#### **Introduction**



Price vs. Valuation (5.11)

Forward price for investment assets

$$\circ F_0 = S_0 e^{rT}$$

$$\circ F_0 = (S_0 - I) e^{rT}$$

$$\circ F_0 = S_0 e^{(r-q)T}$$

**Short Selling** 

### **Determination of Prices**



#### **Determination of Prices** #Question 12



- Suppose that you enter into a 6-month forward contract on a non-dividend-paying stock
- when the stock price is \$30 and
- the risk-free interest rate (with continuous compounding) is 5% per annum.
- What is the forward price?

#### **Determination of Prices** #Question 13



- A stock index currently stands at 350.
- The risk-free interest rate is 4% per annum (with continuous compounding) and
- the dividend yield on the index is 3% per annum.
- What should the futures price for a 4-month contract be?

#### **Determination of Prices** #Question 20



- Suppose that the risk-free interest rate is 6% per annum with continuous compounding and
- that the dividend yield on a stock index is 4% per annum.
- The index is standing at 400, and
- the futures price for a contract deliverable in four months is 405.
- What arbitrage opportunities does this create?

(Skip) Question 5.18, 5.33, 5.19

## **Valuation**



#### Valuation #Question 17



- A 1-year long forward contract on a non-dividend-paying stock is entered into
- when the stock price is \$40 and
- the risk-free rate of interest is 5% per annum with continuous compounding.
  - (a) What are the forward price and the initial value of the forward contract?
  - (b) Six months later, the price of the stock is \$45 and the risk-free interest rate is still 5%. What are the forward price and the value of the forward contract?

(Skip) Question 5.11

# **Foreign Currency**



### **Foreign Currency**



Forward price for investment asset.

$$\circ F_0 = S_0 e^{rT}$$

$$\circ F_0 = (S_0 - I) e^{rT}$$
$$\circ F_0 = S_0 e^{(r-q)T}$$

$$\circ F_0 = S_0 e^{(r-q)T}$$



• Explain why a foreign currency can be treated as an asset providing a known yield.



- The 2-month interest rates in Switzerland and the United States are, respectively,
- 1% and 2% per annum with continuous compounding.
- The spot price of the Swiss franc is \$1.0500.
- The futures price for a contract deliverable in 2 months is \$1.0500.
- What arbitrage opportunities does this create?



- The spot exchange rate between the Swiss franc and U.S. dollar is 1.0404 (\$ per franc).
- Interest rates in the United States and Switzerland are 0.25% and 0% per annum, respectively, with continuous compounding.
- The 3-month forward exchange rate was 1.0300 (\$ per franc).
- What arbitrage strategy was possible?
- How does your answer change if the forward exchange rate is 1.0500 (\$ per franc).



- Suppose the current USD/euro exchange rate is 1.2000 dollars per euro.
- The six-month forward exchange rate is 1.1950.
- The six-month USD interest rate is 1% per annum continuously compounded.
- Estimate the six-month euro interest rate.



• Estimate the difference between short-term interest rates in Japan and the United States on May 21, 2020, from the information in Table 5.4.

	Open	High	Low	Prior settlement	Last trade	Change	Volume
Australian	Dollar, U	SD per A	UD, 100,	000 AUD			
June 2020	0.6597	0.6599	0.6549	0.6601	0.6567	-0.0034	92,674
Sept. 2020	0.6598	0.6598	0.6549	0.6602	0.6563	-0.0039	316
British Pou	ind, USD	per GBP	, 62,500 G	BP			
June 2020	1.2235	1.2250	1.2186	1.2231	1.2219	-0.0012	69,100
Sept. 2020	1.2217	1.2253	1.2191	1.2236	1.2246	+0.0010	388
Canadian I	Dollar, US	D per C	AD, 100,0	00 CAD			
June 2020	0.71930	0.71985	0.71575	0.71990	0.71705	-0.00285	51,980
Sept. 2020	0.71915	0.71910	0.71665	0.72000	0.71720	-0.00280	562
Dec. 2020	0.71790	0.71905	0.71680	0.72015	0.71680	-0.00335	164
Euro, USD	per EUF	2, 125,000	EUR				
June 2020	1.09840	1.10140	1.09415	1.09915	1.09510	-0.00405	136,609
Sept. 2020	1.10050	1.10320	1.09650	1.10120	1.09750	-0.00370	1,013
Dec. 2020	1.10190	1.10550	1.09850	1.10350	1.10100	-0.00250	277
Japanese Y	en, USD	per 100 y	en, 12.5 m	illion yen			
June 2020	0.93015	0.93035	0.92745	0.93070	0.92970	-0.00100	61,018
Sept. 2020	0.93040	0.93125	0.92895	0.93200	0.93125	-0.00075	453
Swiss Fran	c, USD po	er CHF, 1	25,000 CF	IF			
June 2020	1.0371	1.0374	1.0303	1.0374	1.0304	-0.0070	18,155
Sept. 2020	1.0397	1.0397	1.0336	1.0402	1.0342	-0.0060	55
Bitcoin, US	SD per B7	C, 5 BTC					
May 2020	9585	9610	8815	9570	9050	-520	8,738
June 2020	9655	9680	8900	9640	9165	-475	1,50
July 2020	9710	9710	8930	9685	9105	-580	130

# **Consumption Asset**



#### **Consumption Asset** #Question 23



- The spot price of silver is \$25 per ounce.
- The storage costs are \$0.24 per ounce per year payable quarterly in advance.
- Assuming that interest rates are 5% per annum for all maturities,
- calculate the futures price of silver for delivery in 9 months.

#### **Consumption Asset** #Question 35



- The spot price of oil is \$50 per barrel and
- the cost of storing a barrel of oil for one year is \$3, payable at the end of the year.
- The risk-free interest rate is 5% per annum continuously compounded.
- What is an upper bound for the one-year futures price of oil?

#### **Consumption Asset** #Question 14



• Explain carefully why the futures price of gold can be calculated from its spot price and other observable variables whereas the futures price of copper cannot.

# **Cost of Carry**



### Cost of Carry #Question 31



- What is the cost of carry for:
  - (a) a non-dividend-paying stock
  - (b) a stock index
  - (c) a commodity with storage costs
  - (d) a foreign currency.

## **Bankers**



#### Bankers #Question 36



- A company that is uncertain about the exact date when it will pay or receive a foreign currency may try to negotiate with its bank a forward contract that specifies a period during which delivery can be made.
- The company wants to reserve the right to choose the exact delivery date to fit in with its own cash flows.
- Put yourself in the position of the bank. How would you price the product that the company wants?

#### Bankers #Question 37



- A company enters into a forward contract with a bank to sell a foreign currency for  $K_1$  at time  $T_1$ .
- The exchange rate at time  $T_1$  proves to be  $S_1$  (> $K_1$ ).
- The company asks the bank if it can roll the contract forward until time  $T_2$  (>  $T_1$ ) rather than settle at time  $T_1$ .
- The bank agrees to a new delivery price,  $K_2$ .
- Explain how  $K_2$  should be calculated.

### Show that ...



#### Show that ... #Question 24



- Suppose that  $F_1$  and  $F_2$  are two futures contracts on the same commodity with times to maturity,  $t_1$  and  $t_2$ , where  $t_2 > t_1$
- Prove that  $F_2 \leq F_1 e^{r(t_2-t_1)}$
- where r is the interest rate (assumed constant), and there are no storage costs.
- For the purposes of this problem, assume that a futures contract is the same as a forward contract.

## **Arithmetic vs. Geometric Average**



#### **Arithmetic vs. Geometric Average** #Question 29



- The Value Line Index is designed to reflect changes in the value of a portfolio of over 1,600 equally weighted stocks.
- Prior to March 9, 1988, the change in the index from one day to the next was calculated as the geometric average of the changes in the prices of the stocks underlying the index.
- In these circumstances, does equation (5.8) correctly relate the futures price of the index to its cash price?
- equation (5.8):  $F_0 = S_0 e^{(r-q)T}$
- If not, does the equation overstate or understate the futures price?

# **Effects of Daily Settlement**



## **Effects of Daily Settlement**



(Skip) Question 5.25, 5.30

# **Futures Price and Expected Spot Price**



# **Futures Price and Expected Spot Price**



(Skip) Question 5.26, 5.16, 5.28