour operator?

A) 935 B) 950 C) 967 D) 976 E) 985

12. A fair die is tossed until a 2 is obtained. If X is the number of trials required to obtain the first 2, what is the smallest value of x for which $P[X \leq x] \geq \frac{1}{2}$?

A) 2 B) 3 C) 4 D) 5 E) 6

13. (SOA) A large pool of adults earning their first driver's license includes 50% low-risk drivers, 30% moderate-risk drivers, and 20% high-risk drivers. Because these drivers have no prior driving record, an insurance company considers each driver to be randomly selected from the pool. This month, the insurance company writes 4 new policies for adults earning their first driver's license. What is the probability that these 4 will contain at least two more high-risk drivers than low-risk drivers?

A) 0.006 B) 0.012 C) 0.018 D) 0.049 E) 0.073

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14. A box contains 10 white and 15 black marbles. Let X denote the number of white marbles in a selection of 10 marbles selected at random and without replacement. Find $\frac{Var[X]}{E[X]}$.

A) $\frac{1}{8}$ B) $\frac{3}{16}$ C) $\frac{2}{8}$ D) $\frac{5}{16}$ E) $\frac{3}{8}$

15. A multiple choice test has 10 questions, and each question has 5 answer choices (exactly one of which is correct). A student taking the test guesses randomly on all questions. Find the probability that the student will actually get at least as many correct answers as she would expect to get with the random guessing approach.

A) 0.624 B) 0.591 C) 0.430 D) 0.322 E) 0.302

16. An analysis of auto accidents shows that one in four accidents results in an insurance claim. In a series of independent accidents, find the probability that the first accident resulting in an insurance claim is one of the first 3 accidents.

A) 0.50 B) 0.52 C) 0.54 D) 0.56 E) 0.58

17. An insurer has 5 independent one-year term life insurance policies. The face amount on each policy is 100,000. The probability of a claim occurring in the year for any given policy is 0.2. Find the probability the insurer will have to pay more than the total expected claim for the year.

A) 0.06 B) 0.11 C) 0.16 D) 0.21 E) 0.26

18. The number of claims per year from a particular auto insurance policy has a Poisson distribution with a mean of 1, and probability function p_k. Based on a number of years of experience, the insurer decides to change the distribution, so that the new probability of 0 claims is $p_0^* = 0.5$, and the new probabilities p_k^* for $k \geq 1$ are proportional to the old (Poisson) probabilities according to the relationship $p_k^* = c \cdot p_k$ for $k \geq 1$. Find the mean of the new claim number distribution.

A) 0.79 B) 0.63 C) 0.5 E) 0.37 E) 0.21

19. The probability generating function of a discrete non-negative integer valued random variable N is a function of the real variable t: $P(t) = \sum_{k=0}^{\infty} t^k \cdot P[N = k] = E[t^N]$.

Which of the following is the correct expression for the probability generating function of the Poisson random variable with mean 2?

A) e^{-2t} B) e^{1-2t} C) e^{2t} D) e^{2t-1} E) $e^{2(t-1)}$

20. An insurer issues two independent policies to individuals of the same age.
The insurer models the distribution of the completed number of years until death for each individual, and uses the geometric distribution $P[N = k] = (0.99)^k \times 0.01$, where $k = 0, 1, 2, \ldots$ and N is the completed number of years until death for each individual.
Find the probability that the two individuals die in the same year.

A) .001 B) .003 C) .005 D) .007 E) .009

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21. An insurer uses the Poisson distribution with mean 4 as the model for the number of warranty claims per month on a particular product. Each warranty claim results in a payment of 1 by the insurer. Find the probability that the total payment by the insurer in a given month is less than one standard deviation above the average monthly payment.

A) 0.9 B) 0.8 C) 0.7 D) 0.6 E) 0.5

22. As part of the underwriting process for insurance, each prospective policyholder is tested for high blood pressure. Let X represent the number of tests completed when the first person with high blood pressure is found. The expected value of X is 12.5.
Calculate the probability that the sixth person tested is the first one with high blood pressure.

A) 0.000 B) 0.053 C) 0.080 D) 0.316 E) 0.394

23. Let X be a random variable with moment generating function
$M(t) = \left(\frac{2+e^t}{3}\right)^9, \quad -\infty < t < \infty$
Calculate the variance of X.

A) 2 B) 3 C) 8 D) 9 E) 11

24. According to the house statistician, a casino estimates that it has a 51% chance of winning on any given hand of blackjack. The casino also assumes that blackjack hands are independent of one another. The casino randomly monitors its blackjack dealers, and as soon as a dealer is found to lose 5 hands in a row, the casino stops the game at that dealer's table and checks the deck of cards that the dealer is using. The casino has just started monitoring a dealer. What is the chance that the game will be stopped at the table sometime within the next 8 hands of blackjack?

A) 0.07 B) 0.09 C) 0.11 D) 0.13 E) 0.15

25. (SOA) A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is $\frac{3}{5}$. The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months. Calculate the probability that there will be at least four months in which no accidents occur before the fourth month in which at least one accident occurs.

A) 0.01 B) 0.12 C) 0.23 D) 0.29 E) 0.41

26. (SOA) Each time a hurricane arrives, a new home has a 0.4 probability of experiencing damage. The occurrences of damage in different hurricanes are independent. Calculate the mode of the number of hurricanes it takes for the home to experience damage from two hurricanes.

A) 2 B) 3 C) 4 D) 5 E) 6

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27. For a certain discrete random variable on the non-negative integers, the probability function satisfies the relationships

    $P(0) = P(1)$ and $P(k+1) = \frac{1}{k} \times P(k)$ for $k = 1, 2, 3, \ldots$ Find $P(0)$.

    A) $ln\,e$ B) $e-1$ C) $(e+1)^{-1}$ D) e^{-1} E) $(e-1)^{-1}$

28. (SOA) Let X represent the number of customers arriving during the morning hours and let Y represent the number of customers arriving during the afternoon hours at a diner. You are given

    i) X and Y are Poisson distributed.

    ii) The first moment of X is less than the first moment of Y by 8.

    iii) The second moment of X is 60% of the second moment of Y.

    Calculate the variance of Y.

    A) 4 B) 12 C) 16 D) 27 E) 35

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PROBLEM SET 6 SOLUTIONS

1. $Var[X] = \frac{(n+1)^2-1}{12}$ and $Var[Y] = \frac{n^2-1}{12}$ so that
$Var[X] - Var[Y] = \frac{2n+1}{12}$ Answer: A

2. Since the breakdowns from one day to another are independent, the number of breakdowns (successes) in 10 days, X, has a binomial distribution with $n = 10$ and $p = 0.2$.
$$P[X \geq 2] = 1 - (P[X=0] + P[X=1])$$
$$= 1 - \binom{10}{0}(0.2)^0(0.8)^{10} - \binom{10}{1}(0.2)^1(0.8)^9 = .6242\,.$$ Answer: D

3. The number of hurricanes N in 20 years will have a binomial distribution with
$n = 20$ (years) and $p = .05$ (chance of hurricane in any one year).
$P[N < 3] = P[N=0] + P[N=1] + P[N=2]$
$$= \binom{20}{0}(0.05)^0(0.95)^{20} + \binom{20}{1}(0.05)^1(0.95)^{19} + \binom{20}{2}(0.05)^2(0.95)^{18} = 0.9245$$
Note that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Answer: E

4. For each group, the number who complete the study is a binomial random variable with $n = 10$ trials (10 people in the group, each is a "trial") and probability $p = 0.8$ of any individual in the group completing the study (probability of success for one trial).

The probability of at least 9 completing the study in group 1 is $P[N=9] + P[N=10]$.
This is $\binom{10}{9}(0.8)^9(0.2)^1 + \binom{10}{10}(0.8)^{10}(0.2)^0 = (10)(0.13422)(.2) + (1)(0.10737) = 0.376$.
The probability that less than 9 complete the study in group 1 is $1 - 0.376 = 0.624$.
The same is true for group 2.
We are asked to find the probability that at least 9 participants complete the study in one of the two groups, but not in both of the groups. Since the two groups are independent, this will be
P[at least 9 complete study in group 1] $\times$ P[less than 9 complete study in group 2]
$+ P$[less than 9 complete study in group 1] $\times$ P[at least 9 complete study in group 2]
$= 0.376 \times 0.624 + 0.624 \times 0.376 = 0.469$. Answer: E

5. Event X - vaccine is from company X,
$\overline{X}$ - vaccine is from a company other than X. $P(X) = \frac{1}{5}$, $P(\overline{X}) = \frac{4}{5}$.
In a sample of 30 vials from Company X, the number of ineffective vials has a binomial distribution with $n = 30$ trials (vials) and probability $p = 0.10$ of any particular one being ineffective. In a sample of 30 vials from a Company other than X, the number of ineffective vials has a binomial distribution with $n = 30$ trials (vials) and probability $p = 0.02$ of any particular one being ineffective.

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Therefore, $P[1 \text{ ineffective}|\text{shipment is from Company } X] = \binom{30}{1}(0.1)^1(0.9)^{29} = 0.1413$, and
$P[1 \text{ ineffective}|\text{shipment is from Company other than } X] = \binom{30}{1}(0.02)^1(0.98)^{29} = 0.3340$.

We wish to find $P[\text{shipment is from Company } X|1 \text{ ineffective}]$

$$= \frac{P[(\text{shipment is from Company } X) \cap (1 \text{ ineffective})]}{P[1 \text{ ineffective}]}.$$

$P[(\text{shipment is from Company } X) \cap (1 \text{ ineffective})]$

$= P[1 \text{ ineffective}|\text{shipment is from Company } X] \times P[\text{shipment is from Company } X]$

$= (0.1413)(\frac{1}{5})$

$P[1 \text{ ineffective}] = P[(1 \text{ ineffective}) \cap X] + P[(1 \text{ ineffective}) \cap \overline{X}]$

$= P[(1 \text{ ineffective})|X] \times P[X] + P[(1 \text{ ineffective})|\overline{X}] \times P[\overline{X}] = (0.1413)(\frac{1}{5}) + (0.3340)(\frac{4}{5})$

Then, $P[\text{shipment is from Company } X|1 \text{ ineffective}] = \frac{(0.1413)(\frac{1}{5})}{(0.1413)(\frac{1}{5})+(0.3340)(\frac{4}{5})} = 0.096$

This can be described by the following probability table.

	Company X	Other Companies
	$P[X] = 0.2$, given	$P[X'] = 0.8$
1 Ineff	$P[1 \text{ Ineff}|X] = 0.1413$ given (calc. for binomial dist.)	$P[1 \text{ Ineff}|\text{other}] = 0.3340$ given (calc. for binomial dist.)
	$\Downarrow$	$\Downarrow$
	$P[1 \text{ Ineff} \cap X]$ $= (0.1413)(0.2)$	$P[1 \text{ Ineff} \cap \text{Other}]$ $= (0.3340)(0.8)$

$\Downarrow$

$P[\text{I ineff}] = (0.1413)(0.2) + (0.3340)(0.8)$

$\Downarrow$

$P[X|1 \text{ ineff}] = \frac{(01413)(0.2)}{(0.1413)(0.2)+(0.3340)(0.8)} = 0.096$ Answer: A

6. N is the number of people who achieve high performance. N has a binomial distribution with $n = 20$ and $p = 0.02$. We wish to find the largest C for which
$P[NC > 120] < 0.01$. From the binomial distribution, we have
$P[N = 0] = \binom{20}{0}(0.02)^0(0.98)^{20} = 0.6676$
$P[N = 1] = \binom{20}{1}(0.02)^1(0.98)^{19} = 0.2725$
$P[N = 2] = \binom{20}{2}(0.02)^2(0.98)^{18} = 0.0528$
Therefore, $P[N > 1] = 0.0599$, $P[N > 2] = 0.0071$.

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If $\frac{120}{C} \geq 2$ then $P[NC > 120] = P[N > \frac{120}{C}] \leq P[N > 2] = 0.0071 < 0.01$,
but if $\frac{120}{C} < 2$ then $P[NC > 120] = P[N > \frac{120}{C}] \leq P[N \geq 2] = 0.0599 > 0.01$.
In order to satisfy $P[NC > 120] < 0.01$ we must have $\frac{120}{C} \geq 2$, or equivalently, $C \leq 60$.

An alternative approach to this problem is to look at each possible value of C in the answers, and find the probability $P[NC > 120]$ for each. Starting with the largest possible value, we get for answer E,

$$P[120N > 120] = P[N > 1] = 1 - P[N = 0] - P[N = 1]$$
$$= 1 - 0.6676 - 0.2725 = 0.0599 > 0.01,$$

so $C = 120$ does not satisfy the probability requirement. Then for answer D, we get

$$P[60N > 120] = P[N > 2] = 1 - P[N = 0] - P[N = 1] - P[N = 2]$$
$$= 1 - 0.6676 - 0.2725 - 0.0528 = 0.0071 < 0.01,$$

so that $C = 60$ does satisfy the requirement. Answer: D

7. Since X has a Poisson distribution, it can take on the non-negative integer values $0, 1, 2, \ldots$ With $E[X] = ln(2)$, the probability function of X is $P[X = x] = \frac{e^{-ln(2)}[ln(2)]^x}{x!} = \frac{1}{2} \times \frac{[ln(2)]^x}{x!}$
The transformed random variable $cos(\pi X)$ can take on the values
$cos(0) = 1,\ cos(\pi) = -1,\ cos(2\pi) = 1,\ -1,\ 1,\ -1, \ldots$ Then

$$E[cos(\pi X)] = \sum_{x=0}^{\infty} cos(\pi x) \times \frac{1}{2} \cdot \frac{[ln(2)]^x}{x!} = \frac{1}{2} \times \sum_{x=0}^{\infty} (-1)^x \times \frac{[ln(2)]^x}{x!} = \frac{1}{2} \times \sum_{x=0}^{\infty} \frac{[-ln(2)]^x}{x!}$$
$$= \frac{1}{2} \times e^{-ln(2)} = \frac{1}{4} \quad \text{(we use the identity, } \sum_{x=0}^{\infty} \frac{a^x}{x!} = e^a\text{)}$$

Answer: B

8. A flawed product is approved if exactly two or if all three inspectors approve it. The number of inspectors approving a flawed product has a binomial distribution with $n = 3$ trials and probability $p = 0.02$ of approval per trial. The probability of exactly two approvals is $\binom{3}{2}(0.02)^2(0.98) = 0.001176$, and the probability of exactly three aprovals is $\binom{3}{3}(0.02)^3 = 0.000008$. The probability of at least two approvals is $0.001176 + 0.000008 = 0.001184$. Answer: C

9. N has a Poisson distribution with mean λ. $P[N = 2] = e^{-\lambda} \times \frac{\lambda^2}{2!}$ and $P[N = 4] = e^{-\lambda} \times \frac{\lambda^4}{4!}$. We are told that $P[N = 2] = 3P[N = 4]$, so that $e^{-\lambda} \times \frac{\lambda^2}{2} = 3e^{-\lambda} \times \frac{\lambda^4}{24} \Rightarrow \lambda = 2$.
The variance of the Poisson distribution is equal to the mean, $\lambda = 2$. Answer: D

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10. The probability that the device fails in year $n = 1, 2, 3, ...,$ is $(0.6)^{n-1}(0.4)$ ($n - 1$ years of non-failure followed year of failure).
This is a version of the geometric distribution.

Year of Failure	Prob.	Amount Paid
1	0.4	4000
2	$(0.6)(0.4) = 0.24$	3000
3	$(0.6)^2(0.4) = 0.144$	2000
4	$(0.6)^3(0.4) = 0.0864$	1000
5		0

The expected amount paid is

$$(4000)(0.4) + (3000)(0.24) + (2000)(0.144) + (1000)(0.0864) = 2694.4\,.$$

Answer: E

11. The tour operator collects 21 fares, $21 \times 50 = 1050$. Let N denote the number of ticket holders who show up. The tour operator does not have to make a refund if $N \leq 20$. If $N = 21$, the tour operator must pay 100. The number of ticket holders that show up has a binomial distribution based on $n = 21$ (ticket holders) and $p = .98$ (probability of any particular ticket holder showing up). Then
$P[N = 21] = \binom{21}{21}(0.98)^{21}(0.02)^0 = 0.65426.$
The expected amount the tour operator must pay in refund and penalty is
$0 \times P[N \leq 20] + 100 \times P[N = 21] = 100 \times 0.65426 = 65.43$.
The expected revenue (after refund and penalty) is $1050 - 65.43 = 984.57$. Answer: E

12. $P[X = k] = (\frac{5}{6})^{k-1} \times \frac{1}{6}$
$P[X \leq x] = \sum_{k=1}^{x} P[X = k] = \frac{1}{6} \times \left[\frac{1-(\frac{5}{6})^x}{1/6}\right] = 1 - (\frac{5}{6})^x \quad \frac{1}{2} \rightarrow (\frac{5}{6})^x \leq \frac{1}{2} \rightarrow x \geq 4.$
Note that $X - 1$ has a geometric distribution with $p = \frac{1}{6}$. Answer: C

13. This problem involves the multinomial distribution. The multinomial distribution with parameters n, p_1, $p_2, \ldots, p_k$ (where n is a positive integer and $0 \leq p_i \leq 1$ for all $i = 1, 2, ..., k$ and $p_1 + p_2 + \cdots p_k = 1$) is defined in the following way. Suppose that an experiment has k possible outcomes, with probabilities p_1, $p_2, ..., p_k$ respectively. If the experiment is performed n successive times (independently), let X_i denote the number of experiments that resulted in outcome i, so that $X_1 + X_2 + \cdots + X_k = n$.
The multinomial probability function is $f(x_1, x_2, ..., x_k) = \frac{n!}{x_1! \cdot x_2! \cdots x_k!} \times p_1^{x_1} \times p_2^{x_2} \times \cdots \times p_k^{x_k}$.

In this problem, the "experiment" outcome is the type of driver, which has three outcomes. These are "low-risk", with probability $p_1 = 0.5$, "moderate-risk", with probability $p_2 = 0.3$, and "high-risk", with probability $p_3 = 0.2$. The "experiment" (choosing a driver) is performed $n = 4$ times.

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We want to find the probability that $X_3 \geq X_1 + 2$ (number of high-risk drivers at least two more than the number of low-risk drivers). We can look at the outcomes that result in this event:

X_1, number of low-risk drivers	X_2, number of moderate-risk rivers	X_3, number of high-risk drivers
0	0	4
0	1	3
0	2	2
1	0	3

These are the only outcomes that result in this event.

$f(0,0,4) = \frac{4!}{0!\times 0!\times 4!} \times (0.5)^0 \times (0.3)^0 \times (0.2)^4 = 0.0016$

$f(0,1,3) = \frac{4!}{0!\times 1!\times 3!} \times (0.5)^0 \times (0.3)^1 \times (0.2)^3 = 0.0096$

$f(0,2,2) = \frac{4!}{0!\times 2!\times 2!} \times (0.5)^0 \times (0.3)^2 \times (0.2)^2 = 0.0216$

$f(1,0,3) = \frac{4!}{1!\times 0!\times 3!} \times (0.5)^1 \times 0(.3)^0 \times (0.2)^3 = 0.0160$

The total probability of this event is then $0.0016 + 0.0096 + 0.0216 + 0.0160 = 0.0488$.

Answer: D

14. X has a hypergeometric distribution with $M = 25$ marbles, $K = 10$ white marbles, and $n = 10$ marbles chosen. Then

$$E[X] = \frac{nK}{M} = \frac{10\times 10}{25} = 4,$$

and

$$Var[X] = \frac{nK(M-K)(M-n)}{M^2(M-1)} = \frac{3}{2} \ \Rightarrow\ \frac{Var[X]}{E[X]} = \frac{(M-K)(M-n)}{M(M-1)} = \frac{3}{8}$$

Answer: E

15. The probability of a correct guess is 0.2 on any particular question. The number of correct guesses forms a binomial distribution based on $n = 10$ trials (10 questions), with a probability of $p = 0.2$ of success (correct answer) on each trial. The expected number of correct guesses is $np = 10 \times 0.2 = 2$. The probability of getting at least 2 correct is $P[X \geq 2]$. The binomial distribution probability is $P[X = k] = \binom{n}{k}(0.2)^k(0.8)^{10-k}$. Then,

$P[X \geq 2] = 1 - (P[X = 0] + P[X = 1]) = 1 - \binom{10}{0}(0.2)^0(0.8)^{10} - \binom{10}{1}(0.2)^1(0.8)^9 = 0.624$

Answer: A

16. Out of 3 independent accidents, the number that result in a claim has a binomial distribution with $n = 3$ and $p = \frac{1}{4}$. The probability that none of the 3 accidents result in a claim is $\binom{3}{0}(\frac{1}{4})^0(\frac{3}{4})^3 = 0.422$. The probability that there is a least one claim in the 3 accidents is

$1 - 0.422 = 0.578$.

Alternatively, the probability that the first accident resulting in a claim is the k-th accident is $(0.75)^{k-1} \times 0.25$ (geometric distribution). Thus, the probability is

$(0.75)^0 \times 0.25 + (0.75)^1 \times 0.25 + (0.75)^2 \times 0.25 = 0.578$. Answer: E

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17. The expected claim from any one policy is $100,000 \times 0.2 = 20,000$, so the overall expected claim from all 5 policies is 100,000. The total claim for the year will be more than 100,000 if there are 2 or more claims. This probability is $P[N \geq 2] = 1 - P[N = 0] - P[N = 1]$, where N is the number of claims.
N has a binomial distribution with $n = 5$, $p = 0.2$.
$P[N = 0] + P[N = 1] = \binom{5}{0}(0.2)^0(0.8)^5 + \binom{5}{1}(0.2)^1(0.8)^4 = 0.73728$
$\Rightarrow P[N \geq 2] = 1 - 0.73728 = 0.26272.$ Answer: E

18. $1 = p_0^* + \sum_{k=1}^{\infty} p_k^*$, so that since $p_0^* = 0.5$, we must have $\sum_{k=1}^{\infty} p_k^* = c\sum_{k=1}^{\infty} p_k = 0.5$.
However, $1 = p_0 + \sum_{k=1}^{\infty} p_k = e^{-1} + \sum_{k=1}^{\infty} p_k \rightarrow \sum_{k=1}^{\infty} p_k = 1 - e^{-1} = 0.6321$,
so that $c = \frac{0.5}{0.6321} = 0.7910$. Then, the new expectation is
$\sum_{k=0}^{\infty} kp_k^* = \sum_{k=1}^{\infty} kp_k^* = \sum_{k=1}^{\infty} kcp_k = c\sum_{k=1}^{\infty} kp_k = c\sum_{k=0}^{\infty} kp_k = c \times \text{old expectation} = c \times 1 = 0.7910$.
Answer: A

19. $P(t) = \sum_{k=0}^{\infty} t^k \times P[N = k] = E[t^N] = \sum_{k=0}^{\infty} t^k \times e^{-2} \cdot \frac{2^k}{k!}$
$= \sum_{k=0}^{\infty} e^{-2} \times \frac{(2t)^k}{k!} = e^{-2} \times \sum_{k=0}^{\infty} \frac{(2t)^k}{k!} = e^{-2} \times e^{2t} = e^{2(t-1)}$. Answer: E

20. $P[\text{death in same year}] = \sum_{k=0}^{\infty} P[\text{both die after } k \text{ complete years}]$
$= \sum_{k=0}^{\infty} P[\text{person 1 dies after } k \text{ complete years}] \times P[\text{person 2 dies after } k \text{ complete years}]$
$= \sum_{k=0}^{\infty} (0.99)^k(0.01)(0.99)^k(0.01) = (0.0001)\sum_{k=0}^{\infty} [(0.99)^2]^k = \frac{0.0001}{1-(0.99)^2} = 0.005$. Answer: C

21. Average monthly payment is 4, variance is 4 (variance of Poisson is equal to mean).
Probability that total payment is less than $4 + 2 = 6$ is
$P[N \leq 5] = e^{-4}[1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!}] = 0.785$. Answer: B

22. This problem makes use of the geometric distribution. The experiment being performed is the blood pressure test on an individual. We define "success" of the experiment to mean that the individual has high blood pressure. We denote the probability of a success occurring in a particular trial by p. Since X is the number of persons tested until the first person with high blood pressure is found, it is like the version of the geometric distribution described as Y earlier in the study guide, where Y is the trial number of the first success (the trial number of the first success is 1, or 2, or 3, ...). The mean of this form of the geometric distribution is $\frac{1}{p}$, so that $\frac{1}{p} = 12.5$ and therefore $p = .08$. The probability that the first success occurs on the 6th trial (first case of high blood pressure is the 6th individual) is $(1-p)^5 p$, since there will be 5 failures and then the first success. This probability is $(0.92)^5 \times 0.08 = 0.0527$. Answer: B

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23. One of the applications of the moment generating function $M_X(t)$ for the random variable X is to calculate the moments of X - for an integer $k \geq 1$,
$E[X^k] = \frac{d^k}{dt^k} M_X(t)\Big|_{t=0}$. Therefore, $Var[X] = E[X^2] - (E[X])^2 = M_X^{(2)}(0) - [M_X'(0)]^2$.
In this problem, $M_X'(t) = 9\left(\frac{2+e^t}{3}\right)^8 (\frac{e^t}{3})$, so that $M_X'(0) = 3$, and
$M_X^{(2)}(t) = 72\left(\frac{2+e^t}{3}\right)^7 (\frac{e^t}{3})^2 + 9\left(\frac{2+e^t}{3}\right)^8 (\frac{e^t}{3})$, so that $M_X^{(2)}(0) = 11$,
and then, $Var[X] = 11 - 3^2 = 2$.
Alternatively, $Var[X] = \frac{d^2}{dt^2} [ln\, M_X(t)]\Big|_{t=0}$. In this problem, $ln\, M_X(t) = 9\, ln(2 + e^t) - 9\, ln\, 3$, so that

$$\frac{d}{dt} [ln\, M_X(t)] = \frac{9e^t}{2+e^t} \text{ , and } \frac{d^2}{dt^2} [ln\, M_X(t)] = \frac{(2+e^t)(9e^t)-(9e^t)(e^t)}{(2+e^t)^2},$$

and then

$$Var[X] = \frac{(3)(9)-(9)(1)}{(3)^2} = 2.$$

A much faster solution is based on the following fact. The moment generating function of the binomial random variable with parameters n (number of trials) and p (probability of success) is $(1 - p + pe^t)^n$. In this case, the mgf corresponds to the binomial distribution with $n = 9$ and $p = \frac{1}{3}$, and therefore the variance is $np(1-p) = 9 \times \frac{1}{3} \times \frac{2}{3} = 2$. Answer: A

24. The game will be stopped only under the following circumstances:
L L L L L , W L L L L L , L W L L L L L , W W L L L L L ,
L L W L L L L L , W L W L L L L L, L W W L L L L L, W W W L L L L L,
where W refers to win and L refers to loss. The sum of the probabilities is

$$(0.49)^5 + (0.51)(0.49)^5 + (0.49)(0.51)(0.49)^5 + (0.51)^2(0.49)^5 + (0.49)^2(0.51)(0.49)^5$$
$$+ (0.51)(0.49)(0.51)(0.49)^5 + (0.49)(0.51)^2(0.49)^5 + (0.51)^3(0.49)^5 = 0.0715.$$ Answer: A

25. We define the random variable X to be the number of months in which no accidents have occurred when the fourth month of accidents has occurred. We wish to find $P[X \geq 4]$. This can be written as
$1 - P[X = 0, 1, 2 \text{ or } 3]$.
$P[X = 0] = P[\text{first 4 months all have accidents}] = (\frac{3}{5})^4 = .01296$.
$P[X = 1] = P[$1 of the first 4 months has no accidents and 3 have accidents
and 5th month has accidents$] = \binom{4}{1}(\frac{2}{5})^1(\frac{3}{5})^3(\frac{3}{5}) = 4(\frac{2}{5})(\frac{3}{5})^4 = 0.2074$.
$P[X = 2] = P[$2 of the first 5 months has no accidents and 3 have accidents

and 6th month has accidents$] = \binom{5}{2}(\frac{2}{5})^2(\frac{3}{5})^3(\frac{3}{5}) = 10(\frac{2}{5})^2(\frac{3}{5})^4 = 0.2074$.
$P[X = 3] = P[$3 of the first 6 months has no accidents and 3 have accidents
and 7th month has accidents$] = \binom{6}{3}(\frac{2}{5})^3(\frac{3}{5})^3(\frac{3}{5}) = 20(\frac{2}{5})^3(\frac{3}{5})^4 = 0.1659$.
$P[X \geq 4] = 1 - 0.1296 - 0.2074 - 0.2074 - 0.1659 = 0.29$.
X has a negative binomial distribution with $r = 4$ and $p = \frac{3}{5}$. Answer: D

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