



3. Consumption-Saving Models: Extensions

Adv. Macro: Heterogenous Agent Models

Jeppe Druedahl & Patrick Moran

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Introduction

- Note: The views expressed in this presentation are those of the author and do not represent the views of the Federal Reserve Board or Federal Reserve System.

Consumption-Saving Models

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 2. How to distinguish between habits vs inattention?
 3. What if households have trouble saving due to temptation?

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- **Central economic questions:**
 1. How do we explain excess smoothness in consumption data?
 2. How to distinguish between habits vs inattention?
 3. What if households have trouble saving due to temptation?
- **Plan for today:**
 1. Discuss the problem of “excess smoothness” of consumption
 2. Study the model of sticky expectations (Carroll et al, 2019)
 3. Study the model of temptation and commitment (Attanasio et al, 2021)

Excess Smoothness

- One of the key puzzles in consumption-saving models is the “excess smoothness” of consumption
 - Theory: Consumption responds instantly, completely to shock
 - Evidence: Consumption is too smooth

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- Campbell and Deaton (1989): Consumption does not react sufficiently to innovations to the permanent component of income

Theory: Simple Model with Quadratic Utility

Hall (1978) Random Walk

- Household utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(\mathbf{c}_t)$$

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But in the data: permanent income much noisier than consumption

Excess Smoothness

One Explanation: Habit Formation

Popular solution in DSGE models: habit formation

- Household utility depends on both $c_{i,t}$ and $c_{i,t-1}$

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(\tilde{c}_{i,t})$$

- where

$$\tilde{c}_{i,t} = c_{i,t} - \chi c_{i,t-1}$$

- χ is positive if goods provide services across periods
- zero if goods are fully non-durable, non-habit forming

Testable implications of habit formation

- Consumption Euler equation (for full derivation, see Dynan 2000):

$$E_t \left[(1 + r) \beta \frac{MU_{i,t+1}}{MU_{i,t}} \right] = 1$$

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- If we assume CRRA utility function, $u(c) = \frac{c^{1-\rho}}{1-\rho}$ then:

$$(1+r)\beta \left(\frac{\tilde{c}_{i,t}}{\tilde{c}_{i,t-1}} \right)^{-\rho} = 1 + \varepsilon_{i,t}$$

where $\varepsilon_{i,t}$ is the expectational error

Testable implications of habit formation

- Taking logs and substituting for \tilde{c} gives:

$$\Delta \ln(c_{i,t} - \chi c_{i,t-1}) = \frac{1}{\rho} [\ln(1+r) + \ln(\beta)] - \frac{1}{\rho} \ln(1 + \varepsilon_{i,t})$$

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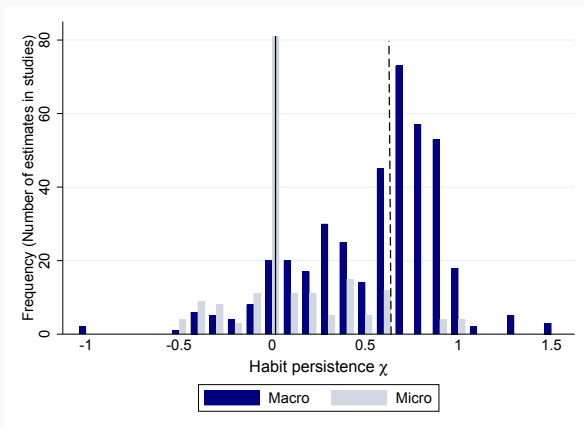
- This yields the following estimable equation:

$$\Delta \ln(c_{i,t}) = \gamma_0 + \chi \Delta \ln(c_{i,t-1}) + e_{i,t}$$

which can be estimated on either micro data or macro data
(if macro data, then just remove the i subscript)

Empirical estimates of habit persistence

- χ has been estimated by over 597 different papers



- Mean χ in macro studies: 0.6
- Mean χ in micro studies: 0.0-0.1

Why the disagreement between macro and micro studies?

Macro: Representative Agent Models

- Theory: C responds instantly, completely to shock
- Evidence: Consumption is too smooth (Campbell & Deaton, 1989)
- Solution: **“Habits” parameter** $\chi^{\text{Macro}} \approx 0.6 \sim 0.8$

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Micro: Heterogeneous Agent Models

- **Uninsurable risk is essential, changes everything**
- Var of micro income shocks much larger than of macro shocks:
$$\text{var}(\Delta \log \mathbf{p}) \approx 100 \times \text{var}(\Delta \log \mathbf{P})$$
- Evidence: **“Habits” parameter** $\chi^{\text{Micro}} \approx 0.0 \sim 0.1$

Macro Inattention

Alternative Explanation: Inattention to Macro Aggregates

Carroll, Crawley, Slacalek, Tokuoka, White (2019):

- Income Has Idiosyncratic and Aggregate Components
- Idiosyncratic Component Is Perfectly Observed
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Utility Cost of Inattention Small

- Micro: Critical (and Easy) To Notice You're Unemployed
- Macro: *Not* Critical To *Instantly* Notice If $U \uparrow$

Hall (1978) Random Walk

- **Total Wealth** (Human + Nonhuman):

$$\mathbf{o}_{t+1} = (\mathbf{o}_t - \mathbf{c}_t)R + \zeta_{t+1}$$

- **C Euler Equation:**

$$u'(\mathbf{c}_t) = R\beta\mathbb{E}_t[u'(\mathbf{c}_{t+1})]$$

- \Rightarrow **Random Walk** (for $R\beta = 1$):

$$\Delta\mathbf{c}_{t+1} = \epsilon_{t+1}$$

- **Expected Wealth:**

$$\mathbf{o}_t = \mathbb{E}_t[\mathbf{o}_{t+1}] = \mathbb{E}_t[\mathbf{o}_{t+2}] = \dots$$

Sticky Expectations—Individual c

- Consumer who happens to update at t and $t + n$

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- Implies that $\Delta^n \mathbf{o}_{t+n} \equiv \mathbf{o}_{t+n} - \mathbf{o}_t$ is white noise
- So **individual c** is RW across updating periods:

$$\mathbf{c}_{t+n} - \mathbf{c}_t = (r/R) \underbrace{(\mathbf{o}_{t+n} - \mathbf{o}_t)}_{\Delta^n \mathbf{o}_{t+n}}$$

Sticky Expectations—Aggregate C

- Population normed to one, uniformly dist on $[0, 1]$: $\mathbf{C}_t = \int_0^1 \mathbf{c}_{t,i} di$

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- **Calvo-Type Updating of Expectations:** (Probability $\Pi = 0.25$, per quarter)
- Economy composed of many sticky- \mathbb{E} consumers:

$$\begin{aligned}\mathbf{C}_{t+1} &= (1 - \Pi) \underbrace{\mathbf{C}_{t+1}^{\pi}}_{=\mathbf{C}_t} + \Pi \mathbf{C}_{t+1}^{\pi} \\ \Delta \mathbf{C}_{t+1} &\approx \underbrace{(1 - \Pi)}_{\equiv \chi=0.75} \Delta \mathbf{C}_t + \epsilon_{t+1}\end{aligned}$$

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- **Substantial persistence ($\chi = 0.75$) in aggregate C growth**

One More Ingredient: Idiosyncratic Uncertainty ...

- **Differences: Idiosyncratic vs Aggregate shocks**
 - **Idiosyncratic shocks:** Frictionless observation
 - I notice if I am fired, promoted, somebody steals my wallet
 - True RW with respect to these
 - **Aggregate shocks:** Sticky observation
 - May not instantly notice changes in aggregate productivity

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- **Result:**
 - **Idiosyncratic Δc :** dominated by frictionless RW part
 - **Aggregate ΔC :** highly serially correlated
Law of large numbers \Rightarrow idiosyncratic part vanishes

Macro Inattention

Full Heterogeneous Agent Model

Full Heterogeneous Agent Model

Partial Equilibrium

- CRRA Utility
- Idiosyncratic Shocks Calibrated From Micro Data
- Aggregate Shocks Calibrated From Macro Data
- Markov Process (Discrete RW) for Aggr Income Growth
- Liquidity Constraint
- Mildly Impatient Consumers

- Individual's labor productivity is

$$\ell_{t,i} = \overbrace{\theta_{t,i}}^{\equiv \theta_{t,i}} \overbrace{p_{t,i}}^{\equiv p_{t,i}} \Theta_t P_t$$

- Idiosyncratic and aggregate p evolve according to

$$\begin{aligned} p_{t+1,i} &= p_{t,i} \psi_{t+1,i} \\ P_{t+1} &= \Phi_{t+1} P_t \Psi_{t+1} \end{aligned}$$

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- Φ is Markov 'underlying' aggregate pty growth
 - Discrete (bounded) random walk
 - Calibrated to match postwar US pty growth variation
 - Generates predictability in income growth (for IV regressions)

Blanchard (1985) Model of “Perpetual Youth”

- Household survives from t to $t + 1$ with probability $(1 - D)$:

$$p_{t+1,i} = \begin{cases} 1 & \text{for newborns} \\ p_{t,i} \psi_{t+1,i} & \text{for survivors} \end{cases}$$

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- Blanchardian scheme:

$$\mathbf{k}_{t+1,i} = \begin{cases} 0 & \text{if HH } i \text{ dies, is replaced by newborn} \\ \mathbf{a}_{t,i} / (1 - D) & \text{if household } i \text{ survives} \end{cases}$$

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- Why useful? Allows us to have mortality without an additional state variable:

$$v(\cdot) = \max_c u(c) + \beta \mathbb{E}_t[(1 - D)v(\cdot)]$$

- Market resources:

$$\mathbf{m}_{t,i} = \underbrace{W_t \ell_{t,i}}_{\equiv y_t} + \underbrace{R_t}_{1+r_t} \mathbf{k}_{t,i}$$

- End-of-Period ‘Assets’—Unspent resources:

$$\mathbf{a}_{t,i} = \mathbf{m}_{t,i} - \mathbf{c}_{t,i}$$

- Capital transition depends on prob of survival $1 - D$:

$$\mathbf{k}_{t+1,i} = \mathbf{a}_{t,i} / (1 - D)$$

Frictionless Solution

- Normalize everything by $\mathbf{p}_{t,i} \equiv p_{t,i}P_t$, e.g. $m_{t,i} = \mathbf{m}_{t,i}/(p_{t,i}P_t)$
- $c(m, \Phi)$ is the function that solves:

$$v(m_{t,i}, \Phi_t) = \max_c u(c) + (1-D)\beta \mathbb{E}_t \left[(\Phi_{t+1} \psi_{t+1,i})^{1-\rho} v(m_{t+1,i}, \Phi_{t+1}) \right]$$

- Level of consumption:

$$\mathbf{c}_{t,i} = c(m_{t,i}, \Phi_t) \times p_{t,i}P_t$$

Calvo Updating of Perceptions of Aggregate Shocks

1. *True* Permanent income: $P_{t+1} = \Phi_{t+1} P_t \Psi_{t+1}$
2. Tilde (\tilde{P}) denotes perceived variables
3. Perception for consumer who has not updated for n periods:

$$\tilde{P}_{t,i} = \mathbb{E}_{t-n}[P_t | \Omega_{t-n}] = \Phi_{t-n}^n P_{t-n}$$

because Φ is random walk

Sequence Within Period

1. Income shocks are realized and every individual sees her true \mathbf{y} and \mathbf{m} , i.e. $\mathbf{y}_{t,i} = \tilde{\mathbf{y}}_{t,i}$ and $\mathbf{m}_{t,i} = \tilde{\mathbf{m}}_{t,i}$ for all t and i
2. Updating shocks realized: i observes true P_t, Φ_t w/ prob Π ; forms perceptions of her normalized market resources $\tilde{m}_{t,i}$

Sticky Expectations about Aggregate Income

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3. Consumes based on her perception, using $c(\tilde{m}_{t,i}, \tilde{\Phi}_{t,i})$

Key Assumption:

- People act as if their perceptions about aggregate state $\{\tilde{P}_{t,i}, \tilde{\Phi}_{t,i}\}$ are the true aggregate state $\{P_t, \Phi_t\}$

Behavior under Sticky Expectations

- Normalized resources:
 - $m_{t,i} \equiv \mathbf{m}_{t,i} / (p_{t,i} P_t)$ is *actual*
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 - in levels: $\tilde{\mathbf{m}}_{t,i} = \mathbf{m}_{t,i}$; but normalized: $\tilde{m}_{t,i} \neq m_{t,i}$

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- Correctly perceive level of their own spending $\mathbf{c}_{t,i}$

Macro Inattention

Taking the Model to the Data

Dynan (2000) Specification:

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \mathbb{E}[\Delta \log \mathbf{C}_t] + \eta \mathbb{E}[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

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- χ : **Extent of habits**

Data: Micro: $\chi^{\text{Micro}} = 0.1$ (EER 2017 paper)

Macro: $\chi^{\text{Macro}} = 0.6$

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- **η : Fraction of \mathbf{Y} going to 'rule-of-thumb' $\mathbf{C} = \mathbf{Y}$ types**

Data: Micro: $0 < \eta^{\text{Micro}} < 1$ (Depends ...)

Macro: $\eta^{\text{Macro}} \approx 0.5$ (Campbell and Mankiw (1989))

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- **α : Precautionary saving (micro) or IES (Macro)**

Data: Micro: $\alpha^{\text{Micro}} < 0$ (Zeldes (1989))

Macro: $\alpha^{\text{Macro}} < 0$ (but small)

Micro vs Macro: Theory and Empirics

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

	χ	η	α
Micro			
Data	≈ 0	$0 < \eta < 1$	< 0
Theory: Traditional RA Model	≈ 0	$0 < \eta < 1$	< 0
Macro			
Data	≈ 0.75	≈ 0	< 0
Theory: Traditional RA Model	≈ 0	≈ 0	< 0

Traditional RA model = one without consumption habits

Model with ‘Sticky Expectations’ of aggregate variables can match both micro and macro consumption dynamics

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

	χ	η	α
Micro			
Data	≈ 0	$0 < \eta < 1$	< 0
Theory: Habits	≈ 0.75	$0 < \eta < 1$	< 0
Theory: Sticky Expectations	≈ 0	$0 < \eta < 1$	< 0
Macro			
Data	≈ 0.75	≈ 0	< 0
Theory: Habits	≈ 0.75	≈ 0	< 0
Theory: Sticky Expectations	≈ 0.75	≈ 0	< 0

Temptation & Commitment

Temptation & Commitment

A Model of Hand-to-Mouth Behavior

Why do households choose to be wealthy hand to mouth?

- It prevents consumption smoothing over income shocks

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- It prevents consumption smoothing over income shocks

Traditional explanation (Kaplan and Violante, 2014)

- Illiquid assets give large excess returns relative to all liquid assets
- But this is a controversial assumption
- There exists a high return liquid asset: publicly traded equities

Our goal: develop a new model of the wealthy hand to mouth

- In our model, HHs face temptation, making it difficult to save
- Households can overcome these issues through illiquid housing

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This view of WH2M behavior helps us understand other important policies

- We study housing subsidies and mandatory amortization
- Do policies simply encourage substitution from liquid to illiquid assets?

Model with commitment obtains a good fit of the empirical evidence

- Matches large share of WH2M despite high return liquid asset
- Restricted model cannot match WH2M using housing utility alone
- MPC declines slowly with the size of income shocks

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Subsidies to commitment devices can increase overall savings

- Housing subsidies generate mild substitution from liquid assets to housing, but nevertheless boost overall wealth accumulation by 7%
- Mortgage amortization also increases net wealth accumulation by 10%
- The two policies have little effect on the share of WH2M households

Temptation & Commitment

The Model

Life cycle model of consumption and savings

- Demographics: households work for \bar{T} years, then retired for \tilde{T} years
- Choices: consumption, housing
- Assets: Liquid assets, housing, and mortgages

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- Assets: Liquid assets, housing, and mortgages

Novel features

- Temptation preferences make it costly to hold liquid assets
- A commitment device (housing) can reduce temptation

Standard model

- Households are committed to their choices
- No need for commitment

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Hyperbolic discounting model (Strotz, 1956 and Laibson, 1997)

- Relaxes the assumption of standard model on **discounting**
- Different discount rates, time inconsistent
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Temptation preferences (Gul and Pesendorfer, 2001 and 2004)

- Tempting, feasible alternative that is not chosen
- This tempting alternative impacts your utility
- Axiomatic, time consistent
- Commitment: reduce temptation by restricting choice set

$$\max_{\{c_t, h_t\}_{t=0, \dots, T}} \mathbb{E}_0 \sum_{t=0}^T \beta^t U(c_t, h_t, \tilde{c}_t, \tilde{h}_t)$$

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$$U(c_t, h_t, \tilde{c}_t, \tilde{h}_t) = u(c_t, h_t) - \underbrace{\lambda [u(\tilde{c}_t, \tilde{h}_t) - u(c_t, h_t)]}_{\text{utility cost of self-control}}$$

- c_t : nondurable consumption
- h_t : housing status
- λ : degree of temptation

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Most tempting alternative: maximize current period utility

$$[\tilde{c}_t, \tilde{h}_t] = \arg \max_{c_t, h_t \in \mathcal{A}_t} u(c_t, h_t)$$

- \tilde{c}_t : most tempting consumption
- \tilde{h}_t : most tempting housing status

1. Liquid asset (a_t)

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- Discrete asset with N different sizes (rental, flat, house, mansion, etc)
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- Transaction costs generate **commitment benefit**

Assets and Mortgages

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3. Mortgages (m_t)

- Buying a home automatically comes with a mortgage
- Downpayment of ψ percent of the house price
- Fixed-rate mortgage, r^M
- Fixed repayment each period until retirement or house sale

Housing Preferences

Functional form follows Attanasio et al (2012)

$$u(c_t, h_t) = \underbrace{\frac{c_t^{1-\gamma}}{1-\gamma}}_{\text{consumption utility}} \underbrace{e^{\theta\phi(h_t)}}_{\text{multip housing utility}} + \underbrace{\mu\phi(h_t)}_{\text{additive housing utility}} - \underbrace{\chi\mathbb{I}_{h_t \neq h_{t-1}}}_{\text{utility cost of moving}}$$

- γ : coefficient of relative risk aversion
- θ and μ : housing preference parameters
- ϕ : relative utility of house choice h_t
- χ : utility cost of housing adjustment (only applies if $h_t \neq h_{t-1}$)

$$\ln y_t = g_t + z_t$$

- g : Deterministic age profile for income (third order polynomial)
- z : Idiosyncratic income process
 - Exogenous AR(1) process

$$z_t = \rho z_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$z_0 \sim N(0, \sigma_0^2)$$

Given state variables $\Omega_t = \{a_t, z_t, m_t, h_{t-1}\}$

$$V_t(\Omega_t) = \max \left\{ V_t^0(\Omega_t), V_t^1(\Omega_t) \right\}$$

where $V_t^0(\Omega_t)$ and $V_t^1(\Omega_t)$ are the value functions conditional on not adjusting and adjusting housing.

Those who choose not to adjust in period t :

$$V_t^0(\Omega_t) = \max_{\{c_t, a_{t+1}\}} U(c_t, h_t, \tilde{c}_t, \tilde{h}_t) + \beta \mathbb{E}_t V_{t+1}(\Omega_{t+1}), \quad (1)$$

subject to:

$$\begin{aligned} a_{t+1} &= (1+r) \left[a_t + \tilde{y}_t - c_t - \mathbb{I}_t^{own} m p_t - (1 - \mathbb{I}_t^{own}) rent_t \right] \\ \tilde{y}_t &= \begin{cases} \exp(g_t + z_t), & \text{if } t \leq W \\ \text{SS Benefit}(y_W), & \text{if } t > W \end{cases} \\ z_t &= \rho z_{t-1} + \varepsilon_t \quad \text{and} \quad c_t > 0 \end{aligned} \quad (2)$$

Those who choose to adjust housing in period t :

$$V_t^1(\Omega_t) = \max_{\{c_t, h_t, m_{t+1}, a_{t+1}\}} U(c_t, h_t, \tilde{c}_t, \tilde{h}_t) + \beta \mathbb{E}_t V_{t+1}(\Omega_{t+1}), \quad (3)$$

subject to:

$$\begin{aligned} a_{t+1} &= (1+r) \left[a_t + \tilde{y}_t - c_t - (1+F)p_t(h_t) + \frac{m_{t+1}}{(1+r^M)} \right. \\ &\quad \left. + (1-F)p_t(h_{t-1}) - m_t \right] \\ m_{t+1} &\leq (1-\psi^{\min})p_t(h_t)(1+r^M) \\ y_t &= \begin{cases} \exp(g_t + z_t), & \text{if } t \leq W \\ \text{SS Benefit}(y_W), & \text{if } t > W \end{cases} \\ z_t &= \rho z_{t-1} + \varepsilon_t \quad \text{and} \quad c_t > 0 \end{aligned} \quad (4)$$

Temptation & Commitment

Model Calibration

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 - Semi-structural Euler equation approach to estimate λ using CEX data

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 - But this is a controversial assumption, which we choose to relax (e.g. Flavin and Yamashita 2002; Goetzmann and Spiegel 2002; Piazzesi, Schneider, and Tuzel 2007)
- Remaining preference parameters are calibrated internally
 - Parameters: time preference, risk aversion, housing utility parameters, utility cost of moving
 - Target a combination of life-cycle and aggregate moments

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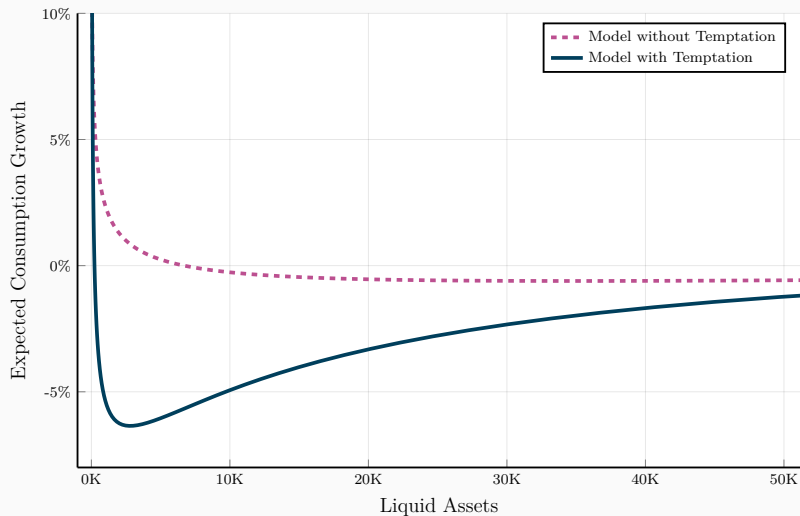
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This insight allows us to identify λ separately from β using data on consumption and assets (see Kovacs, Low, Moran, 2021)

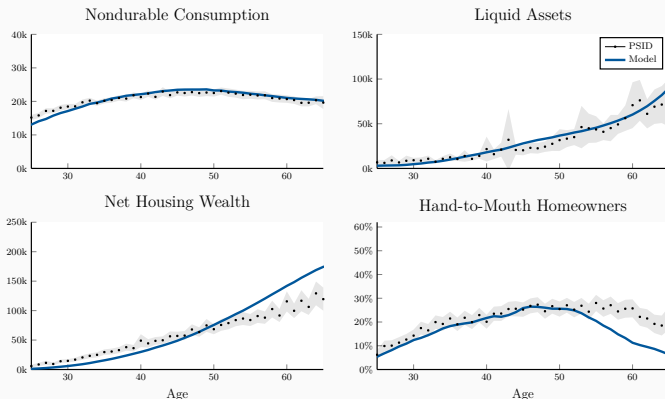
Identification



Temptation & Commitment

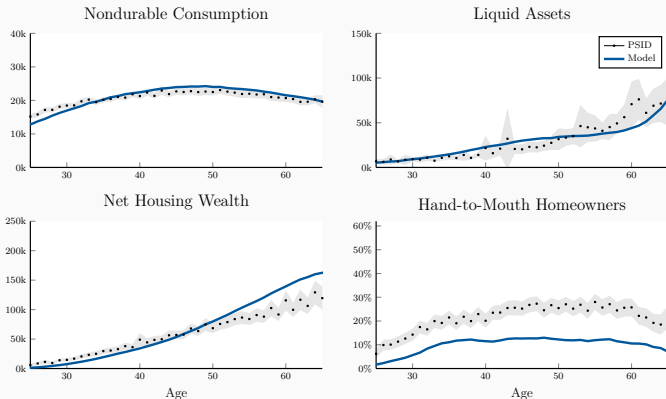
Model Fit

Baseline Model



Baseline model generates good fit, despite presence of high-return liquid asset

Restricted Model



Restricted model predicts 50% less hand-to-mouth homeowners

In addition, the model with temptation & commitment matches recent empirical evidence showing

1. The average MPC remains large even in response to large income shocks (e.g. Fuster, Kaplan, Zafar 2018; Kueng 2018; Fagereng, Holm, Natvik 2021)
2. Households have a demand for illiquidity (Beshears et al, 2021)
3. Mandatory amortization increases wealth accumulation (Bernstein and Koudijs, 2021)

Temptation & Commitment

Implications for Policy

1. Substantial tax benefits to homeownership in the U.S.

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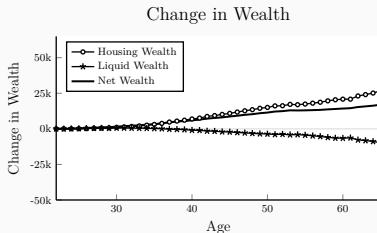
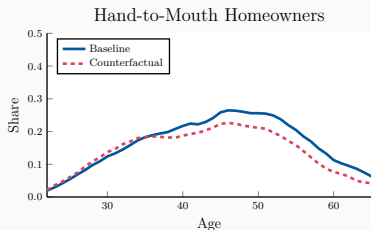
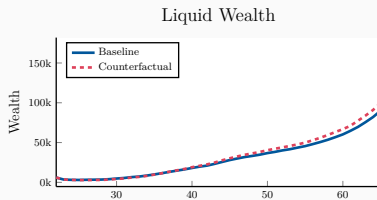
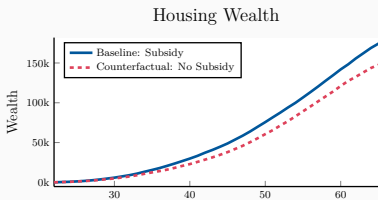
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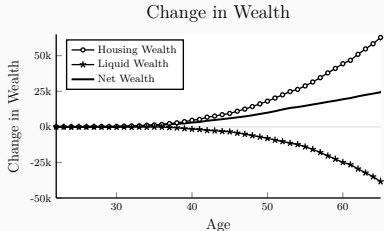
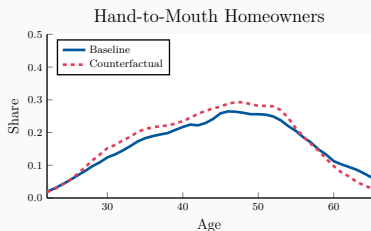
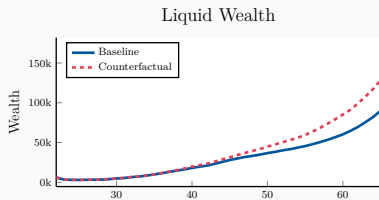
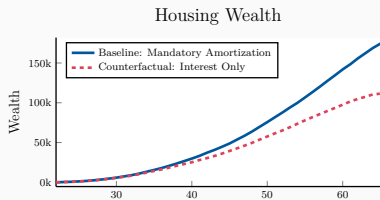
We evaluate two opposing views of such illiquid saving incentives

- May induce portfolio rebalancing from liquid to illiquid assets
- May improve access to commitment, potentially helping HHs accumulate wealth

Policy 1: Housing Subsidies



Policy 2: Mandatory Amortization



Conclusion

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Conclusion

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 - Large share of WH2M, while relaxing strong assumptions on returns
 - Generates good out-of-sample fit of MPCs and recent evidence
- Understanding WH2M behavior has important implications for policy
 - Subsidies to commitment can increase overall savings
 - Mortgage amortization can boost wealth accumulation

Summary

Summary and next week

- **Today:** Two applications of dynamic programming to understand household spending dynamics
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*Biased beliefs = when solving the value function, HHs expect a job loss probability that is different than what happens in the simulation.

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