The Unemployment-Risk Channel in Business-Cycle Fluctuations

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December 2022

Abstract

This note describes the updated HANK-SAM model with positive liquidity.

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1 Model

Time is discrete and indexed by $t \in \{0, 1, \dots\}$.

1.1 Demographics

The economy is inhabited by infinitely-lived workers indexed by $i \in [0, 1]$, infinitely-lived capitalists, a government and a central bank.

1. The workers supply labor exogenously and receive wages when working and unemployment insurance when unemployed. The workers choose consumption and can save in government bonds. Borrowing is not allowed.
2. The capitalists own all firms and consume the profits period-by-period.
3. The government finances the unemployment insurance with taxes and debt.
4. The central bank sets the nominal interest rate following a Taylor rule.

1.2 Production structure

Production has three layers:

1. Intermediate-good producers hire labor in a frictional labor market with search and matching frictions. Matches produce a homogeneous good sold in a perfectly competitive market.
2. Wholesale firms buy intermediate goods and produce differentiated goods that they sell in a market with monopolistic competition. The wholesale firms set their prices subject to a Rotemberg adjustment cost.
3. Final-good firms buy goods from wholesale firms and bundle them in a final good, which is sold in a perfectly competitive market.

1.3 Timing and labor-market dynamics

Step 0: Stocks and productivity. At the beginning of each period t, all aggregate shocks are revealed. The endogenous state variables are the (beginning-of-period) stocks of unemployed workers u_{t-1} and of vacancies v_{t-1}.

Step 1: Separations and entry. Firms are exposed to an idiosyncratic continuation cost shock. After observing the shock they decide whether to continue or exit, which implies an endogenous, time-varying separation rate δ_t in a manner that we describe below. Vacancies are destroyed with rate δ_{ss}, which for simplicity we assume to be constant and exogenous, and have the same value as the steady state separation rate. Firm-specific costs of entering the labor market are realized. Firms that pay the cost post a new vacancy. The endogenous, time-varying vacancy entry rate is denoted ι_t. The resulting stocks of unemployment and vacancies are given by

$$\tilde{u}_t = u_{t-1} + \delta_t(1 - u_{t-1}), \tag{1}$$

$$\tilde{v}_t = (1 - \delta_{ss})v_{t-1} + \iota_t. \tag{2}$$

Step 2: Search and match. Unemployed workers and vacancies randomly match. The matching technology is Cobb-Douglas with matching elasticity α. Denoting market tightness by

$$\theta_t = \frac{\tilde{v}_t}{\tilde{u}_t}, \tag{3}$$

the job-filling rate λ_t^v and job-finding rate λ_t^u are

$$\lambda_t^v = A\theta_t^{-\alpha}, \tag{4}$$

$$\lambda_t^u = A\theta_t^{1-\alpha}. \tag{5}$$

The labor-market stocks after matches are formed are

$$u_t = (1 - \lambda_t^u)\tilde{u}_t, \tag{6}$$

$$v_t = (1 - \lambda_t^v)\tilde{v}_t. \tag{7}$$

Step 3: Production. Production takes place. Wages and profits are paid out.

Step 4: Consumption and saving. All capitalists and workers, both employed and unemployed, make their consumption-and-saving decisions.

1.4 Intermediate-good firms, vacancy creation and job separations

There is a continuum of intermediate-good firms producing a homogeneous good X_t sold in a competitive market. The real price of the intermediate good is P_t^x and one unit of labor produces Z_t units of the intermediate good. The total production of intermediate goods is thus given by

$$X_t = Z_t(1 - u_t), \tag{8}$$

where the log of total factor productivity Z_t is subject to AR(1)-innovations ν_t^Z,

$$Z_t = Z_{ss}\nu_t^Z, \tag{9}$$

$$\log \nu_t^Z = \rho_A \log \nu_{t-1}^Z + \epsilon_t^Z, \tag{10}$$

where σ_Z is the standard deviation of ϵ_t^Z.

To hire labor the firms must post vacancies which are filled with probability λ_t^v, taken as given by each one-worker firm. We denote by V_t^v the value of a vacancy and by V_t^j the value of a match for the firm.

Separations. At the beginning of the period, a firm must pay a continuation cost $\chi_t \sim G$ or else the job match is destroyed.[1] There is no additional heterogeneity and consequently there exists a common cost cutoff $\chi_{c,t} = V_t^j$, such that for all $\chi_t > \chi_{c,t}$, the firm chooses to separate. Accordingly, the Bellman equation for the value of a job after the separation decision is

$$\begin{aligned} V_t^j &= p_t^x Z_t - (w_t - \text{wage subsidy}_t) + \beta \mathbb{E}_t \left[\int^{\chi_{c,t+1}} (V_{t+1}^j - \chi_{t+1}) dG(\chi_{t+1}) \right] \\ &= p_t^x Z_t - (w_t - \text{wage subsidy}_t) + \beta \mathbb{E}_t \left[(1 - \delta_{t+1}) V_{t+1}^j - \mu_{t+1} \right], \end{aligned} \tag{11}$$

where w_t is the real wage, δ_{t+1} is the endogenous separation probability given by $\delta_{t+1} = \int_{V_t^j}^{\infty} G(\chi_t) d(\chi_t)$, and μ_{t+1} is the average continuation cost paid.

[1] Following Mortensen and Pissarides (1994), separation decisions are typically modeled as a result of idiosyncratic productivity shocks, such that low-productivity firms optimally decide to exit. Our simplified assumptions have similar material consequences, but avoid ex-post heterogeneity in firm outcomes.

The continuation-cost distribution G is a mixture of a point mass and a Pareto distribution with shape parameter ψ, location parameter Υ and mixture parameter p. We choose p and Υ so that in steady state, job separations are δ_{ss} and the continuation costs are small. See Appendix A for details. Out of steady state, the endogenous separation probability δ_t are then given by

$$\delta_t = \delta_{ss} \left(\frac{V_t^j}{V_{ss}^j} \right)^{-\psi}, \tag{12}$$

and the average continuation cost, μ_t, is a non-negative increasing function of the job value

$$\mu_t = \mu(V_t^j), \;\; \mu(\bullet) \geq 0, \mu'(\bullet) \geq 0. \tag{13}$$

The idiosyncratic continuation cost implies that the elasticity of job separations to the value of a job is ψ. In the special case where $\psi = 0$ separations occur exogenously at rate δ_{ss}.

Vacancy creation. The Bellman equation for the value of a vacancy is given by

$$V_t^v = -\kappa + \lambda_t^v (V_t^j + \text{hiring subsidy}_t) + (1 - \lambda_t^v)(1 - \delta_{ss})\beta \mathbb{E}_t[V_{t+1}^v], \tag{14}$$

where κ is the flow cost of the vacancy, to be paid every period, and . Vacancies are not subject to the stochastic continuation cost, and are instead destroyed with exogenous probability δ_{ss}. In contrast to the standard assumption of free entry to vacancy creation, we assume that there is a constant mass F of prospective firms drawing a stochastic idiosyncratic entry cost c following a distribution H. The prospective firm posts a vacancy if and only if the value of a vacancy is larger than the entry cost. The total number of vacancies created is therefore $\iota_t = F \cdot H(V_t^v)$. Following Coles and Kelishomi (2018), the entry-cost distribution has a cumulative distribution function $H(c) = F \cdot (c/h)^{\xi}$ on $c \in [0, h]$. With the parameter h sufficiently large so that $h > V_t^v$, the resulting number of vacancies created is $\iota_t = F \cdot (V_t^v)^{\xi}$. Expressing vacancy creation in relation to steady state gives us

$$\iota_t = \iota_{ss} \left(\frac{V_t^v}{V_{ss}^v} \right)^{\xi}. \tag{15}$$

The stochastic-cost entry assumption implies that the elasticity of vacancy creation to the value of a vacancy is ξ. In the limit where $\xi \to \infty$, we must have $V_t^v = V_{ss}^v$ so that all entrants pay the same deterministic entry cost. We set $V_{ss}^v = \kappa_0$ and treat κ_0 as a free parameter. The free entry model is the double limit $\xi \to \infty$ and $\kappa_0 \to 0$, which implies $V_t^v = 0$. To facilitate comparisons with the free entry model we fix κ at a small positive value across all calibrations, $\kappa_0 = 0.1$.

Wage setting. We assume the real wage is determined by a fixed rule,

$$w_t = \left(\frac{u_t}{u_{ss}}\right)^{-\eta_u}. \tag{16}$$

In the baseline model, we follow Hall (2005) and set $\eta_u = w$ so the real wage is constant. A recent body of research has documented that downward nominal wage rigidity is pervasive in the US labor market (Dupraz et al., 2021; Grigsby et al., 2021; Hazell and Taska, 2022). A fixed real wage is therefore likely a weak assumption in the context of studying contractionary shocks, as it implies more wage flexibility than a fully rigid nominal wage with pro-cyclical inflation.

1.5 The final-good sector and the wholesale sector

The representative final-good firm has the production function $Y_t = \left(\int Y_{kt}^{\frac{\epsilon_p - 1}{\epsilon_p}} dk\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ where Y_{kt} is the quantity of the input of wholesale firm k's output used in production. The implied demand curve is $Y_{kt} = \left(\frac{P_{kt}}{P_t}\right)^{-\epsilon_p} Y_t$ where $P_t = \left(\int P_{kt}^{1-\epsilon_p} dk\right)^{\frac{1}{1-\epsilon_p}}$ is the aggregate price level. There is a continuum of wholesale firms indexed by $k \in [0,1]$ producing differentiated goods using the production function $Y_{kt} = X_{kt}$ where X_{kt} is the amount of the intermediate good purchased by firm k at the intermediate-good price P_t^X. The wholesale firms face Rotemberg price adjustment costs, with scale factor ϕ. Since production is linear, the marginal cost of production is the input price P_t^X. In a symmetric equilibrium, optimal price setting implies a standard Rotemberg Phillips curve

$$1 - \epsilon_p + \epsilon_p \cdot p_t^x = \varphi(\Pi_t - 1)\Pi_t - \beta\varphi\mathbb{E}_t\left[(\Pi_{t+1} - \Pi_{ss})\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right], \tag{17}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate, and total output is given by

$$Y_t = X_t = Z_t(1 - u_t). \tag{18}$$

1.6 Workers

The workers are *ex ante* heterogeneous in terms of discount factors, β_i, and *ex post* heterogeneous in terms of months in unemployment, u_{it}, and lagged savings, a_{it-1}. The employed have zero months in unemployment. Income, y_t, is determined by

$$y_t = \begin{cases} w_t & \text{if } u_{it} = 0 \\ \mathrm{UI}_{it}\overline{\phi}_t w_t + (1 - \mathrm{UI}_{it})\underline{\phi} w_t & \text{else} \end{cases} \tag{19}$$

$$\mathrm{UI}_{it} = \begin{cases} 1 & \text{if } u_{it} \leq \overline{u}_t \\ u_{it} - \overline{u}_t & \text{if } u_{it} \in (\overline{u}_t, \overline{u}_t + 1) \\ 0 & \text{if } u_{it} \geq \overline{u}_t + 1, \end{cases} \tag{20}$$

where $\mathrm{UI}_{it} = 1$ denotes high unemployment insurance of $\overline{\phi}_t$, and $\mathrm{UI}_{it} = 0$ denotes low unemployment insurance of $\underline{\phi}$. The maximum duration of unemployment is denoted $\overline{u}_t$ and can take decimal values.

The transition for u_{it} is exogenous, but time-varying, and given by

$$\Pr[u_{it+1}|u_{it} = 0] = \begin{cases} 1 - \delta_t(1 - \lambda_t^u) & \text{if } u_{it+1} = 0 \\ \delta_t(1 - \lambda_t^u)\pi^{\mathrm{UI}} & \text{if } u_{it+1} = 1 \\ \delta_t(1 - \lambda_t^u)(1 - \pi^{\mathrm{UI}}) & \text{if } u_{it+1} = \#_u \\ 0 & \text{else} \end{cases} \tag{21}$$

$$\Pr[u_{it+1}|u_{it} > 0] = \begin{cases} \lambda_t^u & \text{if } u_{it+1} = 0 \\ 1 - \lambda_t^u & \text{if } u_{it+1} = \min\{u_{it} + 1, \#_u\} \\ 0 & \text{else,} \end{cases} \tag{22}$$

where $\#_u$ is a technical maximum counted duration state. With probability π^{UI} the worker transition directly to the maximum duration state, and thus only receives low unemployment insurance.

The recursive problem of the workers is,

$$
\begin{aligned}
V_t^w(\beta_i, u_{it}, a_{it-1}) = & \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_i \mathbb{E}_t \left[V_{t+1}^w \left(\beta_i, u_{it+1}, a_{it} \right) \right] \\
\text{s.t.} \qquad a_{it} + c_{it} = & R_t^{\text{real}} a_{it-1} + \text{transfer}_t + (1-\tau_t) y_t \\
a_{it} \geq & \, 0,
\end{aligned}
$$

where R_t^{real} is the real interest rate from $t-1$ to t, and τ_t is the tax rate. The distribution of workers over β_i, u_{it} and a_{it-1} is denote $\boldsymbol{D}_t$. For later reference, aggregate household pre-tax income is

$$
Y_t^{hh} = w_t(1-u_t) + \overline{\phi}_t w_t \text{UI}_t^{hh} + \underline{\phi} w_t \left(u_t - \text{UI}_t^{hh} \right), \tag{23}
$$

where $\text{UI}_t^{hh} = \int \mathbb{1}\{u_{it} > 0\} \text{UI}_{it} d\boldsymbol{D}_t$ is the aggregate share of workers receiving high unemployment insurance.

1.7 Government

The government follows the fiscal tax rule

$$
\tau_t = \tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}^{hh}}, \tag{24}
$$

where ω determines response of taxes to fluctuations in debt level, B_t.

Government debt is long term with persistence $\delta_q \in [0, 1]$, and priced at q_t. This implies the following government budget equation

$$
\begin{aligned}
q_t(B_t - \delta_q B_{t-1}) = & B_{t-1} \\
& + (1-\tau_t) \left(\overline{\phi}_t \text{UI}_t^{hh} + \underline{\phi} \left(u_t - \text{UI}_t^{hh} \right) \right) w_t \\
& - \tau_t (1-u_t) w_t \\
& + \text{wage subsidy}_t \cdot (1-u_t) \\
& + \text{hiring subsidy}_t \cdot \lambda_t^v ((1-\delta_{ss}) v_{t-1} + \iota_t) \\
& + \text{public spending}_t \\
& + \text{public transfer}_t.
\end{aligned} \tag{25}
$$

1.8 Central bank

The central sets monetary policy according to the following Taylor rule,

$$R_t = R_{ss}\Pi_t^{\phi_\pi}, \tag{26}$$

where R_t is the nominal interest rate from period t to period $t+1$. The Fisher equation states,

$$R_t^{\text{real}} = R_{t-1}/\Pi_t, \tag{27}$$

where R_t^{real} is the real return from period $t-1$ to t.

1.9 Equilibrium

Arbitrage implies that the price of government bonds must imply

$$\frac{1+\delta_q q_{t+1}}{q_t} = R_{t+1}^{\text{real}} \tag{28}$$

The bond market must clear

$$q_t B_t = \int a_t^\star(\beta_i, u_{it}, a_{it-1}) d\boldsymbol{D}_t \tag{29}$$

2 Calibration

The calibration method and the steady state is described in Appendix B.

References

Coles, M. G. and Kelishomi, A. M. (2018). Do job destruction shocks matter in the theory of unemployment? *American Economic Journal: Macroeconomics*, 10(3):118–36.

Dupraz, S., Nakamura, E., and Steinsson, J. (2021). A Plucking Model of Business Cycles. NBER Working Paper 26351.

Grigsby, J., Hurst, E., and Yildirmaz, A. (2021). Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data. *American Economic Review*, 111(2):428–71. Publisher: American Economic Association.

Hall, R. E. (2005). Employment Fluctuations with Equilibrium Wage Stickiness. *American Economic Review*, 95(1):50–65.

Hazell, J. and Taska, B. (2022). Downward Rigidity in the Wage for New Hires. Working Paper.

Mortensen, D. T. and Pissarides, C. A. (1994). Job Creation and Job Destruction in the Theory of Unemployment. *The Review of Economic Studies*, 61(3):397–415.

A Separation decision

In Equation (11), we assume that G is a mixture of a point mass at 0 and a Pareto distribution with location parameter $\Upsilon > 0$ and shape parameter ψ,

$$G(\chi_t) = \begin{cases} 0 & \chi_t < 0, \\ 1-p & 0 \le \chi_t < \Upsilon, \\ (1-p) + p(1-(\chi_t/\Upsilon)^{-\psi}) & \chi_t \ge \Upsilon, \end{cases} \tag{30}$$

This implies

$$\begin{aligned} \delta_t &= \int_{V_t^j}^{\infty} G(\chi_t) d(\chi_t) \\ &= \begin{cases} p & \text{if } V_t^j \le \Upsilon \\ p\left(\frac{V_t^j}{\Upsilon}\right)^{-\psi} & \text{else,} \end{cases} \end{aligned} \tag{31}$$

and

$$\begin{aligned} \mu_t &= \int_0^{V_t^j} \chi_t dG(\chi_t) \\ &= \frac{\mathbb{E}[\chi_t] - \text{Prob.}[\chi_t > V_t^j]\mathbb{E}[\chi_t | \chi_t > V_t^j]}{1 - \text{Prob.}[\chi_t > V_t^j]} \\ &= \begin{cases} 0 & \text{if } V_t^j \le \Upsilon \\ \frac{p\frac{\psi\Upsilon}{\psi-1} - p\left(\frac{V_t^j}{\Upsilon}\right)^{-\psi} \frac{\psi V_t^j}{\psi-1}}{(1-p)+p(1-(\chi_t/\Upsilon)^{-\psi})} & \text{else} \end{cases} \\ &= \begin{cases} 0 & \text{if } V_t^j \le \Upsilon \\ \frac{p\frac{\psi}{\psi-1}\Upsilon\left[1-\left(\frac{V_t^j}{\Upsilon}\right)^{1-\psi}\right]}{1-p\left(\frac{V_t^j}{\Upsilon}\right)^{-\psi}} & \text{else} \end{cases} \\ &= \mu(V_t^j). \end{aligned} \tag{32}$$

We always choose $\Upsilon = \left(\frac{\delta_{ss}}{p}\right)^{\frac{1}{\psi}} V_{ss}^j$, which then implies Equation (12) in the main text.

Furthermore, with $p = \delta_{ss}$ we have $Y = V^j_{ss}$ which implies $\delta_t = \delta_{ss}$ when $V^j_t \leq V^j_{ss}$. Instead we set $p = (1 + \Delta_\delta)\delta_{ss}$ where $\Delta_\delta > 0$ is a small positive number. This implies that δ_t can rise above δ_{ss} when V^j_t falls below V^j_{ss}. It also implies that μ_{ss} is a small positive number.

B Steady state

The following parameters and steady state variables are chosen:

1. **SAM-parameters:** β^{firm} , α, ρ_Z, σ_Z, ψ, ξ, η_u
2. **HANK-parameters:** β_i, $\Pr[\beta_i]$, ϵ_p, φ, δ_π, ω, δ_q, π^{UI}
3. **Government:** $\overline{\phi}_{ss}$, $\underline{\phi}$, $\overline{u}_{ss}$
4. **Steady state targets from data:** δ_{ss}, λ^u_{ss}, θ_{ss}
5. **Steady state choices:** w_{ss}, qB_{ss}
6. **Auxiliary parameters:** $\kappa_0 = 0.1$ and $\Delta_\delta = 0.2$

Technology shock, polices and inflation are

$$Z_{ss} = 1 \tag{33}$$

$$\text{wage subsidy}_t = \text{hiring subsidy}_t = \text{publictranfser}_t = \text{public spending}_t = 0 \tag{34}$$

$$\Pi_{ss} = 1, \tag{35}$$

From the matching function, we directly have

$$A = \frac{\lambda^u_{ss}}{\theta^\alpha_{ss}}. \tag{36}$$

This implies that the steady states of labor markets stocks and flows can be found by

$$
\begin{aligned}
\lambda^v_{ss} &= A\theta_{ss}^{-\alpha}, && (37)\\
u_{ss} &= \frac{\delta_{ss}(1-\lambda^u_{ss})}{\lambda^u_{ss}+\delta_{ss}(1-\lambda^u_{ss})}, && (38)\\
\tilde{u}_{ss} &= \frac{u_{ss}}{1-\lambda^u_{ss}}, && (39)\\
\tilde{v}_{ss} &= \tilde{u}_{ss}\theta_{ss}, && (40)\\
v_{ss} &= (1-\lambda^v_{ss})\tilde{v}_{ss}, && (41)\\
\iota_{ss} &= \tilde{v}_{ss}-(1-\delta_{ss})v_{ss}. && (42)
\end{aligned}
$$

From the Philips curve we have

$$
p^x_{ss} = \frac{\epsilon_p - 1}{\epsilon_p}, \tag{43}
$$

and can then infer p, Y, V^j_{ss} and μ_{ss} by

$$
\begin{aligned}
p &= (1+\Delta_\delta)\delta_{ss} && (44)\\
\tilde{Y} &\equiv \left(\frac{\delta_{ss}}{p}\right)^{\frac{1}{\psi}} && (45)\\
\tilde{\mu} &\equiv \frac{p\tilde{Y}^{-1}}{\frac{\psi}{\psi-1}\left(1-\tilde{Y}\right)^{1-\psi}} && (46)\\
V^j_{ss} &= \frac{P^X_{ss}Z_{ss}-(w_{ss}-\text{wage subsidy}_{ss})}{1+\beta^{\text{firm}}\overline{\mu}-\beta^{\text{firm}}(1-\delta_{ss})} && (47)\\
Y &= \frac{V^j_{ss}}{\tilde{Y}} && (48)\\
\mu_{ss} &= \overline{\mu}V^j_{ss}. && (49)
\end{aligned}
$$

We can also infer V^v_{ss}, κ and F by

$$
\begin{aligned}
V^v_{ss} &= \kappa_0 && (50)\\
\kappa &= \lambda^v_{ss}(V^j_{ss}+\text{hiring subsidy}_{ss})-(1-\beta(1-\lambda^u_{ss})(1-\delta_{ss}))V^v_{ss} && (51)\\
F &= \iota_{ss}(V^v_{ss})^{-\xi}. && (52)
\end{aligned}
$$

Finally, guess on R_{ss}^{real} and calculate

$$
\begin{aligned}
R_{ss} &= R_{ss}^{\text{real}} && (53)\\
q_{ss} &= \frac{1}{R_{ss}^{\text{real}} - \delta_q} && (54)\\
B_{ss} &= \frac{qB_{ss}}{q_{ss}} && (55)\\
\tau_{ss} &= \frac{(1+\delta_q q_{ss})B_{ss} + w_{ss}(1-u_{ss}) + \overline{\phi} w_{ss}\text{UI}_{ss} + \underline{\phi} w_{ss}(u_{ss} - \text{UI}_{ss}) - q_{ss}B_{ss}}{w_{ss}(1-u_{ss}) + \overline{\phi} w_{ss}\text{UI}_{ss} + \underline{\phi} w_{ss}(u_{ss} - \text{UI}_{ss})}. && (56)
\end{aligned}
$$

Update guess of R_{ss}^{real} until the bond market clears by

$$
q_{ss}B_{ss} = \int a_{ss}^{\star}(\beta_i, u_{it}, a_{it-1}) d\boldsymbol{D}_{ss} \tag{57}
$$