CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

11. Introducing HANK

Adv. Macro: Heterogenous Agent Models

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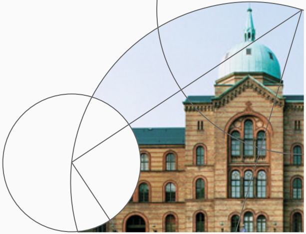
2023

Danmarks
Grundforskningsfond
Danish National
Research Foundation



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- **Code:**
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  2. Example from GEModelToolsNotebooks/HANK
- **Literature:**
  1. Auclert et. al. (2021), »Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models«
  2. Documentation for GEModelTools

IRFs and simulation

Reminder of model class

- **Unknowns:** $\boldsymbol{U}$
- **Shock:** Z
- **Additional variables:** $\boldsymbol{X}$
- **Target equation system;**

$$\boldsymbol{H}(\boldsymbol{U}, \boldsymbol{Z}) = \boldsymbol{0}$$

- **Auxiliary model equations**

$$\boldsymbol{X} = \boldsymbol{M}(\boldsymbol{U}, \boldsymbol{Z})$$

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2. Derive all other IRFs for

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  1. Imprecise for *large* shocks
  2. Imprecise in models with *aggregate non-linearities* (direct in aggregate equations or through micro-behavior)

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- **Insight:** *The IRF from an MIT shock is equivalent to the IRF in a model with aggregate risk, which is linearized in the aggregate variables* (Boppart et. al., 2018)
- **Perspectives:**
    1. **State-space approach with linearization**: Ahn et al. (2018); Bayer and Luetticke (2020); Bhandari et al. (2023); Bilal (2023).
    2. **Why not full *global* solution?** The distribution of households is a state variable for each household $\Rightarrow$ explosion in complexity
       Original: Krusell and Smith (1997, 1998); Algan et al. (2014);
       Deep learning: Fernández-Villaverde et al. (2021); Maliar et al. (2021); Han et al. (2021); Kase et al. (2022); Azinovic et al. (2022); Gu et al. (2023); Chen et al. (2023)

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- **Equivalence:**
  1. Same result if we linearize all aggregated equations and write the model in MA(∞) form
  2. The state space form can also be recovered (not needed)

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  1. The IRF for grid point i_g in a policy function can be calculated as

$$da^*_{i_g,s} = \sum_{s'=s}^{T-1} \sum_{X^{hh} \in \boldsymbol{X}^{hh}} \frac{\partial a^*_{i_g}}{\partial X^{hh}_{s'-s}} dX^{hh}_{s'}.$$

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  3. Distribution can then be simulated forwards

The HANC example from GEModelToolsNotebooks

- **Presentation:** I go through the code for *finding the linearized IRFs and simulating the model*

Calculating moments

- **Calculating moments such as var(dC_t):**
  1. From the simulation, or
  2. From the IRFs,

$$\text{var}(dC_t) = \sum_i \sigma_i^2 \sum_{s=0}^{i} \left(dC_s^i\right)^2$$

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- **Speed-up:** Use Fast Fourier Transform (FFT), see SSJ

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  2. Supply labor and choose consumption
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- **Central bank**: Set nominal interest rate

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for given output price, P_t, and input prices, p_{jt}

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- **Note:** Zero profits (can be used to derive price index)

Derivation of demand curve

- **FOC wrt.** y_{jt}

$$
\begin{aligned}
0 &= P_t \mu \left(\int_0^1 y_{jt}^{\frac{1}{\mu}} dj \right)^{\mu - 1} \frac{1}{\mu} y_{jt}^{\frac{1}{\mu}-1} - p_{jt} \Leftrightarrow \\
\frac{p_{jt}}{P_t} &= \left(\int_0^1 y_{jt}^{\frac{1}{\mu}} dj \right)^{\mu - 1} y_{jt}^{\frac{1-\mu}{\mu}} \Leftrightarrow \\
\left(\frac{p_{jt}}{P_t} \right)^{\frac{\mu}{\mu-1}} &= \left(\int_0^1 y_{jt}^{\frac{1}{\mu}} dj \right)^{\mu} y_{jt}^{-1} \Leftrightarrow \\
y_{jt} &= \left(\frac{p_{jt}}{P_t} \right)^{-\frac{\mu}{\mu-1}} Y_t
\end{aligned}
$$

Intermediary goods firms

- ***Dynamic*** **problem for intermediary goods firms:**

$$J_t(p_{jt-1}) = \max_{y_{jt}, p_{jt}, n_{jt}} \left\{ \frac{p_{jt}}{P_t} y_{jt} - w_t n_{jt} - \Omega(p_{jt}, p_{jt-1}) Y_t + \frac{J_{t+1}(p_{jt})}{1 + r_{t+1}} \right\}$$

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- **Implied dividends:** $d_t = Y_t - w_t N_t - \frac{\mu}{\mu-1} \frac{1}{2\kappa} \left[\log(1 + \pi_t) \right]^2 Y_t$

Derivation of NKPC

- **FOC wrt.** p_{jt}**:**

$$0 = \left(1 - \frac{\mu}{\mu - 1}\right) \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\mu}{\mu-1}} \frac{Y_t}{P_t} + \frac{\mu}{\mu - 1} \frac{w_t}{Z_t} \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\mu}{\mu-1}} \frac{Y_t}{p_{jt}}$$
$$- \frac{\mu}{\mu - 1} \frac{1}{\kappa} \frac{\log\left(\frac{p_{jt}}{p_{jt-1}}\right)}{p_{jt}} Y_t + \frac{J'_{t+1}(p_{jt})}{1 + r_{t+1}}$$

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- **FOC + Envelope + Symmetry +** $\pi_t = P_t/P_{t-1} - 1$

$$0 = \left(1 - \frac{\mu}{\mu - 1}\right)\frac{Y_t}{P_t} + \frac{\mu}{\mu - 1}\frac{w_t}{Z_t}\frac{Y_t}{P_t}$$
$$+ \frac{\mu}{\mu - 1}\frac{1}{\kappa}\log\left(1 + \pi_t\right)\frac{Y_t}{P_t} + \frac{\frac{\mu}{\mu-1}\frac{1}{\kappa}\log\left(1 + \pi_{t+1}\right)\frac{Y_{t+1}}{P_t}}{1 + r_{t+1}}$$

Households

- **Household problem**: Distribution, $\boldsymbol{D}_t$, over z_t and a_{t-1}

$$v_t(z_t, a_{t-1}) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}\left[v_{t+1}(z_{t+1}, a_t) \,|\, z_t, a_t\right]$$

$$\text{s.t. } a_t = (1+r_t)a_{t-1} + (w_t \ell_t - \tau_t + d_t) z_t - c_t \geq 0$$

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- **Effective labor-supply:** $n_t = z_t \ell_t$

EGM I

- **Beginning-of-period value function:**

$$\underline{v}_{a,t}(z_{t-1}, a_{t-1}) = \mathbb{E}_t \left[v_{a,t}(z_t, a_{t-1}) \right] = \mathbb{E} \left[(1 + r_t) c_t^{-\sigma} \right]$$

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$$\begin{aligned} c_t &= \left(\beta \underline{v}_{a,t+1}(z_t, a_t)\right)^{-\frac{1}{\sigma}} \\ \ell_t &= \left(\frac{w_t z_t}{\varphi} c_t^{-\sigma}\right)^{\frac{1}{\nu}} \\ m_t &= c_t + a_t - (w_t \ell_t - \tau_t + d_t) z_t \end{aligned}$$

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$$c^*(z_t, a_{t-1}) \text{ and } \ell^*(z_t, a_{t-1}) \text{ with } m_t = (1 + r_t) a_{t-1}$$

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$$
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$$

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     $$c^* = (1 + r_t)a_{t-1} + (w_t \ell^* - \tau_t + d_t)z_t$$
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  2. Set

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  3. Return to step 1

Government and central bank

- **Monetary policy:** Folow Taylor-rule:

$$i_t = i_t^* + \phi \pi_t$$

where i_t^* is a shock

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$$r_t = (1 + i_{t-1})/(1 + \pi_t) - 1$$

- **Government:** Choose τ_t to keep debt constant and finance exogenous public consumption

$$\tau_t = r_t B_{ss} + G_t$$

Market clearing

1. Assets: $B_{ss} = \int a_t^*(z_t, a_{t-1}) d\boldsymbol{D}_t$
2. Labor: $N_t = \int n_t^*(z_t, a_{t-1}) d\boldsymbol{D}_t$ (in effective units)
3. Goods: $Y_t = \int c_t^*(z_t, a_{t-1}) d\boldsymbol{D}_t + G_t + \frac{\mu}{\mu-1}\frac{1}{2\kappa}\left[\log\left(1+\pi_t\right)\right]^2 Y_t$

As an equation system

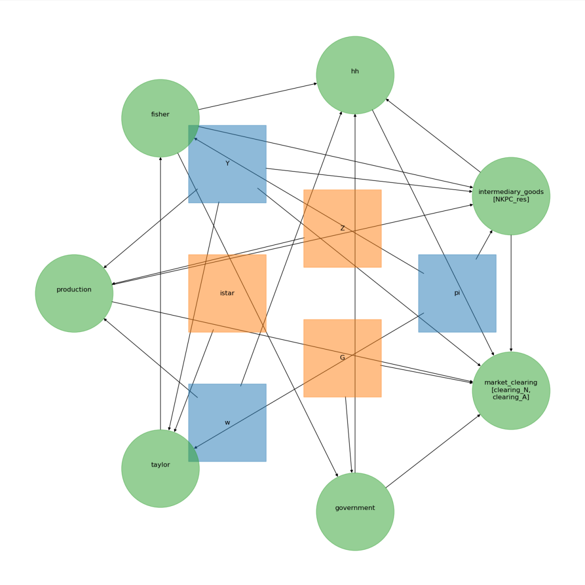
$$\boldsymbol{H}\left(\boldsymbol{\pi}, \boldsymbol{w}, \boldsymbol{Y}, \boldsymbol{i}^{*}, \boldsymbol{Z}, \underline{\boldsymbol{D}}_{0}\right) = \mathbf{0}$$

$$\begin{bmatrix} \log(1+\pi_t) - \left[\kappa\left(\frac{w_t}{Z_t} - \frac{1}{\mu}\right) + \frac{Y_{t+1}}{Y_t}\frac{\log(1+\pi_{t+1}}{1+r_{t+1}})\right] \\ N_t - \int n_t^*(z_t, a_{t-1}) d\boldsymbol{D}_t \\ B_{ss} - \int a_t^*(z_t, a_{t-1}) d\boldsymbol{D}_t \end{bmatrix} = \mathbf{0}$$

The rest of the model is given by

$$\boldsymbol{X} = \boldsymbol{M}(\boldsymbol{\pi}, \boldsymbol{w}, \boldsymbol{Y}, \boldsymbol{i}^{*}, \boldsymbol{Z})$$

As a DAG



hh
fisher
Y
intermediary_goods
[NKPC_res]
Z
production
istar
pi
G
market_clearing
[clearing_N,
clearing_A]
w
taylor
government

Steady state

- **Chosen:** B_{ss}, G_{ss}, r_{ss}
- **Analytically:**
  1. **Normalization:** $Z_{ss} = N_{ss} = 1$
  2. **Zero-inflation:** $\pi_{ss} = 0 \Rightarrow i_{ss} = i^*_{ss} = (1 + r_{ss})(1 + \pi_{ss}) - 1$
  3. **Firms:** $Y_{ss} = Z_{ss} N_{ss}$, $w_{ss} = \frac{Z_{ss}}{\mu}$ and $d_{ss} = Y_{ss} - w_{ss} N_{ss}$
  4. **Government:** $\tau_{ss} = r_{ss} B_{ss} + G_{ss}$
  5. **Assets:** $A_{ss} = B_{ss}$
- **Numerically:** Choose β and φ to get market clearing

The HANK example from GEModelToolsNotebooks

- We go through the code

Transmission mechanism to monetary policy shock

1. **Monetary policy shock:** $i_t^* \downarrow \Rightarrow i_t = i_t^* + \phi\pi_t \downarrow$
2. **Real interest rate:** $r_t = \frac{1+i_{t-1}}{1+\pi_t} \downarrow$
3. **Taxes:** $\tau_t = r_t B_{ss} \downarrow$
4. **Household consumption**, $C_t^{hh} \uparrow$, due to $r_t \downarrow$ and $\tau_t \downarrow$
5. **Firms production,** $Y_t\uparrow$, and **labor demand**, $N_t\uparrow$
6. **Inflation,** $\pi_t \uparrow$, and **wage**, $w_t\uparrow$ and **dividends**, $d_t\downarrow$
7. **Household labor supply**, $N_t^{hh}\uparrow$, due to $w_t \uparrow$ and $d_t \downarrow$, but dampened $\tau_t \downarrow$
8. **Nominal rate**, $i_t \uparrow$ due to $\pi_t \uparrow$ implying $r_t \uparrow$
9. **Household consumption**, $C_t^{hh}\uparrow$, due to $w_t \uparrow$ but dampened by $d_t \downarrow$ and $r_t \uparrow$

Exercise

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1. Compute the non-linear response to a temporary increase in government spending
2. Compute the linearized IRF to the same shock and compare
3. Sketch the transmission mechanism of government spending
4. Analyze how the aggressiveness of monetary policy affects the effectiveness of fiscal policy
5. Compare you previous results with the effects of a public transfer

Summary

Summary and next week

- **Today:**
  1. Linearized IRFs and simulation
  2. A baseline HANK model
- **Next week:** More on HANK models
- **Homework:**
  1. Work on exercise
  2. Skim-read Auclert et al. (2023), »The Intertemporal Keynesian Cross«