Basic Idea of Hedging

Basic Idea of Hedging #Question 19.13

- "The procedure for creating an option position synthetically is the reverse of the procedure for hedging the option position."
- Explain this statement.

Basic Idea of Hedging #Question 19.14

- The Black–Scholes–Merton price of an out-of-the-money call option with an exercise price of $40 is $4.
- A trader who has written the option plans to use a stop-loss strategy.
- The trader's plan is to buy at $40.10 and to sell at $39.90.
- Estimate the expected number of times the stock will be bought or sold.

Basic Idea of Hedging #Question 19.15

- Suppose that a stock price is currently $20 and that
- a call option with an exercise price of $25 is created synthetically using a continually changing position in the stock.
- Consider the following two scenarios:

  (a) Stock price increases steadily from $20 to $35 during the life of the option;

  (b) Stock price oscillates wildly, ending up at $35.
- Which scenario would make the synthetically created option more expensive?
- Explain your answer.

Delta Hedging

Delta Hedging #Question 19.11

- How can a short position in 1,000 options be made delta neutral
- when the delta of each option is 0.7?

Delta Hedging #Question 19.12

- Calculate the delta of an at-the-money six-month European call option on a non-dividend-paying stock
- when the risk-free interest rate is 10% per annum and
- the stock price volatility is 25% per annum.

Delta Hedging #Question 19.26

- Suppose that $70 billion of equity assets are the subject of portfolio insurance schemes.
- Assume that the schemes are designed to provide insurance against the value of the assets declining by more than 5% within 1 year.
- Making whatever estimates you find necessary,
- use the DerivaGem software to calculate the value of the stock or futures contracts that
- the administrators of the portfolio insurance schemes will attempt to sell if the market falls by 23% in a single day.

Gamma issue of Delta Heding

Gamma issue of Delta Heding #Question 19.18

- A company uses delta hedging to hedge a portfolio of long positions in put and call options on a currency.
- Which would give the most favorable result:
- (a) a virtually constant spot rate or (b) wild movements in the spot rate?
- Explain your answer.

Gamma issue of Delta Heding #Question 19.19

- Repeat Problem 19.18 for a financial institution with
- a portfolio of short positions in put and call options on a currency.

Gamma issue of Delta Heding #Question 19.28

- A bank's position in options on the dollar/CAD exchange rate has
- a delta of 30,000 and a gamma of -80,000.
- Explain how these numbers can be interpreted.
- The exchange rate (dollars per CAD) is 0.90.
- What position would you take to make the position delta neutral?
- After a short period of time, the exchange rate moves to 0.93.
- Estimate the new delta.
- What additional trade is necessary to keep the position delta neutral?
- Assuming the bank did set up a delta-neutral position originally, has it gained or lost money from the exchange rate movement?

Various Greeks

Various Greeks #Question 19.20

- A financial institution has just sold 1,000 7-month European call options on the Japanese yen.
- Suppose that the spot exchange rate is 0.80 cent per yen,
- the exercise price is 0.81 cent per yen,
- the risk-free interest rate in the United States is 8% per annum,
- the risk-free interest rate in Japan is 5% per annum, and
- the volatility of the yen is 15% per annum.
- Calculate the delta, gamma, vega, theta, and rho of the financial institution's position.
- Interpret each number.

Various Greeks #Question 19.21

- Under what circumstances is it possible to make a European option on a stock index both gamma neutral and vega neutral
- by adding a position in one other European option?

Various Greeks #Question 19.24

- Show by substituting for the various terms in equation (19.4) that the equation is true for:
- (a) A single European call option on a non-dividend-paying stock
- (b) A single European put option on a non-dividend-paying stock
- (c) Any portfolio of European put and call options on a non-dividend-paying stock.

Various Greeks #Question 19.25

- What is the equation corresponding to equation (19.4) for
- (a) a portfolio of derivatives on a currency and
- (b) a portfolio of derivatives on a futures price?

Various Greeks #Question 19.29

- Use the put–call parity relationship to derive, for a non-dividend-paying stock,
- the relationship between:
- (a) The delta of a European call and the delta of a European put
- (b) The gamma of a European call and the gamma of a European put
- (c) The vega of a European call and the vega of a European put
- (d) The theta of a European call and the theta of a European put.

Various Greeks #Question 19.30

- A financial institution has the following portfolio of over-the-counter options on sterling

Type	*Position*	*Delta of option*	*Gamma of option*	*Vega of option*
Call	−1,000	0.50	2.2	1.8
Call	−500	0.80	0.6	0.2
Put	−2,000	−0.40	1.3	0.7
Call	−500	0.70	1.8	1.4

- A traded option is available with a delta of 0.6, a gamma of 1.5, and a vega of 0.8.
- (a) What position in the traded option and in sterling would make the portfolio both gamma neutral and delta neutral?
- (b) What position in the traded option and in sterling would make the portfolio both vega neutral and delta neutral? Assume that all implied volatilities change by the same amount so that vegas can be aggregated

Various Greeks #Question 19.31

- Consider again the situation in Problem 19.30.
- Suppose that a second traded option with a delta of 0.1, a gamma of 0.5, and a vega of 0.6 is available.
- How could the portfolio be made delta, gamma, and vega neutral?

Various Greeks #Question 19.34

YONSEI UNIVERSITY

- Use DerivaGem to check that equation (19.4) is satisfied for the option considered in Section 19.1.
- (Note: DerivaGem produces a value of theta “per calendar day.” The theta in equation (19.4) is “per year.”)

Delta of non-option derivatives

Delta of non-option derivatives #Question 19.16

- What is the delta of a short position in 1,000 European call options on silver futures?
- The options mature in 8 months, and
- the futures contract underlying the option matures in 9 months.
- The current 9-month futures price is $8 per ounce,
- the exercise price of the options is $8,
- the risk-free interest rate is 12% per annum, and
- the volatility of silver futures prices is 18% per annum.

Delta of non-option derivatives #Question 19.17

- In Problem 19.16, what initial position in 9-month silver futures is necessary for delta hedging?
- If silver itself is used, what is the initial position?
- If 1-year silver futures are used, what is the initial position?
- Assume no storage costs for silver.

Delta of non-option derivatives #Question 19.27

- Does a forward contract on a stock index have the same delta as the corresponding futures contract?
- Explain your answer.

Portfolio Insurance

Portfolio Insurance #Question 19.22

- A fund manager has a well-diversified portfolio that mirrors the performance of an index and is worth $360 million.
- The value of the index is 1,200, and the portfolio manager would like to buy insurance against a reduction of more than 5% in the value of the portfolio over the next 6 months.
- The risk-free interest rate is 6% per annum.
- The dividend yield on both the portfolio and the index is 3%, and the volatility of the index is 30% per annum.

(Next page)

Portfolio Insurance #Question 19.22

- (a) If the fund manager buys traded European put options, how much would the insurance cost?
- (b) Explain carefully alternative strategies open to the fund manager involving traded European call options, and show that they lead to the same result.
- (c) If the fund manager decides to provide insurance by keeping part of the portfolio in risk-free securities, what should the initial position be?
- (d) If the fund manager decides to provide insurance by using 9-month index futures, what should the initial position be?

Portfolio Insurance #Question 19.23

- Repeat Problem 19.22 on the assumption that
- the portfolio has a beta of 1.5.
- Assume that the dividend yield on the portfolio is 4% per annum.

Miscellaneous

Miscellaneous #Question 19.32

- A deposit instrument offered by a bank guarantees that investors will receive a return during a 6-month period that is the greater of (a) zero and (b) 40% of the return provided by a market index.
- An investor is planning to put $100,000 in the instrument.
- Describe the payoff as an option on the index.
- Assuming that the risk-free rate of interest is 8% per annum,
- the dividend yield on the index is 3% per annum, and
- the volatility of the index is 25% per annum,
- is the product a good deal for the investor?

Miscellaneous #Question 19.33

- The formula for the price c of a European call futures option in terms of the futures price F_0 is given in

Chapter 18 as

- $c = e^{-\mathrm{rT}}\,[F_0\, N(d_1) - KN(d_2)]$
- where $d_1 = \frac{\ln(F_0/K)+\sigma^2 T/2}{\sigma\sqrt{T}}$ and
- $d_2 = d_1 - \sigma\sqrt{T}$
- and $K, r, T,$ and σ are the strike price, interest rate, time to maturity, and volatility, respectively.
- (a) Prove that $F_0 N'(d_1) = KN'(d_2)$.
- (b) Prove that the delta of the call price with respect to the futures price is $e^{-\mathrm{rT}}\ N(d_1)$.
- (c) Prove that the vega of the call price is $F_0\sqrt{T}N'(d_1)\ e^{-\mathrm{rT}}$
- (d) Prove the formula for the rho of a call futures option given in Section 19.12.

(Next page)

Miscellaneous #Question 19.33

- $\frac{\partial c}{\partial r} = -Te^{-\mathrm{rT}}[FN(d_1) - KN(d_2)] = \mathrm{cT}$
- The delta, gamma, theta, and vega of a call futures option are the same as those for a call option on a stock paying dividends at rate q, with q replaced by r and S_0replaced by F_0.
- Explain why the same is not true of the rho of a call futures option.