Biometrika (1996), **83**, 4, *pp.* 715–726
Printed in Great Britain

The multivariate skew-normal distribution

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SUMMARY

The paper extends earlier work on the so-called skew-normal distribution, a family of distributions including the normal, but with an extra parameter to regulate skewness. The present work introduces a multivariate parametric family such that the marginal densities are scalar skew-normal, and studies its properties, with special emphasis on the bivariate case.

Some key words: Bivariate distribution; Multivariate normal distribution; Specified marginal; Skewness.

1. THE SKEW-NORMAL DISTRIBUTION

1·1. *Introduction*

The term skew-normal ($\mathcal{SN}$) refers to a parametric class of probability distributions which includes the standard normal as a special case. A random variable Z is said to be skew-normal with parameter λ, written $Z \sim \mathcal{SN}(\lambda)$, if its density function is

$$\phi(z;\lambda) := 2\phi(z)\Phi(\lambda z) \quad (z \in \mathbb{R}), \tag{1·1}$$

where $\phi(z)$ and $\Phi(z)$ denote the $\mathcal{N}(0,1)$ density and distribution function, respectively; the parameter λ which regulates the skewness varies in $(-\infty, \infty)$, and $\lambda = 0$ corresponds to the $\mathcal{N}(0,1)$ density.

The density (1·1) has appeared at various places in the literature, sometimes in a somewhat casual manner. A systematic treatment of this distribution, developed independently from earlier work, has been given by Azzalini (1985, 1986).

The interest in density (1·1) comes from two directions. On the theoretical side, it enjoys a number of formal properties which reproduce or resemble those of the normal distribution and appear to justify its name skew-normal; in particular, $Z^2 \sim \chi^2_1$. From the applied viewpoint, (1·1) is suitable for the analysis of data exhibiting a unimodal empirical distribution but with some skewness present, a situation often occurring in practical problems. See Hill & Dixon (1982) for a discussion and numerical evidence of the presence of skewness in real data. Arnold et al. (1993) include an application of the $\mathcal{SN}$ distribution to real data.

The purpose of the present paper is to introduce a multivariate version of the skew-normal density. Such an extension is potentially relevant for practical applications, since in the multivariate case there are far fewer distributions available for dealing with non-normal data than in the univariate case, especially for the problem of moderate skewness of the marginals.

A multivariate version of (1·1) has been discussed briefly in Azzalini (1985), but it has the disadvantage of being a purely formal extension of the mathematical expression (1·1),