

# Bernhard Böhmler

- Curriculum vitae -

## Biographical information

Place and date of birth: 28<sup>th</sup> July 1988, Leonberg, Germany  
Nationality: German

## Language skills

Mother tongue: German  
Foreign languages: English (level B2), Latin (high school level), Spanish (level A2), French (level A1)

## Education

08/2018 - 09/2023 **Doctoral student in mathematics, TU Kaiserslautern** (funded by the DFG)  
Topic: Trivial source character tables of small finite groups  
Supervisor: Jun.-Prof. Dr. Caroline Lassueur

11/2011 - 09/2016 **Master of Science in mathematics, University of Stuttgart**

Specialisations:

- Representation theory of finite-dimensional algebras
- Homological algebra
- Functional analysis
- Theoretical physics (minor subject)
- Master's Thesis (04/2016 - 09/2016)

Title: Contributions to the representation theory of gendo-symmetric algebras  
Supervisor: Prof. Dr. Wolfgang Rump

10/2008 - 11/2011 **Bachelor of Science in mathematics, University of Stuttgart**

Specialisations:

- Representation theory of finite groups
- Algebra
- Experimental physics (minor subject)
- Bachelor's Thesis (05/2011 - 10/2011)

Title: Influence of character degrees on the group structure

09/1999 - 07/2008 **Abitur, Albert-Schweizer-Gymnasium Leonberg**

## Research interests

- Group theory
- Modular and ordinary representation theory of finite groups
- Representation theory of finite-dimensional algebras and related combinatorial structures
- Auslander-Reiten theory
- Homological algebra

## Publications & preprints

In preparation *An algorithm to compute trivial source character tables of finite groups*

06/2022 *On the extension-closed property for the subcategory  $\text{Tr}(\Omega^2(\text{mod} - A))$*   
DOI: <https://doi.org/10.1007/s10468-022-10140-7>  
*Algebr. Represent. Theory* (2022) (with René Marczinkik)

05/2022 *Trivial source character tables of  $\text{SL}_2(q)$*   
*J. Algebra* **598** (2022), 308-350 (with Niamh Farrell and Caroline Lassueur)

01/2022 *A cluster tilting module for a representation-infinite block of a group algebra*  
*J. Algebra* **589** (2022), 483-494 (with René Marczinkik)

01/2018 *On a conjecture about Morita algebras*  
*J. Algebra* **508** (2018), 569-574 (with René Marczinkik)

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## Research stays

- 10/2021 - 01/2022 4 months at the *University of California, Santa Cruz*, scholarship granted by the German Academic Exchange Service (DAAD)
- 03/2021 2 weeks at *Mathematisches Forschungsinstitut Oberwolfach*, Oberwolfach Research Fellowship
- 01/2020 1 week at the *Isaac Newton Institute for Mathematical Sciences*, University of Cambridge
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## Contributed talks

### Research talks

- 09/2022 **On the computation of trivial source character tables using computer algebra** during the conference *Representation Theory at the Villa Denis* in Kaiserslautern
- 08/2022 **On the computation of trivial source character tables using computer algebra** during the conference *Structure of Group Algebras over Local Rings* in Ambleside (England)
- 09/2021 **On the computation of trivial source character tables** during the *Young Algebraists' Conference* at the EPFL (Switzerland)
- 02/2021 **Broué's abelian defect group conjecture via  $p$ -permutation equivalences** during the seminar *Promotionen und Postdocs in Darstellungstheorie* (online)
- 02/2020 **Charaktertafeln von Moduln kleiner endlicher Gruppen mit trivialen Quellen** during the *Grüppchen* at the Martin-Luther-University in Halle/Saale
- 01/2020 **Trivial source character tables of small finite groups** during the *IRTG-seminar* at the Saarland University
- 09/2019 **Representation type and combinatorics** at the 83<sup>rd</sup> *Séminaire Lotharingien de Combinatoire* in Bad Boll

### Working group talks

- 06/2021 **The Littlewood-Richardson rule** during the seminar *Representation Theory of the Symmetric Group* at the TU Kaiserslautern
- 01/2021  **$p$ -permutation equivalences** during the seminar *Equivalences of block algebras* at the TU Kaiserslautern
- 06/2019 **Blocks with cyclic defect groups I** during the seminar *Block Theory* at the TU Kaiserslautern
- 04/2019 **On a counter-example to a conjecture about Morita algebras** during the seminar *Groups and Representations* at the TU Kaiserslautern
- 10/2018 **Projective and injective modules** during the seminar *Perverse equivalences and applications* at the TU Kaiserslautern

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## Selected attended conferences and workshops

- 10/2022 Darstellungstheorietage in Kaiserslautern
- 09/2022 Representation Theory at the Villa Denis in Kaiserslautern
- 08/2022 Structure of Group Algebras over Local Rings in Ambleside (England)
- 09/2021 Young Algebraists' Conference at the EPFL (Switzerland)
- 02/2021 Promotionen und Postdocs in Darstellungstheorie (online)
- 12/2020 Nikolaus conference (online)
- 09/2020 SFB-TRR 195: Annual conference (online)
- 02/2020 Grüppchen at the Martin-Luther-University in Halle/Saale
- 12/2019 Nikolaus conference in Aachen
- 10/2019 Darstellungstheorietage in Jena
- 06/2019 Norddeutsches Gruppentheorie-Kolloquium in Halle
- 06/2019 Groups, Rings and Associated Structures in Spa (Belgium)
- 12/2018 Nikolaus conference in Aachen
- 09/2018 Darstellungstheorietage in Hannover
- 03/2016 Workshop on Brauer Graph Algebras at the University of Stuttgart (as aide)
- 03/2016 Conference on Triangulated Categories in Algebra, Geometry and Topology University of Stuttgart (as aide)
- 09/2015 Third GAP Days 2015 at the NTNU in Trondheim (Norway)

## University teaching

- 04/2022 - 09/2022 B.Sc. lecture: Character Theory of Finite Groups, assistant  
 04/2022 - 09/2022 B.Sc. lecture: Complex Analysis for Engineers, assistant  
 04/2021 - 09/2021 M.Sc. lecture: Cohomology of Groups, assistant  
 04/2021 - 09/2021 B.Sc. lecture: Complex Analysis for Engineers, assistant  
 10/2020 - 03/2021 M.Sc. lecture: Representation Theory of Finite Groups, assistant  
 04/2019 - 09/2019 B.Sc. lecture: Character Theory of Finite Groups, assistant  
 10/2016 - 03/2017 B.Sc. lecture: Higher Mathematics 3 for Engineers, tutor  
 10/2015 - 12/2015 B.Sc. lecture: Higher Mathematics 3 for Electrical Engineering Technicians, tutor  
 10/2014 - 03/2015 B.Sc. lecture: Higher Mathematics 3 for Engineers, tutor  
 04/2014 - 09/2014 B.Sc. lecture: Higher Mathematics 2 for Engineers, tutor  
 10/2013 - 03/2014 B.Sc. lecture: Higher Mathematics 3 for Engineers, tutor  
 04/2013 - 09/2013 B.Sc. lecture: Algebra for Mathematicians, tutor  
 10/2012 - 03/2013 B.Sc. lecture: Higher Mathematics 1 for Engineers, tutor  
 04/2012 - 09/2012 B.Sc. lecture: Geometry for Mathematicians, tutor

## Work experience

- 09/2017 - 07/2018 High school teacher for mathematics and physics at the Stetten-Institute in Augsburg  
 05/2017 - 08/2017 Freelance collaborator at Schülerhilfe Leonberg (tutor for mathematics)

## Computer skills

- Typesetting L<sup>A</sup>T<sub>E</sub>X (very good)  
 Computer languages GAP (very good), MAGMA (good), SAGE (basics)

## Extracurricular activities

### Further training

- 04/2018 DeltaPlus-course *Mathematik sprachsensibel unterrichten* in Augsburg  
 01/2018 Course *Die Kunst des guten Unterrichtens* with Dr. Siegfried Rodehau in Nürnberg  
 11/2017 Colloquium *Warum Geraden nicht Gerade sein müssen* about synthetic geometry with Prof. Wolfgang Schneider in Augsburg  
 11/2017 Courses "*Sprachsensibler Unterricht*", "*Umgang mit Notfallsituationen*", "*Gesprächsführung*" in the retreat house of St. Paulus in Leitershofen

### Special qualifications

- 10/2011 - 03/2012 Key qualification course for tutors of Higher Mathematics at the University of Stuttgart

### Voluntary services

- 04/2019 - today Clarinet player at the *university orchestra* of the TU Kaiserslautern  
 09/2019 - 03/2020 Tutor for mathematics at the *learning centre* of the TU Kaiserslautern  
 1999 - today Member of (the executive board of) several *chess clubs*  
 1996 - 2016 Member of the wind band *Musikverein Stadtkapelle Leonberg* and leader of the clarinets

### Awards

- 03/2008 1<sup>st</sup> price at the federal state music competition *Jugend musiziert* (solo piano)  
 03/2007 1<sup>st</sup> price at the federal state music competition *Jugend musiziert* (Duo: clarinet & piano)  
 05/2006 3<sup>rd</sup> price at the national music competition *Jugend musiziert* (solo clarinet)  
 03/2006 1<sup>st</sup> price at the federal state music competition *Jugend musiziert* (solo clarinet)  
 10/2005 2<sup>nd</sup> price at the *Matthaes* piano competition in Stuttgart (solo piano)

## Referees

Caroline Lassueur, lassueur@mathematik.uni-kl.de, +49 (0)631 205 2515

Robert Boltje, boltje@ucsc.edu, +1 (0)831 459 5001

Wolfgang Kimmerle, wolfgang.kimmerle@mathematik.uni-stuttgart.de, +49 (0)711 685 65323

## Overview of previous research work & research plan

In my research projects, I use computer algebra systems in order to discover homological and representation theoretic conjectures. I try to prove or disprove these conjectures with the assistance of various computer algebra systems. This approach is promising, since it is possible to do many detailed calculations with the computer which improves the understanding of and the insight into a given mathematical problem. In the following, a selection of projects I plan to work on is presented. All occurring algebras are finite-dimensional algebras over a field.

### Broué's abelian defect group conjecture via $p$ -permutation equivalences

This project is concerned with the modular representation theory of finite groups. Trivial source modules, also known as  $p$ -permutation modules, arise naturally in this context. In order to do calculations with trivial source modules the ordinary characters of their lifts from positive characteristic  $p$  to characteristic zero are of particular interest. The "trivial source character tables" collect information about the character values of trivial source modules with all possible vertices, as well as those of their Brauer constructions.

During my doctoral thesis I examined trivial source character tables both theoretically and with the help of computer algebra. The results obtained so far naturally lead me to  $p$ -permutation equivalences.

Broué's abelian defect group conjecture (ADGC) has generated a lot of interest in recent years. It predicts a categorical equivalence between a block with abelian defect groups and its Brauer correspondent. When formulated in terms of  $p$ -permutation equivalences, Broué's ADGC predicts the following: let  $G$  be a finite group and let  $p$  be a prime number dividing  $|G|$ . Suppose  $k$  is a large enough field. Assume  $b$  is a block of  $kG$  with an abelian defect group  $D$ . Let  $H$  be the normaliser of  $D$  in  $G$ . Let  $c$  be the Brauer correspondent of  $b$  in  $H$ . Then, there exists a  $p$ -permutation equivalence between  $b$  and  $c$ .

The aim of this project is to find all  $p$ -permutation equivalences between two arbitrary blocks  $b$  and  $c$  having the aforementioned properties. Since the number of such equivalences is finite when  $G$  and  $p$  are fixed, this is indeed feasible.

### Classification of representation-finite gendo-symmetric algebras

An algebra  $B$  is called *gendo-symmetric*, if it has the form  $B = \text{End}_A(A \oplus M)$  for a symmetric algebra  $A$  and some  $A$ -module  $M$ . Gendo-symmetric algebras are a generalisation of symmetric algebras and contain many important classes of algebras such as Schur algebras and blocks of category  $\mathcal{O}$ .

The goal of this project is to classify all representation-finite gendo-symmetric algebras. This was started in my master's thesis. The problem is reduced to a Dynkin type classification where some extra exceptional cases appear.

The strategy to solve the remaining open cases is threefold: we plan to employ results about translation quivers by Gabriel and Riedmann, calculate small examples for every family of algebras with GAP, and the theory of universal coverings is again available.

### Testing homological conjectures via local algebras

This project is joint work with René Marcenzik (University of Bonn). The finitistic dimension conjecture is one of the most important homological conjectures for finite-dimensional algebras. It states that the supremum of all projective dimensions of modules with finite projective dimension is finite.

A consequence of the finitistic dimension conjecture is Tachikawa's conjecture which states that a finite-dimensional algebra  $A$  is selfinjective if and only if  $\text{Ext}_A^i(D(A), A) = 0$  for all  $i \geq 1$ . The latter conjecture is open even for local algebras. There are many related problems. For example, consider the following questions.

- 1) Is there a non-selfinjective local algebra  $A$  with  $\text{Ext}_A^1(D(A), A) = \text{Ext}_A^2(D(A), A) = 0$  ?
- 2) If  $A$  is commutative and not selfinjective, can we then have  $\text{Ext}_A^1(D(A), A) = 0$  ?
- 3) When do there exist  $d$ -cluster tilting modules for local algebras where  $d \geq 2$  ?
- 4) If  $A$  is local and self-injective and  $M$  is a non-projective module, do we have  $\text{Ext}_A^1(M, M) \neq 0$  ?

We remark that Asashiba and Hoshino treated questions 1 and 2 in the case that the Loewy length is at most 3 where they have proved that there are no such examples. But it seems that nothing is known when the Loewy length is greater than 3. For point 3 we remark that by now there is only one local algebra  $A$  known with a  $d$ -cluster tilting module  $M$  if  $d \geq 2$ , namely the algebra  $A := k\langle x, y \rangle / (x^2, x \cdot y + y^2 + y^2 \cdot x)$ .

This algebra was found by Jan Geuenich using QPA as an example of a non-selfinjective local algebra with  $\text{Ext}^1(D(A), A) = 0$ . René Marcenzik was then able to construct a 2-precluster tilting object for  $A$  and conjectured that this is even 2-cluster tilting which was subsequently proved by Oeyvind Solberg using QPA.

Apart from this example no other local algebras having a  $d$ -cluster-tilting module are known for  $d \geq 2$ . We plan to attack those 4 questions with QPA / MAGMA by generating all local quiver algebras of a given dimension over a small finite field.