## Tarea 3

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#### Pregunta 1

```
a)
                       P_{i,j} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & 0 & q_i & 1 - q_i - p_i & p_i & 0 & \dots \\ \dots & 0 & 0 & q_i & 1 - q_i - p_i & p_i & \dots \\ \dots & 0 & 0 & 0 & q_i & 1 - q_i - p_i & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}
Para X_1 = 0, q_i = 0.5, p_i = 0.2
x1 <- 0
x <- x1
for(i in 2:500){
  x[i] \leftarrow sample(c(x[i-1]-1,x[i-1],x[i-1]+1),1, prob = c(0.5,0.2,0.3))
table(x)
## x
## -98 -97 -96 -95 -94 -93 -92 -91 -90 -89 -88 -87 -86 -85 -84 -83 -82 -81
        2 1 6 19 10 2
                                  1 3 13 16 14
                                                         9 9 12 10 1
## -80 -79 -78 -77 -76 -75 -74 -73 -72 -71 -70 -69 -68 -67 -66 -65 -64 -63
        4 3 5 5 4 1
                                       4 7 11
                                                    5 1 3
                                  2
                                                                 1 1 4
## -62 -61 -60 -59 -58 -57 -56 -55 -54 -53 -52 -51 -50 -49 -48 -47 -46 -45
        2 1 1 1 2
                               6 16 21 18 17 19
                                                         7 2 5
                                                                      4
## -44 -43 -42 -41 -40 -39 -38 -37 -36 -35 -34 -33 -32 -31 -30 -29 -28 -27
                                       5 2 3 3 2 1
        1 1
                  1 1
                           2
                               6 11
                                                                 1 1
## -26 -25 -24 -23 -22 -21 -20 -19 -18 -17 -16 -15 -14 -13 -12 -11 -10 -9
            1
                                  5
                                                1 2 6 11 13 10
        1
                  2 3
                          2
                               2
                                       8
                                           3
   -8 -7 -6 -5 -4 -3 -2 -1
                           2
         1
              1
                  1
                       1
                               5 12 12
x1 <- 0
x <- x1
for(i in 2:500){
  x[i] \leftarrow sample(c(x[i-1]-1,x[i-1],x[i-1]+1),1, prob = c(0.5,0.2,0.3))
table(x)
## -116 -115 -114 -113 -112 -111 -110 -109 -108 -107 -106 -105 -104 -103 -102
## 2
         4
               8
                    13
                           15
                                7
                                       2
                                          2
                                                2
                                                     1
                                                           3
                                                                5
                                                                        4
                                                                              2
                                                                                   2
## -101 -100 -99 -98 -97 -96 -95 -94 -93 -92 -91 -90 -89 -88 -87
```

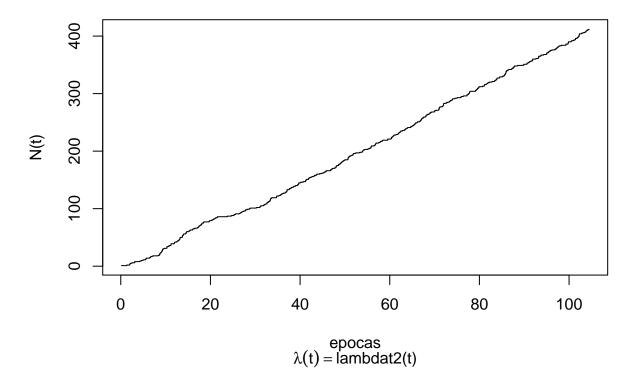
```
5
                                                       7
                                                                   2
##
             5
                         5
                               5
                                    13
                                           8
                                                 9
##
     -86
          -85
                 -84
                      -83
                             -82
                                  -81
                                        -80
                                              -79
                                                     -78
                                                           -77
                                                                -76
                                                                       -75
                                                                             -74
                                                                                   -73
                                                                                         -72
##
       5
             4
                         1
                               3
                                     1
                                           2
                                                       1
                                                             1
                                                                   2
                                                                         6
                                                                               4
                                                                                     2
                                                                                           1
    -71
          -70
                -69
                      -68
                            -67
                                        -65
                                                                -61
                                                                                         -57
##
                                  -66
                                              -64
                                                     -63
                                                           -62
                                                                       -60
                                                                             -59
                                                                                   -58
##
       4
             3
                   5
                         3
                               3
                                     3
                                           2
                                                 9
                                                       8
                                                             3
                                                                   3
                                                                         6
                                                                               5
                                                                                     5
                                                                                           8
##
    -56
          -55
                -54
                      -53
                             -52
                                  -51
                                        -50
                                              -49
                                                     -48
                                                           -47
                                                                -46
                                                                       -45
                                                                             -44
                                                                                   -43
                                                                                         -42
##
      7
             8
                   8
                         7
                               8
                                     7
                                           6
                                                 7
                                                      10
                                                             8
                                                                   6
                                                                               1
                                                                                     1
                                                                                           1
          -40
                -39
                      -38
                                               -34
                                                     -33
                                                           -32
                                                                 -31
                                                                       -30
                                                                             -29
                                                                                   -28
                                                                                         -27
##
    -41
                             -37
                                   -36
                                         -35
##
      1
             4
                   9
                         8
                               7
                                     9
                                          10
                                                 9
                                                       8
                                                             2
                                                                   3
                                                                         5
                                                                               1
                                                                                     5
                                                                                          10
##
    -26
          -25
                -24
                      -23
                             -22
                                   -21
                                         -20
                                              -19
                                                     -18
                                                           -17
                                                                -16
                                                                       -15
                                                                                         -12
                                                                             -14
                                                                                   -13
##
       7
             1
                   1
                         1
                              1
                                     3
                                           5
                                                 3
                                                       2
                                                             3
                                                                   4
                                                                         3
                                                                                     2
                                                                                           1
                              -7
                                    -6
                                          -5
                                                                         0
##
          -10
                  -9
                        -8
                                                -4
                                                      -3
                                                            -2
                                                                  -1
    -11
             2
                                     2
                                                 4
                                                                   2
##
       2
                   1
                         1
                               1
                                           1
                                                       5
                                                                         1
```

## Pregunta 2

a) Grafiquen una ejemplo del proceso considerando el intervalo de tiempo [0,100].

```
lambdat2 <- function(t){</pre>
  x <- paste("","{",0,"<= t & t <",1,"}",sep="")
  for(i in seq(2,100,2)){
    x \leftarrow paste(x,paste("","{",i," \leftarrow t & t < ",i+1,"}",sep=""),sep="")
  return(ifelse(eval(parse(text=x)),3,5))
poissonnohomogeneo<-function(lambdat,n,pic=T){</pre>
  lambda <- 5
  TT <- rexp(n,lambda)
  s <- cumsum(TT)
  u <- runif(n)
  ss <- s[u <= lambdat(s)/lambda]</pre>
  Ns <- 1:length(ss)
  if(pic==T){
  plot(ss, Ns, type = "s", xlab = "epocas", ylab = "N(t)", main = "Simulacion de un proceso Poisson no
  return(list(epocas = ss, cuenta= Ns))
}
poissonnohomogeneo (lambdat2,510)
```

### Simulacion de un proceso Poisson no homogéneo



```
$epocas
##
     [1]
##
           0.1473285
                         1.3434066
                                      1.9958099
                                                   2.0068195
                                                                2.1241860
     [6]
            2.4852113
                         3.1003639
                                      3.1263395
                                                   4.2618953
                                                                4.6161450
##
##
    [11]
            4.9599940
                         5.4973092
                                      5.6271955
                                                   5.6819631
                                                                6.4832790
                                                                8.6657825
    [16]
            6.6243726
                         6.7506420
                                                   8.3607006
##
                                      7.1314181
##
    [21]
           8.6810731
                         8.7355483
                                      8.7959236
                                                   8.9799949
                                                                9.0902188
##
    [26]
           9.1576623
                         9.2494974
                                      9.3000602
                                                   9.3573979
                                                                9.4047602
                        10.2667657
##
    [31]
           9.6138306
                                     10.2782420
                                                  10.3148810
                                                               10.6266578
    [36]
##
           10.8577012
                        11.1916088
                                     11.2002002
                                                  11.2167751
                                                               11.9165843
                        12.0414843
##
    [41]
          11.9910995
                                     12.4945857
                                                  12.6468025
                                                               12.9130058
##
    [46]
          13.0137531
                        13.0928439
                                     13.2107954
                                                  13.2385123
                                                               13.2431111
    [51]
##
          13.7131573
                       13.7529647
                                     13.7611934
                                                  13.8368695
                                                               13.9895930
##
    [56]
           14.0145575
                        14.4588879
                                     14.5882628
                                                  14.6178431
                                                               14.6605043
##
    [61]
          15.2169415
                        15.3234708
                                     15.5200269
                                                  16.0310458
                                                               16.1225363
##
    [66]
           16.4937864
                        17.0802384
                                     17.2772650
                                                  17.4126778
                                                               17.4765966
    [71]
                                     18.0392252
##
          17.6627496
                        17.8407030
                                                  18.0748778
                                                               18.1485987
    [76]
          18.4784687
                        18.5039711
                                     19.6842383
                                                  19.7683056
##
                                                               20.2177806
##
    [81]
          20.4923569
                       20.6869061
                                     20.9874866
                                                  21.0575321
                                                               21.4284172
##
    [86]
          21.5891592
                        23.7417209
                                     24.6555970
                                                  25.2466755
                                                               25.3179497
    [91]
          25.5903530
                        26.4956234
                                     26.7047079
                                                  26.9550338
                                                               27.0814838
##
    [96]
          27.4235452
                                     27.8065215
##
                        27.7822415
                                                  28.2020787
                                                               28.6918352
   [101]
          28.8808582
                       30.2402669
                                     30.9588496
                                                               31.2244607
##
                                                  31.1814263
   [106]
          31.8162593
                       32.0109799
                                     32.1943900
                                                  32.5957029
                                                               32.6326852
##
   [111]
          32.6720124
                       32.9135539
                                     33.1087403
                                                  33.2945071
                                                               33.3232800
##
   [116]
          33.3838582
                       33.4132316
                                     33.4838398
                                                  33.6627841
                                                               34.7406632
  [121]
          34.7892348
                       34.8018956
                                     35.2946524
                                                  35.6308983
                                                               35.7820462
```

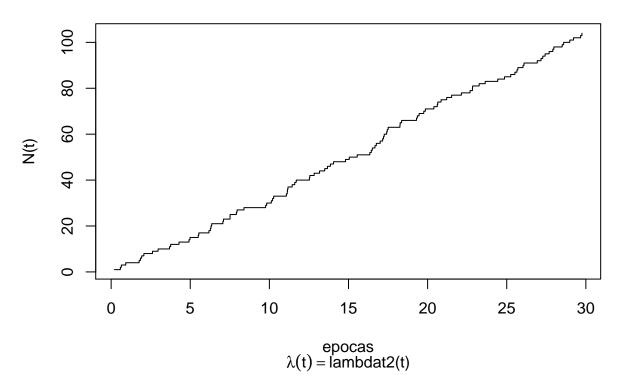
```
## [126]
          35.9439134
                       36.2358156
                                    36.6629980
                                                 36.8045720
                                                              37.0322353
##
   [131]
          37.0347562
                       37.0919999
                                    37.1145069
                                                 37.5550553
                                                              37.6431121
   [136]
          37.9676955
                       38.0734716
                                    38.4497311
                                                 38.6460201
                                                              38.9432985
   [141]
##
          39.3709461
                       39.5021936
                                    39.6499955
                                                 39.6949951
                                                              39.7474091
##
   [146]
          40.1671328
                       40.6910543
                                    41.2175910
                                                 41.2665943
                                                              41.3076061
##
   [151]
          41.4606335
                       41.8782985
                                    41.9143742
                                                 42.0887696
                                                              42.4243261
##
   [156]
          42.6820694
                       43.1123054
                                    43.2622764
                                                 43.6122121
                                                              43.7763157
##
   [161]
           44.4082108
                       44.8813514
                                    45.2864426
                                                 45.4920130
                                                              45.7224043
##
   [166]
          45.8437792
                       46.8436981
                                    46.8438075
                                                 47.0937187
                                                              47.1933088
##
   [171]
           47.7006876
                       48.0042751
                                    48.0330082
                                                 48.0521460
                                                              48.1960111
   [176]
          48.5885028
                       48.5936092
                                    48.8398628
                                                 49.0158681
                                                              49.1695564
   [181]
##
           49.2108424
                       49.4692883
                                    49.5956225
                                                 49.7571572
                                                              49.9986236
##
   [186]
          50.6559832
                       50.7086804
                                    50.7100529
                                                 50.7429254
                                                              50.8554262
##
   [191]
          51.0123227
                       51.1580079
                                    51.5561248
                                                 51.8206598
                                                              51.8277626
   [196]
                                    53.2982788
##
          51.9369536
                       52.4123110
                                                 53.8471991
                                                              53.9193173
   [201]
           54.0276751
                       54.1180730
                                    54.3776329
                                                 55.1380149
                                                              55.4035640
   [206]
##
          55.7858253
                       55.7929976
                                    55.8849885
                                                 55.8935246
                                                              56.5277294
   [211]
          56.6880381
                       56.8183115
                                    56.8642747
                                                 56.9702101
                                                              57.5453840
   [216]
##
          57.8315248
                       57.9607072
                                    58.3144975
                                                 58.4111935
                                                              59.5612529
##
   [221]
          59.5640358
                       60.3095427
                                    60.3710864
                                                 60.3768747
                                                              60.5879685
##
   [226]
          60.7271279
                       60.8291599
                                    60.9496112
                                                 61.4201970
                                                              61.8000736
   [231]
                                                 62.4005717
                                                              62.4756986
##
          61.9455289
                       62.0369323
                                    62.1760804
   [236]
          62.8172584
                       63.2876758
                                    63.4370703
                                                 63.5359542
                                                              63.8564803
##
   [241]
##
          63.9745983
                       64.6953266
                                    64.7146842
                                                 65.0130838
                                                              65.2543935
##
   [246]
          65.4141546
                       65.6060872
                                    65.6527135
                                                 65.7239113
                                                              66.0737762
   [251]
          66.2286802
                       66.6324416
                                    66.8499528
                                                 66.9092540
                                                              67.0502279
   [256]
##
          67.1036801
                       67.1747961
                                    67.4405505
                                                 67.4525250
                                                              67.8864556
   [261]
##
          68.0819982
                       68.1056662
                                    68.1736869
                                                 68.4635261
                                                              68.7509892
   [266]
          69.0036038
##
                       69.0070296
                                    69.2687841
                                                 69.9172766
                                                              69.9683742
                                                              71.0315008
##
   [271]
           70.2514658
                       70.8737455
                                    71.0026544
                                                 71.0221264
##
   [276]
           71.0665286
                       71.2209977
                                    71.7106002
                                                 71.8841226
                                                              71.9647199
##
   [281]
          71.9900353
                       72.0074861
                                    72.0556833
                                                 72.5800960
                                                              72.8010270
##
   [286]
           73.2437439
                       73.3741664
                                    73.5639542
                                                 73.8674835
                                                              73.9843059
   [291]
##
          74.0271959
                       74.5439446
                                    75.0457043
                                                 75.8663275
                                                              75.9660834
   [296]
          76.4450444
                       77.2119848
                                    77.2379493
                                                 77.4942130
                                                              77.6140319
##
                                                 77.9093342
   [301]
##
          77.6928332
                       77.7940509
                                    77.8088142
                                                              79.1380131
   [306]
          79.1707538
                       79.3907777
                                    79.5080635
                                                 79.5604383
                                                              79.7315783
   [311]
          79.8779482
                                    80.7060763
##
                       79.9367479
                                                 81.1164258
                                                              81.1372401
                       81.5817514
   [316]
          81.2045565
                                    81.7686396
                                                 81.9261255
##
                                                              82.2164595
   [321]
##
          82.8233062
                       83.3518454
                                    83.4860340
                                                 83.6571884
                                                              83.8490430
   [326]
##
          83.8923863
                       83.9294443
                                    84.4788328
                                                 84.4961179
                                                              85.1213529
   [331]
                                    85.6211747
##
          85.4419949
                       85.5334214
                                                 85.7609571
                                                              85.8073163
##
   [336]
          85.8835140
                       85.9255129
                                    85.9309831
                                                 86.0138298
                                                              86.1098916
##
   [341]
          86.3052015
                                    87.1865799
                                                 87.3050127
                                                              87.5533508
                       86.5537153
##
   [346]
          87.5605708
                       87.7851362
                                    87.8175174
                                                 88.5410846
                                                              89.7865547
   [351]
##
          89.9428561
                       90.6066428
                                    90.6342409
                                                 90.8987163
                                                              91.0973728
##
   [356]
          91.2460816
                       91.5492017
                                    91.8542516
                                                 91.8648773
                                                              91.9200255
   [361]
##
           92.6144504
                       93.0944552
                                    93.1999873
                                                 93.2250129
                                                              93.2449848
##
   [366]
          93.6860614
                       93.7979920
                                    94.3449083
                                                 94.9943436
                                                              95.0271138
##
   [371]
          95.0706357
                       95.3533878
                                    95.4731914
                                                 95.7419294
                                                              95.7655243
##
   [376]
          96.1548873
                       96.8183865
                                    97.0403029
                                                 97.1455555
                                                              97.3409180
   [381]
          97.4460422
                       97.6582007
                                    97.7688039
                                                 98.2431839
                                                              99.2966319
   [386]
          99.3030841
                       99.7473021
                                    99.7988656
##
                                                 99.8447689
                                                              99.8670275
## [391] 100.4196639 100.6009118 101.0189099 101.2886138 101.4539273
```

```
## [396] 101.4546546 101.8045272 101.8982900 102.1243577 102.1776597
  [401] 102.1955615 102.2746214 102.2939833 102.3212197 102.7824864
  [406] 103.0942733 103.5409467 103.7344912 103.7735479 103.9806584
  [411] 104.1570088 104.4427747
##
  $cuenta
##
     [1]
           1
               2
                   3
                       4
                           5
                                6
                                    7
                                        8
                                            9
                                               10
                                                   11
                                                       12
                                                            13
                                                                14
                                                                    15
##
    [18]
          18
              19
                  20
                      21
                          22
                               23
                                   24
                                       25
                                           26
                                               27
                                                   28
                                                       29
                                                            30
                                                                31
                                                                    32
                                                                        33
                                                                            34
##
    [35]
          35
              36
                  37
                      38
                          39
                               40
                                   41
                                       42
                                           43
                                               44
                                                   45
                                                       46
                                                            47
                                                                48
                                                                    49
                                                                        50
                                                                            51
                      55
##
    [52]
          52
              53
                  54
                          56
                               57
                                   58
                                       59
                                           60
                                               61
                                                   62
                                                       63
                                                            64
                                                                65
                                                                    66
                                                                        67
                                                                            68
    [69]
          69
              70
                  71
                      72
                          73
                               74
                                   75
                                       76
                                           77
                                               78
                                                   79
                                                       80
                                                            81
                                                                82
                                                                    83
                                                                        84
                                                                            85
    [86]
                      89
                          90
                                               95
                                                       97
##
         86
              87
                  88
                               91
                                   92
                                       93
                                           94
                                                   96
                                                           98
                                                                99 100 101
                                                                           102
## [103] 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119
## [120] 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136
## [137] 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153
## [154] 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170
  [171] 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186 187
  [188] 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204
## [205] 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221
## [222] 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237
## [239] 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255
## [256] 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272
## [273] 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289
## [290] 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306
## [307] 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323
## [324] 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340
## [341] 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357
## [358] 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374
## [375] 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391
## [392] 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408
## [409] 409 410 411 412
```

b) Grafiquen el proceso hasta obtener 100 eventos,

poissonnohomogeneo(lambdat2,140)

### Simulacion de un proceso Poisson no homogéneo



```
$epocas
##
##
     [1]
          0.2080757
                      0.5835279
                                  0.6447513
                                             0.9246542
                                                         1.7705570
                                                                     1.8498910
                      2.0686980
                                              2.9683592
                                                         3.6894892
                                                                     3.7609414
##
     [7]
          1.9294436
                                  2.6141757
##
    [13]
          4.2913056
                      4.9193415
                                  4.9834981
                                             5.5040734
                                                         5.5416336
                                                                     6.1836585
          6.2737461
                      6.3142413
                                  6.3542869
                                             7.0622936
    [19]
                                                         7.0984685
                                                                     7.5141674
##
##
    [25]
          7.5170382
                      7.9145339
                                  7.9579008
                                             8.3953913
                                                         9.7630804
                                                                     9.8282833
##
    [31] 10.1252755 10.2021718 10.2746793 11.0744535 11.1297185 11.1425118
##
    [37] 11.1747264 11.4389256 11.6047076 11.6984836 12.5311384 12.5538819
##
    [43] 12.8293268 13.1690716 13.4973293 13.6715984 13.8569682 14.0540887
    [49] 14.8106359 15.0490310 15.5513225 16.3483907 16.4420692 16.5103966
##
##
    [55] 16.6720982 16.7578597 17.0037909 17.1561692 17.2154549 17.2569732
##
    [61] 17.4091748 17.4486170 17.5063376 18.2508101 18.2652623 18.3582517
##
    [67] 19.2988796 19.3524465 19.4614607 19.7571988 19.8589985 20.3955439
##
    [73] 20.6122015 20.6403151 20.8494720 21.2052695 21.5174456 22.1352028
    [79] 22.6841815 22.8330305 22.8436210 23.2571108 23.6397698 24.4324434
    [85] 24.8595308 25.2534079 25.5166746 25.6454404 25.7039898 26.0290678
##
    [91] 26.0925103 26.9302635 27.1666340 27.2777642 27.4255524 27.6808222
##
    [97] 27.9122410 27.9771215 28.5047693 28.5953526 28.9858693 29.2219154
##
   [103] 29.6706686 29.7554069
##
##
   $cuenta
##
                2
     [1]
                            5
                                 6
                                     7
                                              9
                                                 10
                                                                           16
           1
                    3
                        4
                                         8
                                                     11
                                                         12
                                                              13
                                                                  14
                                                                      15
                                                                               17
##
    [18]
          18
              19
                   20
                       21
                           22
                                23
                                    24
                                        25
                                             26
                                                 27
                                                     28
                                                         29
                                                              30
                                                                  31
                                                                      32
                                                                           33
                                                                               34
##
    [35]
          35
              36
                   37
                       38
                           39
                                40
                                    41
                                        42
                                             43
                                                 44
                                                     45
                                                         46
                                                              47
                                                                  48
                                                                      49
                                                                          50
                                                                               51
##
    [52]
          52
              53
                   54
                       55
                           56
                                57
                                    58
                                        59
                                             60
                                                 61
                                                     62
                                                         63
                                                              64
                                                                  65
                                                                      66
                                                                          67
                                                                               68
##
    [69]
          69
              70
                   71
                       72
                           73
                                74
                                    75
                                        76
                                            77
                                                 78
                                                     79
                                                         80
                                                              81
                                                                  82
                                                                      83
                                                                               85
```

```
## [86] 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 ## [103] 103 104
```

c) Estimen la probabilidad de que el n $\tilde{A}^{o}$ mero de eventos observados en el periodo de tiempo (1.25,3] es mayor que 2.

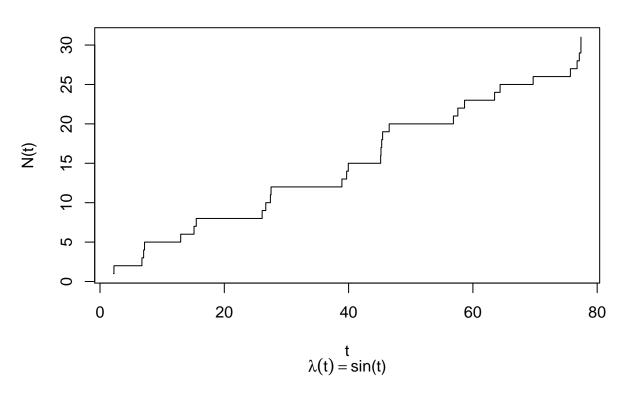
```
fr <- NULL
N <- 10000
for(i in 1:N){
x <- poissonnohomogeneo(lambdat2,10,pic=F);
fr <- c(fr,ifelse(1.25 < x$epocas[x$cuenta==2] & x$epocas[x$cuenta==2] <3,1,0))
}
sum(fr)/N</pre>
## [1] 0
```

### Pregunta 3

Simular un proceso Poisson no homogéneo con función de intensidad dada por  $\lambda(t) = \sin(t)$ 

```
lambdat <- function(t){sin(t)}
poisson.nohomogeneo<-function(lambdat,n){
    lambda <- 1 #mayoriza la función lambdat
    TT <- rexp(n,lambda) #genera variables exponenciales para los tiempos.
    s <- cumsum(TT) #acumula los tiempos en el vector s
    u <- runif(n) #obten n uniformes
    ss <- s[u <= lambdat(s)/lambda]
    Ns <- 1:length(ss) # Conteo
    plot(ss, Ns, type = "s", xlab = "t", ylab = "N(t)", main = "Simulacion de un proceso Poisson no homog
    return(list(epocas = ss, cuenta= Ns))
}
x <- poisson.nohomogeneo(lambdat,100)</pre>
```

## Simulacion de un proceso Poisson no homogeneo



### Pregunta 4

• 
$$P(B_2 \le 1)$$
 
$$P(B_2 \le 1) = P(\frac{B_2}{\sqrt{2}} \le \frac{1}{\sqrt{2}}) = \Phi(\frac{1}{\sqrt{2}}) = 0.3106966$$

• 
$$E[B_4|B_1 = x]$$
  
 $E[B_4|B_1 = x] = E[B_4 - B_1 + B_1|B_1 = x] = E[B_4 - B_1|B_1 = x] + E[B_1|B_1 = x] = E[B_4 - B_1] + E[B_1|B_1 = x] = 0 + x = x$ 

• 
$$Corr(B_{t+s}, B_s)$$
  

$$Corr(B_{t+s}, B_s) = \frac{Cov(B_{t+s}, B_s)}{\sqrt{Var(B_{t+s})}\sqrt{Var(B_s)}} = \frac{E[B_{t+s}B_s] - E[B_{t+s}]E[B_s]}{\sqrt{t+s}\sqrt{s}} = \frac{E[(B_{t+s} - B_s + B_s)B_s] - 0}{\sqrt{t+s}\sqrt{s}}$$

$$= \frac{E[(B_{t+s} - B_s)B_s + B_s^2]}{\sqrt{t+s}\sqrt{s}} = \frac{E[B_{t+s} - B_s]E[B_s] + E[B_s^2]}{\sqrt{t+s}\sqrt{s}} = \frac{0 + Var(B_s) + E[B_s]^2}{\sqrt{t+s}\sqrt{s}} = \frac{s+0}{\sqrt{t+s}\sqrt{s}} = \frac{s}{\sqrt{t+s}\sqrt{s}}$$

• 
$$Var(B_4|B_1)$$
  
 $Var(B_4|B_1) = Var(B_4 - B_1 + B_1|B_1) = Var(B_4 - B_1|B_1) + Var(B_1|B_1) + 2Cov(B_4 - B_1, B_1)$   
 $= Var(B_4 - B_1) + 0 + 2 * 0 = 3$ 

• 
$$P(B_3 \le 5|B_1 = 2)$$
  
 $P(B_3 \le 5|B_1 = 2) = P(B_3 - B_1 \le 5 - 2|B_1 = 2) = P(B_3 - B_1 \le 3) = P(\frac{B_3 - B_1}{\sqrt{2}} \le \frac{3}{\sqrt{2}}) = \Phi(\frac{3}{\sqrt{2}}) = 0.0420482$ 

#### Pregunta 6

Sea S(t) un movimiento Browniano geométrico con drift  $\mu=-0.25$  y  $\sigma=0.5$  Calcular

$$P\{S(90+t) \ge 100 | S(t) = 80\} = P\{\frac{S(90+t)}{S(t)} \ge \frac{100}{80}\} = P\{\ln[\frac{S(90+t)}{S(t)}] \ge \ln[\frac{100}{80}]\}$$

Como  $\ln\left[\frac{S(90+t)}{S(t)}\right] \sim N(90(0.10), 90(0.5)^2)$ 

Entonces

$$P\left\{\frac{\ln\left[\frac{S(90+t)}{S(t)}\right] - 90(0.10)}{\sqrt{90(0.5)^2}} \ge \frac{\ln\left[\frac{100}{80}\right] - 90(0.10)}{\sqrt{90(0.5)^2}}\right\}$$

$$= P\left\{\frac{\ln\left[\frac{S(90+t)}{S(t)}\right] - 90(0.10)}{\sqrt{90(0.5)^2}} \ge -1.850324\right\} = 1 - \phi(-1.850324) = 0.9678666$$

#### Pregunta 7

Sea S(t) un movimiento Browniano geométrico con drift  $\mu = -0.25$  y  $\sigma = 0.4$  Se busca el valor esperado del precio de esta acción en 6 meses.

$$E[S(t+6)|S(t)=35] = E\left[\frac{S(t+180)}{S(t)}S(t)\right] = 35E\left[\frac{S(t+180)}{S(t)}\right] = 35e^{6(-0.25 + \frac{0.4^2}{2})} = 12.62$$

Entonces la ganancia de la acción será 40 - 12.62 = 27.38

#### Pregunta 12

Se asignan  $\mu = 0.2, \, \sigma = 0.3, \, h = 1/52$ 

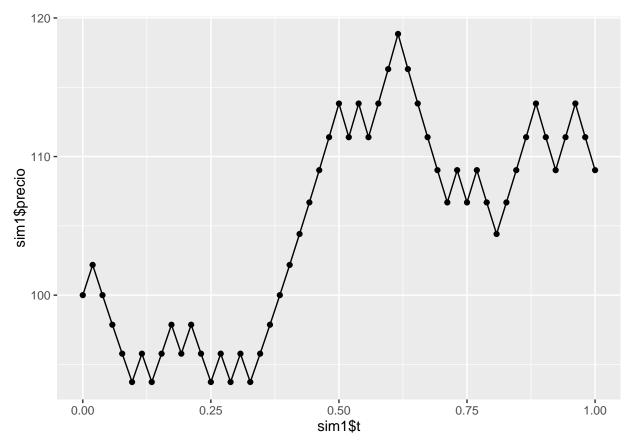
```
s0<-100
mu<-0.2
sig<-0.3
h<-1/52
```

Se calculan u, v, p

Se simula el modelo

```
modelobinom <- function(u, v, p, m, anio, s0){
h <- 1/m
t <- seq(0,anio,h)
precio <- NULL
precio[1]<- s0
for (i in 2:(m*anio+1)){
   precio[i] <- precio[i-1]*sample(c(u,v),1,prob = c(p,1-p))
}
return(list(t=t,precio=precio))
}</pre>
```

```
sim1 <- modelobinom(u, v, p, 52, 1, 100)
ggplot(data=data.frame(sim1$precio, sim1$t), aes(x=sim1$t, y=sim1$precio, group=1)) +
   geom_line()+
   geom_point()</pre>
```

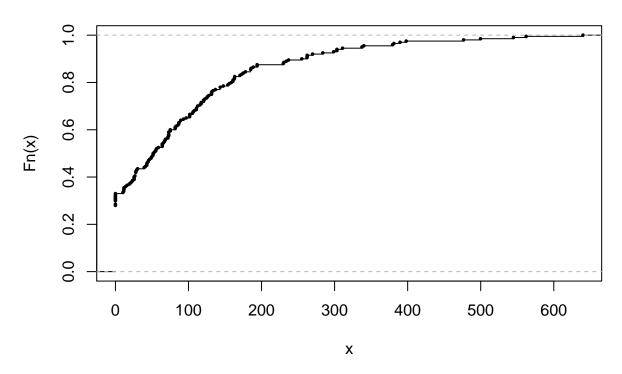


```
Ahora h = 1/252, y se define S_{max} = \{S_j | j = 0, 1, ..., 759\}
```

```
perdida <- NULL
for (i in 1:200){
    sim2 <- modelobinom(u,v,p,252,3,100)
    smax <- max(sim2$precio)
    perdida[i] <- smax - sim2$precio[757]
}

Fn <- ecdf(perdida)
plot(Fn, main ="Distribución Empírica de la pérdida", xlim= c(min(perdida), max(perdida)), pch = 16, cello
</pre>
```

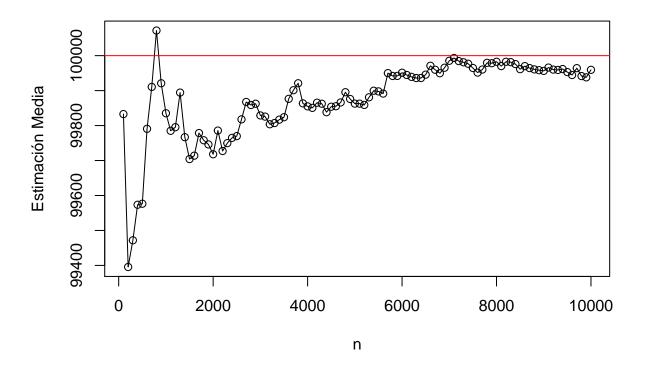
# Distribución Empírica de la pérdida



### Pregunta 13

```
PPCompuesto <- function(lambda,alfa,beta,t){</pre>
  N <- rpois(1, lambda*t)</pre>
  Yis <- rgamma(n=N, shape=alfa, rate=beta)
  Xt <- sum(Yis)</pre>
  return(Xt)
}
Si se hace una muestra con los parámetros \lambda=10,\,\alpha=1000,\,\beta=1 para X_{10}
E[X_{10}] = \lambda * 10 * \alpha * \beta = 10 * 10 * 1000 * 1 = 100,000
Var(X_{10}) = \lambda * 10 * (\alpha \beta^2 + \alpha^2 \beta^2) = 100, 100, 000
PPCompuesto(10,1000,1,10)
## [1] 97889.67
muestra <- NULL
for (i in 1:10000){
  muestra <- c(PPCompuesto(10,1000,1,10), muestra)</pre>
}
n \leftarrow seq(100,10000,by=100)
X <- NULL
Y <- NULL
```

```
for(i in n) {
   X[match(i,n)] <- mean(muestra[1:n[match(i,n)]])
   Y[match(i,n)] <- var(muestra[1:n[match(i,n)]])
   }
plot(n,X[1:length(n)], type = "o", ylab = "Estimación Media")
abline(h = 100000, col="red")</pre>
```



```
plot(n,Y[1:length(n)], type = "o", ylab = "Estimación Varianza")
abline(h = 100100000, col="red")
```

