CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

13. I-HANK

Adv. Macro: Heterogenous Agent Models

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Danmarks
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I-HANK

International HANK

- **Baseline RANK model:** Gali and Monacelli (2005)

  1. Small open economy (SOE) $\rightarrow$ foreign economy exogenous
  2. Demand for home goods declining in relative price
  3. Floating exchange rate
  4. Free capital mobility $\rightarrow$ uncovered interest parity (UIP)

- **Exchange rate shocks:**
  Auclert, Rognlie, Souchier and Straub (2021),
  »Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel«

- **Foreign demand shocks:**
  Druedahl, Ravn, Sunder-Plassmann, Sundram, Waldstrøm (2023),
  »The Transmission of Foreign Demand Shocks«

Exchange rate shocks

Overview of model

- **Small-open-economy** $\Rightarrow$ *trading partner is exogenous*
- **Goods:** *Home* and *foreign*
- **Households:** Standard + CES demand

  (initially households only hold domestic assets $\Rightarrow$ no reevaluation effects)
- **Production of home goods:** Flexible prices with mark-up μ
  1. Production: $Y_t = Z_t N_t$
  2. Wage from FOC: $W_t = \frac{1}{\mu} P_{H,t} Z_t$
  3. Dividends: $D_t = P_{H,t} Y_t - W_t N_t$
- **Unions:** Sticky wages $\Rightarrow$ NKWPC
- **Financial markets:** Floating exchange rate + UIP condition
- **Central bank:** Constant real rate rule

Household problem

- **Nominal exchange rate:** E_t (domestic per foreign currency)
- **Real exchange rate:** $Q_t = \frac{E_t P_t^*}{P_t}$ (depreciation $\equiv Q_t \uparrow$)
- **Domestic CES demand:**

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C^{hh}\left(\boldsymbol{Y}^{hh}\right)$$

$$C_{F,t} = \alpha\left(\frac{E_t P_t^*}{P_t}\right)^{-\eta} C^{hh}\left(\boldsymbol{Y}^{hh}\right)$$

$$P_t = \left[\alpha\left(E_t P_t^*\right)^{1-\eta} + (1-\alpha) P_{H,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

- From demand of $C_{H,t}$:

$$dC_{H,t} = (1-\alpha)\eta\left(dP_t - dP_{H,t}\right) + (1-\alpha)dC_t^{hh}$$

Real income

- From price level;

$$0 = \alpha \left(\frac{E_t P_t^*}{P_t} \right)^{1-\eta} + (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{1-\eta} \Rightarrow$$

$$0 = \alpha(1-\eta) dQ_t + (1-\alpha)(1-\eta)(dP_{H,t} - dP_t) \Leftrightarrow$$

$$dP_t - dP_{H,t} = \frac{\alpha}{1-\alpha} dQ_t$$

- **Real income:**

$$Y_t^{hh} = \frac{W_t N_t + D_t}{P_t} = Y_t \frac{P_{H,t}}{P_t} \Rightarrow$$

$$dY_t^{hh} = dY_t - (dP_t - dP_{H,t}) = dY_t - \frac{\alpha}{1-\alpha} dQ_t$$

Foreign economy and market clearing

- **Armington demand for home goods:**

$$C^*_{H,t} = \alpha \left(\frac{P_{H,t}}{E_t P^*_t} \right)^{-\gamma} C^*_t$$
$$= \alpha \left(\frac{1}{Q_t} \frac{P_{H,t}}{P_t} \right)^{-\gamma} C^*_t$$

- **Market clearing:**

$$Y_t = C_{H,t} + C^*_{H,t} \Rightarrow$$
$$dY_t = (1-\alpha)\eta \left(dP_t - dP_{H,t} \right) + (1-\alpha) dC^{hh}_t + \alpha dC^*_t + \alpha\gamma (dQ_t + dP_t - dP_{H,t})$$
$$= (1-\alpha) dC^{hh}_t + \alpha dC^*_t + \alpha\eta dQ_t + \left(\frac{\alpha(1-\alpha)}{1-\alpha} + \frac{\alpha^2}{1-\alpha} \right) \gamma dQ_t$$
$$= (1-\alpha) dC^{hh}_t + \alpha dC^*_t + \frac{\alpha}{1-\alpha} \chi dQ_t$$

Composite trade elasticity: $\chi \equiv \eta(1-\alpha) + \gamma$

Real exchange rate shock

- **Real exchange rate shock:** $d\boldsymbol{Q}_t$
- **Consumption satisfies:**

$$d\boldsymbol{C}^{hh} = \underbrace{\boldsymbol{M}d\boldsymbol{Y}}_{\text{multiplier}} - \underbrace{\frac{\alpha}{1-\alpha}\boldsymbol{M}d\boldsymbol{Q}}_{\text{real income}}$$

- **Intertemporal Keynesian Cross:**

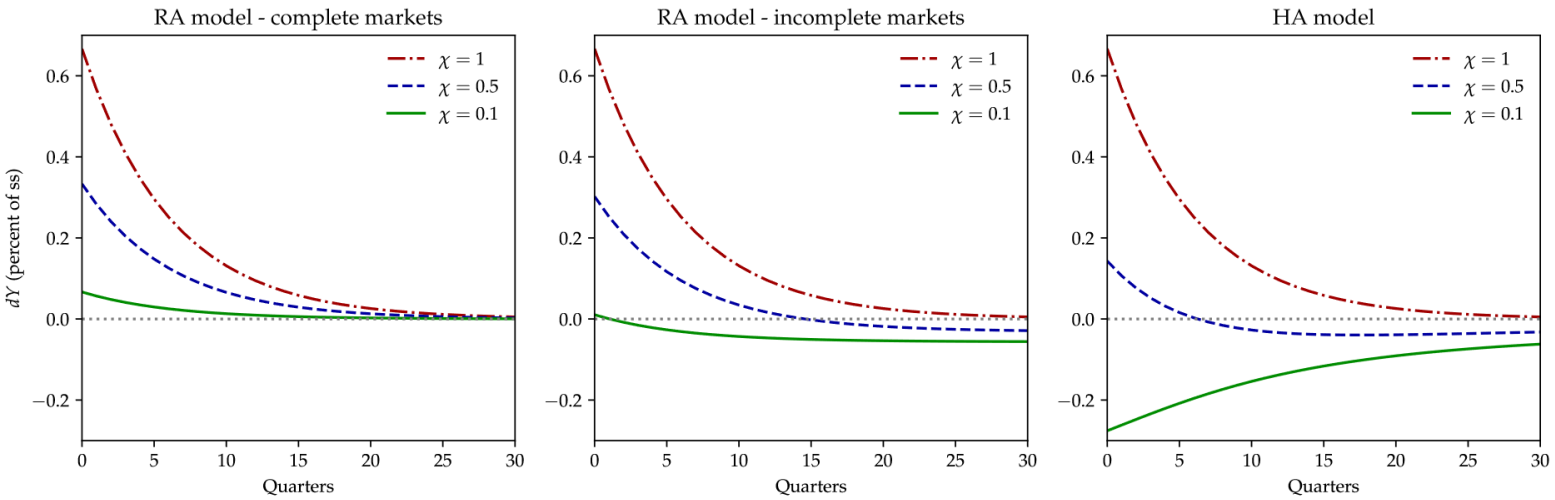
$$d\boldsymbol{Y} = \underbrace{(1-\alpha)\,\boldsymbol{M}d\boldsymbol{Y}}_{\text{multiplier}} \underbrace{-\alpha\boldsymbol{M}d\boldsymbol{Q}}_{\text{real income}} \underbrace{+\frac{\alpha}{1-\alpha}\chi d\boldsymbol{Q}}_{\text{expenditure switching}}$$

$$= \mathcal{M}\left(\frac{\alpha}{1-\alpha}\chi - \alpha\boldsymbol{M}\right) d\boldsymbol{Q}$$

  - Expansion: For high χ
  - Contraction: For low χ, easier if $\boldsymbol{M}$ is »large«

Contractionary depreciation

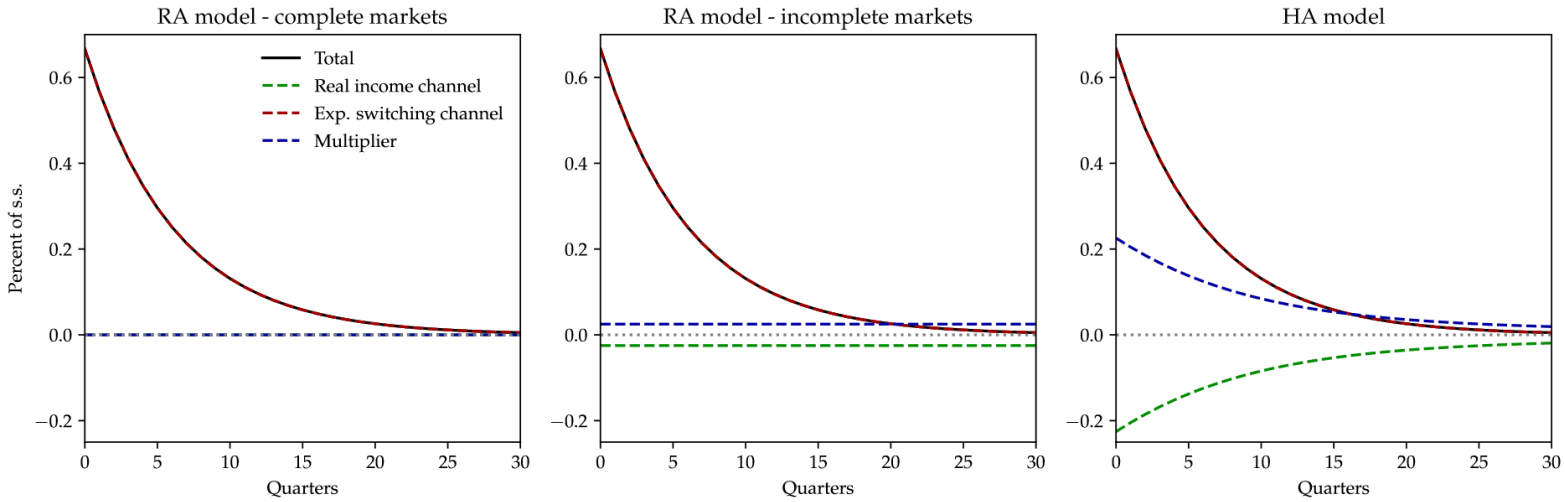
Figure 2: Effect of exchange rate shocks on output for various χ's



Note: impulse response in all three models to the shock to i_t^* displayed in Figure 1. χ is the trade elasticity (the sum of the import and export elasticity to the exchange rate). The HA model generates a contraction on impact for $\chi < \chi^* = 0.37$.

Equivalence and decomposition with $\chi = 1$

Figure 3: Exchange rate shock when $\chi = 1$ and its transmission channels



Note: impulse response in all three models to the shock to i_t^* displayed in Figure 1, with decomposition from proposition 2.

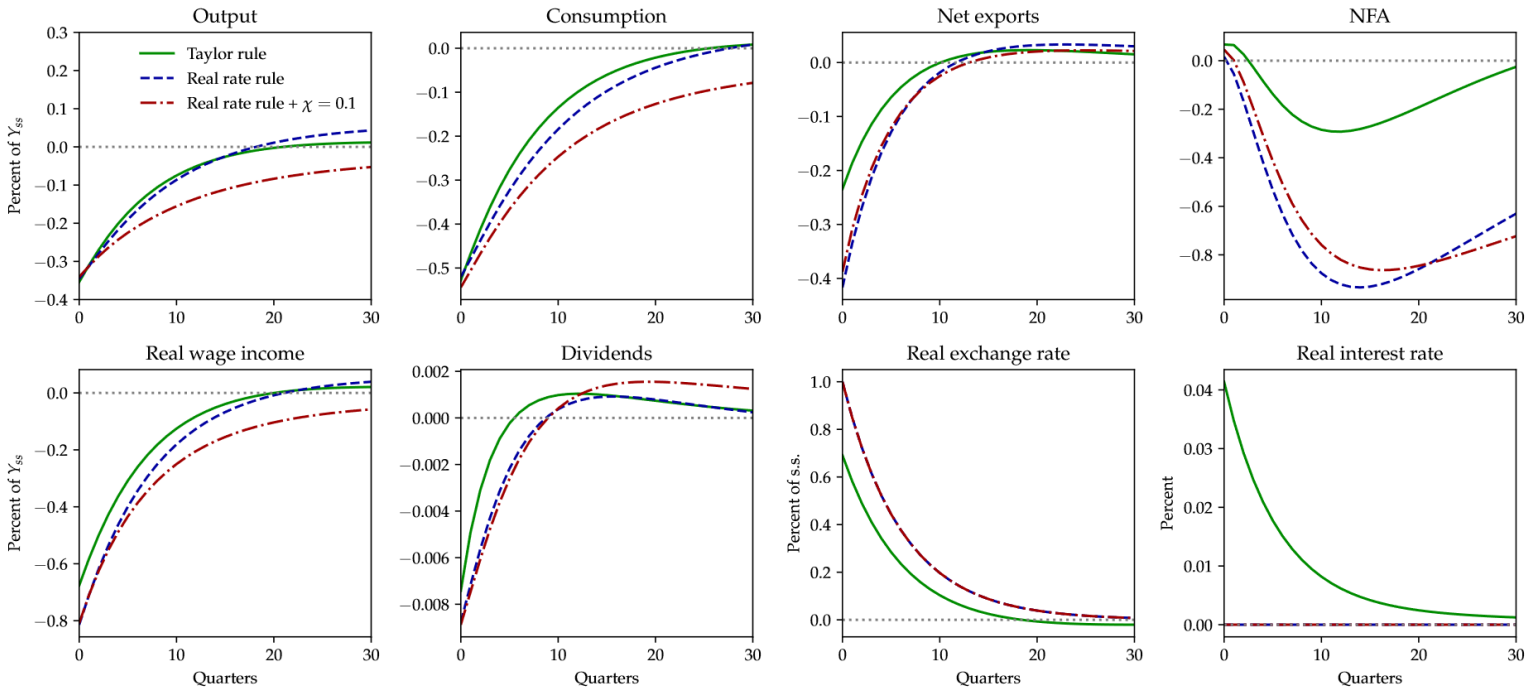
Solution: Guess $d\boldsymbol{Y} = \frac{\alpha}{1-\alpha} d\boldsymbol{Q}$ and verify for any $\boldsymbol{M}$

$$d\boldsymbol{Y} = (1-\alpha)\,\boldsymbol{M} d\boldsymbol{Y} - \alpha \boldsymbol{M} d\boldsymbol{Q} + \frac{\alpha}{1-\alpha} d\boldsymbol{Q} = \frac{\alpha}{1-\alpha} d\boldsymbol{Q}$$

Zero consumption: $d\boldsymbol{C}^{hh} = \boldsymbol{M} d\boldsymbol{Y} - \frac{\alpha}{1-\alpha} \boldsymbol{M} d\boldsymbol{Q} = \boldsymbol{0}$

Quantitative model

Figure 9: Contractionary depreciations



Note: impulse response in the quantitative model to the shock to i_t^* displayed in Figure 1. The model with Taylor rule is our quantitative model; the one with real rate rule is our quantitative model without the Taylor rule; the model with real rate and $\chi = 0.1$ drops delayed substitution and allows households to immediately adjust their consumption baskets across countries.

Add-ons: Non-homothethic preferences, sticky prices , imperfect exchange rate pass-through, delayed substitution, dollar currency pricing, UIP deviations

Demand shocks

Foreign demand shocks

The Transmission of Foreign Demand Shocks

Jeppe Druedahl Søren Hove Ravn Laura Sunder-Plassmann
Jacob Marott Sundram Nicolai Waldstrøm

September, 2023

Simplified model: I-HANK/

Regional dynamics

Regional Keynesian Cross

The Regional Keynesian Cross

Proposition

The first-order response of employment $d\boldsymbol{L}_j$ to a monetary shock $d\boldsymbol{r}_j$ and tradable goods demand shock $d\boldsymbol{C}^T$ solves

$$d\boldsymbol{L}_j = \underbrace{\rho_j \left(\boldsymbol{M}_j^r d\boldsymbol{r}_j + \boldsymbol{M}_j d\boldsymbol{L}_j \right)}_{\text{Regional exposure}} + \underbrace{(1-\rho_j) d\boldsymbol{C}^T}_{\text{National exposure}} - \underbrace{\frac{\nu}{\eta}(1-\rho_j)\left(d\boldsymbol{L}_j - d\boldsymbol{C}^T \right)}_{\text{Expenditure switching}}$$

ν: elasticity of subs. between c^{NT} & c^T

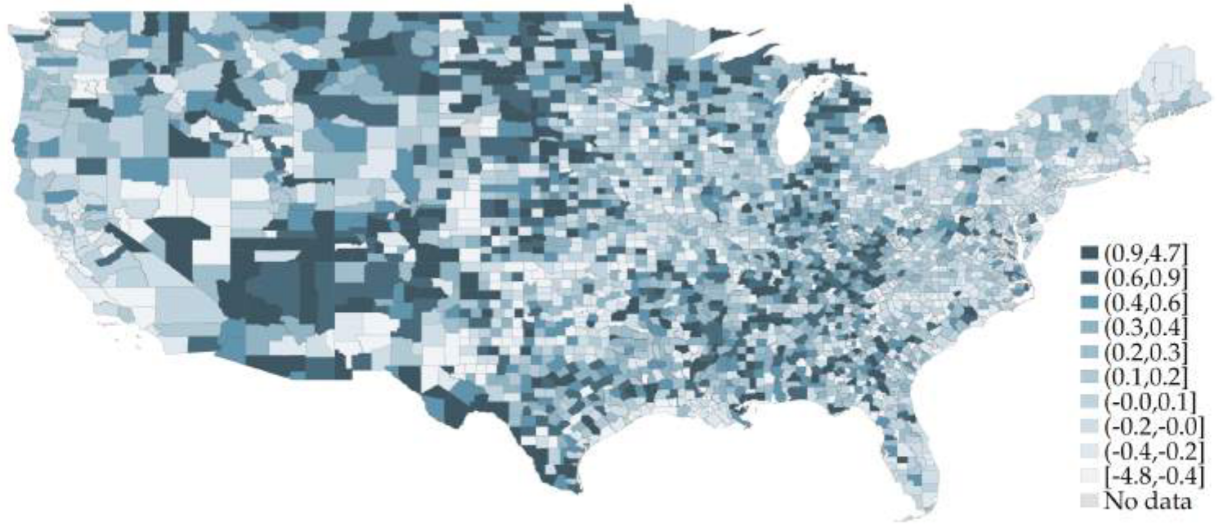
η: elasticity of subs. between ℓ^{NT} & ℓ^T

Source: Bellifemine, Couturier, and Jamilov (2023),

»The Regional Keynesian Cross«

Regional employment effects of MP

Figure 1: Regional Heterogeneity in the Effects of U.S. Monetary Policy



Note: This figure plots the 3-year ahead county-specific cumulative employment responses to a 1 standard deviation expansionary monetary policy shock $\beta_{j,36}$, estimated from the panel local projection (1). The coefficients are in percentage points and represent deviations from the (population weighted) average response.

Summary

Summary and next week

- **Today:** International HANK
- **Next week:** HANK with Search-And-Matching (SAM)
- **Homework:**
  - Paper: Broer et al. (2023),
    »The Unemployment-Risk Channel in Business-Cycle Fluctuations«
  - Slides: Broer et al. (2023),
    »Fiscal stimulus policies according to HANK-SAM«
  - GEModelTools: Model description of HANK-SAM
  - Ask questions regarding the exam