Precautionary borrowing and the credit card debt puzzle

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This paper addresses the credit card debt puzzle using a generalization of the buffer-stock consumption model with long-term revolving debt contracts. Closely resembling actual US credit card law, we assume that card issuers can always deny their cardholders access to new debt, but that they cannot demand immediate repayment of the outstanding balance. Hereby, current debt can potentially soften a household's borrowing constraint in future periods, and thus provides extra liquidity. We show that for some intermediate values of liquid net worth it is indeed optimal for households to simultaneously hold positive gross debt and positive gross assets even though the interest rate on the debt is much higher than the return rate on the assets. Including a risk of being excluded from new borrowing which is positively correlated with unemployment, we are able to simultaneously explain a substantial share of the observed borrower-saver group and match a broad range of percentiles from the empirical distributions of credit card debt and liquid assets.

Keywords. Credit card debt puzzle, precautionary saving, consumption.

JEL classification. D14, D91, E21.

1. Introduction

Beginning with Gross and Souleles (2002) it has been repeatedly shown that many households persistently have both expensive credit card debt and hold low return liquid assets. This apparent violation of the no-arbitrage condition has been termed the "credit card debt puzzle," and no resolution has yet been generally accepted (see, e.g., the surveys by Tufano (2009) and Guiso and Sodini (2013)).

This paper suggests a new explanation of the puzzle based on *precautionary borrowing*. We begin from the observation that credit card debt is actually a long-term revolving debt contract. Specifically, under current US law the card issuer can cancel a credit card at any time, and thus instantly stop the cardholder from accumulating additional debt. However, the card issuer *cannot* force the cardholder to immediately pay back the remaining balance on the credit card. The cardholder is only required to pay the interest on the outstanding balance, and the so-called minimum monthly payment (typically 1% of the outstanding balance). Depending on the specific credit card agreement, the issuer might be able to increase this minimum monthly payment somewhat, but basically the credit card debt is transformed into an installment loan.[1]

We add such long-term revolving debt contracts, which are partially irrevocable from the lender side, to an otherwise standard buffer-stock consumption model in the tradition of Carroll (1992, 1997, 2012). Hereby, households gain a motive for *precautionary borrowing* because current debt potentially relaxes the borrowing constraint in future periods. For equal (and risk-less) interest rates on debt and assets, households will always accumulate as much debt as possible, thus maximizing the option value of having a large gross debt. In the more plausible case of a higher interest rate on debt than on assets, there exists a trade-off between the benefit of the extra liquidity provided by the debt and the net cost of the balance sheet expansion.

We further amplify the motive for precautionary borrowing by including risk regarding access to credit in the model. Specifically, we assume that households with an outstanding balance face an exogenous risk of being excluded from new borrowing, and that this risk increases under unemployment. The US *Consumer Financial Protection Bureau (CFBP)* shows in its "CARD Act Report" that "over 275 million accounts were closed[2] from July 2008 to December 2012, driving a $1.7 trillion reduction in the total [credit] line" (p. 56, CFPB (2013)). It is not clear to which extent this was a demand or supply effect, but anecdotal evidence suggests that the credit card companies unilaterally changed their lending during the Great Recession, and that the supply effect thus dominated. Thus, having a credit card closed seems to be something a rational household should fear. Naturally, households might have an outside option of opening a new credit card at another issuer, but if a household is simultaneously hit by unemployment, this might prove impossible.

Based on a careful calibration which matches the distributions of credit card debt and liquid assets observed in the *Survey of Consumer Finance* 1989–2013 up to and beyond the 85th percentiles, we show numerically that there exists a range of intermediate values of liquid net worth for which it is indeed optimal for a household to simultaneously hold positive gross debt and positive gross assets, even though the interest rate on the debt is much higher than the return rate on the assets. This is especially true when we assume that negative income shocks are positively correlated with a high risk of a fall in the availability of new credit. The parametric robustness of our results is rather strong, indicating that precautionary borrowing is central in understanding the credit

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[1]We thank the *National Consumer Law Center* and the *Consumer Financial Protection Bureau* for help in clarifying the rules for us.

[2]In the sense that no new credit could be accumulated on these account.

card debt puzzle. If credit access risk and/or income risk is calibrated to be negligible it should, however, be noted that the precautionary borrowing channel can only explain a small share of the puzzle group.

We are somewhat cautious in precisely quantifying the importance of precautionary borrowing because our model for computational reasons does not include illiquid assets (e.g., houses). The model is thus not able to match the empirical facts on *total* net worth without muting the precautionary motive completely. Note, however, that Kaplan and Violante (2014) have recently shown that a buffer-stock model with an illiquid asset, subject to transaction costs, can generate a significant share of *wealthy hand-to-mouth* households while still matching total net worth moments. We hypothesize that both poor and wealthy hand-to-mouth households would also rely on precautionary borrowing, and that our results are thus at least qualitatively robust to extending our model in this direction, but fundamentally this remains an open question.

The importance of going beyond one-period debt contracts has naturally been discussed before. Closest to our paper are Attanasio, Leicester, and Wakefield (2011), Attanasio, Bottazzi, Low, Nesheim and Wakefield (2012), Halket and Vasudev (2014) and Chen, Michaux, and Roussanov (2015) who all introduce long-term mortgage contracts, and Alan, Crossley, and Low (2012) who model the "credit crunch" of 2008 in terms of a drying up of new borrowing (a flow constraint) instead of a recall of existing loans (the typical change in the stock constraint).[3]

To the best of our knowledge, Fulford (2015) is the only other paper that investigated the importance of multiperiod debt contracts for the credit card debt puzzle.[4] Our approach differs from his in a number of important ways. First and foremost, we have a much richer specification of the risk regarding access to credit the households face. In particular, we do not rely on the unrealistic assumption that all households face the same risk of being excluded from new borrowing. Instead, we show that a sizable puzzle group can be explained even though only households not repaying their credit card bill in full face a risk of being excluded from new borrowing. This furthermore helps to explain why we empirically observe that a large share of households have neither credit card debt nor liquid assets. Additionally, we allow the risk of losing access to new borrowing to be positively correlated with unemployment and show that this is empirically relevant and quantitatively important for explaining a sizable puzzle group. This is especially important because we, in contrast to Fulford (2015), account for the forced monthly repayments contained in standard credit card contracts and a more realistic income process—with both permanent and transitory shocks and nonzero growth—which otherwise considerably weakens the ability of the precautionary borrowing motive to explain a sizable puzzle group.

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[3]Note that Alan, Crossley, and Low (2012) use the term "precautionary borrowing" (borrowing for a rainy day) in a somewhat different fashion than we do because the second asset in their model is a high return risky asset. This, for example, implies that wealthy households also blow up their balance sheet by taking loans to invest in the risky asset.

[4]We were only made aware of the working paper version of his paper after writing the first draft of the present paper.

On a more technical note, the model we present in this paper explicitly, in contrast to Fulford's (2015) model, nests the canonical buffer-stock consumption model as a limiting case, and can be solved by a state-of-the-art endogenous grid point method for models with nonconvex choice sets. This is first beneficial because it allows us to conduct a range of important robustness tests, such as showing which preferences must characterize puzzle households and that a sizable puzzle group can be explained for a wide range of estimates for the risk of losing and regaining access to new borrowing. Second, it prepares the ground for future research studying the precautionary borrowing motive in more general consumption-saving models.

The paper is structured as follows. Section 2 discusses the related literature. Section 3 presents the model and describes the solution algorithm briefly. Some stylized facts are presented in Section 4 to which the model is calibrated in Section 5. Section 6 presents the central results, and central robustness tests regarding the specification of credit access risk. The welfare gain of the potential for precautionary borrowing is quantified in Section 7, and a battery of additional robustness checks are performed in Section 8. Section 9 concludes. Some details are relegated to the Appendix.

2. Related literature

2.1 *Empirical evidence*

Gross and Souleles (2002) showed that in the 1995 *Survey of Consumer Finance* (SCF) and in a monthly sample of credit card holders from 1995–1998, almost all households with credit card debt held low return liquid assets (e.g., they had funds in checking or saving accounts). In itself, this might not be an arbitrage violation but could be a pure timing issue if the interview took place just after pay day and just before the credit card bill was due. However, a third of their sample held liquid assets larger than one month's income; without any further explanation this certainly seems to be an arbitrage violation.

Their result has been found to be robust to alternative definitions of the puzzle group[5] and stable across time periods (see Telyukova and Wright (2008), Telyukova (2013), Bertaut, Haliassos, and Reiter (2009), Kaplan, Violante, and Weidner (2014) and Fulford (2015)). Telyukova (2013), for example, utilized certain questions in the SCF to ensure that the households in the puzzle group had credit card debt left over after the last statement was paid, and that they either only occasionally or never repay their balance in full. Recently, Gathergood and Weber (2014) have shown that the puzzle is also present in UK data, and that the puzzle group also has many and large expensive installment loans (e.g., car loans).[6]

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[5]We denote the group of households simultaneously holding both liquid assets and credit card debt as *the puzzle group*.

[6]Looking over the life cycle, the puzzle group is smallest among the young (below 30) and old (above 60). Puzzle households are typically found to be in the middle of the income distribution and have at least average education and financial literacy. Many have sizeable illiquid wealth (e.g., housing and retirement accounts). There is also some evidence of persistence in puzzle status, and in total it thus seems hard to explain the puzzle as a result of simple mistakes or financial illiteracy.

Across samples and time periods, the interest rate differential between the credit card debt and the liquid assets considered has typically been around 8–12 percentage points, and thus economically very significant. Depending on the correction for timing, this implies that the net cost of the expanded balance sheets of the puzzle group has been calculated to be in the range of 0.5–1.5% of household income.

2.2 *Other theoretical explanations*

A number of different rational and behavioral explanations of the credit card debt puzzle has been suggested in the literature. First, Gross and Souleles (2002) informally suggested that a behavioral model of either self/spouse-control or mental accounting might be necessary to explain the puzzle.[7] Bertaut and Haliassos (2002), Haliassos and Reiter (2007), and Bertaut, Haliassos, and Reiter (2009) formalized this insight into an accountant-shopper model where a fully rational accountant tries to control an impulsive (i.e., more impatient) fully rational shopper (a different self or a spouse). The shopper can only purchase goods with a credit card, which has an upper credit limit, and the accountant thus has a motive to not use all liquid assets to pay off the card balance in order to limit the consumption possibilities of the shopper. Gathergood and Weber (2014) provided some empirical evidence that a large proportion of households in the puzzle group appeared to be impulsive spenders and a heavy discounter of the future. A fundamental problem with this solution of the puzzle, however, is that it is not clear why the accountant cannot utilize cheaper control mechanisms such as adjusting the credit limit or limiting the shopper's access to credit cards. Furthermore, many households with credit cards also have debit cards, which imply that the shopper in practice has direct access to at least some of the household's liquid assets.

Second, beginning with Lehnert and Maki (2007), and continuing with Lopes (2008) and latest Mankart (2014), it was suggested that US bankruptcy laws might make it optimal for households to strategically accumulate credit card debt in order to purchase exemptible assets in the run-up to a bankruptcy filing. Even though state level variation in the size of the puzzle group and exemption levels seems to support this explanation, the empirical power seems limited because it is only relevant relatively shortly before a filling.[8] Moreover, many households in the puzzle group have both significant financial assets (e.g., bonds and stocks) and nonfinancial assets (e.g., cars and houses), and generally few households ever file for bankruptcy. Finally, it is far from obvious that such a motive for strategic accumulation of exemptible assets can explain the evidence from the UK (see Gathergood and Weber (2014)), which generally has more creditor friendly bankruptcy laws.

A third resolution of the puzzle has been presented by Telyukova (2013) (see also Telyukova and Wright (2008) and Zinman (2007)). She argues that many expenditures

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[7]Note that behavioral models with hyperbolic discounting and a present bias such as Laibson, Repetto, and Tobacman (2003) can explain that households with credit card debt has illiquid assets, but not that they hold fully liquid assets.

[8]Mankart (2014) noted that debt and cash-advances made shortly before the bankruptcy filing (60 or 90 days depending on the time period) are not dischargeable above a rather low threshold.

(e.g., rent and mortgage payments) can only be paid for by using cash, and that house-holds thus have a classical Hicksian motive for holding liquid assets despite having expensive credit card debt. The strength of this demand for liquidity is amplified in her model by rather volatile taste shocks for goods that can only be paid for with cash (e.g., many home and auto repairs). It is, naturally, hard to identify these fundamentally unobserved shocks, and their size in the data. A more serious empirical problem is that the use of credit cards has become much more widespread in the last 30 years; in the model this should imply a sharp fall in the size of the puzzle group not seen in the data. Adding a (costly) cash-out option on the credit card to the model, as is now common, could also further reduce the implied size of the puzzle group. In total, this demand for cash might certainly be a contributing factor, but it seems unlikely that it is the central explanation of the credit card debt puzzle. Finally, note that in a model with both a Hicksian motive for holding liquid assets and a precautionary borrowing motive, the two would reinforce each other.

3. Model

3.1 *Bellman equation*

We consider a continuum of potentially infinitely lived households, each characterized by a vector, $\mathbf{S}_t$, of the following state variables: end-of-period gross debt (D_{t-1}), end-of-period gross assets (A_{t-1}), market income (Y_t), permanent income (P_t), an unemployment indicator, $u_t \in \{0, 1\}$, and an indicator for whether the household is currently excluded from new borrowing, $x_t \in \{0, 1\}$. In each period, the households choose consumption, C_t, and debt, D_t, to maximize expected discounted utility.

Postponing the specification of the exogenous and stochastic income process of Y_t and P_t to Section 3.3, the household optimization problem is given in recursive form as

$$V(\mathbf{S}_t) = \max_{D_t, C_t} \frac{C_t^{1-\rho}}{1 - \rho} + \beta \cdot \mathbb{E}_t\big[V(\mathbf{S}_{t+1})\big], \tag{3.1}$$

s.t.

$$A_t = (1 + r_a) \cdot A_{t-1} + Y_t - C_t,$$
$$\underbrace{- r_d \cdot D_{t-1}}_{\text{interest}} - \underbrace{\lambda \cdot D_{t-1}}_{\text{installment}} + \underbrace{(D_t - (1 - \lambda) \cdot D_{t-1})}_{\text{new debt}}, \tag{3.2}$$

$$N_t = A_t - D_t, \tag{3.3}$$

$$D_t \leq \max\big\{ \underbrace{(1 - \lambda) \cdot D_{t-1}}_{\text{old contract}}, \mathbf{1}_{x_t=0} \cdot \underbrace{(\eta \cdot N_t + \varphi \cdot P_t)}_{\text{new contract}} \big\}, \tag{3.4}$$

$$A_t, D_t, C_t \geq 0, \tag{3.5}$$

where ρ is the relative risk aversion coefficient, β is the discount factor, r_a is the (real) interest rate on *assets*, r_d is the (real) interest rate on *debt*, and $\lambda \in [0, 1]$ is the *minimum payment due rate*. The discount factor includes an exogenous quarterly death probability of 1%; having mortality is technically necessary to ensure that the cross-sectional

distribution of income is finite because there are permanent income shocks (see below). Equation (3.2) is the budget constraint, (3.3) defines end-of-period (financial) net worth, and (3.4) is the borrowing constraint. We only cover the case $r_d > r_a$, and denote the optimal debt and consumption functions by $D^\star(\mathbf{S}_t)$ and $C^\star(\mathbf{S}_t)$.

Note that $x_t = 1$ is denoted as having *lost access to new borrowing* because the borrowing constraint then becomes $D_t \le (1 - \lambda)D_{t-1}$, such that the household cannot increase its stock of debt beyond what previously has been accumulated, net of the forced repayment. We assume that the risk of losing access and the chance of regaining it are given by respectively

$$\Pr[x_{t+1} = 1 | x_t = 0] = \begin{cases} 0 & \text{if } D_t = 0, \\ \pi_{x,w}^{\text{lose}} & \text{if } D_t > 0 \text{ and } u_{t+1} = 0, \\ \pi_{x,u}^{\text{lose}} & \text{if } D_t > 0 \text{ and } u_{t+1} = 1, \end{cases} \tag{3.6}$$

and

$$\Pr[x_{t+1} = 0 | x_t = 1] = \pi_{x,*}^{\text{gain}}. \tag{3.7}$$

We thus assume that the risk of losing access to new borrowing is zero as long as the household pays off its credit card bill in full each period, $D_t = 0$. Otherwise, for $D_t > 0$, we assume that the risk of losing access to new borrowing is given by $\pi_{x,w}^{\text{lose}}$ if the household is working, and $\pi_{x,u}^{\text{lose}} = \chi_{\text{lose}} \cdot \pi_{x,w}^{\text{lose}}$ if the household is unemployed. For the calibration, we denote the risk of losing access to new borrowing before conditioning on unemployment by $\pi_{x,*}^{\text{lose}}$. The chance of regaining access to new borrowing, once access has been lost, is always given by $\pi_{x,*}^{\text{gain}}$.

3.2 *The borrowing constraint*

Our specification of the debt contract is obviously simplistic, but it serves our purpose and only adds one extra state variable to the standard model. If $\eta > 0$, asset-rich households are allowed to take on more debt, even though there is no formal collaterization. We allow for *gearing* in this way to be as general as possible, and we use end-of-period timing and update the effect of income on the borrowing constraint period-by-period following the standard approach in buffer-stock models.[9]

The crucial departure from the canonical buffer-stock model is that we assume the debt contract is *partially irrevocable from the lender side*. This provides the first term ("old contract") in the maximum operator in the borrowing constraint (3.4), implying that a households can always continue to borrow up to the *remaining principal* of their current debt contract (i.e., up to $(1-\lambda)D_{t-1}$). The second term ("new contract") is a more

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[9]Note that the alternative borrowing constraint $D_t \le A_t + \alpha P_t$ has the undesirable implication that the household in principle can choose an infinite large level of debt, D_t, if it keeps its consumption level, C_t, fixed, and saves all the borrowed funds in liquid assets, A_t. Similarly, the borrowing constraint $D_t \le A_{t-1} + \alpha P_t$ has the undesirable implication that the same accumulation of larger and larger balance sheets can happen gradually over time; in this case, however, $r_d - r_a > 0$ ensures that we cannot have $\lim_{t\to\infty} D_t = \infty$.

standard borrowing constraint and only needs to be satisfied if the households want to take on *new debt* ($D_t > (1 - \lambda)D_{t-1}$). Hereby, current debt can potentially relax the borrowing constraint in future periods, and it thus provides extra liquidity. This implies that it might be optimal for a household to make choices such that both $D_t > 0$ and $A_t > 0$, that is, to simultaneously be a borrower and a saver.

In a model of one-period debt (i.e., $\lambda = 1$), it would never be optimal for a household to simultaneously hold both positive assets and positive debt because the option value of borrowing today would disappear. Consequently, it would not be necessary to keep track of assets and debts separately and the model could be written purely in terms of net worth.[10] This would also imply that (3.4) could be rewritten as

$$N_t \geq -\frac{\mathbf{1}_{x_t=0} \cdot \varphi}{1 + \eta} \cdot P_t \tag{3.8}$$

showing that our model nests the canonical buffer-stock consumption model of Carroll (1992, 1997, 2012) in the limit case $\lambda \to 1$.

3.3 *Income*

The income process is given by

$$Y_{t+1} = \tilde{\xi}(u_{t+1}, \xi_{t+1}) \cdot P_{t+1}$$

$$P_{t+1} = \Gamma \cdot \psi_{t+1} \cdot P_t$$

$$\tilde{\xi}(u_{t+1}, \xi_{t+1}) = \begin{cases} \mu & \text{if } u_{t+1} = 1, \\ \dfrac{\xi_{t+1} - u_* \cdot \mu}{1 - u_*} & \text{if } u_{t+1} = 0, \end{cases}$$

$$u_{t+1} = \begin{cases} 1 & \text{with probability } u_*, \\ 0 & \text{else,} \end{cases}$$

where ξ_t and ψ_t are respectively *transitory* and *permanent* mean-one log-normal income shocks[11] (truncated with finite lower and upper supports), and u_* is the unemployment rate.[12]

3.4 *Solution algorithm*

As the model has four continuous states, two discrete states, and two continuous choices, it is not easy to solve, even numerically. We use a novel trick by defining the following auxiliary variables:

$$M_t \equiv (1 + r_a) \cdot A_{t-1} - (r_d + \lambda) \cdot D_{t-1} + Y_t, \tag{3.9}$$

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[10]If $N_t \geq 0$, then $A_t = N_t$ and $D_t = 0$, and if $N_t < 0$, then $D_t = -N_t$ and $A_t = 0$.

[11]Note that the expectation of Y_{t+1} conditional on information at time t thus is $\Gamma \cdot P_t$.

[12]Throughout the paper, we will continue to interpret u_t as unemployment, but it could also proxy for a range of other large shocks to both income and consumption. This would relax the model's tight link between unemployment and a higher risk of a negative shock to the availability of new borrowing.
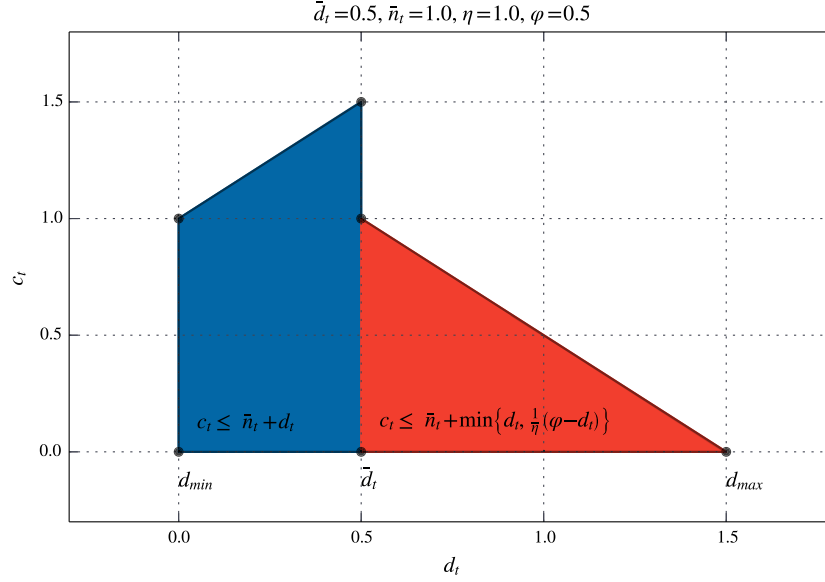
$$\bar{d}_t = 0.5, \, \bar{n}_t = 1.0, \, \eta = 1.0, \, \varphi = 0.5$$

FIGURE 1. Choice set (*example of nonconvexity*).

$$\overline{D}_t \equiv (1 - \lambda) \cdot D_{t-1}, \tag{3.10}$$

$$\overline{N}_t \equiv N_{t|C_t=0} = M_t - \overline{D}_t, \tag{3.11}$$

where M_t is *market resources*, $\overline{D}_t$ is the *beginning-of-period debt principal*, and $\overline{N}_t$ is *beginning-of-period net worth*. Also, using the standard trick of normalizing the model by permanent income[13]—denoting normalized variables with lower cases—we make $\overline{n}_t$ a *state variable* instead of m_t (the standard choice). This speeds up the solution algorithm substantially because a change in $\overline{d}_t$ then only affects the set of feasible debt choices. We hereby get that if the optimal debt choice is smaller than the current debt principal, then all households with smaller debt principals will make the same choice if it is still feasible, that is,

$$k < 1 : d^{\star}(\overline{d}_t, \overline{n}_t) = d \leq k \cdot \overline{d}_t \quad \Rightarrow \quad \forall \overline{d} \in [k \cdot \overline{d}_t, \overline{d}_t] : d^{\star}(\overline{d}, \overline{n}_t) = d.$$

Further, if the new debt contract depends on current net wealth, $\eta \neq 0$, the choice set might be nonconvex, as illustrated in Figure 1, using the following characterization of the choice set:

$$d_t \in \left[\max\{-\overline{n}_t, 0\}, \max\{\overline{d}_t, \eta \cdot \overline{n}_t + \mathbf{1}_{x_t=0} \cdot \varphi\} \right], \tag{3.12}$$

$$c_t \in \left[0, \overline{c}(x_t, \overline{d}_t, \overline{n}_t, d_t) \right], \tag{3.13}$$

$$\overline{c}(x_t, \overline{d}_t, \overline{n}_t, d_t) \equiv \begin{cases} \overline{n}_t + d_t & \text{if } d_t \leq \overline{d}_t, \\ \overline{n}_t + \min\left\{ d_t, \frac{1}{\eta}(\mathbf{1}_{x_t=0} \cdot \varphi - d_t) \right\} & \text{if } d_t > \overline{d}_t. \end{cases}$$

[13]See the Appendix for the normalized model equations and details on the solutions algorithm for the discretized model.

This possible *nonconvexity* of the choice set and the general *nonconcavity* of the value function due to the maximum operator in the borrowing constraint (3.4) imply that the value function might not be everywhere differentiable. Despite this, a standard variational argument implies that given a feasible debt choice, $d_t = d$, any optimal interior consumption choice (i.e., where $c_t < \overline{c}(\bullet)$) must *necessarily* satisfy the standard Euler-equation, that is,[14]

$$c_t^\star(\bullet)^{-\rho} = \beta \cdot (1 + r_a) \cdot \mathbb{E}_t\big[\big(\Gamma \cdot \psi_{t+1} \cdot c_{t+1}^\star(\bullet)\big)^{-\rho}\big] \tag{3.14}$$

Sufficiency of the Euler-equation can then be ensured by numerically checking that it does not have multiple solutions (see the Appendix).

Similar to Barillas and Fernández-Villaverde (2007), Hintermaier and Koeniger (2010), Kaplan and Violante (2014), Iskhakov, Jørgensen, Rust, and Schjerning (2017), and especially Fella (2014), the endogenous grid points method originally developed by Carroll (2006) can thus be *nested* inside a *value function iteration* algorithm, with a grid search for the optimal debt choice, speeding up the solution algorithm.[15] The full solution algorithm is presented in the Appendix.

3.5 *Policy functions*

Based on the calibration in Section 5, Figure 2 shows in which disjoint sets of states the household with median preferences chooses to hold which types of portfolios.

The household chooses to hold assets only ($a_t > 0$, $d_t = 0$) if its beginning-of-period liquid net worth ($\overline{n}_t$) is sufficiently high and to hold debt only ($a_t = 0$, $d_t > 0$) if it is sufficiently low. A high level of liquid net worth implies that the option value of holding debt is zero as liquidity is not a problem. In contrast, with a low level of liquid net worth the household has already borrowed so much that it either cannot borrow any more, or the option value of more debt is not large enough to cover the net cost of expanding the balance sheet.

The household chooses to have both debt and assets ($a_t > 0$ and $d_t > 0$) or neither of both ($a_t = 0$ and $d_t = 0$) if its beginning-of-period liquid net worth is in between the two extremes mentioned above. For high levels of liquid net worth within this range, the household is at the zero kink in both dimensions because of the jumps in the interest

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[14]The variational argument can be briefly presented as follows: Think of a household following the optimal plans for consumption and debt. If the current consumption choice is interior, $0 < c_t < \overline{c}(\bullet)$, then the household can adjust consumption today by a small amount $\Delta \neq 0$ and adjust it tomorrow by $-(1 + r_a)\Delta$ without violating neither the borrowing constraint nor making the debt choices today and tomorrow infeasible. In the limit for $\Delta \to 0$, the implied change in the expected discounted utility seen from period t is the difference between the LHS and RHS of the Euler-equation (3.14) multiplied by the sign of Δ. If this difference is not exactly zero, it implies that there exists a small $\Delta \neq 0$, which can increase the household's expected discounted utility. This would violate the assumption that the household is following the optimal plans for consumption and debt. In sum, the Euler-equation (3.14) must necessarily hold with equality for all optimal interior consumption choices. The necessity of the Euler-equation can alternatively be proven using recent results from Clausen and Strub (2013) on envelope theorems for models with nonconvexities. This approach is used by, for example, Fella (2014).

[15]On the precision and speed-up benefits of using EGM, see Jørgensen (2013).
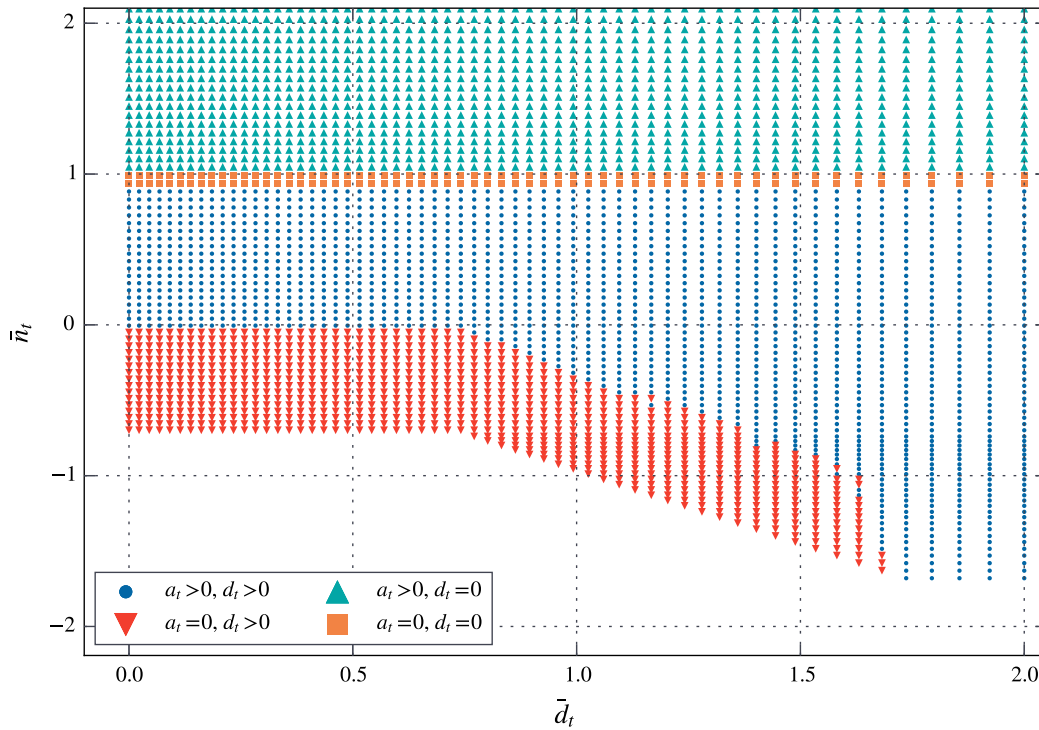
FIGURE 2. Implied group choice ($u_t = 0$, $x_t = 0$). *Note*: Using the calibration in Section 5, and for a household with median preferences.

rate and in the risk of being excluded from new borrowing, and thus chooses to have neither debt nor assets. For lower levels of liquid net worth, the household instead finds it optimal to have both debt and assets because it is in a more fragile situation where it needs to secure some precautionary funds. The utility value of this "puzzle" choice is weakly increasing in the beginning-of-period debt principal ($\overline{d}_t$) because the household can then easily accumulate more debt in excess of what it needs to accumulate for consumption purposes.

4. STYLIZED FACTS

In order to match our model to the data, Tables 1 and 2 present the central stylized facts on the credit card debt puzzle using a methodology similar to Telyukova (2013). In contrast to Telyukova, however, we use the combined versions of the *Survey of Consumer Finance* (SCF) from 1989 to 2013 instead of only focusing on 2001 as she does. We include all working age households with heads of ages 25–64 in our sample, and handle inflation and real growth by expressing all monetary variables relative to the mean quarterly after-tax income in the survey year. *Credit card debt* (D_t) is defined as the balance due on the credit card *after* the last statement was paid. *Liquid assets* (A_t) are defined as the sum of checking and savings accounts plus idle money in brokerage accounts. *Liquid net worth* is defined as liquid assets minus credit card debt ($N_t = A_t - D_t$).

TABLE 1. Stylized facts—shares, debt, and liquid assets.

	Puzzle	Borrower	Saver	Corner	All
	Percent				
Share	25.1	4.8	46.0	24.0	100.0
	Relative to Mean Quarterly Income				
Credit card debt, D_t (mean)	0.51	0.40	0.02	0.05	0.17
5th percentile	0.05	0.06	0.00	0.00	0.00
15th percentile	0.09	0.08	0.00	0.00	0.00
25th percentile	0.13	0.11	0.00	0.00	0.00
50th percentile	0.30	0.25	0.00	0.00	0.00
75th percentile	0.62	0.47	0.00	0.00	0.14
85th percentile	0.90	0.67	0.02	0.03	0.33
95th percentile	1.62	1.22	0.09	0.27	0.85
Liquid assets, A_t (mean)	0.60	0.01	1.88	0.03	1.02
5th percentile	0.05	0.00	0.06	0.00	0.00
15th percentile	0.08	0.00	0.10	0.00	0.01
25th percentile	0.11	0.00	0.15	0.00	0.03
50th percentile	0.24	0.01	0.43	0.00	0.17
75th percentile	0.56	0.02	1.22	0.02	0.60
85th percentile	0.86	0.03	2.12	0.03	1.13
95th percentile	1.93	0.04	6.11	0.10	3.33
Liquid net worth, N_t (mean)	0.09	−0.39	1.86	−0.03	0.86
5th percentile	−1.31	−1.20	0.05	−0.12	−0.60
15th percentile	−0.60	−0.65	0.09	−0.00	−0.10
25th percentile	−0.33	−0.45	0.14	0.00	0.00
50th percentile	−0.03	−0.23	0.41	0.00	0.07
75th percentile	0.23	−0.10	1.18	0.01	0.49
85th percentile	0.52	−0.07	2.09	0.02	1.02
95th percentile	1.55	−0.05	6.10	0.03	3.24

Source: Survey of Consumer Finance 1989–2013, all households with heads of ages 25–64 and nonnegative income. All calculations are survey weighted. The groups are defined in the text. Credit card debt is the balance due after the last statement was paid. Liquid assets include checking and savings accounts plus idle money in brokerage accounts. Liquid net worth is defined as credit card debt minus liquid assets. Income is after federal taxes calculated using the NBER TAXSIM program. All monetary variables are expressed relative to the mean quarterly income of the survey year.

All working age households are divided into *four* subgroups. Households are included in the "*puzzle*" group (or interchangeably the "*borrower-saver*" group) if they report repaying their credit card balance off in full "*only sometimes*" or "*never,*" and have both credit card debt and liquid assets in excess of a cut-off given by $500 in 2001 (as in Telyukova) and otherwise adjusted with the change in mean after-tax income.[16] In contrast, households are defined as pure "*borrowers*" if they report repaying their credit card balance off in full "*only sometimes*" or "*never,*" and have credit card debt exceeding their liquid assets plus the cut-off. Households are defined as pure "*savers*" if they have liquid net worth in excess of their credit card debt plus the cut-off, while the remaining households are included in the "*corner*" group because they have both low levels of both debt

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[16]This implies that the cut-off is constant at around 3.7% of mean quarterly after-tax income.

TABLE 2. Stylized facts—additional dimensions.

	Puzzle	Borrower	Saver	Corner	All
	Relative to Mean Quarterly Income				
After-tax income (mean)	1.14	0.63	1.52	0.49	1.13
25th percentile	0.67	0.36	0.59	0.21	0.44
50th percentile	0.99	0.56	0.99	0.37	0.79
75th percentile	1.40	0.83	1.57	0.62	1.29
Installment loans (mean)	0.89	0.67	0.53	0.34	0.58
25th percentile	0.00	0.00	0.00	0.00	0.00
50th percentile	0.42	0.21	0.00	0.00	0.01
75th percentile	1.26	0.98	0.68	0.30	0.77
Home value (mean)	10.61	5.03	13.99	3.18	10.11
25th percentile	1.92	0.00	0.00	0.00	0.00
50th percentile	8.47	2.98	8.93	0.00	6.15
75th percentile	14.66	7.55	17.41	4.26	13.51
Home equity (mean)	4.84	2.17	8.76	1.71	5.76
25th percentile	0.00	0.00	0.00	0.00	0.00
50th percentile	2.46	0.37	3.97	0.00	1.83
75th percentile	6.36	2.62	10.40	1.54	6.76
Total net worth (mean)	15.68	3.95	45.40	4.05	26.00
25th percentile	1.96	0.00	3.28	0.00	0.76
50th percentile	6.92	1.24	12.09	0.45	5.47
75th percentile	15.98	4.46	33.72	2.94	18.20

Source: Survey of Consumer Finance 1989–2013; see Table 1. Mortgages are not included installment loans.

and assets (or have similar amounts of both and report normally repaying their credit card balance in full).[17]

The first row of Table 1 shows that approximately one in four households are in the puzzle group according to our definition. Increasing or decreasing the cut-off naturally reduces or enlarges the size of the puzzle, while dropping the requirement that the households should report paying the balance off in full "only sometimes" or "never" increases the puzzle group to about 31% of the sample. About 40% of the puzzle households have liquid assets in excess of their monthly after-tax income, and 70% have liquid assets in excess of half of their monthly after-tax income.[18] Figure 3 shows that the puzzle group share in the period 1989 to 2013 has fluctuated between 20 and 30% with some evidence for a downward trend since 1998, though the Great Recession could be muddying the picture somewhat.[19]

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[17]In the working paper version, we only divided the households into three groups as in Telyukova (2013). Here, we also include the "corner" group following the approach in Gorbachev and Luengo-Prado (2016). Hereby, our results are also more comparable with theirs.

[18]See Telyukova (2013) and Fulford (2015) for further discussions of the size of the puzzle group. Fulford (2015) estimated a puzzle group share of 40% looking only at households with a credit card and using a low cut-off of 0.01% of income.

[19]The downward trend could be due to both measurement problems, changes in institutions such as the increasing importance of electronic payments, or changes in interest rates and credit card laws and agreements. We do not aim to explain this potential trend in the present paper.
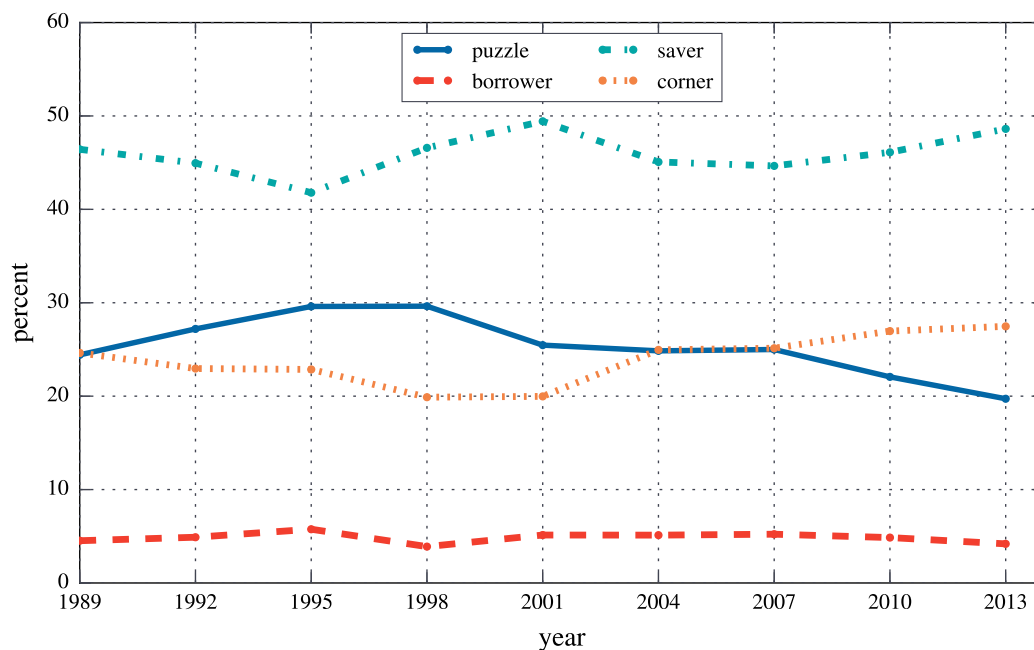
FIGURE 3. Group shares over time. *Source*: Survey of Consumer Finances 1989–2013; see Table 1.

Taking a closer look at the puzzle households, Table 1 shows that the median puzzle household has approximately zero net worth, and credit card debt and liquid assets equal to about one month's of after-tax income. Table 2 additionally shows that the average puzzle household has a bit higher income than the average income of the full population, and that this is even more true for the median puzzle household compared to the median of the full population. Table 2 also shows that the puzzle households have sizable *installment loans* (mortgages not included), the larger part of which are *vehicle loans*. The interest rates on these loans are typically significantly lower than on credit cards, and there can be some contractual terms that disincentivize premature repayment. Nonetheless, it is an indication that the puzzle households are also using other precautionary borrowing channels than credit cards.

As also noted by Telyukova (2013), the puzzle households are often rather wealthy measured in *total net worth* (thus also including all illiquid assets). This is to a large degree explained by *housing equity*, with almost 80% of the puzzle group being home owners. For computational reasons, our model does not include an illiquid asset, but as shown in Kaplan and Violante (2014), a buffer stock model with an illiquid asset, and a transaction cost for tapping into this wealth can imply that households between adjustments act as hand-to-mouth households. In a similar way, *hyperbolic discounting* such as in Laibson, Repetto, and Tobacman (2003) might further imply that households "*over*"-accumulate illiquid assets in order to strengthen their self-control abilities and better counteract the present bias of their future selves.

Table 3. Calibration.

Parameter	Value	Note/Source
Income		
Permanent inc. growth, Γ (annual)	1.02	Avg. US GDP per capita growth rate 1947–2014
Unemployment rate, u_*	0.07	Carroll, Slacalek, and Tokuoka (2015)
Variance of permanent shock, σ_ψ^2	$0.01 \cdot \frac{4}{11}$	Carroll, Slacalek, and Tokuoka (2015)
Variance of transitory shock, σ_ξ^2	$0.01 \cdot 4$	Carroll, Slacalek, and Tokuoka (2015)
Unemployment benefit, μ	0.30	As share of permanent income, Martin (1996)
Borrowing and saving		
Real interest rate on assets, r_a	-1.48%	Annual, Kaplan and Violante (2014)
Interest rate spread, $r_d - r_a$	12.36%	Annual, Telyukova (2013) and Edelberg (2006)
Credit limit, φ	0.74	Kaplan and Violante (2014)
Collateral limit, η	0.00	Standard buffer-stock model
Minimum repayment share, λ	0.03	Standard credit card contract (quarterly freq.)
Credit access risk		
Unconditional prob. of losing access, $\pi_{x,*}^{\text{lose}}$	2.63%	Fulford (2015)
Prob. of regaining access, $\pi_{x,*}^{\text{gain}}$	6.07%	Fulford (2015)
Extra risk for unemployed, χ_{lose}	4	See text

5. Calibration

5.1 *First step: Calibration*

The calibrated parameters are presented in Table 3. The model is simulated at a *quarterly* frequency, but we discuss interest and growth rates in annualized terms. In Section 8, we present a detailed discussion of how robust the results are to changing each single parameter.

The gross income growth factor $\Gamma = 1.02$ is chosen to match US trend growth in GDP per capita. The variances of the income shocks and the unemployment rate are all taken from Carroll, Slacalek, and Tokuoka (2015) who showed that they parsimoniously match central empirical facts from the literature on estimating uncertain income processes. In annual terms, the variance of both the permanent and the transitory shock are 0.01.[20] The unemployment replacement rate μ is set to 0.30 as documented in Martin (1996); we find the choice of $\mu = 0.15$ in Carroll, Slacalek, and Tokuoka (2015) to be too extreme.[21]

Regarding *borrowing and saving*, we first follow Kaplan and Violante (2014) who, based on SCF data, set the real interest rate on liquid wealth to -1.48% (annually) and found that the borrowing constraint binds at 74% of quarterly income. We thus set $\varphi = 0.74$, and choose $\eta = 0$ to stay as close as possible to these results (and the standard

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[20]For the transitory shock, the variance at a quarterly frequency is simply $4 \times$ annual transitory variance, while Carroll, Slacalek, and Tokuoka (2015) showed that for the permanent shock the conversion factor should be $\frac{4}{11}$.

[21]Following Carroll, Slacalek, and Tokuoka (2015), we assume no persistence in unemployment, which seems to be a valid assumption for a quarterly model given that the median unemployment duration historically has been well below 12 weeks (Source: UEMPMED from the FRED database). Even the mean unemployment duration has an historical average of only 15 weeks (Source: UEMPMEAN from the FRED database).

parametrization of the buffer-stock model). The interest rate on credit card debt is taken from Telyukova (2013); she found that the mean nominal interest rate in the borrower-saver group is 14% which we then adjust for 2.5 percentage points of inflation and a 0.62 percentage points default risk (see Edelberg (2006)). In total, this implies an interest rate spread of 12.4%, which is a bit lower than the 13.2% spread in Fulford (2015), but larger than the 10.0% spread in Telyukova (2013). We set the quarterly repayment rate λ to 0.03 because many credit card companies use a minimum payment rate of 1% on a monthly basis.

For the *credit access risk*, we broadly follow Fulford (2015), who utilizes a proprietary data set containing a representative sample of 0.1% of all individuals with a credit report at the credit-reporting agency *Equifax* from 1999 to 2013. He estimates that each quarter the risk of losing access to credit is 2.63% while the chance of regaining access is 6.07%. Unfortunately, Fulford is only able to condition on general covariates such as age, year, credit risk, geographical location, and reported number of cards. Specifically, he does not condition on neither the actual debt accumulated nor income shocks, such as unemployment. For our model, we need the risk of losing credit access conditional on both the last period's outstanding balance and unemployment status. This is furthermore necessary in order to avoid the undesirable situation that a household repaying its credit card bill in full each month faces the same risk of losing access to new borrowing as a household who continuously keeps rolling over a positive outstanding balance.[22] An additional challenge for Fulford's estimates is that they do not account for voluntary closures of credit card accounts, which might be especially prevalent among households without an outstanding balance. As a stark assumption, we use Fulford's estimates for the risk of losing access to new borrowing for a household with a strictly positive outstanding balance. Section 8 includes a detailed discussion of the robustness of our results with respect to this choice.

To calibrate χ_{lose}, the factor determining the excess credit access risk for the unemployed, we instead turn to the *Survey of Consumer Finance* (SCF) 2007–2009 panel where households were asked whether or not they have a credit card in 2007 and then again in 2009. This measure of credit card access is inferior to Fulford's, but we believe that the two measures are rather closely related. We restrict attention to households between ages 25 and 59, with positive income, and who in 2007 had a credit card with an outstanding balance after the last repayment. Table 4 shows that 12.0% of these households when reinterviewed in 2009 reported *not* having a credit card anymore; we denote this as having "lost access." Conditional on experiencing any weeks of unemployment, the fraction of those who have lost access increases to 22.0%. Similar to Fulford, we are not able to determine whether this indicates voluntary choices made by the households, but Table 5 reports the odds-ratios from logit estimations controlling for both various *background* variables (age, age squared, minority, household size) and *economic* variables (home ownership, log (normal) income, liquid assets, self-employment, education). The effect from unemployment remains significant even when all controls are used. This finding is in line with Crossley and Low (2013) who showed using the 1995 *Canadian Out*

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[22]We thank a referee for stressing this point.

TABLE 4. Lost access and unemployment—raw.

	Lost Access[1]	Share of Sample
	Percent	
All	12.0	–
No unemployment over last year[2]	9.1	77.5
Any unemployment	22.0	22.5
Some unemployment ($\geq$ 1 month)	23.5	20.6
Deep unemployment ($\geq$ 3 months)	25.2	14.6

Source: SCF panel 2007–2009; Households between ages 25 and 59, positive income, had a credit card in 2007 with an outstanding balance after the last repayment. Adjusted for survey weights and multiple imputations.

[1] *Lost Access*: Report not having a credit card in 2009.

[2] *Unemployment*: Sum of head and spouse over the last 12 months.

TABLE 5. Lost Access and unemployment—logit.

	Any Unemployment			Deep Unemployment		
	(1)	(2)	(3)	(1)	(2)	(3)
	Odds-Ratio (s.e.)					
Unemployment	2.83***	3.30***	2.68***	3.12***	3.30***	2.78***
	(0.94)	(1.06)	(0.88)	(0.94)	(1.06)	(1.02)
Single		4.45***	2.88**		4.39***	2.85**
		(1.55)	(1.26)		(1.53)	(1.24)
Home owner			0.37***			0.39**
			(0.14)			(0.14)
Background controls[1]		✓	✓		✓	✓
Economic controls[2]			✓			✓
Observations	695	695	695	695	695	695

Source: See Table 4. *: $p < 0.10$, **: $p < 0.05$, ***: $p < 0.01$.

[1] *Background controls*: Age, age squared, minority, household size.

[2] *Economic controls*: Home ownership, log (normal) income, liquid assets, self-employment, education (none, high school, college).

of Employment Panel that current unemployment is important for explaining the share of households answering "no" to the question "[i]f you needed it, COULD you borrow money from a friend, family, or a financial institution in order to increase your household expenditures."

Choosing an exact number for χ_{lose} given these observations, is, however, not straightforward. Note, however, that if we consider a household who had access 2 years ago (8 quarters), and was treated with some unemployment in the last year (4 quarters), then we can calculate the *theoretical* odds-ratio of losing access to new borrowing if we assume that the household never repays its credit card in full, that is, always have $d_t > 0$. Given the Markov processes of x_t and u_t, we can then easily calculate

$$f(\hat{x}, \hat{u}) \equiv \mathbb{E}\big[x_8 = \hat{x} | x_1 = 0, \exists k \in \{5, 6, 7, 8\} : u_k = \hat{u}\big], \tag{5.1}$$

such that the theoretical odds-ratio becomes

$$\text{odds-ratio} = \frac{f(1,1)/f(0,1)}{f(1,0)/f(0,0)}. \tag{5.2}$$

For $\chi_{\text{lose}} = 4$, we get a theoretical odds-ratio of 1.8 for these households, which does not seem extreme in light of the empirical evidence. Due to Fulford's very low estimate of gaining access, however, we still have that the risk of losing access conditional on being affected by unemployment is 19.9%; if not affected by unemployment, the probability is 12.4%, and in percentage points the increase is thus 7.5, which is again not extreme compared to the increase we see in Table 4. We thus stick with the choice of $\chi_{\text{lose}} = 4$, and perform an extensive robustness analysis of this calibrations in Section 8.

5.2 *Second step: Moment matching*

In order to calibrate the preference parameters β (the discount factor) and ρ (the relative risk aversion coefficient), we match selected percentiles of the distributions of liquid assets and credit card debt. Specifically, we match the 5th, 15th, 25th, 50th, 75th, and 85th percentiles of both distributions. We do not target the top of the distributions because our model is not designed to fit these, for example, matching the top of the debt distribution would probably require introducing idiosyncratic borrowing constraints, while additional saving motives must be introduced to explain why some households hold such large amounts of liquid assets.[23]

In order to achieve a good fit of both the debt and liquid asset distributions, we need to introduce preference heterogeneity. We follow Alan and Browning (2010) and assume that the discount factor and relative risk aversion coefficient both follows translated logistic models. We assume that households have constant preferences over time, but that there is heterogeneity across households. Specifically, we have

$$\beta_{i,j} = 0.8 + 0.2 \cdot \frac{\exp(\mu_\beta + \exp \phi_\beta \cdot i)}{1 + \exp(\mu_\beta + \exp \phi_\beta \cdot i)}, \quad i \sim \mathcal{U}[0,1],$$

$$\rho_{i,j} = 1 + 10 \cdot \frac{\exp(\mu_\rho + \exp \phi_\rho \cdot j)}{1 + \exp(\mu_\rho + \exp \phi_\rho \cdot j)}, \quad j \sim \mathcal{U}[0,1],$$

where μ_β and μ_ρ are location parameters, and ϕ_β and ϕ_ρ are dispersion parameters.[24] For computational feasibility, we approximate the preference distribution with $n = 5$ equiprobable points in each dimension such that, $(i,j) \in [\frac{1/2}{n}, \frac{1+1/2}{n}, \ldots, \frac{n-1+1/2}{n}]^2$. We then calibrate the preference parameters by solving the following method of simulated moments problem:

$$(\hat{\mu}_\beta, \hat{\mu}_\rho, \hat{\phi}_\beta, \hat{\phi}_\rho) = \arg \min_{\mu_\beta, \mu_\rho, \phi_\beta, \phi_\rho} (\Lambda_{\text{sim}} - \Lambda_{\text{data}})'(\Lambda_{\text{sim}} - \Lambda_{\text{data}}), \tag{5.3}$$

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[23]Alternatively, it could be a timing issue in the data in the sense that the households reporting high levels of liquid assets are in the process of investing these funds in more illiquid assets with higher risk-adjusted rates of return.

[24]We have also experimented with $\rho_{i,j} = 1 + 10 \frac{\exp(\mu_\rho + \omega_{\beta\rho} i + \exp \phi_\rho j)}{1 + \exp(\mu_\rho + \omega_{\beta\rho} i + \exp \phi_\rho j)}$, where $\omega_{\beta\rho}$ is a correlation parameter, but we did not find that it added any new insights.

where Λ_{data} contains the selected moments from the data, and Λ_{sim} is the cross-sectional moments from a simulation with 50,000 households in each preference category after an initial burn-in-period.[25]

The result is

$$\beta \in \{0.951, 0.954, 0.958, 0.964, 0.971\},$$

$$\rho \in \{1.04, 1.16, 1.62, 3.11, 6.19\},$$

which implies annual discount rates in the range 0.82 to 0.89. This is certainly in the lower end of what is usually estimated, but higher than the calibrated choice of 0.79 in Fulford (2015). The risk aversion coefficients lies in the range typically considered.

The good fit of the model is seen in the two last columns of Table 6. In all cases, the simulated debt and liquid asset percentiles are very close to what we observe in the data except at the extreme top, where the tails of both the credit card debt and the liquid asset distributions are too small. This also implies that the mean levels of credit card debt and especially liquid assets are underestimated.

6. Results

In Table 6, we see that the model, under the chosen parametrization and with a cut-off for the grouping of 3.7% of mean quarterly income, can explain that about 13% of households choose to be borrower-savers. This is substantially below, and close to half, of the empirical estimate of 25% (see Table 1), but still a substantial share. This shows that precautionary borrowing is at least one of the central explanations of the credit card debt puzzle. It is especially important that such a large proportion of the puzzle group can be explained even when the model also very closely matches the distribution of liquid assets up to the 85th percentile. Furthermore, the implied size of the balance sheets of the puzzle households is also rather large; the median puzzle household, for example, has a credit card debt of 30% of mean quarterly income just like in the data. Unlike the data, the debt of the median puzzle household is, however, a bit larger than its assets, but already at the 75th percentile the simulated puzzle households have substantial positive liquid net worth.

Table 6 also shows that 12.1% of the households in the simulation are in the corner group with neither debts nor assets. This is a substantial share though only approximately half of what we see in the the data. The too small puzzle and corner groups are instead counterbalanced by a larger borrowing group (19.3% in the simulation, 4.8% in the data), and a larger saver group (55.5% in the simulation, 46% in the data).

Overall, we consider the fit of the model to be rather good, and note that the size of both the puzzle group and the corner group could be overestimated empirically. Empirically, the puzzle group is, for example, extended at expense of the borrower group by including households who only accidentally hold some liquid assets at the survey date

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[25]When a household dies, it is replaced with a new household without any debt and assets equal to 1 week's permanent income, and with the same lagged permanent income as the *mean* of the current population. The burn-in period is 500 periods; see, for example, McKay (2017) for a similar approach.

TABLE 6. Results.

	Puzzle	Borrower	Saver	Corner	All	
			Percent			
Share	13.1	19.3	55.5	12.1	100.0	SCF
			Relative to Mean Quarterly Income			
Credit card debt, D_t (mean)	0.38	0.35	0.00	0.00	0.12	0.17
5th percentile	0.06	0.06	0.00	0.00	0.00	0.00
15th percentile	0.11	0.11	0.00	0.00	0.00	0.00
25th percentile	0.16	0.15	0.00	0.00	0.00	0.00
50th percentile	0.30	0.27	0.00	0.00	0.00	0.00
75th percentile	0.50	0.46	0.00	0.00	0.14	0.14
85th percentile	0.64	0.58	0.00	0.00	0.31	0.33
95th percentile	0.92	0.85	0.00	0.03	0.60	0.85
Liquid assets, A_t (mean)	0.25	0.00	1.00	0.01	0.59	1.02
5th percentile	0.04	−0.00	0.06	0.00	0.00	0.00
15th percentile	0.06	−0.00	0.12	0.00	0.00	0.01
25th percentile	0.07	0.00	0.19	0.00	0.01	0.03
50th percentile	0.17	0.00	0.47	0.00	0.17	0.17
75th percentile	0.33	0.00	1.21	0.01	0.60	0.60
85th percentile	0.46	0.01	1.92	0.02	1.12	1.13
95th percentile	0.74	0.03	3.62	0.03	2.68	3.33
Liquid net worth, N_t (mean)	−0.13	−0.34	1.00	0.00	0.47	0.86
5th percentile	−0.69	−0.84	0.06	−0.02	−0.52	−0.60
15th percentile	−0.43	−0.58	0.12	0.00	−0.24	−0.10
25th percentile	−0.31	−0.45	0.19	0.00	−0.09	0.00
50th percentile	−0.14	−0.26	0.47	0.00	0.12	0.07
75th percentile	0.06	−0.14	1.21	0.01	0.57	0.49
85th percentile	0.20	−0.10	1.92	0.02	1.11	1.02
95th percentile	0.49	−0.06	3.62	0.03	2.68	3.24

Puzzle group: D_t, $A_t > 0.037$. Borrower group: $D_t - A_t > 0.037$. Saver group: $A_t - D_t > 0.037$. Corner group: rest.

because they have, for example, either just received their paycheck or have some ex-penses in the near future, which are not payable by credit card (e.g., rent). On the other hand, households in the corner group might hold nonobserved liquid assets (such as cash) or formally or informally have access to other liquid funds.

Preference heterogeneity. Figure 4 shows how the size of each group (puzzle, bor-rower, saver, and corner) vary with the preference parameters (β, ρ). We see that the group shares vary a lot across preferences. We see that only relatively nonrisk averse households ever choose to be borrowers, while almost all households with a high enough discount factor and/or risk aversion coefficient choose to be savers. The share of puzzle households is largest for impatient households with medium levels of risk aver-sion. These predictions fit the empirical results in Gorbachev and Luengo-Prado (2016) very well.

Changing group definitions and risk levels. Table 7 shows the effect of changing, re-spectively, the cut-off for the group definitions, and the credit access and income risk parameters.

(a) Puzzle share   (b) Borrower share

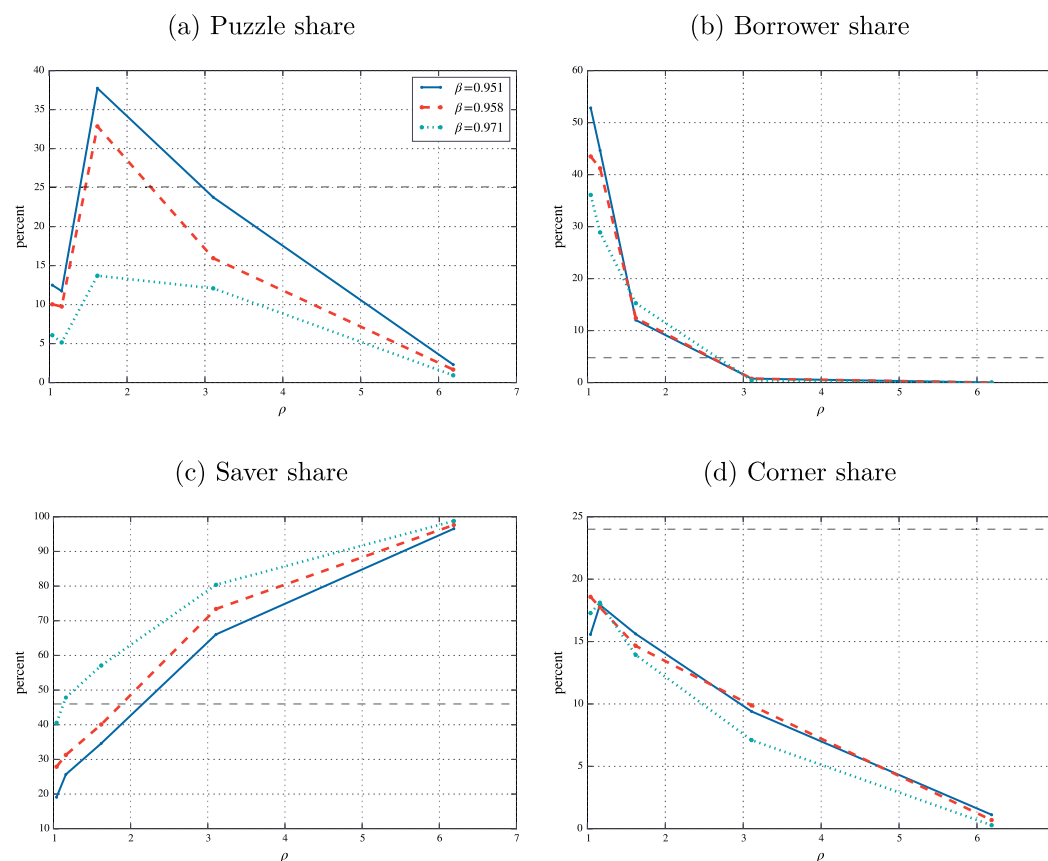(c) Saver share   (d) Corner share

FIGURE 4. Importance of preference heterogeneity. *Note*: Baseline parameter values in Table 3. Vertical lines show the observed household shares from Table 1.

In rows (2) and (3), we see that when the cut-off is reduced to 0.005 the size of the puzzle group increases to about 19%. This, naturally, creates a mismatch between the methodology used on the actual data and the simulated data, but it could be defended using the argument that the actual data contains measurement error, while the simulation data only contains much smaller approximation errors.

Rows (4) to (7), on the other hand, show that the puzzle group is reduced by about a third if χ_{lose} (the factor determining the extra risk of losing access to new borrowing while unemployed) is lowered from 4.0 to 1.0 without any large effects on the accumulation of credit card debt and liquid assets (the three rightmost columns). Allowing for correlation between income risk and credit access risk is thus quantitatively important, implying that the results in Fulford (2015), all else equal, understates the size of the puzzle group.

A lower risk of losing access to new borrowing also substantially reduces the puzzle group, while a larger risk enlarges it (rows (8)–(10)); here, the effects on the debt and asset accumulation are, however, large and a full reestimation could in principle dampen (or amplify) the change in the size of the puzzle group. A higher probability of regaining

Table 7. Changing group definitions and risk levels.

	Puzzle	Borrower	Saver	Corner	A_t	D_t	N_t
	Percent					*Mean*	
Baseline	13.1	19.3	55.5	12.1	0.12	0.59	0.47
cut-off = 0.025	15.2	18.4	56.8	9.5	0.12	0.59	0.47
cut-off = 0.005	18.3	16.4	57.8	7.5	0.12	0.59	0.47
$\chi_{\text{lose}} = 1.0$	9.3	25.0	54.3	11.4	0.11	0.58	0.47
$\chi_{\text{lose}} = 2.0$	9.5	24.0	54.8	11.7	0.12	0.58	0.47
$\chi_{\text{lose}} = 6.0$	14.1	17.2	56.2	12.4	0.12	0.59	0.48
$\pi_*^{\text{lose}} = 0.005$	2.8	37.4	49.4	10.4	0.13	0.53	0.40
$\pi_*^{\text{lose}} = 0.010$	6.5	31.0	51.5	11.0	0.13	0.55	0.43
$\pi_*^{\text{lose}} = 0.040$	18.5	12.0	57.4	12.1	0.12	0.61	0.49
$\pi_*^{\text{gain}} = 0.03$	12.9	15.7	59.9	11.5	0.10	0.62	0.52
$\pi_*^{\text{gain}} = 0.12$	12.2	23.2	52.8	11.8	0.13	0.57	0.44
$\pi_*^{\text{gain}} = 0.24$	9.9	26.1	52.1	11.9	0.13	0.56	0.42
$u_* = 0.04$	10.6	27.9	49.6	12.0	0.14	0.52	0.38
$u_* = 0.05$	11.1	25.0	51.9	12.0	0.13	0.54	0.42
$u_* = 0.06$	12.4	22.0	53.6	12.0	0.12	0.57	0.45

Note: Baseline parameter values in Table 3. For reference, baseline cut-off = 0.037, $\chi_{\text{lose}} = 4.0$, $\pi_*^{\text{lose}} = 0.0263$, $\pi_*^{\text{gain}} = 0.0607$, and $u_* = 0.07$.

access reduces the size of the puzzle group (rows (11)–(13)), although the effect is rather small; even when $\pi_*^{\text{gain}} = 0.24$ the puzzle group is still about 10%. Moreover, it should be noted that experiments, not reported here, shows that it is only the *belief* of a high risk of losing access, or a low chance of regaining it, which create a large puzzle group.

The final three rows of Table 7 show that if the unemployment rate is reduced from 0.07 to 0.04 the size of the puzzle group drops to 10.6%.

Transitions in and out of the puzzle group. Figures 5 and 6 provide further details on what happens before and after a household enters the puzzle group (i.e., the household is in the puzzle group at quarter $k = 0$ but not at $k = -1$). In Figure 5, we see which group the household came from when entering the puzzle group, and which group it exits to afterwards. We see that the *short run* persistence is limited as just above 40% of the households are still in the puzzle group 1 year on (plot I). On the other hand, there is substantial *long run* persistence as more than 20% of households are also in the puzzle group 4 years later compared to the unconditional prediction of 13.1%; this is due to the preference heterogeneity. The origin of the puzzle households are divided between most borrowers (~45%), a third savers (~35%), and fewest from the corner group (~20%).

Figure 5 shows what happens to the liquid net worth and the income of the puzzle households before they enter the puzzle group. We see that the households are deaccumulating liquid net worth at an accelerating speed in the quarters before entering the puzzle group (plot I), and that this is due to both unemployment and negative transitory income shocks (plots II and IV). Continuing large falls in transitory income are necessary to make households choose to be borrower-savers. On the other hand, plot III shows that falls in permanent income are *not* necessary, because such shocks also lowers the
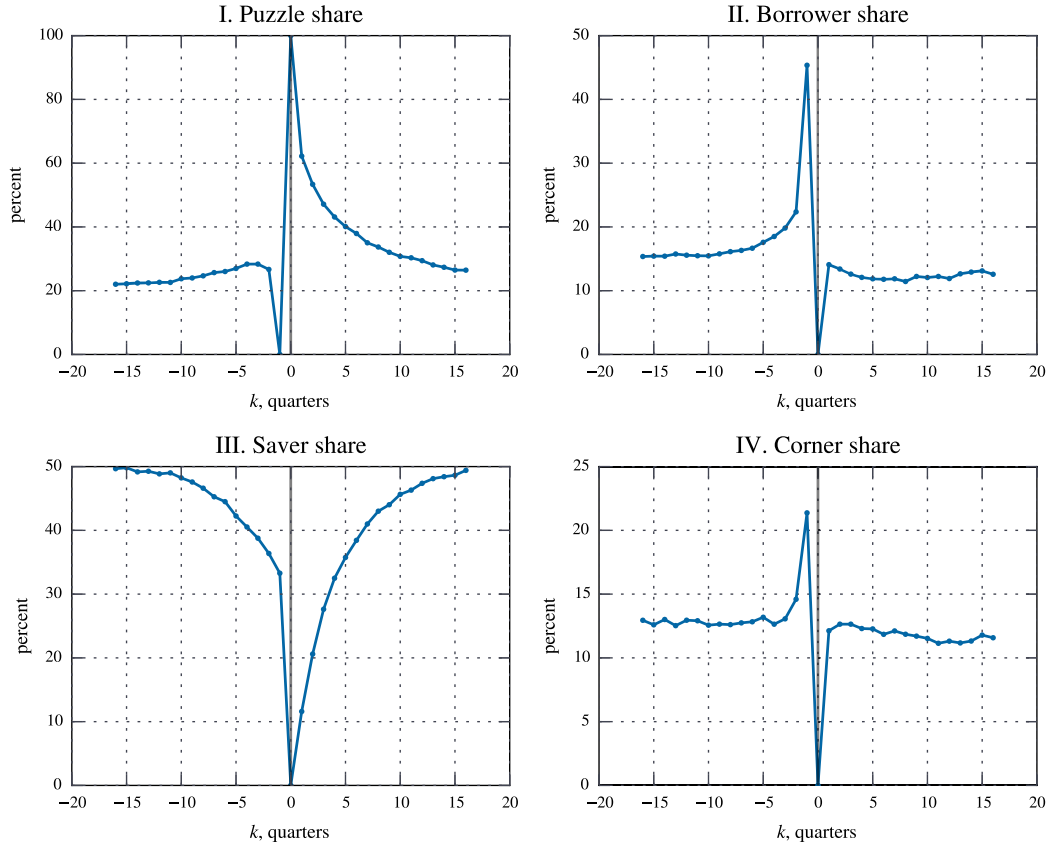
FIGURE 5. Transitions in and out of the puzzle—groups. *Sample*: Households who are in the puzzle group at $k = 0$, but were not so at $k = -1$.

7. THE WELFARE GAIN OF PRECAUTIONARY BORROWING

The welfare of the households can be measured as the ex ante discounted expected utility seen from an initial period. The simulation analog of this measure can be calculated taking the average over a sample of households experiencing different draws of shocks,

$$
\mathcal{U}_0(P_0) = P_0^{1-\rho} \cdot \frac{1}{N} \cdot \sum_{k=0}^{N} \sum_{t=0}^{T} \cdot \beta^t \cdot \frac{\left(c^\star(s_{kt}) \cdot \Gamma^t \cdot \prod_{h=1}^{t} \psi_{kh} \right)^{1-\rho}}{1 - \rho},
$$

$$
\mathcal{T}\big(s_{kt}, d^\star(s_{kt}), c^\star(s_{kt})\big) \Rightarrow s_{k,t+1},
$$

(7.1)

optimal consumption level of the household, and thus does not induce precautionary borrowing.
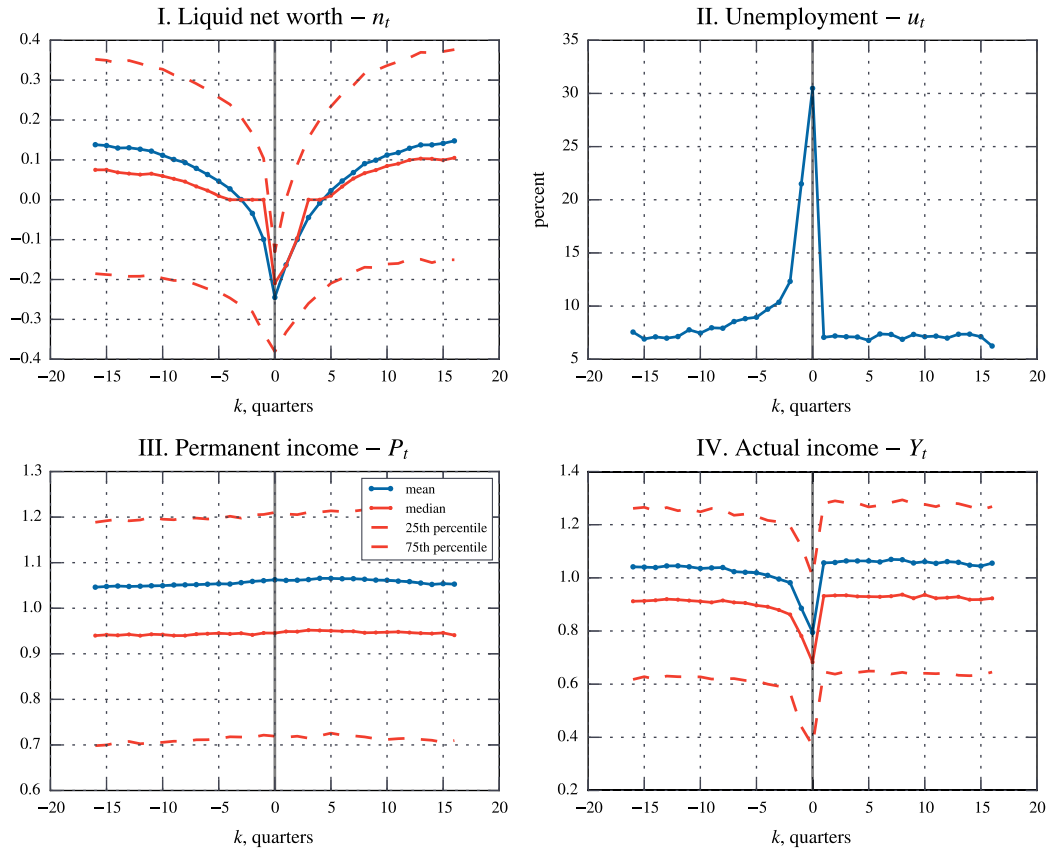
FIGURE 6. Transitions in and out of the puzzle—variables. *Sample*: Households who are in the puzzle group at $k = 0$, but were not so at $k = -1$.

where s_{kt} is the vector of normalized state variables of household k at age t (in quarters) and $\mathcal{T}(\bullet)$ is the stochastic transition function.[26]

We are now interested in the level of welfare across different values of λ, remembering that as $\lambda \to 1$ we return to the canonical buffer-stock model, which does not allow precautionary borrowing. Facilitating these comparisons, we can analytically derive the compensation in terms of a percentage increase (τ) in initial permanent income, and thus the average future path of permanent income a household needs to receive in order to be indifferent to a change in λ relative to the baseline:

$$\mathcal{U}_0(P_0, \lambda_0) = \mathcal{U}_0\left(P_0 \cdot \left(1 + \frac{\tau}{100}\right), \lambda\right) \quad \Leftrightarrow \quad \frac{\tau}{100} = \left(\frac{\mathcal{U}_0(P_0, \lambda_0)}{\mathcal{U}_0(P_0, \lambda)}\right)^{\frac{1}{1-\rho}} - 1. \qquad (7.2)$$

---

[26]In practice, we simulate an economy with 50,000 households over 700 periods, where households who die are replaced by new households. New households are born without debt, assets equal to one week's permanent income, and with the same lagged permanent income as the mean of the current population (exactly as in the calibration exercise above, see Footnote 25). We calculate the ex ante discounted expected utility using the last 200 periods of our simulation sample.
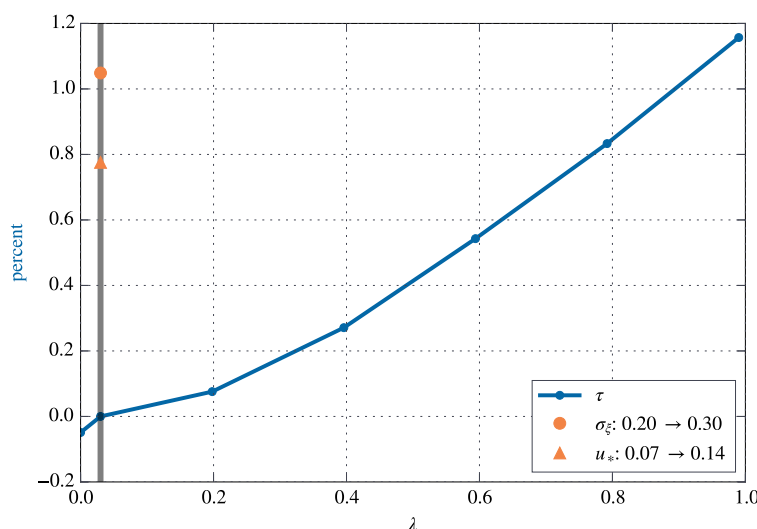
FIGURE 7. Welfare. *Note*: Vertical gray line represents baseline λ value. The results are for the median household. The vertical axis shows the compensation in percentage of initial permanent income needed to be indifferent to the baseline. See Footnote 26 for further details.

It should be noted that this is a *partial equilibrium* exercise in the sense that we keep the initial wealth new born households receive constant across the various parametrizations.[27] Consequently, we will focus on the relative welfare loss from losing access to precautionary borrowing compared to facing either more volatile transitory income shocks or a higher unemployment rate.

The results for a household with median preferences are plotted in Figure 7; as λ increases, the required compensation naturally increases as the choice set of the households only shrinks and the scope for precautionary borrowing becomes more limited.[28] In total, the households need a compensating increase in the path of permanent income of close to 1.2% to be indifferent between $\lambda = 0.03$ (the baseline) and $\lambda = 0.99$. Figure 7 also shows that increasing σ_ξ from 0.20 to 0.30 implies a τ about 1.0%, while increasing u_* from 7 to 14% implies a τ about 0.8%. The welfare loss of losing access to precautionary borrowing is thus larger than the welfare loss from a doubling of the unemployment rate.

Figure 8 shows why the absolute compensating equivalent becomes so large in our exercise when $\lambda \to 1$. Panel (a) firstly shows that the standard deviation of normalized consumption by age (in quarters) increases universally when $\lambda = 0.03$ is increased to $\lambda = 0.99$. The households dislike this additional variation in consumption due to the concavity of the utility function. From a standard Lucas-type back-of-the-envelope calculation it can, however, be concluded that the welfare loss is very large relative to the

---

[27]In a *general equilibrium* exercise, it would be necessary to account for the fact that the buffer-stock of one household is bequeathed and becomes the initial assets of another household. We abstract from this dynamic here.

[28]We discuss welfare in terms of a household with median preferences for ease of exposition. Figure 4 showed that more than 30% of the households with median preferences are in the puzzle group.
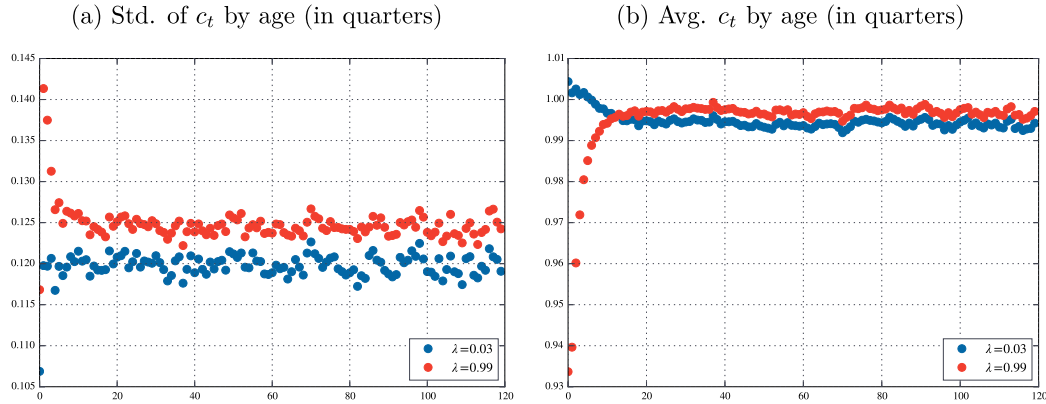
(a) Std. of c_t by age (in quarters)   (b) Avg. c_t by age (in quarters)

FIGURE 8. Explaining the welfare loss. *Note*: Figure 8 shows the standard deviation and average level of consumption by age (in quarters) in the simulation samples used to calculate the compensating equivalents in Figure 7. See Footnote 26 for further details.

increase in the standard deviation of consumption. The central explanation for the welfare loss instead is that the new-born households who do not have access to precautionary borrowing need to build up a much larger buffer-stock. In panel (b) of Figure 8, we thus see that they reduce their initial level of consumption substantially. This is only somewhat counterweighed by a higher long-run level of consumption made possible by the return provided by the increased buffer-stock.

8. Robustness

8.1 *Growth impatience*

Figure 9 shows how the size of the puzzle group (*full line*), the size of the borrower group (*dashed line*), the average credit card debt (*dashed-dotted line*), and the average level of liquid assets (*dotted line*) are affected by changes in the real interest rate, r_a, and the growth rate of permanent income, Γ. As in the previous section, we for simplicity focus on a population of households with median preferences.

In understanding the figure, it is useful to consider *the growth impatience factor* as defined in Carroll (2012),

$$\overline{\beta} \equiv \big(\beta \cdot (1 + r_a)\big)^{\frac{1}{\rho}} \cdot \Gamma^{-1}. \tag{8.1}$$

In the perfect foresight case, a growth impatience factor *less than one* implies that for an unconstrained consumer the ratio of consumption to permanent income will *fall* over time. In general, a larger growth impatience factor induces saving; these savings also satisfy the household's precautionary motive making costly *precautionary borrowing* less needed. Consequently, the puzzle group is *increasing* in Γ and *decreasing* in r_a. Omitting income growth, as in Fulford (2015), can thus have a sizable impact of the model-implied size of the puzzle group for a given choice of preferences parameter.
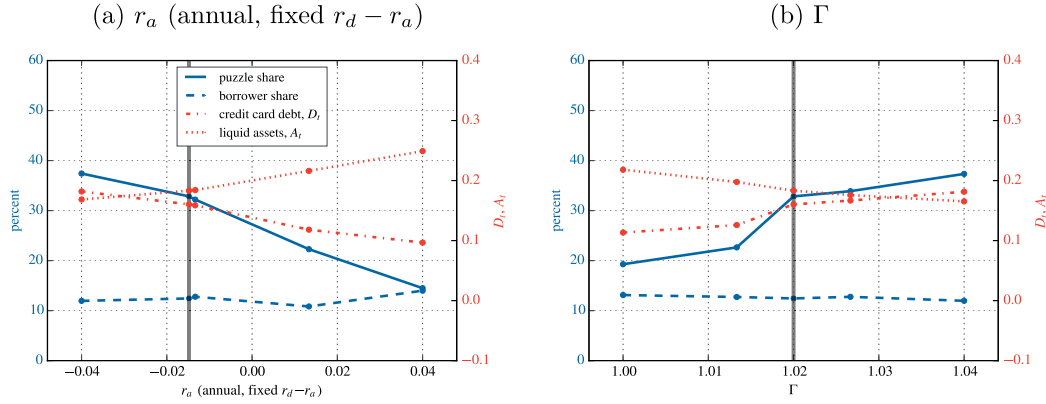
FIGURE 9. Growth impatience. *Note*: Vertical gray line represents baseline parameter value. The results are for a population of households with median preferences.

In Figure 4, we likewise see that the puzzle group is *decreasing* in patience, β, and eventually in risk aversion, ρ (we always have $\beta \cdot (1 + r_a) < 1$). Initially, however, an increase in the curvature of the utility function (ρ) expands the puzzle group because it implies a stronger incentive to smooth consumption, making it relatively more worthwhile for the households to pay the costs of precautionary borrowing.

Summing up, the model can explain a large puzzle group if households are impatient enough, in a growth corrected sense, and are neither too risk neutral nor too risk averse.

8.2 *Income uncertainty*

The underlying motive for precautionary borrowing is to insure against transitory income losses. We therefore see in Figure 10 that the size of the puzzle group is at first increasing in the variance of the *transitory* income shock and risk of unemployment (higher σ_ξ and u_*). At some point, however, larger transitory shocks does not increase the puzzle group because they induce too much precautionary saving. Lowering the unemployment benefits (lower μ) likewise only initially increase the puzzle group.

A larger variance of the *permanent* income shock (higher σ_ψ), in contrast, shrinks the puzzle group because the strengthened incentive to accumulate precautionary funds implies that the average net worth increases so much that the households do not need to rely on precautionary borrowing. This can also be understood as the consequence of an increase in the *uncertainty adjusted* growth impatience factor,

$$\tilde{\beta} \equiv \left(\beta \cdot (1 + r_a)\right)^{\frac{1}{\rho}} \cdot \Gamma^{-1} \cdot \mathbb{E}\left[\psi_{t+1}^{-1}\right] = \overline{\beta} \cdot \mathbb{E}\left[\psi_{t+1}^{-1}\right], \tag{8.2}$$

where the last term is increasing in the variance of the permanent shock due to Jensen's inequality. The same mechanism, moreover, also implies that the puzzle group is *decreasing* in higher unemployment persistence, where $\pi_{u,u}$ is the unemployment risk for the unemployed. Omitting persistent income risk, as in Fulford (2015), can thus have a sizable impact of the model-implied size of the puzzle group for a given choice of preferences parameter.
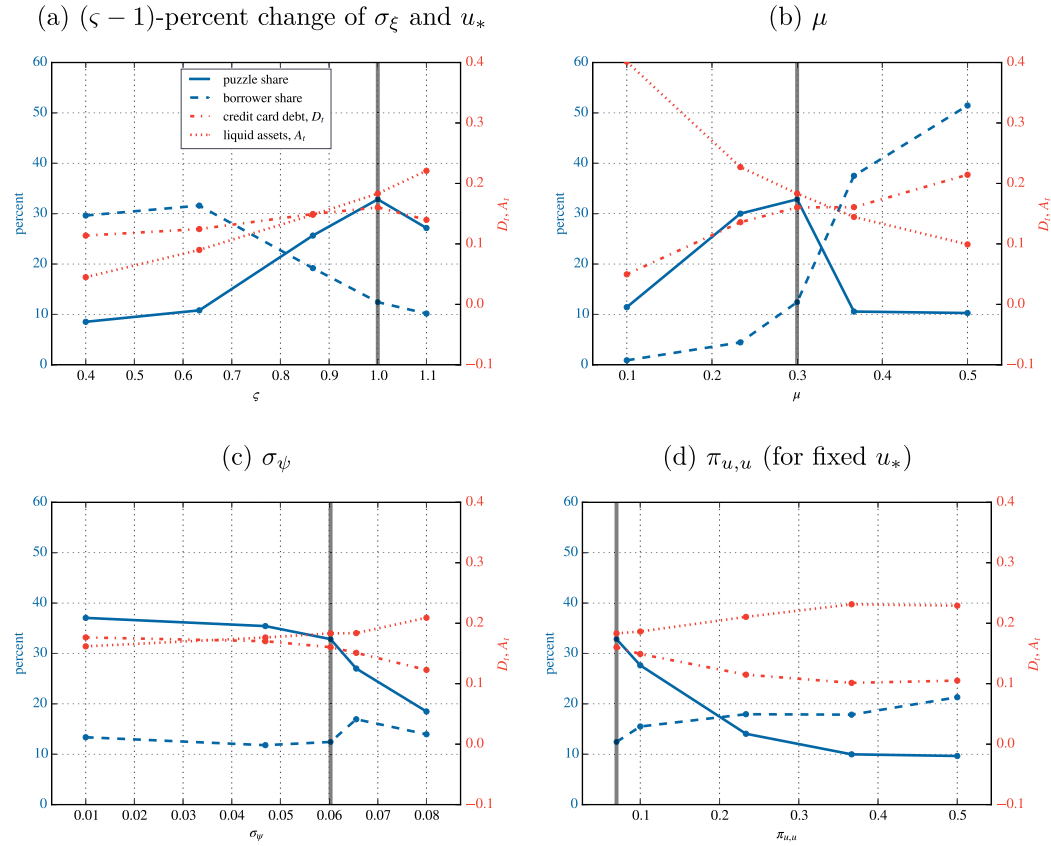
(a) $(\varsigma - 1)$-percent change of σ_ξ and u_*

(b) μ

(c) σ_ψ

(d) $\pi_{u,u}$ (for fixed u_*)

FIGURE 10. Income uncertainty. *Note*: Vertical gray line represents baseline parameter value. The results are for a population of households with median preferences.

8.3 *Terms of borrowing*

Naturally, the size of the puzzle group is decreasing if either the cost of borrowing increases (higher $r_d - r_a$, fixed r_a) or the repayment rate increases (higher λ). This is shown in the two first graphs in Figure 11. Not including forced repayments, as in Fulford (2015), can thus have a sizable impact of the model-implied size of the puzzle group. Furthermore, the puzzle group is relatively small if the "credit limit," φ, is too small, as the extensive potential for precautionary borrowing is then limited. Allowing for gearing in the form of a $\eta > 0$ does almost not affect the results, and our results are thus robust in this direction.

8.4 *Credit access risk*

Figure 12 shows the effects of changing the *unconditional* probabilities for losing ($\pi_{x,*}^{\text{lose}}$) and gaining ($\pi_{x,*}^{\text{gain}}$) access to new debt. In the first graph, we see that the puzzle group is increasing in the risk of losing access, but that the effect is highly nonlinear as a higher risk also induces more saving. The second graph shows that the puzzle group is (per-
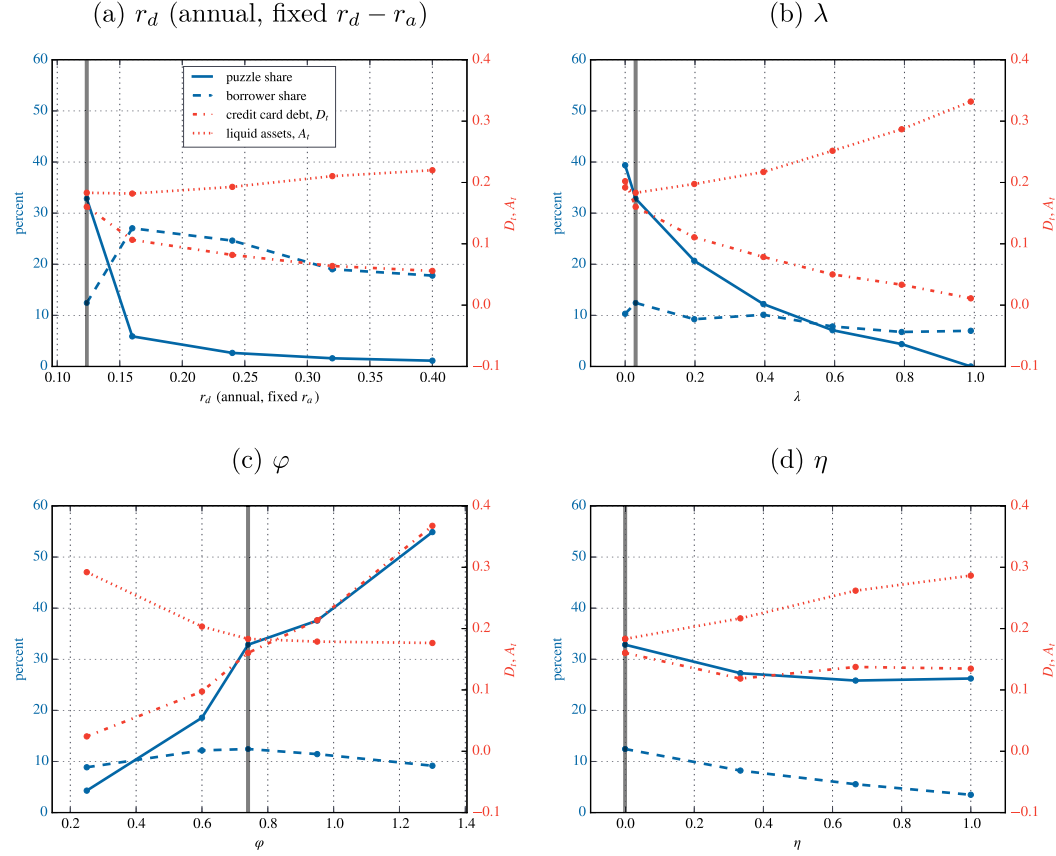
FIGURE 11. Terms of borrowing. *Note*: Vertical gray line represents baseline parameter value. The results are for the households with median preferences.

haps surprisingly) also initially increasing in the probability of regaining access to credit when it is lost; the intuition is that long expected exclusion spells induce more prior saving diminishing the need for precautionary borrowing. Afterwards a higher chance of regaining lowers the size of the puzzle group because it makes expected spells of lost access shorter, and thus less dangerous. It is thus clear that our results do not hinge on the assumption of a very low probability of regaining access.

9. Conclusion

We have shown that precautionary borrowing can explain a large part of the puzzle group of households who simultaneously hold expensive credit card debt and hold low-return liquid assets. We have moreover shown that no knife-edge assumptions on preferences or income uncertainty are needed for this result, that is, we find that the model implies a substantial puzzle group for a broad range of assumptions regarding preferences and income uncertainty. The power of the precautionary borrowing channel is, however, strongest if households are relatively impatient in a growth and uncertainty
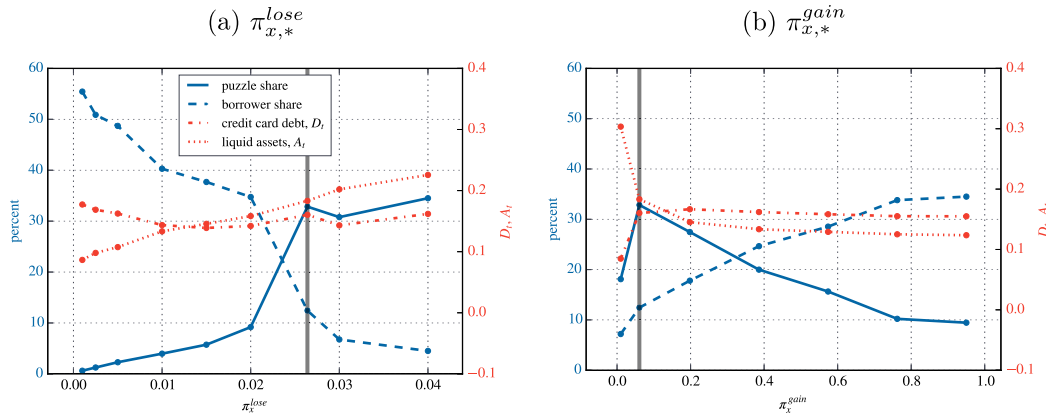
(a) $\pi_{x,*}^{lose}$ (b) $\pi_{x,*}^{gain}$

FIGURE 12. Credit access risk. *Note*: Vertical gray line represents baseline parameter value. The results are for a population of households with median preferences.

adjusted sense, are neither too risk neutral nor too risk averse, and are subject to sizable transitory income shocks. Empirical results in Gorbachev and Luengo-Prado (2016) support this characterization of the puzzle group.

The strongest assumption we need in order to amplify our results is that negative income shocks are perceived to be positively correlated with a higher risk of a fall in the availability of credit. This is not an implausible assumption, and we provide some indicative empirical evidence adding to that of Fulford (2015). More work on disentangling demand and supply effects in these estimates is, however, needed. If credit access risk and/or income risk is assumed to be negligible it should, however, be noted that the precaution borrowing channel can only explain a small share of the puzzle group.

A natural extension of our model would be to include an illiquid asset subject to transaction costs as in Kaplan and Violante (2014). We conjecture that in such a model precautionary borrowing will still be an important tool for both poor and wealthy hand-to-mouth households, but fundamentally it remains an open question. Together with a detailed life-cycle setup, such an extension is probably necessary to empirically estimate the importance of precautionary borrowing with precision. This we leave for future work. Extending the model in this direction would also make it possible to study the implications of precautionary borrowing for the average marginal propensity to consume out of both income and credit shocks. Finally, the concept of precautionary borrowing is also relevant for understanding households utilization of other forms of consumer loans, including car loans and mortgages.

APPENDIX: SOLUTION ALGORITHM

The purpose of the present appendix is to describe the *solution algorithm* in detail.

A.1 *Discretization*

To facilitate solving the model, we consider a *discretized* version with *finite-horizon*:

$$v_t(u_t, x_t, \overline{d}_t, \overline{n}_t) = \max_{d_t, c_t} u(c_t) + \beta \cdot \sum \Omega_{t+1}(\bullet),$$

$$\text{s.t.}$$

$$n_t = \overline{n}_t - c_t,$$

$$\Omega_{t+1}(d_t, n_t; u_+, x_+, \psi, \xi) = (\Gamma\psi)^{1-\rho} \cdot v_{t+1}\big(u_+, x_+, \overline{d}_+(\bullet), \overline{n}_+(\bullet)\big),$$

$$\overline{d}_+(d_t; \psi) = \arg\min_{z \in \overline{\mathcal{D}}} \left| z - \frac{1}{\Gamma\psi} \cdot (1-\lambda) \cdot d_t \right|,$$

$$\overline{\mathcal{D}} = \{0, \ldots, Y\}, \qquad |\overline{\mathcal{D}}| = N_{\overline{d}} \in \mathbb{N}, \quad Y > 0,$$

$$\overline{n}_+(d_t, n_t; u_+, \psi, \xi) = \frac{1}{\Gamma\psi} \cdot \big[(1+r_a) \cdot n_t - (r_d - r_a) \cdot d_t\big] + \tilde{\xi}(u_+, \xi),$$

$$d_t \in \mathcal{D}(u_t, x_t, \overline{d}_t, \overline{n}_t),$$

$$c_t \in \mathcal{C}(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t),$$

$$v_T(\overline{n}_t) = u\big(\max\{\overline{n}_t, 0\}\big),$$

$$\sum \equiv \sum_{U \times X \times \Psi \times \Xi} p(u_+, x_+, \psi, \xi | u_t, x_t) = 1,$$

where $\mathcal{D}(u_t, x_t, \overline{d}_t, \overline{n}_t)$ is the choice set for d_t and $\mathcal{C}(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t)$ is the choice set for c_t:

$$d_t \in \big[\max\{-\overline{n}_t, 0\}, \max\{\overline{d}_t, \eta \cdot \overline{n}_t + \mathbf{1}_{x_t=0} \cdot \varphi\}\big], \tag{A.1}$$

$$c_t \in \big[0, \overline{c}(\overline{d}_t, \overline{n}_t, d_t)\big],$$

$$\overline{c}(\overline{d}_t, \overline{n}_t, d_t) \equiv \begin{cases} \overline{n}_t + d_t & \text{if } d_t \le \overline{d}_t, \\ \overline{n}_t + \min\left\{d_t, \dfrac{1}{\eta}(\mathbf{1}_{x_t=0} \cdot \varphi - d_t)\right\} & \text{if } d_t > \overline{d}_t. \end{cases} \tag{A.2}$$

The critical step is discretizing the $\overline{d}_+(\bullet)$-function, but we can easily verify that both a higher Y and/or a higher $N_{\overline{d}}$ do *not* change the *optimal choice functions* $d_t^\star(u_t, x_t, \overline{d}_t, \overline{n}_t)$ and $c_t^\star(u_t, x_t, \overline{d}_t, \overline{n}_t)$.

The shocks are discretized using *Gauss–Hermite quadrature* with node sets $\Psi = \Psi(N_\psi)$ and $\xi = \xi(N_\xi)$, where N_ψ and N_ξ are the number of nodes for each shock. The lower and upper supports are $\underline{\psi} \equiv \min(\Psi)$, $\overline{\psi} \equiv \max(\Psi)$, $\overline{\xi} \equiv \max(\Xi)$, and $\underline{\xi} \equiv \min(\Xi)$. The shock probabilities naturally sum to one, and are conditional on the u_t and x_t states.

A.2 *State space*

The discretization allows us to construct the *state space* starting from the *terminal period*

$$\mathcal{S}_T(u_T, x_T) = \big\{(\overline{d}_T, \overline{n}_T) : \overline{d}_T \in \overline{\mathcal{D}}, \overline{n}_T \ge \kappa_T(u_T, x_T, \overline{d}_T)\big\},$$

$$\kappa_T(u_T, x_T, \overline{d}_T) = 0 \tag{A.3}$$

and using the *recursion*

$$\mathcal{S}_t(u_t, x_t) = \{(\overline{d}_t, \overline{n}_t) : \overline{d}_t \in \overline{\mathcal{D}}, \overline{n}_t \ge \kappa_t(u_t, x_t, \overline{d}_t)\},$$

$$\kappa_t(u_t, x_t, \overline{d}_t) = \min(\mathcal{Z}), \tag{A.4}$$

$$\mathcal{Z} = \left\{ z : \exists d_t \quad \begin{matrix} d_t \in \mathcal{D}(u_t, x_t, \overline{d}_t, z) \text{ and} \\ \forall(\psi, \xi, u_+, x_+): \\ \overline{n}_+(d_t, z; u_+, \psi, \xi) \ge \kappa_{t+1}(u_+, x_+, \overline{d}_+(d_t, \psi)) \end{matrix} \right\}.$$

This procedure ensures that there for all *interior* points in the state space *exists* a set of choices such that the *value function is finite*. On the contrary, such a set of choices does not exist on the *border* of the state space, and the value function therefore approaches $-\infty$ as $\overline{n}_t \to \kappa_t(u_t, x_t, \overline{d}_t) \ge -\max\{\overline{d}_t, \frac{\mathbf{1}_{x_t=0} \cdot \varphi}{1+\eta}\}$.

A corollary is that the households will always choose d_t and c_t such that

$$n_t > \underline{n}_t(d_t) = \max_{x_+, u_+, \psi, \xi} \frac{\Gamma\psi \cdot \left[\kappa_{t+1}(x_+, u_+, \overline{d}_+(d_t, \psi)) - \tilde{\xi}(u_+, \xi)\right] + (r_d - r_a) \cdot d_t}{1 + r_a}. \tag{A.5}$$

Note that the state space does not seem to have an *analytical* form, but in the limit must satisfy

$$\mathcal{S}_{-\infty}(u_t, x_t) \subseteq \mathcal{S}_L \cap \mathcal{S}_S,$$

$$\mathcal{S}_L = \left\{ (\overline{d}, \overline{n}) : \overline{n} > -\max\left\{\overline{d}, \frac{\mathbf{1}_{x_t=0} \cdot \varphi}{1+\eta}\right\} \right\},$$

$$\mathcal{S}_S = \{(\overline{d}, \overline{n}) : \overline{n} > -(\phi + \phi^2 \dots) \min\{\mu, \underline{\xi}\}\}, \tag{A.6}$$

$$\phi \equiv \frac{\Gamma\underline{\psi}}{1+r_d} < 1.$$

Outside $\mathcal{S}_L$ the household lacks *liquidity* in the current period, and outside $\mathcal{S}_S$ it is *insolvent* under *worst case expectations*. This is also clear from Figures 13 and 14.

The state space grid is constructed beginning with an universal $\overline{d}_t$-vector with $N_{\overline{d}}$ nodes chosen such that there are relative more nodes closer to zero. For each combination of u_t and x_t, we hereafter construct a t-specific $\overline{n}_t$-vector as the union of (a) all unique $\kappa_t(u_t, x_t, \overline{d}_t)$-values, and (b) a $\overline{n}_t$-vector with $N_{\overline{n}}$ nodes beginning in the largest $\kappa_t(u_t, x_t, \overline{d}_t)$-value and chosen such that there are relative more nodes closer to this minimum. The grid values of $\overline{n}_t$ conditional on $\overline{d}_t$ is then the t-specific $\overline{n}_t$-vector excluding all $\overline{n}_t < \kappa_t(u_t, x_t, \overline{d}_t)$, implying a total maximum of $N_{\overline{d}} + N_{\overline{n}}$ nodes in the $\overline{n}_t$-dimension. The grid is illustrated in Figure 15.
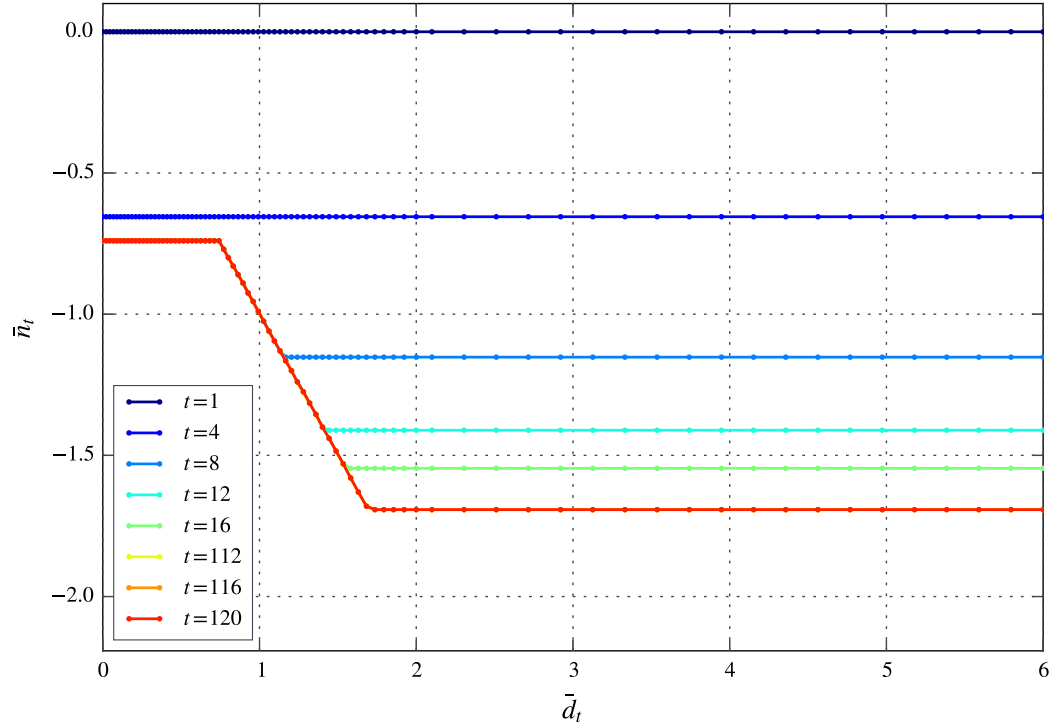
FIGURE 13. State space border, $\kappa_t(0, 0, \overline{d}_t)$.

A.3 *Value function iteration*

The *value function iteration* is now given by $\forall(u_t, x_t), \forall(\overline{d}_t, \overline{n}_t) \in \mathcal{S}_t(u_t, x_t)$

$$v_t(u_t, x_t, \overline{d}_t, \overline{n}_t) = \max_{d_t, c_t} \frac{c_t^{1-\rho}}{1-\rho} + \beta \cdot \sum \Omega_{t+1}(d_t, n_t; u_+, x_+, \psi, \xi), \qquad (A.7)$$

where when t is so low that $\mathcal{S}_t \approx \mathcal{S}_{-\infty}$, we could implement the following *stopping criterion*:

$$\nexists(u, x, \overline{d}, \overline{n}) \in \mathcal{S}_{-\infty} : \left| v_t(u, x, \overline{d}, \overline{n}) - v_{t+1}(u, x, \overline{d}, \overline{n}) \right| \geq \zeta, \qquad (A.8)$$
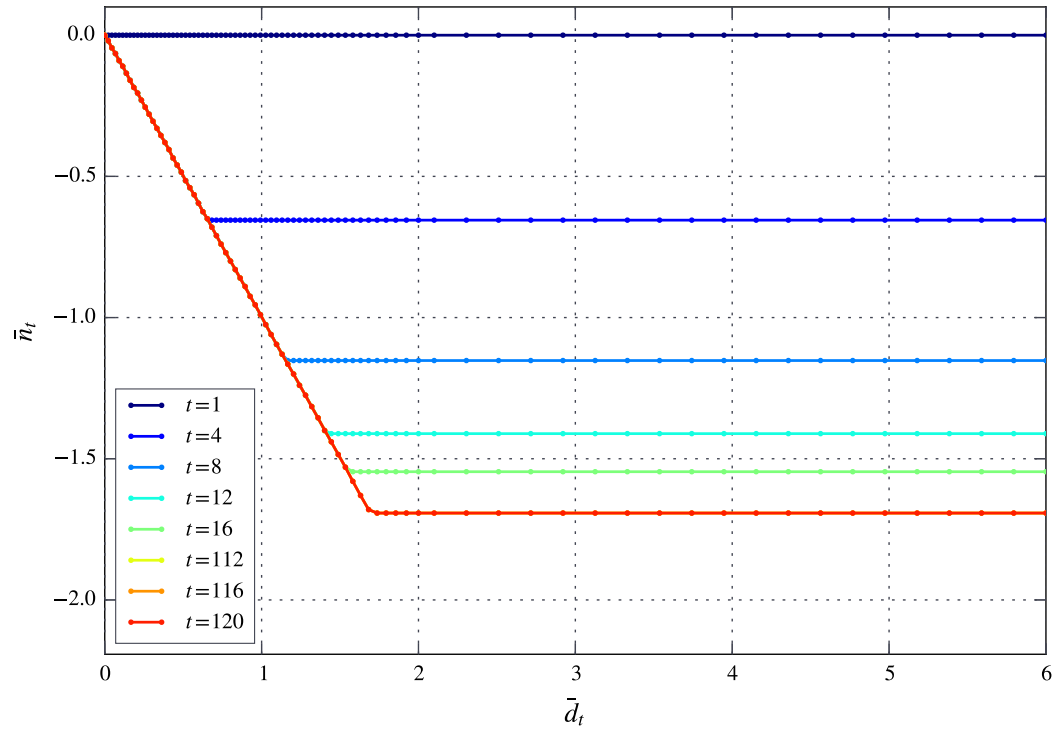
where ζ is a *tolerance parameter*. To simplify matters, we instead always iterate T-periods and check that our results are unchanged when increasing T.

A.4 *Unconstrained consumption function*

Assuming that the debt choice, $d_t = d$, the employment status, $u_t = u$, and the credit market access status, $x_t = x$, are given, the *Euler-equation* for the consumption choice, c_t, is

$$c_t = \left[(1 + r_a) \cdot \beta \cdot \sum (\Gamma \psi \cdot c_{t+1}^\star)^{-\rho} \right]^{-\frac{1}{\rho}}, \qquad (A.9)$$

where $c_{t+1}^\star = c_{t+1}^\star(u_+, x_+, \overline{d}_+(d, \psi), \overline{n}_+(d, n_t, u_+, \psi, \xi))$.

FIGURE 14. State space border, $\kappa_t(1, 1, \overline{d}_t)$.

Assuming that the $c^{\star}_{t+1}$-function is known from earlier iterations, the *endogenous grid point method* can now be used to construct an *unconstrained consumption function*. The steps are:

1. Construct a *grid vector* of n_t-values denoted $\overrightarrow{n}$ with the minimum value $\underline{n}_t(d) + \varepsilon$ (see equation (A.5)) where ε is a small number (e.g., 10^{-8}) and of length N_n with more values closer to the minimum.

2. Construct an *associated consumption vector*

$$\overrightarrow{c} = \left((1 + r_a) \cdot \beta \cdot \sum \left(\Gamma\psi \cdot c^{\star}_{t+1}\big(u_+, x_+, \overline{d}_+(d, \psi), \overline{n}_+(d, \overrightarrow{n}, \psi, \xi)\big)\right)^{-\rho}\right)^{-\frac{1}{\rho}}.$$

3. Construct an *endogenous grid vector* of $\overline{n}_t$-values by

$$\overrightarrow{\overline{n}} = \overrightarrow{n} + \overrightarrow{c}.$$

4. The *unconstrained consumption function*, $c^{\circ}_{u,x,d}(\overline{n}_t)$ can now be constructed from the association between $\{\underline{n}_t, \overrightarrow{\overline{n}}\}$ and $\{0, \overrightarrow{c}\}$ together with *linear interpolation*.

Note that this can be done independently across d_t's and does *not* depend on the states, except for u_t and x_t which affects the expectations. This step speeds up the algorithm tremendously because it avoids root finding completely.
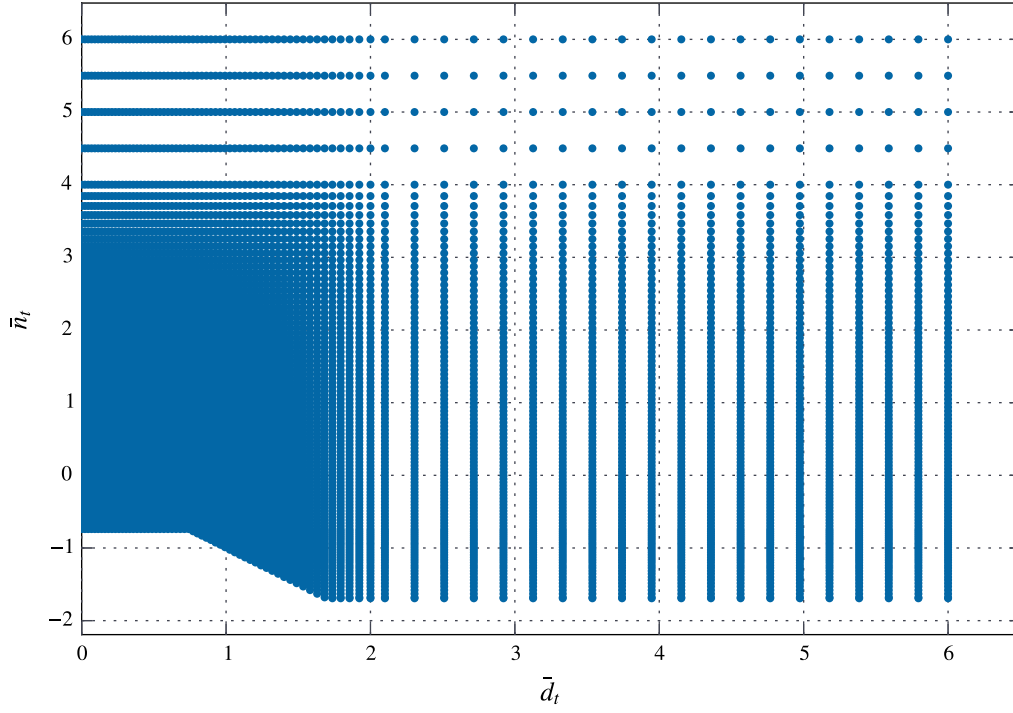
FIGURE 15. State space grid ($u_t = 0$, $x_t = 0$, $t = 0$).

Note that because we lack a proof of sufficiency of the Euler-equation, we cannot be certain that $\overrightarrow{n}$ will be *increasing*, and thus only have *unique values*. If the same value is repeated multiple times in $\overrightarrow{n}$, the EGM-algorithm breaks down, but in practice we find that this is never the case as long as the degree of uncertainty is "large enough."

A.5 *Choice functions*

The consumption choice can now be integrated out, and the household problem written purely in terms of the debt choice, that is,

$$
\begin{aligned}
v(u_t, x_t, \overline{d}_t, \overline{n}_t) &= \max_{d_t \in \mathcal{D}(u_t, x_t, \overline{d}_t, \overline{n}_t)} \frac{\left(c^\bullet(\bullet)\right)^{1-\rho}}{1-\rho} \\
&\quad + \beta \cdot \sum \Omega_{t+1}(d_t, n_t; u_+, x_+, \psi, \xi), \\
\text{s.t.} & \\
n_t &= \overline{n}_t - c^\bullet(\bullet), \\
c^\bullet(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t) &= \min\left\{c^\circ_{u_t, x_t, d_t}(\overline{n}_t), \overline{c}(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t)\right\},
\end{aligned} \tag{A.10}
$$

$$\overline{c}(u_t, x_t, \overline{d}_t, \overline{n}_t, d_t) = \begin{cases} \overline{n}_t + d_t & \text{if } d_t \le \overline{d}_t, \\ \overline{n}_t + \min\left\{ d_t, \dfrac{1}{\eta} \cdot (\mathbf{1}_{x_t=0} \cdot \varphi - d_t) \right\} & \text{if } d_t > \overline{d}_t. \end{cases}$$

This problem can be solved using a *grid search algorithm* over a *fixed d_t-grid* with step-size d_{step}, such that $c^{\circ}_{u_t, x_t, d_t}(\overline{n}_t)$ is a simple look-up table. This has to be done for all possible states, but it is possible to speed this up by utilizing some bounds on the optimal debt choice function. Specifically, we use that given

$$d^{\star}(u_t, x_t, Y, \overline{n}_t) = d_Y, \tag{A.11}$$

$$d^{\star}(u_t, x_t, 0, \overline{n}_t) = d_0, \tag{A.12}$$

$$d^{\star}(u_t, x_t, \overline{d}_{d=d_0}, \overline{n}_t) = d_0, \tag{A.13}$$

we must have

$$\forall \overline{d}_t \in [d_Y : Y] : d^{\star}(u_t, x_t, \overline{d}_t, \overline{n}_t) = d_Y, \tag{A.14}$$

$$\forall \overline{d}_t \in [d_0 : d_Y), \, \varepsilon \ge 0 : d^{\star}(u_t, x_t, \overline{d}_t + \varepsilon, \overline{n}_t) \ge d^{\star}(u_t, x_t, \overline{d}_t, \overline{n}_t), \tag{A.15}$$

$$\forall \overline{d}_t \in (0, d_0) : d^{\star}(u_t, x_t, \overline{d}_t, \overline{n}_t) \le d_0, \tag{A.16}$$

$$\forall \overline{d}_t \in [0 : \overline{d}_{d=d_0}] : d^{\star}(u_t, x_t, \overline{d}_t, \overline{n}_t) = d_0. \tag{A.17}$$

Over u_t, x_t, and $\overline{n}_t$ the problem is jointly parallelizable. The value function is evaluated in the $\overline{n}_{t+1}$-dimension[29] by "negative inverse negative inverse" linear interpolation, where the negative inverse value function is interpolated linearly and the negative inverse of the result is then used; this is beneficial because the value function is then equal to zero on the border of the state space.

Note that the grid search needs to be *global* because we otherwise might find multiple *local* extrema and because there might be *discontinues* due to the nonconvex choice set. This directly give us $d^{\star}(u_t, x_t, \overline{d}_t, \overline{n}_t)$ and, therefore, also

$$c^{\star}(u_t, x_t, \overline{d}_t, \overline{n}_t) = c^{\bullet}(u_t, x_t, \overline{d}_t, \overline{n}_t, d^{\star}(u_t, x_t, \overline{d}_t, \overline{n}_t)). \tag{A.18}$$

A.6 *Implementation*

The algorithm is implemented in *Python 2.7,* but the core part is written in *C* parallelized using *OpenMP* and called from Python using *CFFI*. Only free open source languages and programs are needed to run the code. The code files are available in the supplementary file on the journal website, http://qeconomics.org/supp/604/code_and_data.zip.

Table 8 shows the parametric settings we use. Our results are robust to using even finer grids.

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[29]The other dimensions are fully discretized.

Table 8. Algorithm settings.

Parameter	Value
Nodes for transitory income shock, N_ξ	4
Nodes for permanent income shock, N_ψ	4
Nodes for beginning-of-period debt, $N_{\bar{d}}$	80
Nodes for beginning-of-period net wealth, $N_{\bar{n}}$	80
Nodes for net wealth grid vector ($\vec{n}$), N_n	40
Value used to calculate minimum of net wealth grid vector, ε	10^{-8}
Step-size of fixed debt grid, d_{step}	$5 \cdot 10^{-3}$
Number of iterations, T	120

References

Alan, S. and M. Browning (2010), "Estimating intertemporal allocation parameters using synthetic residual estimation." *The Review of Economic Studies*, 77 (4), 1231–1261. [802]

Alan, S., T. Crossley, and H. Low (2012), "Saving on a rainy day, borrowing for a rainy day." IFS Working Paper W12/11. [787]

Attanasio, O., A. Leicester, and M. Wakefield (2011), "Do house prices drive consumption growth? The coincident cycles of house prices and consumption in the UK." *Journal of the European Economic Association*, 9 (3), 399–435. [787]

Attanasio, O. P, R. Bottazzi, H. W. Low, L. Nesheim, and M. Wakefield (2012), "Modelling the demand for housing over the life cycle." *Review of Economic Dynamics*, 15 (1), 1–18. [787]

Barillas, F. and J. Fernández-Villaverde (2007), "A generalization of the endogenous grid method." *Journal of Economic Dynamics and Control*, 31 (8), 2698–2712. [794]

Bertaut, C. C. and M. Haliassos (2002), "Debt revolvers for self control." Working paper. Available at http://papers.econ.ucy.ac.cy/RePEc/papers/0208.pdf. [789]

Bertaut, C. C., M. Haliassos, and M. Reiter (2009), "Credit card debt puzzles and debt revolvers for self control." *Review of Finance*, 13 (4), 657–692. [788, 789]

Carroll, C. D. (1992), "The buffer-stock theory of saving: Some macroeconomic evidence." *Brookings Papers on Economic Activity*, 2, 61–156. [786, 792]

Carroll, C. D. (1997), "Buffer-stock saving and the life cycle/permanent income hypothesis." *The Quarterly Journal of Economics*, 112 (1), 1–55. [786, 792]

Carroll, C. D. (2006), "The method of endogenous gridpoints for solving dynamic stochastic optimization problems." *Economics Letters*, 91 (3), 312–320. [794]

Carroll, C. D. (2012), "Theoretical foundations of buffer stock saving." Working paper. Available at http://www.econ2.jhu.edu/people/ccarroll/papers/BufferStockTheory/. [786, 792, 810]

Carroll, C. D., J. Slacalek, and K. Tokuoka (2015), "Buffer-stock saving in a Krusell–Smith world." *Economics Letters*, 132, 97–100. [799]

CFPB (2013), "CARD Act report—A review of the impact of the CARD Act on the consumer credit card market." Discussion paper, Consumer Financial Protection Bureau. [786]

Chen, H., M. Michaux, and N. Roussanov (2015), "Houses as ATMs? Mortgage refinancing and macroeconomic uncertainty." NBER Working Paper 19421. [787]

Clausen, A. and C. Strub (2013), "A general and intuitive envelope theorem." Working paper. Available at https://andrewclausen.net/Clausen_strub_envelope.pdf. [794]

Crossley, T. F. and H. W. Low (2013), "Job loss, credit constraints, and consumption growth." *Review of Economics and Statistics*, 96 (5), 876–884. [800]

Edelberg, W. (2006), "Risk-based pricing of interest rates for consumer loans." *Journal of Monetary Economics*, 53 (8), 2283–2298. [799, 800]

Fella, G. (2014), "A generalized endogenous grid method for non-smooth and non-concave problems." *Review of Economic Dynamics*, 17 (2), 329–344. [794]

Fulford, S. L. (2015), "How important is variability in consumer credit limits." *Journal of Monetary Economics*, 72, 42–63. [787, 788, 797, 799, 800, 803, 805, 810, 811, 812, 814]

Gathergood, J. and J. Weber (2014), "Self-control, financial literacy & the co-holding puzzle." *Journal of Economic Behavior & Organization*, 107, Part B, 455–469. [788, 789]

Gorbachev, O. and M. J. Luengo-Prado (2016), "The credit card puzzle: The role of preferences, liquidity risk, and financial literacy." Working paper. Available at http://www.luengoprado.net/pdfs/CC.pdf. [797, 804, 814]

Gross, D. B. and N. S. Souleles (2002), "Do liquidity constraints and interest rates matter for consumer behavior? Evidence from credit card data." *The Quarterly Journal of Economics*, 117 (1), 149–185. [785, 788, 789]

Guiso, L. and P. Sodini (2013), "Chapter 21—Household finance: An emerging field." In *Handbook of the Economics of Finance*, Vol. 2 (G. M. Constantinides, ed.), 1397–1532, Elsevier. Part B. [785]

Haliassos, M. and M. Reiter (2007), "Credit card debt puzzles." CFS Working Paper No. 2005/26. [789]

Halket, J. and S. Vasudev (2014), "Saving up or settling down: Home ownership over the life cycle." *Review of Economic Dynamics*, 17 (2), 345–366. [787]

Hintermaier, T. and W. Koeniger (2010), "The method of endogenous gridpoints with occasionally binding constraints among endogenous variables." *Journal of Economic Dynamics and Control*, 34 (10), 2074–2088. [794]

Iskhakov, F., T. H. Jørgensen, J. Rust, and B. Schjerning (2017), "The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks." *Quantitative Economics*, 8 (2), 317–365. [794]

Jørgensen, T. H. (2013), "Structural estimation of continuous choice models: Evaluating the EGM and MPEC." *Economics Letters*, 119 (3), 287–290. [794]

Kaplan, G., G. Violante, and J. Weidner (2014), "The wealthy hand-to-mouth." Brookings Papers on Economic Activity, 77–138. [788]

Kaplan, G. and G. L. Violante (2014), "A model of the consumption response to fiscal stimulus payments." *Econometrica*, 82 (4), 1199–1239. [787, 794, 798, 799, 814]

Laibson, D., A. Repetto, and J. Tobacman (2003), "A debt puzzle." In *Knowledge, Information and Expectations in Modern Economics: In Honor of Edmund S. Phelps* (P. Aghion, R. Frydman, J. E. Stiglitz, and M. Woodford, eds.), 228–266, Princeton University Press, Princeton, NJ. [789, 798]

Lehnert, A. and D. M. Maki (2007), "Consumption, debt and portfolio choice." In *Financial Instruments for Households: Credit Usage From Mortgages to Credit Cards* (S. Agarwal and B. W. Ambrose, eds.), 55–76, Palgrave Macmillan, New York. [789]

Lopes, P. (2008), "Credit card debt and default over the life cycle." *Journal of Money, Credit and Banking*, 40 (4), 769–790. [789]

Mankart, J. (2014), "The (un-) importance of Chapter 7 wealth exemption levels." *Journal of Economic Dynamics and Control*, 38, 1–16. [789]

Martin, J. P. (1996), "Measures of replacement rates for the purpose of international comparisons: A note." *OECD economic studies*, 26 (1), 99–115. [799]

McKay, A. (2017), "Time-varying idiosyncratic risk and aggregate consumption dynamics." *Journal of Monetary Economics*, 88, 1–14. [803]

Telyukova, I. A. (2013), "Household need for liquidity and the credit card debt puzzle." *The Review of Economic Studies*, 80 (3), 1148–1177. [788, 789, 795, 797, 798, 799, 800]

Telyukova, I. A. and R. Wright (2008), "A model of money and credit, with application to the credit card debt puzzle." *The Review of Economic Studies*, 75 (2), 629–647. [788, 789]

Tufano, P. (2009), "Consumer finance." *Annual Review of Financial Economics*, 1 (1), 227–247. [785]

Zinman, J. (2007), "Household borrowing high and lending low under no-arbitrage." Working paper. Available at https://www.dartmouth.edu/~jzinman/Papers/Zinman_bhll_apr07.pdf. [789]