

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/334888262>

# Multi-Frame Super-Resolution: A Survey

Conference Paper · December 2018

DOI: 10.1109/ICCIC.2018.8782382

CITATIONS

0

READS

28

5 authors, including:



**Mahmoud Khattab**

3 PUBLICATIONS 0 CITATIONS

[SEE PROFILE](#)



**Ali Amer Alwan**

International Islamic University Malaysia

64 PUBLICATIONS 129 CITATIONS

[SEE PROFILE](#)



**Ahmed Badawy**

King Khalid University

13 PUBLICATIONS 16 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:



Information Extraction from Web Pages [View project](#)



Disaster Recovery and Business Continuity for Database Services in Multi-Cloud [View project](#)

# Multi-Frame Super-Resolution: A Survey

Mahmoud M. Khattab<sup>a,b</sup>, Akram M Zeki, Ali A. Alwan

Department of Computer Science

<sup>a</sup>International Islamic University Malaysia, IIUM

Kuala Lumpur, Malaysia

[mmkhattab2000@gmail.com](mailto:mmkhattab2000@gmail.com)

Ahmed S. Badawy

Department of Computer Engineering

<sup>b</sup>King Khalid University, KKU

Abha, Saudi Arabia

[bdwy@kku.edu.sa](mailto:bdwy@kku.edu.sa)

Lalitha Saroja Thota

Department of Computer Science and Engineering

Sree Vaanmayi Inst of Engg and Technology

BibiNagar, Nalgonda Dist, Telangana, India

**Abstract**—One of the primary measurements of image quality is image resolution. High-resolution images are often required and desired for most of applications as they embody supplementary information. However, the best utilization of image sensors and optical technologies to increase the image pixel density is usually restrictive and overpriced. Therefore, the effective use of image processing techniques for acquiring a high-resolution image generated from low-resolution images is an inexpensive and powerful solution. This kind of image improvement is named image super-resolution. This paper undertakes to investigate the current super-resolution approaches adopted to generate a high-resolution image. Furthermore, it highlights the strengths and the limitations of these approaches. More to the point, several image quality metrics are discussed to measure the similarity between the reconstructed image and the original image.

**Keywords**—*Super-resolution, frequency domain, spatial domain, image interpolation, resolution enhancement, regularized framework*

## I. INTRODUCTION

In fact, the global world experienced an enormous advancement in software and hardware technologies within the past decade. Industrial sectors have made the best use of modern technology to generate electronic devices such as computer systems, cellular mobile phones, personal digital assistant (PDA), and innumerable devices at inexpensive costs [1]. Moreover, the manufacturing methods of camera sensor have been highly developed to generate high-quality digital cameras. Many applications of computer vision such as medical imaging, satellite imaging, pattern recognition, surveillance and forensic, astronomical imaging, and target detection are still in an urgent need for high-resolution (HR) image which frequently exceeds the abilities of the HR digital cameras [2, 3].

Optical resolution is certainly a method of measuring the capability of the camera system or an element of the camera system utilized for explaining the image details. Accordingly, there are two primary methods of raising the spatial image resolution: firstly, the technical strategy method that is related to hardware solutions. Secondly, the analytical strategy method that is associated with software solutions [4]. With respect to the technical strategy concept, it identifies either the

improvement of registration device or the replacement with a higher resolution device. However, the use of a highly qualified camera is often limited by its high price, large size, or sensor manufacturing limitations [5]. Concerning the analytical strategy concept, it is usually low-priced and more flexible in comparison to the hardware solutions. The class of resolution improvement methods got the name of super-resolution (SR) image reconstruction [2, 6].

On one hand, SR image reconstruction usually represents a great enhancing and challenging method of digital imaging. The reason is that it attempts to rebuild HR images by combining the partial information presented inside several low-resolution (LR) images of a particular scene through image reconstruction [7]. On the other hand, SR incorporates up-sampling of LR images. Then, it eliminates distortions such as noising and blurring. In comparison to different image improvement techniques, SR not only increases the quality of LR images by improving their particular spatial resolution but also tries to eliminate distortions [2].

This research paper presents a survey of most important multi-frame SR approaches and it is organized as follows. Section 2 illustrates observation model so that it reflects the HR image into the observed LR images. Different multi-frame SR approaches are represented in Section 3. The image quality metrics are discussed in Section 4. A detailed discussion is offered in Section 5, and the paper is concluded in Section 6.

## II. OBSERVATION MODEL

An observation model identifies the true manner where the observed LR images are acquired. Generally, we assume that the procedure of image acquisition consists of warping, blurring, down-sampling, and noise degradations as shown in Figure 1, and the observation model is definitely simulated the following:

$$y_k = DB_k M_k x + n_k \quad (1)$$

where  $k$  is LR images that participated in the reconstruction process and  $x$  is the original image that degraded by warping ( $M$ ), blurring ( $B$ ), down-sampling ( $D$ ), and additive noise ( $n$ ). After the model is well-known, it may be used to inverse the process in order to retrieve the HR image from a various of LR

images. Therefore, it can be said that the observation model is inverted in order that the problem requires a prior information from the HR image to get a reliable and suitable solution. [2, 6].

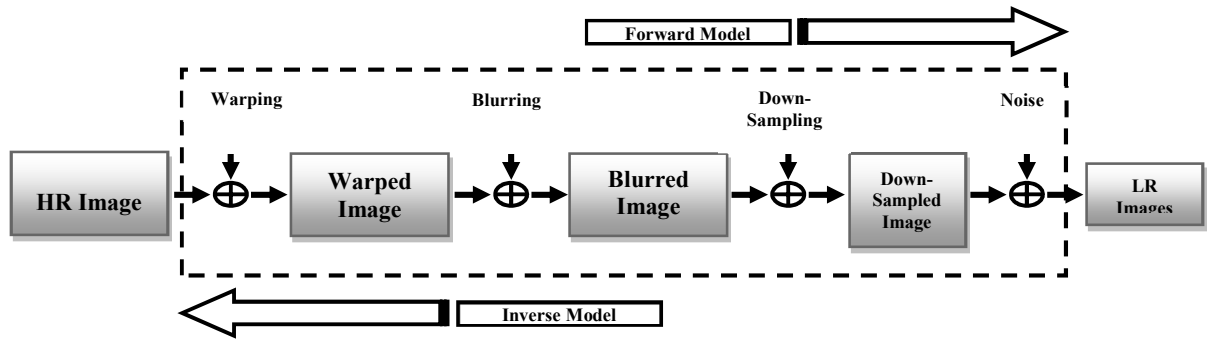


Fig. 1. The observation model employed in most SR techniques

### III. MULTI-FRAME SUPER-RESOLUTION APPROACHES

According to the above-mentioned discussion, the essential intention of SR image reconstruction is certainly to generate a powerful HR image dependent on a few LR images that are captured through the exact scene. There are diverse approaches for rebuilding the new HR image through the observed LR images. These kinds of approaches attempt to address particular aliasing artifacts that are generally contained in LR images due to the down-sampling process by emulating the particular image observation model. In this paper, SR image reconstruction approaches can be categorized into three classes: (i) frequency-domain approaches, (ii) interpolation-based approaches, and (iii) regularization-based approaches as shown in Figure 2. These varieties of approaches are certainly studied within the following subsections.

#### A. Frequency Domain Approaches

The frequency domain approaches are really well-known ways for enhancing image resolution. They can unquestionably rebuild the desired HR image from the aliasing artifacts which are usually present in every LR image. In fact, this is achieved through transforming the input LR images towards the frequency domain. This transformation is followed by an estimation of the reconstructed HR image in frequency domain. Lastly, the reconstructed HR image is converted back again into the spatial domain. Actually, the first SR approach is created by Tsai and Huang [8] based on the frequency domain. They focus on LR satellite images. Therefore, a lot of researchers subsequently extend this method to produce different types of SR approaches. These approaches are generally divided into three categories: discrete Fourier transform (DFT), discrete cosines transform (DCT), and discrete wavelet transform (DWT). These categories are fully described in the following subsections.

- Discrete Fourier Transform

Tsai and Huang [8] suppose that the series of LR images are globally translated and totally freed from distortions such as blurring or noise effects. First, they suggest transforming and merging the LR images information into the DFT domain. This

merging is based on the relationship amongst the aliased DFT parameters of the detected LR images and the unidentified HR image. Second, the mixed data are converted back again to the spatial domain where, in fact, the new image could have an increased resolution in comparison to the LR images. An expansion of this approach [8] is presented by Kim et al. [9]. They present a weighted recursive least square algorithm that relies on the aliasing relationship among both LR images and HR image. According to their approach, they presume that all LR images include the exact blurring and noising characteristics. This technique is further enhanced by Kim and Su [10] to check out distinct blurs for every LR image. Bose et al. [11] propose the recursive total least squares approach for SR reconstruction to minimize the negative effects of registration errors.

Vandewalle et al. [12] typically make use of correlation method in the frequency domain to discover motion parameters among the LR images. According to the frequency domain, these motion parameters are approximated depending on the actual fact of the spatial shift of the LR images. Also, it varies merely with a phase shift among the two images that are actually acquired from their correlation. Utilizing the phase of correlation approach leads to transforming each image rotation and scale into vertical and horizontal shifts. To reduce errors that are generated by aliasing, the small portions of the discrete Fourier parameters are used since they are free from aliasing. Then, the LR images are mixed based on the relationship among the aliased DFT parameters of the noticed LR images and the unidentified HR image. The following fusion is the conversion of information back again into the spatial domain to obtain the reconstructed HR image [12].

- Discrete Cosines Transform

Kang and Rhee develop the DCT approach [13] to reduce the computational costs, where they apply the multi-channel adaptive regularization parameters to eliminate ill-posedness. Park et al. [14] suggest an HR reconstruction technique for DCT depending on compressed images. They concurrently approximate the quantization system noise. To simplify this process, they make the quantization noise in the spatial domain as a colored Gaussian noise process. Furthermore, they obtain the inverse noise covariance matrix to create a multi-channel

smoothing functional. The suggested inverse noise covariance matrix differs from the signal pattern. Additionally, it prevents a demand to make the original DCT coefficients at low bit-rates.

Kumar [5] proposes a DCT approach with the reliable denoising, which regularly rebuilds an HR image from a few of LR images.

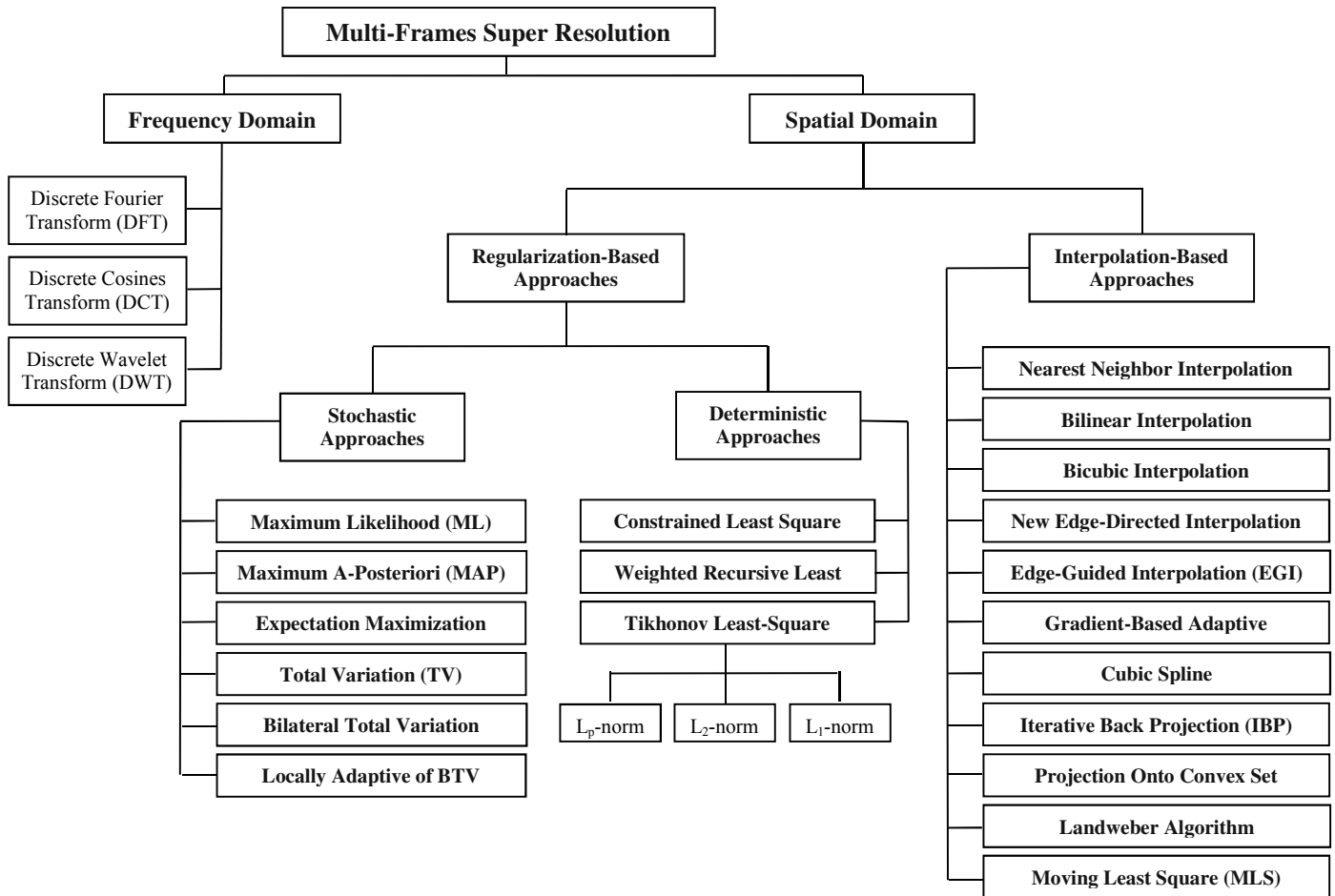


Fig. 2. The most popular multi-frames SR approaches

- Discrete Wavelet Transform

Recently, a large number of researchers start to research the utilization of the wavelet transformation in SR reconstruction. In such a maneuver, they endeavor to handle the SR problem and to extract the complete details which are usually dropped or degraded through the procedure of the image acquisition. This is motivated by the fact that the wavelet transformation offers a strong and effective multi-scale representation of the image for retrieving the high-frequency details [15]. This method commonly treats the LR images as a low-pass filter of the unidentified HR image. The goal of this method is to approximate the scale subband coefficients, which it is accompanied by employing the inverse wavelet transformation to create the HR image.

Nguyen and Milanfar [15] use wavelet interpolation accompanied by restoration technique for SR. They initially suggest to compute the wavelet coefficients of the observed LR images. After that, they interpolate them on the HR grid for treating the blurring values. Through deconvolving, an estimation of HR image is achievable if the interpolated values are well-known blurring. El-Khamy and et al. [16] use the

wavelet domain to execute the registration of several LR images. Wavelet coefficients are denoised and merged after registration by utilizing a regularization method. Interpolation strategies are used to obtain HR wavelet coefficients. Lastly, an inverse wavelet transform is executed to obtain the HR image in the spatial domain. Chappalli and Bose [17] additionally apply very flexible thresholding methods to eliminate the noise from the wavelet coefficients and build up a real-time denoising for SR reconstruction strategy. Ji and Fermuller [18, 19] offer a powerful wavelet SR method to tackle the mistake incurred in both the registration computation and the blurring detection computation. In this way, they break down the wavelet coefficients directly onto two channels. Finally, these coefficients seem to be up-sampled, filtered, and merged to obtain the simulated image. The SR image is gathered using iterative back projection technique with effective regularization conditions at each iteration to eliminate the noise. Li [20] suggests image resolution improvement by extrapolating high-band wavelet coefficients.

Anbarjafari and Demirel [21] recommend an exciting new SR technique depending on the interpolation of the high-

frequency subband images acquired by DWT and the input LR images. The suggested method takes advantage of DWT to split an image to several subband images. After that, these subband images are generally interpolated accompanied by merging them to obtain a new HR image using inverse DWT.

### B. Spatial Domain Approaches

Spatial domain approaches are classified as one of the most popular approaches to develop the SR image. The popularity of these approaches is because the motion is not restricted only to translational shifts. Therefore, a more general global or non-global motion may also be integrated and managed. In this paper, spatial domain approaches are usually split into interpolation-based approaches and regularization-based approaches. They are explained in the following subsections.

- Interpolation-Based Approaches

These approaches are the most intuitive techniques for constructing the SR image. Firstly, they project all the obtained LR images into the reference image to merge all the obtainable details from every image. The reason is that all LR images present quantity of extra details regarding the scene. Lastly, the image is deblurred for creating the SR image. The interpolation-based approaches consist of the following three steps: (i) registration of LR images, (ii) interpolation into the HR grid, and (iii) restoration of HR image [2]. First, for the image registration, it is the procedure of geometrically alignment a group of LR images with regards to one specific LR image named the reference image. LR images contain a distinct sub-pixel shifting and rotation from each other. As a result, it is necessary to obtain a correct approximation of movement parameters before merging them to generate an HR image. Due to the incorrect estimation of movement parameters, it produces a variety of visual artifacts that corrupt the resolution of the reconstructed image. Second, the image interpolation is used for generating an HR image by estimating new pixels in the image based on a group of pixels. Finally, the image restoration is used for improving the reconstructed HR image that is created from the interpolation step [2].

The most simple approach for image interpolation is the nearest neighbor interpolation [22]. For every pixel on the HR grid, the nearest known LR pixel is chosen and the value of this pixel is merely used as the value at the grid point. It is considered the fastest method among other interpolation approaches. Nevertheless, this approach brings the significant distortion, shows up the mosaic, and produces images with a blocky visibility. A second simple and well-known approach is a bilinear interpolation. The bilinear interpolation [23] takes into account the nearest 2x2 neighbors of known pixel values around the unidentified pixel by computing a weighted average of the four pixels to achieve its last interpolated value. This approach leads to much smoother images than the nearest neighbor interpolation approach. However, the bilinear interpolation approach is more complicated than the nearest neighbor interpolation approach. Therefore, it has a high computation cost and does not have sufficient results. Moreover, it has a low pass filtering characteristics, therefore, the high-frequency component is passed and the image contour has some extent of fuzzy.

A bicubic interpolation approach is generally executed in the same manner of the bilinear interpolation approach by taking into consideration the nearest 4x4 neighbors of known pixels. Since they are at different distances from the unidentified pixel so that this approach can relatively obtain a clear image quality. However, it requires a greater amount of computation. Therefore, this approach generates visibly sharper images in comparison to the previous two approaches. It perhaps gets the optimal mixture of processing time and output quality [23]. Additionally, this approach can be widely used in a large number of image processing applications such as Adobe Photoshop, Adobe After Effects, Avid and Macromedia Final Cut Pro etc. A New Edge Directed Interpolation (NEDI) [23] is another approach based on the local covariance parameters of the LR images.

An Edge Guided Interpolation (EGI) approach [24] splits the neighbor of every pixel to make a couple observation subsets through the orthogonal directions and estimate the lacking pixel. This approach merges both of the estimated values into the powerful estimation by applying linear-minimum mean square error estimation. A gradient-based adaptive interpolation [25] takes into consideration the distance among the interpolated pixel and the nearby respected pixel. The results of this suggested technique increase and enhance the quality of recovered images. Furthermore, it is a powerful technique to detect the registration mistake and needs a low-computational cost.

A cubic spline approach [26] meets a piecewise continuing curve and moving through lots of points. The fundamental job of this approach is to compute weights that are used to interpolate the information. The steps of registration, interpolation, and restoration can be executed to accomplish the HR image that comes from a series of LR images through an Iterative Back Projection (IBP) approach [27]. In IBP approach, the reconstructed image is approximated by reducing the error among the simulated LR images and it's observed. The IBP approach is extremely very simple and easy to understand. However, it is not generally going to give a unique result because of the ill-posed trouble. An additional simply implemented SR approach is the Projection Onto Convex Set (POCS) that has been developed by Stark and Oskoui [28]. In POCS approach, a set of restrictions are described to limit the space of HR image. These restriction sets are curved and facilitated the particular attractive of SR image features such as positivity, fineness, bounded energy, and dependability. The intersection coming from all these sets demonstrates the area of the allowable solution. As a result, this problem is minimized for locating the intersection of the restriction sets. The projecting operators are decided for every convex restriction set to get the solution. This operator reflects the primary estimation of the HR image against the relevant restriction set. Repetitively executing this method, a great solution is acquired at the area of intersection of the k convex restriction sets. This approach actually does not integrate any observation noise.

In order to enhance the quality of an image, various methods are suggested to improve the interpolation based approaches such as: Ur and Gross [29] execute a non-uniform interpolation of a couple of spatially shifted LR images. They use the generalized multi-channel sampling theorem. The benefit of

this method is the low-computational cost, which it is actually ideal for real-time applications. However, the ideality of the whole rebuilding process is not assured, because of the interpolation mistakes are not considered. Komatsu et al. [30] show a new scheme to obtain a better resolution image. They apply the Landweber algorithm at multiple images concurrently with multiple cameras. In addition, they make use of the block-matching approach to measure comparative shifts. If the cameras currently have the same aperture, it enforces serious restrictions both in their agreement and in the configuration of the scene.

- Regularization-Based Approaches

Generally, the SR image reconstruction approaches are really an ill-posed problem due to two factors. A first factor is an inadequate number of LR images, while the second factor is the ill-conditioned blur operators. Techniques which usually use to support the inversion of the ill-posed problem are identified as a regularization. The regularization approach takes advantage of the prior knowledge of the unidentified HR image to resolve the SR problem. Therefore, deterministic and stochastic regularization approaches are offered in the following subsection.

### I. Deterministic Approaches

The deterministic approach presents the regularization term which transforms the ill-posed problem into a well-posed one. It happens through the use of prior information about the perfect solution depending on the regularization term ( $R$ ) and regularization constant ( $\lambda$ ) [31]. A constrained least square (CLS) regularization approach incorporates the smoothness constraints as a priori information. In this instance,  $R$  is a high-pass filter which usually reduces the quantity of high-frequency details for the new reconstructed image. While the regularization parameter  $\lambda$  handles the high-frequency details. The larger values of  $\lambda$  may possibly lead to smooth the generated image. These values are a suitable choice if a little amount of LR images are present and/or there is a great deal of noise. While the smaller values of  $\lambda$  may produce a noisy solution which can be applied when a huge quantity of LR images are present and the quantity of noise is little [2]. Tikhonov least-square approach [31] incorporates l2-norm of the second order derivation ( $p=2$ ) of the HR image as a regularization term. The primary benefit of the l2-norm is that it is certainly easy to resolve. On the other hand, The l2-norm does not promise a unique solution. Also, it is optimum when the model error is white-Gaussian distribution [31, 32]. For this reason, Farsiu et al. [33] employ an alternative l1-norm ( $p=1$ ). They verify that the l1-norm works more effectively than the l2-norm when the images consist of non-Gaussian errors for very quickly and powerful SR. Several researchers employ developed approaches [34-36] with combined error modes. Therefore, the lp-norm function ( $1 \leq p \leq 2$ ) may also be utilized as the constraint function due to its convex property and its own convenience for the imaging model errors. When  $1 \leq p \leq 2$ , it leads to a weighted mean of measurements. If the value of  $p$  is near to one, the solution is computed with a greater weight throughout the measurements near to the median value. When the value of  $p$  is near to two, the solution is estimated to the average value [33]. Sometimes, images are infected by

Gaussian and non-Gaussian errors. Therefore, the lp-norm function is recognized to be a highly effective solution [35]. Kim and Bose [9] suggest a weighted recursive least square algorithm for generating the SR image. The weight depends upon the prior information of the image. This algorithm provides higher weights to the LR images. With various weights, the problem basically decreases to the general least square estimation. Finally, interpolation and restoration are adapted to get the HR image. Mallat and Yu [37] recommend a regularization SR approach which employs adaptive estimators acquired by combining a family group of linear inverse estimators.

### II. Stochastic Approaches

Depending on the observation model explained above, the goal is to rebuild the HR image from a collection of warped, blurred, noisy, and under-sampled images. As the model in (2) is an ill-conditioned, SR actually is an ill-posed inverse problem. As a result, the stochastic approaches are well-known especially the Bayesian theorem. The reason is that they present the adaptable, flexible, and convenient way to incorporate a priori information. Moreover, they create a powerful relationship between the LR images and the unidentified HR image [38-44].

Maximum likelihood (ML) is suggested by Tom and Katsaggelos [45]. The goal of this approach is to get the ML estimation of the HR image. On one hand, it only takes into account the relationship amongst the LR images and the primary HR image. On the other hand, Maximum A-Posteriori (MAP) method combines the prior image model to expose the expectancy of the unidentified HR image. ML and MAP are famous approaches because of their flexibility and adaptability for protecting edges and combining parameters [38-44]. Tipping and Bishop [46] employ an expectation-maximization (EM) algorithm to approximate the hyper-parameter value by increasing its misfit likelihood function. However, the EM algorithm produces a huge computational load. Moreover, it does not always meet the global ideal [47]. Pickup et al. [48] develop a new technique by minimizing the computational cost. They alter the prior image model by taking illumination adjustments among the collected LR images. He and Kondi [49] suggest a strategy to simultaneously estimate both hyper-parameter value and the HR image by enhancing the cost function. On the other hand, this approach considers that the shifts between LR images are limited to integer value on the new HR grid. In [42], Woods et al. suggest to use the EM algorithm and calculate the ML approximation of the hyper-parameter.

Total variation (TV) regularization is initially suggested by Osher et al. [1, 50-52] to protect edge information and prevent ringing results. On the other hand, the finding results of the TV prior model is sometimes lead to a "staircase" result with intense noises particularly in flat or smooth areas [53]. Therefore, several researchers suggest spatially adaptive approaches for overcoming the disadvantages of the TV prior model [1, 54]. There are a few approaches that categorize the image into flat and detailed areas by spatial information. They use a greater and a smaller charges parameter for the smooth regions and edges respectively. On the other hand, the spatially

adaptive indications such as the difference curvature, gradients, and structure tensor are usually very critical to the noises. Bilateral total variation (BTV) is utilized for estimating TV, maintaining the flatness of continued areas, and protecting edges in discontinued areas [33]. A locally adaptive version of BTV (LABTV) is presented for giving a stability among the noise reductions and the protection of image information [55].

#### IV. SIMILARITY MEASURE

To be able to assess the fidelity of the image reconstruction procedure, every reconstructed HR image has to match to the original image which is called similarity measurement. In addition, it assists in the monitoring and evaluating the performance of the image reconstruction procedure. There are numerous similarity measures existing in the literature. A few of the most well-known are Peak Signal-to-Noise Ratio (PSNR), Mean Square Error (MSE), and Structural Similarity (SSIM) [56-58].

The PSNR is measured from the MSE, which is the average error amongst the original image and the SR image. Given an SR  $m \times n$  image  $\hat{X}(i, j)$  and its original  $X(i, j)$ , MSE and PSNR are defined as:

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [X(i, j) - \hat{X}(i, j)]^2 \quad (2)$$

$$PSNR = 20 \log_{10} \left( \frac{L}{\sqrt{MSE}} \right) \quad (3)$$

The SSIM index measures the similarity between the SR and original images. The SSIM considers luminance, contrast, and structural adjustments amongst the two images. The SSIM index is defined as:

$$SSIM(X, \hat{X}) = \frac{(2\mu_X \mu_{\hat{X}} + C_1)(2\sigma_{X\hat{X}} + C_2)}{(\mu_X^2 + \mu_{\hat{X}}^2 + C_1)(\sigma_X^2 + \sigma_{\hat{X}}^2 + C_2)} \quad (4)$$

where  $\mu_X$  and  $\mu_{\hat{X}}$  are the means and  $\sigma_X$  and  $\sigma_{\hat{X}}$  are the standard deviations of the original and SR images.  $\sigma_{X\hat{X}}$  is the covariance of  $X$  and  $\hat{X}$ , and  $C_1$  and  $C_2$  are constants. The value of SSIM closes to 1, if the SR image is very similar to the original image.

#### V. DISCUSSION AND ANALYSIS

From this review, many researchers have implemented diverse methods in an effort to produce a high quality image based on the SR image reconstruction approaches [35, 59, 60]. Tsai and Huang [8] and Kim et al. [9] suppose that the series of LR images are globally translated and totally freed from distortions such as blurring or noise effects. Therefore, Kim and Su [10] check out distinct blurs for every LR image. Kang and Rhee [13] and Park et al. [14] develop the DCT approach to reduce the computational costs and approximate the quantization system noise respectively. El-Khamy and et al. [16] use the wavelet domain to execute the registration of several LR images. However, these frequency domain approaches have many problems which prohibited researchers from an advanced development, especially in a case for the sensitivity of model errors and difficulty in dealing with more complex motion models.

Due to the limitations of the frequency domain approaches, the spatial domain approaches are classified as the most popular approach adopted to develop the SR image. The popularity of these approaches springs from the fact that the motion is not restricted to the translational shifts only; therefore, a more general global or non-global motion may also be integrated and managed. The nearest neighbor interpolation [22] is considered the fastest method among other interpolation approaches. However, it brings the significant distortion, shows up the mosaic, and produces images with a blocky visibility. The bilinear interpolation [23] leads to much smoother images than the nearest neighbor interpolation approach. However, it has a high computation cost and does not have sufficient results. While the bicubic interpolation approach [23] can relatively obtain a clear image quality. Also, it generates visibly sharper images in comparison to the previous two approaches. It perhaps gets the optimal mixture of processing time and output quality. However, it requires a greater amount of computation. The edge guided interpolation approach [24] is a powerful technique to detect the registration mistake and needs a low-computational cost. The IBP [27] and POCS [28] approaches are extremely very simple and easy to understand. However, it is not generally going to give a unique result because of the ill-posed trouble. However, interpolation-based approaches regularly generate images with several drawbacks around the object's borders, consisting of zigzag, blurring, and aliasing edges.

The SR image reconstruction approaches are really depicted as an ill-posed problem. The reason is that the inadequate number of LR images and the ill-conditioned blur operators. Techniques which usually manipulate to assist in the inversion of this problem are recognized as regularization. CLS [31] regularization approach incorporates the smoothness constraints as a priori information. But, its results depend on the choice of the regularization parameter. Tikhonov least-square approach [31] incorporates l2-norm as a regularization term because it is easy to resolve. However, the l2-norm does not promise a unique solution. Also, it is optimum when the model error is white-Gaussian distribution. For this reason, Farsiu et al. [33] employ an alternative l1-norm that works more effectively than the l2-norm when the images consist of non-Gaussian errors for very quickly and powerful SR. ML is suggested by Tom and Katsaggelos [45] to get the ML estimation of the HR image. However, it only takes into account the relationship amongst the LR images and the primary HR image. Therefore, MAP method combines the prior image model to expose the expectancy of the unidentified HR image. ML and MAP are famous approaches because of their flexibility and adaptability for protecting edges and combining parameters. TV regularization is suggested to protect edge information and prevent ringing results. On the other hand, the finding results of the TV prior model is sometimes lead to a "staircase" result with intense noises particularly in flat or smooth areas. Table 1 shows the comparative analysis of a common multi-frame SR approaches. In addition, a comparison between frequency and spatial domain approaches is shown in Table 2.

As a result of these previous reviews it is clear that the regularization approaches take advantage of the prior

knowledge to fix the SR problem. Thus, the regularization approaches can be used as an attempt to stabilize the inversion process and compensate for the absent information. Additionally, it is used to represent a prior of the image, remove artifacts, and bring the prior information.

generates a stable solution, improve the convergence rate, and include artificial constraints on the solution such as smoothness and edge-preserving. Therefore, regularization-based approaches are a challenging in SR image reconstruction.

TABLE I. COMPARISON OF A COMMON SR APPROACHES

	Frequency domain	POCS	IBP	ML	MAP	TV
<b>SR method</b>	Frequency	Interpolation	Interpolation	Regularization	Regularization	Regularization
<b>Motion models</b>	Shifts	Any	Any	Any	Any	Any
<b>Noise model</b>	G	G	G	G or L	G or L	G or L
<b>A prior knowledge</b>	No	Yes	No	No	Yes	Yes
<b>Deblurring</b>	No	Yes	Yes	Yes	Yes	Yes
<b>Computation cost</b>	Low	Medium	Low	High	High	High
<b>Solution uniqueness</b>	Unique	Non-unique	Non-unique	Non-unique	Unique	Unique
<b>Applicability</b>	Limited	Limited	Wide	Limited	Very Wide	Very Wide

G= GAUSSIAN, L= LAPLACIAN

TABLE II. COMPARISON BETWEEN FREQUENCY AND SPATIAL DOMAIN APPROACHES

	Frequency Domain	Spatial Domain
<b>Observation model</b>	Frequency domain	Spatial domain
<b>Motion models</b>	Global translation	Almost unlimited
<b>Model errors</b>	Limited	Almost unlimited
<b>Noise model</b>	Limited	Very Flexible
<b>SR Mechanism</b>	De-aliasing	De-aliasing, A-prior information
<b>Computation requirement</b>	Low	High
<b>Regularization</b>	Limited	Excellent
<b>Simplicity</b>	Very simple	Generally complex
<b>Extensibility</b>	Poor	Excellent
<b>Applicability</b>	Limited	Wide
<b>Performance</b>	Good for specific applications	Good

## VI. CONCLUSION

In this paper, a survey of the current multi-frame SR approaches is presented over the previous three decades. The primary improvement of SR approaches can essentially be split into three phases. In the first 10 years phase, researchers shift their focus from the study of frequency domain to spatial domain approaches, especially interpolation-based approaches. In the second phase, regularized SR approaches acquire a primary emphasis. Within the last phase, the Bayesian MAP construction has become the most common approach due to its great performance and flexible properties. Recently, researchers have primarily focused on SR reconstruction in several areas. Nevertheless, the comprehensive practical use of SR still continues to be described as problematic.

## REFERENCES

- [1] Q. Yuan, L. Zhang, and H. Shen, "Multiframe super-resolution employing a spatially weighted total variation model," *IEEE Transactions on Circuits and Systems for Video Technology*, vol. 22, pp. 379-392, 2012.
- [2] S. C. Park, M. K. Park, and M. G. Kang, "Super-resolution image reconstruction: a technical overview," *IEEE signal processing magazine*, vol. 20, pp. 21-36, 2003.
- [3] H. Hou, M. Wang, and X. Wang, "RL-MS-L Filter Function for CT Image Reconstruction," *TELKOMNIKA (Telecommunication Computing Electronics and Control)*, vol. 14, pp. 195-202, 2016.
- [4] L. Yue, H. Shen, J. Li, Q. Yuan, H. Zhang, and L. Zhang, "Image super-resolution: The techniques, applications, and future," *Signal Processing*, vol. 128, pp. 389-408, 2016.
- [5] C. Kumar, "A novel super resolution reconstruction of low resolution images progressively using dct and zonal filter based denoising," *arXiv preprint arXiv:1102.5688*, 2011.
- [6] M. Protter, M. Elad, H. Takeda, and P. Milanfar, "Generalizing the nonlocal-means to super-resolution reconstruction," *IEEE Transactions on image processing*, vol. 18, pp. 36-51, 2009.
- [7] W. Long, Y. Lu, L. Shen, and Y. Xu, "HIGH-RESOLUTION IMAGE RECONSTRUCTION: AN ENVELOPE/TV MODEL AND A FIXED-POINT PROXIMITY ALGORITHM," *International Journal of Numerical Analysis & Modeling*, vol. 14, 2017.
- [8] R. Tsai and T. S. Huang, "Multiframe image restoration and registration," *Advances in computer vision and Image Processing*, vol. 1, pp. 317-339, 1984.
- [9] S. Kim, N. K. Bose, and H. Valenzuela, "Recursive reconstruction of high resolution image from noisy undersampled multiframes," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, pp. 1013-1027, 1990.
- [10] S. P. Kim and W.-Y. Su, "Recursive high-resolution reconstruction of blurred multiframe images," *IEEE Transactions on Image Processing*, vol. 2, pp. 534-539, 1993.
- [11] N. Bose, H. Kim, and H. Valenzuela, "Recursive implementation of total least squares algorithm for image reconstruction from noisy, undersampled multiframes," in *Acoustics, Speech, and Signal Processing, 1993. ICASSP-93., 1993 IEEE, Minneapolis MN, vol. V, pp. 269-272, 1993.*, 1993, pp. 269-272.
- [12] P. Vandewalle, S. Süssstrunk, and M. Vetterli, "A frequency domain approach to registration of aliased images with application to super-resolution," *EURASIP Journal on applied signal processing*, vol. 2006, pp. 233-233, 2006.
- [13] S. Rhee and M. G. Kang, "Discrete cosine transform based regularized high-resolution image reconstruction algorithm," *Optical Engineering*, vol. 38, pp. 1348-1356, 1999.
- [14] S. C. Park, M. G. Kang, C. A. Segall, and A. K. Katsaggelos, "Spatially adaptive high-resolution image reconstruction of DCT-based compressed images," *IEEE transactions on image processing*, vol. 13, pp. 573-585, 2004.
- [15] N. Nguyen and P. Milanfar, "A wavelet-based interpolation-restoration method for superresolution (wavelet superresolution)," *Circuits, Systems and Signal Processing*, vol. 19, pp. 321-338, 2000.



- [16] S. E. El-Khamy, M. M. Haddoud, M. I. Dessouky, B. M. Salam, and F. A. El-Samie, "Regularized super-resolution reconstruction of images using wavelet fusion," *Optical Engineering*, vol. 44, pp. 097001-097001-10, 2005.
- [17] M. B. Chappalli and N. Bose, "Simultaneous noise filtering and super-resolution with second-generation wavelets," *IEEE Signal Processing Letters*, vol. 12, pp. 772-775, 2005.
- [18] H. Ji and C. Fermüller, "Wavelet-based super-resolution reconstruction: theory and algorithm," in *European Conference on Computer Vision—ECCV 2006*, 295-307, 2006, pp. 295-307.
- [19] H. Ji and C. Fermüller, "Robust wavelet-based super-resolution reconstruction: theory and algorithm," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 31, pp. 649-660, 2009.
- [20] X. Li, "Image resolution enhancement via data-driven parametric models in the wavelet space," *EURASIP Journal on Image and Video Processing*, vol. 2007, pp. 1-12, 2007.
- [21] G. Anbarjafari and H. Demirel, "Image super resolution based on interpolation of wavelet domain high frequency subbands and the spatial domain input image," *ETRI journal*, vol. 32, pp. 390-394, 2010.
- [22] N. Nguyen, P. Milanfar, and G. Golub, "A computationally efficient super-resolution image reconstruction algorithm," *IEEE transactions on image processing*, vol. 10, pp. 573-583, 2001.
- [23] X. Li and M. T. Orchard, "New edge-directed interpolation," *IEEE transactions on image processing*, vol. 10, pp. 1521-1527, 2001.
- [24] L. Zhang and X. Wu, "An edge-guided image interpolation algorithm via directional filtering and data fusion," *IEEE transactions on Image Processing*, vol. 15, pp. 2226-2238, 2006.
- [25] J. Chu, J. Liu, J. Qiao, X. Wang, and Y. Li, "Gradient-based adaptive interpolation in super-resolution image restoration," in *2008 9th International Conference on Signal Processing*, 2008, pp. 1027-1030.
- [26] X. Zhang and Y. Liu, "A Computationally Efficient Super-Resolution Reconstruction Algorithm Based On The Hybrid Interpolation," *Journal of Computers*, vol. 5, pp. 885-892, 2010.
- [27] M. Irani and S. Peleg, "Improving resolution by image registration," *CVGIP: Graphical models and image processing*, vol. 53, pp. 231-239, 1991.
- [28] H. Stark and P. Oskoui, "High-resolution image recovery from image-plane arrays, using convex projections," *JOSA A*, vol. 6, pp. 1715-1726, 1989.
- [29] H. Ur and D. Gross, "Improved resolution from subpixel shifted pictures," *CVGIP: Graphical Models and Image Processing*, vol. 54, pp. 181-186, 1992.
- [30] T. Komatsu, K. Aizawa, T. Igarashi, and T. Saito, "Signal-processing based method for acquiring very high resolution images with multiple cameras and its theoretical analysis," *IEE Proceedings I-Communications, Speech and Vision*, vol. 140, pp. 19-24, 1993.
- [31] E. Plenge, D. H. Poot, M. Bernsen, G. Kotek, G. Houston, P. Wielopolski, et al., "Super-resolution methods in MRI: Can they improve the trade-off between resolution, signal-to-noise ratio, and acquisition time?," *Magnetic resonance in medicine*, vol. 68, pp. 1983-1993, 2012.
- [32] H. Song, L. Zhang, P. Wang, K. Zhang, and X. Li, "AN adaptive L1–L2 hybrid error model to super-resolution," in *In Image Processing (ICIP), 2010 17th IEEE International Conference on (pp. 2821-2824)*. IEEE., 2010, pp. 2821-2824.
- [33] S. Farsiu, M. D. Robinson, M. Elad, and P. Milanfar, "Fast and robust multiframe super resolution," *IEEE transactions on image processing*, vol. 13, pp. 1327-1344, 2004.
- [34] L. Yue, H. Shen, Q. Yuan, and L. Zhang, "A locally adaptive L1–L2 norm for multi-frame super-resolution of images with mixed noise and outliers," *Signal Processing*, vol. 105, pp. 156-174, 2014.
- [35] C. Chen, H. Liang, S. Zhao, Z. Lyu, S. Fang, and X. Pei, "Integrating the Missing Information Estimation into Multi-frame Super-Resolution," *Circuits, Systems, and Signal Processing*, vol. 35, pp. 1213-1238, 2016.
- [36] X. Zeng and L. Yang, "A robust multiframe super-resolution algorithm based on half-quadratic estimation with modified BTV regularization," *Digital Signal Processing*, vol. 23, pp. 98-109, 2013.
- [37] S. Mallat and G. Yu, "Super-resolution with sparse mixing estimators," *IEEE Transactions on Image Processing*, vol. 19, pp. 2889-2900, 2010.
- [38] S. Villena, M. Vega, R. Molina, and A. K. Katsaggelos, "Bayesian super-resolution image reconstruction using an  $\ell_1$  prior," in *Image and Signal Processing and Analysis, 2009. ISPA 2009. Proceedings of 6th International Symposium on (pp. 152-157)*. IEEE, 2009, pp. 152-157.
- [39] J. Tian and K.-K. Ma, "Stochastic super-resolution image reconstruction," *Journal of Visual Communication and Image Representation*, vol. 21, pp. 232-244, 2010.
- [40] S. Villena, M. Vega, S. D. Babacan, R. Molina, and A. K. Katsaggelos, "Bayesian combination of sparse and non-sparse priors in image super resolution," *Digital Signal Processing*, vol. 23, pp. 530-541, 2013.
- [41] S. P. Belekos, N. P. Galatsanos, and A. K. Katsaggelos, "Maximum a posteriori video super-resolution using a new multichannel image prior," *IEEE Transactions on Image Processing*, vol. 19, pp. 1451-1464, 2010.
- [42] N. A. Woods, N. P. Galatsanos, and A. K. Katsaggelos, "Stochastic methods for joint registration, restoration, and interpolation of multiple undersampled images," *IEEE Transactions on Image Processing*, vol. 15, pp. 201-213, 2006.
- [43] L. C. Pickup, D. P. Capel, S. J. Roberts, and A. Zisserman, "Bayesian methods for image super-resolution," *The Computer Journal*, vol. 52, pp. 101-113, 2009.
- [44] G. Polatkan, M. Zhou, L. Carin, D. Blei, and I. Daubechies, "A Bayesian nonparametric approach to image super-resolution," *IEEE transactions on pattern analysis and machine intelligence*, vol. 37, pp. 346-358, 2015.
- [45] B. C. Tom and A. K. Katsaggelos, "Reconstruction of a high-resolution image by simultaneous registration, restoration, and interpolation of low-resolution images," in *In Image Processing, 1995. Proceedings., International Conference on (Vol. 2, pp. 539-542)*. IEEE, 1995, pp. 539-542.
- [46] M. E. Tipping and C. M. Bishop, "Bayesian image super-resolution," in *Advances in neural information processing systems (pp. 1303-1310)*. 2002.
- [47] C. J. Wu, "On the convergence properties of the EM algorithm," *The Annals of statistics*, pp. 95-103, 1983.
- [48] L. C. Pickup, D. P. Capel, S. J. Roberts, and A. Zisserman, "Bayesian image super-resolution, continued," in *NIPS*, 2006.
- [49] H. He and L. P. Kondi, "Resolution enhancement of video sequences with simultaneous estimation of the regularization parameter," *Journal of Electronic Imaging*, vol. 13, pp. 586-596, 2004.
- [50] S. Osher, M. Burger, D. Goldfarb, J. Xu, and W. Yin, "An iterative regularization method for total variation-based image restoration," *Multiscale Modeling & Simulation*, vol. 4, pp. 460-489, 2005.
- [51] R. Pan and S. J. Reeves, "Efficient Huber-Markov edge-preserving image restoration," *IEEE Transactions on Image Processing*, vol. 15, pp. 3728-3735, 2006.
- [52] T. Goldstein and S. Osher, "The split Bregman method for L1-regularized problems," *SIAM journal on imaging sciences*, vol. 2, pp. 323-343, 2009.
- [53] Q. Yuan, L. Zhang, and H. Shen, "Regional spatially adaptive total variation super-resolution with spatial information filtering and clustering," *IEEE Transactions on Image Processing*, vol. 22, pp. 2327-2342, 2013.
- [54] Y. Dong, M. Hintermüller, and M. M. Rincon-Camacho, "Automated regularization parameter selection in multi-scale total variation models for image restoration," *Journal of Mathematical Imaging and Vision*, vol. 40, pp. 82-104, 2011.
- [55] X. Li, Y. Hu, X. Gao, D. Tao, and B. Ning, "A multi-frame image super-resolution method," *Signal Processing*, vol. 90, pp. 405-414, 2010.
- [56] Z. Tang, M. Deng, C. Xiao, and J. Yu, "Projection onto convex sets super-resolution image reconstruction based on wavelet bi-cubic interpolation," in *In Electronic and Mechanical Engineering and Information Technology (EMEIT), 2011 International Conference on (Vol. 1, pp. 351-354)*. IEEE., 2011, pp. 351-354.
- [57] I. Begin and F. P. Ferrie, "Comparison of super-resolution algorithms using image quality measures," in *Computer and Robot Vision, 2006. The 3rd Canadian Conference on Computer and Robot Vision (CRV'06) 2006*, pp. 72-72.
- [58] Z. Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, "Image quality assessment: from error visibility to structural similarity," *IEEE transactions on image processing*, vol. 13, pp. 600-612, 2004.
- [59] A. Laghrib, A. Ghazdali, A. Hakim, and S. Raghay, "A multi-frame super-resolution using diffusion registration and a nonlocal variational image restoration," *Computers & Mathematics with Applications*, vol. 72, pp. 2535-2548, 2016.
- [60] R. Nayak and D. Patra, "Super resolution image reconstruction using penalized-spline and phase congruency," *Computers & Electrical Engineering*, 62, 232-248., 2016.