CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

12. More on HANK

Adv. Macro: Heterogenous Agent Models

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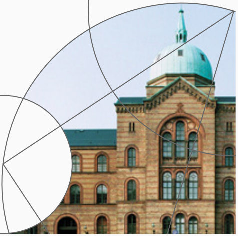

Introduction

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- **Today:** Fiscal policy in a HANK model with sticky wages
  - Some analytical insights
  - Additional numerical results

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- **Today:** Fiscal policy in a HANK model with sticky wages
  - Some analytical insights
  - Additional numerical results
- **Literature:** Auclert et. al. (2023),
  »The Intertemporal Keynesian Cross«
  - Long paper with many (technical) details
  - We will focus on the main results

Model

Households

- **Household problem:**

$$
\begin{aligned}
v_t(z_t, a_{t-1}) &= \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[v_{t+1}(z_{t+1}, a_t) \right] \\
\text{s.t. } a_t + c_t &= (1 + r_t^a) a_{t-1} + (1 - \tau_t) w_t \ell_t z_t + \chi_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \ \ , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \mathbb{E}[z_t] = 1 \\
a_t &\geq 0
\end{aligned}
$$

- **Active decisions:** Consumption-saving, c_t (and a_t)
- **Union decision:** Labor supply, ℓ_t
- **Consumption function:** $C_t^{hh} = C^{hh} \left(\{ r_s^a, \tau_s, w_s, \ell_s, \chi_s \}_{s \geq t} \right)$

Firms

- **Production and profits:**

$$Y_t = \Gamma_t L_t$$
$$\Pi_t = P_t Y_t - W_t L_t$$

- **First order condition:**

$$\frac{\partial \Pi_t}{\partial L_t} = 0 \Leftrightarrow P_t \Gamma_t - W_t = 0 \Leftrightarrow w_t \equiv W_t / P_t = \Gamma_t$$

Zero profits: $\Pi_t = 0$

- **Wage and price inflation:**

$$\pi_t^w \equiv W_t / W_{t-1} - 1$$
$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 = \frac{W_t / \Gamma_t}{W_{t-1} / \Gamma_{t-1}} - 1 = \frac{1 + \pi_t^w}{\Gamma_t / \Gamma_{t-1}} - 1$$

Union

- Everybody work the same:

$$\ell_t = L_t^{hh}$$

- Unspecified *wage adjustment costs* imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left(\varphi \left(L_t \right)^{\nu} - \frac{1}{\mu} \left(1 - \tau_t \right) w_t \left(C_t \right)^{-\sigma} \right) + \beta \pi_{t+1}^w$$

Government

- **Spending:** G_t
- **Tax bill:** T_t

$$T_t = \int \tau_t w_t \ell_t z_t d\boldsymbol{D}_t = \tau_t \Gamma_t L_t = \tau_t Y_t$$

- **One-period bonds:**

$$B_t = (1 + r_t^b) B_{t-1} + G_t + \chi_t - T_t$$

- **Long-term bonds:** Geometrically declining payment stream of $1, \delta, \delta^2, \ldots$ for $\delta \in [0,1]$. The bond price is q_t.

$$q_t(B_t - \delta B_{t-1}) = B_{t-1} + G_t + \chi_t - T_t$$

- Potential **tax-rule:**

$$\tau_t = \tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}}$$

Central bank

- Standard **Taylor rule:**

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left((1 + r_{ss}) \, (1 + \pi_t)^{\phi_\pi} \right)^{1-\rho_i}$$

  Alternative: Real rate rule

$$1 + i_t = (1 + r_{ss})(1 + \pi_{t+1})$$

  Indeterminacy: Consider limit or assume future tightening

- **Fisher-equation:**

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$

Arbitrage

1. One-period *real* bond, $q_t = 1$:

$$t > 0: \ r_t^b = r_t^a = r_{t-1}$$
$$r_0^b = r_0^a = 1 + r_{ss}$$

2. One-period *nominal* bond, $q_t = 1$:

$$t > 0: \ r_t^b = r_t^a = r_{t-1}$$
$$t > 0: \ r_0^b = r_0^a = \left(1 + r_{ss}\right)\left(1 + \pi_{ss}\right) / \left(1 + \pi_0\right)$$

3. Long-term *(real)* bonds:

$$\frac{1 + \delta q_{t+1}}{q_t} = 1 + r_t$$
$$1 + r_t^b = 1 + r_t^a = \frac{1 + \delta q_t}{q_{t-1}} = \begin{cases} 1 + r_{t-1} & \text{for } t > 0 \\ \frac{1+\delta q_0}{q_{-1}} & \text{for } t > 0 \end{cases}$$

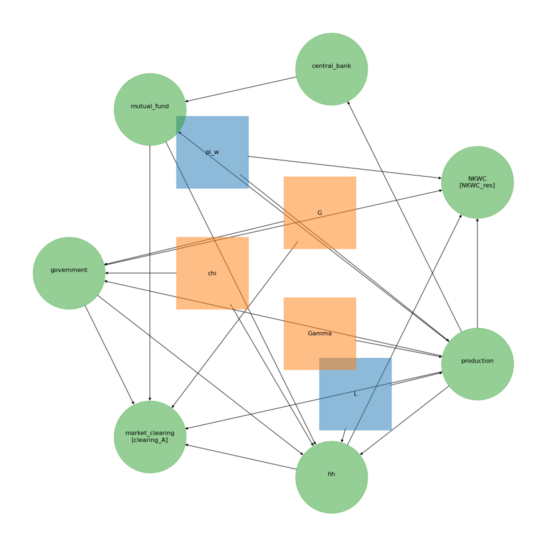
Market clearing

1. Asset market: $q_t B_t = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

Equation system

$$
\boldsymbol{H}(\pi^{\boldsymbol{w}}, \boldsymbol{L}, \boldsymbol{G}, \chi, \boldsymbol{\Gamma}) = \begin{bmatrix}
w_t - \Gamma_t \\
1 + \pi_t - \frac{1+\pi_t^w}{\Gamma_t / \Gamma_{t-1}} \\
Y_t - \Gamma_t L_t \\
1 + i_t - (1 + i_{t-1})^{\rho_i} \left((1 + r_{ss}) (1 + \pi_t)^{\phi_\pi} \right)^{1-\rho_i} \\
1 + r_t - \frac{1+i_t}{1+\pi_{t+1}} \\
\frac{1+\delta q_{t+1}}{q_t} - (1 + r_t) \\
1 + r_t^a - \frac{1+\delta q_t}{q_{t-1}} \\
\tau_t - \left[\tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}} \right] \\
q_t (B_t - \delta B_{t-1}) - [B_{t-1} + G_t + \chi_t - \tau_t Y_t] \\
q_t B_t - A_t^{hh} \\
\pi_t^w - \left[\kappa \left(\varphi \left(L_t^{hh} \right)^{\nu} - \frac{1}{\mu} (1 - \tau_t) w_t \left(C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^W \right]
\end{bmatrix} = \mathbf{0}
$$

DAG



central_bank
mutual_fund
pi_w
NKWC
[NKWC_res]
G
government
chi
Gamma
production
L
market_clearing
[clearing_A]
hh

Analytical insights

Simpler consumption function

- **Assumptions:**
  1. One-period real bond
  2. No lump-sum transfers, $\chi_t = 0$
  3. Real rate rule: $r_t = r_{ss}$
  4. Fiscal policy in terms of dG_t and dT_t satisfying IBC

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (dG_t - dT_t) = 0$$

- **Tax-bill:** $T_t = \tau_t w_t \int \ell_t z_t d\boldsymbol{D}_t = \tau_t \Gamma_t L_t = \tau_t L_t$
- **Household income:** $(1 - \tau_t) w_t \ell_t z_t = (Y_t - T_t) z_t$
- **Consumption function:** Simplifies to

$$C_t^{hh} = C^{hh}(\{Y_s - T_s\}_{s \geq t}) = C^{hh}(\boldsymbol{Y} - \boldsymbol{T})$$

Two-equation version in Y and r

$$\boldsymbol{Y} = C^{hh}(\boldsymbol{r}, \boldsymbol{Y} - \boldsymbol{T}) + \boldsymbol{G}$$
$$\boldsymbol{r} = \mathcal{R}(\boldsymbol{Y}; \boldsymbol{G}, \boldsymbol{T})$$

- **Second equation:**
  1. **Government:** $\boldsymbol{T}, \boldsymbol{Y} \rightarrow \boldsymbol{\tau}$
  2. **Goods market clearing:** $\boldsymbol{G}, \boldsymbol{Y} \rightarrow \boldsymbol{C}$
  3. **Firm behavior I:** $\boldsymbol{\Gamma}, \boldsymbol{Y} \rightarrow \boldsymbol{L}, \boldsymbol{w}$
  4. **NKWC:** $\boldsymbol{L}, \boldsymbol{w}, \boldsymbol{\tau} \rightarrow \boldsymbol{\pi}^{\boldsymbol{w}}$
  5. **Firm behavior II:** $\boldsymbol{\pi}^{\boldsymbol{w}} \rightarrow \boldsymbol{\pi}$
  6. **Central bank:** $\boldsymbol{\pi} \rightarrow \boldsymbol{i}$
  7. **Fisher:** $\boldsymbol{i}, \boldsymbol{\pi} \rightarrow \boldsymbol{r}$
- **Heterogeneity does not enter** $\mathcal{R}(\boldsymbol{Y}; \boldsymbol{G}, \boldsymbol{T})$
- **Real rate rule:** Inflation is a side-show

Intertemporal Keynesian Cross

$$\boldsymbol{Y} = C^{hh}(\boldsymbol{Y} - \boldsymbol{T}) + \boldsymbol{G}$$

- **Total differentiation:**

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} (dY_t - dT_t)$$

IBC implies: $\sum_{t=0}^{\infty}(1 + r_{ss})^{-t} \frac{\partial C_t^{hh}}{\partial Z_s} = (1 + r_{ss})^{-s}$

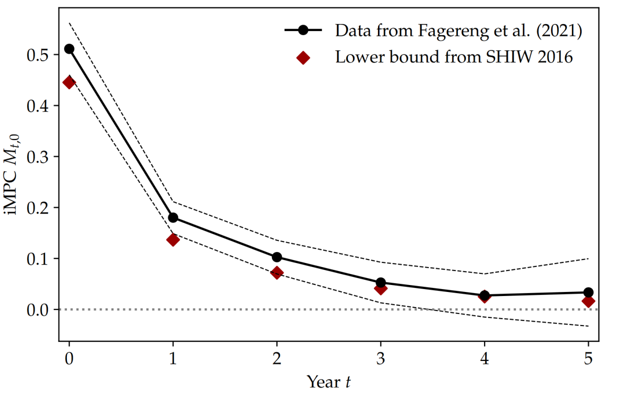
- **Intertemporal Keynesian Cross** in vector form

$$d\boldsymbol{Y} = d\boldsymbol{G} - d\boldsymbol{T} + \boldsymbol{M} d\boldsymbol{Y}$$

where $M_{t,s} = \frac{\partial C_t^{hh}}{\partial Z_s}$ encodes entire *complexity*

iMPCs in the data

Figure 1: iMPCs in the Norwegian and Italian data



Other columns: Druedahl et al. (2023) show in micro-data that consumption responds today to news about future income.

Perspective: Static Keynesian Cross

- **Old Keynesians:** Consumption only depend on current income

$$Y_t = G_t + C^{hh}(Y_t - T_t)$$

- **Total differentiate:**

$$\begin{aligned} dY_t &= dG_t + \frac{\partial C_t^{hh}}{\partial Z_t}(dY_t - dT_t) \\ &= dG_t + \text{mpc}(dY_t - dT_t) \end{aligned}$$

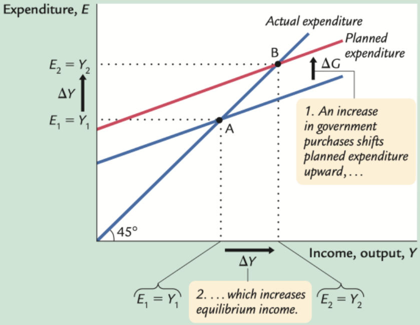
- **Solution**

$$dY_t = \frac{1}{1 - \text{mpc}}(dG_t - \text{mpc}dT_t)$$

from multiplier-process $1 + \text{mpc} + \text{mpc}^2 \cdots = \frac{1}{1-\text{mpc}}$

Static Keynesian Cross

figure 10-5



An Increase in Government Purchases in the Keynesian Cross An increase in government purchases of ΔG raises planned expenditure by that amount for any given level of income. The equilibrium moves from point A to point B, and income rises from Y_1 to Y_2. Note that the increase in income ΔY exceeds the increase in government purchases ΔG. Thus, fiscal policy has a multiplied effect on income.

Intertemporal solution technicalities

- **NPV-vector:** $\boldsymbol{q} = [1, (1+r_{ss})^{-1}, (1+r_{ss})^{-2}, \dots]'$
- **Household IBC:** $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$ and $\boldsymbol{q}'(\boldsymbol{I} - \boldsymbol{M}) = \boldsymbol{0}$
- **Government IBC:** $\boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}) = \boldsymbol{0}$
- **Problem:** $(\boldsymbol{I} - \boldsymbol{M})^{-1}$ cannot exist because

$$
\begin{aligned}
(\boldsymbol{I} - \boldsymbol{M})d\boldsymbol{Y} &= d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T} \Leftrightarrow \\
\boldsymbol{q}'(\boldsymbol{I} - \boldsymbol{M})d\boldsymbol{Y} &= \boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}) \Leftrightarrow \\
\boldsymbol{0} &= \boldsymbol{0}
\end{aligned}
$$

- **Problem:** If unique solution then on the form

$$
\begin{aligned}
d\boldsymbol{Y} &= \mathcal{M}(d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}) \\
\mathcal{M} &= (\boldsymbol{K}(\boldsymbol{I} - \boldsymbol{M}))^{-1}\boldsymbol{K}
\end{aligned}
$$

Indeterminancy: Still work-in-progress (Auclert et. al., 2023)

Intermezzo: Response of consumption

$$
\begin{aligned}
d\boldsymbol{Y} &= d\boldsymbol{G} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{G}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) \Rightarrow \\
d\boldsymbol{C} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})
\end{aligned}
$$

Fiscal multipliers

$$d\boldsymbol{Y} = d\boldsymbol{G} + \underbrace{\mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})}_{d\boldsymbol{C}}$$

- **Balanced budget multiplier:**

$$d\boldsymbol{G} = d\boldsymbol{T} \Rightarrow d\boldsymbol{Y} = d\boldsymbol{G}, d\boldsymbol{C} = 0$$

  Note: Central that income and taxes affect household income in exactly the same way = no redistribution

- **Deficit multiplier:** $d\boldsymbol{G} \neq d\boldsymbol{T}$
  1. Larger effect of $d\boldsymbol{G}$ than $d\boldsymbol{T}$
  2. *Numerical results needed*

Fiscal multiplier

Impact-multiplier:

$$\frac{\partial Y_0}{\partial G_0}$$

Cummulative-multiplier:

$$\frac{\sum_{t=0}^{\infty}(1+r_{ss})^{-1}dY_t}{\sum_{t=0}^{\infty}(1+r_{ss})^{-1}dG_t}$$

Comparison with RA model

- **From lecture 1:** $\beta(1+r_{ss})=1$ implies

$$C_t=(1-\beta)\sum_{s=0}^{\infty}\beta^s Y_{t+s}^{hh}+r_{ss}a_{-1}$$

- The **iMPC-matrix** becomes

$$\boldsymbol{M}^{RA}=\begin{bmatrix}(1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots\end{bmatrix}=(1-\beta)\mathbf{1}\boldsymbol{q}'$$

- **Consumption response** is zero

$$\begin{aligned} d\boldsymbol{C}^{\boldsymbol{RA}} &= \mathcal{M}\boldsymbol{M}^{\boldsymbol{RA}}(d\boldsymbol{G}-d\boldsymbol{T}) \\ &= \mathcal{M}(1-\beta)\mathbf{1}\boldsymbol{q}'(d\boldsymbol{G}-d\boldsymbol{T}) \\ &= \mathbf{0} \Leftrightarrow d\boldsymbol{Y}=d\boldsymbol{G} \end{aligned}$$

Comparison with TA model

- **Hands-too-Mouth (HtM) households:** λ share have $C_t = Y_{t+s}^{hh}$

$$\boldsymbol{M}^{TA} = (1-\lambda)\boldsymbol{M}^{RA} + \lambda \boldsymbol{I}$$

- **Intertemporal Keynesian Cross** becomes

$$(\boldsymbol{I} - \boldsymbol{M}^{TA})d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}^{TA}d\boldsymbol{T}$$

$$(\boldsymbol{I} - \boldsymbol{M}^{RA})d\boldsymbol{Y} = \underbrace{\frac{1}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right]}_{d\tilde{\boldsymbol{G}}_t} - \boldsymbol{M}^{RA}d\boldsymbol{T}$$

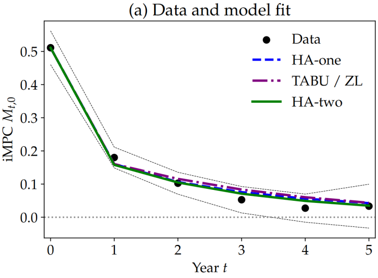
- **Same solution-form as RA:** $d\boldsymbol{Y} = d\tilde{\boldsymbol{G}}_t$

$$d\boldsymbol{Y} = d\tilde{\boldsymbol{G}}_t = d\boldsymbol{G}_t + \frac{\lambda}{1-\lambda}\left[d\boldsymbol{G} - dT\right]$$

- Still a cummulative multiplier is 1 (both for RA and HtM)

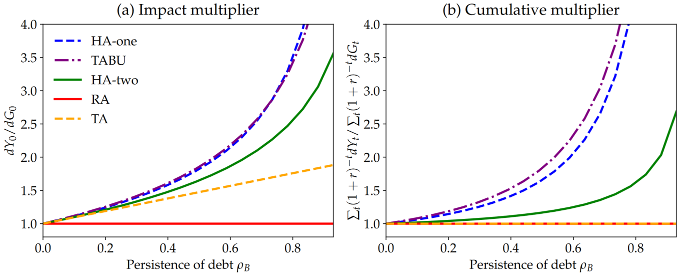
iMPCs in models

Figure 2: iMPCs in the Norwegian data and several models



Multipliers and debt-financing

Figure 5: Multipliers according to the IKC



Note. These figures assume a persistence of government spending equal to $\rho_G = 0.76$, and vary ρ_B in $dB_t = \rho_B(dB_{t-1} + dG_t)$. See section 7.1 for details on calibration choices.

Fiscal multiplier

Generalized IKC

- **Budget constraint** can be written with initial capital gain

$$a_t + c_t = (Y_t - T_t)z_t + \chi_t + \begin{cases} (1 + r_{t-1})a_{t-1} & \text{if } t > 0 \\ (1 + r_{ss} + \text{cap}_0)a_{t-1} & \text{if } t = 0 \end{cases}$$

1. Real bond: $\text{cap}_0 = 0$
2. Nominal bond:

$$\text{cap}_0 = \frac{(1 + r_{ss})(1 + \pi_{ss})}{1 + \pi_0} - (1 + r_{ss})$$

3. Long-term bond:

$$\text{cap}_0 = \frac{1 + \delta q_0}{q_{-1}} - (1 + r_{ss})$$

Generalized IKC

- **Consumption-function** $C_t^{hh} = C^{hh}(\boldsymbol{Y} - \boldsymbol{T}, \chi, \boldsymbol{r}, \mathbf{cap}_0)$ implies

$$d\boldsymbol{C} = \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + \boldsymbol{M}^{\chi} d\boldsymbol{\chi} + \boldsymbol{M}^{r} d\boldsymbol{r} + \boldsymbol{m}^{\text{cap}} \text{cap}_0$$

where

$$\boldsymbol{M}_{t,s}^{\chi} = \left[\frac{\partial C_t^{hh}}{\partial \chi_s}\right], \boldsymbol{M}_{t,s}^{r} = \left[\frac{\partial C_t^{hh}}{\partial r_s}\right], \boldsymbol{m}_t^{\text{cap}} = \left[\frac{\partial C_t^{hh}}{\partial \text{cap}_0}\right]$$

- Why are $\boldsymbol{M}^{\chi}$ and $\boldsymbol{M}$ different?

Exercise

Exercise

Use *HANK-sticky-wages* in sub-folder.

1. Compute *fiscal multipliers* varying:
   1.1 Bond maturity: δ
   1.2 Fiscal aggressiveness: ω
   1.3 Monetary aggressiveness: ϕ_π
2. Does the model match the evidence of intertemporal MPCs?
   What happens to the fiscal multiplier if the fit is improved?

Summary

Summary and next week

- **Today:** Fiscal policy in a HANK model with sticky wages
- **Next week:** I-HANK
- **Homework:**
  1. Work on exercise
  2. Read: Druedahl et al. (2022),
     »The Transmission of Foreign Demand Shocks«