



9. Demographics and global imbalances

Adv. Macro: Heterogenous Agent Models

Jeppe Druedahl & Patrick Moran

2023



Introduction

Disclaimer

- Note: The views expressed in this presentation are those of the author and do not represent the views of the Federal Reserve Board or Federal Reserve System.

Demographics and global imbalances

- **Previously:** Learned how to think about and compute transition paths in HA models

Demographics and global imbalances

- **Previously:** Learned how to think about and compute transition paths in HA models
- **Today:** Apply these methods to macro questions about long-term changes in household saving behavior and interest rates

Demographics and global imbalances

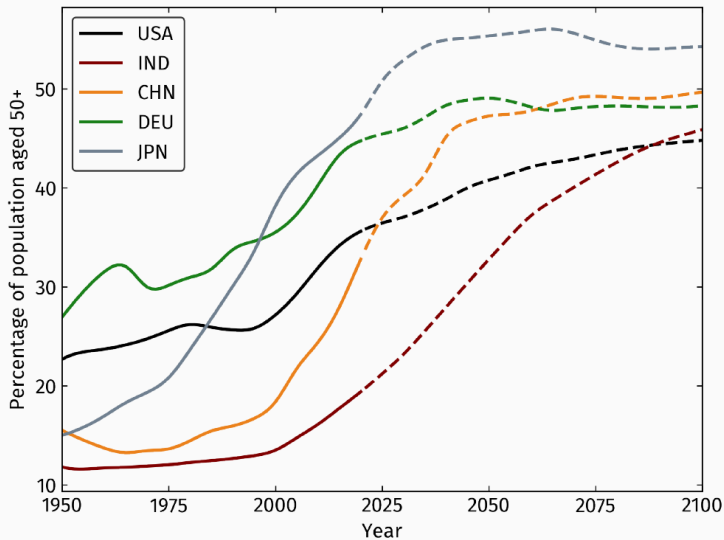
- **Previously:** Learned how to think about and compute transition paths in HA models
- **Today:** Apply these methods to macro questions about long-term changes in household saving behavior and interest rates
- **Central economic questions:**
 1. How do aging populations affect interest rates and global imbalances?
 2. What will happen going forward?
 3. Should we be concerned about an asset market meltdown?

Demographics and global imbalances

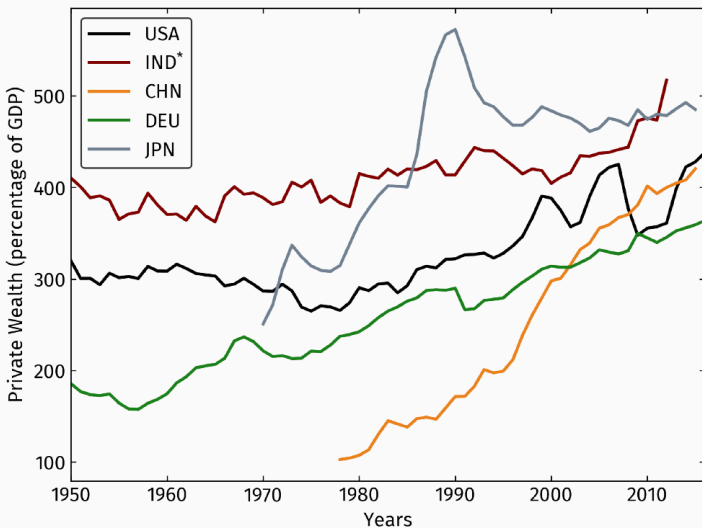
- **Previously:** Learned how to think about and compute transition paths in HA models
- **Today:** Apply these methods to macro questions about long-term changes in household saving behavior and interest rates
- **Central economic questions:**
 1. How do aging populations affect interest rates and global imbalances?
 2. What will happen going forward?
 3. Should we be concerned about an asset market meltdown?
- **Plan for today:** Discuss “Demographics, Wealth, and Global Imbalances in the Twenty-First Century”
 1. Develop a multi-country model with a world interest rate
 2. Quantify the model using a “sufficient statistics” approach
 3. Study the various effects of an aging population

Motivation

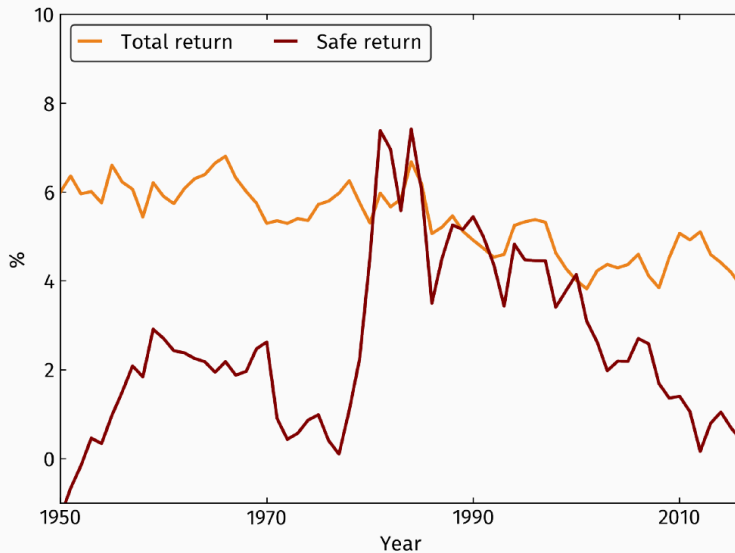
The world population is aging



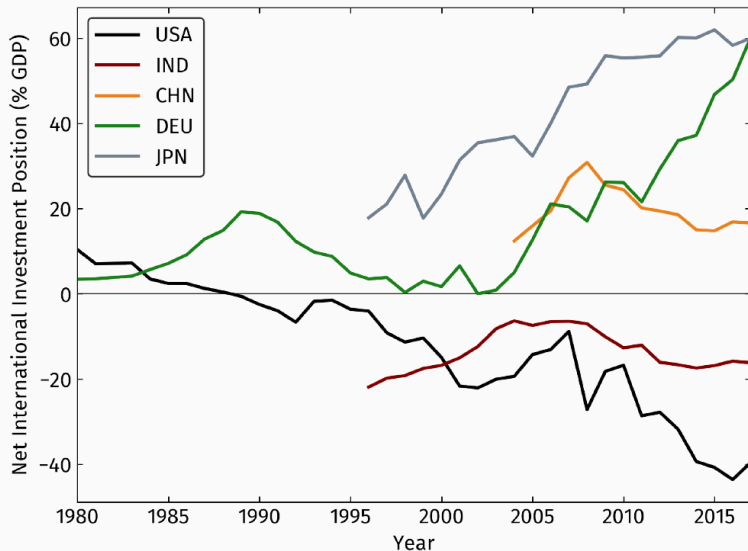
...wealth-to-GDP ratios are increasing...



...rates of return on wealth are falling...



...and “global imbalances” are rising



How have demographics shaped these trends?

- Broad agreement that demographics has contributed to historical trends in W/Y , real returns (r), and NFA imbalances

How have demographics shaped these trends?

- Broad agreement that demographics has contributed to historical trends in W/Y , real returns (r), and NFA imbalances
- Older population saves more, unevenly across countries

How have demographics shaped these trends?

- Broad agreement that demographics has contributed to historical trends in W/Y , real returns (r), and NFA imbalances
- Older population saves more, unevenly across countries
- Much less agreement about how much: $\langle + - \rangle \Delta r$ for 1970-2015 is
 - $> -100\text{bp}$ in Gagnon-Johannsen-Lopez-Salido 2021
 - $< -30\text{bp}$ in Eggertsson-Mehrotra-Robbins 2019

And how will demographics continue to shape these trends?

- Critical question: what will happen going forward?

And how will demographics continue to shape these trends?

- Critical question: what will happen going forward?
- Influential view that these trends will revert:

And how will demographics continue to shape these trends?

- Critical question: what will happen going forward?
- Influential view that these trends will revert:
 - “While a large population cohort that is saving for retirement puts upward pressure on the total savings rate, a large elderly cohort may push down aggregate savings by running down accumulated wealth.” [Lane 2020]

And how will demographics continue to shape these trends?

- Critical question: what will happen going forward?
- Influential view that these trends will revert:
 - “While a large population cohort that is saving for retirement puts upward pressure on the total savings rate, a large elderly cohort may push down aggregate savings by running down accumulated wealth.” [Lane 2020]
 - “asset market meltdown” hypothesis [Poterba 2001]

And how will demographics continue to shape these trends?

- Critical question: what will happen going forward?
- Influential view that these trends will revert:
 - “While a large population cohort that is saving for retirement puts upward pressure on the total savings rate, a large elderly cohort may push down aggregate savings by running down accumulated wealth.” [Lane 2020]
 - “asset market meltdown” hypothesis [Poterba 2001]
 - “great demographic reversal” hypothesis [Goodhart-Pradhan 2020]

And how will demographics continue to shape these trends?

- To answer this question, we will study “Demographics, Wealth, and Global Imbalances in the Twenty-First Century” by Auclert, Malmberg, Martenet and Rognlie (2021)

And how will demographics continue to shape these trends?

- To answer this question, we will study “Demographics, Wealth, and Global Imbalances in the Twenty-First Century” by Auclert, Malmberg, Martenet and Rognlie (2021)
- These authors develop a multi-country model with overlapping generations of households and equilibrium world interest rates

And how will demographics continue to shape these trends?

- To answer this question, we will study “Demographics, Wealth, and Global Imbalances in the Twenty-First Century” by Auclert, Malmberg, Martenet and Rognlie (2021)
- These authors develop a multi-country model with overlapping generations of households and equilibrium world interest rates
 - Demographic change alters demand for assets in each country

And how will demographics continue to shape these trends?

- To answer this question, we will study “Demographics, Wealth, and Global Imbalances in the Twenty-First Century” by Auclert, Malmberg, Martenet and Rognlie (2021)
- These authors develop a multi-country model with overlapping generations of households and equilibrium world interest rates
 - Demographic change alters demand for assets in each country
 - This affects world interest rate & financial flows between countries

And how will demographics continue to shape these trends?

- To answer this question, we will study “Demographics, Wealth, and Global Imbalances in the Twenty-First Century” by Auclert, Malmberg, Martenet and Rognlie (2021)
- These authors develop a multi-country model with overlapping generations of households and equilibrium world interest rates
 - Demographic change alters demand for assets in each country
 - This affects world interest rate & financial flows between countries
- Big challenge: how to take this model to the data to discipline the importance of demographics?

Taking the model to the data

- The authors develop a sufficient statistic approach to answer this question

Taking the model to the data

- The authors develop a sufficient statistic approach to answer this question
- First, they show analytically that the effect of demographic change on W/Y , r , and NFA depends only on:

Taking the model to the data

- The authors develop a sufficient statistic approach to answer this question
- First, they show analytically that the effect of demographic change on W/Y , r , and NFA depends only on:
 1. Age profiles of wealth, labor income, and consumption

Taking the model to the data

- The authors develop a sufficient statistic approach to answer this question
- First, they show analytically that the effect of demographic change on W/Y , r , and NFA depends only on:
 1. Age profiles of wealth, labor income, and consumption
 2. Demographic projections

Taking the model to the data

- The authors develop a sufficient statistic approach to answer this question
- First, they show analytically that the effect of demographic change on W/Y , r , and NFA depends only on:
 1. Age profiles of wealth, labor income, and consumption
 2. Demographic projections
 3. The elasticity of intertemporal substitution

Taking the model to the data

- The authors develop a sufficient statistic approach to answer this question
- First, they show analytically that the effect of demographic change on W/Y , r , and NFA depends only on:
 1. Age profiles of wealth, labor income, and consumption
 2. Demographic projections
 3. The elasticity of intertemporal substitution
 4. The elasticity of substitution between capital and labor

Taking the model to the data

- The authors develop a sufficient statistic approach to answer this question
- First, they show analytically that the effect of demographic change on W/Y , r , and NFA depends only on:
 1. Age profiles of wealth, labor income, and consumption
 2. Demographic projections
 3. The elasticity of intertemporal substitution
 4. The elasticity of substitution between capital and labor
- Second, they use this framework to measure the importance of demographic change

Taking the model to the data

- The authors develop a sufficient statistic approach to answer this question
- First, they show analytically that the effect of demographic change on W/Y , r , and NFA depends only on:
 1. Age profiles of wealth, labor income, and consumption
 2. Demographic projections
 3. The elasticity of intertemporal substitution
 4. The elasticity of substitution between capital and labor
- Second, they use this framework to measure the importance of demographic change
- Admittedly, this approach requires a lot of simplifying assumptions. The authors solve and simulate the full model and show that it gives similar results

Main results

- The authors reject the “great demographic reversal” hypothesis
 - Do not find that aging will decrease savings and increase interest rates
 - Instead, it appears the global savings glut has just begun

Main results

- The authors reject the “great demographic reversal” hypothesis
 - Do not find that aging will decrease savings and increase interest rates
 - Instead, it appears the global savings glut has just begun
- In addition, the authors refute the “asset market meltdown” hypothesis
 - Will dissaving of the old reverse the effects of demographics?
 - Yes, slightly. But it does not cause r to increase
 - As a result, no asset market meltdown

Model

Model: Main Elements

- OLG model, demographic change + multiple countries facing $\{r_t\}$

Model: Main Elements

- OLG model, demographic change + multiple countries facing $\{r_t\}$
- Demographics
 - Exogenous, time-varying sequence of births N_{0t}
 - Exogenous, constant sequence of mortality rates ϕ_j
 - No migration

Model: Main Elements

- OLG model, demographic change + multiple countries facing $\{r_t\}$
- Demographics
 - Exogenous, time-varying sequence of births N_{0t}
 - Exogenous, constant sequence of mortality rates ϕ_j
 - No migration
- Production
 - Aggregate production function with capital and effective labor, with elasticity of substitution η
 - Constant growth rate of labor-augmenting technology γ
 - Perfect competition, free capital adjustment

Model: Main Elements

- OLG model, demographic change + multiple countries facing $\{r_t\}$
- Demographics
 - Exogenous, time-varying sequence of births N_{0t}
 - Exogenous, constant sequence of mortality rates ϕ_j
 - No migration
- Production
 - Aggregate production function with capital and effective labor, with elasticity of substitution η
 - Constant growth rate of labor-augmenting technology γ
 - Perfect competition, free capital adjustment
- Government
 - Flow budget constraint

$$G_t + w_t \sum_{j=0}^T N_{jt} \mathbb{E} tr_j + (1 + r_t) B_t = \tau w_t \sum_{j=0}^T N_{jt} \mathbb{E} l_j + B_{t+1}$$

- Balance budget by changing G_t , not τ_t or tr_{jt} , to keep B_t/Y_t constant

Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort k (age $j \equiv t - k$)

$$\max \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \right],$$

$$\text{s.t } c_{jt} + \phi_j a_{j+1,t+1} \leq w_t \left((1 - \tau) \ell(z_j) + tr(z^j) \right) + (1 + r_t) a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}$$

Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort k (age $j \equiv t - k$)

$$\max \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \right],$$

$$\text{s.t. } c_{jt} + \phi_j a_{j+1,t+1} \leq w_t \left((1 - \tau) \ell(z_j) + tr(z^j) \right) + (1 + r_t) a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}$$

- $\sigma \equiv$ elasticity of intertemporal substitution

Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort k (age $j \equiv t - k$)

$$\max \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right],$$

$$\text{s.t } c_{jt} + \phi_j a_{j+1,t+1} \leq w_t \left((1-\tau)\ell(z_j) + tr(z^j) \right) + (1+r_t) a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}$$

- $\sigma \equiv$ elasticity of intertemporal substitution
- β_j : age-specific discount rate

Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort k (age $j \equiv t - k$)

$$\max \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right],$$

$$\text{s.t } c_{jt} + \phi_j a_{j+1,t+1} \leq w_t \left((1-\tau)\ell(z_j) + tr(z^j) \right) + (1+r_t) a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}$$

- $\sigma \equiv$ elasticity of intertemporal substitution
- β_j : age-specific discount rate
- Φ_j : survival probability by age ($\Phi_j = \prod_j \phi_j$)

Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort k (age $j \equiv t - k$)

$$\max \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right],$$

$$\text{s.t } c_{jt} + \phi_j a_{j+1,t+1} \leq w_t \left((1-\tau)\ell(z_j) + tr(z^j) \right) + (1+r_t) a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}$$

- $\sigma \equiv$ elasticity of intertemporal substitution
- β_j : age-specific discount rate
- Φ_j : survival probability by age ($\Phi_j = \prod_j \phi_j$)
- $\ell(z_t)$: risky labor supply driven by arbitrary stochastic process z_t

Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort k (age $j \equiv t - k$)

$$\max \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right],$$

$$\text{s.t } c_{jt} + \phi_j a_{j+1,t+1} \leq w_t \left((1-\tau)\ell(z_j) + tr(z^j) \right) + (1+r_t) a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}$$

- $\sigma \equiv$ elasticity of intertemporal substitution
- β_j : age-specific discount rate
- Φ_j : survival probability by age ($\Phi_j = \prod_j \phi_j$)
- $\ell(z_t)$: risky labor supply driven by arbitrary stochastic process z_t
- $\tau, tr(z^j)$: taxes and (state-contingent) government transfers

Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort k (age $j \equiv t - k$)

$$\max \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right],$$

$$\text{s.t. } c_{jt} + \phi_j a_{j+1,t+1} \leq w_t \left((1-\tau)\ell(z_j) + tr(z^j) \right) + (1+r_t) a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}$$

- $\sigma \equiv$ elasticity of intertemporal substitution
- β_j : age-specific discount rate
- Φ_j : survival probability by age ($\Phi_j = \prod_j \phi_j$)
- $\ell(z_t)$: risky labor supply driven by arbitrary stochastic process z_t
- $\tau, tr(z^j)$: taxes and (state-contingent) government transfers
- a_{jt} : annuity holdings

Equilibrium

Given demographics and policy, in an integrated world equilibrium:

- Individuals optimize
- Firms optimize
- Global asset markets clear

$$\sum_c W_t^c = \sum_c (K_t^c + B_t^c) \quad \forall t$$

where W_t^c is aggregate household wealth in country c :

$$W_t^c = \sum_{j=0}^J N_{jt}^c a_{jt}^c$$

Next: consider small country aging alone, with world at steady state
→ r constant (will adjust later)

Compositional effects as sufficient statistics

Proposition 1

The wealth-to-GDP ratio of a small country aging alone with constant r and γ follows

$$\frac{W_t}{Y_t} \propto \frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}}$$

where $a_{j0} \equiv \mathbb{E}a_{j,0}$ and $h_{j0} = \mathbb{E}w_0 \ell_{j,0}$ are average initial asset holdings and pretax labor income by age, and $\pi_{jt} = N_{jt}/N_t$ is the share of the population of age j .

In a partial equilibrium world (where r does not adjust to changing demographics) then all changes in W/Y reflect the changing age composition π_{jt} of the population, given fixed profiles of asset holdings by age (a_{j0}) and income by age (h_{j0}).

Compositional effects as sufficient statistics

Based on Proposition 1, we can compute the change in log wealth to GDP ratio as follows:

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_o}{Y_o}\right) = \log\left(\frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum_j \pi_{j0} a_{jo}}{\sum_j \pi_{j0} h_{j0}}\right) \equiv \Delta_t^{comp}$$

- The above is measurable from demographic projections and hh. surveys

Compositional effects as sufficient statistics

Based on Proposition 1, we can compute the change in log wealth to GDP ratio as follows:

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_o}{Y_o}\right) = \log\left(\frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum_j \pi_{j0} a_{jo}}{\sum_j \pi_{j0} h_{j0}}\right) \equiv \Delta_t^{comp}$$

- The above is measurable from demographic projections and hh. surveys
- Why? Demographics do not affect (normalized) individual decisions

Compositional effects as sufficient statistics

Based on Proposition 1, we can compute the change in log wealth to GDP ratio as follows:

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_o}{Y_o}\right) = \log\left(\frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum_j \pi_{j0} a_{jo}}{\sum_j \pi_{j0} h_{j0}}\right) \equiv \Delta_t^{comp}$$

- The above is measurable from demographic projections and hh. surveys
- Why? Demographics do not affect (normalized) individual decisions
- Later: we'll think about how Δ_t^{comp} affects general equilibrium outcomes

Measuring compositional effects

- Calculate Δ_t^{comp} for 25 countries:

$$\Delta_t^{\text{comp}} \equiv \log \left(\frac{\sum \pi_{jt} a_{j0}}{\sum \pi_{jt} h_{j0}} \right) - \log \left(\frac{\sum \pi_{j0} a_{j0}}{\sum \pi_{j0} h_{j0}} \right)$$

- Calculate Δ_t^{comp} for 25 countries:

$$\Delta_t^{comp} \equiv \log \left(\frac{\sum \pi_{jt} a_{j0}}{\sum \pi_{jt} h_{j0}} \right) - \log \left(\frac{\sum \pi_{j0} a_{j0}}{\sum \pi_{j0} h_{j0}} \right)$$

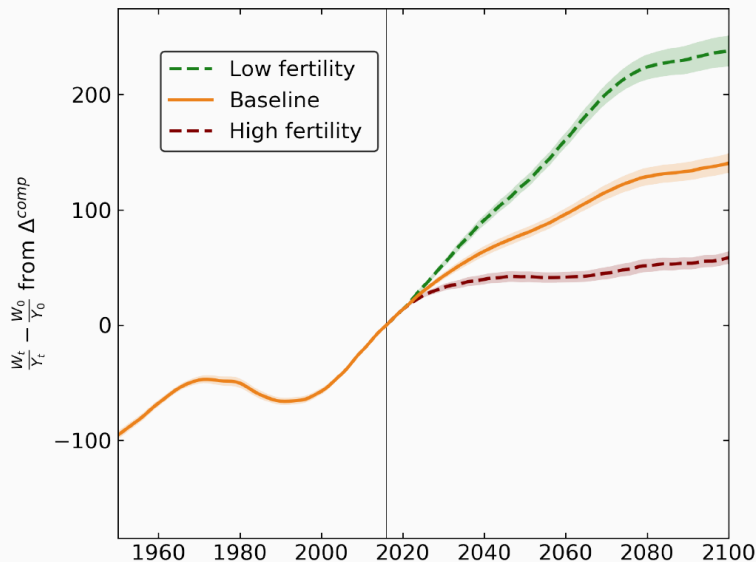
- Data:
 - π_{jt} : projections of age distributions over individuals 2019 UN World Population Prospects
 - a_{j0}, h_{j0} age-wealth and labor income profiles in base year
 - For US: SCF, LIS/CPS, and Sabelhaus-Henriques Volz (2019)
 - a_{j0} includes funded part of DB pensions
 - Household \rightarrow individual (j) by splitting wealth among adults

- Calculate Δ_t^{comp} for 25 countries:

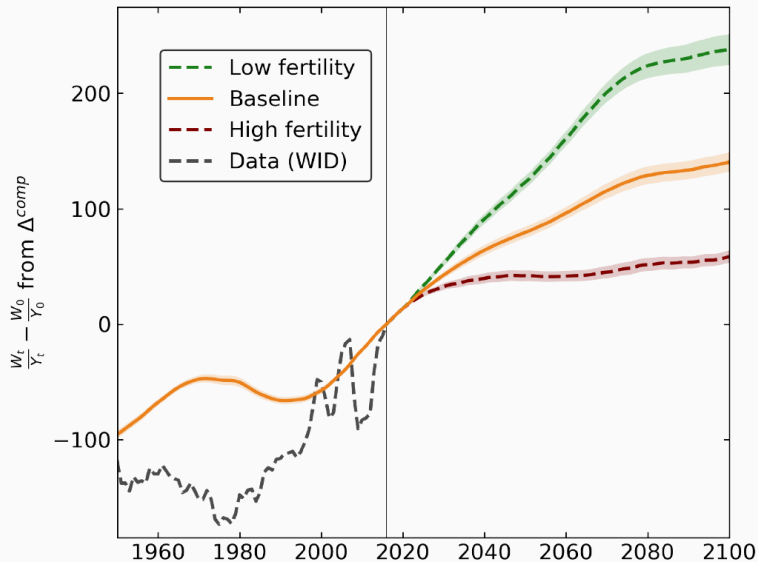
$$\Delta_t^{comp} \equiv \log \left(\frac{\sum \pi_{jt} a_{j0}}{\sum \pi_{jt} h_{j0}} \right) - \log \left(\frac{\sum \pi_{j0} a_{j0}}{\sum \pi_{j0} h_{j0}} \right)$$

- Data:
 - π_{jt} : projections of age distributions over individuals 2019 UN World Population Prospects
 - a_{j0}, h_{j0} age-wealth and labor income profiles in base year
 - For US: SCF, LIS/CPS, and Sabelhaus-Henriques Volz (2019)
 - a_{j0} includes funded part of DB pensions
 - Household \rightarrow individual (j) by splitting wealth among adults
- Report implied level change $\frac{W_t}{Y_t} - \frac{W_0}{Y_0} = \frac{W_0}{Y_0} (\exp \{ \Delta_t^{comp} \} - 1)$

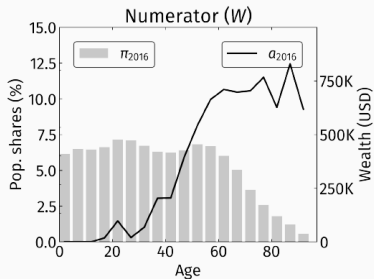
Δ^{comp} in the United States: 1950-2100



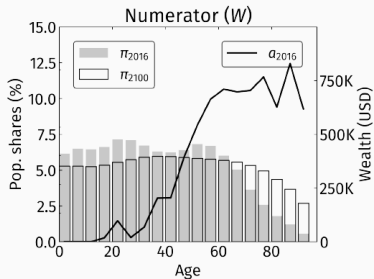
Δ^{comp} in the United States: 1950-2100



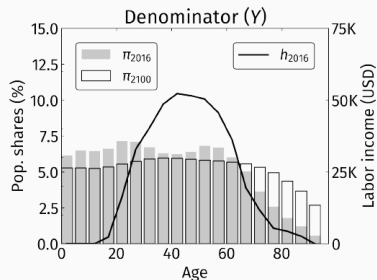
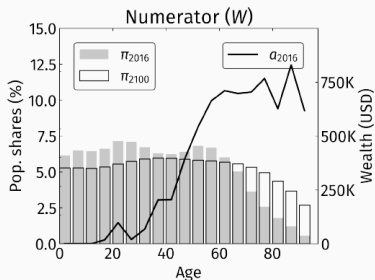
Where do these large effects come from?



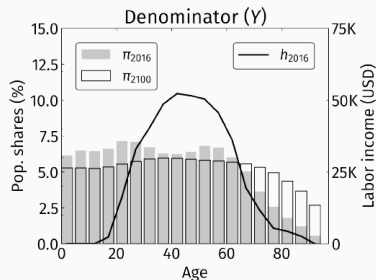
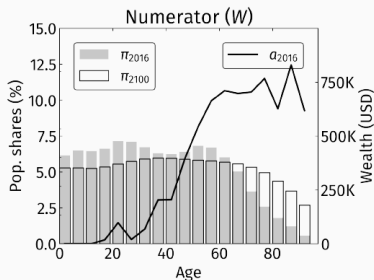
Where do these large effects come from?



Where do these large effects come from?

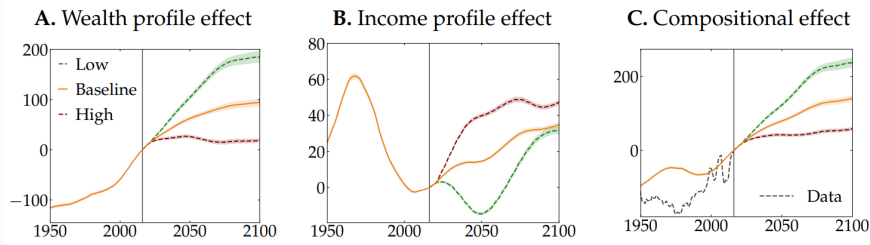


Where do these large effects come from?



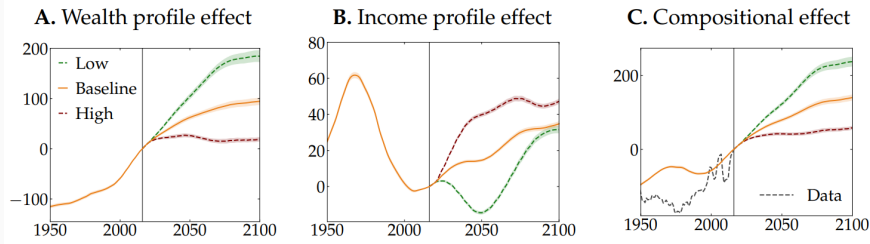
- In paper: separate contribution of numerator (wealth) and denominator (income)
 - Going forward: W contributes $\sim 2/3$, Y contributes $\sim 1/3$
 - Historically demographic dividend pushed Y up, reversed in 2010

Where do these large effects come from?



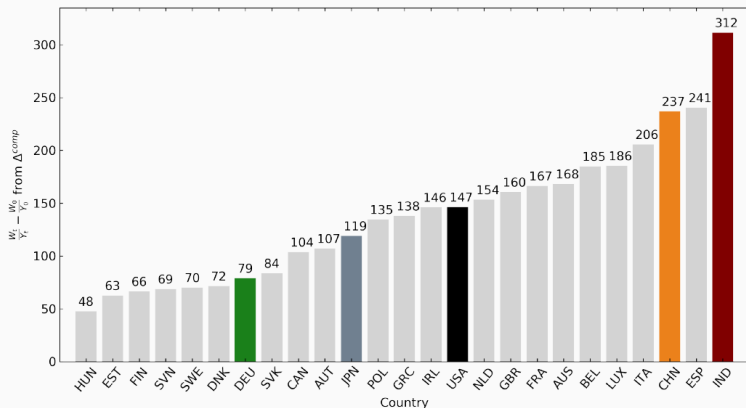
- Historically “demographic dividend” pushed up Y , as a larger share of households were at peak working age

Where do these large effects come from?



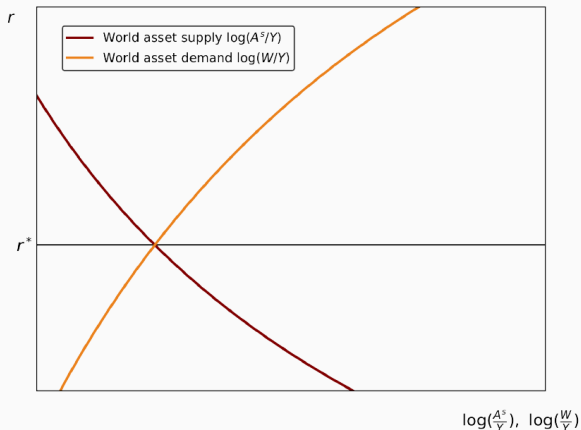
- Historically “demographic dividend” pushed up Y , as a larger share of households were at peak working age
- But this effect has been less pronounced recently, as elderly households earn less

Across countries, Δ^{comp} large and heterogeneous by 2010



General equilibrium implications

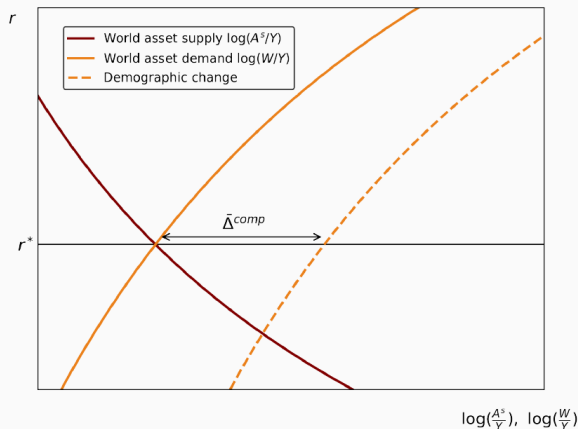
General equilibrium implications



Semielasticity of asset demand $\bar{\epsilon}_d$: depends on σ and observables

Semielasticity of asset supply $\bar{\epsilon}_s$: depends on η and observables

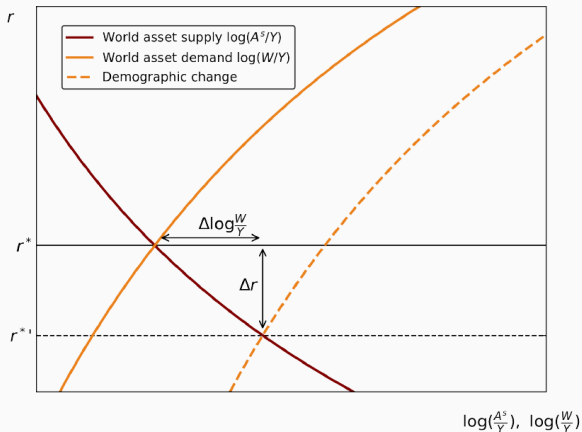
General equilibrium implications



Asset demand shift of $\bar{\Delta}^{comp}$: wealth-weighted average of $\Delta^{comp, c}$

Large and positive in the data.

General equilibrium implications



Proposition 2

If the age profiles of assets and consumption are constant, net foreign assets are zero, and governments maintain constant debt-to-GDP ratios, then the long run change in the rate of return is:

$$\Delta r \approx -\frac{\bar{\Delta}^{\text{comp}}}{\bar{\epsilon}_S + \bar{\epsilon}_d}$$

where $\bar{\epsilon}_S$ is the average semielasticity of asset supply to r , and $\bar{\epsilon}_d$ is the average semielasticity of asset holdings to r , and $\bar{\Delta}^{\text{comp}}$ is the average compositional change.

What determines the asset demand semielasticity?

$$\epsilon^d = \underbrace{\sigma \frac{C}{(1+g)W} \frac{\text{Var } Age_c}{1+r}}_{\equiv \epsilon_{\text{substitution}}^d} + \underbrace{\frac{\mathbb{E}Age_c - \mathbb{E}Age_a}{1+r}}_{\equiv \epsilon_{\text{income}}^d}$$

- Age_a, Age_c : R.V. showing the share of assets and consumption by age

What determines the asset demand semielasticity?

$$\epsilon^d = \underbrace{\sigma \frac{C}{(1+g)W} \frac{\text{Var } Age_c}{1+r}}_{\equiv \epsilon_{\text{substitution}}^d} + \underbrace{\frac{\mathbb{E}Age_c - \mathbb{E}Age_a}{1+r}}_{\equiv \epsilon_{\text{income}}^d}$$

- Age_a, Age_c : R.V. showing the share of assets and consumption by age
- The substitution effect:
 - Proportional to $\text{Var } Age_c$ since there is more scope for intertemporal substitution if consumption is more spread out over the life cycle

What determines the asset demand semielasticity?

$$\epsilon^d = \underbrace{\sigma \frac{C}{(1+g)W} \frac{\text{Var } Age_c}{1+r}}_{\equiv \epsilon_{\text{substitution}}^d} + \underbrace{\frac{\mathbb{E}Age_c - \mathbb{E}Age_a}{1+r}}_{\equiv \epsilon_{\text{income}}^d}$$

- Age_a , Age_c : R.V. showing the share of assets and consumption by age
- The substitution effect:
 - Proportional to $\text{Var } Age_c$ since there is more scope for intertemporal substitution if consumption is more spread out over the life cycle
- The income effect:
 - Reflects the fact that a higher r increases total income, if $\mathbb{E}Age_a < \mathbb{E}Age_c$ (i.e. the extra interest income is saved before it is consumed)

What determines the asset demand semielasticity?

$$\epsilon^d = \underbrace{\sigma \frac{C}{(1+g)W} \frac{\text{Var } Age_c}{1+r}}_{\equiv \epsilon_{\text{substitution}}^d} + \underbrace{\frac{\mathbb{E}Age_c - \mathbb{E}Age_a}{1+r}}_{\equiv \epsilon_{\text{income}}^d}$$

- Age_a, Age_c : R.V. showing the share of assets and consumption by age
- The substitution effect:
 - Proportional to $\text{Var } Age_c$ since there is more scope for intertemporal substitution if consumption is more spread out over the life cycle
- The income effect:
 - Reflects the fact that a higher r increases total income, if $\mathbb{E}Age_a < \mathbb{E}Age_c$ (i.e. the extra interest income is saved before it is consumed)
- The above can be measured assuming fixed Age_a and Age_c
 - The authors find $\epsilon_{\text{substitution}}^d = 39.5$, $\epsilon_{\text{income}}^d = -2$, thus $\epsilon^d > 0$

What determines the asset supply semielasticity?

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \frac{\bar{K}_0}{\bar{W}_0}$$

- η is the elasticity of substitution between capital and labor

What determines the asset supply semielasticity?

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \frac{\bar{K}_0}{\bar{W}_0}$$

- η is the elasticity of substitution between capital and labor
- $r_0 + \delta = 9.7\%$ is the user cost of capital

What determines the asset supply semielasticity?

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \frac{\bar{K}_0}{\bar{W}_0}$$

- η is the elasticity of substitution between capital and labor
- $r_0 + \delta = 9.7\%$ is the user cost of capital
- $\frac{\bar{K}_0}{\bar{W}_0} = 0.78$ is the initial global capital-wealth ratio

What determines the asset supply semielasticity?

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \frac{\bar{K}_0}{\bar{W}_0}$$

- η is the elasticity of substitution between capital and labor
- $r_0 + \delta = 9.7\%$ is the user cost of capital
- $\frac{\bar{K}_0}{\bar{W}_0} = 0.78$ is the initial global capital-wealth ratio
- Based on the above calibration, $\bar{\epsilon}^s > 0$ for any plausible η .

Change in world interest rate

Since $\bar{\epsilon}_S + \bar{\epsilon}_d > 0$, then the change in the world interest rate must be negative:

$$\Delta r \approx -\frac{\bar{\Delta}_{\text{comp}}}{\bar{\epsilon}_S + \bar{\epsilon}_d} < 0$$

With different assumptions on the elasticity of intertemporal substitution (σ) and the elasticity of substitution between capital and labor (η), this gives:

η	σ		
	0.25	0.50	1.00
0.60	-3.24	-1.59	-0.79
1.00	-2.09	-1.25	-0.70
1.25	-1.71	-1.10	-0.65

Change in capital to income ratio

Proposition 2 gives a similar formula for the change in capital to income:

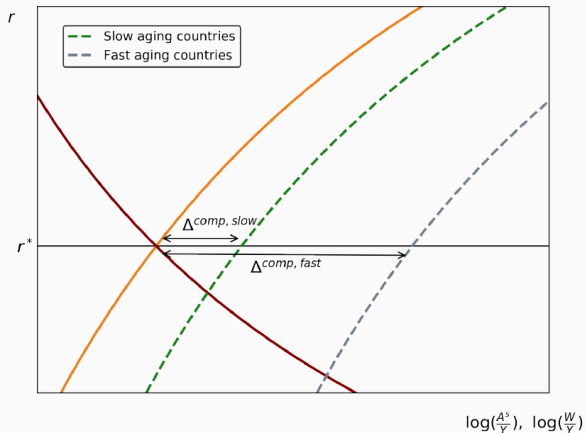
$$\overline{\Delta \log \left(\frac{W}{Y} \right)} \approx \frac{\bar{\epsilon}_S}{\bar{\epsilon}_S + \bar{\epsilon}_d} \bar{\Delta}^{\text{comp}} > 0$$

Again with different assumptions on the IES (σ) and the elasticity of substitution between capital and labor (η)

η	σ		
	0.25	0.50	1.00
0.60	15.6	7.7	3.8
1.00	16.7	10.0	5.6
1.25	17.1	11.1	6.5

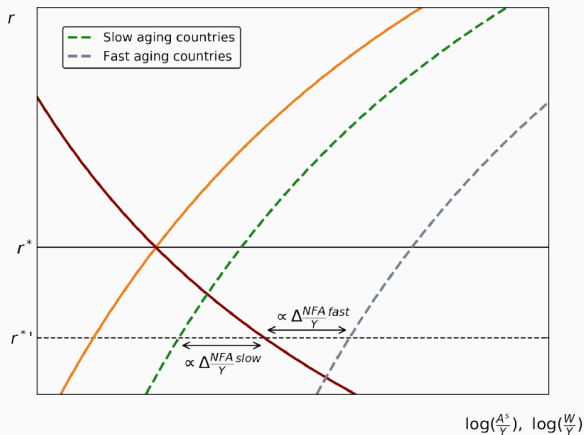
The authors argue that simulations from the general model deliver similar outcomes

Change in net foreign assets



Country specific Δ^{comp} large and heterogeneous in the data

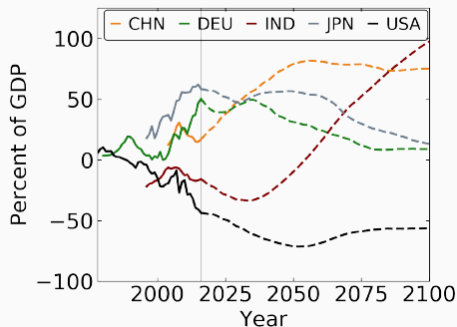
Change in net foreign assets



$$\Delta \left(\frac{NFA}{Y} \right) \approx \frac{W_0}{Y_0} (\Delta^{\text{comp}} - \bar{\Delta}^{\text{comp}})$$

Change in net foreign assets

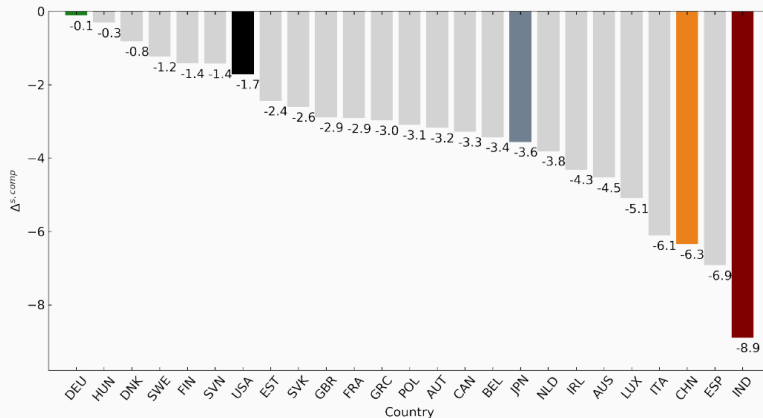
$$\Delta \left(\frac{NFA}{Y} \right) \approx \frac{W_0}{Y_0} (\Delta^{\text{comp}} - \bar{\Delta}^{\text{comp}})$$



→ Data suggest large global imbalances going forward

Change in savings rate

- Perform same exercise as above, but project S/Y from composition



Richer model

Limitation to baseline model

- What are some limitations of their baseline analysis?

Limitation to baseline model

- What are some limitations of their baseline analysis?
 - Demographics have no effect on individual savings

Limitation to baseline model

- What are some limitations of their baseline analysis?
 - Demographics have no effect on individual savings
 - Demographics have no effect on the tax-and-transfer system

Limitation to baseline model

- What are some limitations of their baseline analysis?
 - Demographics have no effect on individual savings
 - Demographics have no effect on the tax-and-transfer system
 - No bequest motives

Limitation to baseline model

- What are some limitations of their baseline analysis?
 - Demographics have no effect on individual savings
 - Demographics have no effect on the tax-and-transfer system
 - No bequest motives
 - No changes in mortality (only birth rates)

Limitation to baseline model

- What are some limitations of their baseline analysis?
 - Demographics have no effect on individual savings
 - Demographics have no effect on the tax-and-transfer system
 - No bequest motives
 - No changes in mortality (only birth rates)
 - Demographics have no effect on TFP growth

Limitation to baseline model

- What are some limitations of their baseline analysis?
 - Demographics have no effect on individual savings
 - Demographics have no effect on the tax-and-transfer system
 - No bequest motives
 - No changes in mortality (only birth rates)
 - Demographics have no effect on TFP growth
- To study some of these changes, the authors extend their baseline model → then simulate the transition path using the techniques we've learned in class

Results from richer model

- Main finding: Δ^{comp} in the richer model is roughly similar to the results from the data

Country	$\Delta^{comp,c}$	
	Model	Data
AUS	30	29
CAN	21	20
CHN	47	45
DEU	21	20
ESP	42	37
FRA	31	30
GBR	27	26
IND	65	56
ITA	34	30
JPN	24	22
NLD	34	33
USA	32	29

Results from richer model

- GE Effects from the model are also roughly similar

	Δr	$\overline{\Delta \log \frac{W}{Y}}$	$\bar{\Delta}^{comp}$	$\bar{\Delta}^{soe}$	$\bar{\epsilon}^d$	$\bar{\epsilon}^s$
Sufficient statistic analysis	-1.23	9.9	31.8		17.8	8.0
Preferred model specification	-1.23	10.3	34.1	30.3	17.1	8.0
<i>Alternative model specifications</i>						
+ Constant bequests	-1.18	10.0	34.1	27.0	14.9	8.0
+ Constant mortality	-1.23	10.9	34.1	27.1	13.8	8.0
+ Constant taxes and transfers	-1.33	11.9	34.1	30.1	14.5	8.0
+ Constant retirement age	-1.49	13.4	34.1	34.1	14.6	8.0
+ No income risk	-1.47	13.2	33.9	33.9	13.8	8.0
+ Annuities	-1.33	11.5	34.2	34.2	17.2	8.0
<i>Alternative fiscal rules</i>						
Only lower expenditures	-1.29	11.0	34.1	32.6	17.9	8.0
Only higher taxes	-0.88	6.7	34.1	19.4	14.6	8.0
Only lower benefits	-1.50	12.9	34.1	39.1	18.4	8.0

Notes: Δr , $\overline{\Delta \log \frac{W}{Y}}$, $\bar{\Delta}^{comp}$, and $\bar{\Delta}^{soe}$ denote the changes in the model simulation between 2016 and 2100, with Δr reported in percentage points and the other three reported in percent ($100 \cdot \log$).

Conclusion

- How does population aging affect wealth-output ratios, real interest rates, and capital flows?
 - what matters is the compositional effect Δ^{comp}
 - large and heterogeneous in the data

Conclusion

- How does population aging affect wealth-output ratios, real interest rates, and capital flows?
 - what matters is the compositional effect Δ^{comp}
 - large and heterogeneous in the data
- The approach developed by the authors:
 - Refutes the asset market meltdown hypothesis: r falls
 - Suggests the global savings glut has just begun

Summary

Summary and next week

- **Previously:** Learned to study transitional dynamics in an Aiyagari economy

Summary and next week

- **Previously:** Learned to study transitional dynamics in an Aiyagari economy
- **Today:** Saw an application to global interest rates and financial flows
 - Rather than explicitly simulating the economy, the authors use a sufficient statistics approach
 - This helps give intuition. Then simulate the full model to test their assumptions

Summary and next week

- **Previously:** Learned to study transitional dynamics in an Aiyagari economy
- **Today:** Saw an application to global interest rates and financial flows
 - Rather than explicitly simulating the economy, the authors use a sufficient statistics approach
 - This helps give intuition. Then simulate the full model to test their assumptions
- **Next lecture:** Learn about the baseline HANK model with Jeppe