# Assignment I

**Vision:** This project teaches you to solve for the *stationary equilibrium* and *transition path* in a heterogeneous agent model and analyze the economic results.

- **Problem:** The problem consists of
  1. A number of questions (page 2)
  2. A model (page 3-4)
- **Code:** The problem is designed to be solved with the *GEModelTools* package.
- **Structure:** Your project should consist of
  1. A single self-contained pdf-file with all results
  2. A single Jupyter notebook showing how the results are produced
  3. Well-documented *.py* files
- **Hand-in:** Upload a single zip-file on Absalon (and nothing else)
- **Deadline:** 6th of October 2023
- **Exam:** Your Assignment I will be a part of your exam portfolio.
  You can incorporate feedback before handing in the final version.

# HANC with multiple types of labor

a) **Setup.** Define the stationary equilibrium, the transition path and create a DAG for the model.

b) **Solve for the stationary equilibrium.** Discuss and illustrate which factors determines wealth inequality.

c) **Compute and inspect the Jacobians of the household block wrt. $\varphi_0$.**

d) **Solve for the transition path when $\varphi_{1t}$ is 10 percent higher for 10 periods.** Discuss which types of agents this benefits.

e) **Solve for the transition path when $\varphi_{1t}$ is *permanently* 10 percent higher.** Discuss which types of agents this benefits.

# 1. Model

**Households.** The model has a continuum of infinitely lived households indexed by $i \in [0,1]$. Households are *ex ante* heterogeneous in terms of their discount factors, $\beta_i$, and their ability, $\chi_i$. The discount factors are drawn with equal probabilities from a three element set, $\beta_i \in \{\breve{\beta} - \sigma_\beta, \breve{\beta}, \breve{\beta} + \sigma_\beta\}$. The abilities are either low or high, $\chi_i \in \{0,1\}$, with probabilities $\frac{2}{3}$ and $\frac{1}{3}$.

Households choose consumption and exogenously supply two types of labor, $\eta_i^j$ for $j \in \{0,1\}$ with associated productivity $\varphi_t^j$. Savings is in terms of capital, which is rented out to firms at the rental rate, $r_t^K$. There are no possibilities to borrow. Households are *ex post* heterogeneous in terms of their stochastic labor productivity, $s_{it}$, and their (end-of-period) savings, $a_{it-1}$. The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before shocks are realized and $\boldsymbol{D}_t$ afterwards. The real wages are $w_t^j$, and real-profits are $\Pi_t$.

The household problem is

$$\begin{aligned}
v_t(s_{it}, a_{it-1}) &= \max_{c_t} \frac{c_{it}^{1-\sigma}}{1-\sigma} - \nu \frac{\left(\sum_{j=0}^{1} \eta_i^j\right)^{1+\varepsilon}}{1+\varepsilon} + \beta_i \mathbb{E}_t \left[ v_{t+1}(s_{it+1}, a_{it}) \right] \qquad (1) \\
\text{s.t. } a_{it} + c_{it} &= (1 + r_t^K - \delta) a_{it-1} + \sum_{j=0}^{1} w_t^j \varphi_t^j \eta_i^j s_{it} + \Pi_t \\
\log s_{it+1} &= \rho_s \log s_{it} + \psi_{it+1}, \;\; \psi_{it+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \; \mathbb{E}[s_{it}] = 1 \\
a_{it} &\geq 0.
\end{aligned}$$

The Euler-equation is

$$c_{it}^{-\sigma} = \beta_i \mathbb{E} \left[ v_{a,it+1}(s_{it+1}, a_{it}) \right] \qquad (2)$$

$$v_{a,it} = (1 + r_t^K - \delta) c_{it}^{-\sigma}. \qquad (3)$$

The aggregate quantities of central interest are

$$C_t^{hh} = \int c_{it} d\boldsymbol{D}_t \qquad (4)$$

$$L_t^{j,hh} = \int \varphi_t^i \eta_i^j s_{it} d\boldsymbol{D}_t \;\; \text{for } j \in \{0,1\} \qquad (5)$$

$$A_t^{hh} = \int a_{it} d\boldsymbol{D}_t \qquad (6)$$

**Firms.** A representative firm rents capital, $K_{t-1}$, and hires both types of labor, $L_t^1$ and $L_t^2$, to produce goods, with the production function

$$Y_t = \Gamma_t K_{t-1}^{\alpha} \Pi_{j=0}^{1} \left( L_t^j \right)^{\frac{1-\alpha}{2}} \tag{7}$$

where $\Gamma_t$ is technology and $\alpha$ is the Cobb-Douglas weight parameter on capital. Capital depreciates with the rate $\delta \in (0, 1)$. The real rental price of capital is $r_t^K$ and the real wages are $w_t^j$. Profits are $\Pi_t = Y_t - \sum_{j=0}^{1} w_t^j L_t^j - r_t^K K_{t-1}$. The households own the representative firm in equal shares.

The law-of-motion for capital is $K_t = (1-\delta)K_{t-1} + I_t$.

**Market clearing.** Market clearing implies

1. Asset market: $K_t = A_t^{hh}$
2. Labor market: $L_t^j = L_t^{j,hh}$
3. Goods market: $Y_t = C_t^{hh} + I_t$

# 2. Calibration

1. **Preferences:** $\sigma = 2$, $\breve{\beta} = 0.975$, $\sigma_\beta = 0.01$, $\nu = 0.5$, $\varepsilon = 1.0$
2. **Labor supply**: $\eta_i^j = \begin{cases} 1 & \text{if } \chi_i = j \\ 0 & \text{else} \end{cases}$, $\varphi_{ss}^0 = 1$, $\varphi_{ss}^1 = 2$
3. **Income process:** $\rho_s = 0.95$, $\sigma_\psi = 0.30\sqrt{(1-\rho_z^2)}$,
4. **Production:** $\Gamma_{ss} = 1$, $\alpha_{ss} = 0.36$, $\delta = 0.10$