

# ASSIGNMENT I

**Vision:** This project teaches you to solve for the *stationary equilibrium* and *transition path* in a heterogeneous agent model and analyze the economic results.

- **Problem:** The problem consists of
  1. A number of questions (page 2)
  2. A model (page 3-4)
- **Code:** The problem is designed to be solved with the *GEModelTools* package.
- **Structure:** Your project should consist of
  1. A single self-contained pdf-file with all results
  2. A single Jupyter notebook showing how the results are produced
  3. Well-documented *.py* files
- **Hand-in:** Upload a single zip-file on Absalon (and nothing else)
- **Deadline:** 6th of October 2023
- **Exam:** Your Assignment I will be a part of your exam portfolio.  
You can incorporate feedback before handing in the final version.

## HANC with multiple types of labor

- a) **Setup.** Define the stationary equilibrium, the transition path and create a DAG for the model.
- b) **Solve for the stationary equilibrium.** Discuss and illustrate which factors determines wealth inequality.
- c) **Compute and inspect the Jacobians of the household block wrt.  $\varphi_0$ .**
- d) **Solve for the transition path when  $\varphi_{1t}$  is 10 percent higher for 10 periods.** Discuss which types of agents this benefits.
- e) **Solve for the transition path when  $\varphi_{1t}$  is *permanently* 10 percent higher.** Discuss which types of agents this benefits.

# 1. Model

**Households.** The model has a continuum of infinitely lived households indexed by  $i \in [0, 1]$ . Households are *ex ante* heterogeneous in terms of their discount factors,  $\beta_i$ , and their ability,  $\chi_i$ . The discount factors are drawn with equal probabilities from a three element set,  $\beta_i \in \{\check{\beta} - \sigma_\beta, \check{\beta}, \check{\beta} + \sigma_\beta\}$ . The abilities are either low or high,  $\chi_i \in \{0, 1\}$ , with probabilities  $\frac{2}{3}$  and  $\frac{1}{3}$ .

Households choose consumption and exogenously supply two types of labor,  $\eta_i^j$  for  $j \in \{0, 1\}$  with associated productivity  $\varphi_t^j$ . Savings is in terms of capital, which is rented out to firms at the rental rate,  $r_t^K$ . There are no possibilities to borrow. Households are *ex post* heterogeneous in terms of their stochastic labor productivity,  $s_{it}$ , and their (end-of-period) savings,  $a_{it-1}$ . The distribution of households over idiosyncratic states is denoted  $\underline{D}_t$  before shocks are realized and  $\mathbf{D}_t$  afterwards. The real wages are  $w_t^j$ , and real-profits are  $\Pi_t$ .

The household problem is

$$\begin{aligned} v_t(s_{it}, a_{it-1}) &= \max_{c_t} \frac{c_{it}^{1-\sigma}}{1-\sigma} - v \frac{\left(\sum_{j=0}^1 \eta_i^j\right)^{1+\varepsilon}}{1+\varepsilon} + \beta_i \mathbb{E}_t [v_{t+1}(s_{it+1}, a_{it})] \\ \text{s.t. } a_{it} + c_{it} &= (1 + r_t^K - \delta)a_{it-1} + \sum_{j=0}^1 w_t^j \varphi_t^j \eta_i^j s_{it} + \Pi_t \\ \log s_{it+1} &= \rho_s \log s_{it} + \psi_{it+1}, \quad \psi_{it+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \quad \mathbb{E}[s_{it}] = 1 \\ a_{it} &\geq 0. \end{aligned} \tag{1}$$

The Euler-equation is

$$c_{it}^{-\sigma} = \beta_i \mathbb{E} [v_{a,it+1}(s_{it+1}, a_{it})] \tag{2}$$

$$v_{a,it} = (1 + r_t^K - \delta) c_{it}^{-\sigma}. \tag{3}$$

The aggregate quantities of central interest are

$$C_t^{hh} = \int c_{it} d\mathbf{D}_t \tag{4}$$

$$L_t^{j, hh} = \int \varphi_t^j \eta_i^j s_{it} d\mathbf{D}_t \text{ for } j \in \{0, 1\} \tag{5}$$

$$A_t^{hh} = \int a_{it} d\mathbf{D}_t \tag{6}$$

**Firms.** A representative firm rents capital,  $K_{t-1}$ , and hires both types of labor,  $L_t^1$  and  $L_t^2$ , to produce goods, with the production function

$$Y_t = \Gamma_t K_{t-1}^\alpha \Pi_{j=0}^1 \left( L_t^j \right)^{\frac{1-\alpha}{2}} \quad (7)$$

where  $\Gamma_t$  is technology and  $\alpha$  is the Cobb-Douglas weight parameter on capital. Capital depreciates with the rate  $\delta \in (0, 1)$ . The real rental price of capital is  $r_t^K$  and the real wages are  $w_t^j$ . Profits are  $\Pi_t = Y_t - \sum_{j=0}^1 w_t^j L_t^j - r_t^K K_{t-1}$ . The households own the representative firm in equal shares.

The law-of-motion for capital is  $K_t = (1 - \delta)K_{t-1} + I_t$ .

**Market clearing.** Market clearing implies

1. Asset market:  $K_t = A_t^{hh}$
2. Labor market:  $L_t^j = L_t^{j, hh}$
3. Goods market:  $Y_t = C_t^{hh} + I_t$

## 2. Calibration

1. **Preferences:**  $\sigma = 2, \check{\beta} = 0.975, \sigma_\beta = 0.01, \nu = 0.5, \varepsilon = 1.0$
2. **Labor supply:**  $\eta_i^j = \begin{cases} 1 & \text{if } \chi_i = j \\ 0 & \text{else} \end{cases}, \varphi_{ss}^0 = 1, \varphi_{ss}^1 = 2$
3. **Income process:**  $\rho_s = 0.95, \sigma_\psi = 0.30\sqrt{(1 - \rho_s^2)},$
4. **Production:**  $\Gamma_{ss} = 1, \alpha_{ss} = 0.36, \delta = 0.10$