The Fellegi-Sunter Model of Record Linkage

Olivier Binette

August 26, 2020

Agenda

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Today's goal: Introduce the Fellegi-Sunter *model* of record linkage.

Focus on:

- Motivating example and some fundamental ideas.
- Databases, records and attributes.
- Pairwise record comparisons.
- Model parameters: m and u distributions, matching weights, and matching configuration matrix.

Afterwards:

- Model estimation and linkage rules.
- Sadinle (2017) and McVeigh (2020) go into more detail here.

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Outline:

1. Newcombe et al. (1959): "Automatic Linkage of Vital Records"
    - Provides a motivating example and introduces key ideas.
2. Fellegi and Sunter (1969): "A Theory for Record Linkage"
    - I will focus on the record linkage *model* rather than the theory.
    - I will deviate a little bit from the original paper in order to introduce the key supplemental idea of "Bayesian FS."

References: see "(Almost) All of Entity Resolution"

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2. pp. 13 — 19

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Newcombe et al. (1959)

Newcombe's Automatic Linkage of Vital Records

Newcombe et al. (1959). Published in *Science*:

Automatic Linkage of Vital Records*

Computers can be used to extract "follow-up" statistics of families from files of routine records.

H. B. Newcombe, J. M. Kennedy, S. J. Axford, A. P. James

Newcombe's Automatic Linkage of Vital Records

What they did:

- Introduced an automatic (probabilistic) record linkage technique and implemented it on the Datatron 205 computer.

Two things here:

- Stated record linkage as a statistical problem and proposed the first unsupervised probabilistic RL approach.
- They showed that a computer could be programed to perform RL.

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Newcombe's Automatic Linkage of Vital Records

Their applied goal: to link **34,138 birth records** from 1955 in British Columbia **to 114,471 marriage records** in the preceding ten year period.

	Marriage record	Birth record
Husband's family name	Ayad	Ayot
Wife's family name	Barr	Barr
Husband's initials	J Z	J Z
Wife's initials	M T	B T
Husband's birth province	AB	AB
Wife's birth province	PE	PE

Table 1: Example of identify information from compared marriage and birth records. This is adapted and translated from Table I of Newcombe (1969). AB and PE represent the Canadian provinces of Alberta and Prince Edward Island.

Newcombe's Automatic Linkage of Vital Records

Newcombe's algorithm:

1. Sort records by the Soundex coding of family names.
2. Where Soundex coding agrees, and informal likelihood ratio test determines whether or not to link.

Soundex coding:

- Olivier $\longrightarrow$ O416
- Oliver $\longrightarrow$ O416
- Olivia $\longrightarrow$ O410
- Rebecca $\longrightarrow$ R120
- Rebbeka $\longrightarrow$ R120
- Beka $\longrightarrow$ B200

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Likelihood ratio test:

- Imagine that two records agree on the husband's first initial J.
- Let p_L be the probability of this given that the records are actually a match, and let p_F be the probability of this given that the records are not a match.
- Let p_R be the proportion of the initial "J" among husbands.

Then

$$p_L \approx p_R, \quad p_F \approx p_R^2$$

so

$$\log(p_L/p_F) \approx -\log(p_R).$$

This is the "matching weight."

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Likelihood ratio test (cont'd):

- If the initial is very common, e.g. $p_R = 0.1$, then

  $$\log(p_L/p_F) \approx -\log(0.1) \approx 2.3$$

  is weights in a little bit for a match.

- If the initial is not at all common, e.g. $p_R = 0.0001$, then

  $$\log(p_L/p_F) \approx -\log(0.0001) \approx 9.2$$

  weights in much more in favor of a match.

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Newcombe's Automatic Linkage of Vital Records

Performance:

- Processed 10 record pairs per minute.
- About 98.3% of the true matches were detected, and about 0.7% of the linked records were not actual matches.
- "by far the largest part of the effort" was the preparation of punched card files reproducing marriage records in an adequate format.

Caveat:

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The Fellegi-Sunter model

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Fellegi and Sunter (1969). Published in JASA:

A THEORY FOR RECORD LINKAGE*

IVAN P. FELLEGI AND ALAN B. SUNTER

Dominion Bureau of Statistics

A mathematical model is developed to provide a theoretical framework for a computer-oriented solution to the problem of recognizing those records in two files which represent identical persons, objects or events (said to be *matched*).

The Fellegi-Sunter model

What this paper does:

- It formalizes the approach of Newcombe et al. (1959) in a decision-theoretic framework.

Given a pair of records, it considers three possible actions:

- to *link* them;
- to call them a *possible link*; or
- to *not link* them.

An "optimal" decision rule is proposed for this.

Here I'm focusing on the model rather than the decision-theoretic framework.

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Basic elements:

- Two *databases* A and B
  - Duplication *accross* but not within databases (bipartite record linkage).
- *Records* with corresponding *attributes* or *fields*
  - Name, age, address, SSN, etc.

What we want to do:

- Figure out which records refer to the same **entity** (a *person*, *object* or *event*.)

How we'll do that:

- We'll compare records in pairs from databases A and B, as to obtain multidimensional measures of similarity.
- Based on the measures of similarity, we'll try to group together the records which refer to the same entity.

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The Fellegi-Sunter model

	Field 1	Field 2	Field 3
Record no.	First name	Last name	Age
1	Olivier	Binette	25
2	Peter	Hoff	NA
$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_1	Beka	Steorts	NA

	Field 1	Field 2	Field 3
Record no.	First name	Last name	Age
1	Oliver	Binette	NA
2	Brian	K	NA
$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_2	Frances	Hung	NA

The Fellegi-Sunter model

Let $i = 1, 2, \ldots, N_1 \times N_2$ enumerate the set of all record pairs in $A \times B$.

Comparison vectors:

- For the ith pair of records, we compute a corresponding *comparison vector*

$$\gamma_i = (\gamma_i^{(1)}, \gamma_i^{(2)}, \ldots, \gamma_i^{(k)}).$$

- Each γ_i^j compares the jth field of the records.
- For example, if the jth field is "age," we could have $\gamma_i^j = 0$ if ages are the same, and $\gamma_i^j = 1$ if ages different.

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Binary comparisons:

- $\gamma_i^j \in \{0, 1\}$

Levels of agreement/disagreement:

- $\gamma_i^j \in \{0, 1, 2, \ldots, L_j\}$

How they're obtained:

- You choose!
- Use string distance functions to compare names.

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The set $\{\gamma_k\}_{j=1}^{N_1 \times N_2}$ of computed comparison vectors becomes the **observed data** for the Fellegi-Sunter model.

Next component of the model:

- The **matching configuration** $r = \{r_j\}_{j=1}^{N_1 \times N_2}$, with $r_j = 1$ if the jth record pair matches, and $r_j = 0$ otherwise.
  - This is the adjacency list representation. We can also use a matching configuration matrix.
- This is not a very efficient representation for bipartite matching. Saindle (2017) instead uses a *matching labeling*.

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The Fellegi-Sunter model

- For record pairs that are a ***m**atch* ($r_j = 1$), we assume that $\gamma \sim m$ independently.
- For record pairs that are ***u**nmatched* ($r_j = 0$), we assume that $\gamma \sim u$ independently.
- More precisely,

$$p\left(\{\gamma_j\}_{j=1}^{N_1 \times N_2} \mid r, m, u\right) = \left(\prod_{j\,:\,r_j=1} m(\gamma_j)\right) \times \left(\prod_{j\,:\,r_j=0} u(\gamma_j)\right).$$

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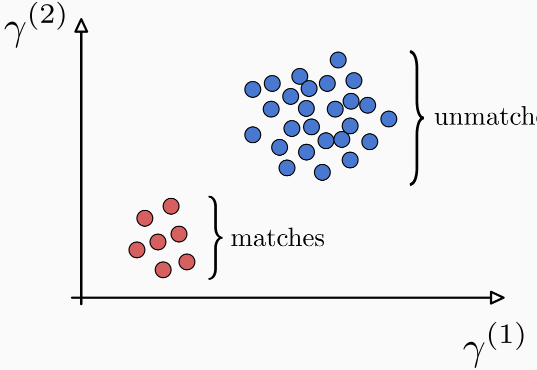

What's left to do?

- Estimate model parameters.
- Define a prior $p(r, m, u)$.
- Obtain a posterior

$$p(r \mid \{\gamma_j\}_{j=1}^{N_1 \times N_2}) = \int p(r, m, u \mid \{\gamma_j\}_{j=1}^{N_1 \times N_2})\, dm\, du$$
$$\propto \int p(\{\gamma_j\}_{j=1}^{N_1 \times N_2} \mid r, m, u) p(r, m, u)\, dm\, du$$

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- This is **not** what Fellegi-Sunter originally proposed
- Originally, FS proposed to estimate m and u on their own.
- Then, define the log-likelihood ratio (**matching weight**)

$$W(\gamma_j) = \log \frac{m(\gamma_j)}{u(\gamma_j)}.$$

- Say that the jth pair is a match if $W(\gamma_j)$ is large, that they're not a match if $W(\gamma_j)$ is small: this is a likelihood ratio test.

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What's the problem with the original FS approach?

- You consider all record pairs independently.
- You could link records a and b, and b and c, and yet say that a and c are not a match. This is incoherent.
- In the *bipartite record linkage* framework, we want to specify a prior on r which reflects the fact that there is duplication across but not within databases.

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- I've introduced the very basic components of the Fellegi-Sunter model
- Sadinle (2017) and McVeigh (2020) provide information about the priors and about model fitting.

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