



6. Consumption-Saving Models: Extensions

Adv. Macro: Heterogenous Agent Models

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Introduction

Consumption-Saving Models

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 2. How to distinguish between habits vs inattention?
 3. What if households have trouble saving due to temptation?

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- **Central economic questions:**
 1. How do we explain excess smoothness in consumption data?
 2. How to distinguish between habits vs inattention?
 3. What if households have trouble saving due to temptation?
- **Plan for today:**
 1. Discuss the problem of “excess smoothness” of consumption
 2. Study the model of sticky expectations (Carroll et al, 2019)
 3. Study the role of housing as a “savings commitment device” (Attanasio et al, 2021)

Important Note

- The views expressed in this presentation are those of the author and do not represent the views of the Federal Reserve Board or Federal Reserve System.

Excess Smoothness

- One of the key puzzles in consumption-saving models is the “excess smoothness” of consumption
 - Theory: Consumption responds instantly, completely to shock
 - Evidence: Consumption is too smooth

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- Campbell and Deaton (1989): Consumption does not react sufficiently to innovations to the permanent component of income

Theory: Simple Model with Quadratic Utility

Hall (1978) Random Walk

- Household utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(\mathbf{c}_t)$$

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$$\mathbf{o}_{t+1} = (\mathbf{o}_t - \mathbf{c}_t)R + \zeta_{t+1}$$

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But in the data: permanent income much noisier than consumption

Excess Smoothness

One Explanation: Habit Formation

Popular solution in DSGE models: habit formation

- Household utility depends on both $c_{i,t}$ and $c_{i,t-1}$

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(\tilde{c}_{i,t})$$

- where

$$\tilde{c}_{i,t} = c_{i,t} - \chi c_{i,t-1}$$

- χ is positive if goods provide services across periods
- zero if goods are fully non-durable, non-habit forming

Testable implications of habit formation

- Consumption Euler equation (for full derivation, see Dynan 2000):

$$E_t \left[(1 + r) \beta \frac{MU_{i,t+1}}{MU_{i,t}} \right] = 1$$

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- If we assume CRRA utility function, $u(c) = \frac{c^{1-\rho}}{1-\rho}$ then:

$$(1+r)\beta \left(\frac{\tilde{c}_{i,t}}{\tilde{c}_{i,t-1}} \right)^{-\rho} = 1 + \varepsilon_{i,t}$$

where $\varepsilon_{i,t}$ is the expectational error

Testable implications of habit formation

- Taking logs and substituting for \tilde{c} gives:

$$\Delta \ln(c_{i,t} - \chi c_{i,t-1}) = \frac{1}{\rho} [\ln(1+r) + \ln(\beta)] - \frac{1}{\rho} \ln(1 + \varepsilon_{i,t})$$

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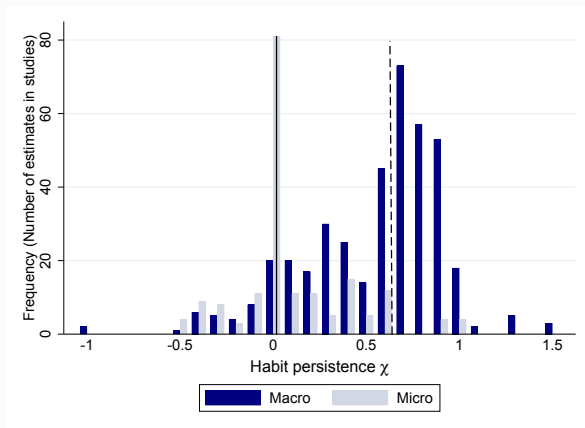
- This yields the following estimable equation:

$$\Delta \ln(c_{i,t}) = \gamma_0 + \chi \Delta \ln(c_{i,t-1}) + e_{i,t}$$

which can be estimated on either micro data or macro data
(if macro data, then just remove the i subscript)

Empirical estimates of habit persistence

- χ has been estimated by over 597 different papers



- Mean χ in macro studies: 0.6
- Mean χ in micro studies: 0.0-0.1

Why the disagreement between macro and micro studies?

Macro: Representative Agent Models

- Theory: C responds instantly, completely to shock
- Evidence: Consumption is too smooth (Campbell & Deaton, 1989)
- Solution: **“Habits” parameter** $\chi^{\text{Macro}} \approx 0.6 \sim 0.8$

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Micro: Heterogeneous Agent Models

- **Uninsurable risk is essential, changes everything**
- Var of micro income shocks much larger than of macro shocks:
$$\text{var}(\Delta \log \mathbf{p}) \approx 100 \times \text{var}(\Delta \log \mathbf{P})$$
- Evidence: **“Habits” parameter** $\chi^{\text{Micro}} \approx 0.0 \sim 0.1$

Macro Inattention

Alternative Explanation: Inattention to Macro Aggregates

Carroll, Crawley, Slacalek, Tokuoka, White (2019):

- Income Has Idiosyncratic and Aggregate Components
- Idiosyncratic Component Is Perfectly Observed
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- Pervasive Lesson of Micro Data

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Utility Cost of Inattention Small

- Micro: Critical (and Easy) To Notice You're Unemployed
- Macro: *Not* Critical To *Instantly* Notice If $U \uparrow$

Hall (1978) Random Walk

- **Total Wealth** (Human + Nonhuman):

$$\mathbf{o}_{t+1} = (\mathbf{o}_t - \mathbf{c}_t)R + \zeta_{t+1}$$

- **C Euler Equation:**

$$u'(\mathbf{c}_t) = R\beta\mathbb{E}_t[u'(\mathbf{c}_{t+1})]$$

- \Rightarrow **Random Walk** (for $R\beta = 1$):

$$\Delta\mathbf{c}_{t+1} = \epsilon_{t+1}$$

- **Expected Wealth:**

$$\mathbf{o}_t = \mathbb{E}_t[\mathbf{o}_{t+1}] = \mathbb{E}_t[\mathbf{o}_{t+2}] = \dots$$

Sticky Expectations—Individual c

- Consumer who happens to update at t and $t + n$

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- Implies that $\Delta^n \mathbf{o}_{t+n} \equiv \mathbf{o}_{t+n} - \mathbf{o}_t$ is white noise
- So **individual c** is RW across updating periods:

$$\mathbf{c}_{t+n} - \mathbf{c}_t = (r/R) \underbrace{(\mathbf{o}_{t+n} - \mathbf{o}_t)}_{\Delta^n \mathbf{o}_{t+n}}$$

Sticky Expectations—Aggregate C

- Population normed to one, uniformly dist on $[0, 1]$: $\mathbf{C}_t = \int_0^1 \mathbf{c}_{t,i} di$

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- **Calvo-Type Updating of Expectations:** (Probability $\Pi = 0.25$, per quarter)
- Economy composed of many sticky- \mathbb{E} consumers:

$$\begin{aligned}\mathbf{C}_{t+1} &= (1 - \Pi) \underbrace{\mathbf{C}_{t+1}^{\pi}}_{=\mathbf{C}_t} + \Pi \mathbf{C}_{t+1}^{\pi} \\ \Delta \mathbf{C}_{t+1} &\approx \underbrace{(1 - \Pi)}_{\equiv \chi=0.75} \Delta \mathbf{C}_t + \epsilon_{t+1}\end{aligned}$$

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- **Substantial persistence ($\chi = 0.75$) in aggregate C growth**

One More Ingredient: Idiosyncratic Uncertainty ...

- **Differences: Idiosyncratic vs Aggregate shocks**
 - **Idiosyncratic shocks:** Frictionless observation
 - I notice if I am fired, promoted, somebody steals my wallet
 - True RW with respect to these
 - **Aggregate shocks:** Sticky observation
 - May not instantly notice changes in aggregate productivity

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- **Result:**
 - **Idiosyncratic Δc :** dominated by frictionless RW part
 - **Aggregate ΔC :** highly serially correlated
Law of large numbers \Rightarrow idiosyncratic part vanishes

Macro Inattention

Full Heterogeneous Agent Model

Full Heterogeneous Agent Model

Partial Equilibrium

- CRRA Utility
- Idiosyncratic Shocks Calibrated From Micro Data
- Aggregate Shocks Calibrated From Macro Data
- Markov Process (Discrete RW) for Aggr Income Growth
- Liquidity Constraint
- Mildly Impatient Consumers

- Individual's labor productivity is

$$\ell_{t,i} = \overbrace{\theta_{t,i}}^{\equiv \theta_{t,i}} \overbrace{p_{t,i}}^{\equiv p_{t,i}} \Theta_t P_t$$

- Idiosyncratic and aggregate p evolve according to

$$\begin{aligned} p_{t+1,i} &= p_{t,i} \psi_{t+1,i} \\ P_{t+1} &= \Phi_{t+1} P_t \Psi_{t+1} \end{aligned}$$

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- Φ is Markov 'underlying' aggregate pty growth
 - Discrete (bounded) random walk
 - Calibrated to match postwar US pty growth variation
 - Generates predictability in income growth (for IV regressions)

Blanchard (1985) Model of “Perpetual Youth”

- Household survives from t to $t + 1$ with probability $(1 - D)$:

$$p_{t+1,i} = \begin{cases} 1 & \text{for newborns} \\ p_{t,i} \psi_{t+1,i} & \text{for survivors} \end{cases}$$

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- Blanchardian scheme:

$$\mathbf{k}_{t+1,i} = \begin{cases} 0 & \text{if HH } i \text{ dies, is replaced by newborn} \\ \mathbf{a}_{t,i} / (1 - D) & \text{if household } i \text{ survives} \end{cases}$$

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- Why useful? Allows us to have mortality without an additional state variable:

$$v(\cdot) = \max_c u(c) + \beta \mathbb{E}_t[(1 - D)v(\cdot)]$$

- Market resources:

$$\mathbf{m}_{t,i} = \underbrace{W_t \ell_{t,i}}_{\equiv y_t} + \underbrace{R_t}_{1+r_t} \mathbf{k}_{t,i}$$

- End-of-Period ‘Assets’—Unspent resources:

$$\mathbf{a}_{t,i} = \mathbf{m}_{t,i} - \mathbf{c}_{t,i}$$

- Capital transition depends on prob of survival $1 - D$:

$$\mathbf{k}_{t+1,i} = \mathbf{a}_{t,i} / (1 - D)$$

Frictionless Solution

- Normalize everything by $\mathbf{p}_{t,i} \equiv p_{t,i}P_t$, e.g. $m_{t,i} = \mathbf{m}_{t,i}/(p_{t,i}P_t)$
- $c(m, \Phi)$ is the function that solves:

$$v(m_{t,i}, \Phi_t) = \max_c u(c) + (1-D)\beta \mathbb{E}_t \left[(\Phi_{t+1} \psi_{t+1,i})^{1-\rho} v(m_{t+1,i}, \Phi_{t+1}) \right]$$

- Level of consumption:

$$\mathbf{c}_{t,i} = c(m_{t,i}, \Phi_t) \times p_{t,i}P_t$$

Calvo Updating of Perceptions of Aggregate Shocks

1. *True* Permanent income: $P_{t+1} = \Phi_{t+1} P_t \Psi_{t+1}$
2. Tilde (\tilde{P}) denotes perceived variables
3. Perception for consumer who has not updated for n periods:

$$\tilde{P}_{t,i} = \mathbb{E}_{t-n}[P_t | \Omega_{t-n}] = \Phi_{t-n}^n P_{t-n}$$

because Φ is random walk

Sequence Within Period

1. Income shocks are realized and every individual sees her true \mathbf{y} and \mathbf{m} , i.e. $\mathbf{y}_{t,i} = \tilde{\mathbf{y}}_{t,i}$ and $\mathbf{m}_{t,i} = \tilde{\mathbf{m}}_{t,i}$ for all t and i

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2. Updating shocks realized: i observes true P_t, Φ_t w/ prob Π ; forms perceptions of her normalized market resources $\tilde{m}_{t,i}$

Sticky Expectations about Aggregate Income

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3. Consumes based on her perception, using $c(\tilde{m}_{t,i}, \tilde{\Phi}_{t,i})$

Key Assumption:

- People act as if their perceptions about aggregate state $\{\tilde{P}_{t,i}, \tilde{\Phi}_{t,i}\}$ are the true aggregate state $\{P_t, \Phi_t\}$

Behavior under Sticky Expectations

- Normalized resources:
 - $m_{t,i} \equiv \mathbf{m}_{t,i} / (p_{t,i} P_t)$ is *actual*
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- Correctly perceive level of their own spending $\mathbf{c}_{t,i}$

Macro Inattention

Taking the Model to the Data

Dynan (2000) Specification:

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \mathbb{E}[\Delta \log \mathbf{C}_t] + \eta \mathbb{E}[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

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- χ : **Extent of habits**

Data: Micro: $\chi^{\text{Micro}} = 0.1$ (EER 2017 paper)

Macro: $\chi^{\text{Macro}} = 0.6$

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- **η : Fraction of \mathbf{Y} going to 'rule-of-thumb' $\mathbf{C} = \mathbf{Y}$ types**

Data: Micro: $0 < \eta^{\text{Micro}} < 1$ (Depends ...)

Macro: $\eta^{\text{Macro}} \approx 0.5$ (Campbell and Mankiw (1989))

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- **α : Precautionary saving (micro) or IES (Macro)**

Data: Micro: $\alpha^{\text{Micro}} < 0$ (Zeldes (1989))

Macro: $\alpha^{\text{Macro}} < 0$ (but small)

Micro vs Macro: Theory and Empirics

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

	χ	η	α
Micro			
Data	≈ 0	$0 < \eta < 1$	< 0
Theory: Traditional RA Model	≈ 0	$0 < \eta < 1$	< 0
Macro			
Data	≈ 0.75	≈ 0	< 0
Theory: Traditional RA Model	≈ 0	≈ 0	< 0

Traditional RA model = one without consumption habits

Model with ‘Sticky Expectations’ of aggregate variables can match both micro and macro consumption dynamics

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

	χ	η	α
Micro			
Data	≈ 0	$0 < \eta < 1$	< 0
Theory: Habits	≈ 0.75	$0 < \eta < 1$	< 0
Theory: Sticky Expectations	≈ 0	$0 < \eta < 1$	< 0
Macro			
Data	≈ 0.75	≈ 0	< 0
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Temptation & Commitment

Temptation & Commitment

A Model of Hand-to-Mouth Behavior

Why do households choose to be wealthy hand to mouth?

- It prevents consumption smoothing over income shocks

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- It prevents consumption smoothing over income shocks

Traditional explanation (Kaplan and Violante, 2014)

- Illiquid assets give large excess returns relative to all liquid assets
- But this is a controversial assumption
- There exists a high return liquid asset: publicly traded equities

Our goal: develop a new model of the wealthy hand to mouth

- In our model, HHs face temptation, making it difficult to save
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This view of WH2M behavior helps us understand other important policies

- We study housing subsidies and mandatory amortization
- Do policies simply encourage substitution from liquid to illiquid assets?

Model with commitment obtains a good fit of the empirical evidence

- Matches large share of WH2M despite high return liquid asset
- Restricted model cannot match WH2M using housing utility alone
- MPC declines slowly with the size of income shocks

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Subsidies to commitment devices can increase overall savings

- Housing subsidies generate mild substitution from liquid assets to housing, but nevertheless boost overall wealth accumulation by 7%
- Mortgage amortization increases wealth accumulation by 10%
- The policies have little effect on the share of WH2M households

Temptation & Commitment

The Model

Life cycle model of consumption and savings

- Demographics: households work for \bar{T} years, then retired for \tilde{T} years
- Choices: consumption, housing
- Assets: Liquid assets, housing, and mortgages

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- Assets: Liquid assets, housing, and mortgages

Novel features

- Temptation preferences make it costly to hold liquid assets
- A commitment device (housing) can reduce temptation

Standard model

- Households are committed to their choices
- No need for commitment

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Hyperbolic discounting model (Strotz, 1956 and Laibson, 1997)

- Relaxes the assumption of standard model on **discounting**
- Different discount rates, time inconsistent
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Temptation preferences (Gul and Pesendorfer, 2001 and 2004)

- Tempting, feasible alternative that is not chosen
- This tempting alternative impacts your utility
- Axiomatic, time consistent
- Commitment: reduce temptation by restricting choice set

$$\max_{\{c_t, h_t\}_{t=0, \dots, T}} \mathbb{E}_0 \sum_{t=0}^T \beta^t U(c_t, h_t, \tilde{c}_t, \tilde{h}_t)$$

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$$U(c_t, h_t, \tilde{c}_t, \tilde{h}_t) = u(c_t, h_t) - \underbrace{\lambda [u(\tilde{c}_t, \tilde{h}_t) - u(c_t, h_t)]}_{\text{utility cost of self-control}}$$

- c_t : nondurable consumption
- h_t : housing status
- λ : degree of temptation

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Most tempting alternative: maximize current period utility

$$[\tilde{c}_t, \tilde{h}_t] = \arg \max_{c_t, h_t \in \mathcal{A}_t} u(c_t, h_t)$$

- \tilde{c}_t : most tempting consumption
- \tilde{h}_t : most tempting housing status

1. Liquid asset (a_t)

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- Discrete asset with N different sizes (rental, flat, house, mansion)
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- Transaction costs generate **commitment benefit**

Assets and Mortgages

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3. Mortgages (m_t)

- Buying a home automatically comes with a mortgage
- Downpayment of ψ percent of the house price
- Fixed-rate mortgage, r^M
- Fixed repayment each period until retirement or house sale

Housing Preferences

Functional form follows Attanasio et al (2012)

$$u(c_t, h_t) = \underbrace{\frac{c_t^{1-\gamma}}{1-\gamma}}_{\text{consumption utility}} \underbrace{e^{\theta\phi(h_t)}}_{\text{multip housing utility}} + \underbrace{\mu\phi(h_t)}_{\text{additive housing utility}} - \underbrace{\chi\mathbb{I}_{h_t \neq h_{t-1}}}_{\text{utility cost of moving}}$$

- γ : coefficient of relative risk aversion
- θ and μ : housing preference parameters
- ϕ : relative utility of house choice h_t
- χ : utility cost of housing adjustment (only applies if $h_t \neq h_{t-1}$)

$$\ln y_t = g_t + z_t$$

- g : Deterministic age profile for income (third order polynomial)
- z : Idiosyncratic income process
 - Exogenous AR(1) process

$$z_t = \rho z_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

$$z_0 \sim N(0, \sigma_0^2)$$

Given state variables $\Omega_t = \{a_t, z_t, m_t, h_{t-1}\}$

$$V_t(\Omega_t) = \max \left\{ V_t^0(\Omega_t), V_t^1(\Omega_t) \right\}$$

where $V_t^0(\Omega_t)$ and $V_t^1(\Omega_t)$ are the value functions conditional on not adjusting and adjusting housing.

Those who choose not to adjust in period t :

$$V_t^0(\Omega_t) = \max_{\{c_t, a_{t+1}\}} U(c_t, h_t, \tilde{c}_t, \tilde{h}_t) + \beta \mathbb{E}_t V_{t+1}(\Omega_{t+1}), \quad (1)$$

subject to:

$$\begin{aligned} a_{t+1} &= (1+r) \left[a_t + \tilde{y}_t - c_t - \mathbb{I}_t^{own} mp_t - (1 - \mathbb{I}_t^{own}) rent_t \right] \\ \tilde{y}_t &= \begin{cases} \exp(g_t + z_t), & \text{if } t \leq W \\ \text{SS Benefit}(y_W), & \text{if } t > W \end{cases} \\ z_t &= \rho z_{t-1} + \varepsilon_t \quad \text{and} \quad c_t > 0 \end{aligned} \quad (2)$$

Those who choose to adjust housing in period t :

$$V_t^1(\Omega_t) = \max_{\{c_t, h_t, m_{t+1}, a_{t+1}\}} U(c_t, h_t, \tilde{c}_t, \tilde{h}_t) + \beta \mathbb{E}_t V_{t+1}(\Omega_{t+1}), \quad (3)$$

subject to:

$$\begin{aligned} a_{t+1} &= (1+r) \left[a_t + \tilde{y}_t - c_t - (1+F)p_t(h_t) + \frac{m_{t+1}}{(1+r^M)} \right. \\ &\quad \left. + (1-F)p_t(h_{t-1}) - m_t \right] \\ m_{t+1} &\leq (1-\psi^{\min})p_t(h_t)(1+r^M) \\ y_t &= \begin{cases} \exp(g_t + z_t), & \text{if } t \leq W \\ \text{SS Benefit}(y_W), & \text{if } t > W \end{cases} \\ z_t &= \rho z_{t-1} + \varepsilon_t \quad \text{and} \quad c_t > 0 \end{aligned} \quad (4)$$

Temptation & Commitment

Model Calibration

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 - But this is a controversial assumption, which we choose to relax (e.g. Flavin and Yamashita 2002; Goetzmann and Spiegel 2002; Piazzesi, Schneider, and Tuzel 2007)
- Remaining preference parameters are calibrated internally
 - Parameters: time preference, risk aversion, housing utility parameters, utility cost of moving
 - Target a combination of life-cycle and aggregate moments

Key Insight: Temptation alters the relationship between consumption growth and assets

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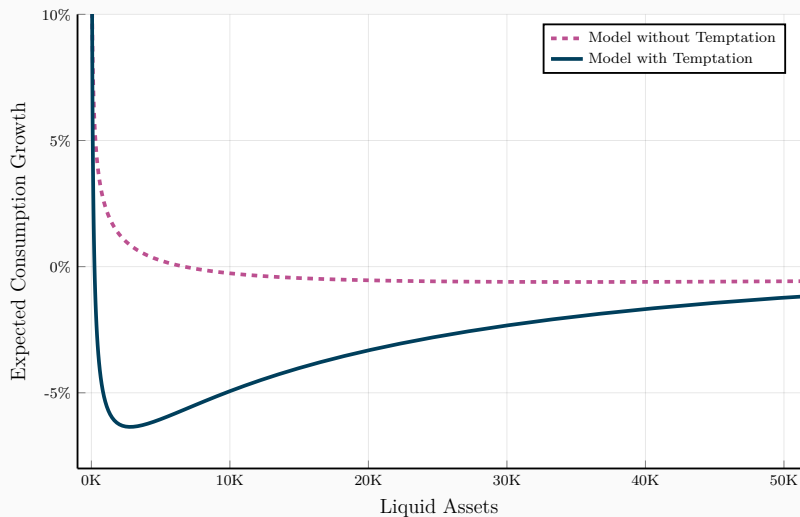
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This insight allows us to identify λ separately from β using data on consumption and assets (see Kovacs, Low, Moran, 2021)

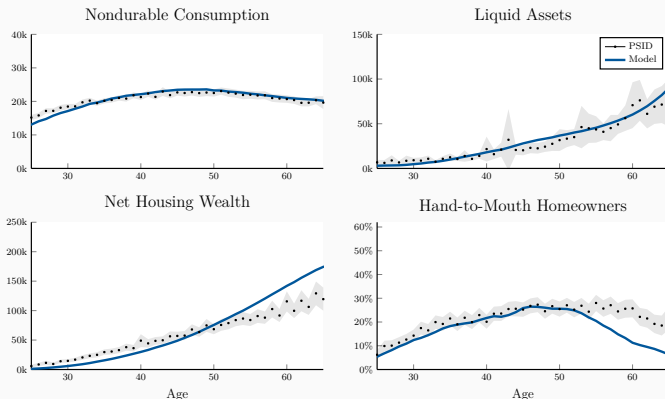
Identification



Temptation & Commitment

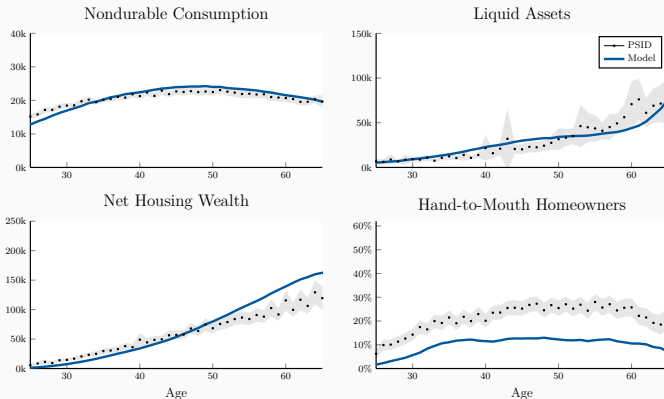
Model Fit

Baseline Model



Baseline model generates good fit, despite presence of high-return liquid asset

Restricted Model



Restricted model predicts 50% less hand-to-mouth homeowners

In addition, the model with temptation & commitment matches recent empirical evidence showing

1. The average MPC remains large even in response to large income shocks (e.g. Fuster, Kaplan, Zafar 2018; Kueng 2018; Fagereng, Holm, Natvik 2021)
2. Households have a demand for illiquidity (Beshears et al, 2021)
3. Mandatory amortization increases wealth accumulation (Bernstein and Koudijs, 2021)

Temptation & Commitment

Implications for Policy

1. Substantial tax benefits to homeownership in the U.S.

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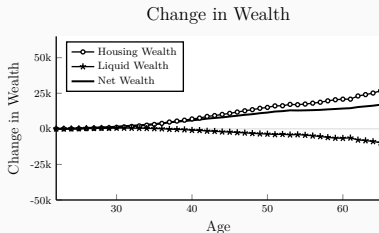
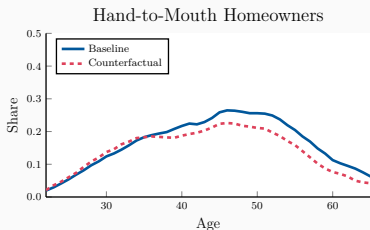
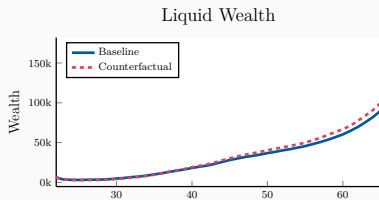
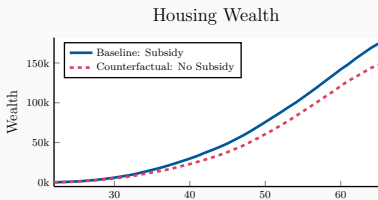
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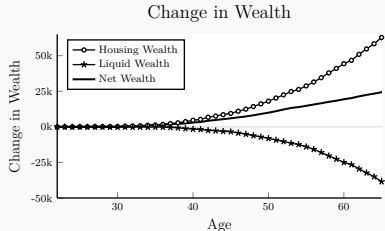
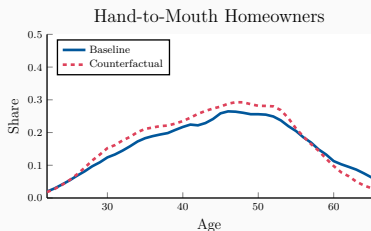
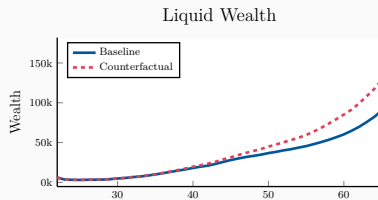
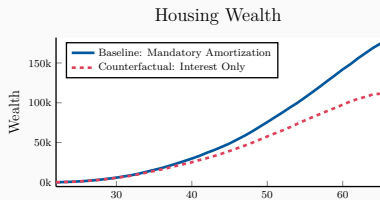
We evaluate two opposing views of such illiquid saving incentives

- May induce portfolio rebalancing from liquid to illiquid assets
- May improve access to commitment, potentially helping HHs accumulate wealth

Policy 1: Housing Subsidies



Policy 2: Mandatory Amortization



Conclusion

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 - Generates good out-of-sample fit of MPCs and recent evidence

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 - Large share of WH2M, while relaxing strong assumptions on returns
 - Generates good out-of-sample fit of MPCs and recent evidence
- Understanding WH2M has important implications for policy
 - Subsidies to commitment can increase overall savings
 - Mortgage amortization can boost wealth accumulation

Summary

Summary and next week

- **Today:** Two applications of dynamic programming to understand household spending dynamics
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*Biased beliefs = when solving the value function, HHs expect a job loss probability that is different than what happens in the simulation.

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