## Introduction



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# Compounding



### **Compounding** #Question 34



- An interest rate is quoted as 5% per annum with semiannual compounding.
- What is the equivalent rate with
  - (a) annual compounding,
  - (b) monthly compounding, and
  - (c) continuous compounding?

• (Skip) Question 4.11, 4.18, 4.31

### **Compounding** #Question 13



- An investor receives \$1,100 in one year in return for an investment of \$1,000 now.
- Calculate the percentage return per annum with:
  - (a) Annual compounding
  - (b) Semiannual compounding
  - (c) Monthly compounding
  - (d) Continuous compounding

## **Compounding** #Question 19



- A deposit account pays 4% per annum with continuous compounding,
- but interest is actually paid quarterly.
- How much interest will be paid each quarter on a \$10,000 deposit?

# **Zero Rates / Bond Pricing**





- A 3-year bond provides a coupon of 8% semiannually and has a cash price of 104.
- What is the bond's yield?



- A 5-year bond provides a coupon of 5% per annum, payable semiannually.
- Its price is 104.
- What is the bond's yield?
- You may find Excel's Solver useful.



- A 5-year bond with a yield of 7% (continuously compounded) pays an 8% coupon at the end of each year.
- (a) What is the bond's price?
- (b) What is the bond's duration?
- (c) Use the duration to calculate the effect on the bond's price of a 0.2% decrease in its yield.
- (d) Recalculate the bond's price on the basis of a 6.8% per annum yield and verify that the result is in agreement with your answer to (c).



- The cash prices of 6-month and 1-year Treasury bills are 94.0 and 89.0.
- A 1.5-year Treasury bond that will pay coupons of \$4 every 6 months currently sells for \$94.84.
- A 2-year Treasury bond that will pay coupons of \$5 every 6 months currently sells for \$97.12.
- Calculate the 6-month, 1-year, 1.5-year, and 2-year Treasury zero rates.



- The 6-month and 1-year zero rates are both 5% per annum.
- For a bond that has a life of 18 months and pays a coupon of 4% per annum (with semiannual payments and one having just been made),
- the yield is 5.2% per annum.
- What is the bond's price?
- What is the 18-month zero rate? All rates are quoted with semiannual compounding.



- Suppose that 6-month, 12-month, 18-month, 24-month, and 30-month zero rates are, respectively, 4%, 4.2%, 4.4%, 4.6%, and 4.8% per annum, with continuous compounding.
- Estimate the cash price of a bond with a face value of 100 that will mature in 30 months and pay a coupon of 4% per annum semiannually.



- The table below gives Treasury zero rates and cash flows on a Treasury bond.
- Zero rates are continuously compounded.
- (a) What is the bond's theoretical price?
- (b) What is the bond's yield assuming it sells for its theoretical price?

Maturity (years)	Zero rate	Coupon payment	Principal
0.5	2.0%	\$20	
1.0	2.3%	\$20	
1.5	2.7%	\$20	
2.0	3.2%	\$20	\$1,000



- Suppose that the 6-month, 12-month, 18-month, and 24-month zero rates are 5%, 6%, 6.5%, and 7%, respectively.
- What is the 2-year par yield?



- A 10-year 8% coupon bond currently sells for \$90.
- A 10-year 4% coupon bond currently sells for \$80.
- What is the 10-year zero rate?
- (*Hint*: Consider taking a long position in two of the 4% coupon bonds and a short position in one of the 8% coupon bonds.)

# **Forward Rates / FRA**



#### Forward Rates / FRA #Question 14



Suppose that risk-free zero interest rates with continuous compounding are as follows:

Maturity (months)	Rate (% per annum)	
3	3.0	
6	3.2	
9	3.4	
12	3.5	
15	3.6	
18	3.7	

- Calculate forward interest rates for the second, third, fourth, fifth, and sixth quarters.
- (Skip) Question 4.23

#### Forward Rates / FRA #Question 15



- Assuming that SOFR rates are as in Problem 4.14,
- what is the value of an FRA where
- the holder will pay SOFR and receive 4.5% (quarterly compounded) for
- a three-month period starting in one year on a principal of \$1,000,000?

Maturity (months)	Rate (% per annum)	
3	3.0	
6	3.2	
9	3.4	
12	3.5	
15	3.6	
18	3.7	

• (Skip) Question 4.30 Swap

# **Duration**



#### **Duration** #Question 17



- What does duration tell you about the sensitivity of a bond portfolio to interest rates?
- What are the limitations of the duration measure?

#### **Duration** #Question 28



- A 5-year bond with a yield of 7% (continuously compounded) pays an 8% coupon at the end of each year.
- (a) What is the bond's price?
- (b) What is the bond's duration?
- (c) Use the duration to calculate the effect on the bond's price of a 0.2% decrease in its yield.
- (d) Recalculate the bond's price on the basis of a 6.8% per annum yield and verify that the result is in agreement with your answer to (c).

#### **Duration** #Question 35



- Portfolio A consists of a 1-year zero-coupon bond with a face value of \$2,000 and a 10-year zero-coupon bond with a face value of \$6,000.
- Portfolio B consists of a 5.95-year zero-coupon bond with a face value of \$5,000.
- The current yield on all bonds is 10% per annum.
- (a) Show that both portfolios have the same duration.
- (b) Show that the percentage changes in the values of the two portfolios for a 0.1% per annum increase in yields are the same.
- (c) What are the percentage changes in the values of the two portfolios for a 5% per annum increase in yields?

(Skip) Question 4.36 DerivaGem

## **Term Structure**



#### **Term Structure** #Question 16



- The term structure of interest rates is upward-sloping.
- Put the following in order of magnitude:
  - (a) The 5-year zero rate
  - (b) The yield on a 5-year coupon-bearing bond
  - (c) The forward rate corresponding to the period between 4.75 and 5 years in the future
- What is the answer when the term structure of interest rates is downward sloping?

#### **Term Structure** #Question 25



- Explain carefully why liquidity preference theory is consistent with
- the observation that the term structure of interest rates tends to be upward-sloping more often than it is downward-sloping.

#### **Term Structure** #Question 26



- When the zero curve is upward-sloping,
  the zero rate for a particular maturity is greater than the par yield for that maturity.
- When the zero-curve is downward-sloping, the reverse is true.
- Explain why this is so.

## Miscellaneous #Question 27



• Why does a loan in the repo market involve very little credit risk?