Computational details of *The Transmission of Foreign Demand Shocks*

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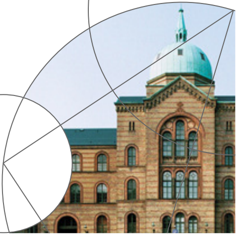

Overview

- This part of the lecture will cover how to solve the model in Druedahl et al. (2022)
- Nothing fundamentally new:
  - Solve the household problem using EGM
  - Utilize the fake-news method to get Jacobian of HH problem
  - ... and sequence-space formulation to get GE Jacobians (Auclert et al. (2019))
  - ... and solve for full non-linear transition path in response to shocks using Broyden's method
- But still some model elements you might not have seen:
  - Open economy
  - Multiple sectors
  - Multiple countries
  - Different household blocks across model versions

Overview

- The main body of the paper concerns a small open economy (SOE) HANK model that trades with a foreign, large open economy
- Blocks in domestic HANK model:
  - Prices (UIP and law-of-one-price) & Exports
  - Firm block
    - Production function, labor demand, IO, profits, Philip-curves etc.
    - ... $\times 2$ since we have 2 sectors
  - Asset pricing (essentially one arbitrage condition)
  - Public sectors (Taxes and bonds)
  - Monetary policy (floating or fixed exchange rate)
  - Households

Code for IHANK model class

Steady state

- Calibrate steady state to match a bunch of empirical targets
- Some moments we can directly impose in the steady state (markup etc.) but ultimately we need a root finder to match some targets using parameters:
    - Goods market clearing in both sectors (solving steady state)
    - **wealth/income ratio** (χ) (calibration)
    - $\text{Profits}_S/\text{Profits} = \text{GDP}_S/\text{GDP}$ (calibration)
    - Household Imports/GDP ratio (calibration)
- Solve by guessing on sectoral production, sectoral fixed costs, and firm imports shares to match these targets/residuals

Steady state - two step solution procedure

- Note that because we calibrate to $\frac{A}{\mathcal{Y}^{HH}}$ target χ we can solve the steady state without solving the household block:

$$C + A = (1 + r^a)A + \mathcal{Y}^{HH}$$
$$\Leftrightarrow C = (1 + r^a \times \chi)\,\mathcal{Y}^{HH}$$

- and $A = \chi\mathcal{Y}^{HH}$
- Implies a **two step** calibration procedure:
  1. Calibrate steady state independently of household block conditional on $A/GDP = \chi$ (fast step)
  2. Calibrate chosen household block (HA/RA) to match $A/GDP = \chi$ and other targets (MPCs etc.) (potentially slow)

Code for steady-state calibration I

HA Household problem

- The household problem:

$$V_t^{s,k}(e_t, a_{t-1}) = \max_{c_t, a_t} u\left(c_t\right) - \nu\left(n_t\right) + \beta_t^k \mathbb{E}_t\left[V_{t+1}^{s,k}(e_{t+1}, a_t)\right] \tag{1}$$

s.t.

$$c_t + a_t = \left(1 + r_t^a\right) a_{t-1} + w_{s,t} n_{s,t} e_t + T_t - \tau\left(\tau_t, e_t\right), \tag{2}$$

$$\ln e_t = \rho_e \ln e_{t-1} + \epsilon_t^e, \quad \epsilon_t^e \sim \mathcal{N}\left(0, \sigma_e^2\right), \tag{3}$$

$$a_t \geq 0, \tag{4}$$

- with 4 states:
    - Assets a_{t-1} (endogenous, 300 grid points)
    - Earnings e_t (exogenous, 6 grid points)
    - Discount factor β^k (exogenous, 3 grid points)
    - Sector: s (exogenous, 2 states)

Code for HA block

RA Household problem

- In the RA model C, A are characterized by the budget constraint and Euler equation:

$$c_t + a_t = (1 + r_t^a)\, a_{t-1} + w_t n_t + T_t - \tau \tag{5}$$

$$u'(C_t) = \beta \left(1 + r_{t+1}^a\right) u'(C_{t+1}) \tag{6}$$

Household calibration

- In the HA model:
  - Calibrate average β to match wealth-income ratio χ
  - Calibrate dispersion/variance of β to match $MPC = 0.55$
  - Note: If we only wanted to match the aggregate MPC we could do without β-heterogeneity
- In the RA model:
  - We know C, A from step one of the calibration
  - Set $\beta = \frac{1}{1+r^a}$ to satisfy the Euler equation and we're done

Code for steady-state calibration II

Solving for transition path

- We have the steady state
- Now: Interested in solving for the **transition path** to exogenous shock (i.e. foreign demand shock)
- We utilize the standard methods available in **GEModelTools**
  - Calculate Jacobian of HA-block using the Fake-news algorithm from Auclert et al. (2019)
  - Calculate Jacobian of a set of **targets** to **unknowns** using the household Jacobian when evaluating consumption/savings etc.
  - Supply Broyden solver with Jacobian to solve for full non-linear transition path

Solving for transition path

- Write the transition path as a DAG that maps **unknowns** to **targets**
- Unknowns and targets (i.e. model residuals):

$$\text{Targets} = \begin{bmatrix} \text{Goods market T} \\ \text{Goods market NT} \\ \text{NKWPC T} \\ \text{NKWPC NT} \\ \text{Monetary policy} \\ \text{Government budget} \end{bmatrix} \qquad \text{Unknowns} = \begin{bmatrix} Z_T \\ Z_{NT} \\ \pi_F \\ \pi \\ \pi_H^* \\ B \end{bmatrix}$$

Model solution - DAG

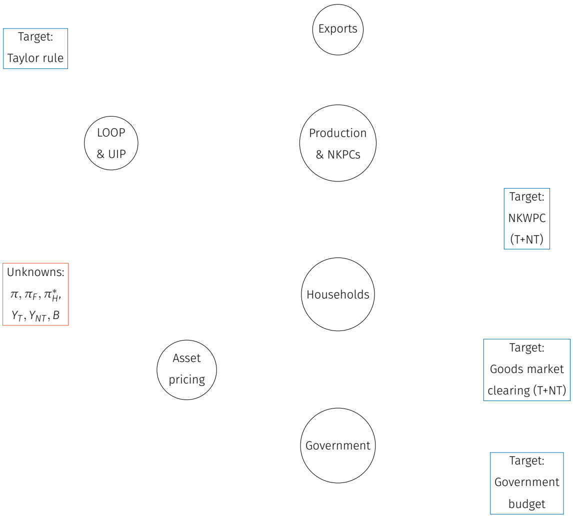

Figure 1: DAG of the main model.

Model solution - DAG

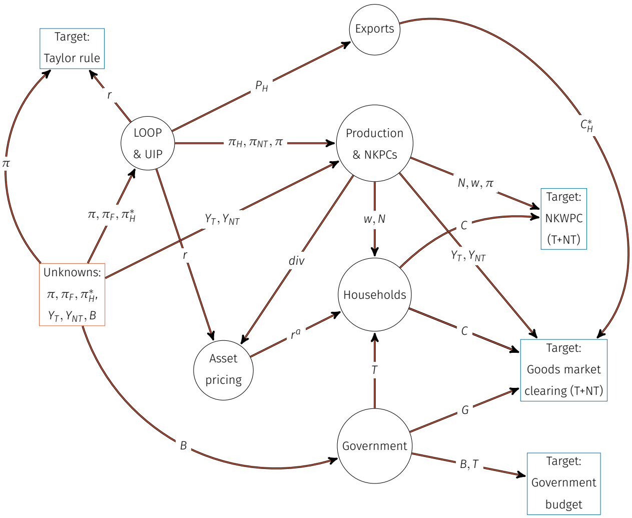

Figure 2: DAG of the main model.

Model solution with RANK

- With a representative agent the model is not **stationary** in response to shocks (Schmitt-Grohé and Uribe (2003)) because there is a unit root in the Euler equation $\Rightarrow$ Temporary shocks have permanent effects
- Can be seen clearly by considering the Euler equation in steady state: where $r^a_{t+1} = r^*_{ss}$ and $C_t = C_{t+1}$:

$$u'(C_t) = \beta\left(1 + r^a_{t+1}\right) u'(C_{t+1})$$
$$\Rightarrow 1 = \beta\left(1 + r^*_{ss}\right)$$

- Everything here is **exogenous** to the SOE $\Rightarrow$ we are missing one equation to pin A, C in the long run!

Model solution with RANK - inducing stationarity

- One ad-hoc solution from Schmitt-Grohé and Uribe (2003): Add risk premium in UIP:

$$1 + r_t = (1 + r_t^*) \frac{Q_{t+1}}{Q_t} \Gamma_t$$
$$\Gamma_t \equiv \exp\left\{ -\varepsilon^D \left(\frac{Q_t B_t^*}{GDP_{ss}} - \frac{Q_{ss} B_{ss}^*}{GDP_{ss}} \right) \right\}$$

- If you accumulate a lot foreign bonds $\frac{Q_t B_t^*}{GDP_{ss}} > \frac{Q_{ss} B_{ss}^*}{GDP_{ss}}$ then the return r decrease and HHs consume more and save less, thus driving down the stock of foreign bonds

Stationarity with HANK

- No stationary issues in HANK version of model
    - No unit root in Euler equation:

$$u'(c) = \beta(1 + r^*_{ss})\,\mathbb{E}\,u'(c)$$

    - since $u'(c) \neq \mathbb{E}\,u'(c)$
- The presence of a precautionary savings motive generates a wealth-target (Carroll, Hall, and Zeldes (1992))

Code for transition path

Foreign demand shocks

- The foreign demand shock we consider is the GE outcome of a canonical NK model:

$$\pi_t^* = \kappa\left(mc_t^* - \frac{1}{\mu^*}\right) + \frac{1}{1+r_{t+1}^*}\pi_{t+1}^*$$

$$u'\left(C_t^*\right) = \beta^*\left(1+r_{t+1}^*\right)u'\left(C_{t+1}^*\right)$$

$$r_t^* = r^* + \phi^*\pi_t^*$$

- We solve this as a separate model in GEModelTools
- Gives us responses for π^*, C^*, r^* to a β^* shock
- We then feed π^*, C^*, r^* into the SOE HANK model simultaneously to simulate the effects of a demand shock in the foreign economy

Code for foreign demand shock

References

Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub. 2019. *Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models.* Technical report w26123. Cambridge, MA: National Bureau of Economic Research, July. Accessed September 2, 2021. http://www.nber.org/papers/w26123.pdf.

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Schmitt-Grohé, Stephanie, and Martın Uribe. 2003. "Closing small open economy models." *Journal of international Economics* 61 (1): 163–185.