PRESENT VALUE

PRESENT VALUE DEFINITION

If I have a contract to receive $1 at time T from now, what is the value of that contract now (i.e. its present value)? Such future payments are “discounted” to reflect the time value of money.

Alternatively, how much do I need to invest now to generate a future value of $1

PRESENT VALUE: SIMPLE INTEREST

Assuming a simple interest rate r_s, \$1 invested now for a period T would be worth $1 + r_sT$ at the end of the period. Hence, $1/(1 + r_sT)$ dollars invested now would generate \$1 at the end of the interest period.

NOTE: Assumes interest proportional to principal

EXAMPLE

If I am to receive $1 a year from now and I can invest at 2%/year, I should be willing to receive a "discounted value" of

$$\$1/[1 + 0.02\ (1)] = \$0.98039$$

today.

PRESENT VALUE: COMPOUND INTEREST

Assuming a compound interest rate r_c, \$1 invested now for N periods would be worth $(1 + r_c)^N$ at the end of the N periods. Hence, $1/ (1 + r_c)^N$ dollars invested now would generate \$1 at the end of the N periods.

EXAMPLE

If I am to receive $1 a year from now and I can invest at 2%/year, compounded quarterly, I should be willing to receive

$$\frac{\$1}{\left(1+{}^{0.02}\!/_{4}\right)^{4}} = \$0.98025$$

today.

THE PRESENT VALUE OF AN ANNUAL ANNUITY

An annuity that pays an annual coupon c (a percentage) on a principal, P (a dollar amount), discounted at a rate y, for N years has the present value

$$\frac{cP}{1+y}+\frac{cP}{(1+y)^2}+\ldots+\frac{cP}{(1+y)^N}$$

YIELD TO MATURITY OR INTERNAL RATE OF RETURN (IRR)

YIELD TO MATURITY DEFINED

The yield to maturity of a bond is it's market internal rate of return. In other words, if the future cash flows of a bond are discounted at the yield to maturity and summed, the resulting value is the market price of the bond.

YIELD TO MATURITY AT ANNUAL COUPON DATE

At a coupon date, a bond is an annuity plus a single "Balloon payment" of the principal at the end. If M is the market price, we have, if N is the number of remaining periods

$$M = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \ldots + \frac{C}{(1+y)^N} + \frac{1}{(1+y)^N}$$

YIELD TO MATURITY FOR SEMI-ANNUAL COUPONS

A bond paying semi-annual coupons for Y years will have N = 2Y coupon payments, and

$$M = \frac{c/2}{1+y/2} + \frac{c/2}{\left(1+y/2\right)^2} + \ldots + \frac{1+c/2}{\left(1+y/2\right)^N}$$

YIELD TO MATURITY: SUMMATION CONVENTION

The previous formula can also be written

$$M = \sum_{i=1}^{N-1} \frac{c/2}{(1+y/2)^i} + \frac{1+c/2}{(1+y/2)^N}$$

YIELD TO MATURITY: OBSERVATIONS

- There is no known formula for finding the yield to maturity for a given market price for $N > 4$ periods; numerical methods must then be used
- EXCEL functions PRICE and YIELD compute bond price, yield to maturity
- If $c < y$, market price (M) < 100
- If $c > y$, market price (M) > 100

NEWTON'S METHOD FOR FINDING YTM

Want to find y such that $PV(y) = M$.

1. Set $y_0 = 0$
2. For $i > 0$, compute

$$y_i = y_{i-1} - [PV(y_{i-1}) - M]/PV'(y_{i-1}),$$

where $PV'(y) = d\,PV(y)/dy$

3. Repeat step 2 until $|y_i - y_{i-1}| < 1.0E\text{-}14$
4. Return y_i

Guaranteed to converge!

PRICES IN PRACTICE

Price Quotations: Treasury Notes and Bonds

- Quotes as percentage of par (face value) and 32nd's of a percent, e.g.

  100-24

  would imply a price of 100 $^{24}/_{32}$ of par, or 100.75% of par.
- A quote of 100-24+ implies a price of 100 $^{24}/_{32}$ + $^{1}/_{64}$ = 100 $^{49}/_{64}$% of par
- The quote of 100-24++ = 100 $^{99}/_{128}$%

WHAT HAPPENS IN BETWEEN COUPON DATES?

- The actual value of the bond will change from day to day as interest accrues, even if the bond's yield to maturity doesn't change
- To facilitate pricing and trading, accrued interest is subtracted from the quoted price of the bond. The result is the "clean price".
- The actual amount paid for the bond, which includes accrued interest, is the "dirty price"

DIRTY PRICE COMPUTATION FOR N > 1 COUPONS

z = days between settlement and next coupon

x = days between prev and next coupon

N = number of coupons remaining (> 1)

$$P = \sum_{i=0}^{N-1} \frac{c/2}{(1+y/2)^{i+z/x}} + \frac{1}{(1+y/2)^{N-1+z/x}}$$

DIRTY PRICE COMPUTATION FOR N = 1 COUPON

z = days between settlement and coupon

x = days between prev and next coupon

$$D = \frac{1 + c/2}{1 + \left(\frac{y}{2}\right)\left(\frac{z}{x}\right)}$$

COMPUTATION OF CLEAN PRICE, *C*

z = days between prev coupon and settlement

x = days between prev and next coupon

c = annualized coupon rate

$$C = D - (c/2)(z/x)$$