19. With a model population of 10,000, we have $\#B = 10,000 \times 0.05 = 500$ faulty components and $\#B' = 9,500$ working components. We also have $\#A \cap B = \#B \times P[A|B] = 475$ devices that are faulty and that test as faulty, and we have $\#A' \cap B' = \#B' \times P[A'|B'] = 9,025$ components that are working and do not test faulty. Therefore, there are
$\#A \cap B' = \#B' - \#A' \cap B' = 9,500 - 9,025 = 475$ components that are working and test faulty. The total number of components that test faulty is $\#A = \#A \cap B + \#A \cap B' = 950$.
The probability that a component is faulty given that it test faulty is the proportion $\frac{\#A \cap B}{A} = \frac{1}{2}$.
The conditional probability approach to solving the problem is as follows. We can calculate entries in the following table in the order indicated.

	A	A'
B	$P[A\|B] = 0.95$ (given)	
$P[B] = 0.05$ (given)	**1.** $P[A \cap B] = P[A\|B] \cdot P[B] = 0.0475$	
B'	**3.** $P[A \cap B']$	**2.** $P[A' \cap B']$
$P[B']$ $= 1 - P[B]$ $= 0.95$	$= P[B'] - P[A' \cap B']$ $= 0.0475$	$= P[A'\|B'] \cdot P[B']$ $= 0.9025$

4. $P[A] = P[A \cap B] + P[A \cap B'] = 0.095$

5. $P[B|A] = \frac{P[B \cap A]}{P[A]} = \frac{0.0475}{0.095} = 0.5$. Answer: E

20. $P[X = 1|X \leq 1] = \frac{P[X=1]}{P[X=0]+P[X=1]} = \frac{e^{-\lambda} \times \lambda^1/1!}{(e^{-\lambda} \times \lambda^0/0!)+(e^{-\lambda} \times \lambda^1/1!)} = \frac{\lambda}{\lambda+1} = 0.8$
$\rightarrow \lambda = 4$ Answer: A

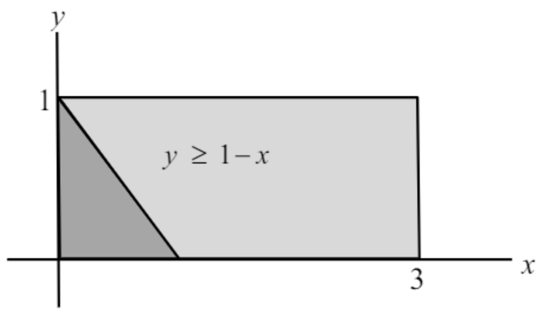
21. $f_X(x) = \int_0^1 (x + y)\, dy = x + \frac{1}{2}$ Answer: E

22. The variance of a uniform random variable on the interval $[a, b]$ is $\frac{(b-a)^2}{12}$.

$$\begin{aligned} Var[X_1 + 2X_2 - X_3] &= Var[X_1] + 4Var[X_2] + Var[X_3] \\ &\quad + 2 \times 2Cov[X_1, X_2] - 2Cov[X_1, X_3] - 2 \times 2Cov[X_2, X_3] \\ &= \frac{1}{12} + \frac{4}{12} + \frac{1}{12} + \frac{4}{24} - \frac{2}{24} - \frac{4}{24} = \frac{5}{12} \end{aligned}$$

Answer: C

23. The region of probability is the lightly shaded region below. It is the complement of the darkly shaded region. The probability of the darkly shaded region is $\int_0^1 \int_0^{1-y} f(x,y)\,dx\,dy$.
Therefore, the probability of the lighter region is $1 - \int_0^1 \int_0^{1-y} f(x,y)\,dx\,dy$.



Answer: D

24. We are given $P[A] = 0.25$, $P[B] = 0.2$, $P[C] = 0.1$, $P[B \cap C] = 0$, and from independence, we have $P[A \cap B] = 0.25 \times 0.2 = 0.05$, $P[A \cap C] = 0.25 \times 0.1 = 0.025$.
Using the probability rule for the union of events, we have
$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$.
Since no people have both genes B and C, it is also true that no one has all three genes.
Thus, $P[A \cup B \cup C] = 0.25 + 0.2 + 0.1 - 0.05 - 0.025 - 0 + 0 = 0.475$. Answer: B

25. In order to be a probability distribution, we must have $\Sigma\Sigma\Sigma p(x,y,z) = 1$:
$k[0+1+2+3+4+5+1+2+3+4+5+6+2+3+4+5+6+7] = 1$
$\rightarrow k = \frac{1}{63}$. Given that $X = 0$, the conditional distribution of Y and Z is $p(y,z|X=0) = \frac{p(0,y,z)}{p_X(0)}$
$p(0,y,z) = \frac{1}{63}(2y+z)$, and
$p_X(0) = \sum_y \sum_z p(0,y,z) = \frac{1}{63}(0+2+4+1+3+5+2+4+6) = \frac{3}{7}$
The conditional probabilities for Y, Z are

(y,z)	$(0,0)$	$(1,0)$	$(2,0)$	$(0,1)$	$(1,1)$	$(2,1)$	$(0,2)$	$(1,2)$	$(2,2)$
$p(y,z\|X=0)$	0	$\frac{2/63}{3/7}$	$\frac{4/63}{3/7}$	$\frac{1/63}{3/7}$	$\frac{3/63}{3/7}$	$\frac{5/63}{3/7}$	$\frac{2/63}{3/7}$	$\frac{4/63}{3/7}$	$\frac{6/63}{3/7}$
Number of unreimbursed accidents	0	0	0	0	0	1	0	1	2

Expected number of unreimbursed accidents is $1 \times \frac{5/63}{3/7} + 1 \times \frac{4/63}{3/7} + 2 \times \frac{6/63}{3/7} = \frac{7}{9}$.
Answer: E

26. $E[X] = \int_0^2 x \times \frac{x}{2}\,dx = \frac{4}{3}$. Then, $X - E[X] = X - \frac{4}{3}$, which is negative for $0 \le x \le \frac{4}{3}$ and is positive for $\frac{4}{3} \le x \le 2$. Thus, $|X - E[X]| = \frac{4}{3} - X$ if $0 \le X \le \frac{4}{3}$ and $|X - E[X]| = X - \frac{4}{3}$ if $\frac{4}{3} \le \mathrm{x} \le 2$. Then,
$E[\,|X - E[X]|\,] = \int_0^{4/3} (\frac{4}{3} - x) \times \frac{x}{2} dx + \int_{4/3}^2 (x - \frac{4}{3}) \times \frac{x}{2} dx = \frac{32}{81}$. Answer: C

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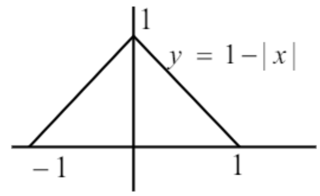
27. The marginal distribution of X is

$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$

$\int_0^{1-|x|} 1\,dy = 1-|x|$ for $-1 \le x \le 1$.

$E[X] = \int_{-1}^{1} x(1-|x|)\,dx$

$= \int_{-1}^{0} x(1+x)\,dx + \int_0^1 x(1-x)\,dx = 0,$



$E[X^2] = \int_{-1}^{1} x^2(1-|x|)\,dx = \int_{-1}^{0} x^2(1+x)\,dx + \int_0^1 x^2(1-x)\,dx$

$= \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$. $Var[X] = E[X^2] - (E[X])^2 = \frac{1}{6}$. Answer: B

28. The probability of tossing a total of 2 is $p = \frac{1}{36}$ (both dice have to turn up "1"). The number of "2"s occurring in $n = 2160$ tosses of the dice has a binomial distribution with mean $np = 60$ and variance $np(1-p) = 58.33$. Applying the normal approximation with continuity correction, we have

$P[X < 55] = P\left[\frac{X-60}{\sqrt{58.33}} < \frac{54.5-60}{\sqrt{58.33}}\right] = \Phi(-0.72) = 1 - \Phi(0.72) = 1 - 0.7642 = 0.2358.$ Answer: A

29. $T =$ time of husband's death , $U =$ time of wife's death , $W = max(T,U)$.

$Cov(T,W) = E[TW] - E[T]E[W]$. $E[T] = 20$, expectation of uniform $[0,40]$.

$$F_W(t) = P[W \le t] = P[(T \le t) \cap (U \le t)] = P[T \le t] \times P[U \le t]$$
$$= \frac{t}{40} \times \frac{t}{40} = \frac{t^2}{1600} \text{ for } 0 \le t \le 40$$

The pdf of W is $f_W(t) = F_W'(t) = \frac{t}{800}$, and $E[W] = \int_0^{40} t \times \frac{t}{800}\,dt = \frac{80}{3}$.

Alternatively,

$$E[W] = E[max(T,U)] = \int_0^{40}\int_0^{40} max(t,u) \times \frac{1}{40} \times \frac{1}{40})\,du\,dt$$
$$= \int_0^{40}\int_0^{t} t \times \frac{1}{1600}\,du\,dt + \int_0^{40}\int_t^{40} u \times \frac{1}{1600}\,du\,dt = \frac{40}{3} + \frac{40}{3} = \frac{80}{3}.$$

Let $h(t,u) = t \times max(t,u)$. Then,

$$E[TW] = E[h(T,U)] = \int_0^{40}\int_0^{40} t \times max(t,u) \times \frac{1}{40} \times \frac{1}{40}\,du\,dt$$
$$= \int_0^{40}\int_0^{t} t^2 \times \frac{1}{1600}\,du\,dt + \int_0^{40}\int_t^{40} tu \times \frac{1}{1600}\,du\,dt = 400 + 200 = 600$$

$Cov(T,W) = 600 - 20 \times \frac{80}{3} = \frac{200}{3}$. Answer: C

30. The density function for the time of breakdown t for a particular transistor is ce^{-ct} for $t > 0$. Thus, the cumulative distribution function for the break down time of transistor k is $P[T_k \leq t] = \int_0^t ce^{-cs}\,ds = 1 - e^{-ct}$ for $t > 0$. Let W denote the break down time of the last (10th) transistor. Then the event that the 10th transistor breaks down by time t is equivalent to the event that all transistors break down by time t (if the last one breaks down by time t then all the others have already broken down by that time). Thus,

$$\begin{aligned} G(t) = P[W \leq t] &= P[(T_1 \leq t) \cap (T_2 \leq t) \cap \cdots \cap (T_{10} \leq t)] \\ &= P[T_1 \leq t] \times P[T_2 \leq t] \times \cdots \times P[T_{10} \leq t] = (1 - e^{-ct})^{10} \end{aligned}$$

The second last equality is a consequence of the assumption of independence of the T_k's. Thus, the density function of W is $g(t) = G'(t) = 10(1 - e^{-ct})^9 ce^{-ct}$. Answer: A

PRACTICE EXAM 3

1. Let X_1 and X_2 form a random sample from a Poisson distribution. The Poisson distribution has a mean of 1. If $Y = min[X_1, X_2]$, then $P[Y{=}1] =$

A) $\frac{2e-1}{e^2}$ B) $\frac{2e-3}{e^2}$ C) $\frac{e-1}{e}$ D) $\frac{3-e}{e}$ E) $\frac{1}{e}$

2. X and Y are random losses with the following joint density function:

$f(x,y) = \frac{3}{4}x$ for $0 < x < y < 2$, and 0 elsewhere. Find the probability that the total loss $X+Y$ is no greater than 2.

A) $\frac{1}{12}$ B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) $\frac{1}{3}$ E) $\frac{1}{2}$

3. An insurer classifies flood hazard based on geographical areas, with hazard categorized as low, medium and high. The probability of a flood occurring in a year in each of the three areas is

Area Hazard	low	medium	high
Prob. of Flood	0.001	0.02	0.25

The insurer's portfolio of policies consists of a large number of policies with 80% low hazard policies, 18% medium hazard policies and 2% high hazard policies. Suppose that a policy had a flood claim during a year. Find the probability that it is a high hazard policy.

A) 0.50 B) 0.53 C) 0.56 D) 0.59 E) 0.62

4. Let X and Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} 6x \text{ for } 0<x<y<1 \\ 0, \text{otherwise} \end{cases}.$$

Note that $E[X] = \frac{1}{2}$ and $E[Y] = \frac{3}{4}$. What is $Cov[X,Y]$?

A) $\frac{1}{40}$ B) $\frac{1}{20}$ C) $\frac{1}{10}$ D) $\frac{1}{5}$ E) 1

5. A marketing survey indicates that 60% of the population owns an automobile, 30% owns a house, and 20% owns both an automobile and a house. Calculate the probability that a person chosen at random owns an automobile or a house, but not both.

A) 0.4 B) 0.5 C) 0.6 D) 0.7 E) 0.9

6. The number of injury claims per month is modeled by a random variable N with

$$P[N=n] = \frac{1}{(n+1)(n+2)}, \quad \text{where } n \geq 0.$$

Determine the probability of at least one claim during a particular month, given that there have been at most four claims during that month.

A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{1}{2}$ D) $\frac{3}{5}$ E) $\frac{5}{6}$

7. A survey of a large number randomly selected males over the age of 50 shows the following results:

- the proportion found to have diabetes is 0.02
- the proportion found to have heart disease is 0.03
- the proportion having neither heart disease nor diabetes is 0.96.

Find the proportion that have both diabetes and heart disease.

A) 0 B) 0.001 C) 0.006 D) 0.01 E) 0.05

8. Customers at Fred's Cafe win a 100 dollar prize if their cash register receipts show a star on each of the five consecutive days Monday,..., Friday in any one week. The cash register is programmed to print stars on a randomly selected 10% of the receipts. If Mark eats at Fred's once each day for four consecutive weeks and the appearance of stars is an independent process, what is the standard deviation of X, where X is the number of dollars won by Mark in the four-week period?

A) 0.61 B) 0.62 C) 0.63 D) 0.64 E) 0.65

9. Let (X,Y) have joint density function $f(x,y) = \begin{cases} 2 \text{ for } 0<x<y<1 \\ 0, \text{otherwise} \end{cases}$.

For $0 < x < 1$, what is $Var[Y|X = x]$?

A) $\frac{1}{18}$ B) $\frac{(1-x)^2}{12}$ C) $\frac{1+x}{2}$ D) $\frac{1}{3}$

E) Cannot be determined from the given information

10. A health insurer finds that health claims for an individual in a one year period are random and depend upon whether or not the individual is a smoker. For a smoker, the expected health claim in a year is \$500 with a standard deviation of \$200, and for a non-smoker, the expected health claim is \$200 with a standard deviation of \$100. The insurer estimates that 30% of the population are smokers. The insurer accepts a group health insurance policy with a large number of members in the group. Find the standard deviation for the aggregate claims for a randomly selected member of the group.

A) 184.1 B) 186.8 C) 189.5 D) 192.1 E) 194.7

11. At a certain large university the weights of male students and female students are approximately normally distributed with means and standard deviations of (180,20) and (130,15), respectively. If a male and female are selected at random, what is the probability that the sum of their weights is less than 280?

A) 0.1587 B) 0.1151 C) 0.0548 D) 0.0359 E) 0.0228

12. A loss distribution is uniformly distributed on the interval from 0 to 100.

Two insurance policies are being considered to cover part of the loss.

Insurance policy 1 insures 80% of the loss.

Insurance policy 2 covers the loss up to a maximum insurance payment of $L < 100$.

Both policies have the same expected payment by the insurer.

Find the ratio

$$\frac{Var[\text{insurer payment under policy 2}]}{Var[\text{insurer payment under policy 1}]} \text{ (nearest .1).}$$

A) 1.5 B) 1.2 C) 0.9 D) 0.6 E) 0.3

13. The random variable X has an exponential distribution with mean $\frac{1}{b}$.

It is found that $M_X(-b^2) = 0.2$.

Find b.

A) 1 B) 2 C) 3 D) 4 E) 5

14. An inspector has been informed that a certain gambling casino uses a "fixed" deck of cards one-quarter of the time in its blackjack games. With a fair deck, the probability of the casino winning a particular hand of blackjack is 0.52, but with a fixed deck the probability of the casino winning a particular hand is 0.75. The inspector visits the casino and plays 3 games of blackjack (from the same deck of cards), losing all of them. Find the conditional probability that the deck was fixed given that the inspector lost all 3 games.

A) 0 B) 0.25 C) 0.50 D) 0.75 E) 1

15. Let X and Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} xy \text{ for } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1 \\ 0, \text{otherwise} \end{cases}.$$

What is $P[\frac{X}{2} \leq Y \leq X]$?

A) $\frac{3}{32}$ B) $\frac{1}{8}$ C) $\frac{1}{4}$ D) $\frac{3}{8}$ E) $\frac{3}{4}$

16. The life (in days) of a certain machine has an exponential distribution with a mean of 1 day. The machine comes supplied with one spare. Find the density function (t measure in days) of the combined life of the machine and its spare if the life of the spare has the same distribution as the first machine, but is independent of the first machine.

A) te^{-t} B) $2e^{-t}$ C) e^{-t} D) $(t-1)e^{-t}$ E) $2te^{-t}$

17. An insurer finds that the time until occurrence of a claim from its property insurance division is exponentially distributed with a mean of 1 unit of time, and the time until occurrence of a claim from its life insurance division is exponentially distributed with a mean of 2 units of time. Claims occur independently in the two divisions. Find the expected time until the first claim occurrence, property or life.

A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$ E) $\frac{5}{6}$

18. A factory makes three different kinds of bolts: Bolt A, Bolt B and Bolt C. The factory produces millions of each bolt every year, but makes twice as many of Bolt B as it does Bolt A. The number of Bolt C made is twice the total of Bolts A and B combined. Four bolts made by the factory are randomly chosen from all the bolts produced by the factory in a given year. Which of the following is most nearly equal to the probability that the sample will contain two of Bolt B and two of Bolt C?

A) $\frac{8}{243}$ B) $\frac{96}{625}$ C) $\frac{384}{2410}$ D) $\frac{32}{243}$ E) $\frac{1}{6}$

19. Events X, Y and Z satisfy the following relationships:

$X \cap Y' = \phi$, $Y \cap Z' = \phi$, $P(X' \cap Y) = a$, $P(Y' \cap Z) = b$, $P(Z) = c$.

Find $P(X)$ in terms of a, b and c.

A) $a+b+c$ B) $a+b-c$ C) $c+b-a$ D) $c+a-b$ E) $c-b-a$

20. Let Z_1, Z_2, Z_3 be independent random variables each with mean 0 and variance 1, and let $X = 2Z_1 - Z_3$ and $Y = 2Z_2 + Z_3$. What is ρ_{XY}?

A) -1 B) $-\frac{1}{3}$ C) $-\frac{1}{5}$ D) 0 E) $\frac{3}{5}$

21. Suppose that X has a binomial distribution based on 100 trials with a probability of success of 0.2 on any given trial. Find the approximate probability $P[15 \leq X \leq 25]$ using the integer correction.

A) 0.17 B) 0.34 C) 0.50 D) 0.67 E) 0.83

22. The model for the amount of damage to a particular property during a one-month period is as follows: there is a .99 probability of no damage, there is a 0.01 probability that damage will occur, and if damage does occur, it is uniformly distributed between 1000 and 2000. An insurance policy pays the amount of damage up to a policy limit of 1500. It is later found that the original model for damage when damage does occur was incorrect, and should have been uniformly distributed between 1000 and 5000. Find the amount by which the insurer's expected payment was understated when comparing the original model with the corrected model.

A) $\frac{11}{16}$ B) $\frac{13}{16}$ C) $\frac{15}{16}$ D) $\frac{17}{16}$ E) $\frac{19}{16}$

23. A coin is twice as likely to turn up tails as heads. If the coin is tossed independently, what is the probability that the third head occurs on the fifth toss?

A) $\frac{8}{81}$ B) $\frac{40}{243}$ C) $\frac{16}{81}$ D) $\frac{80}{243}$ E) $\frac{3}{5}$

24. The repair costs for boats in a marina have the following characteristics:

Boat Type	Number of boats	Probability that repair is needed	Mean of repair cost given a repair	Variance of repair cost given a repair
Power Boats	100	0.3	300	10,000
Sailboats	300	0.1	1000	400,000
Luxury Yachts	50	0.6	5000	2,000,000

At most one repair is required per boat each year. The marina budgets an amount, Y, equal to the aggregate mean repair costs plus the standard deviation of the aggregate repair costs. Calculate Y.

A) 200,000 B) 210,000 C) 220,000 D) 230,000 E) 240,000

25. A writes to B and does not receive an answer. Assuming that one letter in n is lost in the mail, find the chance that B received the letter. It is to be assumed that B would have answered the letter if he had received it.

A) $\frac{n}{n-1}$ B) $\frac{n-1}{n^2}+\frac{1}{n}$ C) $\frac{n-1}{2n-1}$ D) $\frac{n}{2n-1}$ E) $\frac{n-1}{n^2}$

26. A study is done of people who have been charged by police on a drug-related crime in a large urban area. A conviction must take place in order for there to be a sentence of jail time.

The following information is determined:

(a) 75% are convicted.

(b) 10% of those convicted actually did not commit the crime.

(c) 25% of those not convicted actually did commit the crime.

(d) 2% of those who actually did not commit the crime are jailed.

(e) 20% of those who actually did commit the crime are not jailed.

Find the probability that someone charged with a drug-related crime who is convicted but not sentenced to jail time actually did not commit the crime.

A) 0.35 B) 0.40 C) 0.45 D) 0.50 E) 0.55

27. An insurance policy covers losses incurred by Jim and Bob who work at ABC Company. Jim and Bob each have a probability of 40% of incurring a loss during a year, and their losses are independent of one another. Jim is allowed at most one loss per year, and so is Bob. The policy reimburses the full amount of the total losses of Jim and Bob combined up to an annual maximum of 8000. If Jim has a loss, the amount is uniformly distributed on [1000, 5000], and the same is true for Bob. Given that Jim has incurred a loss in excess of 2000, determine the probability that total losses will exceed reimbursements made by the policy.

A) $\frac{1}{20}$ B) $\frac{1}{15}$ C) $\frac{1}{10}$ D) $\frac{1}{8}$ E) $\frac{1}{6}$

28. Let Y be a continuous random variable with cumulative distribution function

$$F(y) = \begin{cases} 0 \text{ for } y \le a \\ 1 - e^{-\frac{1}{2}(y-a)^2} \text{ otherwise} \end{cases},$$

where a is a constant. Find the 75th percentile of Y.

A) $F(.75)$ B) $a - \sqrt{2\,ln\,2}$ C) $a + \sqrt{2\,ln\,2}$ D) $a - 2\sqrt{ln\,2}$ E) $a + 2\sqrt{ln\,2}$

29. Let X be a continuous random variable with density function $f(x) = \begin{cases} 2x^{-2} \text{ for } x \ge 2 \\ 0, \text{otherwise} \end{cases}$.

Determine the density function of $Y = \frac{1}{X-1}$ for $0 < y \le 1$.

A) $\frac{1}{y^2}$ B) $\frac{2}{(y+1)^2}$ C) $\frac{2}{(y-1)^2}$ D) $2(\frac{y}{y+1})^2$ E) $2(\frac{y+1}{y})^2$

30. X and Y are loss random variables, with X discrete and Y continuous. The joint density function of X and Y is $f(x,y) = \frac{(x+1)e^{-y/2}}{12}$ for $x = 0, 1, 2$ and $0 < y < \infty$.

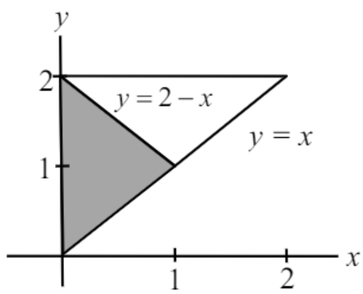
Find the probability that the total loss, $X + Y$ is less than 2.

A) $\frac{1}{6}(3 - 2e^{-1/2} - e^{-1})$ B) $\frac{1}{6}(3 - e^{-1/2} - 2e^{-1})$ C) $\frac{1}{3}(3 - 2e^{-1/2} - e^{-1})$
D) $\frac{1}{3}(3 - e^{-1/2} - e^{-1})$ E) $\frac{1}{2}(3 - 2e^{-1/2} - e^{-1})$

PRACTICE EXAM 3 - SOLUTIONS

1. $$\begin{aligned} P[Y=1] &= P[(X_1=1)\cap(X_2\geq 1)] + P[(X_2=1)\cap(X_1\geq 2)] \\ &= P[X_1=1]\times P[X_2\geq 1] + P[X_2=1]\times P[X_1\geq 2] \\ &= P[X_1=1]\times\left(1-P[X_2=0]\right) + P[X_2=1]\times\left(1-P[X_1\leq 1]\right) \\ &= e^{-1}(1-e^{-1}) + e^{-1}(1-2e^{-1}) \;=\; 2e^{-1}-3e^{-2} \;=\; \tfrac{2e-3}{e^2}. \end{aligned}$$ Answer: B

2. The event $X+Y\leq 2$ is equivalent to the event $Y\leq 2-X$. The region of probability is shaded in the graph at the right. The probability is found by integrating the joint density function over the two-dimensional region. $\int_0^1\int_x^{2-x}\frac{3}{4}x\,dy\,dx = \int_0^1\frac{3}{4}x(2-2x)\,dx = \frac{1}{4}$.



Answer: C

3. This is a classical Bayesian probability situation. Let C denote the event that a flood claim occurred. We wish to find $P(H|C)$. With a model populatiojn of 10,000 we have $\#L = 8,000$, $\#M = 1800$ and $\#H = 200$. Also, $\#C\cap L = \#L\times P(C\cap L) = 8$, and similarly, $\#C\cap M = 36$ and $\#C\cap H = 50$. The probability that a policy is high hazard given that there was a claim is $\frac{\#C\cap H}{C} = \frac{50}{8+36+50} = \frac{50}{94} = 0.532$.

The conditional probability approach to solving the problem is as follows.

We can summarize the information in the following table, with the order of calculations indicated.

	L, $P(L)=0.8$ (given)	M, $P(M)=0.18$ (given)	H, $P(H)=0.02$ (given)
C	$P(C|L)=0.001$ (given)	$P(C|M)=0.02$ (given)	$P(C|H)=0.25$ (given)
	1. $P(C\cap L)$ $=P(C|L)\times P(L)$ $=0.0008$	**2.** $P(C\cap M)$ $=P(C|M)\times P(M)$ $=0.0036$	**3.** $P(C\cap H)$ $=P(C|H)\times P(H)$ $=0.005$

4. $P(C) = P(C\cap L)+P(C\cap M)+P(C\cap H) = 0.0094$.

5. $P(H|C) = \frac{P(H\cap C)}{P(C)} = \frac{0.005}{0.0094} = 0.532$. Answer: B

4. $Cov[X,Y] = E[XY] - E[X] \times E[Y]$

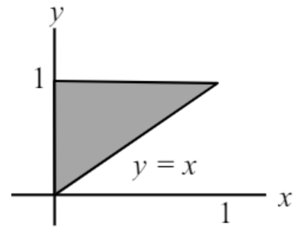
The region of probability is the triangle above the line $y = x$ in the unit square $0 \le x \le 1$, $0 \le y \le 1$.

$E[XY] = \int_0^1 \int_0^y xy \times 6x \, dx \, dy = \frac{2}{5}$

$\rightarrow Cov[X,Y] = \frac{2}{5} - \frac{1}{2} \times \frac{3}{4} = \frac{1}{40}$.

Alternatively,

$E[XY] = \int_0^1 \int_x^1 xy \times 6x \, dy \, dx = \frac{2}{5}$.



Answer: A

5. We identify the events A and H: $A =$ a randomly chosen person owns an automobile,
$H =$ a randomly chosen person owns a house.

We are given $P[A] = 0.60$, $P[H] = 0.30$, $P[A \cap H] = 0.20$.

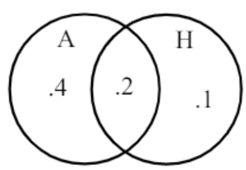
We wish to find $P[A \cap H'] + P[A' \cap H]$

(the event $A \cap H'$ is the event that a randomly chosen person owns an automobile but does not own a house, and the reverse for the event $A' \cap H$).

In a population of 10, 6 would own an automobile, 3 would own a house, and 2 would own at least one is $6 + 3 - 2 = 7$ and the number owning exactly one is $7 - 2 = 5$, so the probability of owning exactly one is $\frac{5}{10} = 0.5$.

The event probabliity approach to solving the problem is as follows.

The diagram at the right indicates the breakdown of the components of the two events. We can see that $P[A' \cap H] = 0.1$ and $P[A \cap H'] = 0.4$, so the desired probability is 0.5.



Also, from rules of probability we have $P[A \cap H'] = P[A] - P[A \cap H] = 0.60 - 0.20 = 0.40$, and similarly, $P[A' \cap H] = P[H] - P[A \cap H] = 0.30 - 0.20 = 0.10$. Therefore, the probability in question is $0.40 + 0.10 = 0.50$. An alternative way to consider the problem is to start with the event $A \cup H$, which is the event that a randomly chosen person either owns an automobile, or owns a house (or owns both), and then note that this event is the disjoint union of the event in question and the event $A \cap H$, so that "either A or H or both" = "exactly one of A or H" $\cup$ "both A and H".

It follows that the probability in question is

$P[A \text{ or } H, \text{but not both}] = P[A \cup H] - P[A \cap H]$

$= \left(P[A] + P[H] - P[A \cap H]\right) - P[A \cap H] = (0.60 + 0.30 - 0.20) - 0.20 = 0.50$.

The following table also summarizes the calculations in a simple way.

	$P(A) = 0.6$, given	$\Rightarrow$	$P(A') = 1 - 0.6 = 0.4$
$P(H) = 0.3$	$P(A \cap H) = 0.2$	$\Rightarrow$	$P(A' \cap H)$
given	given		$= P(H) - P(A \cap H)$
$\Downarrow$	$\Downarrow$		$= 0.3 - 0.2 = 0.1$
$P(H')$	$P(A \cap H')$		
$= 1 - 0.3 = 0.7$	$= P(A) - P(A \cap H)$		
	$= 0.6 - 0.2 = 0.4$		

Then $P(A \cap H') + P(A' \cap H) = 0.4 + 0.1 = 0.5.$ Answer: B

6. We are asked to find $P[N \geq 1|N \leq 4] = \frac{P[1 \leq N \leq 4]}{P[N \leq 4]}$.

$P[1 \leq N \leq 4] = P[N = 1] + P[N = 2] + P[N = 3] + P[N = 4]$

$= \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} + \frac{1}{5\times 6} = \frac{1}{3}$.

$P[N \leq 4] = P[N = 0] + P[1 \leq N \leq 4] = \frac{1}{1\times 2} + \frac{1}{3} = \frac{5}{6}$.

$P[N \geq 1|N \leq 4] = \frac{1/3}{5/6} = 0.4.$ Answer: B

7. We identify events as follows:

D : randomly chosen individual has diabetes

H : randomly chosen individual has heart disease

We are given $P[D] = 0.02$, $P[H] = 0.03$, $P[D' \cap H'] = 0.96$.

Using rules of probability, we have

$.98 = P[D'] = P[D' \cap H] + P[D' \cap H'] \rightarrow P[D' \cap H] = 0.98 - 0.96 = 0.02$, and

$.03 = P[H] = P[H \cap D] + P[H \cap D'] \rightarrow P[H \cap D] = 0.03 - 0.02 = 0.01$.

These calculations are summarized in the following table.

	$P(D) = 0.02$, given	$\Rightarrow$	$P(D') = 1 - 0.02 = 0.98$
	$\Downarrow$		
$P(H) = 0.03$	$P(H \cap D)$		
given	$= P(D) - P(H' \cap D) = 0.02 - 0.01 = 0.01$		
$\Downarrow$	$\Uparrow$		
$P(H') = 0.97$	$P(D \cap H')$	$\Leftarrow$	$P(D' \cap H') = 0.96$, given
$= 1 - 0.03$	$= P(H') - P(D' \cap H') = 0.97 - 0.96 = 0.01$		Answer: D

8. $P[\text{win in a given week}] = (0.1)^5 = p$. Then the number of wins in four weeks, N, has a binomial distribution $B(4, p)$, and $X = 100N$ dollars is the amount won in 4 weeks. Then,

$Var[X] = 100^2 Var[N] = 100^2 \times 4 \times p \times (1 - p) \Rightarrow$ the standard deviation of X is

$200\sqrt{p(1 - p)} = 0.63.$ Answer: C

9. $Var[Y|X=x] = E[Y^2|X=x] - (E[Y|X=x])^2$.
$f_{Y|X}(y|X=x) = \frac{f(x,y)}{f_X(x)}$, where $f_X(x) = \int_x^1 2\,dy = 2(1-x)$.

Thus, $f_{Y|X}(y|X=x) = \frac{1}{1-x}$ so that $E[Y|X=x] = \int_x^1 y \times \frac{1}{1-x}\,dy = \frac{1+x}{2}$
and $E[Y^2|X=x] = \int_x^1 y^2 \times \frac{1}{1-x}\,dy = \frac{1+x+x^2}{3}$, and then
$Var[Y|X] = \frac{1+x+x^2}{3} - [\frac{1+x}{2}]^2 = \frac{(1-x)^2}{12}$.

Alternatively, note that given any joint uniform distribution, any related conditional distribution is also uniform. Given $X=x$, Y has a uniform distribution on $(x,1)$ and thus has a variance of $\frac{(1-x)^2}{12}$.
Answer: B

10. This is an example of a mixture of distributions. X_1 is the annual claim amount for a smoker and X_2 is the annual claim amount for a non-smoker. We are given
$E[X_1] = 500$, $\sqrt{Var[X_1]} = 200$, $E[X_2] = 200$, $\sqrt{Var[X_1]} = 100$.

We are also given the mixing weights $\alpha_1 = 0.3$ (proportion of the population that are smokers), and $\alpha_2 = 0.7$. The distribution of the annual claim amount for a randomly chosen individual from the group is X, which is a mixture of X_1 and X_2.
$E[X] = (.3)E[X_1] + (.7)E[X_2] = 290$.
$E[X^2] = 0.3 \times E[X_1^2] + 0.7 \times E[X_2^2]$.

We know that $Var[X_1] = 40,000 = E[X_1^2] - (E[X_1])^2 = E[X_1^2] - (500)^2$
$\rightarrow E[X_1^2] = 290,000$, and
$Var[X_2] = 10,000 = E[X_2^2] - (E[X_2])^2 = E[X_2^2] - (200)^2 \rightarrow E[X_2^2] = 50,000$.

Then, $E[X^2] = 0.3 \times E[X_1^2] + 0.7 \times E[X_2^2] = 122,000$.

Finally, $Var[X] = E[X^2] - (E[X])^2 = 122,000 - (290)^2 = 37,900$
and the standard deviation is $\sqrt{37,500} = 194.7$. Answer: E

11. Let X and Y denote the random variables of the weights of the male and female students respectively. Since the students are chosen at random, X and Y are independent. But then
$W = X + Y$ is normal with mean $\mu_W = \mu_X + \mu_Y = 310$ and variance
$\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 = 625$. Thus, $P[W < 280] = P\left[\frac{W-310}{\sqrt{625}} < \frac{280-310}{\sqrt{625}}\right] = P[Z < -1.2]$,
(where Z has a standard normal distribution) $= 0.1151$. Answer: B

12. Expected payment under policy 2 is
$\int_0^L y(.01)dy + L \times P[X > L] = .005L^2 + L \times (\frac{100-L}{100}) = L - .005L^2$.

This is equal to the expected payment under policy 1, which is
$0.8 \times E[X] = 0.8 \times 50 = 40$. Solving $L - .005L^2 = 40$ results in
$L = 55.28$, 144.72. We discard 144.72 as a limit since it is larger than the maximum loss amount. Thus, $L = 55.28$.

The variance of insurer payment under policy 1 is
$Var[0.8X] = 0.64Var[X] = 0.64 \times (\frac{100^2}{12}) = 533.33$.
Under policy 2, $E[(\text{insurer payment})^2] = \int_0^{55.28} y^2 \times 0.01\, dy + (55.28)^2 \times P[X > 55.28]$
$= 563.10 + (55.28)^2[\frac{100-55.28}{100}] = 1929.69$, and
$Var[\text{insurer payment}] = 1929.69 - 40^2 = 329.69$.
$\frac{Var[\text{insurer payment under policy 2}]}{Var[\text{insurer payment under policy 1}]} = \frac{329.69}{533.33} = 0.618$. Answer: D

13. $M_X(t) = \frac{b}{b-t} \Rightarrow M_X(-b^2) = \frac{b}{b-(-b^2)} = \frac{b}{b+b^2} = \frac{1}{1+b} = 0.2 \Rightarrow b = 4$. Answer: D

14. Let $A =$ 'deck is fixed', $X =$ number of games lost out of 3 games.

We wish to find $P[A|X = 3]$.

X has a binomial distribution with $n = 3$ and p depends on whether or not the deck is fixed.

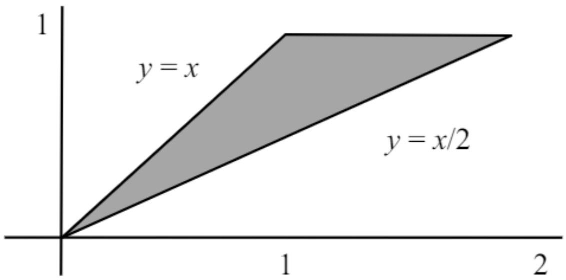
We use the usual Bayesian approach.
$P[A|X = 3] = \frac{P[X=3|A] \times P[A]}{P[X=3]} = \frac{P[X=3|A] \times P[A]}{P[X=3|A] \times P[A] + P[X=3|A'] \times P[A']}$.
We are given that $P[A] = 0.25$ (the casino uses a fixed deck one-quarter the time).

Also, if the deck is fixed then $p = 0.75$, and $P[X = 3|A] = (0.75)^3 = 0.421875$.

If the deck is fair, then $p = 0.52$, and $P[X = 3|A'] = (0.52)^3 = 0.140608$.

Then $P[A|X = 3] = \frac{0.421875 \times 0.25}{0.421875 \times 0.25 + 0.140608 \times 0.75} = 0.50$. Answer: C

15. The region of probability is shown in the shaded figure below



The probability is $\int_0^1 \int_{x/2}^{x} xy\,dy\,dx + \int_1^2 \int_{x/2}^{1} xy\,dy\,dx = \frac{3}{32} + \frac{9}{32} = \frac{3}{8}$.
Alternatively, the probability is $\int_0^1 \int_y^{2y} xy\,dx\,dy = \frac{3}{8}$. Answer: D

16. $T = T_1 + T_2$, where T_i is the random lifetime of machine i (n days). Since T_1 and T_2 are independent, the joint density of $T_1$and T_2 is $f(t_1, t_2) = e^{-t_1}e^{-t_2}$.
Applying the convolution method for the sum of random variables results in
$f_T(t) = \int_0^t f(s, t-s)\,ds = \int_0^t e^{-s}e^{-(t-s)}ds = te^{-t}$. Answer: A

17. $X =$ time until property claim, $Y =$ time until life claim.
$f(x) = e^{-x}$, $g(y) = \frac{1}{2}e^{-y/2}$.
$T =$ time until next claim $= min(X, Y)$.
$P[T > t] = P[X > t] \times P[Y > t] = e^{-t} \cdot e^{-t/2} = e^{-3t/2}$.
Pdf of T is $h(t) = \frac{d}{dt}P[T \leq t] = -\frac{d}{dt}P[T > t] = -\frac{d}{dt}(e^{-3t/2}) = \frac{3}{2}e^{-3t/2}$.
This is the pdf of an exponential random variable with mean $\frac{2}{3}$. Answer: D

18. Because of the proportions in which the bolts are produced, a randomly selected bolt will have a $\frac{1}{9}$ chance of being of type A, a $\frac{2}{9}$ chance of being of type B, and a $\frac{2}{3}$ chance of being of type C. A random selection of size n from the production of bolts will have a multinomial distribution with parameters n , $p_A = \frac{1}{9}$, $p_B = \frac{2}{9}$ and $p_C = \frac{2}{3}$, with probability function

$$P[N_A = n_A, N_B = n_b, N_C = n_c] = \frac{n!}{n_A!\,n_B!\,n_C!}\left(\frac{1}{9}\right)^{n_A}\left(\frac{2}{9}\right)^{n_B}\left(\frac{2}{3}\right)^{n_C}$$

With $n = 4$, $P[N_A = 0, N_B = 2, N_C = 2] = \frac{4!}{0!\,2!\,2!}\left(\frac{1}{9}\right)^0\left(\frac{2}{9}\right)^2\left(\frac{2}{3}\right)^2 = \frac{32}{243}$ Answer: D

19. $X = (X \cap Y) \cup (X \cap Y') \rightarrow X = X \cap Y \rightarrow P(Y) = P(Y \cap X) + P(Y \cap X') = P(X) + a.$
$Y = (Y \cap Z) \cup (Y \cap Z') \rightarrow Y = Y \cap Z \rightarrow c = P(Z) = P(Z \cap Y) + P(Z \cap Y') = P(Y) + b.$

Then, $P(X) + a + b = c \rightarrow P(X) = c - b - a$. It is also true that $X \subset Y \subset Z$, so that

$$\begin{aligned} c = P(Z) &= P(X) + P(Y - X) + P(Z - Y) \\ &= P(X) + P(Y \cap X') + P(Z \cap Y') = P(X) + a + b. \end{aligned}$$ Answer: E

20. $\rho_{XY} = \frac{Cov[X,Y]}{\sigma_X \sigma_Y} = \frac{Cov[2Z_1 - Z_3, 2Z_2 + Z_3]}{\sqrt{Var[2Z_1 - Z_3] \cdot Var[2Z_2 + Z_3]}}$.
$Cov[2Z_1 - Z_3, 2Z_2 + Z_3]$
$= 4Cov[Z_1, Z_2] + 2Cov[Z_1, Z_3] - 2Cov[Z_3, Z_2] - Cov[Z_3, Z_3]$
$= 4(0) + 2(0) - 2(0) - Var[Z_3] = -1$ (since $Cov[Z_3, Z_3] = Var[Z_3]$ and independent random variables have covariance of 0).

$Var[2Z_1 - Z_3] = 4Var[Z_1] + Var[Z_3] - 2(2Cov[Z_1, Z_3]) = 5$
$Var[2Z_2 + Z_3] = 4Var[Z_2] + Var[Z_3] + 2(2Cov[Z_2, Z_3]) = 5$

Thus, the correlation is $\rho_{XY} = \frac{-1}{\sqrt{5 \times 5}} = -\frac{1}{5}$. Answer: C

21. The mean and variance of X are $E[X] = 100 \times 0.2 = 20$, $Var[X] = 100 \times 0.2 \times 0.8 = 16$.
Using the normal approximation with integer correction, we assume that X is approximately normal and find

$$P[14.5 \leq X \leq 25.5] = P\left[\frac{14.5 - 20}{\sqrt{16}} \leq \frac{X - 20}{\sqrt{16}} \leq \frac{25.5 - 20}{\sqrt{16}}\right] = P[-1.375 \leq Z \leq 1.375],$$

where Z has a standard normal distribution.
$$\begin{aligned} P[-1.375 \leq Z \leq 1.375] &= \Phi(1.375) - \Phi(-1.375) = \Phi(1.375) - [1 - \Phi(1.375)] \\ &= 2\Phi(1.375) - 1. \end{aligned}$$

From the standard normal table we have $\Phi(1.3) = 0.9032$ and $\Phi(1.4) = 0.9192$. Using linear interpolation (since 1.375 is $\frac{3}{4}$ of the way from 1.3 to 1.4) we have

$\Phi(1.375) = 0.25 \times \Phi(1.3) + 0.75 \times \Phi(1.4) = 0.9152$, and then the probability in question is $2 \times 0.9152 - 1 = 0.8304$. Answer: E

22. When damage occurs, the pdf of the amount of damage is 0.001 for the uniform distribution on the interval from 1000 to 2000.

Expected insurer payment $= 0.01 \times$ Expected payment given that damage occurs
$= 0.01 \times [\int_{1000}^{1500} x(0.001)\,dx + 1500 \times P(\text{damage exceeds 1500 when damage occurs})]$
$= 0.01 \times [625 + (1500)(\frac{2000-1500}{2000-1000})] = 13.75$. For the corrected model,

Expected insurer payment $= 0.01 \times$ Expected payment given that damage occurs
$= 0.01 \times [\int_{1000}^{1500} x(.00025)\,dx + 1500 \times P(\text{damage exceeds 1500 when damage occurs})]$
$= 0.01 \times [156.25 + (1500)(\frac{5000-1500}{5000-1000})] = 14.6875$.

Increase in expected value is 0.9375 . Answer: C

23. $P[\text{head}] = \frac{1}{3}$, $P[\text{tail}] = \frac{2}{3}$

$P[\text{3rd head on 5th toss}] = P[(\text{2 heads in first 4 tosses}) \cap (\text{head on 5th toss})]$
$= P[\text{2 heads in first 4 tosses}] \times P[\text{head on 5th toss}]$
$= \binom{4}{2}(\frac{1}{3})^2(\frac{2}{3})^2(\frac{1}{3}) = \frac{8}{81}$

Note that the number of tails, X, that are tossed until the 3rd head occurs can also be regarded as negative binomial distribution with $p = \frac{1}{3}$ and $r = 3$, and we are finding $P[X = 2]$. Answer: A

24. For each boat type we find the mean and the variance of the repair cost. The mean of the aggregate repair costs is the sum of the mean repair costs for the 450 boats, and assuming independence of boat repair costs for all 450 boats, the variance of the aggregate cost is the sum of the variances for the 450 boats.

For each type of boat, the repair cost is a mixture of 0 (if no repair is needed) and X_i (repair cost variable for boat i if a repair is needed). The mean repair cost for boat i is
$E[X_i] \times$ prob. repaid is needed for boat i , and the second moment of the repair cost for boat i is
$E[X_i^2] \times$ prob. repaid is needed for boat i (note that $E[X_i^2] = Var[X_i] + (E[X_i])^2$).
The variance of the repaid cost for boat i is the second moment minus the square of the first moment.

Power boats: Mean repair cost for one boat $= 300 \times 0.3 = 90$,
second moment of repair cost for one boat $= [10,000 + 300^2] \times 0.3 = 30,000$.
Variance of repair cost for one power boat $= 30,000 - 90^2 = 21,900$.

Sailboats: Mean repair cost for one boat $= 1000 \times 0.1 = 100$,

second moment of repair cost for one boat $= [400,000 + 1000^2] \times 0.1 = 140,000$.

Variance of repair cost for one power boat $= 140,000 - 100^2 = 130,000$.

Luxury Yachts: Mean repair cost for one boat $= 5000 \times 0.6 = 3000$,

second moment of repair cost for one boat $= [2,000,000 + 5000^2] \times 0.6 = 16,200,000$.

Variance of repair cost for one power boat $= 16,200,000 - 3000^2 = 7,200,000$.

The mean of the aggregate repair cost is $100 \times 90 + 300 \times 100 + 50 \times 3000 = 189,000$,

and the variance is $100 \times 21,900 + 300 \times 130,000 + 50 \times 7,200,000 = 401,190,000$.

The amount budgeted by the marina is $189,000 + \sqrt{401,190,000} = 209,030$. Answer: B

25. $P[B \text{ received the letter} | A \text{ did not receive an answer after writing to } B]$

$$= \frac{P[(B \text{ received the letter}) \cap (A \text{ did not receive an answer after writing to } B)]}{P[A \text{ did not receive an answer after writing to } B]}.$$

But, $P[A \text{ does not receive a reply after writing to } B]$

$$= P[(A \text{ does not receive a reply after writing to } B) \cap (B \text{ received } A\text{'s letter})]$$
$$+ P[(A \text{ does not receive a reply after writing to } B) \cap (B \text{ did not receive } A\text{'s letter})]$$

$P[(A \text{ does not receive a reply after writing to } B) \cap (B \text{ received } A\text{'s letter})]$

$$= P[A \text{ does not receive a reply after writing to } B | B \text{ received } A\text{'s letter}]$$
$$\times P[B \text{ received } A\text{'s letter}] = \frac{1}{n} \times \frac{n-1}{n} \quad \text{and}$$

$P[(A \text{ does not receive a reply after writing to } B) \cap (B \text{ did not receive } A\text{'s letter})]$

$$= P[A \text{ does not receive a reply after writing to } B | B \text{ did not receive } A\text{'s letter}]$$
$$\times P[B \text{ did not receive } A\text{'s letter}] = 1 \times \frac{1}{n}$$

Therefore,

$P[A \text{ does not receive a reply after writing to } B] = \frac{n-1}{n} \times \frac{1}{n} + \frac{1}{n} = \frac{2n-1}{n^2}$ and

$P[B \text{ received the letter} | A \text{ did not receive an answer after writing to } B]$

$$= \frac{P[(B \text{ received the letter}) \cap (A \text{ did not receive an answer after writing to } B)]}{P[A \text{ did not receive an answer after writing to } B]}$$

$$= \frac{(n-1)/n^2}{(2n-1)/n^2} = \frac{n-1}{2n-1}.$$ Answer: C

26. Our "probability space" consists of all people who have been charged with a drug-related offence. We define the following events:

C - the person is convicted , T - the person is sentenced to jail time

D - the person did actually commit the crime.

Since jail time is sentenced only to those who are convicted, we have

$T \subset C$, so that $P(T \cap C) = P(T)$. We are also given the following information:

$P(C) = 0.75$, $P(D'|C) = 0.10$, $P(D|C') = 0.25$, $P(T|D') = 0.02$, $P(T'|D) = 0.20$

We wish to find $P(D'|C \cap T')$. In the model population approach, $P(D'|C \cap T') = \frac{\#D' \cap C \cap T'}{\#C \cap T'}$.

With a model population of 100,000 we have $\#C = 75,000$ and $\#C' = 25,000$.

From this we get $\#D' \cap C = \#C \times P(D'|C) = 75,000 \times 0.1 = 7,500$ so that

$\#D \cap C = \#C - \#D' \cap C = 67,500$. We also get $\#D \cap C' = 25,000 \times 0.25 = 6,250$.

Then, $\#D = \#D \cap C + \#D \cap C' = 67,500 + 6,250 = 73,750$ and $\#D' = 26,250$.

We now get $\#T \cap D' = 26,250 \times 0.02 = 525$ and $\#T' \cap D = 73,750 \times 0.2 = 14,750$,

and then $\#T \cap D = \#D - \#T' \cap D = 73,750 - 14,750 = 59,000$ and

$\#T = \#T \cap D' + \#T \cap D = 525 + 59,000 = 59,525$ and $\#T' = 40,475$.

Since $T \subset C$ we have $\#T = \#T \cap C$ and $\#D' \cap C \cap T = \#D' \cap T$ so that

$\#C \cap T' = \#C - \#C \cap T = \#C - \#T = 15,475$

and $\#D' \cap C \cap T = \#D' \cap T = 525$.

Then $\#D' \cap C \cap T' = \#D' \cap C - \#D' \cap C \cap T = 7,500 - 525 = 6,975$.

Finally, $P(D'|C \cap T') = \frac{\#D' \cap C \cap T'}{\#C \cap T'} = \frac{6,975}{15,475} = 0.4507$.

The conditional probability solution is as follows. From the given information, we get

$P(D' \cap C) = P(D'|C) \times P(C) = 0.1 \times 0.75 = 0.075$, and

$P(D \cap C') = P(D|C') \times P(C') = 0.25 \times 0.25 = 0.0625$.

Since $P(C) = 0.75$ and $P(C') + 0.25$, we get

$P(D \cap C) = P(C) - P(D' \cap C) = 0.75 - 0.075 = 0.675$.

Then $P(D) = P(D \cap C) + P(D \cap C') = 0.675 + 0.0625 = 0.7375$, and $P(D') = 0.2625$.

Then, $P(T \cap D') = P(T|D') \times P(D') = 0.02 \times 0.2625 = 0.00525$ and

$P(T' \cap D) = P(T'|D) \times P(D) = 0.20 \times 0.7375 = 0.1475$, so that

$P(T \cap D) = P(D) - P(T' \cap D) = 0.7375 - 0.1475 = 0.59$ and

$P(T) = P(T \cap D) + P(T \cap D') = 0.59 + 0.00525 = 0.59525$, and $P(T') = 0.40475$.

Then $P(C \cap T') = P(C) - P(C \cap T) = P(C) - P(T) = 0.75 - 0.59525 = 0.15475$, and

$P(D' \cap C \cap T') = P(D' \cap C) - P(D' \cap C \cap T) = 0.075 - 0.00525 = 0.06975$

(note that $P(D' \cap C \cap T) = P(D' \cap T)$ because $T \subset C$).

Finally, $P(D'|C \cap T') = \frac{P(D' \cap C \cap T')}{P(C \cap T')} = \frac{0.06975}{0.15475} = 0.4507$. Answer: C