Linear Algebra - Worksheet

Read this article before beginning the exercises: Linear Algebra Explained in 4 Pages

This assignment consists of 3 parts:

- Matrix Dimensions
- Vector Operations
- Matrix Operations

After completing the exercises by hand, use Python to check your work.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix}$$
 $v = \begin{bmatrix} 3 & 5 & -1 & 4 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 8 \\ 0 \\ 5 \end{bmatrix}$

1. Matrix Dimensions

Write the dimensions of each matrix.

1.5)
$$u \sim [1 \times 4]$$

$$1.6)$$
 w in $[74 \times 1]$

2. Vector Operations

Perform the following operations. Assume $\alpha = 6$.

$$(2.1) \vec{u} + \vec{v} = [6+3, 2+5, -3-1, 5+4] = [9, 7, -4, 9]$$

$$(2.2) \ \vec{u} - \vec{v} = [6-3, 2-5, -3+1, 5-4] = [3, -3, -2, 1]$$

$$2.3) \alpha \vec{u} = 6 \times [6, 7, -3, 5] = [36, 17, -18, 30]$$

$$2.4) \vec{u} \cdot \vec{v} = (6 \times 3) + (2 \times 6) + (-3 \times -1) + (5 \times 4) = 51$$

$$2.5) \|\vec{u}\| = \sqrt{\frac{18}{4}} + 10 + 3 + 20$$

3. Matrix Operations

Evaluate each of the following expressions, if it is defined; else fill in with "not defined." Do your work by hand on scratch paper.

$$3.1) A + C = \text{ not defined}.$$

$$3.2) A - C^{T} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -4 & -7 & -3 \\ 3 & 6 & 4 \end{bmatrix}$$

$$3.3) C^{T} + 3D = \begin{bmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{bmatrix} + 3 \cdot \begin{bmatrix} 3 & -2 & -3 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 9 & 6 \\ -1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 9 & -6 & -3 \\ 3 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 14 & 3 & 3 \\ 2 & 7 & 9 \end{bmatrix}$$

$$3.4) BA = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -5 & -4 \\ 2 & 7 & 4 \end{bmatrix}$$

$$3.5)$$
 $BA^T = not$ defined

Optional

$$3.6) BC = wot defined$$

$$3.7) CB = \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -6 \\ 9 & -8 \\ 6 & -6 \end{bmatrix}$$

3.8)
$$B^4 = \left(\begin{bmatrix} B^z \end{bmatrix}^2 : \left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right)^2 : \left(\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \right)^2 :$$

$$3.9) AA^T =$$

$$3.10) D^T D =$$