CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

13a. Global solution methods with aggregate risk

Adv. Macro: Heterogenous Agent Models

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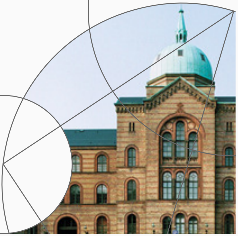

Danmarks Grundforskningsfond
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  1. Non-linear solution
  2. Linear solution $\rightarrow$ simulation with aggregate risk

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    1. **Local solution in state space** (pertubation)
       Higher order terms can (in principle) be included
       Reference: Bayer and Luetticke (2020)

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       Problem: *The distribution of households is a state variable*
       References: Krusell and Smith (1998); Algan et al. (2014); Proehl (2019); Maliar et al. (2021); Azinovic et al. (2022); Kase et al. (2022)

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  github.com/JeppeDruedahl/GlobalHA

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- **Disclaimer:** *This is advanced stuff!*

Household problem with aggregate risk

$$
\begin{aligned}
v(Z_t, \boldsymbol{D}_t, z_{it}, a_{it-1}) &= \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[v(Z_{t+1}, \boldsymbol{D}_{t+1}, z_{it+1}, a_{it}) \right] \\
\text{s.t. } a_{it} &= (1+r_t) a_{it-1} + w_t z_{it} - c_{it} \geq 0 \\
\log z_{it+1} &= \rho_z \log z_{it} + \psi_{it+1} \ \ , \psi_{it+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \, \mathbb{E}[z_{it}] = 1 \\
K_{t-1} &= \int a_{it-1} d\boldsymbol{D}_t \\
L_t &= \int z_{it} d\boldsymbol{D}_t = 1 \\
r_t &= \alpha Z_t (K_{t-1}/L_t)^{\alpha\text{-}1} - \delta \\
w_t &= (1-\alpha) Z_t (K_{t-1}/L_t)^{\alpha} \\
\boldsymbol{D}_{t+1} &= \Gamma(Z_t, \boldsymbol{D}_t) \\
Z_{t+1} &\sim \Gamma_Z(Z_t)
\end{aligned}
$$

Method

Assuming (strong) approximate aggregation

$$
\begin{aligned}
v(Z_t, K_{t-1}, z_{it}, a_{it-1}) &= \max_{c_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[v(Z_{t+1}, K_t, z_{it+1}, a_{it}) \right] \\
\text{s.t. } a_{it} &= (1+r_t) a_{it-1} + w_t z_{it} - c_{it} \geq 0 \\
\log z_{it+1} &= \rho_z \log z_{it} + \psi_{it+1} \quad , \psi_{it+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi),\ \mathbb{E}[z_{it}] = 1 \\
r_t &= \alpha Z_t (K_{t-1})^{\alpha\text{-}1} - \delta \\
w_t &= (1-\alpha) Z_t (K_{t-1})^{\alpha} \\
K_t &= \text{PLM}(Z_t, K_{t-1}) \\
Z_{t+1} &\sim \Gamma_Z(Z_t)
\end{aligned}
$$

PLM $\equiv$ »perceived law of motion«

Weak approximation in general

$$
\begin{aligned}
v(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) &= \max_{c_t} u(c_t) + \beta \mathbb{E}_t \left[v(\boldsymbol{Z}_{t+1}, \boldsymbol{S}_t, \boldsymbol{z}_{t+1}, a_t) \right] \\
\text{s.t. } \boldsymbol{S}_t, \boldsymbol{P}_t &= \text{PLM}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}) \\
a_t + c_t &= m(\boldsymbol{z}_t, a_{t-1}, \boldsymbol{P}_t) \\
\boldsymbol{z}_{t+1} &\sim \Gamma_z(\boldsymbol{z}_t) \\
\boldsymbol{Z}_{t+1} &\sim \Gamma_Z(\boldsymbol{Z}_t) \\
a_t &\geq -b
\end{aligned}
$$

1. $\boldsymbol{Z}_t$ are exogenous aggregate shocks.
2. $\boldsymbol{S}_{t-1}$ are pre-determined (finite dimensional) aggregate states.
3. $\boldsymbol{P}_t$ are »prices«.
4. PLM($\bullet$) is the *Perceived-Law-of-Motion*.
5. $\boldsymbol{z}_t$ is stochastic and exogenous idiosyncratic states.
6. c_t is consumption providing utility $u(c_t)$ discounted by β.
7. a_t is end-of-period assets (borrowing constraint given by b).
8. $m(\bullet)$ is cash-on-hand with $\frac{\partial m(\bullet)}{\partial a_{t-1}} > 0$.

EGM can still be used

1. **EGM**

$$
\begin{aligned}
q(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_t) &= \mathbb{E}\left[v_a(\boldsymbol{Z}_{t+1}, \boldsymbol{S}_t, \boldsymbol{z}_{t+1}, a_t)\right] \\
\tilde{c}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_t) &= (\beta q(\bullet))^{-\frac{1}{\sigma}} \\
\tilde{m}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_t) &= a_t + c(\bullet) \\
c^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) &= \text{interp } \tilde{m}(\bullet) \to \tilde{c}(\bullet) \text{ at } m(\boldsymbol{z}_t, a_{t-1}, \boldsymbol{P}_t) \\
a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) &= m(\bullet) - c^\star(\bullet) \\
v_a(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) &= \frac{\partial m(\bullet)}{\partial a_{t-1}} c^\star(\bullet)^{-\sigma}
\end{aligned}
$$

2. **Implied savings:**

$$
\begin{aligned}
a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, m_t) &= a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) \\
a_{t-1} &= m^{-1,a}(m_t, \boldsymbol{z}_t, \boldsymbol{P}_t)
\end{aligned}
$$

Simulation

1. Draw $\boldsymbol{Z}_t$ given $\boldsymbol{Z}_{t-1}$
2. Find
$$a_t^\star(\boldsymbol{z}_t, m_t) = a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, m_t)$$
   by interpolation over $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$
3. Search for $\boldsymbol{P}_t$ so
$$\int a_t^\star(\boldsymbol{z}_t, m(\boldsymbol{z}_t, a_{t-1}, \boldsymbol{P}_t))d\boldsymbol{D}_t$$
   clears the savings market

Fixed-point iteration for PLM

1. Draw shocks to be used in all simulations

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4. Given the PLM compute $\breve{\boldsymbol{S}}^0$ and $\breve{\boldsymbol{P}}^0$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$

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5. Set the PLM convergence iteration counter $n = 0$

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7. Simulate the model given household behavior

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Fixed-point iteration for PLM

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6. Solve the approximate household problem given $\breve{\boldsymbol{S}}^n$ and $\breve{\boldsymbol{P}}^n$
7. Simulate the model given household behavior
8. Estimate the PLM on the simulated data
9. Given the PLM compute $\breve{\boldsymbol{S}}_{NEW}$ and $\breve{\boldsymbol{P}}_{NEW}$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$

Fixed-point iteration for PLM

1. Draw shocks to be used in all simulations
2. Solve and simulate a linearized version of the model
3. Estimate the PLM on the simulated data
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5. Set the PLM convergence iteration counter $n = 0$
6. Solve the approximate household problem given $\breve{\boldsymbol{S}}^n$ and $\breve{\boldsymbol{P}}^n$
7. Simulate the model given household behavior
8. Estimate the PLM on the simulated data
9. Given the PLM compute $\breve{\boldsymbol{S}}_{NEW}$ and $\breve{\boldsymbol{P}}_{NEW}$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$
10. Stop if $|\breve{\boldsymbol{S}}_{NEW} - \breve{\boldsymbol{S}}^n|_\infty <$ tol. and $|\breve{\boldsymbol{P}}_{NEW} - \breve{\boldsymbol{P}}^n|_\infty <$ tol.

Fixed-point iteration for PLM

1. Draw shocks to be used in all simulations
2. Solve and simulate a linearized version of the model
3. Estimate the PLM on the simulated data
4. Given the PLM compute $\breve{\boldsymbol{S}}^0$ and $\breve{\boldsymbol{P}}^0$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$
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10. Stop if $|\breve{\boldsymbol{S}}_{NEW} - \breve{\boldsymbol{S}}^n|_\infty <$ tol. and $|\breve{\boldsymbol{P}}_{NEW} - \breve{\boldsymbol{P}}^n|_\infty <$ tol.
11. Update $\breve{\boldsymbol{S}}$ and $\breve{\boldsymbol{P}}$ by relaxation with $\omega \in (0,1)$

$$\breve{\boldsymbol{S}}^{s+1} = \omega \breve{\boldsymbol{S}}_{NEW} + (1-\omega)\breve{\boldsymbol{S}}^s$$
$$\breve{\boldsymbol{P}}^{s+1} = \omega \breve{\boldsymbol{P}}_{NEW} + (1-\omega)\breve{\boldsymbol{P}}^s.$$

Fixed-point iteration for PLM

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2. Solve and simulate a linearized version of the model
3. Estimate the PLM on the simulated data
4. Given the PLM compute $\breve{\boldsymbol{S}}^0$ and $\breve{\boldsymbol{P}}^0$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$
5. Set the PLM convergence iteration counter $n = 0$
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7. Simulate the model given household behavior
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10. Stop if $|\breve{\boldsymbol{S}}_{NEW} - \breve{\boldsymbol{S}}^n|_\infty <$ tol. and $|\breve{\boldsymbol{P}}_{NEW} - \breve{\boldsymbol{P}}^n|_\infty <$ tol.
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$$\breve{\boldsymbol{S}}^{s+1} = \omega \breve{\boldsymbol{S}}_{NEW} + (1-\omega)\breve{\boldsymbol{S}}^s$$
$$\breve{\boldsymbol{P}}^{s+1} = \omega \breve{\boldsymbol{P}}_{NEW} + (1-\omega)\breve{\boldsymbol{P}}^s.$$

12. Increment n and return to step 6

PLMs

- **Input:** $X_{it} \in \mathbf{Z}_t, \mathbf{S_{t-1}}$, i'th input to the PLM for $i \in \{1, \ldots, \#_{ZS}\}$

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- **Linear:** Estimated by OLS

$$Y_{jt} = \Psi_{j0} + \sum_{i=1}^{\#_{ZS}} \Psi_{ji} X_{it}$$

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$$Y_{jt} = \Psi_{j0} + \sum_{i=1}^{\#_{ZS}} \Psi_{ji} X_{it}$$

- **Non-linear:** Estimated with *Radial Basis Functions* (RBF)

$$Y_{jt} = \Psi_{j00} + \sum_{i=1}^{\#_{ZS}} \Psi_{j0i} X_{it} + \sum_{\tau=1}^{\mathcal{T}} \Psi_{jk} \phi \left(\sum_{i=1}^{\#_{ZS}} \sqrt{\left(X_{it} - X_{i\tau}^{\text{sim}}\right)^2} \right)$$

$X_{i\tau}^{\text{sim}}$ is simulation outcome

$\phi(x) = x^2 \log x$

PLMs

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$\phi(x) = x^2 \log x$

- **Paper:** Also neural net

Model

Firms

- **Technology shocks:**

$$Z_{t+1} - Z_{ss} = \rho_Z(Z_t - Z_{ss}) + \epsilon^Z_{t+1}, \;\; \epsilon^Z_{t+1} \sim \mathcal{N}(0, \sigma^2_Z)$$

- **Production firm problem:**

$$\max_{L_t, K_{t-1}, u_t} u_t Z_t K^{\alpha}_{t-1} L^{1-\alpha}_t - w_t L_t - r^k_t K_{t-1} - \chi_1 (u_t - \tilde{u}) - \frac{\chi_2}{2} (u_t - \tilde{u})^2$$
$$\text{s.t. } u_t \leq \bar{u}.$$

implies

$$\begin{aligned}
r^k_t &= \alpha u_t Z_t (K_{t-1}/L_t)^{\alpha\text{-}\mathbf{1}} \equiv r^k(u_t, Z_t, K_{t-1}, L_t) \\
w_t &= (1-\alpha) u_t Z_t (K_{t-1}/L_t)^{\alpha} \equiv w(u_t, Z_t, K_{t-1}, L_t) \\
u_t &= \max\left[\frac{Z_t K^{\alpha}_{t-1} L^{1-\alpha}_t - \chi_1 + \chi_2 \tilde{u}}{\chi_2}, \bar{u}\right] \equiv u(Z_t, K_{t-1}, L_t)
\end{aligned}$$

Capital producers

- **Capital producer problem**

$$\max_{\{I_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t I_t \left\{ q_t \left[1 - \frac{\phi}{2} \left(\log \frac{I_t}{I_{t-1}} \right)^2 \right] - 1 \right\}$$

implies

$$q_t \left[1 - \phi \log \frac{I_t}{I_{t-1}} \right] = 1 - \beta \mathbb{E}_t \left[q_{t+1} \phi \log \left(\frac{I_{t+1}}{I_t} \right) \right]$$

- **Accumulation:** $K_t = I_t + (1-\delta) K_{t-1}$
- **Real interest rate:**

$$r_t = r_t^k - q_t \delta = r^k(u_t, Z_t, K_{t-1}, L_t) - q_t \delta \equiv r(u_t, Z_t, K_{t-1}, L_t, q_t).$$

Households

$$v(Z_t, K_{t-1}, I_{t-1}, z_t, a_{t-1}) = \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[v(Z_{t+1}, K_t, I_t, z_{t+1}, a_t) \right]$$

s.t.

$$\begin{aligned}
r_t, w_t, K_t, I_t &= \mathrm{PLM}(Z_t, K_{t-1}, I_{t-1}) \\
a_t + c_t &= (1 + r_t) a_{t-1} + w_t z_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \quad , \psi_{t+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \; \mathbb{E}[z_{t+1}] = 1 \\
Z_{t+1} &= Z_{ss} + \rho_Z (Z_t - Z_{ss}) + \epsilon^Z_{t+1}, \;\; \epsilon^Z_{t+1} \sim \mathcal{N}(0, \sigma_Z^2) \\
a_t &\geq 0
\end{aligned}$$

Solution method (1/2)

1. The shocks are $\boldsymbol{Z}_t = \{Z_t\}$.
2. The aggregate states are $\boldsymbol{S}_t = \{K_t, I_t\}$.
3. The »prices« are $\boldsymbol{P}_t = \{r_t, w_t\}$,
4. The PLM is

$$
\begin{aligned}
K_t &= \mathrm{PLM}_K(Z_t, I_{t-1}, K_{t-1}; \Psi) \\
q_t &= \mathrm{PLM}_q(Z_t, I_{t-1}, K_{t-1}; \Psi) \\
u_t, w_t, r_t^k, r_t &= u(\bullet), w(\bullet), r_t^k(\bullet), r(\bullet)
\end{aligned}
$$

5. The cash-on-hand function is

$$
m(z_t, a_{t-1}, \boldsymbol{P}_t) = (1 + r_t)a_{t-1} + w_t z_t.
$$

Solution method (2/2)

6. The market clearing condition is

$$\int a_t^\star(\boldsymbol{z}_t, m(\boldsymbol{z}_t, a_{t-1}, w_t, r_t))dD_t = K_t,$$

where we guess on I_t and get r_t from

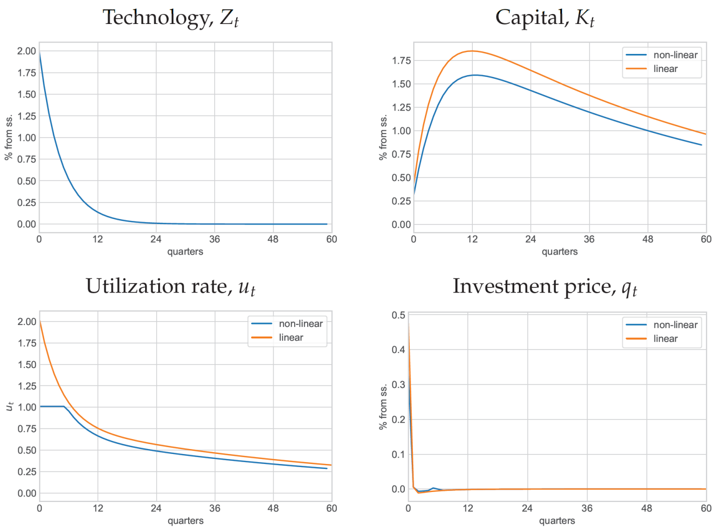
$$\begin{aligned} q_t &= \frac{1-\beta\mathbb{E}_t\left[q_{t+1}\phi\log\left(\frac{I_{t+1}}{I_t}\right)\right]}{1-\phi\log\left(\frac{I_t}{I_{t-1}}\right)} \\ K_{t+1} &= \mathrm{PLM}_K(Z_{t+1}, I_t, K_t; \Psi) \\ I_{t+1} &= K_{t+1} - (1-\delta)K_t \\ q_{t+1} &= \mathrm{PLM}_q(Z_{t+1}, I_t, K_t; \Psi) \\ u_t, w_t, r_t^k, r_t &= u(\bullet), w(\bullet), r_t^k(\bullet), r(\bullet) \end{aligned}$$

where expectations are evaluated using Gauss-Hermite quadrature

Results

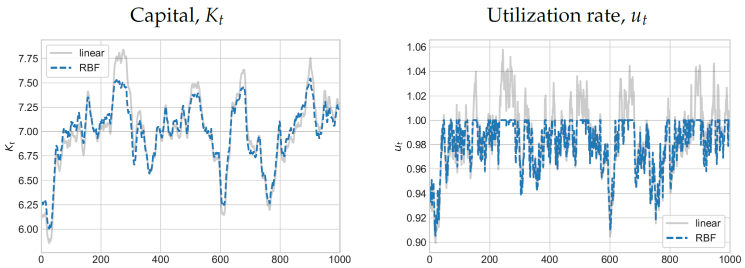
Linear is wrong - IRF

Figure 1: Perfect foresight and linearized impulse responses



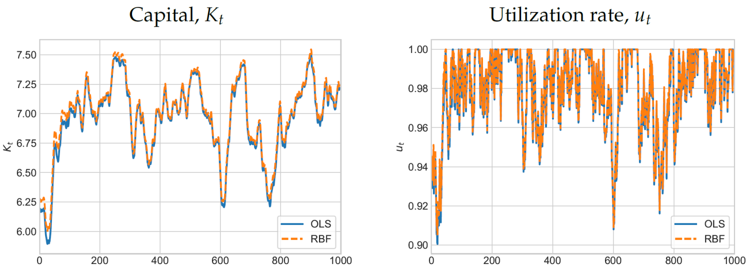
Linear is wrong - simulation

Figure 2: Simulation (in-sample): RBF vs. linear



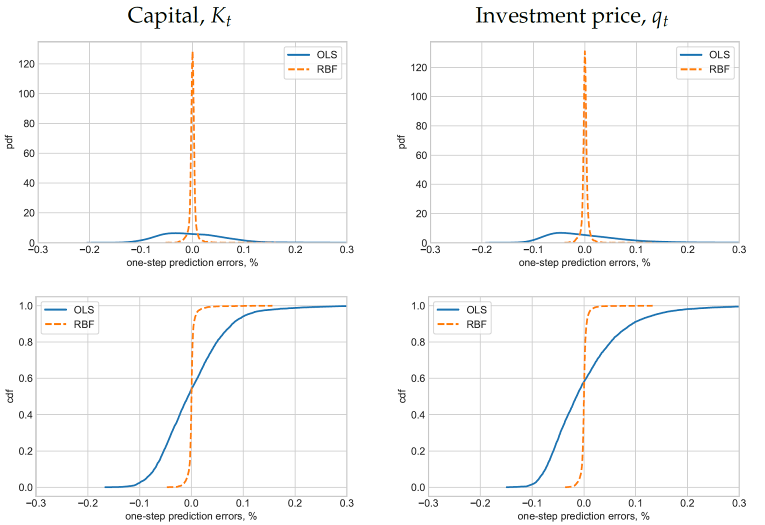
OLS vs. RBF: Some differences

Figure 3: Simulation (out-of-sample): PLM methods



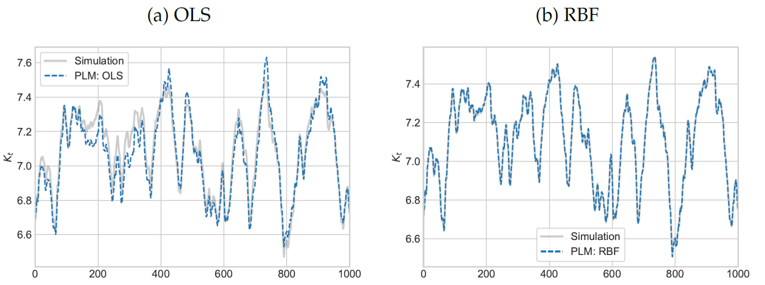
RBF much more precise (1/3)

Figure 4: One-step ahead PLM errors



RBF much more precise (2/3)

Figure 5: Dynamic PLM errors



Pure PLM simulation from $K_{-1}^{\text{PLM}} = K_{-1}$ and $I_{-1}^{\text{PLM}} = I_{-1}$

$$
\begin{aligned}
K_t^{\text{PLM}} &= \text{PLM}_K(Z_t, K_{t-1}^{PLM}, I_{t-1}^{PLM}) \\
q_t^{\text{PLM}} &= \text{PLM}_K(Z_t, K_{t-1}^{PLM}, I_{t-1}^{PLM}) \\
I_t^{\text{PLM}} &= K_t^{\text{PLM}} - (1-\delta)K_{t-1}^{PLM}.
\end{aligned}
$$

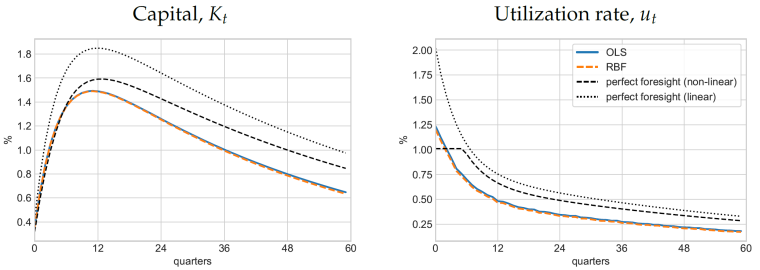
RBF much more precise (3/3)

Table 1: HANC: Prediction Errors

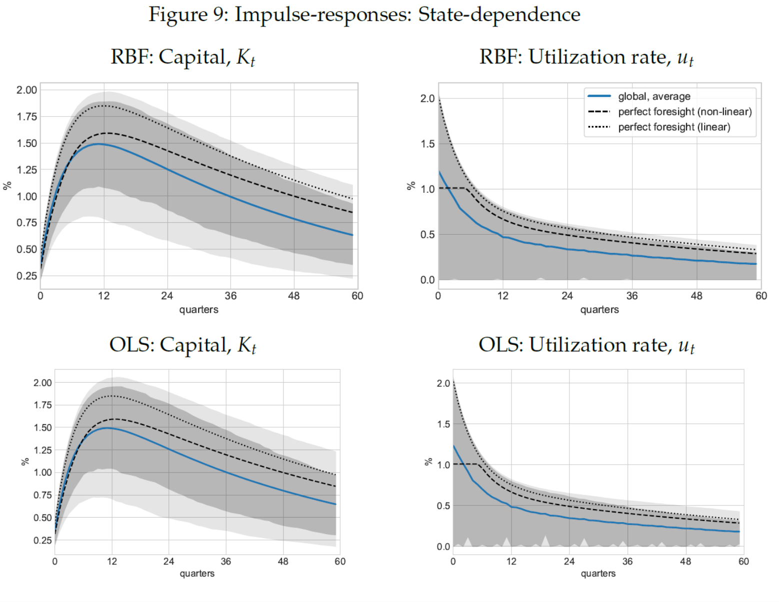
	OLS	RBF	NN
	dynamic log prediction errors $\times$ *100*		
max	3.75	0.26	
mean	0.88	0.04	
median	0.79	0.03	
99th perc.	3.16	0.18	
90th perc.	1.62	0.07	
		timings (secs.)	
total	666.3	722.2	
- solve household problem	356.2	315.4	
- simulate with market clearing	310.0	356.3	
- estimate PLMs	0.0	50.5	
iterations	13	14	

Some differences in IRFs

Figure 8: Impulse-responses: Global vs. perfect foresight



A lot of state dependence



Conclusion

Conclusion (1/2)

- **Expanding literature:**
  1. **Hardware:** Graphic cards
  2. **Software:** Automatic differentiation
  3. **Discretization:** Polynomial chaos
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- **Agent-based models:** $\div$ intention, $\div$ forward-looking

Conclusion (2/2)

My sapere aude project: **»Modeling economic agents as deep reinforcement learners«** (optimizing $\rightarrow$ »satisfying«)