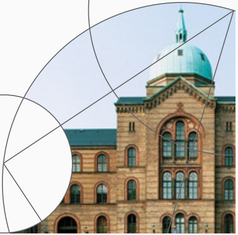
CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

12. Analytical HANK

Adv. Macro: Heterogenous Agent Models

Jeppe Druedahl & Patrick Moran

2022



Danmarks
Grundforskningsfond
Danish National
Research Foundation

Introduction

Disclaimer

- Note: The views expressed in this presentation are those of the author and do not represent the views of the Federal Reserve Board or Federal Reserve System.

Analytical HANK

- **Previously:** Learned to solve and simulate quantitative HANK models with rich household heterogeneity

Analytical HANK

- **Previously:** Learned to solve and simulate quantitative HANK models with rich household heterogeneity

- **Challenge:** In large models with many moving parts, often difficult to understand the main mechanisms driving the results

- **Previously:** Learned to solve and simulate quantitative HANK models with rich household heterogeneity

- **Challenge:** In large models with many moving parts, often difficult to understand the main mechanisms driving the results

- **Goal for today:** Derive analytical solutions to simplified HANK models to better understand the main transmission mechanisms

- **Previously:** Learned to solve and simulate quantitative HANK models with rich household heterogeneity

- **Challenge:** In large models with many moving parts, often difficult to understand the main mechanisms driving the results

- **Goal for today:** Derive analytical solutions to simplified HANK models to better understand the main transmission mechanisms

- **Central economic questions:**
  1. What are the main mechanisms driving monetary policy transmission in standard HANK and RANK models?
  2. Does it matter whether we include price rigidities or wage rigidities in the model?
  3. Is the fiscal multiplier greater than or less than one? What mechanisms drive this?

- **Today:** To answer these questions, helpful to look at solutions coming from simplified models solved with paper and pencil

Analytical HANK

- **Today:** To answer these questions, helpful to look at solutions coming from simplified models solved with paper and pencil

- **Key insight:** Possible to turn off liquidity in simple HANK models, so that there is no risk sharing (in contrast: RANK models have full risk sharing, quantitative HANK models have partial risk sharing)

- **Today:** To answer these questions, helpful to look at solutions coming from simplified models solved with paper and pencil

- **Key insight:** Possible to turn off liquidity in simple HANK models, so that there is no risk sharing (in contrast: RANK models have full risk sharing, quantitative HANK models have partial risk sharing)

- **Plan for today:**
  1. Learn to solve the zero-liquidity analytical HANK model
  2. Study monetary transmission, the role of firm profits, and the distinction between price and wage rigidities
  3. Study fiscal policy and the role of intertemporal MPCs

Zero Liquidity HANK Model

Model framework

- We'll begin with a simple HANK model following Broer et al (2020)

Model framework

- We'll begin with a simple HANK model following Broer et al (2020)
- We'll include two types of households: workers ands capitalists
  - Ex ante identical except that capitalists own the firms

Model framework

- We'll begin with a simple HANK model following Broer et al (2020)
- We'll include two types of households: workers ands capitalists
  - Ex ante identical except that capitalists own the firms
- Workers face idiosyncratic productivity risk
  - Workers ex ante identical, ex post different because of different realizations of productivity shocks

Model framework

- We'll begin with a simple HANK model following Broer et al (2020)
- We'll include two types of households: workers ands capitalists
  - Ex ante identical except that capitalists own the firms
- Workers face idiosyncratic productivity risk
  - Workers ex ante identical, ex post different because of different realizations of productivity shocks
- All households can trade in riskless bond subject to a "very tight" borrowing constraint

Simple HANK model

- Household side: workers and capitalists
  - Capitalists collects firm dividends, workers do not
  - Idiosyncratic productivity risk
  - Participation cost of working
  - Choose how much to work, consume and save
  - In equilibrium: capitalists choose not to work

Simple HANK model

- Household side: workers and capitalists
  - Capitalists collects firm dividends, workers do not
  - Idiosyncratic productivity risk
  - Participation cost of working
  - Choose how much to work, consume and save
  - In equilibrium: capitalists choose not to work

- Firm side: closely follows Galí (2009)
  - Monopolistic firms
  - Use labor inputs
  - Set prices subject to the Calvo friction

Simple HANK model

- Household side: workers and capitalists
  - Capitalists collects firm dividends, workers do not
  - Idiosyncratic productivity risk
  - Participation cost of working
  - Choose how much to work, consume and save
  - In equilibrium: capitalists choose not to work

- Firm side: closely follows Galí (2009)
  - Monopolistic firms
  - Use labor inputs
  - Set prices subject to the Calvo friction

- Motivation: Tractable form of type-heterogeneity that matches
  1. A small share of the households own almost all financial wealth (Piketty-Zucman, 2015)
  2. At the top of the wealth distribution, labor income is a small share of total income (Gornemann-Kuester-Nakajima, 2016)

- Household side: workers and capitalists
    - Capitalists collects firm dividends, workers do not
    - Idiosyncratic productivity risk
    - Participation cost of working
    - Choose how much to work, consume and save
    - In equilibrium: capitalists choose not to work

- Firm side: closely follows Galí (2009)
    - Monopolistic firms
    - Use labor inputs
    - Set prices subject to the Calvo friction

- Motivation: Tractable form of type-heterogeneity that matches
    1. A small share of the households own almost all financial wealth (Piketty-Zucman, 2015)
    2. At the top of the wealth distribution, labor income is a small share of total income (Gornemann-Kuester-Nakajima, 2016)

- Later: compare the solution to a textbook RANK model

HANK model: Households

- Worker $j \in [0,1]$ solves:

$$\max_{C_{jt},N_{jt},B_{jt}} \quad E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{jt}) - \frac{N_{jt}^{1+\varphi}}{1+\varphi} - \vartheta * \mathbb{I}_{N_{jt}>0} \right)$$

$$\text{s.t.} \quad P_t C_{jt} + Q_t B_{jt} \leq W_{jt} N_{jt} + B_{jt-1}$$

$$B_{jt} \geq 0$$

HANK model: Households

- Worker $j \in [0,1]$ solves:

$$\begin{aligned} \max_{C_{jt},N_{jt},B_{jt}} \quad & E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{jt}) - \frac{N_{jt}^{1+\varphi}}{1+\varphi} - \vartheta * \mathbb{I}_{N_{jt}>0} \right) \\ \text{s.t.} \quad & P_t C_{jt} + Q_t B_{jt} \leq W_{jt} N_{jt} + B_{jt-1} \\ & B_{jt} \geq 0 \end{aligned}$$

- Capitalist $j \in (1, m_c]$ solves:

$$\begin{aligned} \max_{C_{jt},N_{jt},B_{jt}} \quad & E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{jt}) - \frac{N_{jt}^{1+\varphi}}{1+\varphi} - \vartheta * \mathbb{I}_{N_{jt}>0} \right) \\ \text{s.t.} \quad & P_t C_{jt} + Q_t B_{jt} \leq W_{jt} N_{jt} + B_{jt-1} + P_t D_{jt} \\ & B_{jt} \geq 0 \end{aligned}$$

- Worker $j \in [0, 1]$ solves:

$$\max_{C_{jt}, N_{jt}, B_{jt}} \quad E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{jt}) - \frac{N_{jt}^{1+\varphi}}{1+\varphi} - \vartheta * \mathbb{I}_{N_{jt}>0} \right)$$
$$\text{s.t.} \quad P_t C_{jt} + Q_t B_{jt} \leq W_{jt} N_{jt} + B_{jt-1}$$
$$B_{jt} \geq 0$$

- Capitalist $j \in (1, m_c]$ solves:

$$\max_{C_{jt}, N_{jt}, B_{jt}} \quad E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{jt}) - \frac{N_{jt}^{1+\varphi}}{1+\varphi} - \vartheta * \mathbb{I}_{N_{jt}>0} \right)$$
$$\text{s.t.} \quad P_t C_{jt} + Q_t B_{jt} \leq W_{jt} N_{jt} + B_{jt-1} + P_t D_{jt}$$
$$B_{jt} \geq 0$$

- Assumption 1: capitalists split the well-diversified portfolio of firm claims equally, $\rightarrow D_{jt} = \frac{D_t}{m_c}$

HANK model: Households

- Worker $j \in [0,1]$ solves:

$$\max_{C_{jt},N_{jt},B_{jt}} \quad E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{jt}) - \frac{N_{jt}^{1+\varphi}}{1+\varphi} - \vartheta * \mathbb{I}_{N_{jt}>0} \right)$$

$$\text{s.t.} \quad P_t C_{jt} + Q_t B_{jt} \leq W_{jt} N_{jt} + B_{jt-1}$$

$$B_{jt} \geq 0$$

- Capitalist $j \in (1, m_c]$ solves:

$$\max_{C_{jt},N_{jt},B_{jt}} \quad E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{jt}) - \frac{N_{jt}^{1+\varphi}}{1+\varphi} - \vartheta * \mathbb{I}_{N_{jt}>0} \right)$$

$$\text{s.t.} \quad P_t C_{jt} + Q_t B_{jt} \leq W_{jt} N_{jt} + B_{jt-1} + P_t D_{jt}$$

$$B_{jt} \geq 0$$

- Assumption 1: capitalists split the well-diversified portfolio of firm claims equally, $\rightarrow D_{jt} = \frac{D_t}{m_c}$
- Assumption 2: m_c small $\rightarrow$ capitalists choose not to work

HANK model: Production side

- CES demand for intermediate goods :

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} Y_t$$

HANK model: Production side

- CES demand for intermediate goods :

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} Y_t$$

- Intermediate goods producer i uses production technology

$$Y_{it} = \int_{j=0}^{1} A_{jt} N_{jit} dj$$

HANK model: Production side

- CES demand for intermediate goods :

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} Y_t$$

- Intermediate goods producer i uses production technology

$$Y_{it} = \int_{j=0}^{1} A_{jt} N_{jit} dj$$

- where A_{jt} is the productivity of household j, $A_j \sim F$ with finite support and $E(A_j) = 1$

HANK model: Production side

- CES demand for intermediate goods :

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} Y_t$$

- Intermediate goods producer i uses production technology

$$Y_{it} = \int_{j=0}^{1} A_{jt} N_{jit} dj$$

- where A_{jt} is the productivity of household j, $A_j \sim F$ with finite support and $E(A_j) = 1$

- Calvo friction and intermediate firm maximization problem otherwise identical

- Government
  - Fiscal authority does nothing – no taxation nor government debt
  - Central bank follows Taylor rule:

$$\frac{1}{Q_t} = \frac{1}{\beta}\Pi_t^{\phi_\pi} e^{\nu_t}$$

$$\Rightarrow \qquad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t$$

HANK model

- Government
  - Fiscal authority does nothing – no taxation nor government debt

  - Central bank follows Taylor rule:

$$\begin{aligned} \frac{1}{Q_t} &= \frac{1}{\beta}\Pi_t^{\phi_\pi} e^{\nu_t} \\ \Rightarrow \qquad \hat{i}_t &= \phi_\pi \pi_t^p + \nu_t \end{aligned}$$

- Equilibrium conditions:

$$\begin{aligned} \int_{j=0}^{1+m_c} C_{jt}\, dj &= Y_t \\ \int_0^{1+m_c} B_{jt}\, dj &= 0 \end{aligned}$$

Solution of the simple HANK model

- Most HANK models do not admit an analytical solution
  - Why? The wealth distribution is an endogenous state variable

Solution of the simple HANK model

- Most HANK models do not admit an analytical solution
  - Why? The wealth distribution is an endogenous state variable
- In contrast, this simple HANK model is possible to solve
  - Zero borrowing constraint $\rightarrow$ Degenerate equilibrium bond wealth distribution
    - Also used in Krusell-Mukoyama-Smith (2011), Werning (2015); McKay-Reis (2016); Ravn-Sterk (2016)
    - "Autarky solution" $\rightarrow$ Workers consume labor income hand-to-mouth
    - Similarly, capitalists consume profit income hand-to-mouth

Solution of the simple HANK model

- Most HANK models do not admit an analytical solution
  - Why? The wealth distribution is an endogenous state variable

- In contrast, this simple HANK model is possible to solve
  - Zero borrowing constraint $\rightarrow$ Degenerate equilibrium bond wealth distribution
    - Also used in Krusell-Mukoyama-Smith (2011), Werning (2015); McKay-Reis (2016); Ravn-Sterk (2016)
    - "Autarky solution" $\rightarrow$ Workers consume labor income hand-to-mouth
    - Similarly, capitalists consume profit income hand-to-mouth
  - Individual worker consumption is linear in aggregate worker consumption
  - Individual capitalist consumption is linear in agg. capitalist consumption
  - Reduced form representation: two-agent model

Solution of the simple HANK model

- Most HANK models do not admit an analytical solution
  - Why? The wealth distribution is an endogenous state variable
- In contrast, this simple HANK model is possible to solve
  - Zero borrowing constraint $\rightarrow$ Degenerate equilibrium bond wealth distribution
    - Also used in Krusell-Mukoyama-Smith (2011), Werning (2015); McKay-Reis (2016); Ravn-Sterk (2016)
    - "Autarky solution" $\rightarrow$ Workers consume labor income hand-to-mouth
    - Similarly, capitalists consume profit income hand-to-mouth
  - Individual worker consumption is linear in aggregate worker consumption
  - Individual capitalist consumption is linear in agg. capitalist consumption
  - Reduced form representation: two-agent model
- Next: Derivation of the equilibrium in our simple HANK model

Aggregation I - Labour supply

- No borrowing $+$ zero net supply of assets $\rightarrow$ The equilibrium must coincide with autarky

Aggregation I - Labour supply

- No borrowing + zero net supply of assets → The equilibrium must coincide with autarky

- Worker consumption:

$$C_{jt} = \frac{W_{jt}}{P_t} N_{jt} \tag{1}$$

Aggregation I - Labour supply

- No borrowing + zero net supply of assets → The equilibrium must coincide with autarky

- Worker consumption:

$$C_{jt} = \frac{W_{jt}}{P_t} N_{jt} \tag{1}$$

- Intratemporal first order condition:

$$\frac{W_{jt}}{P_t} = MRS_{jt} = N_{jt}^{\varphi} C_{jt} \tag{2}$$

Aggregation I - Labour supply

- No borrowing + zero net supply of assets $\rightarrow$ The equilibrium must coincide with autarky

- Worker consumption:

$$C_{jt} = \frac{W_{jt}}{P_t} N_{jt} \tag{1}$$

- Intratemporal first order condition:

$$\frac{W_{jt}}{P_t} = MRS_{jt} = N_{jt}^{\varphi} C_{jt} \tag{2}$$

- (1) + (2) $\Rightarrow N_{it} = N_{jt} \quad \forall i, j \in [0, 1]$

Aggregation I - Labour supply

- No borrowing + zero net supply of assets → The equilibrium must coincide with autarky

- Worker consumption:

$$C_{jt} = \frac{W_{jt}}{P_t} N_{jt} \tag{1}$$

- Intratemporal first order condition:

$$\frac{W_{jt}}{P_t} = MRS_{jt} = N_{jt}^{\varphi} C_{jt} \tag{2}$$

- $(1) + (2) \Rightarrow N_{it} = N_{jt} \quad \forall i, j \in [0, 1]$

- **Workers all supply the same amount of labour**

- Define the aggregate per efficiency unit wage and aggregate supply of labor efficiency units:

$$W_t = \int_{j=0}^{1} \frac{W_{jt}}{A_{jt}} dj \qquad N_t = \int_{j=0}^{1} A_{jt} N_{jt} dj$$

- Define the aggregate per efficiency unit wage and aggregate supply of labor efficiency units:

$$W_t = \int_{j=0}^{1} \frac{W_{jt}}{A_{jt}} dj \qquad N_t = \int_{j=0}^{1} A_{jt} N_{jt} dj$$

- Because labor inputs are perfectly substitutable:

$$W_t = \frac{W_{jt}}{A_{jt}} \quad \forall j.$$

- Define the aggregate per efficiency unit wage and aggregate supply of labor efficiency units:

$$W_t = \int_{j=0}^{1} \frac{W_{jt}}{A_{jt}} dj \qquad N_t = \int_{j=0}^{1} A_{jt} N_{jt} dj$$

- Because labor inputs are perfectly substitutable:

$$W_t = \frac{W_{jt}}{A_{jt}} \quad \forall j.$$

- Because workers all supply the same amount of labour:

$$N_t = N_{jt} \quad \forall j.$$

Aggregation II - Consumption

- Define the aggregate per efficiency unit wage and aggregate supply of labor efficiency units:

$$W_t = \int_{j=0}^{1} \frac{W_{jt}}{A_{jt}} dj \qquad N_t = \int_{j=0}^{1} A_{jt} N_{jt} dj$$

- Because labor inputs are perfectly substitutable:

$$W_t = \frac{W_{jt}}{A_{jt}} \quad \forall j.$$

- Because workers all supply the same amount of labour:

$$N_t = N_{jt} \quad \forall j.$$

- **Worker j consumption is proportional to aggregate worker consumption:**

$$C_{jt} = \frac{W_{jt}}{P_t} N_{jt} = A_{jt} \frac{W_{jt}}{A_{jt} P_t} N_{jt} = A_{jt} \frac{W_t}{P_t} N_t = A_{jt} C_t$$

where $C_t \equiv \frac{W_t}{P_t} N_t$

Aggregate Euler equation

- Q_t must adapt so that no household chooses to save in equilibrium.

Aggregate Euler equation

- Q_t must adapt so that no household chooses to save in equilibrium.

- Household j intertemporal optimality condition:

$$Q_t = \beta E_t \left\{ \left(\frac{C_{jt+1}}{C_{jt}} \right)^{-1} \frac{P_t}{P_{t+1}} + \upsilon_{jt} \right\}.$$

Aggregate Euler equation

- Q_t must adapt so that no household chooses to save in equilibrium.

- Household j intertemporal optimality condition:

$$Q_t \quad = \quad \beta E_t \left\{ \left(\frac{C_{jt+1}}{C_{jt}} \right)^{-1} \frac{P_t}{P_{t+1}} + \upsilon_{jt} \right\}.$$

- The household with the lowest consumption growth, "the marginal saver", must be indifferent between saving and borrowing with $\upsilon_{jt} = 0$

Aggregate Euler equation

- Q_t must adapt so that no household chooses to save in equilibrium.
- Household j intertemporal optimality condition:

$$Q_t \quad = \quad \beta E_t \left\{ \left(\frac{C_{jt+1}}{C_{jt}} \right)^{-1} \frac{P_t}{P_{t+1}} + \upsilon_{jt} \right\}.$$

- The household with the lowest consumption growth, “the marginal saver”, must be indifferent between saving and borrowing with $\upsilon_{jt} = 0$
- All other households are constrained with $\upsilon_{jt} > 0$.

Aggregate Euler equation

- Q_t must adapt so that no household chooses to save in equilibrium.
- Household j intertemporal optimality condition:

$$Q_t \quad = \quad \beta E_t \left\{ \left(\frac{C_{jt+1}}{C_{jt}} \right)^{-1} \frac{P_t}{P_{t+1}} + \upsilon_{jt} \right\}.$$

- The household with the lowest consumption growth, "the marginal saver", must be indifferent between saving and borrowing with $\upsilon_{jt} = 0$
- All other households are constrained with $\upsilon_{jt} > 0$.
- Who is the marginal saver?

Aggregate Euler equation cont.

- Capitalist expected consumption growth: $E_t \frac{D_{t+1}}{D_t}$

Aggregate Euler equation cont.

- Capitalist expected consumption growth: $E_t \frac{D_{t+1}}{D_t}$
- Worker expected consumption growth: $E_t \frac{A_{jt+1} C_{t+1}}{A_{jt} C_t}$

Aggregate Euler equation cont.

- Capitalist expected consumption growth: $E_t \frac{D_{t+1}}{D_t}$
- Worker expected consumption growth: $E_t \frac{A_{jt+1} C_{t+1}}{A_{jt} C_t}$
- Assume aggregate shocks are small in relation to idiosyncratic shocks

Aggregate Euler equation cont.

- Capitalist expected consumption growth: $E_t \frac{D_{t+1}}{D_t}$
- Worker expected consumption growth: $E_t \frac{A_{jt+1} C_{t+1}}{A_{jt} C_t}$
- Assume aggregate shocks are small in relation to idiosyncratic shocks
  $\rightarrow$ **the "marginal saver" is the worker with lowest expected productivity growth:**

$$
\begin{aligned}
Q_t &= \beta^{eff} E_t \left\{ \frac{C_{t+1}^{-1}}{C_t^{-1}} \frac{P_t}{P_{t+1}} \right\} \\
\beta^{eff} &= \beta \max \left\{ E_t \left[\left(\frac{A_{jt+1}}{A_{jt}} \right)^{-1} \right] \right\} > \beta
\end{aligned}
$$

Aggregate Euler equation cont.

- Capitalist expected consumption growth: $E_t \frac{D_{t+1}}{D_t}$
- Worker expected consumption growth: $E_t \frac{A_{jt+1} C_{t+1}}{A_{jt} C_t}$
- Assume aggregate shocks are small in relation to idiosyncratic shocks
  $\rightarrow$ **the "marginal saver" is the worker with lowest expected productivity growth:**

$$
\begin{aligned}
Q_t &= \beta^{eff} E_t \left\{ \frac{C_{t+1}^{-1}}{C_t^{-1}} \frac{P_t}{P_{t+1}} \right\} \\
\beta^{eff} &= \beta \max \left\{ E_t \left[\left(\frac{A_{jt+1}}{A_{jt}} \right)^{-1} \right] \right\} > \beta
\end{aligned}
$$

- Note: steady state interest rate smaller but elasticity of aggregate consumption to interest rate is *as if* in RANK (Werning, 2015)

Aggregate Euler equation cont.

- Capitalist expected consumption growth: $E_t \frac{D_{t+1}}{D_t}$
- Worker expected consumption growth: $E_t \frac{A_{jt+1} C_{t+1}}{A_{jt} C_t}$
- Assume aggregate shocks are small in relation to idiosyncratic shocks
  $\rightarrow$ **the "marginal saver" is the worker with lowest expected productivity growth:**

$$
\begin{aligned}
Q_t &= \beta^{eff} E_t \left\{ \frac{C_{t+1}^{-1}}{C_t^{-1}} \frac{P_t}{P_{t+1}} \right\} \\
\beta^{eff} &= \beta \max \left\{ E_t \left[\left(\frac{A_{jt+1}}{A_{jt}} \right)^{-1} \right] \right\} > \beta
\end{aligned}
$$

- Note: steady state interest rate smaller but elasticity of aggregate consumption to interest rate is *as if* in RANK (Werning, 2015)
- Log-linearize Euler equation and aggregation result around steady state:

$$
\begin{aligned}
\hat{c}_t &= E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\hat{c}_t &= \hat{\omega}_t + \hat{n}_t
\end{aligned}
$$

Other equilibrium conditions

- On firm side, log-linearization of first order condition implies the standard Phillips curve:

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{mc}_t$$

  - where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$
  - with CRS production technology, $\hat{mc}_t = \hat{\omega}_t$

- On firm side, log-linearization of first order condition implies the standard Phillips curve:

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{mc}_t$$

  - where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$
  - with CRS production technology, $\hat{mc}_t = \hat{\omega}_t$

- Intratemporal optimality condition:

$$\begin{aligned} & \frac{W_{jt}}{P_t} = C_{jt} N_{jt}^{\varphi} \\ \Leftrightarrow \quad & \frac{W_t}{P_t} = A_{jt} C_t N_t^{\varphi} \\ \Rightarrow \quad & \hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t \end{aligned}$$

Summary of log-linearized equilibrium

- Our simple HANK model:

$$
\begin{aligned}
\text{Phillips}: &\quad \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{IS}: &\quad \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: &\quad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Labor supply}: &\quad \hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t \\
\text{Market clearing}: &\quad \hat{c}_t = \hat{\omega}_t + \hat{n}_t
\end{aligned}
$$

Summary of log-linearized equilibrium

- Our simple HANK model:

$$\begin{aligned}
\text{Phillips}: &\quad \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{IS}: &\quad \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: &\quad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Labor supply}: &\quad \hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t \\
\text{Market clearing}: &\quad \hat{c}_t = \hat{\omega}_t + \hat{n}_t
\end{aligned}$$

- Next step: how does this compare with a textbook RANK model?

Zero Liquidity HANK Model

Comparison with RANK Model

Textbook RANK model

- Departure point: Galí (2009), Ch. 3
- Household side: representative agent
  - collects labor and profit income
  - chooses how much to work, consume and save each period
- Firm side: Monopolistic firms
  - use labor inputs
  - set prices subject to the Calvo friction

Textbook model: Households

- The representative agent solves:

$$\max_{C_t, B_t, N_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$
$$\text{s.t.} \quad P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + P_t D_t$$

Textbook model: Production side

- A competitive final goods producer assembles intermediate goods using the Dixit-Stiglitz aggregator $\rightarrow$ CES demand for intermediate goods:

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} Y_t$$

Textbook model: Production side

- A competitive final goods producer assembles intermediate goods using the Dixit-Stiglitz aggregator $\rightarrow$ CES demand for intermediate goods:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} Y_t$$

- Intermediate goods producer i uses production technology

$$Y_{it} = N_{it}$$

Textbook model: Production side

- A competitive final goods producer assembles intermediate goods using the Dixit-Stiglitz aggregator $\rightarrow$ CES demand for intermediate goods:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} Y_t$$

- Intermediate goods producer i uses production technology

$$Y_{it} = N_{it}$$

- Calvo fricition: Intermediate goods firms can only reset prices with probability $1 - \theta$

Textbook model: Production side

- A competitive final goods producer assembles intermediate goods using the Dixit-Stiglitz aggregator $\rightarrow$ CES demand for intermediate goods:

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} Y_t$$

- Intermediate goods producer i uses production technology

$$Y_{it} = N_{it}$$

- Calvo fricition: Intermediate goods firms can only reset prices with probability $1 - \theta$

- A resetting firm maximizes the sum of expected discounted profits subject to the demand function

Textbook model: other conditions

- Fiscal authority does nothing – no taxation nor government debt

Textbook model: other conditions

- Fiscal authority does nothing – no taxation nor government debt

- Central bank follows Taylor rule:

$$\frac{1}{Q_t} = \frac{1}{\beta}\Pi_t^{\phi_\pi} e^{\nu_t}$$

$$\Rightarrow \qquad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t$$

Textbook model: other conditions

- Fiscal authority does nothing – no taxation nor government debt

- Central bank follows Taylor rule:

$$\frac{1}{Q_t} = \frac{1}{\beta}\Pi_t^{\phi_\pi} e^{\nu_t}$$
$$\Rightarrow \qquad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t$$

- Equilibrium conditions:

$$\begin{aligned} C_t &= Y_t \\ B_t &= 0 \end{aligned}$$

Textbook model: other conditions

- Fiscal authority does nothing – no taxation nor government debt

- Central bank follows Taylor rule:

$$\frac{1}{Q_t} = \frac{1}{\beta}\Pi_t^{\phi_\pi} e^{\nu_t}$$

$$\Rightarrow \qquad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t$$

- Equilibrium conditions:

$$\begin{aligned} C_t &= Y_t \\ B_t &= 0 \end{aligned}$$

- Textbook RANK model is easy to solve
  - Up to the first order, the state space consist only of aggregate variables
  - (Fluctuations in price dispersion are second order)

Summary of log-linearized equilibrium

- Textbook RANK model:

$$
\begin{aligned}
\text{Phillips}: \quad & \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{IS}: \quad & \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: \quad & \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Labor supply}: \quad & \hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t \\
\text{Market clearing}: \quad & \hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t
\end{aligned}
$$

where $\bar{S} = \frac{W_t N_t}{Y_t P_t} = \frac{\epsilon_p - 1}{\epsilon_p}$ is the steady state labor share

Summary of log-linearized equilibrium

- Textbook RANK model:

$$\begin{aligned}
\text{Phillips}: &\quad \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{IS}: &\quad \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: &\quad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Labor supply}: &\quad \hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t \\
\text{Market clearing}: &\quad \hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t
\end{aligned}$$

- Our simple HANK model:

$$\begin{aligned}
\text{Phillips}: &\quad \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{IS}: &\quad \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: &\quad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Labor supply}: &\quad \hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t \\
\text{Market clearing}: &\quad \hat{c}_t = \hat{\omega}_t + \hat{n}_t
\end{aligned}$$

where $\hat{c}_t$ is now the deviation in the aggregate consumption of workers

HANK vs RANK

- Key difference: which consumption aggregate enters into the IS curve

HANK vs RANK

- Key difference: which consumption aggregate enters into the IS curve
- Textbook RANK model: the consumption aggregate depends on both labor income and firm profits

$$\hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t$$

HANK vs RANK

- Key difference: which consumption aggregate enters into the IS curve
- Textbook RANK model: the consumption aggregate depends on both labor income and firm profits

$$\hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t$$

- Simple HANK model: the consumption aggregate in the IS curve depends only on labor income (firm profits irrelevant)

$$\hat{c}_t = \hat{\omega}_t + \hat{n}_t$$

HANK vs RANK

- Key difference: which consumption aggregate enters into the IS curve

- Textbook RANK model: the consumption aggregate depends on both labor income and firm profits

$$\hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t$$

- Simple HANK model: the consumption aggregate in the IS curve depends only on labor income (firm profits irrelevant)

$$\hat{c}_t = \hat{\omega}_t + \hat{n}_t$$

- Should we be concerned about the fact that firm profits are so important in the RANK model? Most households do not own firms...

Monetary Policy Transmission

Our goal

Inspect the monetary transmission mechanism in simple HANK model

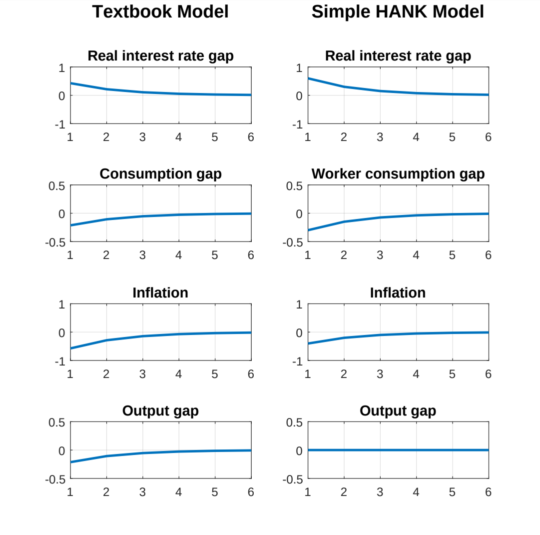
- Tractable model that admits analytical solutions
- Compare response to monetary shock to textbook RANK model
- Compare under two forms of nominal rigidities: rigid prices and rigid wages

- Let's feed in a shock to the Taylor Rule:

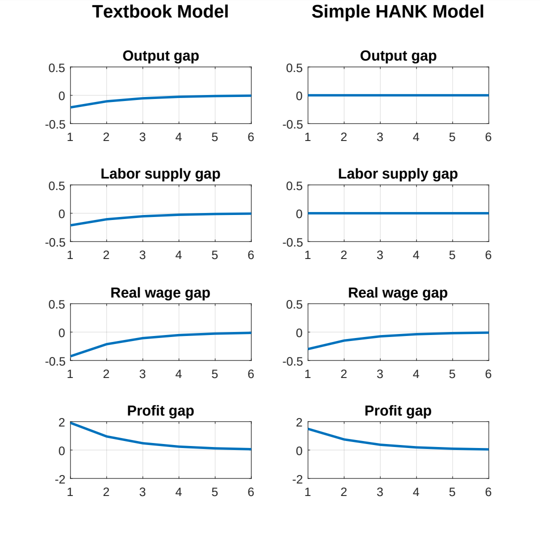
$$\hat{i}_t = \phi_\pi \pi_t^p + \nu_t$$

- Assume AR(1): $\nu_t = \rho_\nu \nu_{t-1} + \epsilon_{\nu t}$
- Feed in a 25 basis point shock with $\rho_\nu = 0.5$
- How do the two models respond?
- Other parameters follow Galí (2008)

Monetary Shock: Consumption, Output and Inflation



Monetary Shock: Labor supply, wages and profits



The determination of labor supply

- Textbook RANK model – intratemporal optimality and market clearing:

$$\hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$
$$\hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t$$

The determination of labor supply

- Textbook RANK model – intratemporal optimality and market clearing:

$$\hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$
$$\hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t$$

- Equilibrium labor supply:

$$\hat{n}_t = \frac{1 - \bar{S}}{\varphi + \bar{S}}(\hat{\omega}_t - \hat{d}_t)$$

The determination of labor supply

- Textbook RANK model – intratemporal optimality and market clearing:

$$\hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$
$$\hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t$$

- Equilibrium labor supply:

$$\hat{n}_t = \frac{1 - \bar{S}}{\varphi + \bar{S}}(\hat{\omega}_t - \hat{d}_t)$$

- Higher steady state profits means lower steady state labor share $\bar{S} \rightarrow$ income effect of wages dampened

The determination of labor supply

- Textbook RANK model – intratemporal optimality and market clearing:

$$\hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$
$$\hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t$$

- Equilibrium labor supply:

$$\hat{n}_t = \frac{1 - \bar{S}}{\varphi + \bar{S}}(\hat{\omega}_t - \hat{d}_t)$$

- Higher steady state profits means lower steady state labor share $\bar{S} \rightarrow$ income effect of wages dampened

- Countercylical response in profits: direct income effect, offsetting that of procyclical wages

The determination of labor supply

- simple HANK model – intratemporal optimality and market clearing:

$$\hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$

$$\hat{c}_t = \hat{\omega}_t + \hat{n}_t$$

The determination of labor supply

- simple HANK model – intratemporal optimality and market clearing:

$$\hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$
$$\hat{c}_t = \hat{\omega}_t + \hat{n}_t$$

- Equilibrium labor supply:

$$\hat{n}_t = 0$$

The determination of labor supply

- simple HANK model – intratemporal optimality and market clearing:

$$\hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$
$$\hat{c}_t = \hat{\omega}_t + \hat{n}_t$$

- Equilibrium labor supply:

$$\hat{n}_t = 0$$

- No profits $\rightarrow$ income and substitution effect cancel

The determination of labor supply

- simple HANK model – intratemporal optimality and market clearing:

$$\hat{\omega}_t = \varphi \hat{n}_t + \hat{c}_t$$
$$\hat{c}_t = \hat{\omega}_t + \hat{n}_t$$

- Equilibrium labor supply:

$$\hat{n}_t = 0$$

- No profits $\rightarrow$ income and substitution effect cancel

- The zero result in the simple HANK model is due to KPR preferences, generally depends on strength of income vis-a-vis substitution effect

Explanation

- Key finding from our simple HANK model: monetary policy does not affect output

Explanation

- Key finding from our simple HANK model: monetary policy does not affect output
- Why active monetary transmission in RANK but not in HANK?
  - Key difference: in RANK, working households receive profit income
  - RANK: profits respond countercyclically and dampens the relative income effect of wage fluctuations
  - HANK: income and substitution effects from wage changes cancel

Explanation

- Key finding from our simple HANK model: monetary policy does not affect output
- Why active monetary transmission in RANK but not in HANK?
  - Key difference: in RANK, working households receive profit income
  - RANK: profits respond countercyclically and dampens the relative income effect of wage fluctuations
  - HANK: income and substitution effects from wage changes cancel
- In other words, HANK model undoes the influence of profits on monetary transmission mechanism

Explanation

Take-aways

- What does our analysis say about the textbook RANK model?
    - Without rigid wage setting, transmission mechanism relies on
        - profits being distributed to working households
        - profits responding countercyclically

Explanation

Take-aways

- What does our analysis say about the textbook RANK model?
  - Without rigid wage setting, transmission mechanism relies on
    - profits being distributed to working households
    - profits responding countercyclically

- What does our analysis say about quantitative HANK models?
  - Heterogeneity/non-insurable income risk does not necessarily alter the equilibrium dynamics
  - The distribution of profits does alter equilibrium dynamics

Explanation

Take-aways

- What does our analysis say about the textbook RANK model?
  - Without rigid wage setting, transmission mechanism relies on
    - profits being distributed to working households
    - profits responding countercyclically
- What does our analysis say about quantitative HANK models?
  - Heterogeneity/non-insurable income risk does not necessarily alter the equilibrium dynamics
  - The distribution of profits does alter equilibrium dynamics

Does the transmission mechanism in RANK seem plausible?

- No. First, very few households own firms in the real world
- Second, most empirical evidence says that profits are procyclical: expansionary monetary policy $\rightarrow$ greater firm profits

How to resolve this problem?

- We need a model where firm profits are procyclical, not countercyclical

How to resolve this problem?

- We need a model where firm profits are procyclical, not countercyclical
- Let's think about the form of nominal rigidities in our model
  - Rigid prices:
    expansionary policy $\rightarrow$ wages rise faster than prices $\rightarrow$ profits fall
  - Rigid wages:
    expansionary policy $\rightarrow$ prices rise faster than wages $\rightarrow$ profits rise

How to resolve this problem?

- We need a model where firm profits are procyclical, not countercyclical
- Let's think about the form of nominal rigidities in our model
  - Rigid prices:
    expansionary policy $\rightarrow$ wages rise faster than prices $\rightarrow$ profits fall
  - Rigid wages:
    expansionary policy $\rightarrow$ prices rise faster than wages $\rightarrow$ profits rise
- Next step: Introduce rigid wages to our simple HANK model, again comparing its predictions to the corresponding textbook RANK model

Monetary Policy Transmission

Introducing Rigid Wages

- Households are differentiated by type and face CES-demand curve

Setup

- Households are differentiated by type and face CES-demand curve
- Households can only reset their wages subject to a quadratic adjustment cost (Rotemberg, 1987)

Setup

- Households are differentiated by type and face CES-demand curve
- Households can only reset their wages subject to a quadratic adjustment cost (Rotemberg, 1987)
- Timing within period:
  1. Aggregate shock is realized
  2. Households choose whether to participate and set their wages
  3. Idiosyncratic shocks are realized
  4. Trade in goods and bond markets

- Households are differentiated by type and face CES-demand curve
- Households can only reset their wages subject to a quadratic adjustment cost (Rotemberg, 1987)
- Timing within period:
  1. Aggregate shock is realized
  2. Households choose whether to participate and set their wages
  3. Idiosyncratic shocks are realized
  4. Trade in goods and bond markets
- Idiosyncratic shocks are iid + $B_{jt} = 0$, all HHs set the same wage

Setup

- Households are differentiated by type and face CES-demand curve
- Households can only reset their wages subject to a quadratic adjustment cost (Rotemberg, 1987)
- Timing within period:
  1. Aggregate shock is realized
  2. Households choose whether to participate and set their wages
  3. Idiosyncratic shocks are realized
  4. Trade in goods and bond markets
- Idiosyncratic shocks are iid + $B_{jt} = 0$, all HHs set the same wage
- $\rightarrow$ we can once more aggregate the model analytically

Setup

- Households are differentiated by type and face CES-demand curve
- Households can only reset their wages subject to a quadratic adjustment cost (Rotemberg, 1987)
- Timing within period:
  1. Aggregate shock is realized
  2. Households choose whether to participate and set their wages
  3. Idiosyncratic shocks are realized
  4. Trade in goods and bond markets
- Idiosyncratic shocks are iid $+ \; B_{jt} = 0$, all HHs set the same wage
- $\rightarrow$ we can once more aggregate the model analytically
- Why not Calvo friction as in Erceg-Henderson-Levin (2000)?
  - produces observationally equivalent wage Phillips curve
  - but the wage distribution depends on the aggregate state $\rightarrow$ aggregation of the Euler equation fails

Production technology

- Households are differentiated by type, aggregated by intermediate goods firms using CES production function

Production technology

- Households are differentiated by type, aggregated by intermediate goods firms using CES production function

- $\rightarrow$ Downward-sloping demand curve:

$$N_{jt} = \frac{1}{A_{jt}} \left(\frac{\frac{W_{jt}}{A_{jt}}}{W_t} \right)^{-\epsilon_w} N_t$$

Production technology

- Households are differentiated by type, aggregated by intermediate goods firms using CES production function

- $\rightarrow$ Downward-sloping demand curve:

$$N_{jt} = \frac{1}{A_{jt}} \left(\frac{\frac{W_{jt}}{A_{jt}}}{W_t} \right)^{-\epsilon_w} N_t$$

- and wage index:

$$W_t = \left[\int_{j=0}^{1} \left(\frac{W_{jt}}{A_{jt}} \right)^{1-\epsilon_w} dj \right]^{\frac{1}{1-\epsilon_w}}$$

Worker problem

- Participation utility cost ϑ

Worker problem

- Participation utility cost ϑ

- Conditional on particpating, worker j chooses $C_{jt+k}, N_{jt+k}, W_{jt+k}$ to maximize :

$$
\begin{aligned}
& E_t \sum_{k=0}^{\infty} \beta^k \left(\log C_{jt+k} - \frac{N_{jt+k}^{1+\varphi}}{1+\varphi} - \vartheta \right) \\
\text{s.t.} \quad & P_{t+k} C_{jt+k} + Q_{t+k} B_{jt+k} = \\
& W_{jt+k} N_{jt+k} - \frac{\xi}{2} \left(\frac{W_{jt+k}}{W_{jt+k-1}} - 1 \right)^2 W_{jt+k} N_{jt+k} + B_{jt+k-1} \\
& B_{jt+k} \geq 0 \\
& N_{jt} = \frac{1}{A_{jt}} \left(\frac{\frac{W_{jt}}{A_{jt}}}{W_t} \right)^{-\epsilon_w} N_t
\end{aligned}
$$

Worker problem

- Participation utility cost ϑ
- Conditional on particpating, worker j chooses $C_{jt+k}, N_{jt+k}, W_{jt+k}$ to maximize :

$$
\begin{aligned}
& E_t \sum_{k=0}^{\infty} \beta^k \left(\log C_{jt+k} - \frac{N_{jt+k}^{1+\varphi}}{1+\varphi} - \vartheta \right) \\
\text{s.t.} \quad & P_{t+k} C_{jt+k} + Q_{t+k} B_{jt+k} = \\
& W_{jt+k} N_{jt+k} - \frac{\xi}{2} \left(\frac{W_{jt+k}}{W_{jt+k-1}} - 1 \right)^2 W_{jt+k} N_{jt+k} + B_{jt+k-1} \\
& B_{jt+k} \geq 0 \\
& N_{jt} = \frac{1}{A_{jt}} \left(\frac{\frac{W_{jt}}{A_{jt}}}{W_t} \right)^{-\epsilon_w} N_t
\end{aligned}
$$

- As before, we set parametric conditions so that the capitalists choose not to participate.

Equilibrium implications I

- Becuase idiosyncratic shocks are iid and realized after wages are set, all households set the same wage $\rightarrow$ individual worker income is proportional to average worker income:

$$W_{jt+k}N_{jt+k} = \frac{A_{jt+k}^{\epsilon_w - 1}}{\left[\int_{s=0}^{1} A_{st+k}^{\epsilon_w - 1} ds\right]} W_{t+k}N_{t+k}$$

Equilibrium implications I

- Becuase idiosyncratic shocks are iid and realized after wages are set, all households set the same wage $\rightarrow$ individual worker income is proportional to average worker income:

$$W_{jt+k}N_{jt+k} = \frac{A_{jt+k}^{\epsilon_w - 1}}{\left[\int_{s=0}^{1} A_{st+k}^{\epsilon_w - 1} ds\right]} W_{t+k}N_{t+k}$$

- Since all households set the same wage, the adjustment cost is indentical to all workers $\rightarrow$ individual worker consumption is still proportional to average worker consumption:

- Becuase idiosyncratic shocks are iid and realized after wages are set, all households set the same wage $\rightarrow$ individual worker income is proportional to average worker income:

$$W_{jt+k}N_{jt+k} = \frac{A_{jt+k}^{\epsilon_w - 1}}{\left[\int_{s=0}^{1} A_{st+k}^{\epsilon_w - 1} ds\right]} W_{t+k}N_{t+k}$$

- Since all households set the same wage, the adjustment cost is indentical to all workers $\rightarrow$ individual worker consumption is still proportional to average worker consumption:

$$\begin{aligned}
C_{jt+k} &= \left(1 - \frac{\xi}{2}\left(\Pi_{t+k}^{w} - 1\right)^2\right)\frac{W_{jt+k}}{P_{t+k}}N_{jt+k} \\
&= \left(1 - \frac{\xi}{2}\left(\Pi_{t+k}^{w} - 1\right)^2\right)\frac{A_{jt+k}^{\epsilon_w - 1}}{\left[\int_{s=0}^{1} A_{st+k}^{\epsilon_w - 1} ds\right]}\frac{W_{t+k}}{P_{t+k}}N_{t+k} \\
&= \frac{A_{jt+k}^{\epsilon_w - 1}}{\left[\int_{s=0}^{1} A_{st+k}^{\epsilon_w - 1} ds\right]} C_{t+k}
\end{aligned}$$

- Since all households set the same wage, and worker consumption is proportional to aggregate consumption, we retrieve a standard wage Phillips curve:

$$\begin{aligned}\pi_t^w &= \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - \hat{mrs}_t) \\ &= \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - (\hat{c}_t + \varphi \hat{n}_t))\end{aligned}$$

where $\lambda_w = \frac{\epsilon_w - 1}{\xi}$

Equilibrium implications II

- Since all households set the same wage, and worker consumption is proportional to aggregate consumption, we retrieve a standard wage Phillips curve:

$$\begin{aligned}\pi_t^w &= \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - \hat{mrs}_t) \\ &= \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - (\hat{c}_t + \varphi \hat{n}_t))\end{aligned}$$

  where $\lambda_w = \frac{\epsilon_w - 1}{\xi}$

- Because idiosyncratic shocks are iid and worker consumption is proportional to aggregate consumption, the euler equation aggregates as before:

$$\hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1})$$

Summary of log-linearized equilibrium

- Textbook RANK model:

$$\begin{aligned}
\text{Phillips}: \quad & \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{Wage Phillips}: \quad & \pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - (\hat{c}_t + \varphi \hat{n}_t)) \\
\text{Wage accounting}: \quad & \hat{\omega}_t = \hat{\omega}_{t-1} + \pi_t^w - \pi_t^p \\
\text{IS}: \quad & \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: \quad & \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Market clearing}: \quad & \hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t
\end{aligned}$$

- Our simple HANK model:

$$\begin{aligned}
\text{Phillips}: \quad & \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{Wage Phillips}: \quad & \pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - (\hat{c}_t + \varphi \hat{n}_t)) \\
\text{Wage accounting}: \quad & \hat{\omega}_t = \hat{\omega}_{t-1} + \pi_t^w - \pi_t^p \\
\text{IS}: \quad & \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: \quad & \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Market clearing}: \quad & \hat{c}_t = \hat{\omega}_t + \hat{n}_t
\end{aligned}$$

Summary of log-linearized equilibrium

- Textbook RANK model:

$$
\begin{aligned}
\text{Phillips}: &\quad \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{Wage Phillips}: &\quad \pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - (\hat{c}_t + \varphi \hat{n}_t)) \\
\text{Wage accounting}: &\quad \hat{\omega}_t = \hat{\omega}_{t-1} + \pi_t^w - \pi_t^p \\
\text{IS}: &\quad \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: &\quad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Market clearing}: &\quad \hat{c}_t = \bar{S}(\hat{\omega}_t + \hat{n}_t) + (1 - \bar{S})\hat{d}_t
\end{aligned}
$$

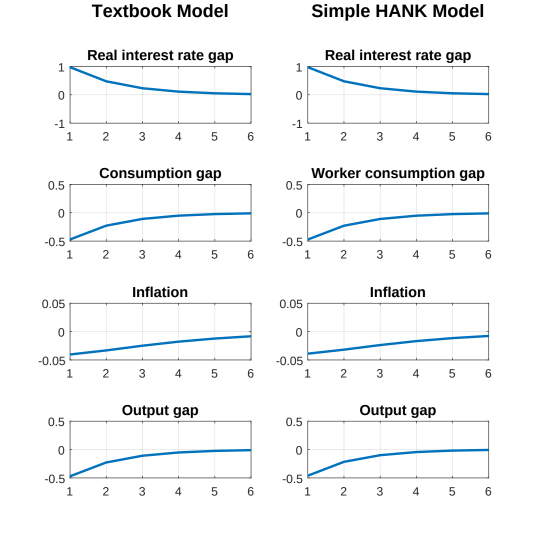
- Our simple HANK model:

$$
\begin{aligned}
\text{Phillips}: &\quad \pi_t^p = \beta E_t \pi_{t+1}^p + \lambda_p \hat{\omega}_t \\
\text{Wage Phillips}: &\quad \pi_t^w = \beta E_t \pi_{t+1}^w - \lambda_w(\hat{\omega}_t - (\hat{c}_t + \varphi \hat{n}_t)) \\
\text{Wage accounting}: &\quad \hat{\omega}_t = \hat{\omega}_{t-1} + \pi_t^w - \pi_t^p \\
\text{IS}: &\quad \hat{c}_t = E_t \hat{c}_{t+1} - (\hat{i}_t - E_t \pi_{t+1}) \\
\text{Taylor rule}: &\quad \hat{i}_t = \phi_\pi \pi_t^p + \nu_t \\
\text{Market clearing}: &\quad \hat{c}_t = \hat{\omega}_t + \hat{n}_t
\end{aligned}
$$

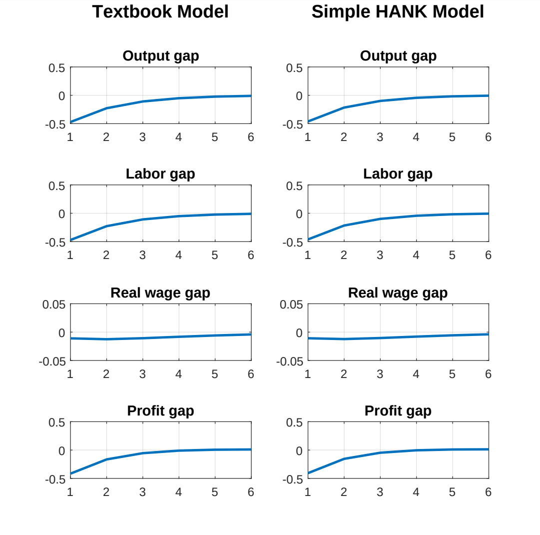
A monetary experiment

- Assume AR(1): $\nu_t = \rho_\nu \nu_{t-1} + \epsilon_{\nu t}$
- Feed in a 25 basis point shock with $\rho_\nu = 0.5$
- How do the two models respond?
- Parameterization: standard, we set ξ so that the wage Phillips curve has the same slope as the wage Phillips curve derived with Calvo friction, using resetting probability from Galí (2008)

Monetary Shock: Consumption, Output and Inflation



Monetary Shock: Labor supply, wages and profits



Rigid wages: Interpretation

- As before, real interest rate increase and worker consumption demand fall

Rigid wages: Interpretation

- As before, real interest rate increase and worker consumption demand fall
- With sufficiently rigid wage setting, nominal wages, inflation and real wages are all non-responsive

Rigid wages: Interpretation

- As before, real interest rate increase and worker consumption demand fall
- With sufficiently rigid wage setting, nominal wages, inflation and real wages are all non-responsive
- Because real wages do no respond, profits, and thus capitalist consumption demand is alinged with worker consumption

Rigid wages: Interpretation

- As before, real interest rate increase and worker consumption demand fall
- With sufficiently rigid wage setting, nominal wages, inflation and real wages are all non-responsive
- Because real wages do no respond, profits, and thus capitalist consumption demand is alinged with worker consumption
- Goods market can only clear if labor usage respond procyclically

Rigid wages: Interpretation

- As before, real interest rate increase and worker consumption demand fall
- With sufficiently rigid wage setting, nominal wages, inflation and real wages are all non-responsive
- Because real wages do no respond, profits, and thus capitalist consumption demand is alinged with worker consumption
- Goods market can only clear if labor usage respond procyclically
- Labor usage becomes "demand-determined"

Rigid wages: Interpretation

- As before, real interest rate increase and worker consumption demand fall
- With sufficiently rigid wage setting, nominal wages, inflation and real wages are all non-responsive
- Because real wages do no respond, profits, and thus capitalist consumption demand is alinged with worker consumption
- Goods market can only clear if labor usage respond procyclically
- Labor usage becomes "demand-determined"
- No role for income and substitution effects

Summary

1. What does our analysis say about the textbook RANK model?
    - Transmission mechanism in textbook RANK model does not square well with the data
    - Monetary policy affects output because 1) profits are distributed to working households and 2) profits respond countercyclically

Summary

1. What does our analysis say about the textbook RANK model?
    - Transmission mechanism in textbook RANK model does not square well with the data
    - Monetary policy affects output because 1) profits are distributed to working households and 2) profits respond countercyclically

2. What does our analysis say about quantitative HANK models?
    - Non-insurable labor income risk does not necessarily alter the equilibrium dynamics
    - The distribution of profits does when labor markets are flexible

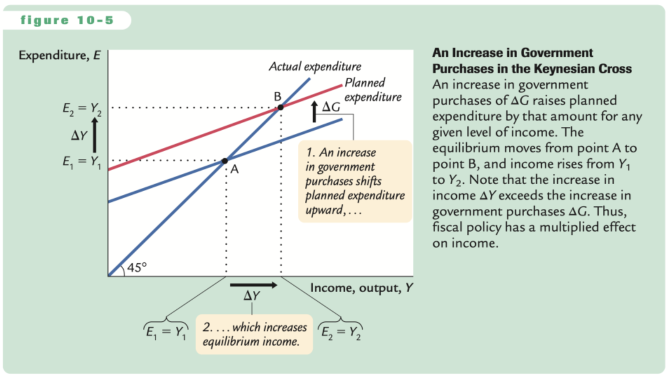
Summary

1. What does our analysis say about the textbook RANK model?
    - Transmission mechanism in textbook RANK model does not square well with the data
    - Monetary policy affects output because 1) profits are distributed to working households and 2) profits respond countercyclically
2. What does our analysis say about quantitative HANK models?
    - Non-insurable labor income risk does not necessarily alter the equilibrium dynamics
    - The distribution of profits does when labor markets are flexible
3. What does our analysis say about the real world?
    - Monetary transmission mechanism only active when wages are sufficiently rigid
    - Consistent with evidence from calendar-varying VARs (Olivei-Tenreyro, 2007, 2010; Bjorklund-Carlsson-Skans, 2016)

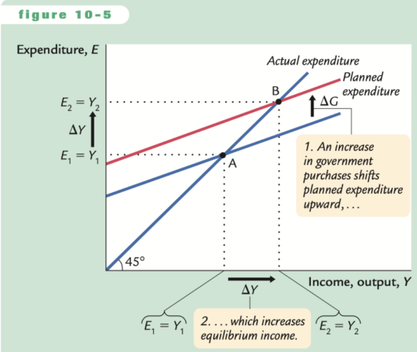
Fiscal Policy Transmission

Motivation

- Long-standing open question: what determines the fiscal multiplier?



- Long-standing open question: what determines the fiscal multiplier?



An Increase in Government Purchases in the Keynesian Cross An increase in government purchases of ΔG raises planned expenditure by that amount for any given level of income. The equilibrium moves from point A to point B, and income rises from Y_1 to Y_2. Note that the increase in income ΔY exceeds the increase in government purchases ΔG. Thus, fiscal policy has a multiplied effect on income.

- To answer this question: will turn to Auclert, Rognlie, and Straub (2018)

Summary

Summary

- **Previously:** Learned to solve and simulate quantitative HANK models

Summary

- **Previously:** Learned to solve and simulate quantitative HANK models
- **Today:** Discussed two stylized HANK models that allow us to derive analytical insights
  - Monetary policy: key role of profits in RANK and HANK models, important to fix the model to make the mechanism more realistic
  - Fiscal policy: derived the intertemporal Keynesian cross, showing what conditions are necessary for a fiscal multiplier greater than one

Summary

- **Previously:** Learned to solve and simulate quantitative HANK models
- **Today:** Discussed two stylized HANK models that allow us to derive analytical insights
  - Monetary policy: key role of profits in RANK and HANK models, important to fix the model to make the mechanism more realistic
  - Fiscal policy: derived the intertemporal Keynesian cross, showing what conditions are necessary for a fiscal multiplier greater than one
- **Important tool:** altering the degree of risk sharing in the economy
  - RANK models: full risk sharing
  - Quantitative HANK models: partial risk sharing
  - Zero liquidity HANK model: no risk sharing

Summary

- **Previously:** Learned to solve and simulate quantitative HANK models

- **Today:** Discussed two stylized HANK models that allow us to derive analytical insights
  - Monetary policy: key role of profits in RANK and HANK models, important to fix the model to make the mechanism more realistic
  - Fiscal policy: derived the intertemporal Keynesian cross, showing what conditions are necessary for a fiscal multiplier greater than one

- **Important tool:** altering the degree of risk sharing in the economy
  - RANK models: full risk sharing
  - Quantitative HANK models: partial risk sharing
  - Zero liquidity HANK model: no risk sharing

- **Next time:** global solution methods