Optimal Placement of Phasor Measurement Units with New Considerations

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Abstract—**Conventional phasor measurement unit (PMU) placement methods normally use the number of PMU installations as the objective function which is to be minimized. However, the cost of one installation of PMU is not always the same in different locations. It depends on a number of factors. One of these factors is taken into account in the proposed PMU placement method in this paper, which is the number of adjacent branches to the PMU located buses. The concept of full topological observability is adopted and a version of binary particle swarm optimization (PSO) algorithm is utilized. Results from the test on a 17-bus system are demonstrated.**

Keywords-PMU, placement, topological observability, binary PSO, adjacent branches

I. INTRODUCTION

Phasor measurement units (PMU) can provide time-synchronized data of bus voltages and branches currents at different locations and the sampling time is sub-second (typically 20, 30 or 60 samples/s). This makes PMU qualified to provide a wide-area dynamic view of a power system which might help improve the performance of various power system control and operation applications. A description of phasor measurement technology and its present and possible future applications in power system is well documented in [1] and [2].

One of the key issues of PMU applications is the selection of sites, i.e. the placement, which is an optimization problem. An optimization problem always consists of an iterative process, an objective function and some constraint conditions. For PMU placement, the constraint condition is the observability of the power system and the objective function is minimum cost of PMU systems. In the literature, the cost is normally represented as the number of PMUs. Reference [3] utilized a nondominated sorting genetic algorithm to get pareto-optimal solutions for PMU placement problem with two conflicting objectives, i.e. minimum number of PMUs and maximum topological observability redundancy. In [4] and [5], the concept of incomplete observability of the power system was considered, and the PMU placement problem was solved respectively by tree search method and binary particle swarm optimization (PSO) algorithm with the objective of minimum number of PMUs. With the same objective, [6] used binary PSO combined with immune algorithm while taking into account the N-1 observability redundancy, [7] utilized integer linear programming and [8] utilized binary PSO combined with genetic algorithm.

However, one fact which has been neglected in the literature is that the cost of one installation of PMU is not always equal to another. The cost could highly depend on some factors such as the number of adjacent branches of the PMU located buses. This is taken into account in the PMU placement method discussed in this paper.

In this paper, the concept of full topological observablity of the power system is adopted as the constraint conditions with its principle described in section II. One version of binary PSO algorithm with the objective of minimum cost of PMU installations is introduced in section III. In section IV, a case study on a 17-bus test system is presented. Section V gives the conclusions and suggests the possible future work.

II. FULL TOPOLOGICAL OBSERVABILITY

In a power system, if the node voltage of one bus can be directly measured or calculated by using other known node voltages and branch currents, the bus is defined as observable. If all buses are observable in the power system, then the power system can be defined as with a full observablity [6].

There are two methods to assess the observability of a power system: numerical and topological. Numerical observability might involve large amount of computation and can be easily influenced by cumulative error, while topological observability method is fast and practical [6]. A power system with at least one spanning measurement tree of full rank can be defined as topologically observable [7].

As in a PMU placement problem, three rules can be used to analyze the topological observability of each bus within the target power system:

Rule 1: a bus with PMU installation is observable, and its adjacent buses are all observable because their voltages can be calculated by Ohm's law with the help of the PMU measurement;

Rule 2: if a bus is adjacent to an observable zero-injection bus (a bus with zero net injection of power from the connected loads and generators) to which all other adjacent buses are

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observable, then the bus is observable because its voltage can be calculated by KCL and Ohm's law;

Rule 3: if all buses adjacent to a zero-injection bus are observable, then the zero-injection bus is observable because its voltage can be calculated by KCL and Ohm's law.

According to these rules, the observability of a power system can be directly analyzed with its topology information known, including power system adjacency information, zero-injection bus location and PMU installation location [6].

III. Binary Particle Swarm Optimization

A. Particle Swarm Optimization

PSO was first developed by Kennedy and Eberhart in 1995 and it was inspired by the behaviour of bird blocks and fish schools.

PSO deals with a swarm of particles that fly through the problem solution hyperspace. Each particle in the swarm involves a position array and a velocity array. The position array is a possible solution to the problem, while the velocity array is used to adjust the position array in each iteration, and the velocity itself is also adjusted in each iteration. Equations (1) and (2) show the iterative mechanism of conventional PSO.

$$\begin{aligned} V_i(t+1) &= \omega \times V_i(t) + c_1 \times ran_1 \times (p_i - X_i(t)) \\ &+ c_2 \times ran_2 \times (p_g - X_i(t)) \end{aligned} \tag{1}$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \tag{2}$$

In (1) and (2) V is the velocity array, X is the position array, i is particle number, t is iteration number, c_1 and c_2 are acceleration coefficients, ran_1 and ran_2 are stochastic numbers arranged between 0 and 1, p_i is the historical best position of particle i, p_g is the historical best position of the whole swarm. In each iteration, constraint conditions are assessed for each particle and the objective function is calculated for those satisfying particles. The best positions p_i and p_g here are derived according to this calculation. When a new position that can minimize the objective function better than p_i is derived by particle i, p_i should be replaced with that position. Similarly, if a new position that can minimize the objective function better than p_g is derived by any particle in the swarm, p_g should be replaced with that position. In this iterative method, the movement of each particle in the solution hyperspace evolves to an optimal or near-optimal solution [9].

B. Binary Particle Swarm Optimization

For PMU placement problem, the position array X of a particle in PSO algorithm gives the information of PMU installation location. This is a binary bit array with the length of the number of buses in the target power system. Value '1' of one bit means an installation of PMU at the corresponding bus, while value '0' means no PMU installation at the corresponding bus. In this case, instead of conventional PSO, a version of binary PSO algorithm should be used, which is interpreted by (3), (4) and (5) [8].

$$\begin{aligned} V_{ij}(t+1) &= \omega \times V_{ij}(t) + c_1 \times ran_1 \times (p_{ij} - X_{ij}(t)) \\ &+ c_2 \times ran_2 \times (p_{gj} - X_{ij}(t)) \end{aligned} \tag{3}$$

$$X_{ij}(t+1) = \begin{cases} 0, \text{ if: } \rho \geq \mathrm{Sig}(V_{ij}(t+1)) \\ 1, \text{ if: } \rho < \mathrm{Sig}(V_{ij}(t+1)) \end{cases} \tag{4}$$

$$\mathrm{Sig}(x) = \frac{1}{1+\exp(-x)} \tag{5}$$

Here j denotes the number of the element in an array, which is as well the number of the bus in the target power system. ρ is a stochastic number arranged between 0 and 1.

To take into account the difference of cost of PMU installation at different buses, the objective function in binary PSO algorithm is defined as:

$$\min\left(\sum_j (X_{ij} * weightb_j)\right) \tag{6}$$

In (6), *weightb* involves the impact of the number of adjacent branches to each bus. Since PMUs with more measurement channels are more expensive, *weightb* is assigned bigger values for buses adjacent to more branches.

The flowchart of this binary PSO application is illustrated in Fig.1.

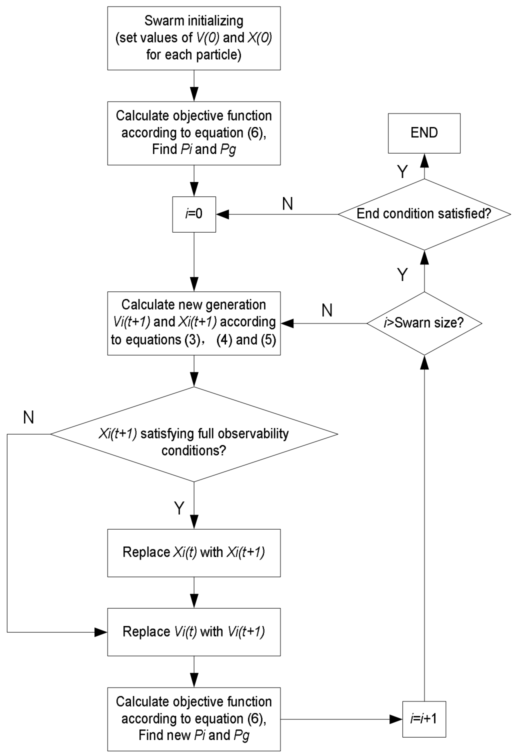

Figure 1. Flowchart of the binary PSO

This process is programmed in MATLAB. A maximum iteration number is used as the 'end condition'.

IV. CASE STUDY

A. Observability analysis

A 17-bus test system with different voltage levels [10] is selected as the target system and the topology is shown in Fig.2.

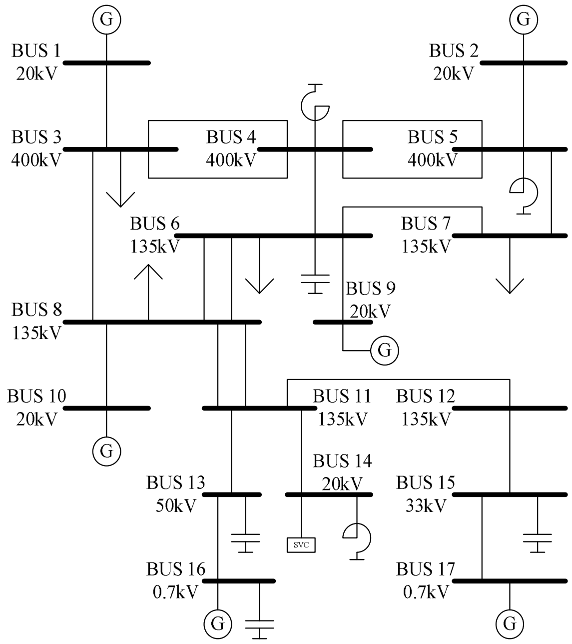

Figure 2. Topology of the 17-bus test system

$$ADJ = \begin{bmatrix}
1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&1&1&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1
\end{bmatrix} \quad (7)$$

ADJ in (7) is the adjacency matrix of this power system. In the matrix, '1' means there is at least one connection between the two buses respectively denoted by the column number and the row number of the element, while '0' means there is no connection.

The zero-injection bus array of this power system is:

$$zero = [0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,0], \quad (8)$$

In (8) '1' means that the corresponding bus is a zero-injection bus, while '0' means the other way.

The adjacency matrix *ADJ*, zero-injection bus array *zero* and the PMU installation array *X* participate in the target power system observability assessment for each particle in each PSO iteration according to the three rules mentioned in section II.

B. Objective function calculation

To involve the impact of adjacent branch number in the objective cost function, *weightb* is defined in (9) according to [11].

$$weightb_j = \begin{cases} 1.0, \text{ if } NUM_j < 3 \\ 1.5, \text{ if } NUM_j \geq 3 \end{cases} \quad (9)$$

In (9), NUM_j stands for the adjacent branch number of bus *j*.

So for the target 17-bus test system, the *weightb* array can be illustrated in Fig. 3.

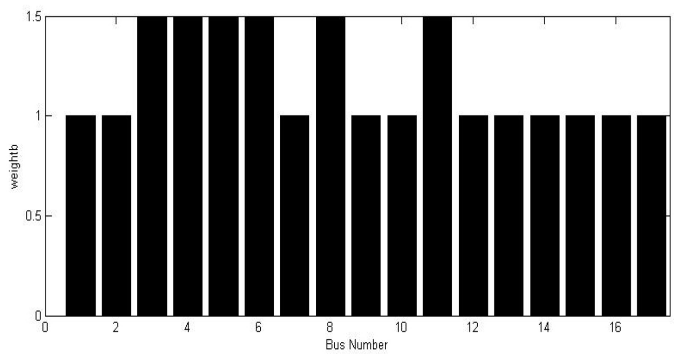

Figure 3. *weightb* in the 17-bus system

Then in the PSO process, the objective function can be calculated according to (6) with this array and also the PMU installation array *X* for each particle obtained in each iteration. Those PMU installation arrays resulting in small objective function values could be recorded as *pi* and *pg*, while *pi* and *pg* would be replaced when new PMU installation arrays with smaller objective function values appear in the following iterations.

C. Results

With maximum iteration number set to 1000 and swarm size set to 50, feasible solutions of the proposed PMU placement method are shown in table 1. To compare with the conventional method which considers only the number of PMUs as the objective function, some solutions of the conventional method are shown in table 2.

TABLE I. RESULTS FROM THE PROPOSED METHOD

PMU number	PMU location	Cost
6	1,2,6,10,13,17	6.5
6	1,2,6,10,15,16	6.5
6	1,2,6,10,13,15	6.5
6	1,5,9,10,13,15	6.5
6	1,5,9,10,15,16	6.5
6	1,5,9,10,13,17	6.5

TABLE II. RESULTS FROM THE CONVENTIONAL METHOD

PMU number	PMU location	Cost
6	2,3,6,10,13,15	7.0
6	3,5,8,9,13,15	7.5
6	2,3,6,8,15,16	7.5
6	1,2,6,10,15,16	6.5
6	1,5,9,10,15,16	6.5
6	1,2,6,10,13,15	6.5
6	3,5,6,9,15,16	7.5
6	3,5,9,10,15,16	7.0
6	1,5,8,9,13,17	7.0
6	3,5,6,10,13,17	7.5
6	1,2,6,8,13,17	7.0

In these two tables, the 'cost' values are calculated using (6). The solutions in table 1 are the only six solutions that can be obtained by the proposed method after 100 tests, while the solutions of the conventional method could be more than those shown in table 2. Both methods are able to find the smallest number of PMU installations for full topological observability, which is 6. However, when considering the difference of cost of PMU installations at different locations with regard to different numbers of adjacent branches, only the proposed method is qualified to target to the best solutions with the cost function value of 6.5.

V. CONCLUSIONS

From a more practical point of view, the cost of one PMU installation at different locations could be very different. A number of factors could influence the cost, such as the communication condition of the located bus and the number of adjacent branches at the bus. The latter factor is considered in the proposed PMU placement optimization method in this paper and the method is proved to be more qualified than the conventional methods to find the best solutions in the sense of full power system observability with lowest cost which is not only represented as the number of PMUs.

Nowadays, more and more applications of PMUs in power system control and operation are emerging. PMU placement problem in real situation could involve more than full observability and lowest cost. Requirement from the specific applications of PMUs should be taken into account, for example, observability redundancy, influence of PMU device malfunction or communication failure, the performance of corresponding power system operation and control schemes. These factors should be considered in the future work in this area.

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