CENTER FOR ECONOMIC BEHAVIOR & INEQUALITY

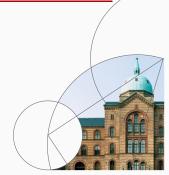


# 14a. HANK-SAM

Adv. Macro: Heterogenous Agent Models

Jeppe Druedahl 2023







Introduction

#### Introduction

#### Further advanced topics:

- 1. Policy analysis
- 2. Life-cycle
- 3. Endogenous idiosyncratic risk (time-varying)
- 4. Discrete choices (with taste shocks)
- 5. Bounded rationality (non-FIRE)

#### Introduction

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#### Example: HANK-SAM

- Paper: Broer et al. (2023),
   »The Unemployment-Risk Channel in Business-Cycle Fluctuations«
- Slides: Broer et al. (2023),
   »Fiscal stimulus policies according to HANK-SAM«
- GEModelTools: Model description of HANK-SAM

# Policies

#### **Policies**

- Policies as shocks: Choose ARMA(p,q) process and study IRFs
   (also for utility, inequality and social welfare)
   Non-linear transition: Interaction with initial state and other shocks
- 2. **Policies from targets:** Make different policies comparable by achieving the *exact* same path of outcomes
- 3. Policy rules: Parameterize and study effects on IRFs to shocks
- Optimal policy with quadratic loss function (McKay and Wolf, 2023)
  - 4.1 Easy to solve numerically with ad hoc loss function
  - 4.2 Harder to derive loss function from social welfare (Ramsey problem)
- 5. **Optimal policy:** Discretion, commitment, timeless perspective (Dávila and Schaab, 2023)

# Steady state $\rightarrow$ transition path $\rightarrow$ timeless perspective

1. Maximize welfare in steady state:

$$\max_{policies} \int v_{ss}(\boldsymbol{x}_i; \boldsymbol{X}_{ss}^{hh}) d\boldsymbol{D}_{ss} = \max_{policies} \int \sum_{k=0}^{\infty} \beta_i^k u(\boldsymbol{c}_{ss}^{\star}\left(\boldsymbol{x}_i; \boldsymbol{X}_{ss}^{hh}\right)) d\boldsymbol{D}_{ss}$$

2. Maximize welfare along transition path (Ramsey problem)

$$\max_{policies} \int v_0(\boldsymbol{x}_{it}; \left\{\boldsymbol{X}_s^{hh}\right\}_{s=0}^{\infty}) d\boldsymbol{D}_0 = \max_{policies} \sum_{t=0}^{\infty} \int \beta_i^k u(\boldsymbol{x}_{it}; \left\{\boldsymbol{X}_s^{hh}\right\}_{s=t}^{\infty}) d\boldsymbol{D}_t$$

- ÷ same steady state if too costly to get there (accumulation?)
- Weird time-0 policies (tax capital 100% and redistribute?)
- 3. Maximize welfare in timeless perspective

"add penalties such that policy is stationary in absence of shocks"

Life-cycle

# Households with life-cycle

- Age:  $h_{it} \in \{0, 1, \dots, \#_h 1\}$
- Mortality:  $\delta(h_{it}) \in [0,1]$ ,  $\delta(\#_h 1) = 1$ ,  $\zeta(j) = \int \mathbf{1}\{h_{it} = j\}d\mathbf{D}_{ss}$
- Income profile:  $\mathcal{Z}(z_{it}, h_{it+1})$
- Household problem,  $\{r_t, w_t, q_t\} \rightarrow \{a_{it}, b_{it}, c_{it}, \ell_{it}\}$ :

$$\begin{split} v_0(\beta_i, h_{it}, z_{it}, a_{it-1}) &= \max_{c_{it}} u(c_t) + \beta_i (1 - \delta(h_{it})) \mathbb{E}_t \left[ v_{t+1} \left( \beta_i, h_{it+1}, z_{it+1}, a_{it} \right) \right] \\ \text{s.t. } \ell_{it} &= z_{it} \\ a_{it} &= \begin{cases} (1 + r_t) q_t + w_t \ell_{it} - c_{it} + \Pi_t & \text{if } h_{it} = 0 \\ (1 + r_t) a_{it-1} + w_t \ell_{it} - c_{it} + \Pi_t & \text{else} \end{cases} \\ b_{it} &= \delta(h_{it}) a_{it} \\ h_{it+1} &= \begin{cases} 0 & \text{with prob. } \delta(h_{it}) \\ h_{it} + 1 & \text{else} \end{cases} \\ z_{it+1} \sim \mathcal{Z}(z_{it}, h_{it+1}), \ \mathbb{E}\left[z_{it}\right] = 1 \\ a_{it} &> 0 \end{split}$$

# **Equation system**

$$\begin{split} \boldsymbol{H}(\boldsymbol{K},\boldsymbol{q},\boldsymbol{\Gamma},\underline{\boldsymbol{D}}_0) &= \begin{bmatrix} A_t - A_t^{hh} \\ \zeta(0)q_t - B_{t-1}^{hh} \\ \forall t \in \{0,1,\dots,T-1\} \end{bmatrix} = \boldsymbol{0} \end{split}$$
 where  $K_{-1} = \int a_{t-1}d\underline{\boldsymbol{D}}_0$  and 
$$L_t = 1 \\ A_t = K_t \\ r_t^K &= \alpha \Gamma_t (K_{t-1}/L_t)^{\alpha-1} \\ w_t &= (1-\alpha)\Gamma_t (K_{t-1}/L_t)^{\alpha} \\ \boldsymbol{D}_t &= \Pi_z'\underline{\boldsymbol{D}}_t \\ \underline{\boldsymbol{D}}_{t+1} &= \Lambda_t'\boldsymbol{D}_t \\ A_t^{hh} &= a_t^{*\prime}'\boldsymbol{D}_t \\ \forall t \in \{0,1,\dots,T-1\} \end{split}$$

Endogenous idiosyncratic risk

# Consumption problem

Recursive household problem:

$$\begin{aligned} v_t(u_{it}, a_{it-1}) &= \max_{c_{it}} u(c_{it}) + \beta \underline{v}_t \left( u_{it}, a_{it} \right) \\ \text{s.t.} \\ a_{it} &= (1 + r_t) a_{it-1} + y_{it} - c_{it} \\ y_{it} &= (1 - \tau_t) w_t \cdot \begin{cases} 1 & \text{if } u_{it} = 0 \\ \phi \in (0, 1) & \text{if } u_{it} = 1 \end{cases} \\ a_{it} &\geq 0 \end{aligned}$$

- Working if  $u_{it} = 0$ , unemployed if  $u_{it} = 1$
- Solution method: Standard EGM

# External endogenous risk

#### Expectation step:

$$\underline{v}_t(u_{it-1}, a_{it-1}) = \mathbb{E}\left[v_t(u_{it}, a_{it-1}) \mid u_{it-1}, a_{it-1}, \delta_t, \lambda_t^u\right]$$
 s.t. 
$$\pi_t(u_{it} \mid u_{it-1}) = \begin{cases} \delta_t & \text{if } u_{it} = 1 \text{ and } u_{it-1} = 0 \\ 1 - \delta_t & \text{if } u_{it} = 0 \text{ and } u_{it-1} = 0 \\ \lambda_t^u & \text{if } u_{it} = 0 \text{ and } u_{it-1} = 1 \\ 1 - \lambda_t^u & \text{if } u_{it} = 1 \text{ and } u_{it-1} = 1 \end{cases}$$

- Stochastic transition matrix:  $\Pi_{t,z} = \Pi_z(\delta_t, \lambda_t^u)$
- Envelope condition: Nothing changed
- Transition steps:

$$oldsymbol{D}_t = \Pi'_{t,z} \underline{oldsymbol{D}}_t$$
  
 $\underline{oldsymbol{D}}_{t+1} = \Lambda'_t oldsymbol{D}_t$ 

# Internal endogenous risk

#### Expectation step:

$$\underline{v}_t(u_{it-1}, a_{it-1} \mid s_{it}) = \mathbb{E}\left[v_t(u_{it}, a_{it-1}) \mid u_{it-1}, a_{it-1}, s_{it}, \delta_t, \lambda_t^{u,s}\right]$$
s.t.
$$\pi_t(u_{it} \mid u_{it-1}, s_{it}) = \begin{cases} \delta_t & \text{if } u_{it} = 1 \text{ and } u_{it-1} = 0\\ 1 - \delta_t & \text{if } u_{it} = 0 \text{ and } u_{it-1} = 0\\ \lambda_t^{u,s} s_{it} & \text{if } u_{it} = 0 \text{ and } u_{it-1} = 1\\ 1 - \lambda_t^{u,s} s_{it} & \text{if } u_{it} = 1 \text{ and } u_{it-1} = 1 \end{cases}$$

#### Search decision:

- 1. Discrete search choice:  $s_{it} \in \{0, 1\}$
- 2. Search cost:  $\lambda$  if  $s_{it} = 1$
- 3. Taste shocks:  $\varepsilon$  ( $s_{it}$ )  $\sim$  Extreme value (Iskhakov et. al., 2017)
- See also: Bardóczy (2021)

#### Discrete search decision

Standard logit formula:

$$\begin{split} \underline{v}_t(u_{it-1}, a_{it-1}) &= \max_{s_{it} \in \{0, 1\}} \left\{ \underline{v}_t(u_{it-1}, a_{it-1} \mid s_{it}) - \lambda \mathbf{1}_{s_{it}=1} + \sigma_{\varepsilon} \varepsilon \left(s_{it}\right) \right\} \\ &= \sigma_{\varepsilon} \log \left( \exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \mid 0)}{\sigma_{\varepsilon}} + \exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \mid 1)}{\sigma_{\varepsilon}} \right) \end{split}$$

Transition matrix:

$$\Pi_{t,z} = \Pi_z \left( \left\{ r_\tau, w_\tau, \tau_\tau, \delta_\tau, \lambda_\tau^{u,s} \right\}_{\tau \ge t} \right)$$

# **Envelope condition**

Choice probabilities:

$$P_t(s \mid u_{it-1}, a_{it-1}) = \frac{\exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \mid s)}{\sigma_{\xi}}}{\sum_{s' \in \{0,1\}} \exp \frac{\underline{v}_t(u_{it-1}, a_{it-1} \mid s')}{\sigma_{\xi}}}$$

Envelope condition:

$$\underline{v}_{a,t}(u_{t-1}, a_{t-1}) = \sum_{s \in \{0,1\}} P_t(s \mid u_{it-1}, a_{it-1}) \pi_t(u_{it} \mid u_{it-1}, s) v_{a,t}(u_{it}, a_{it-1})$$

$$= \sum_{s \in \{0,1\}} P_t(s \mid u_{it-1}, a_{it-1}) \pi_t(u_{it} \mid u_{it-1}, s) c_t^*(u_{it}, a_{it-1})^{-\sigma}$$

- Break of monotonicity ⇒ FOC still necessary, but not sufficient
  - 1. **Normally:** Savings  $\uparrow \Rightarrow$  future consumption  $\uparrow \Rightarrow$  marginal utility  $\downarrow$
  - Now also: Future search jump ↓ ⇒ future income ↓
     ⇒ future consumption ↓ ⇒ marginal utility ↑

# Upper envelope for given $z^{i_z}$

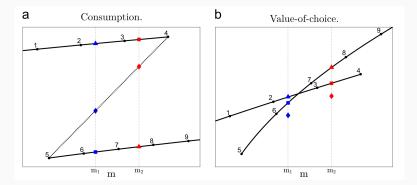
1. Generate candidate points:  $\forall i_a \in \{0, 1, \dots, \#_a - 1\}$ 

$$w^{i_a} = \beta \underline{v}_{t+1}(z^{i_z}, a^{i_a})$$
 $c^{i_a} = u'^{-1} (\beta \underline{v}_{a,t+1}(z^{i_z}, a^{i_a}))$ 
 $m^{i_a} = a^{i_a} + c^{i_a}$ 
 $v^{i_a} = u(c^{i_a}) + w^{i_a}$ 

2. Apply upper-envelope:  $\forall i_{a-} \in \{0, 1, \dots, \#_a - 1\}$ 

$$\begin{split} c^*(a^{i_{3-}}) &= \max_{j \in \{0,1,\dots\#_{s^-}-2\}} u\left(c^{i_{3-}}\right) + w^{i_{3-}} \text{ s.t.} \\ m^{i_{3-}} &= (1+r_t)a^{i_{3-}} + w_tz^{i_z} \in \left[m^j, m^{j+1}\right] \\ c^{i_{3-}} &= \min\left\{\text{interp }\left\{m^{i_3}\right\} \to \left\{c^{i_3}\right\} \text{ at } m^{i_{3-}}, m^{i_{3-}}\right\} \\ a^{i_{3-}} &= m^{i_{3-}} - c^{i_{3-}} \\ w^{i_{3-}} &= \text{interp }\left\{a^{i_3}\right\} \to \left\{w^{i_3}\right\} \text{ at } a^{i_{3-}} \end{split}$$

#### Illustration



- 1. **Numbering:** Different levels of end-of-period assets,  $a^{i_a}$
- 2. **Problem:** Find the consumption function at  $m_1$  and  $m_2$
- 3. Largest value-of-choice: Denoted by the triangles

**Source:** Druedahl and Jørgensen (2017),  $G^2EGM$ 

# **Example**

Beg.-of-period value function:

$$\underline{v}_{t+1}(a_t) = \sqrt{m_{t+1}} + \eta \max{\{m_{t+1} - \underline{m}, 0\}}$$
 where  $m_{t+1} = (1+r)a_t + 1$ 

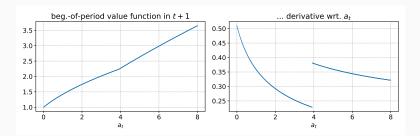
Derivative:

$$\underline{v}_{a,t+1}(a_t) = \frac{1}{2}(1+r)m_{t+1}^{-\frac{1}{2}} + (1+r)\eta \mathbf{1} \{m_{t+1} > \underline{m}\}$$

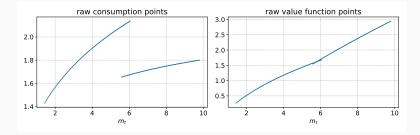
Budget constraint:

$$a_t + c_t = (1+r)a_{t-1} + 1$$

# **Next-period values**

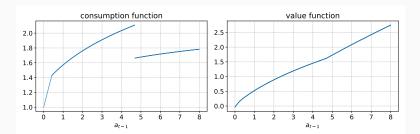


#### Raw values of $c^{i_a}$ and $v^{i_a}$



**Problem:** Overlaps  $\Rightarrow$  not a function  $m_t$ !

# Result after upper envelope



# General problem structure

General problem structure with nesting:

$$\begin{split} \overline{v}_t\left(\overline{x}_t, d_t, e_t, m_t\right) &= \max_{c_t \in [0, m_t]} u(c_t, d_t, e_t) + \beta \underline{v}_{t+1} \left(\underline{\Gamma}_t \left(\overline{x}_t, d_t, e_t, a_t\right)\right) \\ & \text{with } a_t = m_t - c_t \\ v(x_t) &= \max_{d_t \in \Omega^d(x_t)} \overline{v}_t \left(\overline{\Gamma}_t \left(x_t, d_t\right)\right) \\ \underline{v}_t \left(\underline{x}_t\right) &= \max_{e_t \in \Omega^e(\underline{x}_t)} \mathbb{E}\left[v \left(\Gamma \left(\underline{x}_t, e_t\right)\right) \mid \underline{x}_t, e_t\right] \end{split}$$

- Finding c<sub>t</sub>: EGM with upper envelope can (typically) still be used
- Finding  $d_t$  and  $e_t$ :
  - 1. Combination of discrete and continuous choices
  - 2. Typically requires use of numerical optimizer or root-finder
- Druedahl (2021), »A Guide on Solving Non-Convex Consumption-Saving Models« (costly with extra states in v̄)

# Non-FIRE

# Motivating example

- FIRE: Full International Rational Expectations
- **IKC**:  $d\mathbf{Y} = \mathbf{M}^r d\mathbf{r} + \mathbf{M} d\mathbf{Y}$  where  $M_{t,s} = \frac{\partial C_t}{\partial Y_s}$  and similar for  $\mathbf{M}^r$
- Myopic behavior:
  - 1. Agents never thinks about the future
  - 2. Agents gradually observe current aggregate variables

$$m{M}^{
m myopic} = \left[ egin{array}{cccc} M_{0,0} & 0 & 0 & \cdots \ M_{1,0} & M_{0,0} & 0 & \cdots \ M_{2,0} & M_{1,0} & M_{0,0} & \cdots \ dots & dots & dots & dots & dots \end{array} 
ight]$$

#### Consider t = 1:

- M<sub>1,0</sub>dY<sub>0</sub>: Effect from past shock observed
- $M_{0,0}dY_1$ : Effect of unexpected change in period 1

#### Sticky expectations

• Sticky expectations: A fraction  $1-\theta$  updates expectations each period (from Carroll et. al., 2020)

$$m{M}^{
m sticky} = egin{bmatrix} M_{0,0} & (1- heta)M_{0,1} & (1- heta)M_{0,2} & \cdots \ M_{1,0} & (1- heta)M_{1,1} + heta M_{0,0} & (1- heta)M_{1,2} + heta (1- heta)M_{0,1} & \cdots \ M_{2,0} & (1- heta)M_{2,1} + heta M_{1,0} & dots & \ddots \ dots & dots & dots & dots & dots \ \end{pmatrix}$$

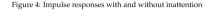
#### Consider t = 0:

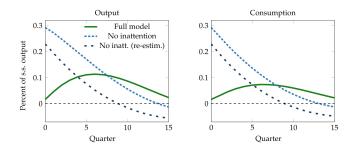
- 1. Non-updaters:  $\theta M_{0,0} dY_0$
- 2. Updaters:  $(1-\theta)\sum_{s>0} M_{0,s} dY_s$

#### Consider t = 1:

- 1. Ingoing updaters:  $(1-\theta)\sum_{s>0} M_{1,s} dY_s$
- 2. Ingoing non-updaters:  $\theta \left( M_{1,0} dY_0 + M_{0,0} dY_1 \right)$
- 3. New updaters:  $\theta(1-\theta)\sum_{s>1} M_{0,s} dY_{s+1}$

# Hump-shaped response to monetary policy





*Note.* This figure shows the general equilibrium paths of output and consumption in our estimated HA model with different assumptions on inattention. The solid green uses our baseline estimates of household inattention. The dashed blue line is the impulse response when the inattention parameter is set to  $\theta = 0$ , holding all other parameters fixed at their estimated value in table 2. The dotted dark blue line reestimates the model parameters without inattention.

**Source:** Auclert et. al. (2020), »Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model«

# UI extensions might be less powerful

Figure 4: Partial-equilibrium Consumption Responses to an UI Extension UI duration in quarters Consumption response to UI duration 0.8 1.0 FIRE Myopic 0.8 0.6 Estimated pct deviation deviation 0.6 0.4 0.4 0.2 0.2 0.0 0.0 -0.212 15 18 12 15 3 9 0 3 9 18 quarters quarters

**Source:** Bardóczy and Guerreiro (2020), »Unemployment Insurance in Macroeconomic Stabilization with Imperfect Expectations«



**HANK-SAM** 

# My ongoing work

#### Zero liqduity:

Broer, Druedahl, Harmenberg and Öberg (2023), »The Unemployment-Risk Channel in Business-Cycle Fluctuations«

#### Positive liqudity:

Broer, Druedahl, Harmenberg and Öberg (2023), »Fiscal stimulus policies according to HANK-SAM«



**Code:** HANK-SAM in GEModelToolsNotebooks

# Household problem

$$\begin{aligned} v_t(\beta_i, u_{it}, a_{it-1}) &= \max_{c_{it}, a_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_i \mathbb{E}_t \left[ v_{t+1} \left( \beta_i, u_{it+1}, a_{it} \right) \right] \\ \text{s.t. } a_{it} + c_{it} &= (1+r_t)a_{it-1} + (1-\tau_t)y_t(u_{it}) + \mathsf{div}_t + \mathsf{transfer}_t \\ a_{it} &\geq 0 \end{aligned}$$

- 1. Dividends and government transfers: div<sub>t</sub> and transfer<sub>t</sub>
- 2. Real wage: w<sub>t</sub>
- 3. Income tax:  $\tau_t$
- 4. **Separation rate** for employed:  $\delta_t$
- 5. **Job-finding rate** for unemployed:  $\lambda_t^{u,s} s(u_{it-1})$  (where  $s(u_{it-1})$  is exogenous search effectiveness)
- 6. US-style duration-dependent **UI system:** 
  - a) High replacement rate  $\overline{\phi}$ , first  $\overline{u}$  months
  - b) Low replacement rate  $\phi\text{,}$  after  $\overline{u}$  months

# Income process

Income is

$$y_{it}(u_{it}) = w_{ss} \cdot egin{cases} 1 & ext{if } u_{it} = 0 \ \overline{\phi} \mathsf{UI}_{it} + (1 - \mathsf{UI}_{it}) \underline{\phi} & ext{else} \end{cases}$$

where share of the month with UI is

$$\mathsf{UI}_{it} = egin{cases} 0 & \text{if } u_{it} = 0 \ 1 & ext{else if } u_{it} < \overline{u}_t \ 0 & ext{else if } u_{it} > \overline{u}_t + 1 \ \overline{u} - \left(u_{it} - 1
ight) & ext{else} \end{cases}$$

■ **Note:** Hereby  $\overline{u}$  becomes a continuous variables

#### Transition probabilities

Beginning-of-period value function:

$$\underline{v}_{t}\left(\beta_{i}, u_{it-1}, a_{it-1}\right) = \mathbb{E}\left[v_{t}(\beta_{i}, u_{it}, a_{it-1}) \mid u_{it-1}, a_{it-1}\right]$$

- **Grids:**  $u_{it} \in \{0, 1, \dots, \#_u 1\}$  for  $\#_u 1$
- Workers with  $u_{it-1} = 0$ :

$$u_{it} = egin{cases} 0 & \text{with } 1 - \delta_t \\ 1 & \text{with } \delta_t \end{cases}, \ \mathsf{UI}_{it} = egin{cases} 1 & \text{with } \pi^{\mathsf{UI}} \\ 0 & \text{with } 1 - \pi^{\mathsf{UI}} \end{cases}$$

• **Unemployed** with  $u_{it-1} = 1$ :

$$u_{it} = \begin{cases} 0 & \text{with } \lambda_t^{u,s} s(u_{it-1}) \\ \min \{u_{it-1} + 1, \#_u - 1\} & \text{with } 1 - \lambda_t^{u,s} s(u_{it-1}) \end{cases}, \, \mathsf{UI}_{it} = \mathsf{UI}_{it-1}$$

# Hiring and firing

Job value:

$$V_t^j = p_t^{\scriptscriptstyle{\mathrm{X}}} Z_t - \mathit{w}_{\mathit{ss}} + eta^{\mathsf{firm}} \mathbb{E}_t \left[ (1 - \delta_{\mathit{ss}}) V_{t+1}^j 
ight]$$

Vacancy value:

$$V_t^{
m v} = -\kappa + \lambda_t^{
m v} V_t^j + (1-\lambda_t^{
m v})(1-\delta_{
m ss})eta^{
m firm} \mathbb{E}_t \left[V_{t+1}^{
m v}
ight]$$

• Free entry implies

$$V_t^v = 0$$

# Labor market dynamics

Labor market tightness is given by

$$\theta_t = \frac{v_t}{S_t}$$

Cobb-Douglas matching function implies:

$$\lambda_t^v = A\theta_t^{-\alpha}$$
$$\lambda_t^{u,s} = A\theta_t^{1-\alpha}$$

Law of motion for unemployment:

$$u_t = u_{t-1} + \delta_t (1 - u_{t-1}) - \lambda_t^{u,s} S_t$$

# Standard New Keynesian block

- Intermediate goods price: p<sub>t</sub><sup>X</sup>
- Dixit-Stiglitz demand curve ⇒ Phillips curve relating marginal cost, MC<sub>t</sub> = p<sub>t</sub><sup>x</sup>, and final goods price inflation, Π<sub>t</sub> = P<sub>t</sub>/P<sub>t-1</sub>,

$$1 - \epsilon + \epsilon p_t^{\mathsf{x}} = \phi \pi_t (1 + \pi_t) - \phi \beta^{\mathsf{firm}} \mathbb{E}_t \left[ \pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right]$$

with output  $Y_t = Z_t(1 - u_t)$ 

- Flexible price limit:  $\phi \to 0$
- Taylor rule:

$$1+i_t = (1+i_{ss})\left(rac{1+\pi_t}{1+\pi_{ss}}
ight)^{\delta_{\pi}}$$

#### Government

- $\qquad \qquad \textbf{Unemployment insurance: } \Phi_t = \textit{w}_{\textit{ss}} \left( \overline{\phi}_t \mathsf{UI}_t^{\textit{hh}} + \underline{\phi} \left( \textit{u}_t \mathsf{UI}_t^{\textit{hh}} \right) \right)$
- Total expenses:  $X_t = \Phi_t + G_t + \text{transfer}_t$
- Total taxes:  $taxes_t = \tau_t (\Phi_t + w_{ss}(1 u_t))$
- Government budget is

$$q_t B_t = (1 + q_t \delta_q) B_{t-1} + X_t - \mathsf{taxes}_t$$

Tax rule:

$$egin{aligned} ilde{ au}_t &= rac{\left(1 + q_t \delta_q
ight) B_{t-1} + X_t - q_{ss} B_{ss}}{\Phi_t + w_{ss} (1 - u_t)} \ au_t &= \omega ilde{ au}_t + (1 - \omega) au_{ss} \end{aligned}$$

# **Equilibrium**

#### 1. Financial markets:

$$\begin{split} \frac{1+\delta_q q_{t+1}}{q_t} &= \frac{1+i_t}{1+\pi_{t+1}} \\ 1+r_t &= \begin{cases} \frac{(1+\delta_q q_0)B_{-1}}{A_{-1}^{hh}} & \text{if } t=0 \\ \frac{1+i_{t-1}}{1+\pi_t} & \text{else} \end{cases} \end{split}$$

#### 2. Market clearing:

$$A_t^{hh} = q_t B_t$$
$$Y_t = C_t^{hh} + G_t$$

**Summary** 

#### Summary

#### Mixed advanced:

- 1. Policies (shocks, targets, rule, optimal)
- 2. Life-cycle (age, mortality, income profile)
- Endogenous idiosyncratic risk (external/internal)
- 4. Discrete choices with taste shocks (upper envelope, non-convex)
- 5. Bounded rationality (manipulation of Jacobian, myopic, sticky)
- HANK-SAM (labor market dynamics, fiscal policy)