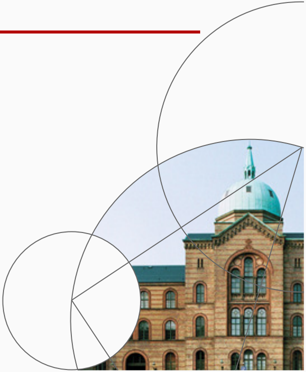
Center for Economic Behavior & Inequality

9. Demographics and global imbalances

Adv. Macro: Heterogenous Agent Models

Jeppe Druedahl & Patrick Moran

2023



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Introduction

Disclaimer

- Note: The views expressed in this presentation are those of the author and do not represent the views of the Federal Reserve Board or Federal Reserve System.

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    2. What will happen going forward?
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- **Central economic questions:**
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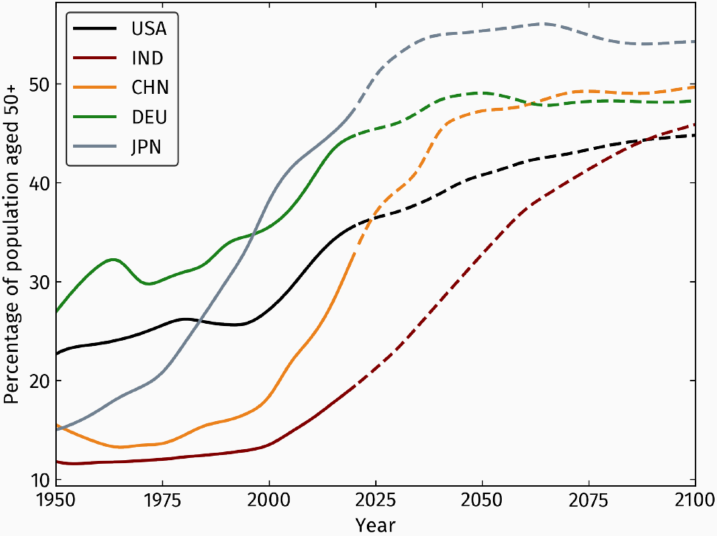
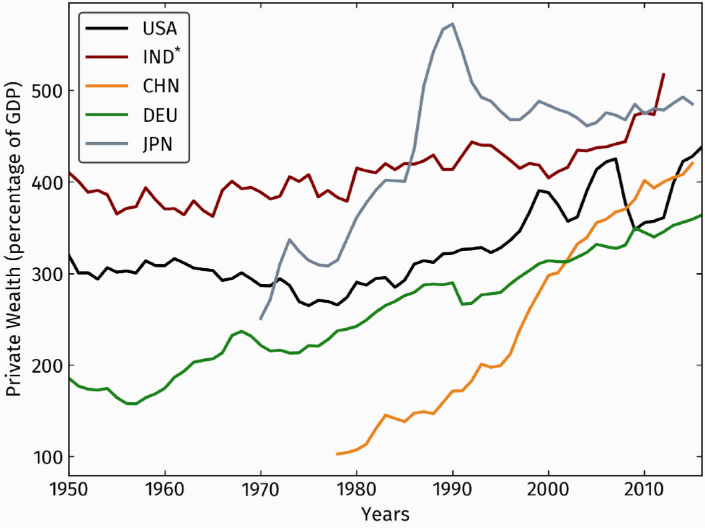
- **Plan for today:** Discuss "Demographics, Wealth, and Global Imbalances in the Twenty-First Century"
  1. Develop a multi-country model with a world interest rate
  2. Quantify the model using a "sufficient statistics" approach
  3. Study the various effects of an aging population

Motivation

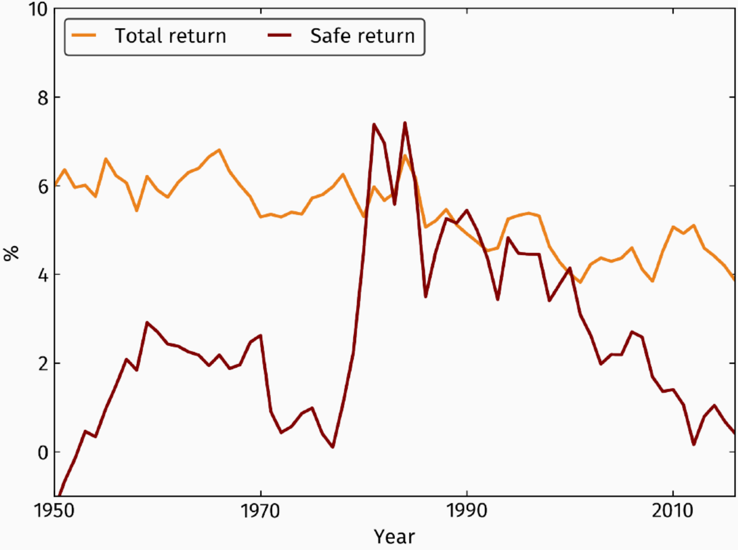
The world population is aging



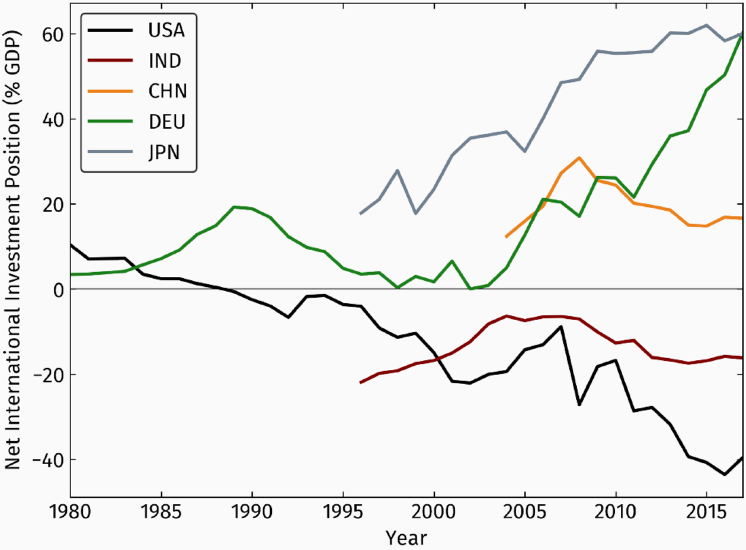
...wealth-to-GDP ratios are increasing...



...rates of return on wealth are falling...



...and "global imbalances" are rising



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- Older population saves more, unevenly across countries

- Much less agreement about how much: <+->Δr for 1970-2015 is

  $> -100\text{bp}$ in Gagnon-Johannsen-Lopez-Salido 2021

  $< -30\text{bp}$ in Eggertsson-Mehrotra-Robbins 2019

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    - "great demographic reversal" hypothesis [Goodhart-Pradhan 2020]

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- Big challenge: how to take this model to the data to discipline the importance of demographics?

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- Second, they use this framework to measure the importance of demographic change
- Admittedly, this approach requires a lot of simplifying assumptions. The authors solve and simulate the full model and show that it gives similar results

Main results

- The authors reject the "great demographic reversal" hypothesis
  - Do not find that aging will decrease savings and increase interest rates
  - Instead, it appears the global savings glut has just begun

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- The authors reject the "great demographic reversal" hypothesis
  - Do not find that aging will decrease savings and increase interest rates
  - Instead, it appears the global savings glut has just begun
- In addition, the authors refute the "asset market meltdown" hypothesis
  - Will dissaving of the old reverse the effects of demographics?
  - Yes, slightly. But it does not cause r to increase
  - As a result, no asset market meltdown

Model

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- Government
  - Flow budget constraint

$$G_t + w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} tr_j + (1 + r_t) B_t = \tau w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} l_j + B_{t+1}$$

  - Balance budget by changing G_t, not τ_t or tr_{jt}, to keep Bt/Yt constant

Environment: heterogeneous agents

- Problem for heterogeneous agents of cohort k (age $j \equiv t-k$)}

$$\max \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right],$$

$$\text{s.t } c_{jt} + \phi_j a_{j+1,t+1} \leq w_t \left((1-\tau)\ell\left(z_j\right) + tr\left(z^j\right) \right) + \left(1+r_t\right) a_{jt}$$

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- a_{jt} : annuity holdings

Equilibrium

Given demographics and policy, in an integrated world equilibrium:

- Individuals optimize
- Firms optimize
- Global asset markets clear

$$\sum_c W_t^c = \sum_c (K_t^c + B_t^c) \quad \forall t$$

where W_t^c is aggregate household wealth in country c:

$$W_t^c = \sum_{j=0}^{J} N_{jt}^c a_{jt}^c$$

Next: consider small country aging alone, with world at steady state
$\rightarrow r$ constant (will adjust later)

Compositional effects as sufficient statistics

Proposition 1

The wealth-to-GDP ratio of a small country aging alone with constant r and γ follows

$$\frac{W_t}{Y_t} \propto \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}}$$

where $a_{j0} \equiv \mathbb{E} a_{j,0}$ and $h_{j0} = \mathbb{E} w_0 \ell_{j,0}$ are average initial asset holdings and pretax labor income by age, and $\pi_{jt} = N_{jt}/N_t$ is the share of the population of age j.

In a partial equilibrium world (where r does not adjust to changing demographics) then all changes in W/Y reflect the changing age composition $\pi_{jt}$of the population, given fixed profiles of asset holdings by age (a_{j0}) and income by age (h_{j0}).

Compositional effects as sufficient statistics

Based on Proposition 1, we can compute the change in log wealth to GDP ratio as follows:

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_o}{Y_o}\right) = \log\left(\frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum_j \pi_{j0} a_{jo}}{\sum_j \pi_{jo} h_{j0}}\right) \equiv \Delta_t^{comp}$$

- The above is measurable from demographic projections and hh. surveys

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- Why? Demographics do not affect (normalized) individual decisions
- Later: we'll think about how Δ_t^{comp} affects general equilibrium outcomes

Measuring compositional effects

Measuring Δ^{comp}

- Calculate Δ_t^{comp} for 25 countries:

$$\Delta_t^{comp} \equiv \log\left(\frac{\sum \pi_{jt} a_{j0}}{\sum \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum \pi_{j0} a_{jo}}{\sum \pi_{j0} h_{j0}}\right)$$

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- Data:
  - π_{jt} : projections of age distributions over individuals 2019 UN World Population Prospects
  - a_{jo}, h_{jo} age-wealth and labor income profiles in base year
    - For US: SCF, LIS/CPS, and Sabelhaus-Henriques Volz (2019)
    - a_{j0} includes funded part of DB pensions
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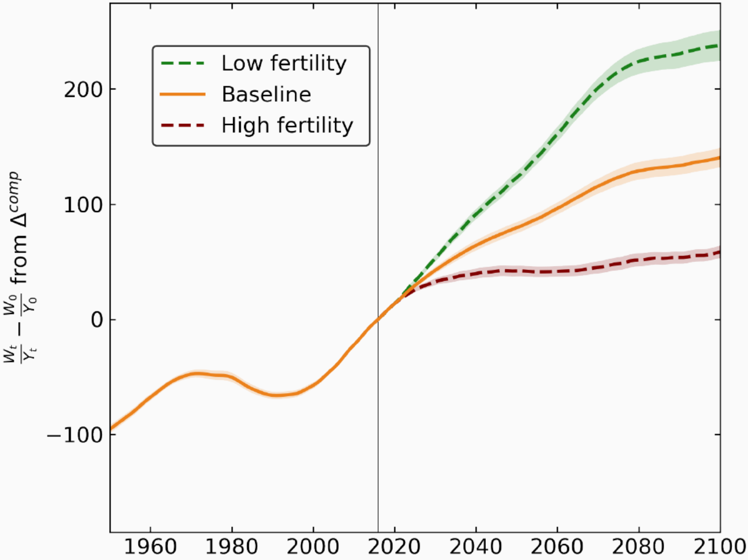
- Report implied level change $\frac{W_t}{Y_t} - \frac{W_0}{Y_0} = \frac{W_0}{Y_0}\left(\exp\left\{\Delta_t^{comp}\right\} - 1\right)$

Δ^{comp} in the United States: 1950-2100



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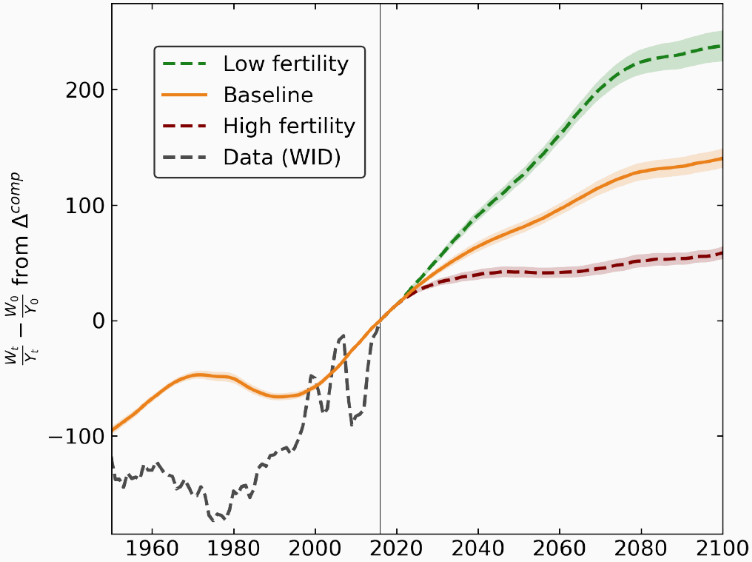
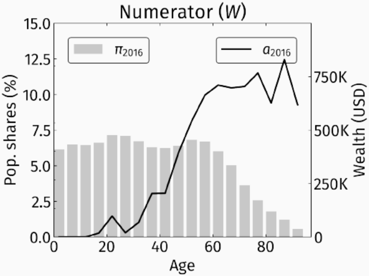


Where do these large effects come from?



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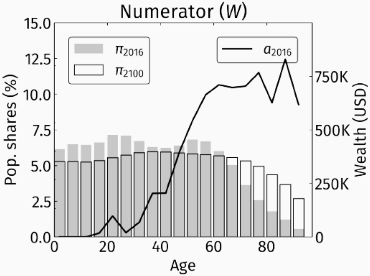

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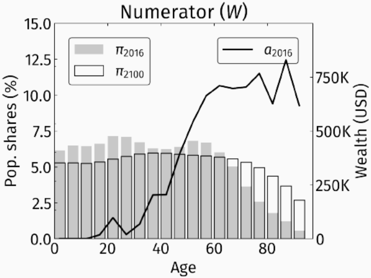

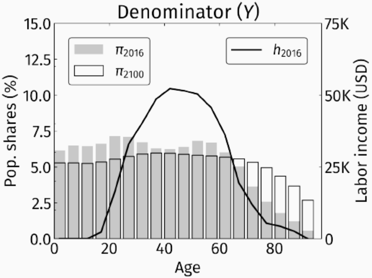

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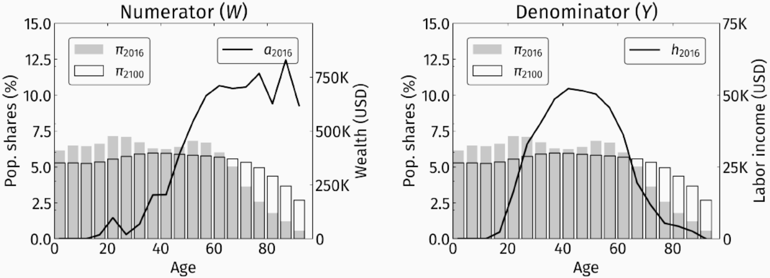

- In paper: separate contribution of numerator (wealth) and denominator (income)
  - Going forward: W contributes $\sim 2/3$, Y contributes $\sim 1/3$
  - Historically demographic dividend pushed Y up, reversed in 2010

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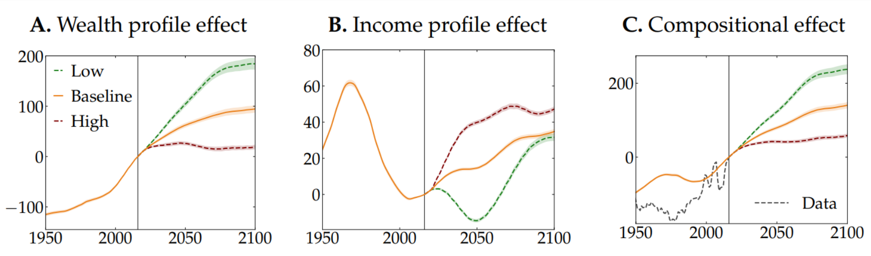

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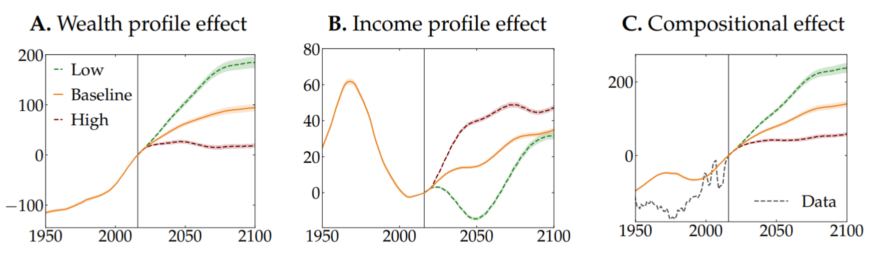
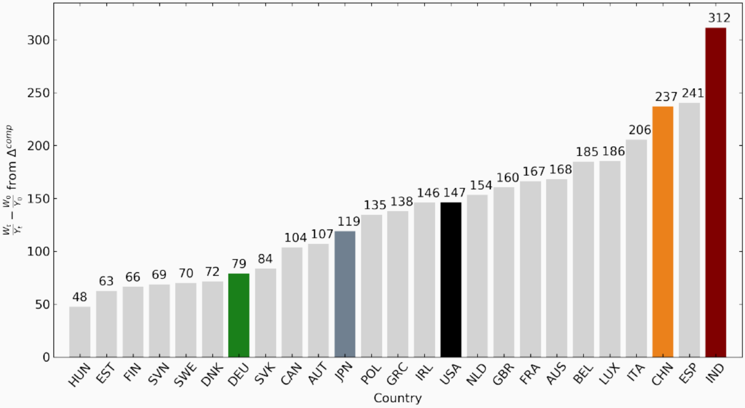


- Historically "demographic dividend" pushed up Y, as a larger share of households were at peak working age
- But this effect has been less pronounced recently, as elderly households earn less

Across countries, Δ^{comp} large and heterogeneous by 2010



General equilibrium implications

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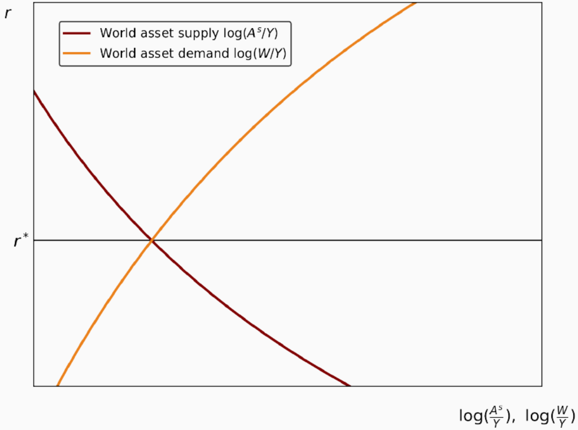
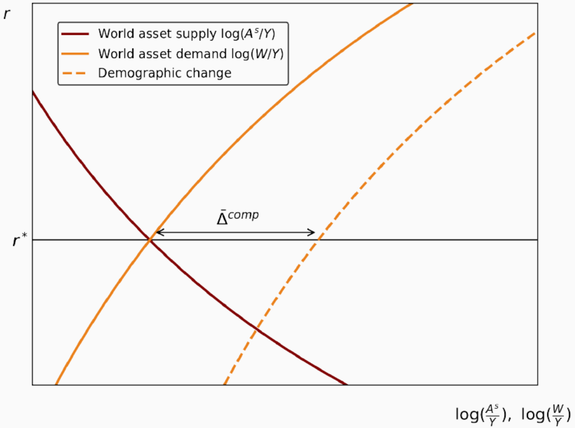


Semielasticity of asset demand $\bar{\epsilon}_d$: depends on σ and observables

Semielasticity of asset supply $\bar{\epsilon}_s$: depends on η and observables

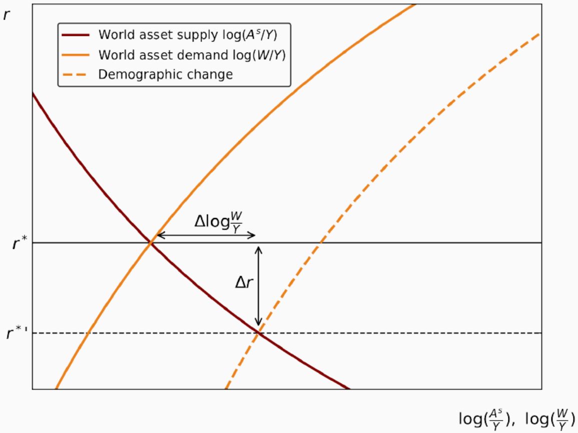
General equilibrium implications



Asset demand shift of $\overline{\Delta}^{\text{comp}}$: wealth-weighted average of $\Delta^{\text{comp, c}}$

Large and positive in the data.

General equilibrium implications



General equilibrium implications

Proposition 2

If the age profiles of assets and consumption are constant, net foreign assets are zero, and governments maintain constaint debt-to-GDP ratios, then the long run change in the rate of return is:

$$\Delta r \approx -\frac{\bar{\Delta}^{\text{comp}}}{\bar{\epsilon}_S + \bar{\epsilon}_d}$$

where $\bar{\epsilon}_S$ is the average semielasticity of asset supply to r,and $\bar{\epsilon}_d$ is the average semielasticity of asset holdings to r, and $\bar{\Delta}^{\text{comp}}$ is the average compositional change.

What determines the asset demand semielasticity?

$$\epsilon^d = \underbrace{\sigma \frac{C}{(1+g)W} \frac{\text{Var}\, Age_c}{1+r}}_{\equiv \epsilon^d_{\text{substitution}}} + \underbrace{\frac{\mathbb{E} Age_c - \mathbb{E} Age_a}{1+r}}_{\equiv \epsilon^d_{\text{income}}}$$

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- The above can be measured assuming fixed Age_a and Age_c
  - The authors find $\epsilon^d_{\text{substitution}} = 39.5$, $\epsilon^d_{\text{income}} = -2$, thus $\epsilon^d > 0$

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- Based on the above calibration, $\bar{\epsilon}^s > 0$ for any plausible η.

Change in world interest rate

Since $\bar{\epsilon}_S + \bar{\epsilon}_d > 0$, then the change in the world interest rate must be negative:

$$\Delta r \approx -\frac{\bar{\Delta}^{\text{comp}}}{\bar{\epsilon}_S + \bar{\epsilon}_d} < 0$$

With different assumptions on the elasticity of intertemporal substitution (σ) and the elasticity of substitution between capital and labor (η), this gives:

	σ		
η	0.25	0.50	1.00
0.60	−3.24	−1.59	−0.79
1.00	−2.09	−1.25	−0.70
1.25	−1.71	−1.10	−0.65

Change in capital to income ratio

Proposition 2 gives a similar formula for the change in capital to income:

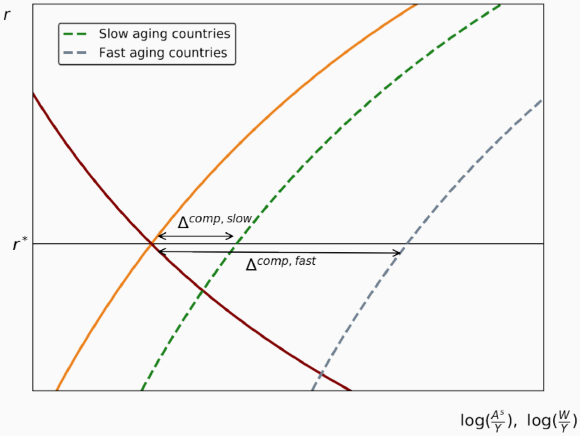
$$\overline{\Delta \log \left(\frac{W}{Y}\right)} \approx \frac{\bar{\epsilon}_{\mathrm{S}}}{\bar{\epsilon}_{\mathrm{S}} + \bar{\epsilon}_{d}} \bar{\Delta}^{\mathrm{comp}} > 0$$

Again with different assumptions on the IES (σ) and the elasticity of substitution between capital and labor (η)

	σ		
η	0.25	0.50	1.00
0.60	15.6	7.7	3.8
1.00	16.7	10.0	5.6
1.25	17.1	11.1	6.5

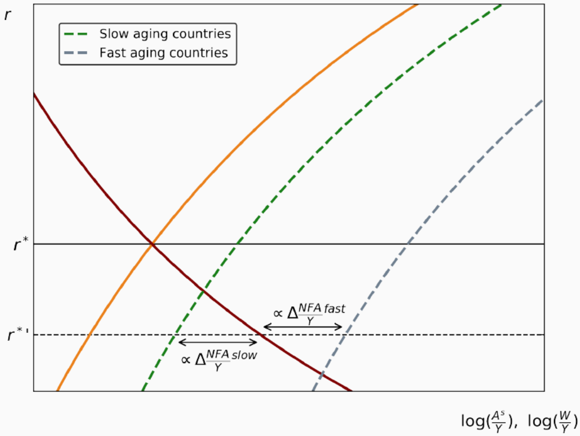
The authors argue that simulations from the general model deliver similar outcomes

Change in net foreign assets



Country specific Δ^{comp} large and heterogeneous in the data

Change in net foreign assets



$$\Delta \left(\frac{NFA}{Y} \right) \approx \frac{W_0}{Y_0} \left(\Delta^{\text{comp}} - \bar{\Delta}^{\text{comp}} \right)$$

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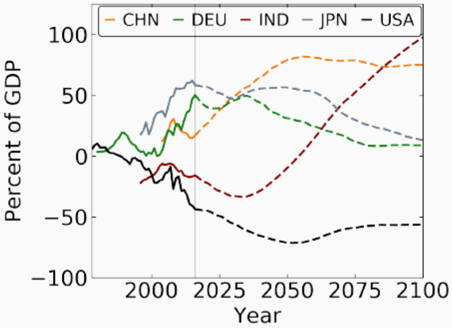
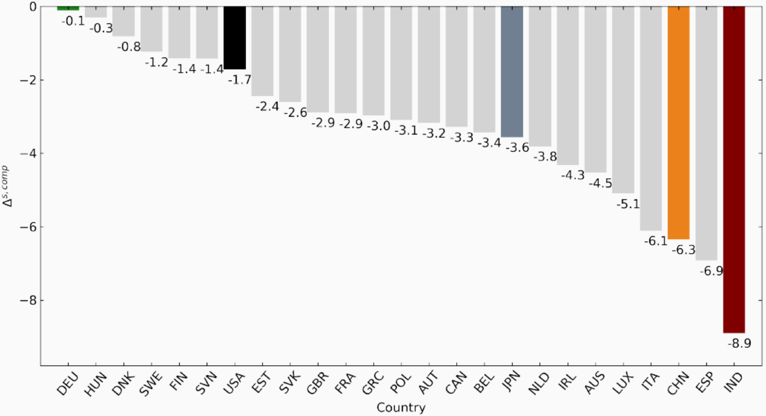


→ Data suggest large global imbalances going forward

Change in savings rate

- Perform same exercise as above, but project S/Y from composition



Richer model

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  - Demographics have no effect on TFP growth
- To study some of these changes, the authors extend their baseline model $\rightarrow$ then simulate the transition path using the techniques we've learned in class

Results from richer model

- Main finding: Δ^{comp} in the richer model is roughly similar to the results from the data

	$\Delta^{comp,c}$	
Country	Model	Data
AUS	30	29
CAN	21	20
CHN	47	45
DEU	21	20
ESP	42	37
FRA	31	30
GBR	27	26
IND	65	56
ITA	34	30
JPN	24	22
NLD	34	33
USA	32	29

Results from richer model

- GE Effects from the model are also roughly similar

	Δr	$\overline{\Delta \log \frac{W}{Y}}$	$\bar{\Delta}^{comp}$	$\bar{\Delta}^{soe}$	$\bar{\epsilon}^{d}$	$\bar{\epsilon}^{s}$
Sufficient statistic analysis	-1.23	9.9	31.8		17.8	8.0
Preferred model specification	-1.23	10.3	34.1	30.3	17.1	8.0
Alternative model specifications						
+ Constant bequests	-1.18	10.0	34.1	27.0	14.9	8.0
+ Constant mortality	-1.23	10.9	34.1	27.1	13.8	8.0
+ Constant taxes and transfers	-1.33	11.9	34.1	30.1	14.5	8.0
+ Constant retirement age	-1.49	13.4	34.1	34.1	14.6	8.0
+ No income risk	-1.47	13.2	33.9	33.9	13.8	8.0
+ Annuities	-1.33	11.5	34.2	34.2	17.2	8.0
Alternative fiscal rules						
Only lower expenditures	-1.29	11.0	34.1	32.6	17.9	8.0
Only higher taxes	-0.88	6.7	34.1	19.4	14.6	8.0
Only lower benefits	-1.50	12.9	34.1	39.1	18.4	8.0

Notes: Δr, $\overline{\Delta \log \frac{W}{Y}}$, $\bar{\Delta}^{comp}$, and $\bar{\Delta}^{soe}$ denote the changes in the model simulation between 2016 and 2100, with Δr reported in percentage points and the other three reported in percent ($100 \cdot \log$).

Conclusion

- How does population aging affect wealth-output ratios, real interest rates, and capital flows?
  - what matters is the compositional effect Δ^{comp}
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  - what matters is the compositional effect Δ^{comp}
  - large and heterogeneous in the data
- The approach developed by the authors:
  - Refutes the asset market meltdown hypothesis: r falls
  - Suggests the global savings glut has just begun

Summary

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