Simulation of Suspension Bridge in Spring System, Responding to Car and Earthquake

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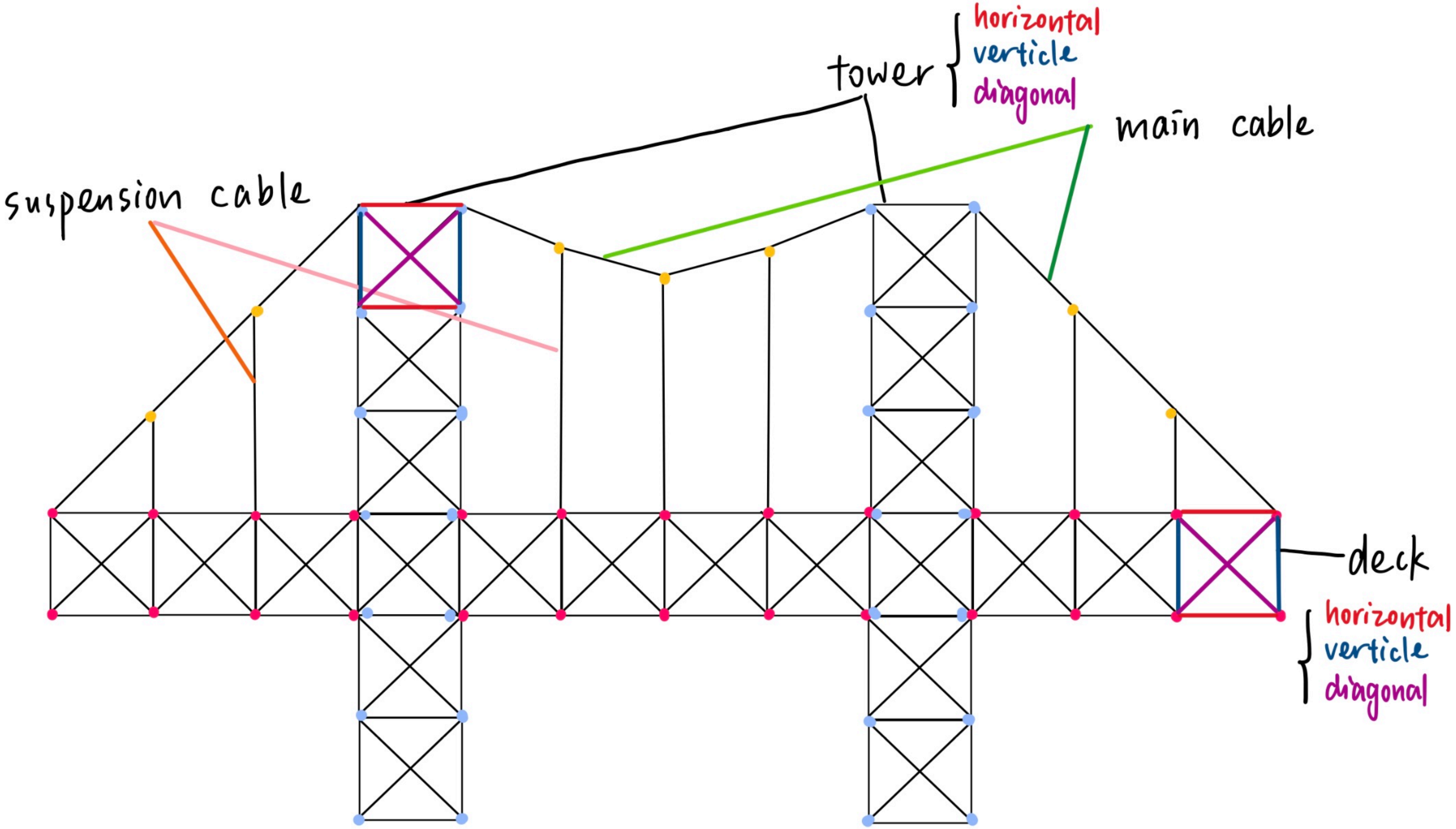
Suspension Bridge Structure (with nodes and links)



Fundamental Equations of Spring System

$$M_k \frac{d\vec{U_k}}{dt} = \sum_{j \in N(k)} T_{jk} \frac{\vec{X_j} - \vec{X_k}}{||\vec{X_j} - \vec{X_k}||}$$

$$\frac{d\vec{X_k}}{dt} = \vec{U_k}$$

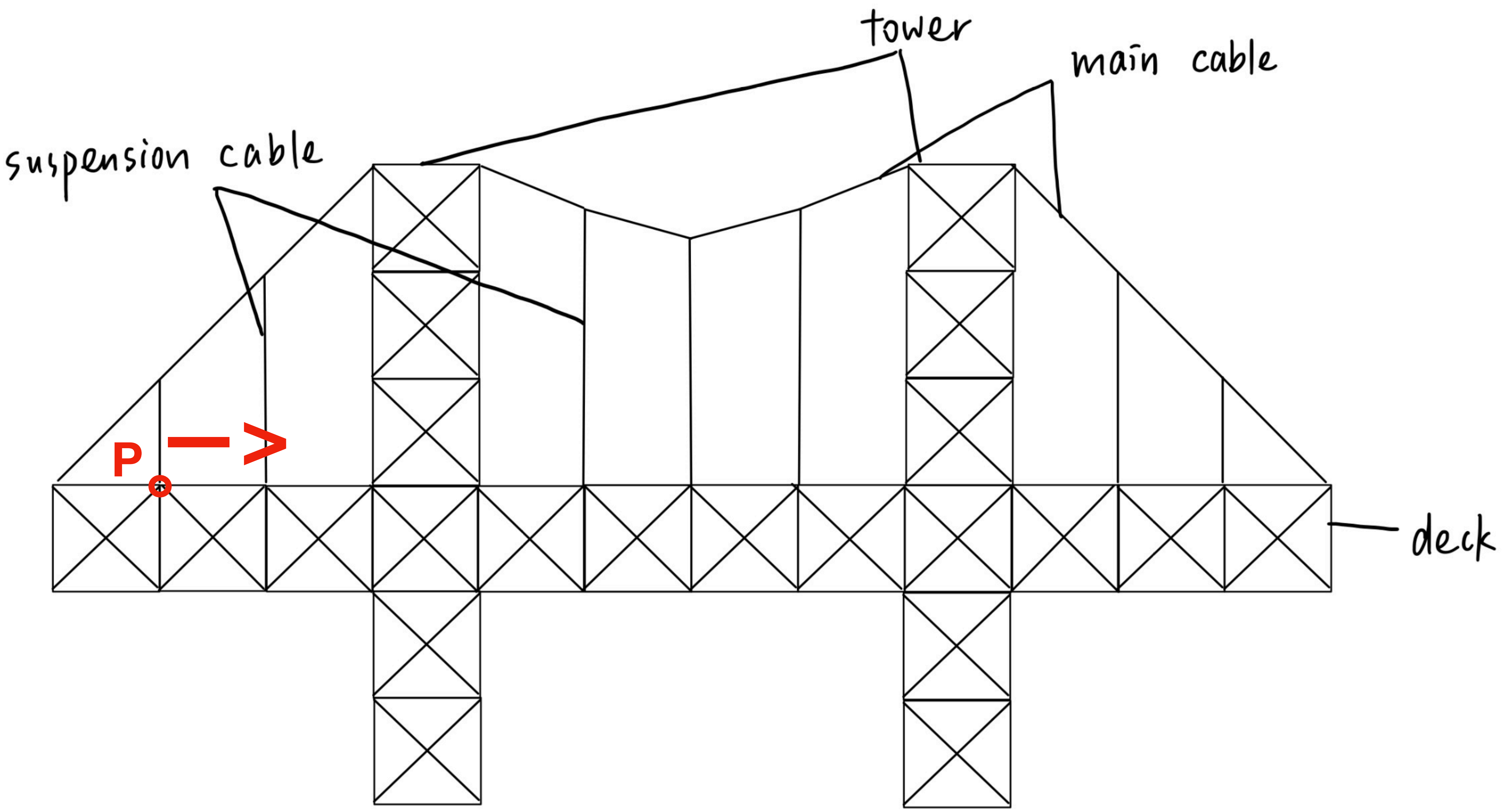
$$T_{jk} = S_{jk}(||\vec{X_j} - \vec{X_k}|| - R^0_{jk}) + D_{jk}\frac{d}{dt}||\vec{X_j} - \vec{X_k}||$$

Note: velocity of the nodes connected to the ground should be 0

When a Car Passes the Suspension Bridge

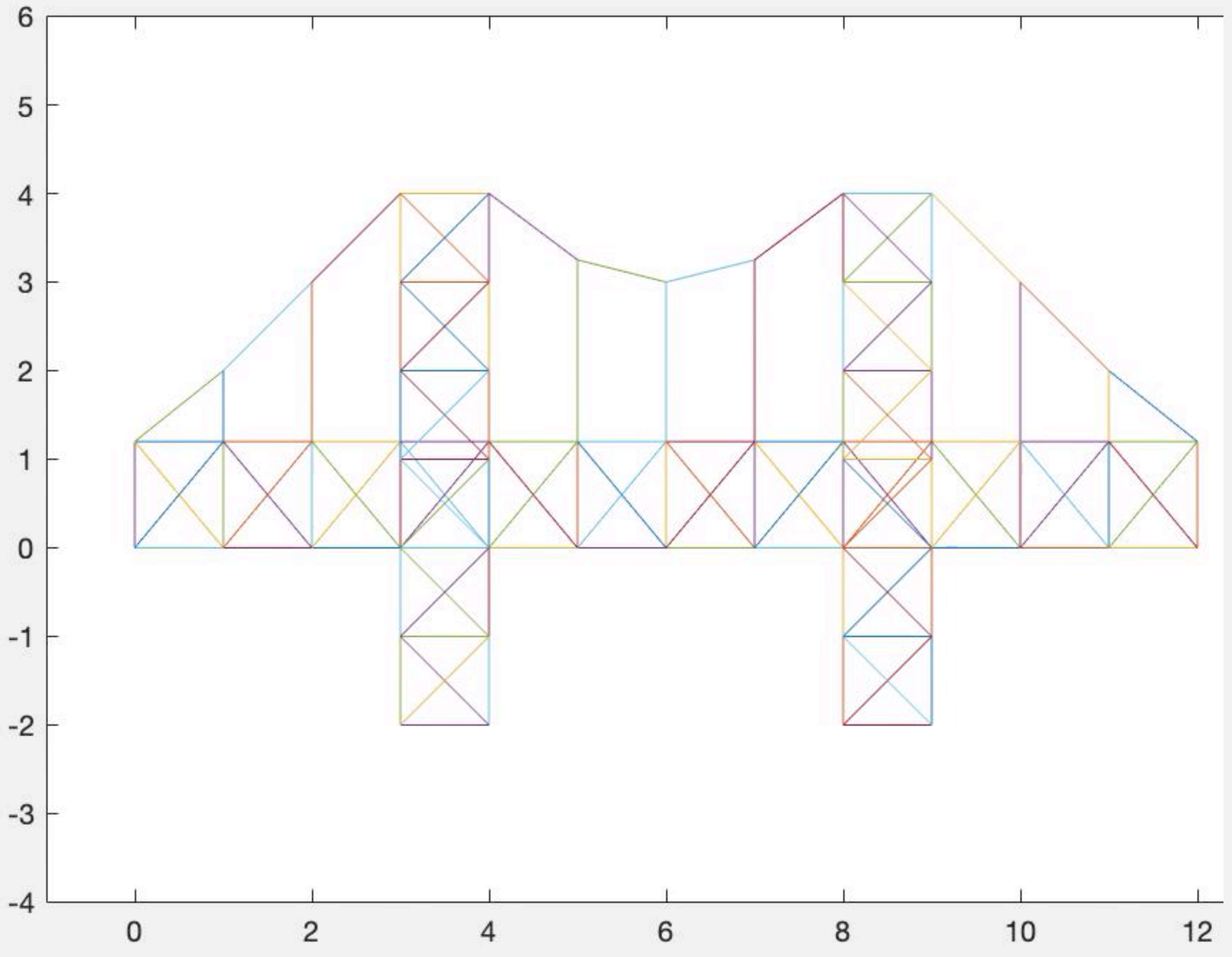
$$\vec{F_p} = \vec{F_p} + \vec{F_{car}}$$

How does a car with different mass influence the suspension bridge?

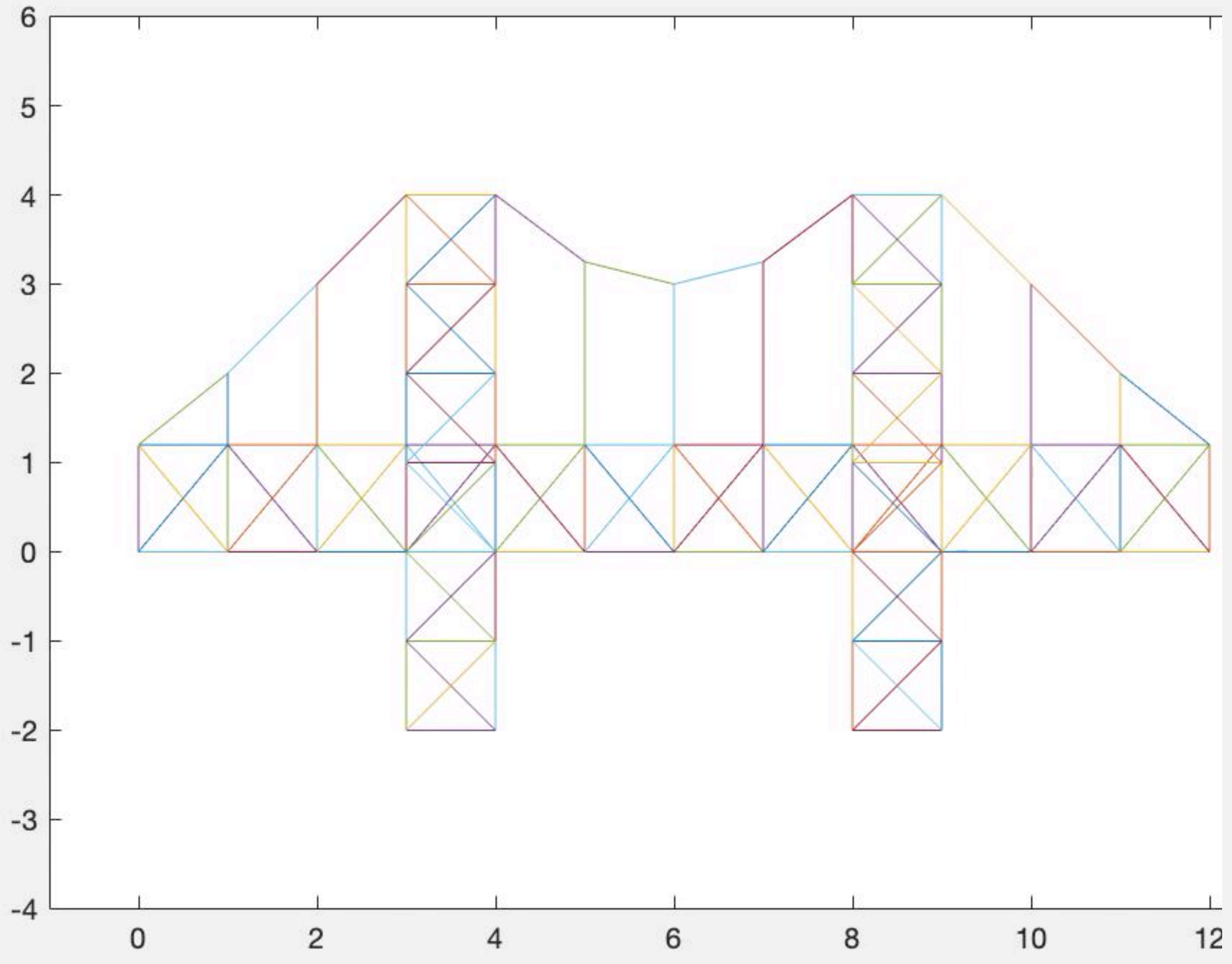


When a Car Passes the Suspension Bridge

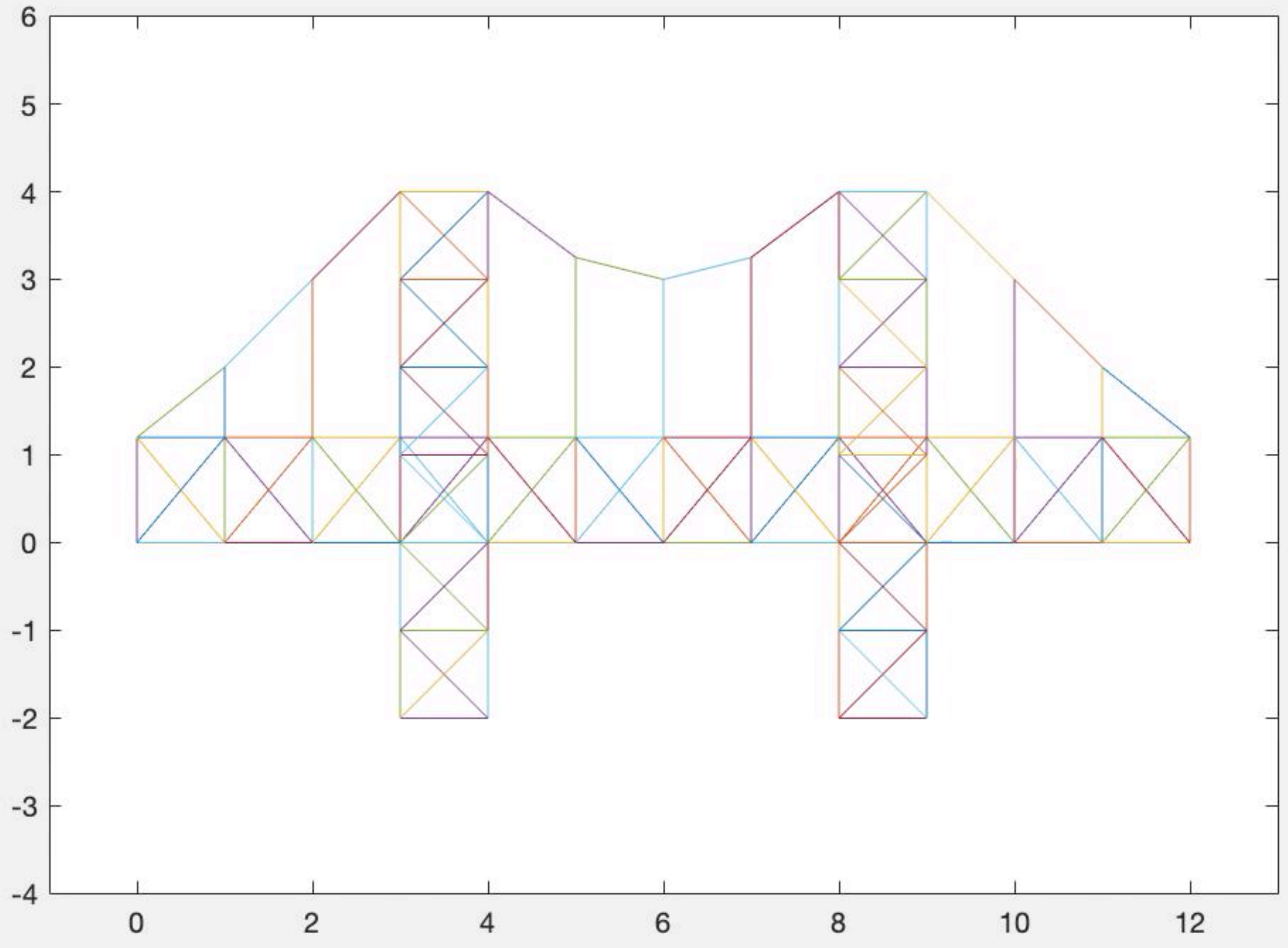
How does a car with different mass influence the suspension bridge?



$m_{car} = 10$



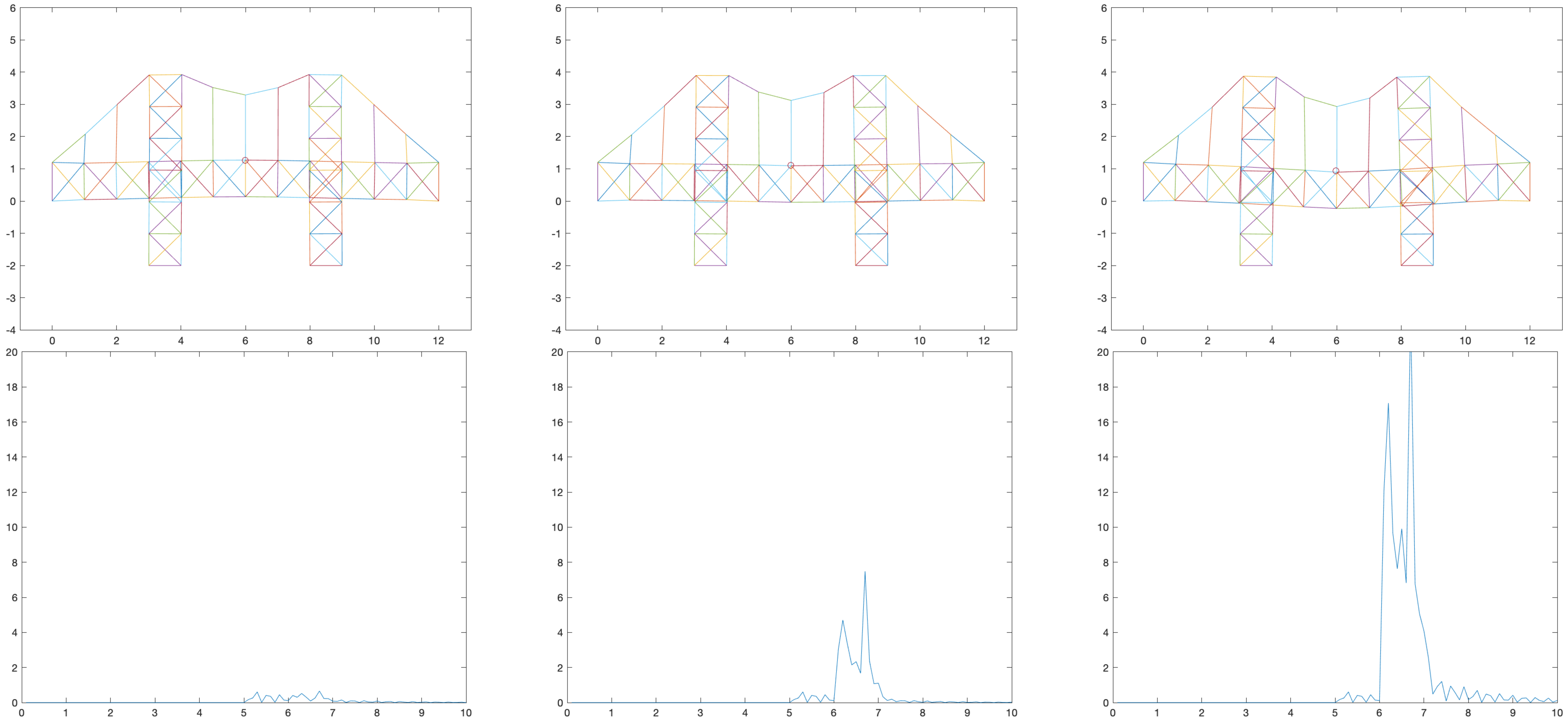
$m_{car} = 50$



$m_{car} = 100$

When a Car Passes the Suspension Bridge

Plot kinetic energy: $E_{kinetic} = \sum_i \frac{1}{2} m_i \vec{U}_i^{\,2}$



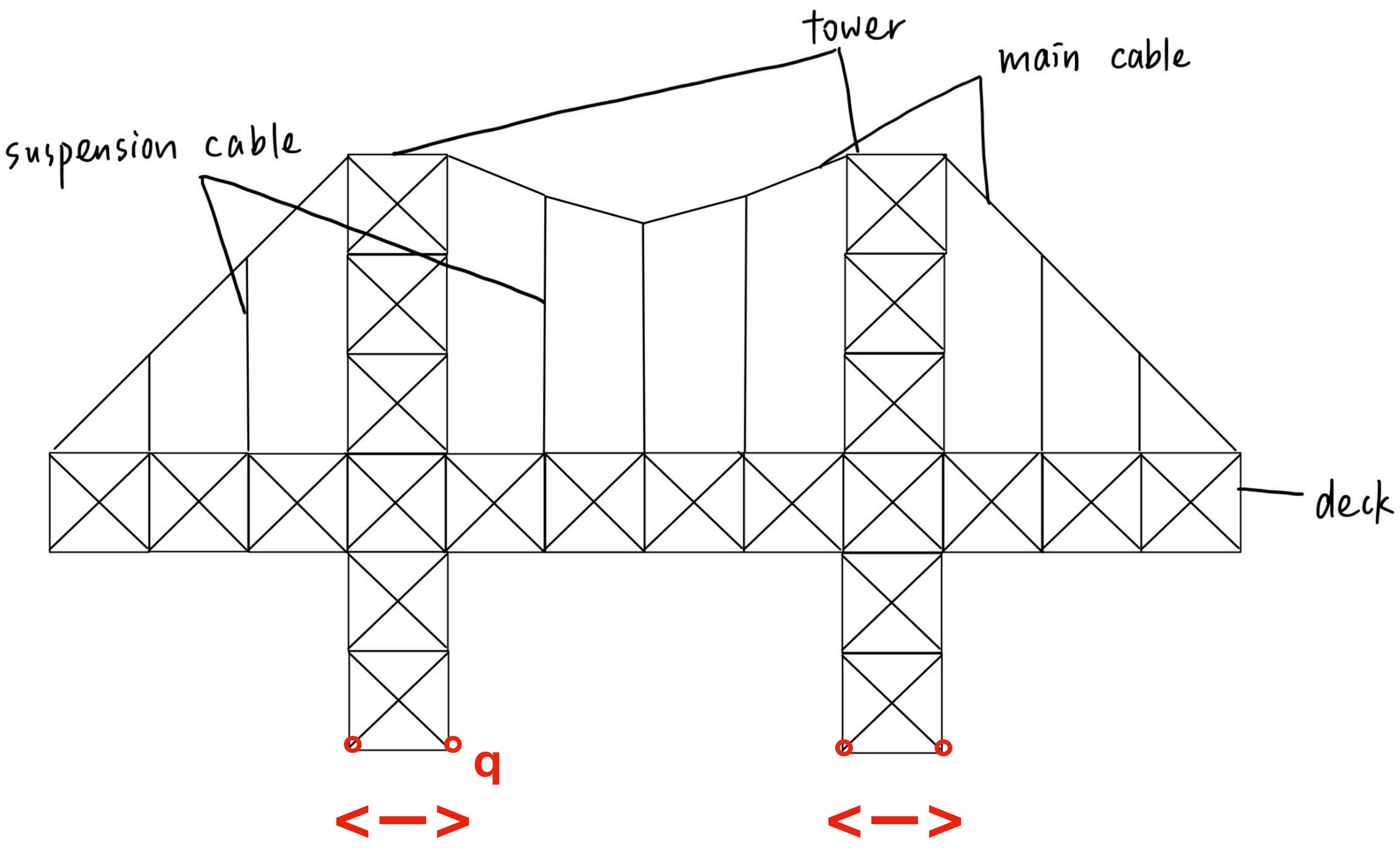
$m_{car} = 10$ $m_{car} = 50$ $m_{car} = 100$

When an Earthquake Happens

$$\vec{X_q}(1) = \vec{X_q}(1) + A\sin(2\pi\omega(t - t_0))$$

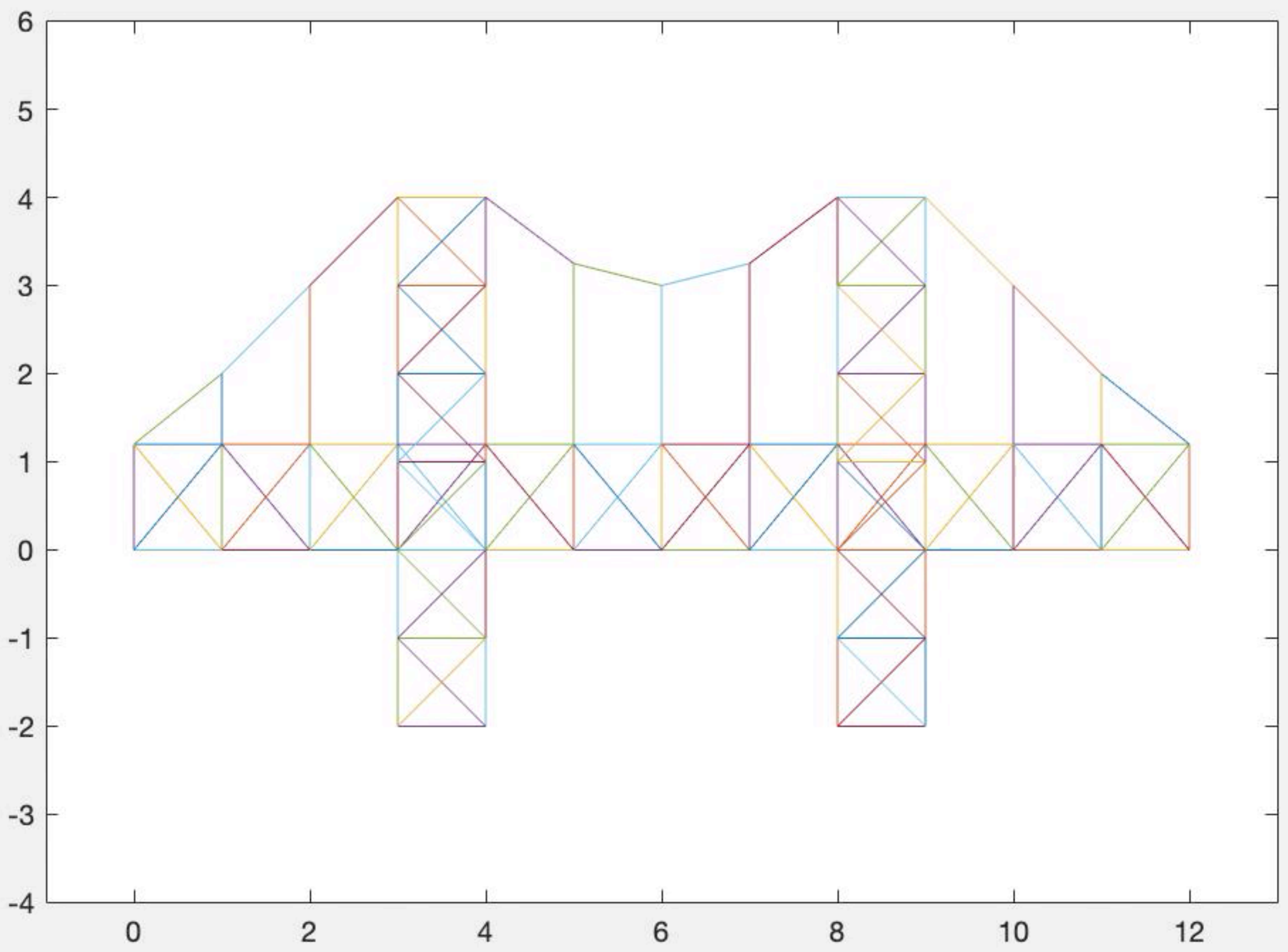
$$\vec{U_q}(1) = \vec{U_q}(1) + 2\pi\omega A\cos(2\pi\omega(t - t_0))$$

How does the earthquake with different frequency and amplitude influence the suspension bridge?

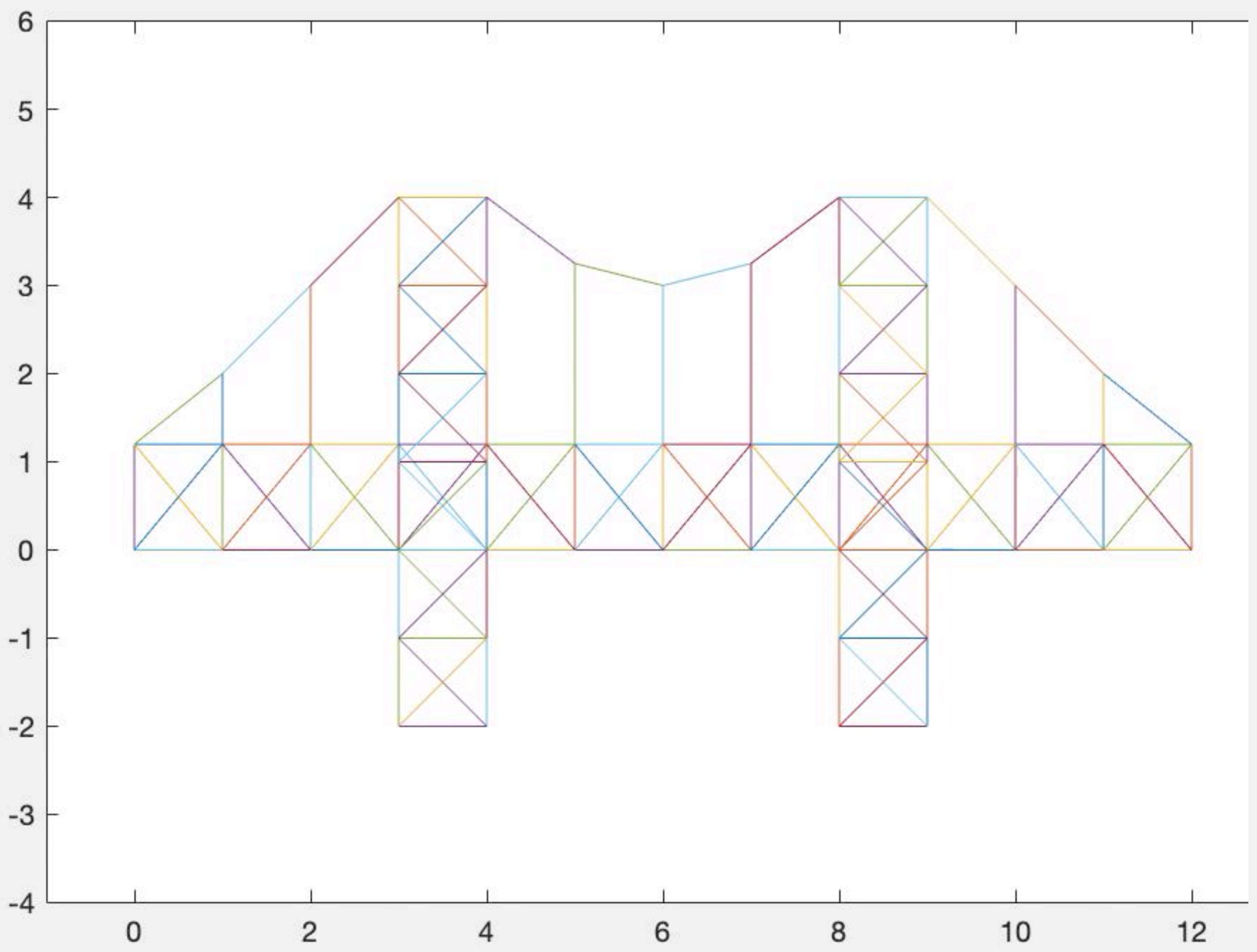


When an Earthquake Happens

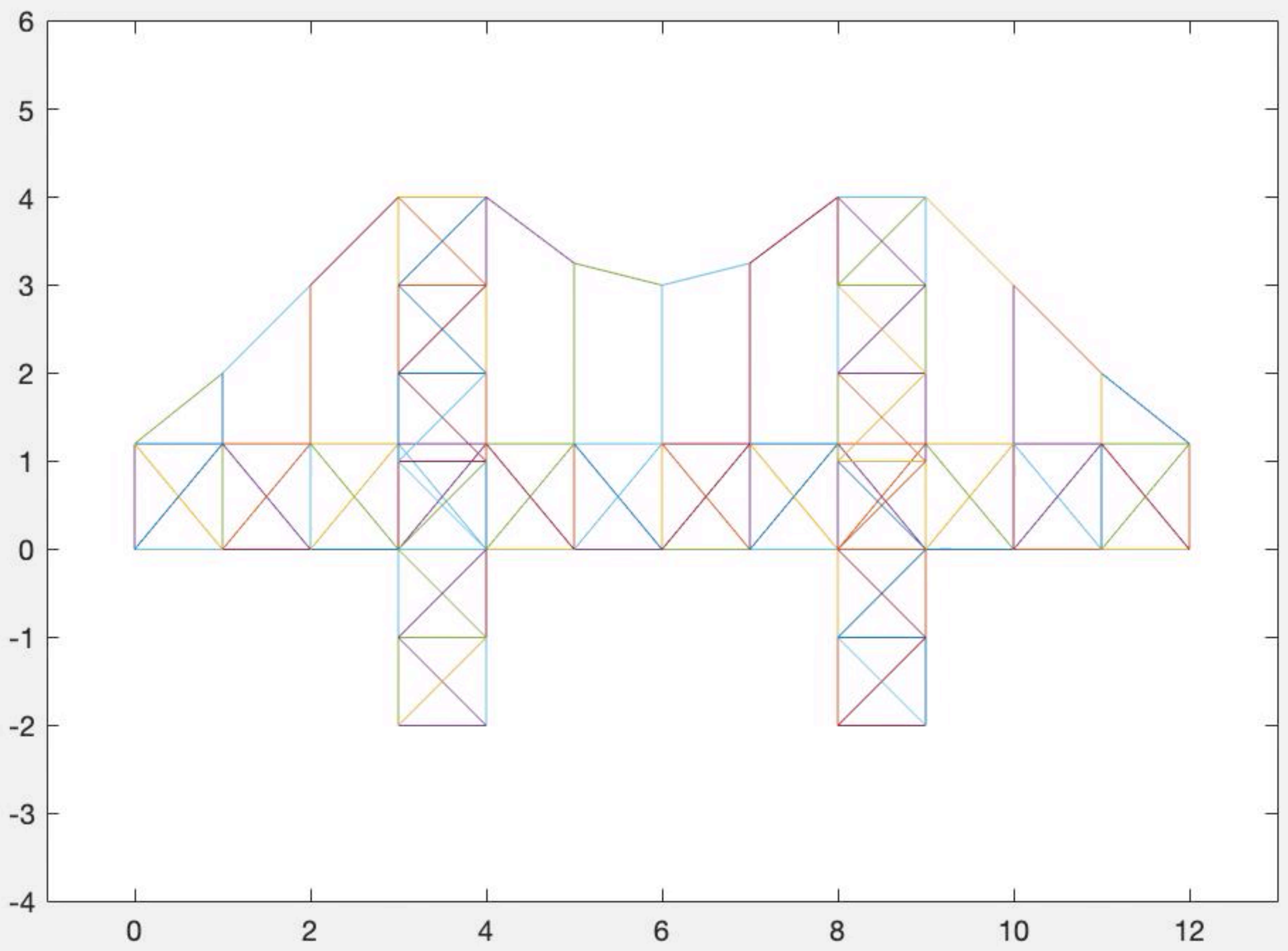
How does the earthquake with same amplitude and different frequency influence the suspension bridge?



$\omega = 2$ and $A = 0.001$



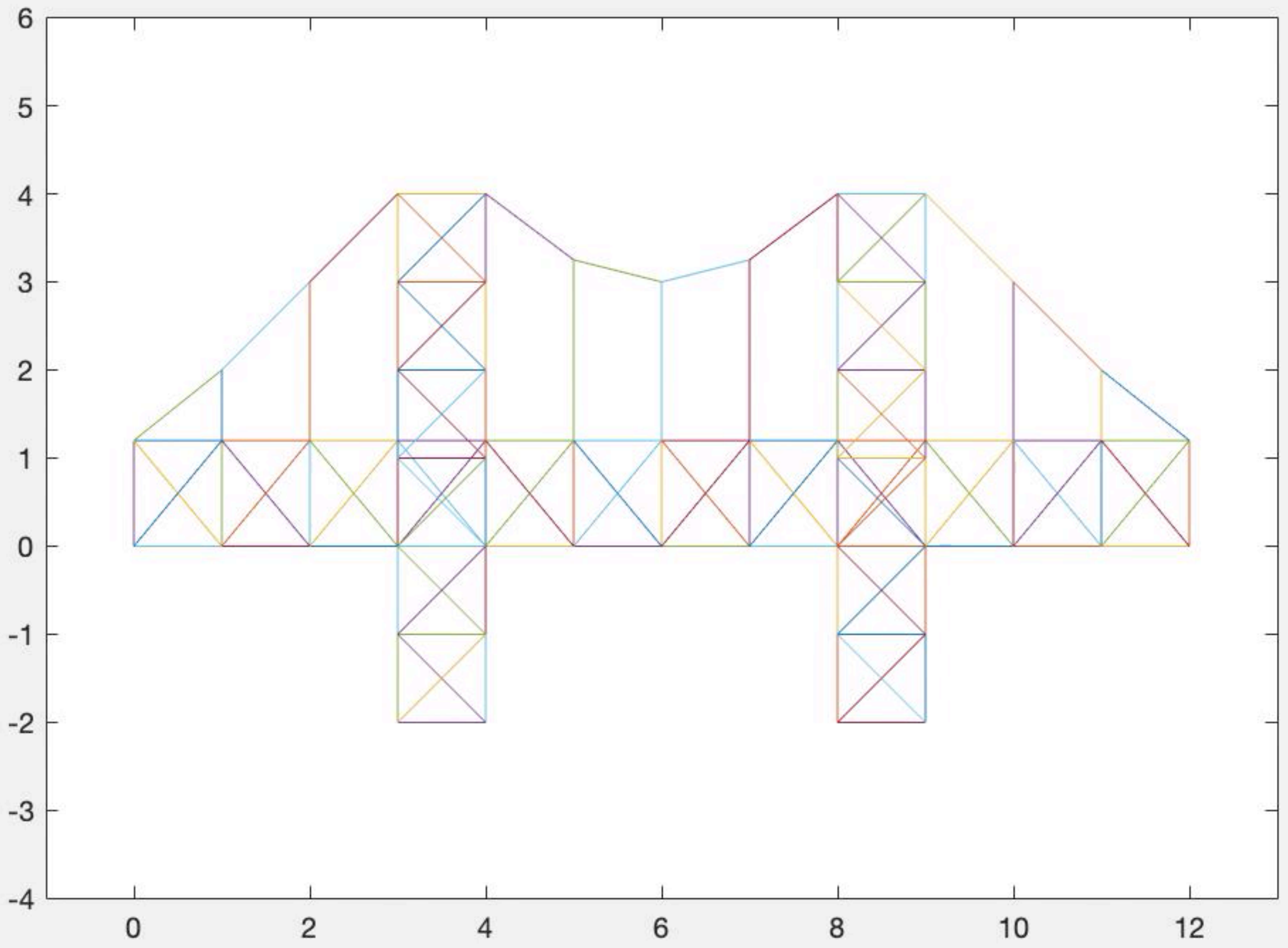
$\omega = 5$ and $A = 0.001$



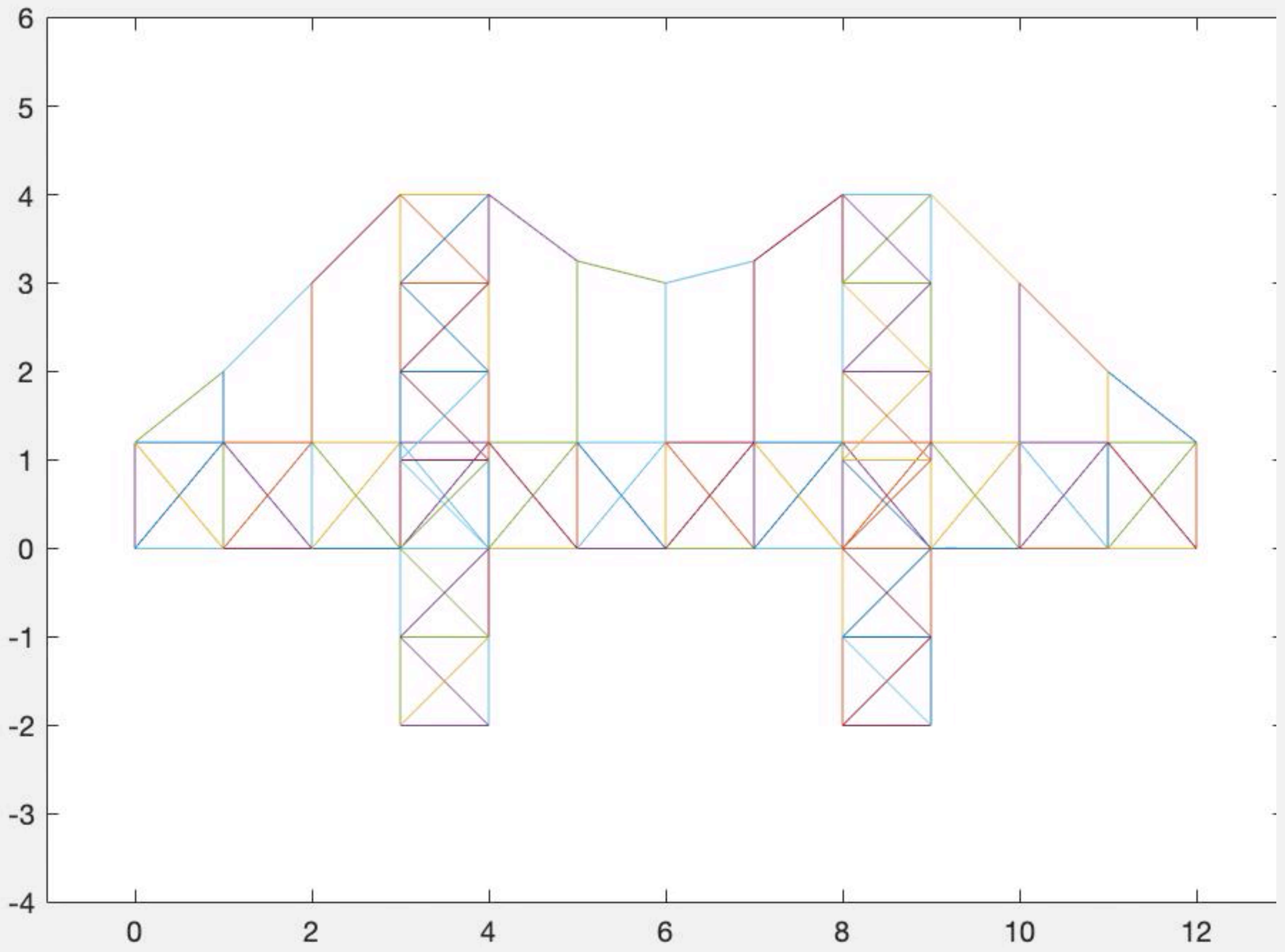
$\omega = 10$ and $A = 0.001$

When an Earthquake Happens

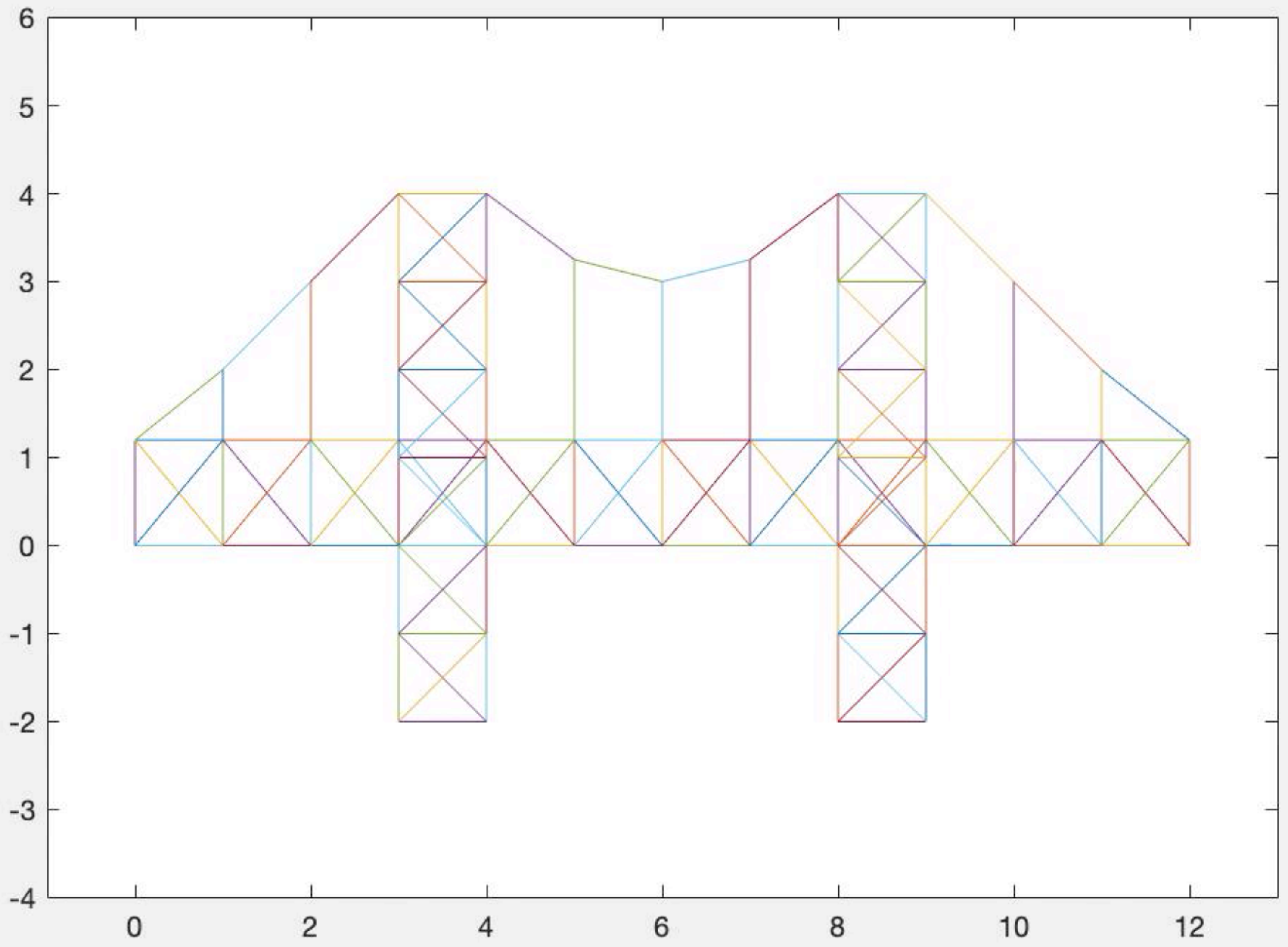
How does the earthquake with same frequency and different amplitude influence the suspension bridge?



$A = 0.0005$ and $\omega = 5$

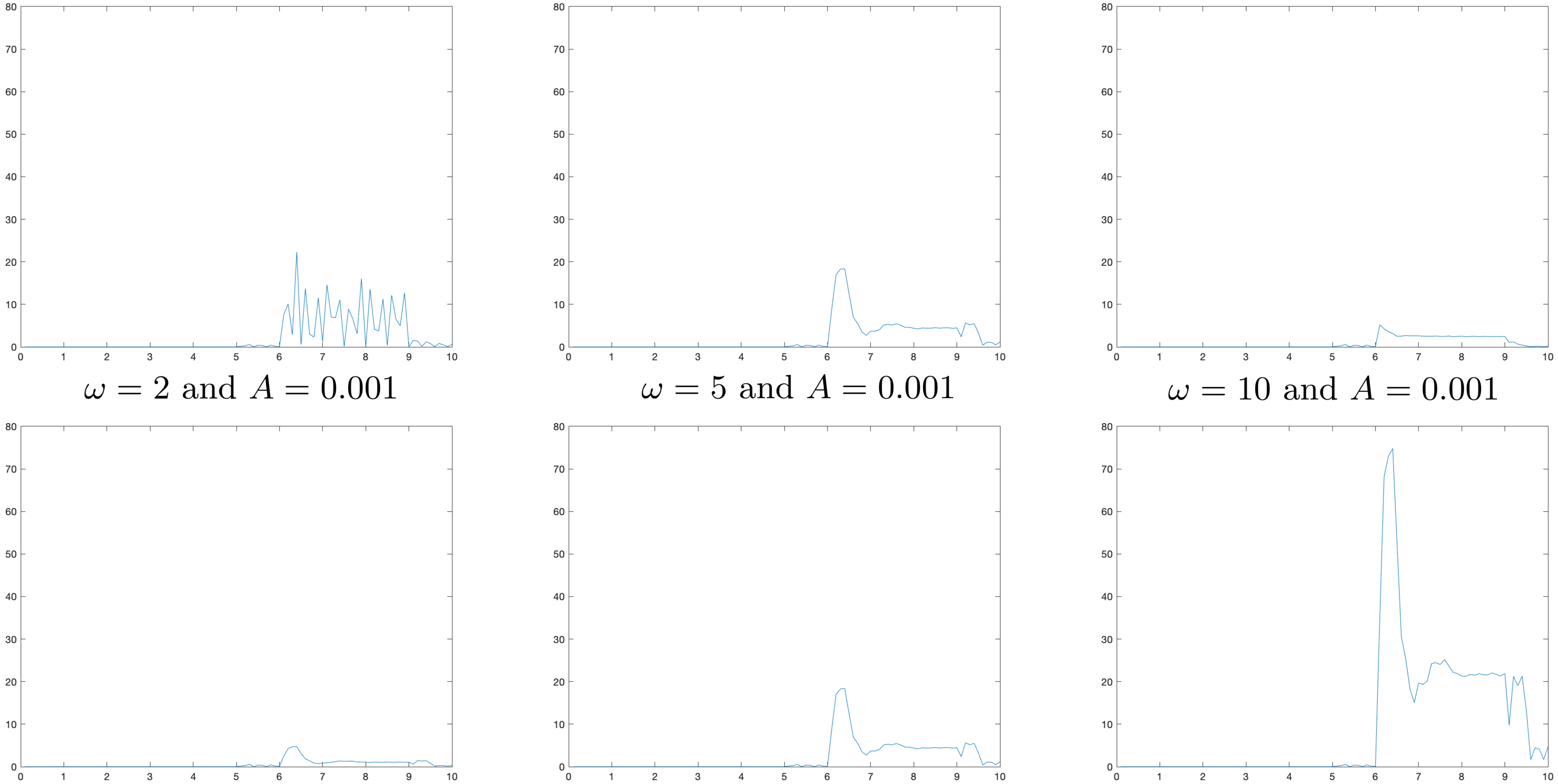


$A = 0.001$ and $\omega = 5$



$A = 0.002$ and $\omega = 5$

When an Earthquake Happens



$\omega = 2$ and $A = 0.001$

$\omega = 5$ and $A = 0.001$

$\omega = 10$ and $A = 0.001$

$A = 0.0005$ and $\omega = 5$

$A = 0.001$ and $\omega = 5$

$A = 0.002$ and $\omega = 5$

When Refine the Suspension Bridge

How does the fineness of the suspension bridge influence itself?

$$m \to \frac{m}{n}$$

$$R^0 \to \frac{R^0}{n}$$

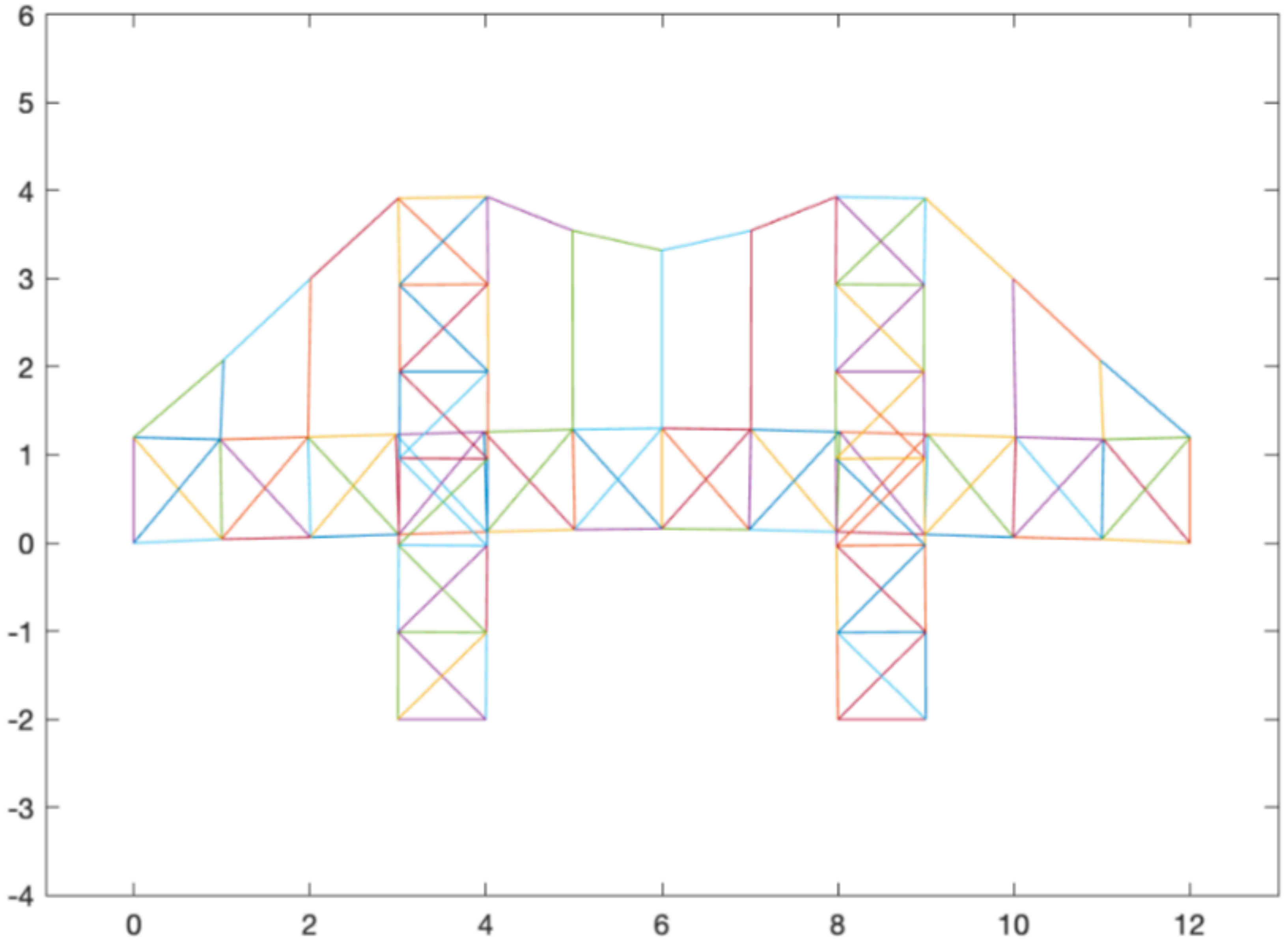
$$S \to S \cdot n$$

$$D \to D \cdot n$$

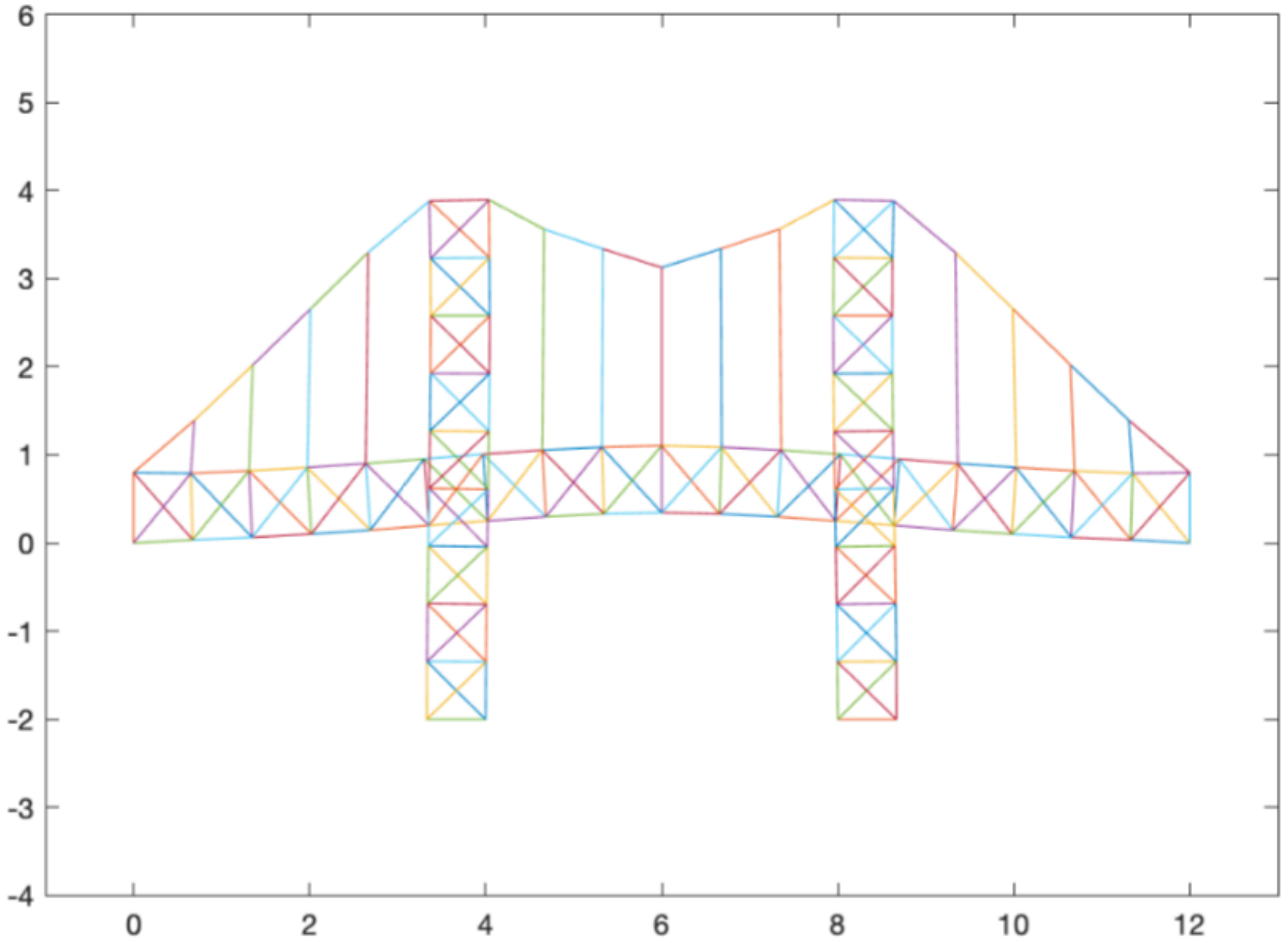
Note: time step should also be refined to ensure numerical stability

When Refine the Suspension Bridge

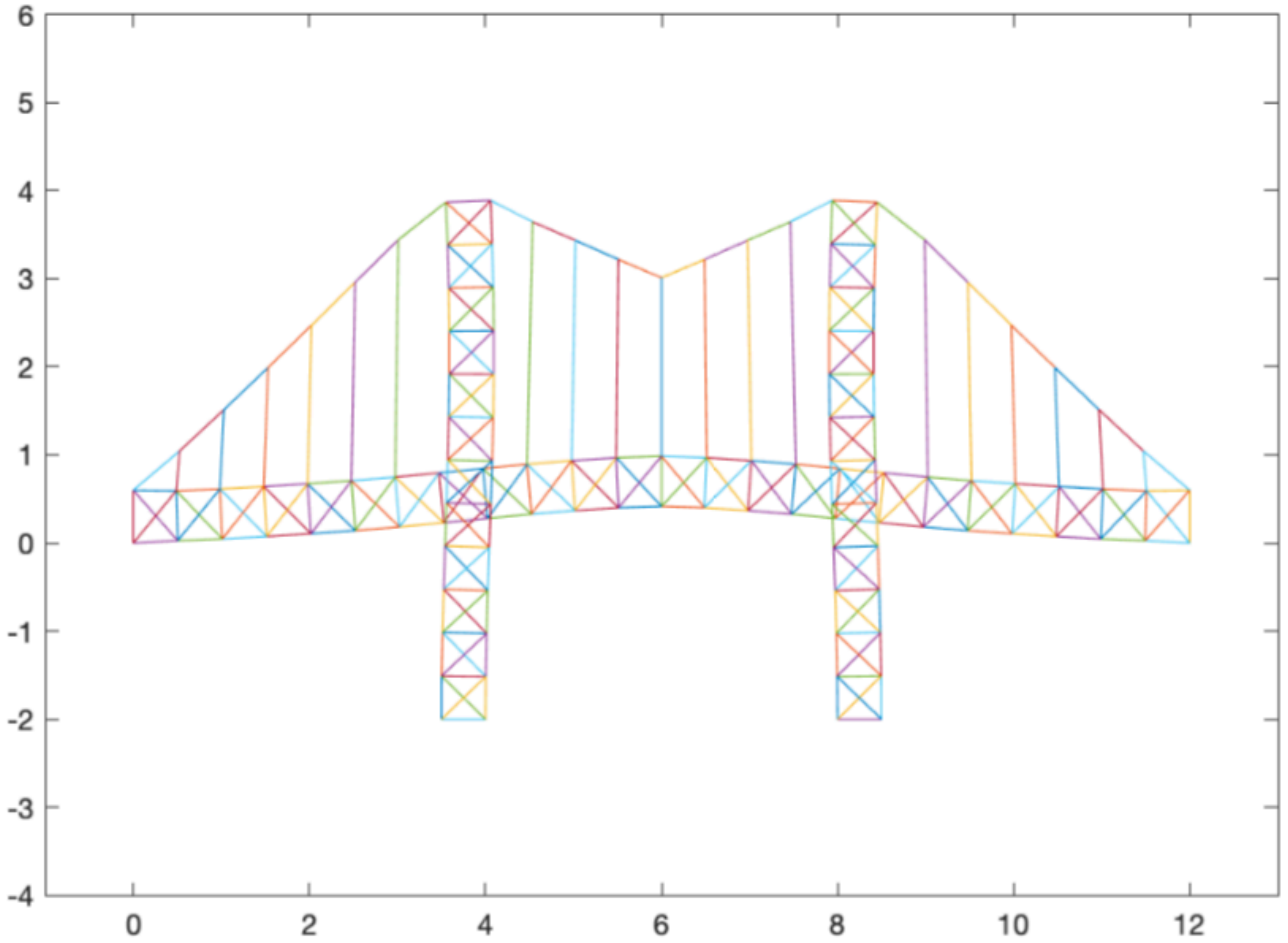
How does the fineness of the suspension bridge influence itself?



n=2



n=3

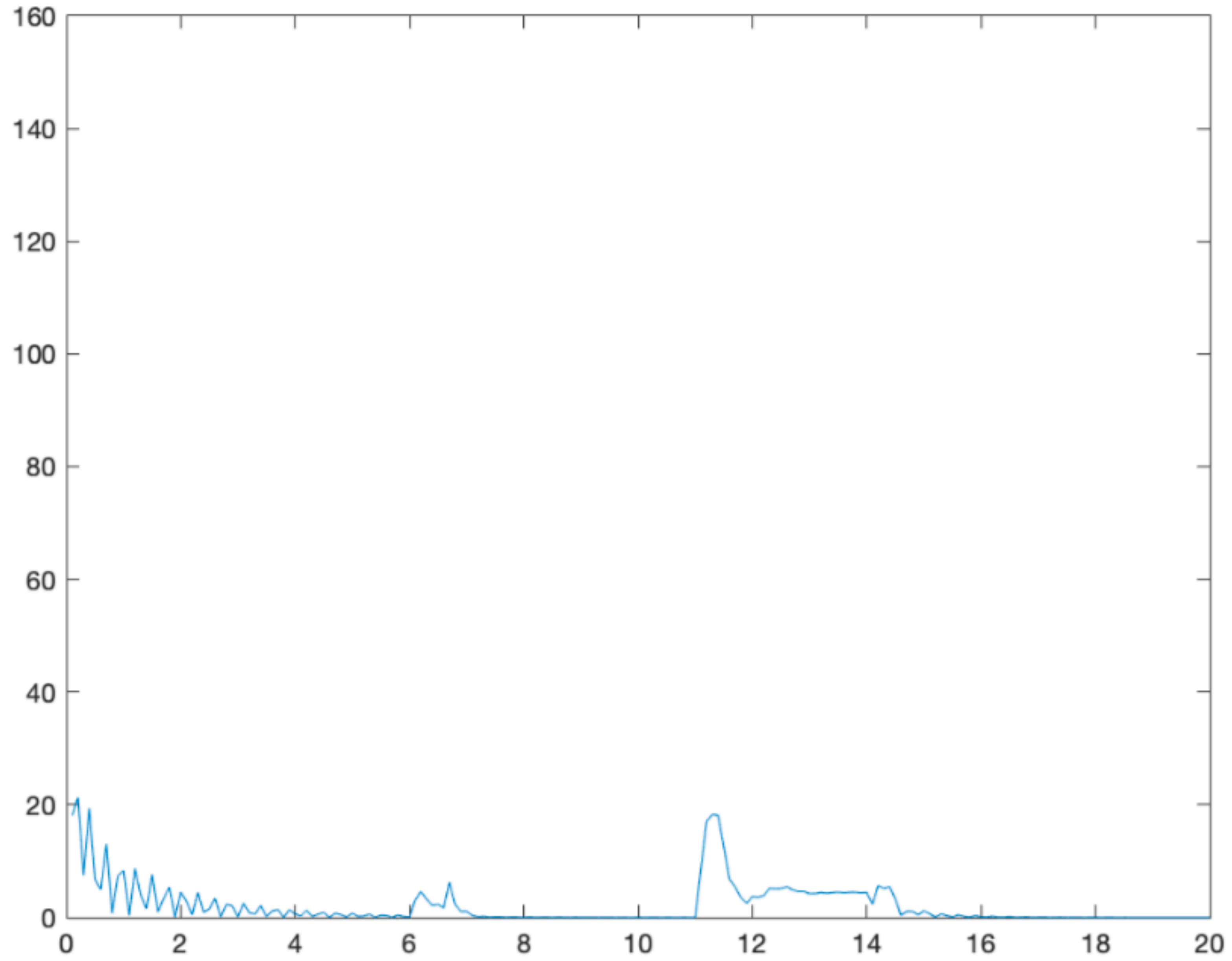


n=4

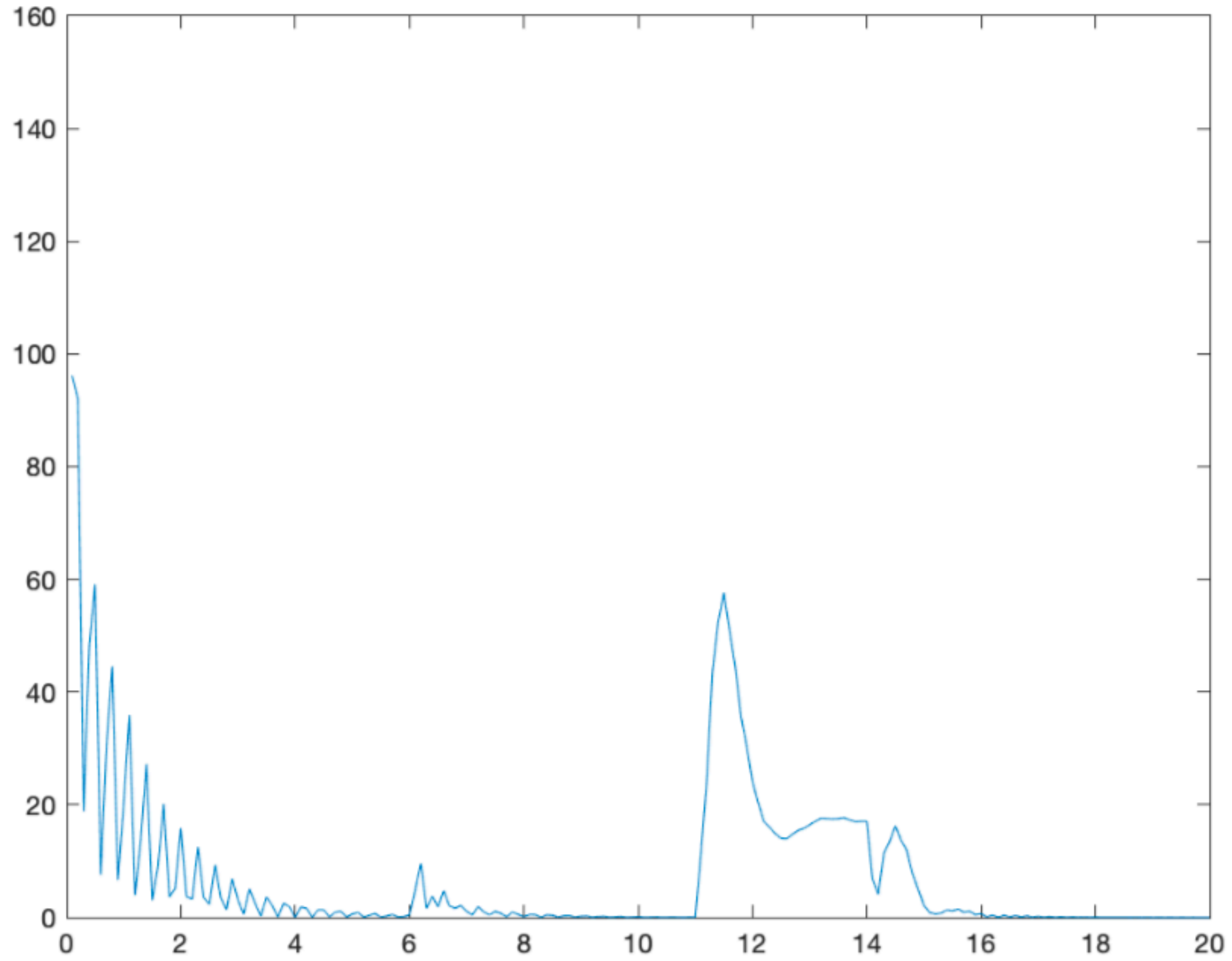
	n=2	**n=3**	**n=4**
Starting kinetic energy	152.8654	180.9286	195.7662
Time to equilibrium	4.6	4	19.6

Table 1: Starting kinetic energy and time to equilibrium

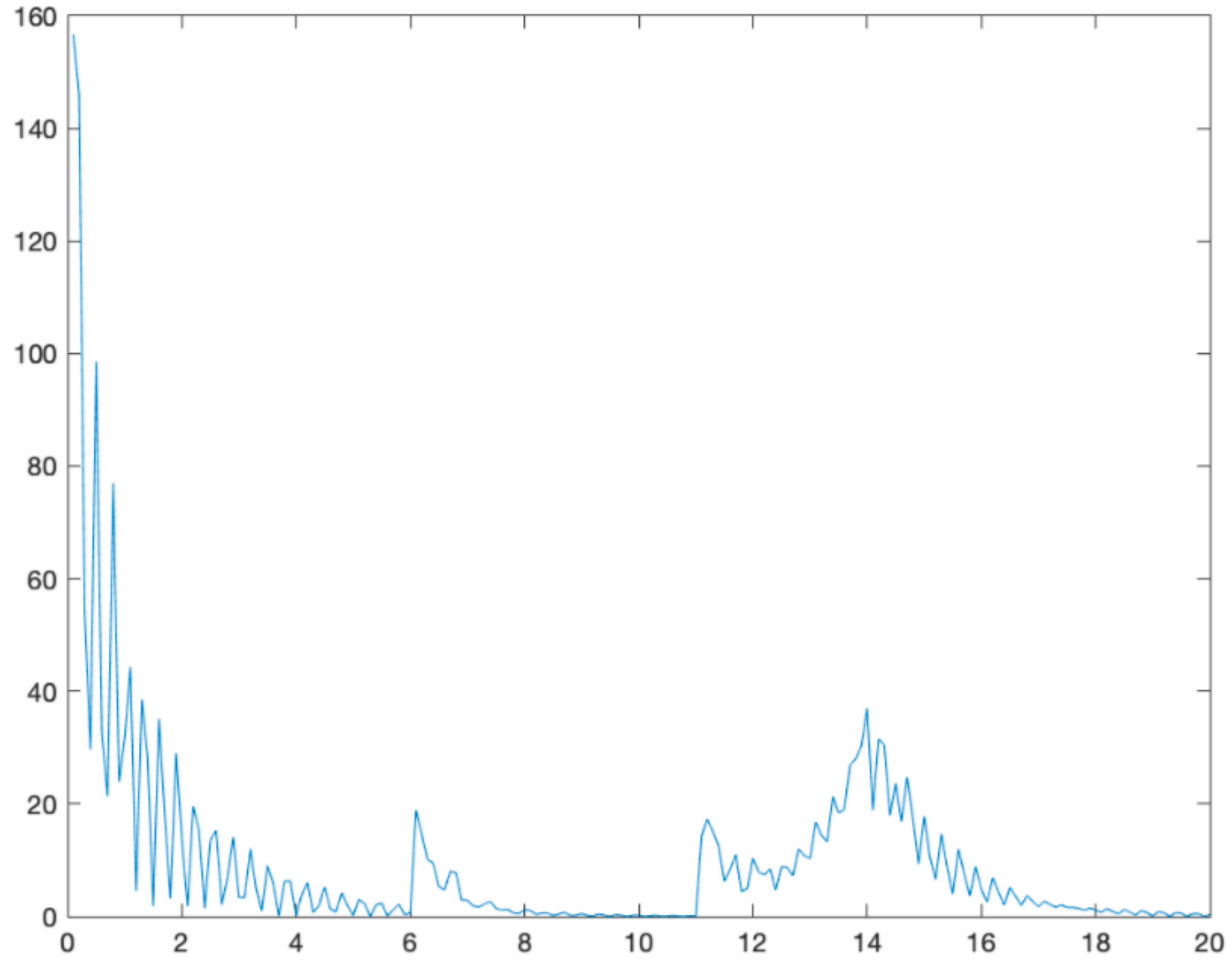
When Refine the Suspension Bridge



(a) The kinetic energy when $n = 2$

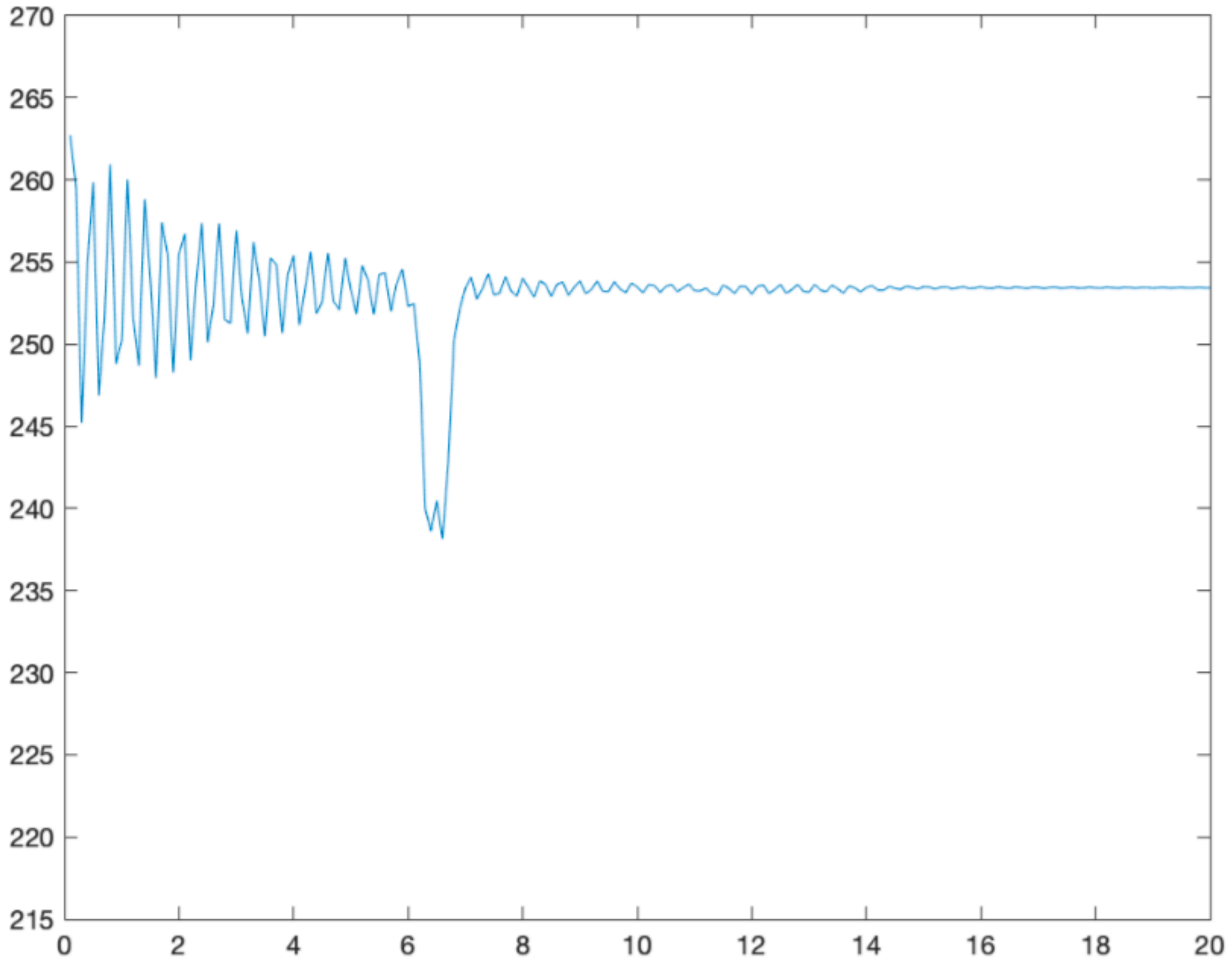


(b) The kinetic energy when $n = 3$

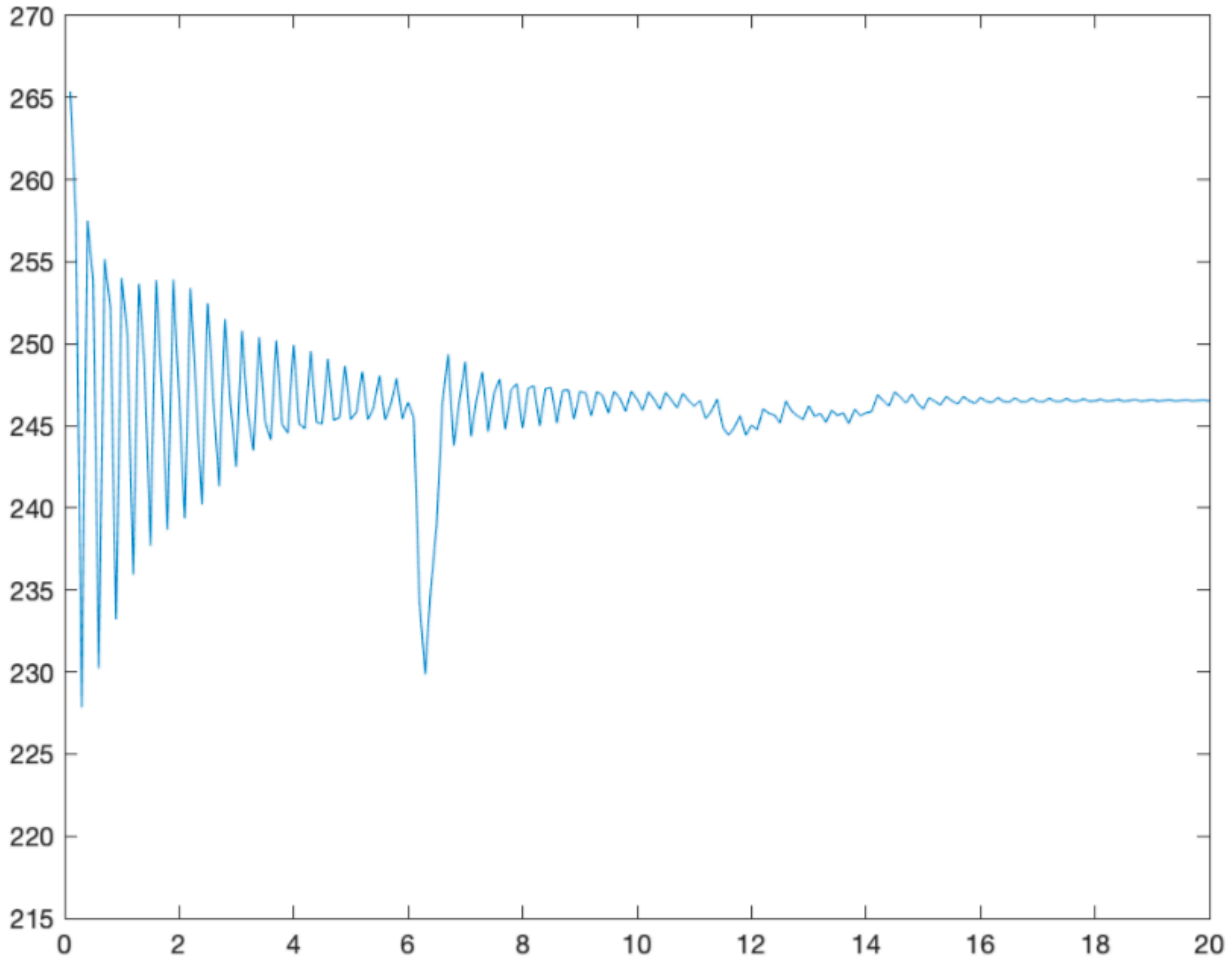


(c) The kinetic energy when $n = 4$

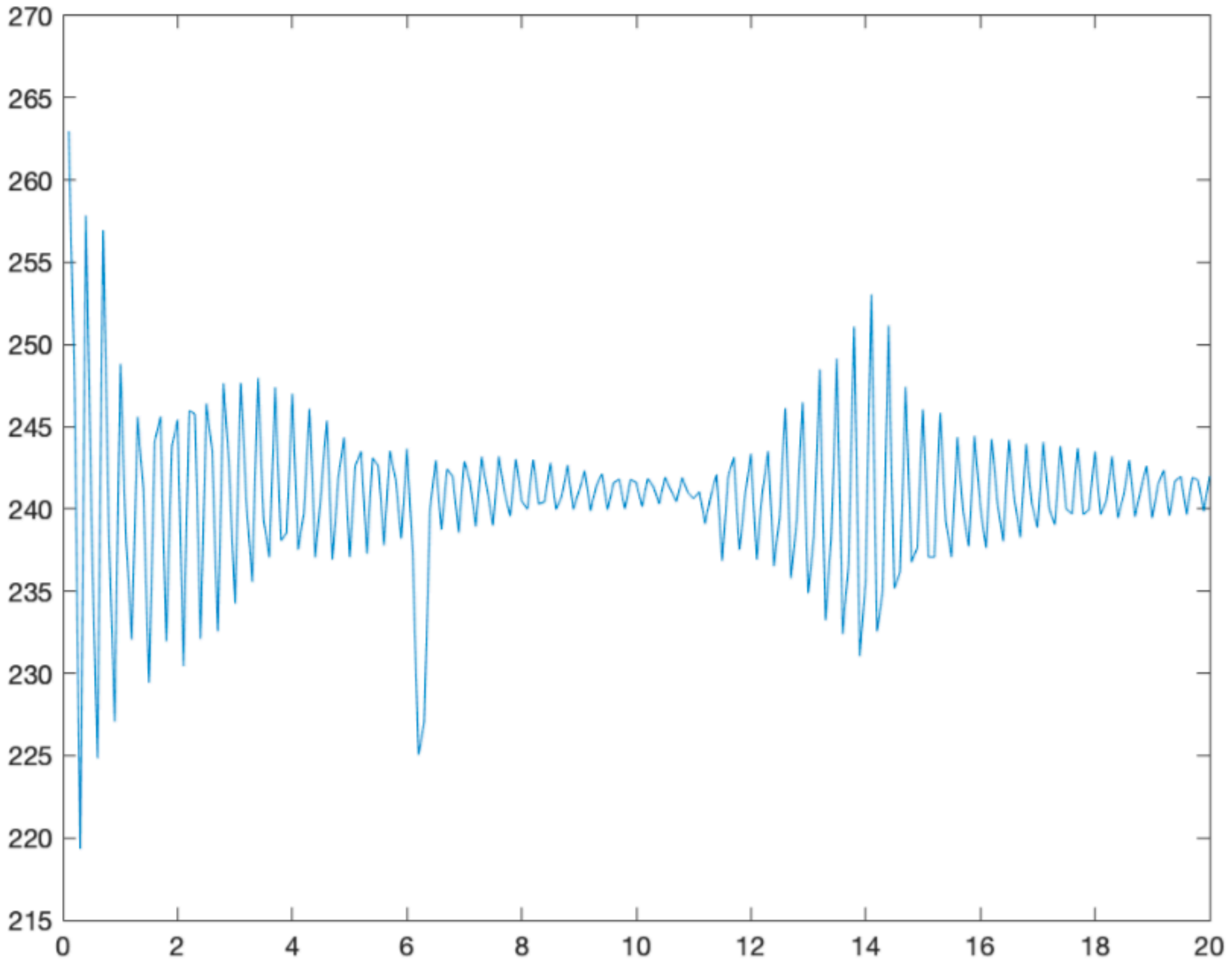
When Refine the Suspension Bridge



(a) The gravitational energy when $n = 2$

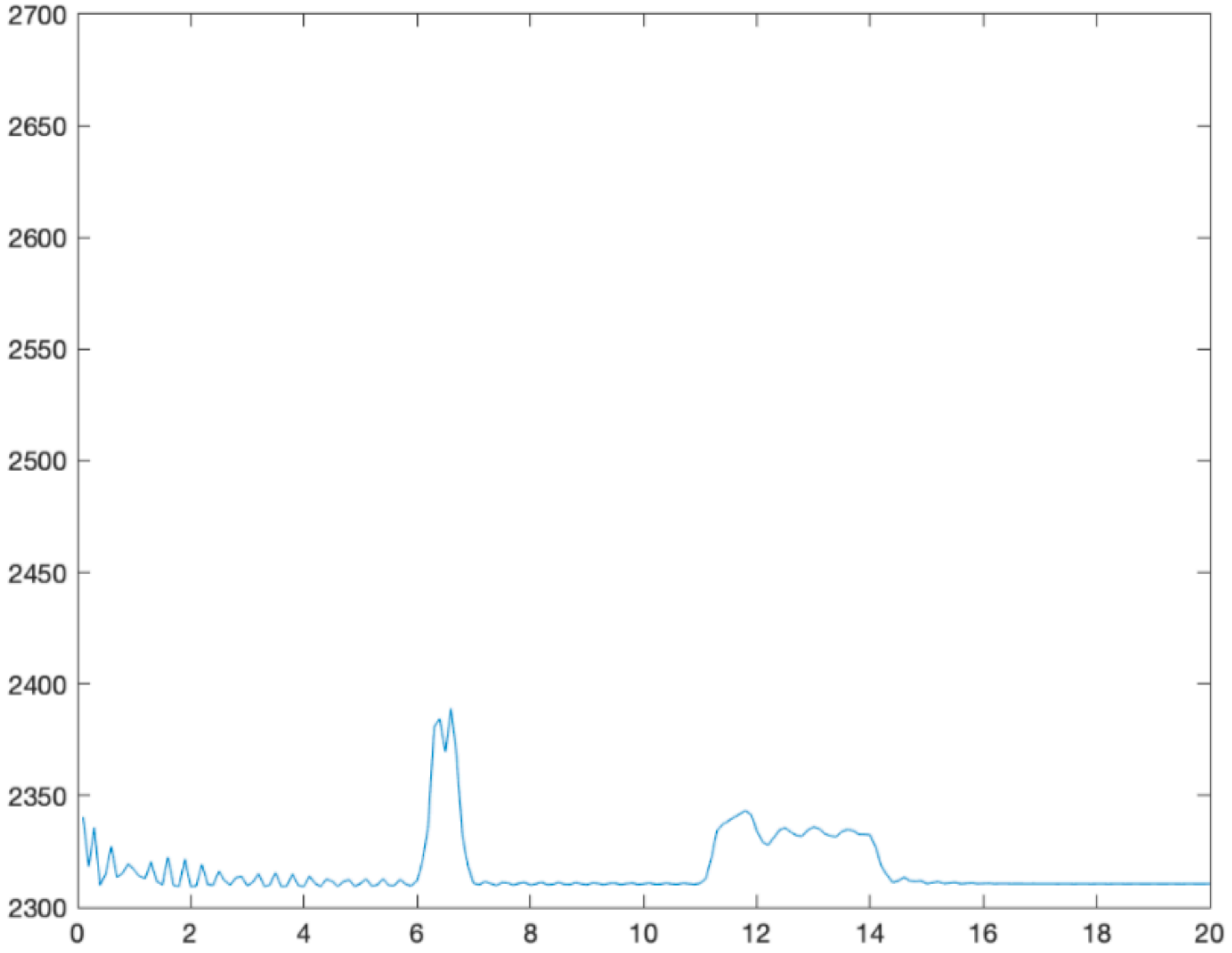


(b) The gravitational energy when $n = 3$

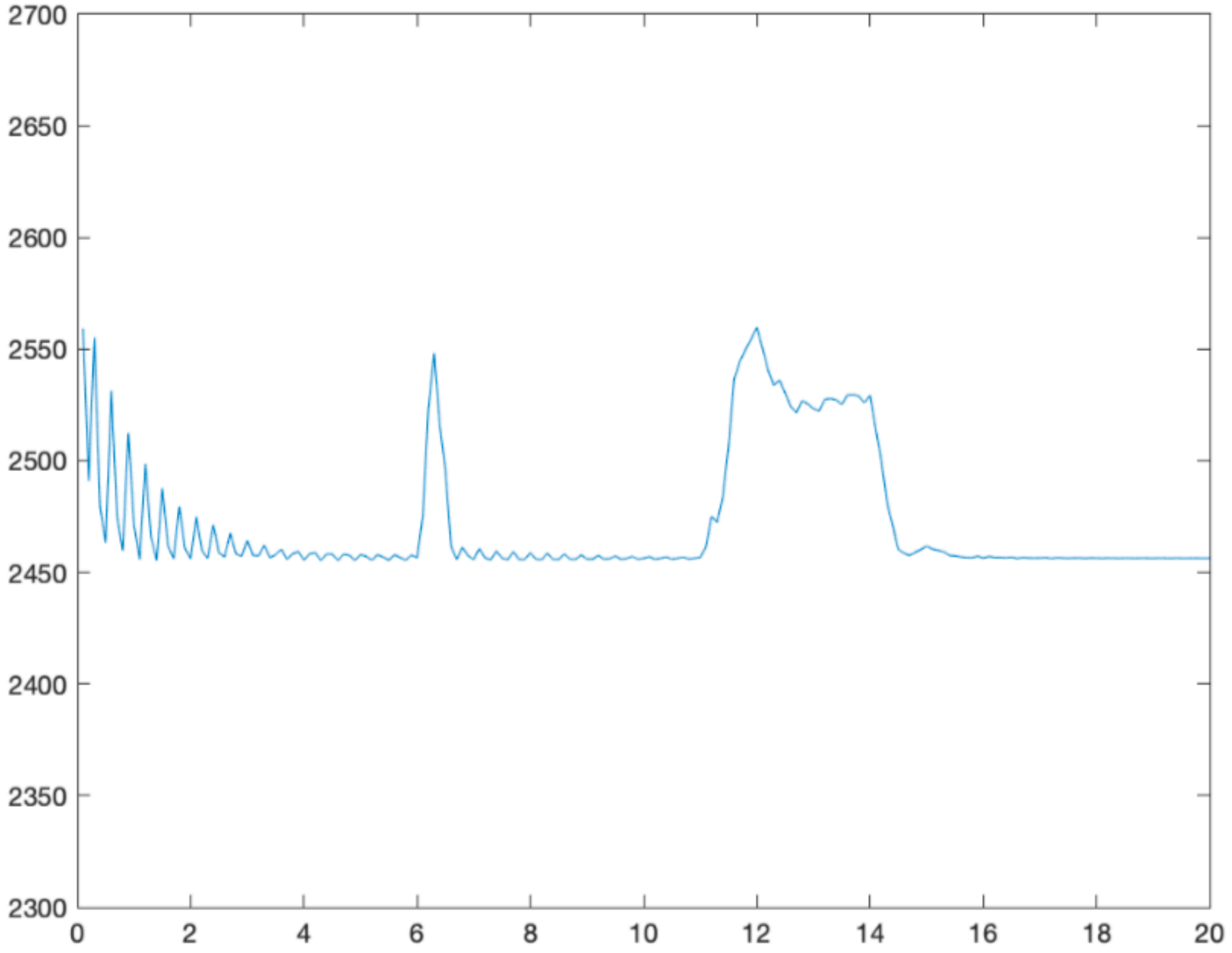


(c) The gravitational energy when $n = 4$

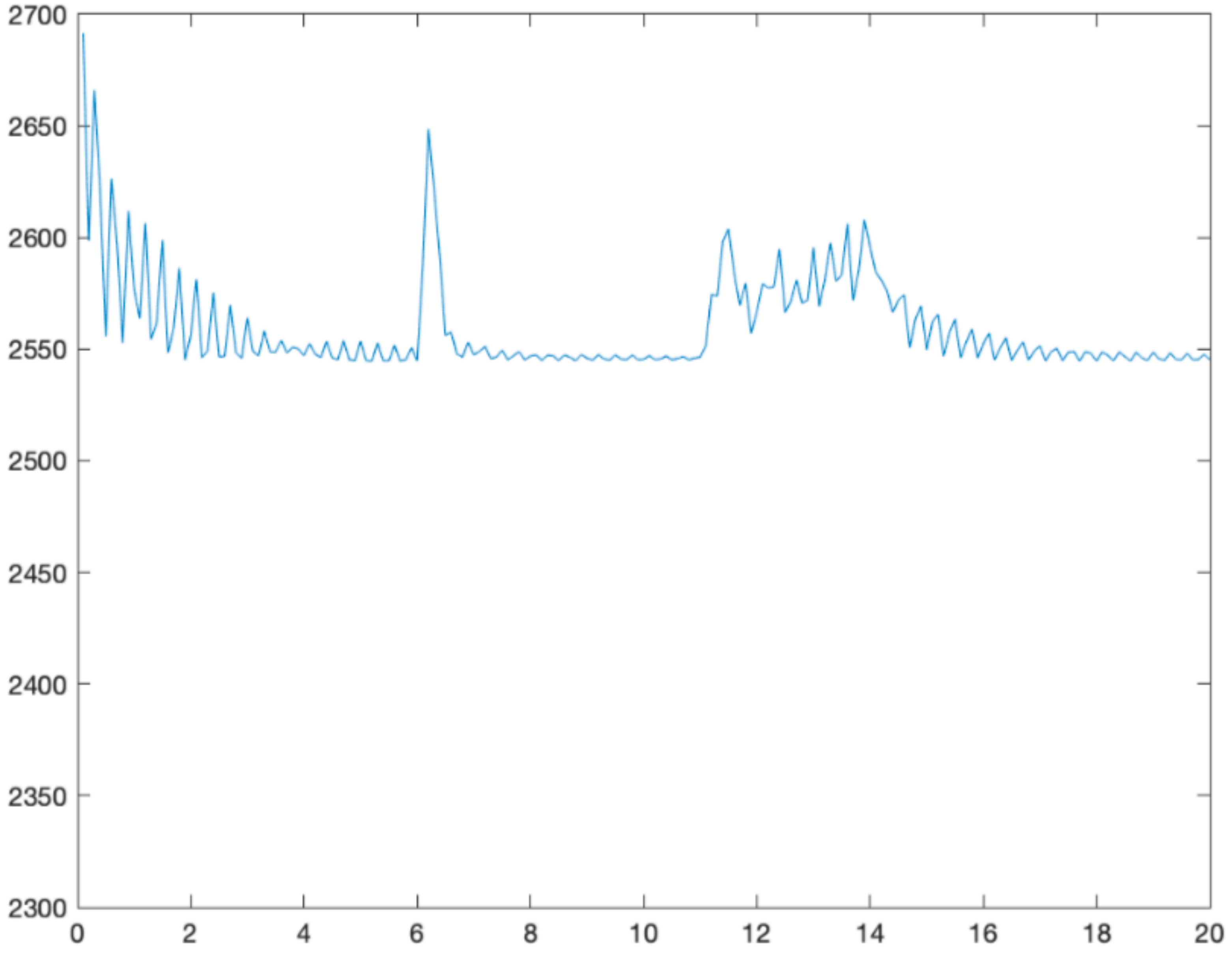
When Refine the Suspension Bridge



(a) The elastic energy when $n = 2$

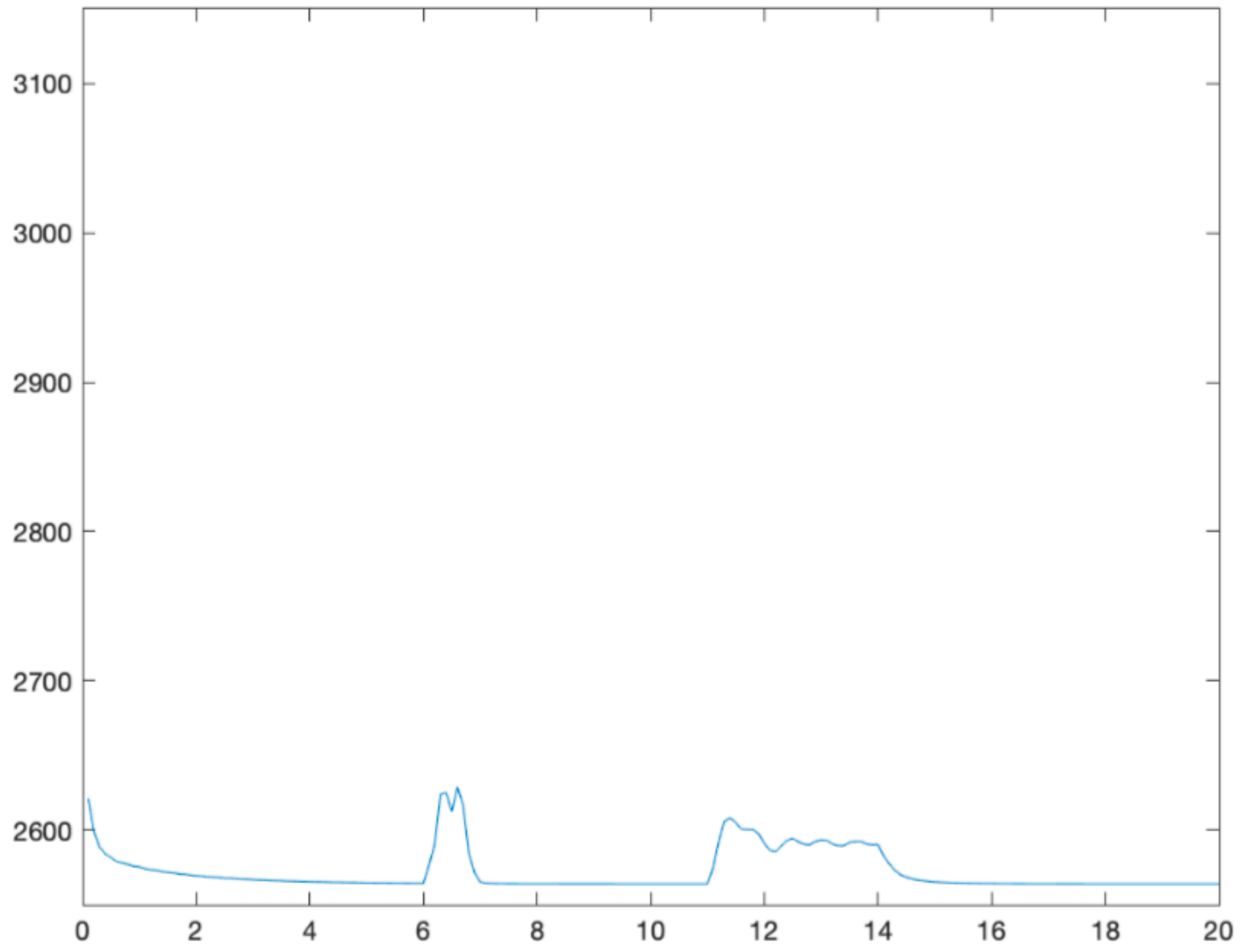


(b) The elastic energy when $n = 3$

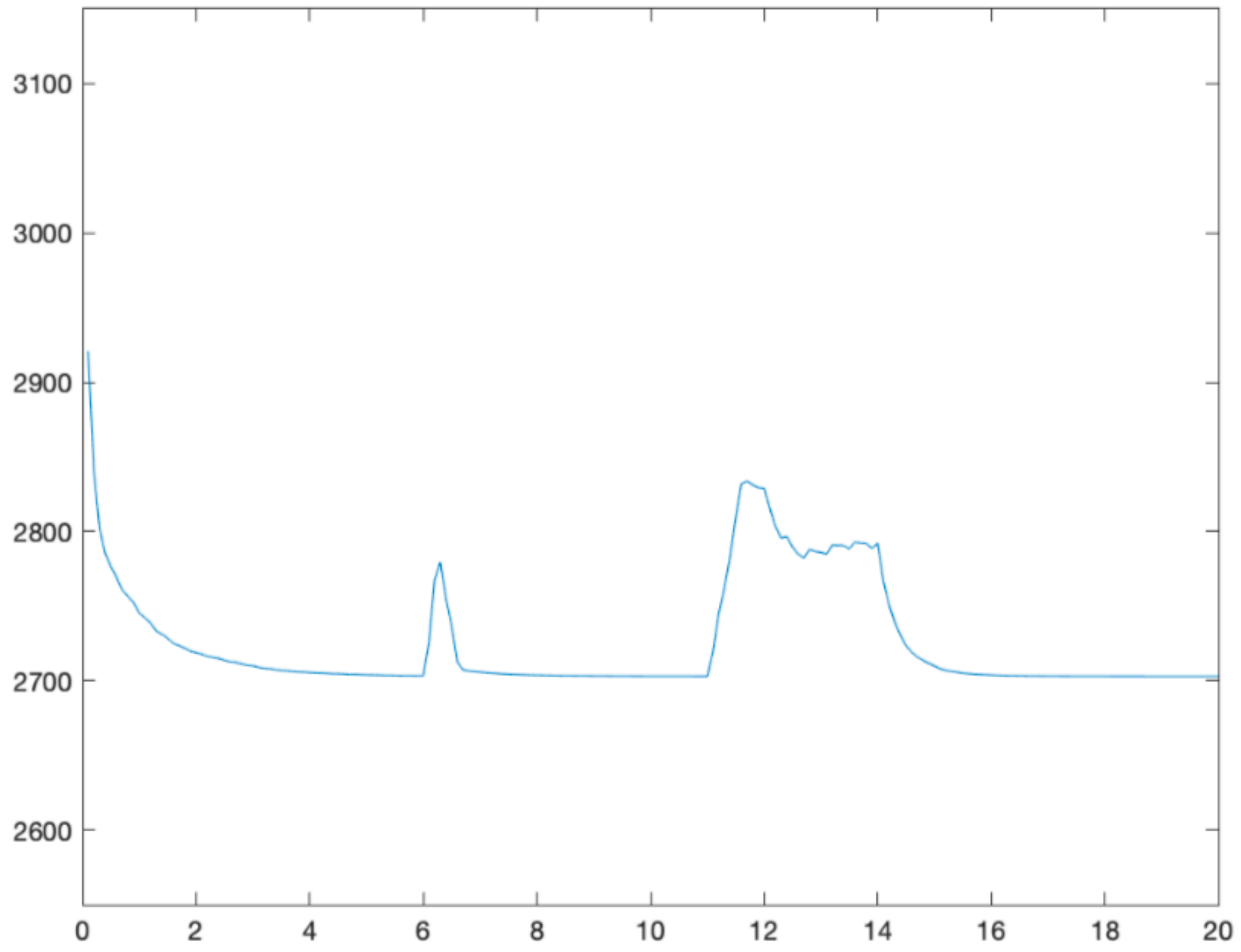


(c) The elastic energy when $n = 4$

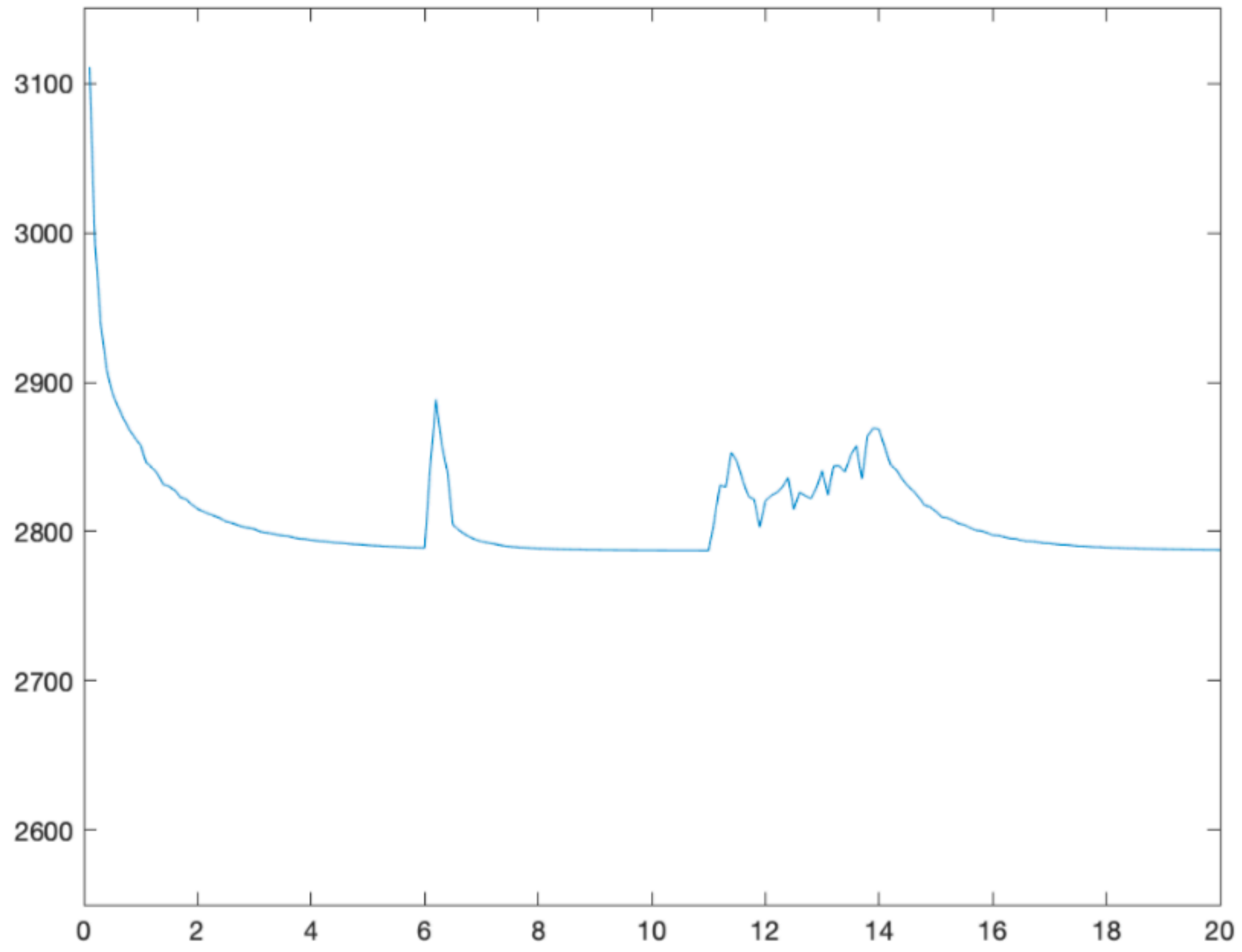
When Refine the Suspension Bridge



(a) The total energy when $n = 2$



(b) The total energy when $n = 3$



(c) The total energy when $n = 4$