GEMODELTOOLS: TWO-SECTOR I-HANK MODEL

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1 Model

We consider a *small open economy* with heterogeneous agents and sticky wages. The *foreign* economy is thus taken as exogenously given.

Households. The home economy has a continuum of infinitely lived households indexed by $i \in [0,1]$. Households are *ex ante* heterogeneous in terms of which sector they work in, $s_i \in \{TH, NT\}$, where TH is the *tradeable* sector (in the home country), and $s_i = NT$ is the *non-tradeable* sector. Households are *ex post* heterogeneous in terms of their time-varying stochastic productivity, z_{it}, and their (end-of-period) savings, a_{it-1}. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. Households supply labor, $n_{s_i,it}$, chosen by a union in each sector, and choose consumption, c_{it}, on their own. Households are not allowed to borrow. The return on savings is r^a_t, the sector-specific real wage is $w_{s_i,t}$, and labor income is taxed with the rate $\tau_t \in [0,1]$.

The household problem in real terms is

$$
\begin{aligned}
v_t(s_i, z_t, a_{t-1}) &= \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu} + \beta_i \mathbb{E}_t \left[v_{t+1}(s_i, z_{t+1}, a_t) \right] \qquad (1) \\
\text{s.t. } a_{it} + c_{it} &= (1 + r^a_t) a_{it-1} + (1-\tau_t) w_{s_i,t} n_{s_i,t} z_{it} \\
\log z_{it+1} &= \rho_z \log z_{it} + \psi_{it+1} \ \ , \psi_{it} \sim \mathcal{N}(\mu_\psi, \sigma_\psi),\ \mathbb{E}[z_{it}] = 1 \\
a_{it} &\geq 0
\end{aligned}
$$

where β is the discount factor, σ is the inverse elasticity of substitution, φ controls the disutility of supplying labor and ν is the inverse of the Frisch elasticity.

Aggregate quantities are

$$A_t^{hh} = \int a_{it} d\boldsymbol{D}_t \tag{2}$$

$$C_t^{hh} = \int c_{it} d\boldsymbol{D}_t \tag{3}$$

$$s \in \{TH, NT\} : \; S_x^{hh} = \int 1\{s_{it} = s\} d\boldsymbol{D}_t \tag{4}$$

An *outer* CES demand system implies that the consumption of tradeable goods, $C_{T,t}$, and non-tradeable goods, $C_{NT,t}$, are given by

$$C_{T,t} = \alpha_T \left(\frac{P_{T,t}}{P_t}\right)^{-\eta_{T,NT}} C_t^{hh} \tag{5}$$

$$C_{NT,t} = (1-\alpha_T) \left(\frac{P_{NT,t}}{P_t}\right)^{-\eta_{T,NT}} C_t^{hh} \tag{6}$$

where α_T is the share of tradeable goods and $\eta_{T,NT}$ is the substitution elasticity. The corresponding price index is

$$P_t = \left[\alpha_T P_{T,t}^{1-\eta_{T,NT}} + (1-\alpha_T) P_{NT,t}^{1-\eta_{T,NT}}\right]^{\frac{1}{1-\eta_{T,NT}}} \tag{7}$$

Am *inner* CES demand system implies that consumption of tradeable goods produced at home, $C_{TH,t}$, and tradeable goods produced in the foreign country, $C_{TF,t}$, are given by

$$C_{TF,t} = \alpha_F \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta_{F,H}} C_{T,t} \tag{8}$$

$$C_{TH,t} = (1-\alpha_F) \left(\frac{P_{TH,t}}{P_{T,t}}\right)^{-\eta_{F,H}} C_{T,t} \tag{9}$$

where α_F is the share of foreign tradeable goods and $\eta_{F,H}$ is the substitution elasticity. The corresponding price index is

$$P_{T,t} = \left[\alpha_F P_{F,t}^{1-\eta_{F,H}} + (1-\alpha_F) P_{TH,t}^{1-\eta_{F,H}}\right]^{\frac{1}{1-\eta_{F,H}}} \tag{10}$$

Firms. A representative firm in each sector, $s \in \{TH, NT\}$, hires labor, $N_{s,t}$, to produce goods, with the production function

$$Y_{s,t} = Z_{s,t} N_{s,t} \tag{11}$$

where Z_t^s is the exogenous technology level. Profits are

$$\Pi_{s,t} = P_{s,t} Y_{s,t} - W_{s,t} N_{s,t} \tag{12}$$

where $P_{TH,t}$ and P_{NT} are the price levels and $W_{s,t}$ are the nominal wage levels. The first order condition for labor implies that

$$P_{s,t} = W_{s,t} / Z_{s,t} \tag{13}$$

The real wage is

$$w_{s,t} = \frac{W_{s,t}}{P_t} \tag{14}$$

Unions. A union in each sector chooses the labor supply of each household and sets wages. Each household is chosen to supply the same amount of labor,

$$n_{s,t} = N_{s,t}^{hh}, \;\; s \in \{T, NT\} \tag{15}$$

Unspecified adjustment costs imply *New Keynesian Wage Philips Curve*s,

$$\pi_{s,t}^{w} = \kappa \int \left(\varphi n_{s,t}^{-\nu} - \frac{1}{\mu} \left(1 - \tau_t\right) w_{s,t} z_{it} c_{it}^{-\sigma} \right) 1\{s_{it} = s\} d\boldsymbol{D}_t + \beta \pi_{s,t+1}^{w} \tag{16}$$

where $1 + \pi_{s,t}^{w} = W_{s,t} / W_{s,t-1}$, κ is the slope parameter and μ is a wage mark-up.

Central bank. The central bank follows a standard Taylor rule

$$1 + i_t = 1 + i_{ss} + \left(\frac{1 + \pi_{t+1}}{1 + \pi_{ss}} \right)^{\phi} \tag{17}$$

where i_t is the nominal return from period t to period $t+1$, $1 + \pi_{t+1} = P_{t+1} / P_t$, and ϕ is the Taylor coefficient on inflation.

The *ex ante* real interest rate from t to $t+1$ is

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{18}$$

The *ex post* real interest rate from $t-1$ to t is

$$1 + r_t^{a} = \frac{1 + i_{t-1}}{1 + \pi_t} \tag{19}$$

Government. The government chooses spending, G_t, and the labor income tax rate, τ_t. The budget constraint for the government then is

$$B_t = (1 + r_t^a)B_{t-1} + \frac{P_{NT,t}}{P_t}G_t - \tau_t \left(w_{T,t}N_{T,t} + w_{NT,t}N_{NT,t}\right)$$

where government consumption is fully in terms of non-tradeable goods.

The tax rule is

$$\tau_t = \tau_{ss} + \omega \frac{B_{t-1} - B_{ss}}{Y_{TH,ss} + Y_{NT,ss}} \tag{20}$$

Foreign economy. The nominal exchange in home currency units per foreign currency unit is denoted E_t. The foreign price level in foreign currency is $P^*_{F,t}$. In home currency, the foreign price level is

$$P_{F,t} = P^*_{F,t}E_t \tag{21}$$

The price of home tradeable goods in foreign currency is

$$P^*_{TH,t} = \frac{P_{TH,t}}{E_t} \tag{22}$$

The real exchange is

$$Q_t = \frac{P_{F,t}}{P_t} = \frac{E_t P^*_{F,t}}{P_t} \tag{23}$$

The foreign demand for the home tradeable goods is

$$C^*_{TH,t} = \left(\frac{P^*_{TH,t}}{P^*_{F,t}}\right)^{-\eta^*} M^*_t = \left(\frac{1}{Q_t}\frac{P_{TH,t}}{P_t}\right)^{-\eta^*} M^*_t \tag{24}$$

where M^*_t is the foreign market size and η^* is the elasticity of foreign demand.

Capital markets are free such that the uncovered interest parity must hold,

$$1 + i_t = \left(1 + i_t^f\right)\frac{E_{t+1}}{E_t} \tag{25}$$

where i_t^f is the foreign nominal interest rate. In real terms this is

$$1 + r_t = \left(1 + r_t^f\right)\frac{Q_{t+1}}{Q_t} \tag{26}$$

where $1 + r_t^f = \frac{1+i_t^f}{1+\pi_{t+1}^f}$ and $1 + \pi_{t+1}^f = P^*_{F,t+1}/P^*_{F,t}$.

Market clearing. The market for home tradeable goods and the market for non-tradeable goods both clear

$$Y_{T,t} = C_{TH,t} + C^*_{TH,t} \tag{27}$$
$$Y_{NT,t} = C_{NT,t} + G_t \tag{28}$$

Accounting. We define the following variables,

$$\text{Gross domestic product: } GDP_t = \frac{P_{TH,t}Y_{TH,t} + P_{NT,t}Y_{NT}}{P_t} \tag{29}$$
$$\text{Net exports: } NX_t = GDP_t - C_t^{hh} - \frac{P_{NT,t}}{P_t}G_t \tag{30}$$
$$\text{Net foreign assets: } NFA_t = A_t^{hh} - B_t \tag{31}$$
$$\text{Current account: } CA_t = NX_t + r_t^a NFA_{t-1} \tag{32}$$

Walras' law then implies

$$NFA_t - NFA_{t-1} = CA_t \tag{33}$$

as shown by

$$\begin{aligned}
\int a_{it} d\mathbf{D}_t &= \int (1+r_t^a)a_{it-1} + (1-\tau_t)w_{s_i,t}n_{s_i,t}z_{it} - c_{it}d\mathbf{D}_t \\
A_t^{hh} &= (1+r_t^a)A_{t-1}^{hh} + (1-\tau_t)\sum_{s\in\{TH,NT\}} w_{s,t}, N_{s,t} - C_t^{hh} \\
&= (1+r_t^a)A_{t-1}^{hh} + GDP_t - C_t^{hh} - \tau_t \sum_{s\in\{TH,NT\}} w_{s,t}, N_{s,t} \\
&= (1+r_t^a)A_{t-1}^{hh} + GDP_t - C_t^{hh} + \left(B_t - (1+r_t^a)B_{t-1} + \frac{P_{NT,t}}{P_t}G_t\right) \\
&= (1+r_t^a)NFA_{t-1} + NX_T + B_t \Leftrightarrow \\
NFA_t - NFA_{t-1} &= r_t^a NFA_{t-1} + NX_t
\end{aligned}$$