The Impact of a Fenestration on Blood Flow in Fontan Circulation

Boning Feng (Bernice)

New York University bf1318@nyu.edu

1 Introduction

There are two ventricles in the heart of a normal person, the left ventricle and the right ventricle. As shown in the figure 1, left ventricle pumps the blood from pulmonary veins to systemic arteries. The blood coming out of the lungs is oxygenated before it goes through the systemic organs. The blood coming out of the systemic organs are deoxygenated and is pumped back to the lungs by the right ventricle. The lungs absorb oxygen and release carbon dioxide, making the blood oxygenated again. The oxygenated blood is in color red, while deoxygenated blood is in blue in the figure 1.

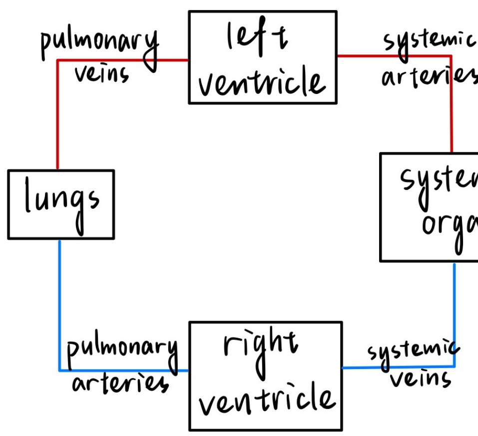

Fig. 1: Graph of Circulation in a Person with Normal Heart

The hypoplastic left heart syndrome, also referred to as HLHS, is defined as underdevelopment of the structures of the left side of the heart[3]. Therefore, the circulation of a patient with HLHS cannot function properly. A common treatment for these patients is Fontan operation, which is a sequence of surgeries during the first several years of life. As a result of the surgeries, a connection between the vena cavae and pulmonary arteries is established. This circulation, known as Fontan circulation, is shown in 2. Because of the low resistence of the surgical connection, we can view the systemic venous pressure and pulmonary artery pressure as approxiamately the same[1].

However, the patients with Fontan circulation still have considerable long-term morbidity. A possible causes of the morbility is low cardiac output because of the serialized organs and lungs leading to higher than normal systemic venous pressure. Since the pulmonary artery pressure approximately equals the systemic venous pressure because of low resistance in the surgical connection of Fontan operation, the systemic venous pressure is elevated compared to a normal circulation [1].

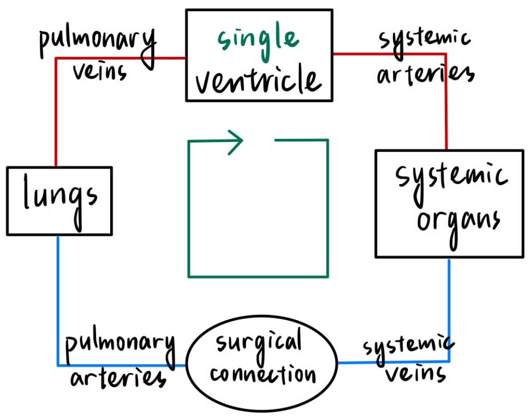

Fig. 2: Graph of Fontan Circulation

In some Fontan patients, a shunt between the systemic veins and the pulmonary veins, known as a fenestration, are introduced. The graph of Fontan circulation with fenestration is shown in 3. The purple blood is a mixture of the oxygenated and deoxygenated blood. However, the benefits of the fenestration is not yet clear [3]. This project models a Fontan circulation with fenestration, and explores the outcome of ferenstration with different resistance. We are particularly interested in the change of flow and pressure and thus if the morbility caused by Fontan criculation can be mitigated.

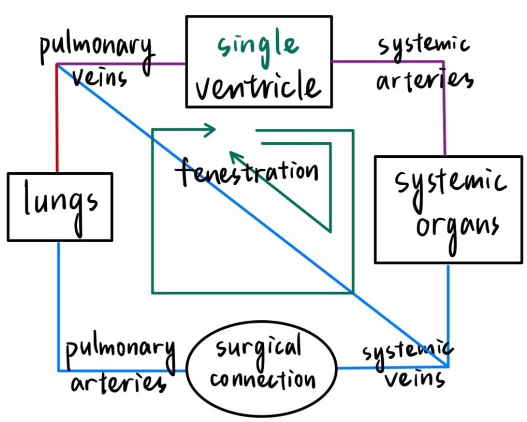

Fig. 3: Graph of Fontan Circulation with Fenestration

2 Equations

2.1 Ventricle

The ventricular compliance is a time-dependent periodic function, with period T corresponding to the duration of the cardiac cycle. We calculate the ventricle compliance $C_{ventricle}(t)$ as $\frac{1}{E_{ventricle}(t)}$, which stands for elastance of the ventricle. $E_{ventricle}(t)$ on the time interval $t \in [0, T]$ is

$$E_{ventricle}(t) = k \frac{g_1(t)}{1 + g_1(t)} \left(\frac{1}{1 + g_2(t)} - \frac{1}{1 + g_2(T)} \right) + E_{min} \tag{1}$$

Where k is a scalar to make $E_{ventricle}(t)$ be fluctuating between parameter E_{max} and E_{min}.

$$g_i(t) = \begin{cases} \left(\frac{t}{\tau_{systole}}\right)^{m_i} & \text{if } i = 1 \\ \left(\frac{t}{\tau_{diatole}}\right)^{m_i} & \text{if } i = 2 \end{cases} \tag{2}$$

Here $\tau_{systole}$ and $\tau_{diatole}$ are systolic and diastolic time constants; m1 and m2 are also parameters that generate a smooth function for the elastance of the single functioning ventricle.

2.2 Circulation

The model in this project is a modification from Peskin and Tu, where the circulation is consisted of N compliance vessels connected by resistance vessels equipped with valves[2]. Because of conservation of volume,

$$\frac{dV_i}{dt} = \sum_{j=1}^{N} Q_{ji} - Q_{ij} \tag{3}$$

where V_i is the volume of compliance chamber i, and where Q_j is the flow from chamber i to chamber j.

The compliance relation for each chamber is

$$V_i = (V_d)_i + C_i P_i \tag{4}$$

C_i, P_i are the compliance and pressure of chamber i, respectively. $(V_d)_i$ is the dead volume of chamber i, i.e., its volume when $P_I = 0$. The compliance of the heart is introduced above, while the C_i for arteries and veins are constants.

The pressure-flow relationship for each pair of chambers i and j is

$$Q_{ij} = S_{ij} G_{ij} (P_i - P_j) \tag{5}$$

Where G_{ij} is the conductance of flow from chamber i to chamber j, which is

$$G_{ij} = \frac{1}{R_{ij}} \tag{6}$$

The reason why we use conductance G instead of resistance R is to cover the case $R = \inf$ using $G = 0$.

S_{ij} is an indicator function:

$$S_{ij} = \begin{cases} 1 & \text{if } P_i > P_j \\ 0 & \text{if } P_i \leq P_j \end{cases} \tag{7}$$

By substituting the above equations we get

$$\frac{d}{dt}(C_i P_i) = \sum_{j=1}^{N} (S_{ij} G_{ij} + S_{ji} G_{ji})(P_j - P_i) \tag{8}$$

3 Numerical Method

We use backward Euler method to solve the equation, where Δt is a small time step.

$$\frac{C_i(t)P_i(t) - C_i(t-\Delta t)P_i(t-\Delta t)}{\Delta t} = \sum_{j=1}^{N} (S_{ij}(t) G_{ij} + S_{ji}(t) G_{ji})(P_j(t) - P_i(t)) \tag{9}$$

4 Result and Discussion

We first check if the heart in Fontan circulation with fenestration work properly. We set the resistence of fenestration as 1, and plot the single ventricle pressure, systemic arteries pressure, and the pressure-time graph. We noticed that the systemic arteries pressure ranges from apporxiamately 67 to 128, which is about the systemic arteries pressure of a normal person, as shown in figure 4

The pressure-time loop 4 is also similar to a normal person, neglecting the first loop when steady state is not yet reached.

We next explore how the flow and pressure in circulation vary as we change the fenestration parameter from big to small, namely weak effects to strong effects. We calculate the flow and pressure as the average ones of a selected period near the end of whole period in our model.

The case when there is no fenestration could be represented as $R_{fenestration} = \inf$, and as we decrease it to the smallest possible value, which means no backflow in the pulmonary arteries.

From 6, 7, and 8, we observe that as the influence of fenestration gets more and more significant, the fenestration flow and systemic flow increase, while pulmonary flow decreases.

We are also interested in knowing whether the abnormally high systemic veins pressure in the Fontan patients are decreased because of fenestration. In figure 9, we find that the systemic veins pressure decreases, but not significantly. At the same time, pulmonary veins pressure increases, not drastically as well.

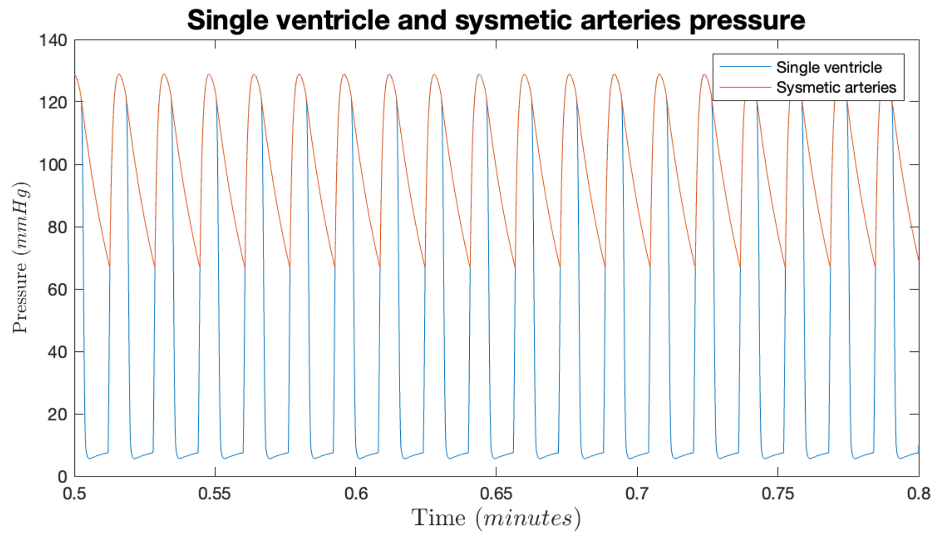

Fig. 4: Graph of Single Ventricle and Systemic Arteries Pressure

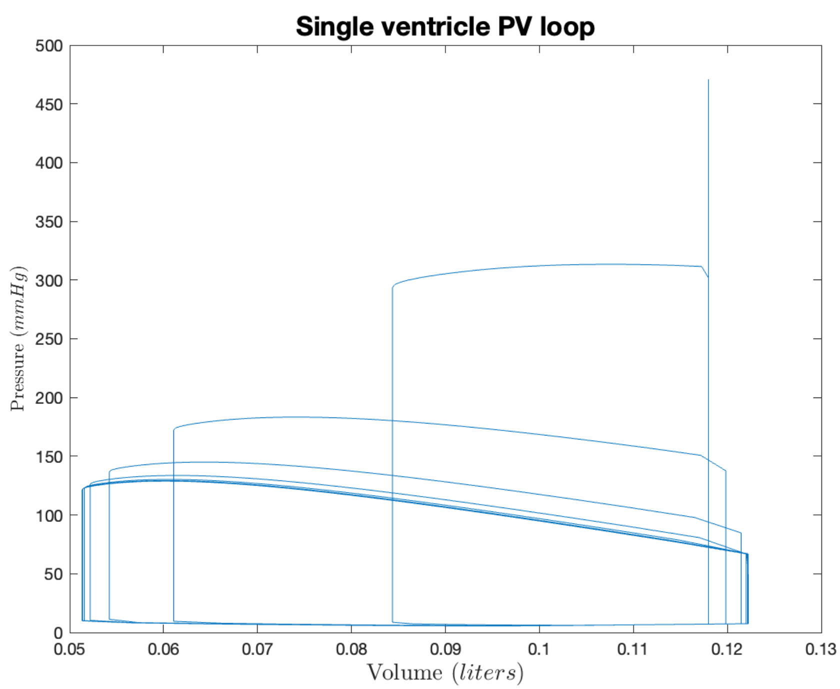

Fig. 5: Single Ventricle PV Loop

However, the differences in the systemic veins pressure and pulmonary veins pressure decreases, and seemingly approaches 0 in responses to smaller resistance of fenestration.

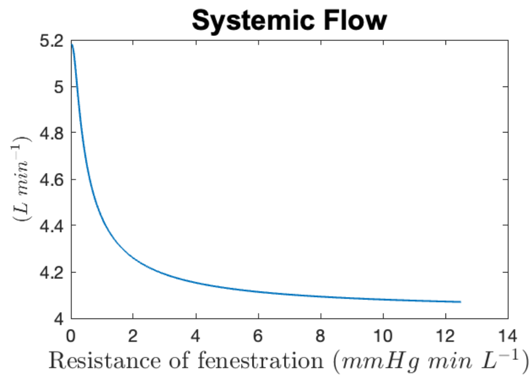

Fig. 6: Systemic Flow in Response to Change of Fenestration Resistance

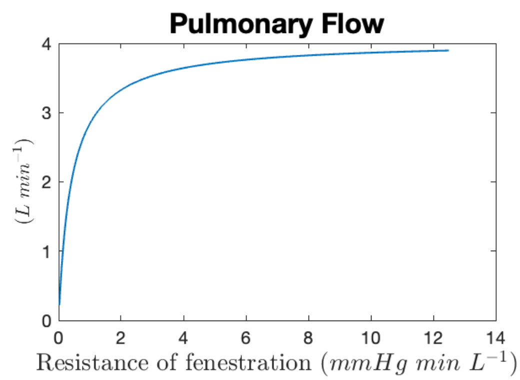

Fig. 7: Pulmonary Flow in Response to Change of Fenestration Resistance

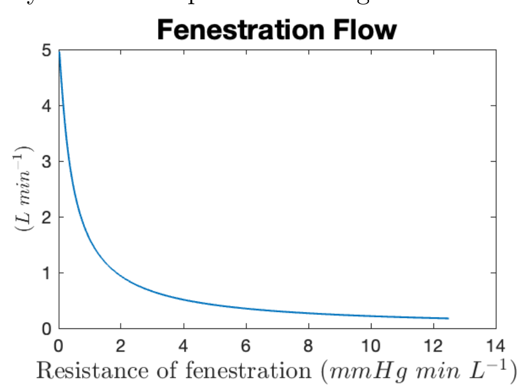

Fig. 8: Fenestration Flow in Response to Change of Fenestration Resistance

5 Summary and Conclusions

In this paper, we model a Fontan circulation and incorporates a fenestration between the systemic and pulmonary veins. We change the fenestration resistence parameter, and observe how flow and pressure in circulation vary accordingly. We find the increase in systemic flow and fenestration flow and pulmonary veins pressure, and decrease in pulmonary flow and systemic veins pressure.

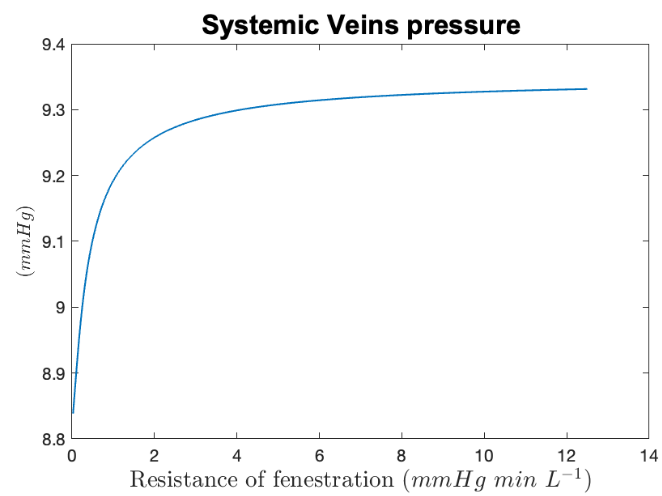

Fig. 9: Systemic Veins Pressure in Response to Change of Fenestration Resistance

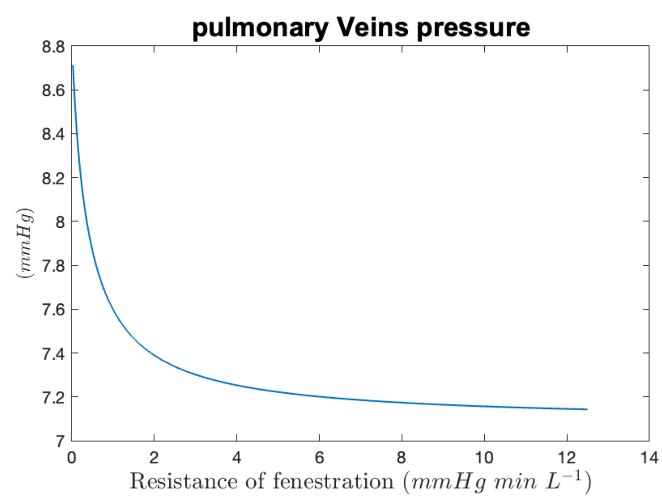

Fig. 10: Pulmonary Veins Pressure in Response to Change of Fenestration Resistance

References

1. Ahmad Z., Jynn L, P.D.R.C.P.C.P.C.: Optimal fenestration of the Fontan circulation (Feb 2022), arXiv:2202.01104
2. Peskin C., C.T.: Computers in Biology and Medicine (1986)
3. Yabrodi M., M.C.: Hypoplastic left heart syndrome: from comfort care to long-term survival. Pediatr Res **81**, 142–149 (2017), `https://doi.org/10.1038/pr.2016.194`

6 Appendix

```
%filename: in_circ.m  (initialization for circ)
T =0.016;     %Duration of heartbeat (minutes)
m1=1.32;
m2=27.4;
tao1=0.269*T;
tao2=0.452*T;
Emin=79.52;
Emax=5232;
TS=0.0050;     %Duration of systole   (minutes)
tauS=0.0025;  %CLV time constant during systole (minutes)
tauD=0.0075;  %CLV time constant during diastole (minutes)
Rs=20.78;       %Systemic resistance  (mmHg/(liter/minute))
Rp= 0.5517;      %Pulmonary resistance (mmHg/(liter/minute))
%Unrealistic valve resistances,
%Chosen small enough to be negligible.
RMi=0.01; %mitral valve resistance (mmHg/(liter/minute))
RAo=0.01; %aortic valve resistance (mmHg/(liter/minute))
RTr=0.01; %tricuspid valve resistance (mmHg/(liter/minute))
RPu=0.01; %pulmonic valve  resistance (mmHg/(liter/minute))
%The following values of Csa and Cpa are approximate.
%They will need adjustment to make the systemic
%blood pressure be roughly 120/80 mmHg
%and to make the pulmonary
%blood pressure be roughly 25/8 mmHg.
Csa=0.0007333;  %Systemic  arterial compliance (liters/mmHg)
Cpa=0.00412;  %Pulmonary arterial compliance (liters/mmHg)
Csv=0.0990;      %Systemic  venous compliance (liters/mmHg)
Cpv=0.01;     %Pulmonary venous compliance (liters/mmHg)
% CLVS=0.001 %Min (systolic)  value of CLV (liters/mmHg)
% CLVD=0.0146  %Max (diastolic) value of CLV (liters/mmHg)
% CRVS=0.0002  %Min (systolic)  value of CRV (liters/mmHg)
% CRVD=0.0365  %Max (diastolic) value of CRV (liters/mmHg)
Vsad=0.7051;   %Systemic arterial volume at P=0 (liters)
Vpad=0.0930;  %Pulmonary arterial volume at P=0 (liters)
Vsvd=2.869;       %Systemic venous volume at P=0 (liters)
Vpvd=0.1475;       %Pulmonary venous volume at P=0 (liters)
VLVd=0.028;   %Left ventricular volume at P=0 (liters)
dt=0.01*T;    %Time step duration (minutes)
%This choice implies 100 timesteps per cardiac cycle.
klokmax=ceil(60*T/dt); %Total number of timesteps
%This choice implies simulation of 15 cardiac cycles.
%Assign an index to each compliance vessel
%of the model circulation:

iLV=1;
isa=2;
```

```
isv=3;
ipa=4;
ipv=5;
N=5;
%Enter parameters and initial values
%into correct slots in arrays.
%Note that the code that follows is independent
%of the specific numbering scheme chosen above.
%Compliance vector:
C=zeros(N,1);
%This makes C a column vector of length N.
fmax=0;
C(iLV)=CV_now(0,fmax);  %initial value
C(isa)=Csa;
C(isv)=Csv;
% C(iRV)=CV_now(0,CRVS,CRVD);  %initial value
C(ipa)=Cpa;
C(ipv)=Cpv;
% C  %This writes the result on the screen.
%Pressure vector (initial values) at end of diastole:
P=zeros(N,1);
%This makes P a column vector of length N.
P(iLV)= 9;
P(isa)=80;
P(isv)= 2;
P(ipa)= 8;
P(ipv)= 5;
% P  %This writes the result on the screen.
%Vector of dead volumes (volume at zero pressure);
%Note: Vd is only needed for output purposes.
%It drops out of the equations we solve for P,
%but we need it if we want to output (e.g., plot)
%the volume of any compliance vessel.
Vd=zeros(N,1);
%This makes Vd a column vector of length N.
Vd(iLV)=VLVd;
Vd(isa)=Vsad;
Vd(isv)=Vsvd;
Vd(ipa)=Vpad;
Vd(ipv)=Vpvd;
% Vd
% Rig the volume
Vset = 5.0;
Vtotal = sum(Vd+C.*P);
P(isa) = P(isa) + (Vset-Vtotal)/C(isa);
newV = sum(Vd+C.*P);
%This writes the results on the screen.
%Conductance matrix:
G=zeros(N,N);
%This makes G an NxN matrix filled with zeros.
```

```
%Any element of G that is not explicitly
%made nonzero remains zero,
%thus modeling an infinite resistance connection,
%that is, no connection at all.
G(iLV,isa)=1/RAo;  %But G(isa,iLV)=0 (no leak)
G(isa,isv)=1/Rs;   %no valve
G(isv,isa)=1/Rs;   %no valve
G(isv,ipa)=1/RTr;  %But G(iRV,isv)=0; (no leak)
G(ipa,isv)=1/RTr;
% G(iRV,ipa)=1/RPu;  %But G(ipa,iRV)=0; (no leak)
G(ipa,ipv)=1/Rp;   %no valve
G(ipv,ipa)=1/Rp;   %no valve
G(ipv,iLV)=1/RMi;  %But G(iLV,ipv)=0; (no leak)

Rpsv = 0.025*loop;
% Rpsv = loop;
% Rpsv = inf;
G(ipv,isv)=1/Rpsv;
G(isv,ipv)=1/Rpsv;

% sG  %This writes the result on the screen.
%Matrix of initial valve states:
S=zeros(N,N);
%This makes S an NxN matrix filled with zeros
%(and writes it on the screen).
%Start with all valves closed.
%Valves will adjust to pressures
%during first time step.
%Initialize arrays to store data for plotting:
t_plot=zeros(1,klokmax);
C_plot=zeros(N,klokmax);
P_plot=zeros(N,klokmax);
%Other variables that we might want to plot
%can be found from these.
%For self-checking in P_new, set CHECK=1.
%To skip self-checking set CHECK=0.
%(should be much faster with CHECK=0)
CHECK=0;
%Initialize flow computation (for output purposes only)
%assign an index to each flow of interest:
jAo=1;
js =2;
jTr=3;
jp =4;
jMi=5;
jpsv=6;
Nflows=6;
%note index of upstream and downstream chamber
%for each flow:
iU=zeros(Nflows,1);
```

```
iD=zeros(Nflows,1);
iU(jAo)=iLV;
iD(jAo)=isa;
iU(js )=isa;
iD(js )=isv;
iU(jTr)=isv;
iD(jTr)=ipa;
iU(jp )=ipa;
iD(jp )=ipv;
iU(jMi)=ipv;
iD(jMi)=iLV;

iU(jpsv)=isv;
iD(jpsv)=ipv;

%extract the conductances from the matrix G:
Gf=zeros(Nflows,1);
Gr=zeros(Nflows,1);
for j=1:Nflows
  Gf(j)=G(iU(j),iD(j)); %forward conductance
  Gr(j)=G(iD(j),iU(j)); %reverse conductance
end
%create arrays to store current pressure differences
%and history over time of the net flows:
Pdiff=zeros(Nflows,1);
Q_plot=zeros(Nflows,klokmax);
```

```


%filename:  circ.m
clear all % clear all variables
clf       % and figures
global T TS tauS tauD;
global G dt CHECK N;
% in_circ  %initialize
pmax = 500;
Nflows=6;
R_plot=zeros(1,pmax);
Qm_plot=zeros(Nflows,pmax);
for loop=1:pmax
    loop
    revised
    R_plot(loop)=Rpsv;
    t_all = [];
    for klok=1:floor(T/dt)
        t=klok*dt;
        t_all = [t_all; t];
    end
```

```
    t_all;
    fmax = 0;
    gT=(T/tao2)^m2;
    for i = 1:length(t_all)
        g1t = (t_all(i)/tao1)^m1;
        g2t = (t_all(i)/tao2)^m2;
        f = (g1t/(1+g1t))*(1/(1+g2t)-1/(1+gT));
        if f>fmax
            fmax=f;
        end
    end

    for klok=1:klokmax
      t=klok*dt;

      P_old=P;
      C_old=C;
      %find current values of left and right
      %ventricular compliance and store each
      %of them in the appropriate slot in the array C:
      C(iLV)=CV_now(t,fmax);
      %find self-consistent valve states and pressures:
      set_valves
      %store variables in arrays for future plotting:
      t_plot(klok)=t;
      C_plot(:,klok)=C;
      P_plot(:,klok)=P;
      V_plot(:,klok)=Vd+C.*P;
      Pdiff=P(iU)-P(iD); %pressure differences
                         %for flows of interest:
      Q_plot(:,klok)=(Gf.*(Pdiff>0)+Gr.*(Pdiff<0)).*Pdiff;

      %(the net flow is computed in each case)
    end
    %plot results:
%     circ_out
    Qm_plot(:,loop)=mean(Q_plot(:,end-2000:end)');
    Pm_plot(:,loop)=mean(P_plot(:,end-2000:end)');
end
figure(1)
subplot(3,1,1),plot(R_plot,Qm_plot(2,:),'LineWidth',1)
title('Systemic Flow','FontSize',16,'FontName','Arial')
xlabel('Resistance of fenestration ($mmHg$ $min$ ...
    $L^{-1}$)','interpreter','latex','fontweight','bold','fontsize',14)
ylabel('($L$ $min^{-1}$)','interpreter','latex')
subplot(3,1,2),plot(R_plot,Qm_plot(3,:),'LineWidth',1)
title('Pulmonary Flow','FontSize',16)
xlabel('Resistance of fenestration ($mmHg$ $min$ ...
    $L^{-1}$)','interpreter','latex','fontweight','bold','fontsize',14)
ylabel('($L$ $min^{-1}$)','interpreter','latex')
```

```
subplot(3,1,3),plot(R_plot,Qm_plot(6,:),'LineWidth',1)
title('Fenestration Flow','FontSize',16)
xlabel('Resistance of fenestration ($mmHg$ $min$ ...
    $L^{-1}$)','interpreter','latex','fontweight','bold','fontsize',14)
ylabel('($L$ $min^{-1}$)','interpreter','latex')
figure(2)
subplot(2,1,1),plot(R_plot,Pm_plot(3,:),'LineWidth',1)
title('Systemic Veins ...
    pressure','FontSize',16,'FontName','Arial')
xlabel('Resistance of fenestration ($mmHg$ $min$ ...
    $L^{-1}$)','interpreter','latex','fontweight','bold','fontsize',14)
ylabel('($mmHg$)','interpreter','latex')
subplot(2,1,2),plot(R_plot,Pm_plot(5,:),'LineWidth',1)
title('pulmonary Veins ...
    pressure','FontSize',16,'FontName','Arial')
xlabel('Resistance of fenestration ($mmHg$ $min$ ...
    $L^{-1}$)','interpreter','latex','fontweight','bold','fontsize',14)
ylabel('($mmHg$)','interpreter','latex')
```

```
function P=P_new(P_old,C_old,C,S)
%filename: P_new.m
global G dt CHECK N;
A=-dt*((S.*G)+(S.*G)');
A=diag(C-(sum(A))')+A;
P=A\(C_old.*P_old);
if(CHECK)
  for i=1:N
    CH(i)=-(C(i)*P(i)-C_old(i)*P_old(i))/dt;
    for j=1:N
      CH(i)=CH(i)+S(j,i)*G(j,i)*(P(j)-P(i));
      CH(i)=CH(i)-S(i,j)*G(i,j)*(P(i)-P(j));
    end
  end
  CH          %Write out the values of CH,
%which should be zero to within roundoff.
end
```

```
function CV=CV_now(t,fmax)
%filename: CV_now.m
global T;
m1=1.32;
m2=27.4;
tao1=0.269*T;
tao2=0.452*T;
Emin=79.52;
Emax=5232;
tc=rem(t,T); %tc=time in the current cycle,
```

```
                %measured from start of systole.
%instead of using the piecewise function, we use a ...
    continuous function
g1=(tc/tao1)^m1;
g2=(tc/tao2)^m2;
gT=(T/tao2)^m2;

if fmax ≠0
    k = (Emax-Emin)/fmax;
    Ev = k*(g1/(1+g1))*(1/(1+g2)-1/(1+gT))+Emin;
    CV = 1/Ev;
else
    CV = 0.01;
end
```

```
%filename: set_valves.m
%script to find self-consistent
%valve states and pressures:
done=0; % not done yet!
while(¬done)  %if not done, keep trying
  S_noted=S;  %note valve states
  %set pressures based on valve states:
  P=P_new(P_old,C_old,C,S);
  %then set valve states based on pressures:
  P_matrix=P*ones(1,N);
  S=((P_matrix) > (P_matrix'));
  %done if all valve states are unchanged:
  done=all(all(S==S_noted));
end
```

```
%filename: circ_out.m
%script to plot results of computer simulation
%of the entire circulation.

figure(1)
subplot(2,1,1),plot(t_plot,P_plot([iLV,isa],:))
xlim([0.5 0.8])
title('Single ventricle and sysmetic arteries ...
    pressure','FontSize',16)
xlabel('Time ...
    ($minutes$)','interpreter','latex','fontweight','bold','fontsize',14)
ylabel('Pressure ($mmHg$)','interpreter','latex')
legend('Single ventricle','Sysmetic arteries')

subplot(2,1,2),plot(t_plot,P_plot(isv,:))
title('Systemic veins pressure','FontSize',16)
xlim([0.5 0.8])
```

```
xlabel('Time ...
    ($minutes$)','interpreter','latex','fontweight','bold','fontsize',14)
ylabel('Pressure ($mmHg$)','interpreter','latex')

%systemic and pulmonary flows:
figure(2)
subplot(3,1,1),plot(t_plot,Q_plot([jAo,js],:))
xlim([0.2 1])
subplot(3,1,2),plot(t_plot,Q_plot([jTr,jp],:))
xlim([0.2 1])
subplot(3,1,3),plot(t_plot,Q_plot([jMi,jpsv],:))
xlim([0.2 1])

figure(3)
plot(V_plot(iLV,:),P_plot(iLV,:))
title('Single ventricle PV loop','FontSize',16)
xlabel('Volume ...
    ($liters$)','interpreter','latex','fontweight','bold','fontsize',14)
ylabel('Pressure ($mmHg$)','interpreter','latex')

% fenestration flow
figure(4)
plot(t_plot,Q_plot(jpsv,:))
xlim([0.2 1])
```