Monte Carlo Methods Homework 6

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Exercise 71. *Use a Metropolized version of the overdamped stochastic Newton scheme, to sample from the Rosenbrock density,*

$$\pi(x) \propto \exp\left(-\frac{100(x_2 - x_1^2)^2 + (1 - x_1)^2}{20}\right).$$

It may be useful to note that when S is a 2×2 matrix with $trace(S) \neq 0$, the matrix

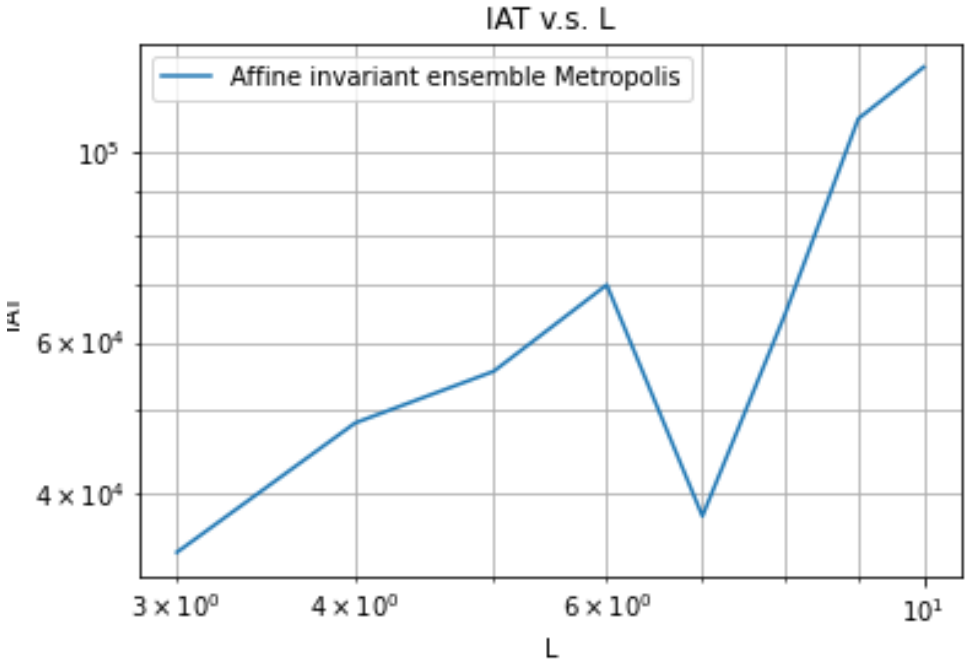
$$R = \frac{S + \sqrt{det(S)}\, I}{\sqrt{trace(S) + 2\sqrt{det(S)}}}$$

is a square root of S. The inverse Hessian, $S = (-D^2 \log \pi(x))^{-1}$, will not be positive definite for all x. When it is not you can replace the inverse Hessian by the square of the matrix obtained by taking absolute values before each square root in the formula for R above. Since you are Metropolizing, you can omit the divergence of S term in the overdamped proposal step without introducing additional bias (though this may increase the rejection rate). Compare the performance of this preconditioned scheme to the Metropolized overdamped scheme with $S = I$.

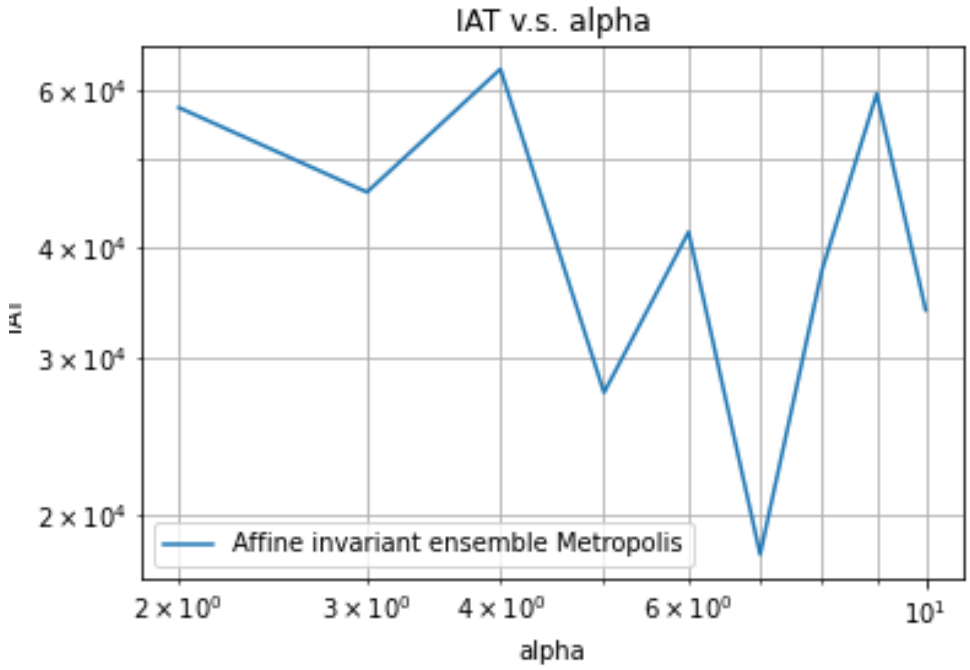
Let $h = 10^{-2}$, maximum of iteration allowed in Stochastic Newton 10^6. I did five The ITA for stochastic Newton are [56088.14836733, 95794.08099281, 3364.16478186, 10580.25259484, 13966.70877991]. The ITA for metropolized Overdamped are [38103.59897241, 60928.39850744, 109908.43178252, 109908.43178252, 22613.8417611]. We can see that, despite significant variations, stochastic Newton performs better than metropolized Overdamped generally.

Exercise 75. *Use the affine invariant ensemble scheme to sample from the Rosenbrock density. Experiment with different values of the parameters α and L in the ensemble scheme and compare to the results in Exercise 71.*

Let $\alpha = \sqrt{2}$ and L in $[3,4,\ldots,10]$. To avoid walkers trapped in a subspace, we start L from 3. The IAT generally increases as L increases.



Let $L = 5$ and α in $[2,3,\ldots,10]$. We don't see much pattern of IAT changes as α increases.


IAT v.s. alpha
6×10^4
4×10^4
3×10^4
2×10^4
IAT
Affine invariant ensemble Metropolis
2×10^0
3×10^0
4×10^0
6×10^0
10^1
alpha