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# 12. More on HANK

## Adv. Macro: Heterogenous Agent Models

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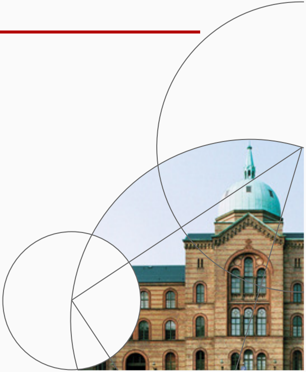

# Introduction

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- **Today:** Fiscal policy in a HANK model with sticky wages
  - Some analytical insights
  - Additional numerical results

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- **Today:** Fiscal policy in a HANK model with sticky wages
  - Some analytical insights
  - Additional numerical results
- **Literature:** Auclert et. al. (2023), »The Intertemporal Keynesian Cross«
  - Long paper with many (technical) details
  - We will focus on the main results

# Sticky wages

# Households

- **Household problem:**

$$
\begin{aligned}
v_t(z_t, a_{t-1}) &= \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} - \varphi \frac{\ell_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t \left[ v_{t+1}(z_{t+1}, a_t) \right] \\
\text{s.t. } a_t + c_t &= (1 + r_t^a) a_{t-1} + (1 - \tau_t) w_t \ell_t z_t + \chi_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \ , \psi_t \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \ \mathbb{E}[z_t] = 1 \\
a_t &\geq 0
\end{aligned}
$$

- **Active decisions:** Consumption-saving, $c_t$ (and $a_t$)
- **Union decision:** Labor supply, $\ell_t$
- **Consumption function:** $C_t^{hh} = C^{hh}\left(\{r_s^a, \tau_s, w_s, \ell_s, \chi_s\}_{s \geq 0}\right)$

# Firms

- **Production and profits:**

$$Y_t = \Gamma_t L_t$$
$$\Pi_t = P_t Y_t - W_t L_t$$

- **First order condition:**

$$\frac{\partial \Pi_t}{\partial L_t} = 0 \Leftrightarrow P_t \Gamma_t - W_t = 0 \Leftrightarrow w_t \equiv W_t / P_t = \Gamma_t$$

Zero profits: $\Pi_t = 0$

- **Wage and price inflation:**

$$\pi_t^w \equiv W_t / W_{t-1} - 1$$
$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 = \frac{W_t / \Gamma_t}{W_{t-1} / \Gamma_{t-1}} - 1 = \frac{1 + \pi_t^w}{\Gamma_t / \Gamma_{t-1}} - 1$$

# Union

- Everybody works the same:

$$\ell_t = L_t^{hh}$$

- Unspecified *wage adjustment costs* imply a **New Keynesian Wage (Phillips) Curve** (NKWPC or NKWC)

$$\pi_t^w = \kappa \left( \varphi \left( L_t^{hh} \right)^{\nu} - \frac{1}{\mu} \left( 1 - \tau_t \right) w_t \left( C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^w$$

# Government

- **Spending:** $G_t$
- **Tax bill:** $T_t$

$$T_t = \int \tau_t w_t \ell_t z_t d\boldsymbol{D}_t = \tau_t \Gamma_t L_t = \tau_t Y_t$$

- If **one-period bonds:**

$$B_t = (1 + r_t^b)B_{t-1} + G_t + \chi_t - T_t$$

- If **long-term bonds:** Geometrically declining payment stream of $1, \delta, \delta^2, \dots$ for $\delta \in [0, 1]$. The bond price is $q_t$.

$$q_t(B_t - \delta B_{t-1}) = B_{t-1} + G_t + \chi_t - T_t$$

- Potential **tax-rule:**

$$\tau_t = \tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}}$$

## Central bank

- Standard **Taylor rule:**

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left( (1 + r_{ss}) (1 + \pi_t)^{\phi_\pi} \right)^{1-\rho_i}$$

**Alternative:** Real rate rule

$$1 + i_t = (1 + r_{ss})(1 + \pi_{t+1})$$

Indeterminacy: Consider limit or assume future tightening

- **Fisher-equation:**

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$

# Arbitrage

1. One-period *real* bond, $q_t = 1$:

$$t > 0: \; r_t^b = r_t^a = r_{t-1}$$
$$r_0^b = r_0^a = 1 + r_{ss}$$

2. or, one-period *nominal* bond, $q_t = 1$:

$$t > 0: \; r_t^b = r_t^a = r_{t-1}$$
$$t > 0: \; r_0^b = r_0^a = (1 + r_{ss})(1 + \pi_{ss}) / (1 + \pi_0)$$

3. or, long-term *(real)* bonds:

$$\frac{1 + \delta q_{t+1}}{q_t} = 1 + r_t$$

$$1 + r_t^b = 1 + r_t^a = \frac{1 + \delta q_t}{q_{t-1}} = \begin{cases} \frac{1+\delta q_0}{q_{-1}} & \text{if } t = 0 \\ 1 + r_{t-1} & \text{else} \end{cases}$$

# Market clearing

1. Asset market: $q_t B_t = A_t^{hh}$
2. Labor market: $L_t = L_t^{hh}$
3. Goods market: $Y_t = C_t^{hh} + G_t$

# Equation system

Taylor-rule and long-term government debt:

$$\begin{bmatrix} w_t - \Gamma_t \\ Y_t - \Gamma_t L_t \\ 1 + \pi_t - \frac{1+\pi_t^w}{\Gamma_t/\Gamma_{t-1}} \\ 1 + i_t - (1 + i_{t-1})^{\rho_i} \left( (1 + r_{ss}) (1 + \pi_t)^{\phi_\pi} \right)^{1-\rho_i} \\ 1 + r_t - \frac{1+i_t}{1+\pi_{t+1}} \\ \frac{1+\delta q_{t+1}}{q_t} - (1 + r_t) \\ 1 + r_t^a - \frac{1+\delta q_t}{q_{t-1}} \\ \tau_t - \left[ \tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}} \right] \\ q_t(B_t - \delta B_{t-1}) - [B_{t-1} + G_t + \chi_t - \tau_t Y_t] \\ q_t B_t - A_t^{hh} \\ \pi_t^w - \left[ \kappa \left( \varphi \left( L_t^{hh} \right)^\nu - \frac{1}{\mu} (1 - \tau_t) w_t \left( C_t^{hh} \right)^{-\sigma} \right) + \beta \pi_{t+1}^W \right] \end{bmatrix} = \mathbf{0}$$

# Reduced equation system with ordered blocks

$$\boldsymbol{H}(\boldsymbol{\pi^w}, \boldsymbol{L}, \boldsymbol{G}, \chi, \boldsymbol{\Gamma}) = \begin{bmatrix} q_t B_t - A_t^{hh} \\ \pi_t^w - \left[\kappa\left(\varphi\left(L_t^{hh}\right)^{\nu} - \frac{1}{\mu}\left(1-\tau_t\right) w_t \left(C_t^{hh}\right)^{\text{-}\sigma}\right) + \beta \pi_{t+1}^W\right] \end{bmatrix} = \boldsymbol{0}$$

$$
\begin{aligned}
\text{Production: } w_t &= \Gamma_t \\
Y_t &= \Gamma_t L_t \\
\pi_t &= \frac{1+\pi_t^w}{\Gamma_t/\Gamma_{t-1}} - 1 \\
\text{Central bank: } i_t &= \left(1+i_{t-1}\right)^{\rho_i} \left(\left(1+r_{ss}\right)\left(1+\pi_t\right)^{\phi_\pi}\right)^{1-\rho_i} - 1 \text{ (forwards)} \\
r_t &= \frac{1+i_t}{1+\pi_{t+1}} - 1 \\
\text{Mutual fund: } q_t &= \frac{1+\delta q_{t+1}}{1+r_t} \text{ (backwards)} \\
r_t^a &= \frac{1+\delta q_t}{q_{t-1}} - 1
\end{aligned}
$$

$$\text{Government: } \begin{bmatrix} \tau_t \\ B_t \end{bmatrix} = \begin{bmatrix} \tau_{ss} + \omega q_{ss} \frac{B_{t-1} - B_{ss}}{Y_{ss}} \\ \frac{(1+\delta q_t) B_{t-1} + G_t + \chi_t - \tau_t Y_t}{q_t} \end{bmatrix} \text{ (forwards)}$$

# DAG


central_bank
mutual_fund
pi_w
NKWC
[NKWC_res]
G
government
chi
Gamma
production
L
market_clearing
[clearing_A]
hh

# Analytical insights

# Simpler consumption function

- **Assumptions:**
  1. One-period real bond
  2. No lump-sum transfers, $\chi_t = 0$
  3. Real rate rule: $r_t = r_{ss}$
  4. Fiscal policy in terms of $dG_t$ and $dT_t$ satisfying IBC

$$\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} (dG_t - dT_t) = 0$$

- **Tax-bill:** $T_t = \tau_t w_t \int \ell_t z_t d\boldsymbol{D}_t = \tau_t \Gamma_t L_t = \tau_t Y_t$
- **Household income:** $(1 - \tau_t) w_t \ell_t z_t = \underbrace{(Y_t - T_t)}_{\equiv Z_t} z_t = Z_t z_t$
- **Consumption function:** Simplifies to

$$C_t^{hh} = C^{hh}(\{Y_s - T_s\}_{s \geq 0}) \Rightarrow \boldsymbol{C}^{hh} = C^{hh}(\boldsymbol{Y} - \boldsymbol{T}) = C^{hh}(\boldsymbol{Z})$$

# Two-equation version in $\boldsymbol{Y}$ and $\boldsymbol{r}$

$$\boldsymbol{Y} = \boldsymbol{G} + C^{hh}(\boldsymbol{r}, \boldsymbol{Y} - \boldsymbol{T})$$
$$\boldsymbol{r} = \mathcal{R}(\boldsymbol{Y}; \boldsymbol{G}, \boldsymbol{T})$$

- **First equation:** Goods market clearing
- **Second equation:**
    1. Government: $\boldsymbol{T}, \boldsymbol{Y} \rightarrow \boldsymbol{\tau}$
    2. Resource constraint: $\boldsymbol{G}, \boldsymbol{Y} \rightarrow \boldsymbol{C}$
    3. Firm behavior I: $\boldsymbol{\Gamma}, \boldsymbol{Y} \rightarrow \boldsymbol{L}, \boldsymbol{w}$
    4. NKWC: $\boldsymbol{L}, \boldsymbol{w}, \boldsymbol{\tau} \rightarrow \boldsymbol{\pi}^{\boldsymbol{w}}$
    5. Firm behavior II: $\boldsymbol{\pi}^{\boldsymbol{w}} \rightarrow \boldsymbol{\pi}$
    6. Central bank: $\boldsymbol{\pi} \rightarrow \boldsymbol{i}$
    7. Fisher: $\boldsymbol{i}, \boldsymbol{\pi} \rightarrow \boldsymbol{r}$
- **Heterogeneity does not enter** $\mathcal{R}(\boldsymbol{Y}; \boldsymbol{G}, \boldsymbol{T})$
- **Real rate rule:** *Inflation is a side-show*

# Intertemporal Keynesian Cross

$$\boldsymbol{Y} = \boldsymbol{G} + C^{hh}(\boldsymbol{Y} - \boldsymbol{T})$$

- **Total differentiation:**

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial C_t^{hh}}{\partial Z_s} (dY_t - dT_t)$$

IBC implies: $\sum_{t=0}^{\infty} (1 + r_{ss})^{-t} \frac{\partial C_t^{hh}}{\partial Z_s} = (1 + r_{ss})^{-s}$

- **Intertemporal Keynesian Cross** in vector form

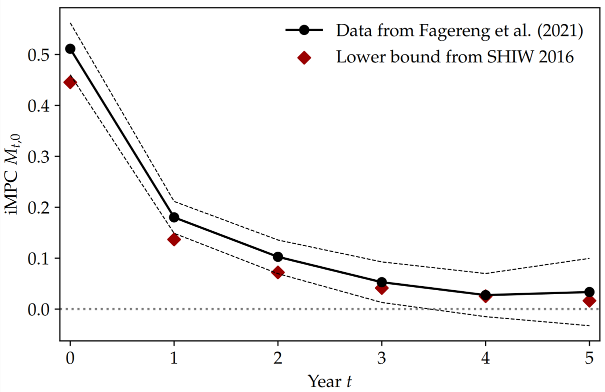
$$d\boldsymbol{Y} = d\boldsymbol{G} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) \Leftrightarrow$$

$$(\boldsymbol{I} - \boldsymbol{M})d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}$$

where $M_{t,s} = \frac{\partial C_t^{hh}}{\partial Z_s}$ encodes the entire *complexity*

# iMPCs in the data

Figure 1: iMPCs in the Norwegian and Italian data




**Other columns:** Druedahl et al. (2023) show in micro-data that consumption responds today to news about future income.

# Perspective: Static Keynesian Cross

- **Old Keynesians:** Consumption only depends on current income

$$Y_t = G_t + C^{hh}(Y_t - T_t)$$

- **Total differentiate:**

$$\begin{aligned} dY_t &= dG_t + \frac{\partial C_t^{hh}}{\partial Z_t}(dY_t - dT_t) \\ &= dG_t + \text{mpc} \cdot (dY_t - dT_t) \end{aligned}$$

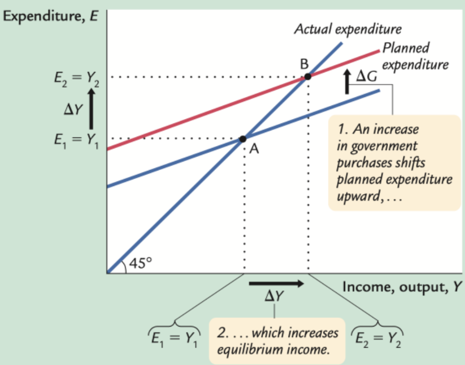
- **Solution**

$$dY_t = \frac{1}{1 - \text{mpc}} (dG_t - \text{mpc} \cdot dT_t)$$

from multiplier-process $1 + \text{mpc} + \text{mpc}^2 \cdots = \frac{1}{1-\text{mpc}}$

# Static Keynesian Cross

figure 10-5




**An Increase in Government Purchases in the Keynesian Cross**
An increase in government purchases of $\Delta G$ raises planned expenditure by that amount for any given level of income. The equilibrium moves from point A to point B, and income rises from $Y_1$ to $Y_2$. Note that the increase in income $\Delta Y$ exceeds the increase in government purchases $\Delta G$. Thus, fiscal policy has a multiplied effect on income.

# Intertemporal solution technicalities

- **IBCs:**
  1. NPV-vector: $\boldsymbol{q} \equiv [1, (1+r_{ss})^{-1}, (1+r_{ss})^{-2}, \ldots]'$
  2. Households: $\boldsymbol{q}'\boldsymbol{M} = \boldsymbol{q}'$ and $\boldsymbol{q}'(\boldsymbol{I}-\boldsymbol{M}) = \boldsymbol{0}$
  3. Government: $\boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}) = \boldsymbol{0}$
- **Problem:** $(\boldsymbol{I}-\boldsymbol{M})^{-1}$ cannot exist because

$$
\begin{aligned}
(\boldsymbol{I}-\boldsymbol{M})d\boldsymbol{Y} &= d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T} \Leftrightarrow \\
\boldsymbol{q}'(\boldsymbol{I}-\boldsymbol{M})d\boldsymbol{Y} &= \boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}) \Leftrightarrow \\
\boldsymbol{0} &= \boldsymbol{0}
\end{aligned}
$$

- **Result:** If unique solution then on the form

$$
\begin{aligned}
d\boldsymbol{Y} &= \mathcal{M}(d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}) \\
\mathcal{M} &= (\boldsymbol{K}(\boldsymbol{I}-\boldsymbol{M}))^{-1}\boldsymbol{K}
\end{aligned}
$$

Indeterminancy: Still work-in-progress (Auclert et. al., 2023)

# Intermezzo: Response of consumption

$$\begin{aligned}
d\boldsymbol{Y} &= d\boldsymbol{G} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{G}) \Leftrightarrow \\
d\boldsymbol{Y} - d\boldsymbol{G} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T}) \Leftrightarrow \\
d\boldsymbol{C} &= \mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})
\end{aligned}$$

# Fiscal multipliers

$$d\boldsymbol{Y} = d\boldsymbol{G} + \underbrace{\mathcal{M}\boldsymbol{M}(d\boldsymbol{G} - d\boldsymbol{T})}_{d\boldsymbol{C}}$$

- **Balanced budget multiplier:**

$$d\boldsymbol{G} = d\boldsymbol{T} \Rightarrow d\boldsymbol{Y} = d\boldsymbol{G}, d\boldsymbol{C} = 0$$

  Note: Central that income and taxes affect household income proportionally in exactly the same way = no redistribution

- **Deficit multiplier:** $d\boldsymbol{G} \neq d\boldsymbol{T}$
  1. Larger effect of $d\boldsymbol{G}$ than $d\boldsymbol{T}$
  2. *Numerical results needed*

# Fiscal multiplier

**Impact-multiplier:**

$$\frac{\partial Y_0}{\partial G_0}$$

**Cummulative-multiplier:**

$$\frac{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}dY_t}{\sum_{t=0}^{\infty}(1+r_{ss})^{-t}dG_t}$$

# Comparison with RA model

- **From lecture 1:** $\beta(1+r_{ss}) = 1$ implies

$$C_t = (1-\beta)\sum_{s=0}^{\infty}\beta^s Y^{hh}_{t+s} + r_{ss}a_{-1}$$

- The **iMPC-matrix** becomes

$$\boldsymbol{M}^{RA} = \begin{bmatrix} (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ (1-\beta) & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} = (1-\beta)\mathbf{1}\boldsymbol{q}'$$

- **Consumption response** is zero

$$\begin{aligned} d\boldsymbol{C}^{\boldsymbol{RA}} &= \mathcal{M}\boldsymbol{M}^{\boldsymbol{RA}}(d\boldsymbol{G} - d\boldsymbol{T}) \\ &= \mathcal{M}(1-\beta)\mathbf{1}\boldsymbol{q}'(d\boldsymbol{G} - d\boldsymbol{T}) \\ &= \mathbf{0} \Leftrightarrow d\boldsymbol{Y} = d\boldsymbol{G} \end{aligned}$$

# Comparison with TA model

- **Hand-to-Mouth (HtM) households:** $\lambda$ share have $C_t = Y_t^{hh}$

$$\boldsymbol{M}^{TA} = (1-\lambda)\boldsymbol{M}^{RA} + \lambda \boldsymbol{I}$$

- **Intertemporal Keynesian Cross** becomes

$$(\boldsymbol{I} - \boldsymbol{M}^{TA})d\boldsymbol{Y} = d\boldsymbol{G} - \boldsymbol{M}^{TA}d\boldsymbol{T}$$
$$(\boldsymbol{I} - \boldsymbol{M}^{RA})d\boldsymbol{Y} = \underbrace{\frac{1}{1-\lambda}\left[d\boldsymbol{G} - \lambda d\boldsymbol{T}\right]}_{d\tilde{\boldsymbol{G}}_t} - \boldsymbol{M}^{RA}d\boldsymbol{T}$$

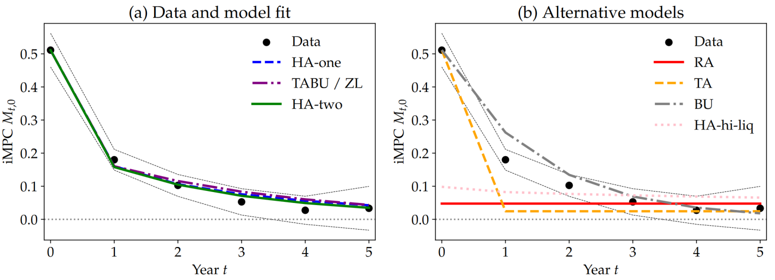
- **Same solution-form as RA:** $d\boldsymbol{Y} = d\tilde{\boldsymbol{G}}_t$

$$d\boldsymbol{Y} = d\tilde{\boldsymbol{G}}_t = d\boldsymbol{G}_t + \frac{\lambda}{1-\lambda}\left[d\boldsymbol{G} - d\boldsymbol{T}\right]$$

- Still a cumulative multiplier of 1 (both for RA and HtM)

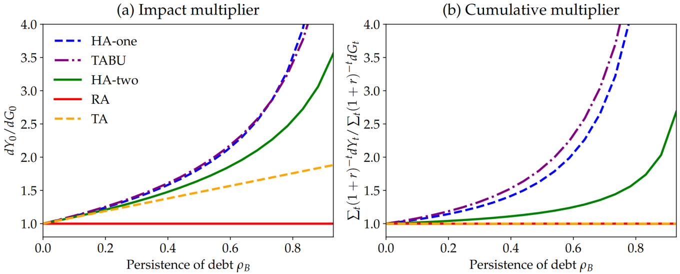
# iMPCs in models

Figure 2: iMPCs in the Norwegian data and several models



# Multipliers and debt-financing

Figure 5: Multipliers according to the IKC




*Note.* These figures assume a persistence of government spending equal to $\rho_G = 0.76$, and vary $\rho_B$ in $dB_t = \rho_B(dB_{t-1} + dG_t)$. See section 7.1 for details on calibration choices.

# Generalized IKC

- **Budget constraint** can be written with initial capital gain

$$a_t + c_t = (Y_t - T_t)z_t + \chi_t + \begin{cases} (1 + r_{t-1})a_{t-1} & \text{if } t > 0 \\ (1 + r_{ss} + \text{cap}_0)a_{t-1} & \text{if } t = 0 \end{cases}$$

1. Real bond: $\text{cap}_0 = 0$
2. Nominal bond:

$$\text{cap}_0 = \frac{(1 + r_{ss})(1 + \pi_{ss})}{1 + \pi_0} - (1 + r_{ss})$$

3. Long-term bond:

$$\text{cap}_0 = \frac{1 + \delta q_0}{q_{-1}} - (1 + r_{ss})$$

# Generalized IKC

- **Consumption-function** $\boldsymbol{C}^{hh} = C^{hh}(\boldsymbol{r}, \boldsymbol{Y} - \boldsymbol{T}, \chi, \mathbf{cap}_0)$ implies

$$d\boldsymbol{C}^{hh} = \boldsymbol{M}^{r} d\boldsymbol{r} + \boldsymbol{M}(d\boldsymbol{Y} - d\boldsymbol{T}) + \boldsymbol{M}^{\chi} d\boldsymbol{\chi} + \boldsymbol{m}^{\text{cap}}\text{cap}_0$$

  where

$$\boldsymbol{M}^{r}_{t,s} = \left[\frac{\partial C^{hh}_t}{\partial r_s}\right], \boldsymbol{M}^{\chi}_{t,s} = \left[\frac{\partial C^{hh}_t}{\partial \chi_s}\right], \boldsymbol{m}^{\text{cap}}_t = \left[\frac{\partial C^{hh}_t}{\partial \text{cap}_0}\right]$$

- Why are $\boldsymbol{M}^{\chi}$ and $\boldsymbol{M}$ different?

# Exercise

# Exercise

Use *HANK-sticky-wages* in sub-folder.

1. Compute *fiscal multipliers* varying:
    1.1 Bond maturity: $\delta$
    1.2 Fiscal aggressiveness: $\omega$
    1.3 Monetary aggressiveness: $\phi_\pi$
2. Does the model match the evidence of intertemporal MPCs? What happens to the fiscal multiplier if the fit is improved?

# Summary

# Summary and next week

- **Today:** Fiscal policy in a HANK model with sticky wages
- **Next week:** I-HANK
- **Homework:**
    1. Work on exercise
    2. Read: Druedahl et al. (2022),
       »The Transmission of Foreign Demand Shocks«