The International Intertemporal Keynesian Cross

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Agenda

1. Recap: The intertemporal Keynesian cross
2. The international intertemporal Keynesian cross
3. Fiscal policy in a small open economy

Recap: The Intertemporal Keynesian Cross (1/4)

- The **static** Keynesian cross is a way of studying fiscal policy in **static** economies:

$$Y = C + G \quad \Leftrightarrow$$

$$Y = a + b(Y - T) + G.$$

  You know this from Principles of Economics B (ØPB) or similar introductory macro courses.

- The **intertemporal** Keynesian cross microfounds and generalizes this to dynamic economies

Recap: The Intertemporal Keynesian Cross (2/4)

- Consider a discrete time, infinite horizon setup: $t = 0, 1, \ldots$
- Start with goods market clearing:

$$Y_t = C_t + G_t.$$

- Take a first-order approximation and write in sequence space:

$$d\boldsymbol{Y} = d\boldsymbol{C} + d\boldsymbol{G}. \tag{1}$$

- The consumption function is (with a constant real rate):

$$C_t = \mathcal{C}_t\left(\{Z_s\}_{s=0}^{\infty}\right),$$

where $Z_t = Y_t - T_t$ is post-tax labor income.

Recap: The Intertemporal Keynesian Cross (3/4)

- Linearize the consumption function and write in sequence space:

$$d\boldsymbol{C} = \boldsymbol{M} d\boldsymbol{Z}, \tag{2}$$

where $\boldsymbol{M} \equiv \partial \boldsymbol{C} / \partial \boldsymbol{Z}$ is the I-MPC matrix.

- Use the consumption function in eq. (2) into goods market clearing in eq. (1):

$$\begin{aligned} d\boldsymbol{Y} &= \boldsymbol{M} d\boldsymbol{Z} + d\boldsymbol{G} \\ &= \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + d\boldsymbol{G} \\ &= \boldsymbol{M} d\boldsymbol{Y} + d\boldsymbol{G} - \boldsymbol{M} d\boldsymbol{T}. \end{aligned}$$

This is the IKC in a closed economy!

Recap: The Intertemporal Keynesian Cross (4/4)

- Re-arrange to set up solution for $d\boldsymbol{Y}$:

$$d\boldsymbol{Y} = \boldsymbol{M}d\boldsymbol{Y} + d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T} \quad \Leftrightarrow$$

$$(\boldsymbol{I} - \boldsymbol{M})d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}.$$

- Tempting to solve for $d\boldsymbol{Y}$ by multiplying by $(\boldsymbol{I} - \boldsymbol{M})^{-1}$ from the left:

$$d\boldsymbol{Y} = (\boldsymbol{I} - \boldsymbol{M})^{-1}\left[d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}\right].$$

- **Warning:** This cannot be done, as $\boldsymbol{I} - \boldsymbol{M}$ is not invertible! (Proof later).
- Reflects that there are **infinitely many solutions** for $d\boldsymbol{Y}$

The Small Open Economy

- Let us now consider a **small open economy**
- **Two goods:** Home (H) and foreign (F)
- Consumption (C_t) is as before, but is now split on the two goods as follows:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \tag{3}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t.$$

- Compared to the closed economy, we need to take a quick detour to handle **relative prices**. Normalize prices to 1 in steady state.

Relative Prices (1/3)

- P_t is the consumer price index:

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{4}$$

- No arbitrage across domestic and foreign bonds yields a real UIP condition:

$$1 + r_t = (1 + r_t^*)\frac{Q_{t+1}}{Q_t},$$

- Q_t is the real exchange rate:

$$Q_t = \frac{E_t P_t^*}{P_t} = \frac{E_t}{P_t}, \tag{5}$$

where the last equality follows from the home economy being small.

Relative Prices (2/3)

- E_t is the nominal exchange rate:

$$E_t = \frac{P_{F,t}}{P^*_{F,t}} = P_{F,t}. \tag{6}$$

- The home economy is small, so $r^*_t = r_{ss}$
- The domestic real rate is fixed by assumption, so $r_t = r_{ss}$
- Inserting in UIP gives

$$1 + r_{ss} = (1 + r_{ss})\frac{Q_{t+1}}{Q_t} \quad \Leftrightarrow$$

$$Q_t = Q_{t+1},$$

implying $Q_t = Q_{ss}$ for all t.

Relative Prices (3/3)

- The definition of the real exchange rate in eq. (5) then implies $E_t = P_t$
- By the definition of the nominal exchange rate in eq. (6), $E_t = P_t$ then implies

  $P_t = P_{F,t}$

- Insert $P_t = P_{F,t}$ in the CPI in eq. (4):

$$
\begin{aligned}
P_t &= \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_t^{1-\eta}\right]^{\frac{1}{1-\eta}} && \Leftrightarrow \\
P_t^{1-\eta} &= (1-\alpha)P_{H,t}^{1-\eta} + \alpha P_t^{1-\eta} && \Leftrightarrow \\
P_t &= P_{H,t}.
\end{aligned}
$$

Consumption Function

- Insert $P_{H,t} = P_t$ into $C_{H,t}$ from eq. (3):

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$$
$$= (1-\alpha)C_t.$$

- Linearize and write in sequence space with $d\boldsymbol{C} = \boldsymbol{M}d\boldsymbol{Z}$:

$$d\boldsymbol{C_H} = (1-\alpha)d\boldsymbol{C}$$
$$= (1-\alpha)\boldsymbol{M}d\boldsymbol{Z}. \tag{7}$$

- This generalizes consumption of home goods from the closed economy to the small open economy (the closed economy is nested for $\alpha = 0$)

IIKC (1/2)

- Linearized sequence space goods market clearing:

$$d\boldsymbol{Y} = d\boldsymbol{C_H} + d\boldsymbol{C^*_H} + d\boldsymbol{G}.$$

- Insert $d\boldsymbol{C_H}$ from eq. (7):

$$\begin{aligned} d\boldsymbol{Y} &= (1-\alpha)\boldsymbol{M}d\boldsymbol{Z} + d\boldsymbol{C^*_H} + d\boldsymbol{G} \\ &= (1-\alpha)\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + d\boldsymbol{G} \\ &= (1-\alpha)\boldsymbol{M}d\boldsymbol{Y} + d\boldsymbol{G} - (1-\alpha)\boldsymbol{M}d\boldsymbol{T}, \end{aligned}$$

using the definition of Z and $d\boldsymbol{C^*_H} = 0$ due to the home economy being small.

IIKC (2/2)

- Try now to solve:

$$\begin{aligned} d\boldsymbol{Y} &= (1-\alpha)\boldsymbol{M}d\boldsymbol{Y} + d\boldsymbol{G} - (1-\alpha)\boldsymbol{M}d\boldsymbol{T} && \Leftrightarrow \\ \left[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}\right]d\boldsymbol{Y} &= d\boldsymbol{G} - (1-\alpha)\boldsymbol{M}d\boldsymbol{T} && \Leftrightarrow \\ d\boldsymbol{Y} &= \left[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}\right]^{-1}\left\{d\boldsymbol{G} - (1-\alpha)\boldsymbol{M}d\boldsymbol{T}\right\}, \end{aligned} \tag{8}$$

  where $\left[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}\right]^{-1}$ **does** exist for $\alpha > 0$.

- Implies that there is a unique solution for $d\boldsymbol{Y}$ in the small open economy
- Let us prove this!

Proof: Key Property of I-MPC Matrix (1/3)

- Start with the aggregate household budget constraint:

$$C_t + A_t = (1+r)A_{t-1} + Z_t.$$

- Solving for assets, recursively inserting, and re-arranging yields:

$$\sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} = (1+r)A_{ss} + \sum_{t=0}^{\infty} \frac{Z_t}{(1+r)^t}.$$

- Take the derivative w.r.t. Z_s (for any $s = 0, 1, \ldots$) on both sides:

$$\sum_{t=0}^{\infty} \frac{M_{t,s}}{(1+r)^t} = \frac{1}{(1+r)^s},$$

  where $M_{t,s} \equiv \partial C_t / \partial Z_s$.

Proof: Key Property of I-MPC Matrix (2/3)

- Define

$$\boldsymbol{q}' = \begin{pmatrix} 1 & \frac{1}{1+r} & \frac{1}{(1+r)^2} & \dots \end{pmatrix}.$$

- Then, for any $s = 0, 1, \dots$:

$$\boldsymbol{q}'\boldsymbol{M}_s = \sum_{t=0}^{\infty} \frac{M_{t,s}}{(1+r)^t} = \frac{1}{(1+r)^s} = \boldsymbol{q}_s,$$

where $\boldsymbol{M}_s = (M_{0,s}, M_{1,s}, \dots)'$ and $\boldsymbol{q}_t$ is element t in q.

Proof: Key Property of I-MPC Matrix (3/3)

- Note that
$$\boldsymbol{M} = \begin{pmatrix} M_{0,0} & M_{0,1} & M_{0,2} & \dots \\ M_{1,0} & M_{1,1} & M_{1,2} & \dots \\ M_{2,0} & M_{2,1} & M_{2,2} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$
such that column s in $\boldsymbol{M}$ is $\boldsymbol{M}_s$. Multiply by $\boldsymbol{q}'$:
$$\begin{aligned} \boldsymbol{q}'\boldsymbol{M} &= \begin{pmatrix} \boldsymbol{q}'\boldsymbol{M}_0 & \boldsymbol{q}'\boldsymbol{M}_1 & \dots \end{pmatrix} \\ &= \begin{pmatrix} \boldsymbol{q}_0 & \boldsymbol{q}_1 & \dots \end{pmatrix} = \boldsymbol{q}'. \end{aligned}$$

Proof: Invertibility (1/2)

- **Key result:** $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$
- Consider now the following linear system of equations in $\boldsymbol{x}$:

$$[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]\,\boldsymbol{x} = \mathbf{0}$$

- Two cases:
  1. $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]$ is invertible $\Leftrightarrow$ Invert to get that $\boldsymbol{x} = 0$ is the unique solution
  2. $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]$ is not invertible $\Leftrightarrow$ There are 0 or infinitely many solutions for $\boldsymbol{x}$
- Re-write to get

$$(1-\alpha)\boldsymbol{M}\boldsymbol{x} = \boldsymbol{x}.$$

Proof: Invertibility (2/2)

- Multiply by $\boldsymbol{q}'$:

$$(1-\alpha)\boldsymbol{q}'\boldsymbol{x} = \boldsymbol{q'x}.$$

- Note: $\boldsymbol{q'x}$ is a scalar
- For $\alpha > 0$: Unique solution is $\boldsymbol{x} = 0$, so we are in case 1, implying that $[\boldsymbol{I} - (1-\alpha)\boldsymbol{M}]$ is invertible
- For $\alpha = 0$: Infinitely many solutions, so we are in case 2, implying that $[\boldsymbol{I} - \boldsymbol{M}]$ is **not** invertible

Summary So Far

Closed economy:

- Cannot invert for $[I - M]$ to solve the IKC
- Infinitely many solutions for $d\boldsymbol{Y}$

Small open economy:

- Can invert $[I - (1 - \alpha)M]$ to solve the IKC
- A unique solution for $d\boldsymbol{Y}$

Fiscal Policy: The Government (1/2)

- Consider the cumulative present-value fiscal multiplier of a shock to G_t:

$$\mathcal{M} = \frac{\sum_{t=0}^{\infty} \frac{dY_t}{(1+r)^t}}{\sum_{t=0}^{\infty} \frac{dG_t}{(1+r)^t}} = \frac{\boldsymbol{q}' d\boldsymbol{Y}}{\boldsymbol{q}' d\boldsymbol{G}}.$$

- The government's budget constraint:

$$B_t = (1+r)B_{t-1} + \mathrm{PD}_t,$$

where $\mathrm{PD}_t \equiv G_t - T_t$ is the primary deficit.

- Iterating (as with the household) yields

$$\frac{B_t}{(1+r)^t} = (1+r)B_{ss} + \sum_{s=0}^{t} \frac{\mathrm{PD}_s}{(1+r)^s}.$$

Fiscal Policy: The Government (2/2)

- Linearize:

$$\frac{dB_t}{(1+r)^t} = \sum_{s=0}^{t} \frac{d\text{PD}_s}{(1+r)^s}.$$

- Use $B_t \to B_{ss}$ for $t \to \infty$:

$$\boldsymbol{q}' d\mathbf{PD} = 0 \quad \Leftrightarrow \quad \boldsymbol{q}' d\boldsymbol{T} = \boldsymbol{q}' d\boldsymbol{G}.$$

  This is the intertemporal budget constraint for the government.

- **Intuition:** In present-value terms, the government has to raise taxes as much as government spending to pay back the debt

Fiscal Policy: Fiscal Multiplier (1/3)

- Start with the IIKC from eq. (8):

$$d\boldsymbol{Y} = (1-\alpha)\boldsymbol{M}d\boldsymbol{Y} + d\boldsymbol{G} - (1-\alpha)\boldsymbol{M}d\boldsymbol{T}.$$

- Multiply by $\boldsymbol{q}'$ from the left:

$$\boldsymbol{q}'d\boldsymbol{Y} = (1-\alpha)\boldsymbol{q}'d\boldsymbol{Y} + \boldsymbol{q}'d\boldsymbol{G} - (1-\alpha)\boldsymbol{q}'d\boldsymbol{T}.$$

- Re-arrange and use $\boldsymbol{q}'d\boldsymbol{G} = \boldsymbol{q}'d\boldsymbol{T}$:

$$\alpha\boldsymbol{q}'\boldsymbol{dY} = \alpha\boldsymbol{q}'d\boldsymbol{G}.$$

- This finally yields the fiscal multiplier when $\alpha > 0$:

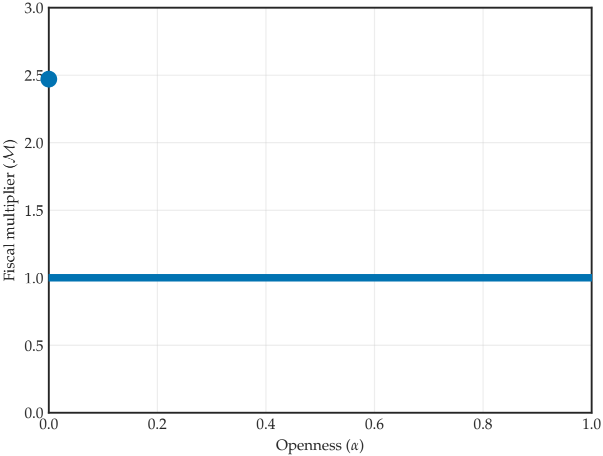
$$\mathcal{M} = 1.$$

Fiscal Policy: Fiscal Multiplier (2/3)

- **Conclusion:** The fiscal multiplier is 1 in the SOE
- Note that the above derivations hold for any $\alpha > 0$, but **not** $\alpha = 0$
- What about $\alpha = 0$? Auclert et al. (2023) show that $\mathcal{M}$ is in general **not** 1. In fact, they usually find **very** large multipliers.

Fiscal Policy: Fiscal Multiplier (3/3)

In other words, there is a discontinuity at $\alpha = 0$ for $\mathcal{M}$ as a function for α:



Fiscal Policy: Intuition (1/3)

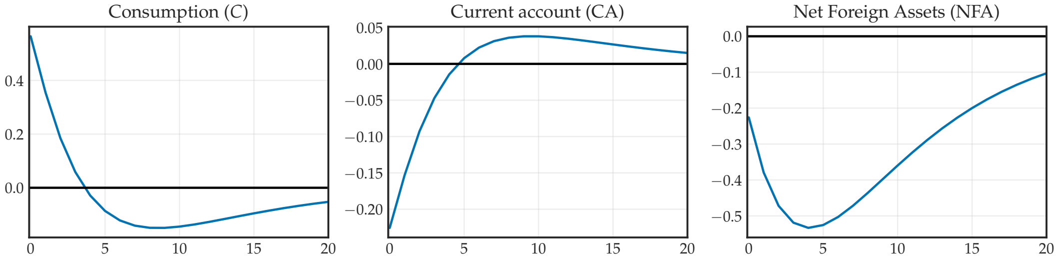
- $\mathcal{M} = 1$ further implies $q' d\boldsymbol{C} = \mathbf{0}$
- To see this, start with linearized consumption and multiply by $\boldsymbol{q}'$:

$$
\begin{aligned}
d\boldsymbol{C} &= (1-\alpha)\boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) \\
\Rightarrow \quad \boldsymbol{q}' d\boldsymbol{C} &= (1-\alpha)\left[\boldsymbol{q}' d\boldsymbol{Y} - \boldsymbol{q}' d\boldsymbol{T}\right] \\
&= (1-\alpha)\left[\boldsymbol{q}' d\boldsymbol{G} - \boldsymbol{q}' d\boldsymbol{T}\right] \\
&= \mathbf{0}.
\end{aligned}
$$

- However, C_t **does** actually increase initially due to front-loaded I-MPCs

Fiscal Policy: Intuition (2/3)

Implies "boom-bust" cycle in consumption:



Consumers go into debt to foreigners, building a negative NFA, which eventually is paid back by cutting consumption.

Fiscal Policy: Intuition (3/3)

Intuition for a multiplier of **exactly** 1 follows from standard Keynesian arguments.

	Higher income	H spending on H	H spending on F
Round 0	1	$1-\alpha$	α
Round 1	$1-\alpha$	$(1-\alpha)^2$	$\alpha(1-\alpha)$
Round 2	$(1-\alpha)^2$	$(1-\alpha)^3$	$\alpha(1-\alpha)^2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Whole increase in government spending ends up abroad:

$$\alpha+\alpha(1-\alpha)+\alpha(1-\alpha)^2+\cdots=\alpha\sum_{s=0}^{\infty}(1-\alpha)^s=\frac{\alpha}{1-(1-\alpha)}=1.$$

Conclusion

- The (international) intertemporal Keynesian cross is a useful tool for analyzing the effects of fiscal policy
- $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$
- Non-trivial to solve IKC in a closed economy — simple in a SOE*
- Fiscal multiplier is 1 in a SOE, not (generally) in a closed economy*

*that is (eventually) funded and with $r_t = r_{ss}$