# BSM Option Pricing

# BSM Option Pricing #Question 15.13

- Calculate the price of a 3-month European put option on
- a non-dividend-paying stock with a strike price of $50
- when the current stock price is $50,
- the risk-free interest rate is 10% per annum, and
- the volatility is 30% per annum.

# BSM Option Pricing #Question 15.21

- What is the price of a European call option on a non-dividend-paying stock
- when the stock price is $52, the strike price is $50,
- the risk-free interest rate is 12% per annum,
- the volatility is 30% per annum, and
- the time to maturity is 3 months?

# BSM Option Pricing #Question 15.22

- What is the price of a European put option on a non-dividend-paying stock
- when the stock price is $69, the strike price is $70,
- the risk-free interest rate is 5% per annum,
- the volatility is 35% per annum, and
- the time to maturity is 6 months?

# BSM Option Pricing #Question 15.26



- Show that the Black–Scholes–Merton formulas for call and put options satisfy put–call parity.

# Dividends

# Dividends #Question 15.14

- A stock price is currently $40.
- It is known that at the end of 3 months it will be either $45 or $35.
- The risk-free rate of interest with quarterly compounding is 8% per annum.
- Calculate the value of a 3-month European put option on the stock
- with an exercise price of $40.
- Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answers.

# Probability Distribution

## **Probability Distribution** #Question 15.11

- What does the Black–Scholes–Merton stock option pricing model assume about the probability distribution of the stock price in one year?
- What does it assume about the probability distribution of the continuously compounded rate of return on the stock during the year?

# More-than-1-Step Binomial Tree #Question 13.15

- A stock price is currently $40.
- Assume that the expected return from the stock is 15% and that its volatility is 25%.
- What is the probability distribution for the rate of return (with continuous compounding) earned over a 2-year period?

## More-than-1-Step Binomial Tree #Question 13.35

- A stock price is currently $50.
- Assume that the expected return from the stock is 18% and its volatility is 30%.
- What is the probability distribution for the stock price in 2 years?
- Calculate the mean and standard deviation of the distribution.
- Determine the 95% confidence interval.

## More-than-1-Step Binomial Tree #Question 13.17

- Using the notation in this chapter,
- prove that a 95% confidence interval for $S_T$ is between
- $S_0 e^{(\mu-\sigma^2/2)T-1.96\sigma\sqrt{T}}$ and $S_0 e^{(\mu-\sigma^2/2)T+1.96\sigma\sqrt{T}}$

# Probability that option will be exercised

## Probability that option will be exercised #Question 15.16

- A stock price follows geometric Brownian motion with an expected return of 16% and a volatility of 35%. The current price is $38.
- (a) What is the probability that a European call option on the stock with an exercise price of $40 and a maturity date in 6 months will be exercised?
- (b) What is the probability that a European put option on the stock with the same exercise price and maturity will be exercised?

# Probability that option will be exercised #Question 15.30

- Show that the probability that a European call option will be exercised in a risk-neutral world is, with the notation introduced in this chapter, $\boldsymbol{N(d_2)}$
- What is an expression for the value of a derivative that pays off $100 if the price of a stock at time T is greater than K?

# Probability that option will be exercised #Question 13.29



- Consider a European call option on a non-dividend-paying stock
- where the stock price is $40, the strike price is $40, the risk-free rate is 4% per annum,
- the volatility is 30% per annum, and the time to maturity is 6 months.
- (a) Calculate u, d, and p for a two-step tree.
- (b) Value the option using a two-step tree.
- (c) Verify that DerivaGem gives the same answer.
- (d) Use DerivaGem to value the option with 5, 50, 100, and 500 time steps.

# Implied Volatility

# Implied Volatility #Question 15.24

- A stock price is currently \$25.
- It is known that at the end of 2 months it will be either \$23 or \$27.
- The risk-free interest rate is 10% per annum with continuous compounding.
- Suppose $S_T$ is the stock price at the end of 2 months.
- What is the value of a derivative that pays off $S_T^2$ at this time?

# Implied Volatility #Question 15.27

- A call option on a non-dividend-paying stock has a market price of $2.5.
- The stock price is $15, the exercise price is $13, the time to maturity is 3 months, and the risk-free interest rate is 5% per annum.
- What is the implied volatility?

# Practical Issues related to Volatility

# Practical Issues related to Volatility #Question 15.12

- The volatility of a stock price is 30% per annum.
- What is the standard deviation of the percentage price change in one trading day?

# Practical Issues related to Volatility #Question 15.34

- If the volatility of a stock is 18% per annum,
- estimate the standard deviation of the percentage price change in
- (a) 1 day, (b) 1 week, and (c) 1 month.

## Practical Issues related to Volatility #Question 15.36

- Suppose that observations on a stock price (in dollars) at the end of each of 15 consecutive weeks are as follows: 30.2, 32.0, 31.1, 30.1, 30.2, 30.3, 30.6, 33.0, 32.9, 33.0, 33.5, 33.5, 33.7, 33.5, 33.2
- Estimate the stock price volatility.
- What is the standard error of your estimate?

# American Options

# American Options #Question 15.23

- Consider an American call option on a stock.
- The stock price is \$70, the time to maturity is 8 months,
- the risk-free rate of interest is 10% per annum,
- the exercise price is \$65, and the volatility is 32%.
- A dividend of \$1 is expected after 3 months and again after 6 months.
- Show that it can never be optimal to exercise the option on either of the two dividend dates.
- Use DerivaGem to calculate the price of the option.

# American Options #Question 15.29

- Consider an American call option on a stock.
- The stock price is \$50, the time to maturity is 15 months,
- the risk-free rate of interest is 8% per annum, the exercise price is \$55, and the volatility is 25%.
- Dividends of \$1.50 are expected in 4 months and 10 months.
- Show that it can never be optimal to exercise the option on either of the two dividend dates.
- Calculate the price of the option.

## American Options #Question 15.31

- Use the result in equation (15.17) to determine the value of a perpetual American put option on a non-dividend-paying stock with strike price K
- if it is exercised when the stock price equals H where H < K.
- Assume that the current stock price S is greater than H.
- What is the value of H that maximizes the option value? Deduce the value of a perpetual American put with strike price K.

# BS Differential Equation

## BS Differential Equation #Question 15.19



- Assume that a non-dividend-paying stock has an expected return of $m$ and a volatility of $s$.
- An innovative financial institution has just announced that it will trade a security that pays off a dollar amount equal to $\ln S_T$ at time $T$, where $S_T$ denotes the value of the stock price at time $T$.
- (a) Use risk-neutral valuation to calculate the price of the security at time $t$ in terms of the stock price, $S$, at time $t$. The risk-free rate is $r$.
- (b) Confirm that your price satisfies the differential equation (15.16).

## BS Differential Equation #Question 15.20

- Consider a derivative that pays off $S_T^n$ at time $T$, where $S_T$ is the stock price at that time.
- When the stock pays no dividends and its price follows geometric Brownian motion, it can be shown that its price at time $t$ $(t \le T)$ has the form $h(t,T)S^n$, where $S$ is the stock price at time $t$ and $h$ is a function only of $t$ and $T$.
- (a) By substituting into the Black–Scholes–Merton partial differential equation, derive an ordinary differential equation satisfied by $h(t,T)$.
- (b) What is the boundary condition for the differential equation for $h(t,T)$ ?
- (c) Show that $h(t,T)$= $e^{0.5\sigma^2}n(n-1)+r(n-1)](T- t)$, where $r$ is the risk-free interest rate and $\sigma$ is the stock price volatility.

## BS Differential Equation #Question 15.25



- With the notation used in this chapter:
- (a) What is $N'(x)$
- (b) Show that $SN'(d_1) = Ke^{-r(T-t)}N'(d_2)$,
- where S is the stock price at time t and
- $d_1 = \frac{\ln(S/K)+(r+\sigma^2/2)(T-t)}{\sigma\sqrt{T}-t}, d_2 = \frac{\ln(S/K)+(r-\sigma^2/2)(T-t)}{\sigma\sqrt{T}-t}$
- (c) Calculate $\frac{\partial d_1}{\partial S}$ and $\frac{\partial d_2}{\partial S}$
- (d) Show that when c = $SN(d_1) = Ke^{-r(T-t)}N'(d_2)$, it follows that $\frac{\partial c}{\partial S} = -rKe^{-r(T-t)}N'(d_2) - SN(d_1)\frac{\sigma}{\sigma\sqrt{T}-t}$,
- where c is the price of a call option on a non-dividend-paying stock.
- (e) Show that $\frac{\partial c}{\partial S} = N(d_1)$
- (f) Show that $c$ satisfies the Black-Scholes-Merton differential equation.
- (g) Show that $c$ satisfies the boundary condition for a European call option,

  i.e, that $c = \max(S-K, 0)$ as $t \quad T$

# Miscellaneous

# Miscellaneous #Question 15.28

- Explain carefully why Black's approach to evaluating an American call option on a dividend-paying stock may give an approximate answer even when only one dividend is anticipated.
- Does the answer given by Black's approach understate or overstate the true option value?
- Explain your answer.

# Miscellaneous #Question 15.18

- A portfolio manager announces that the average of the returns realized in each year of the last 10 years is 20% per annum.
- In what respect is this statement misleading?

# Miscellaneous #Question 15.32

- A company has an issue of executive stock options outstanding.
- Should dilution be taken into account when the options are valued?
- Explain your answer.

# **Miscellaneous** #Question 15.33

- A company's stock price is $50 and 10 million shares are outstanding.
- The company is considering giving its employees 3 million at-the-money 5-year call options.
- Option exercises will be handled by issuing more shares.
- The stock price volatility is 25%, the 5-year risk-free rate is 5%, and the company does not pay dividends.
- Estimate the cost to the company of the employee stock option issue.