1-Step Binomial Tree

1-Step Binomial Tree #Question 13.14

- A stock price is currently \$50.
- It is known that at the end of 2 months it will be either \$53 or \$48.
- The risk-free interest rate is 10% per annum with continuous compounding.
- What is the value of a 2-month European call option with a strike price of \$49?
- Use no-arbitrage arguments.

1-Step Binomial Tree #Question 13.15

- A stock price is currently \$80.
- It is known that at the end of 4 months it will be either \$75 or \$85.
- The risk-free interest rate is 5% per annum with continuous compounding.
- What is the value of a 4-month European put option with a strike price of \$80?
- Use no-arbitrage arguments.

Risk-Neutral Valuation

Risk-Neutral Valuation #Question 13.16

- A stock price is currently \$40.
- It is known that at the end of 3 months it will be either \$45 or \$35.
- The risk-free rate of interest with quarterly compounding is 8% per annum.
- Calculate the value of a 3-month European put option on the stock
- with an exercise price of \$40.
- Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answers.

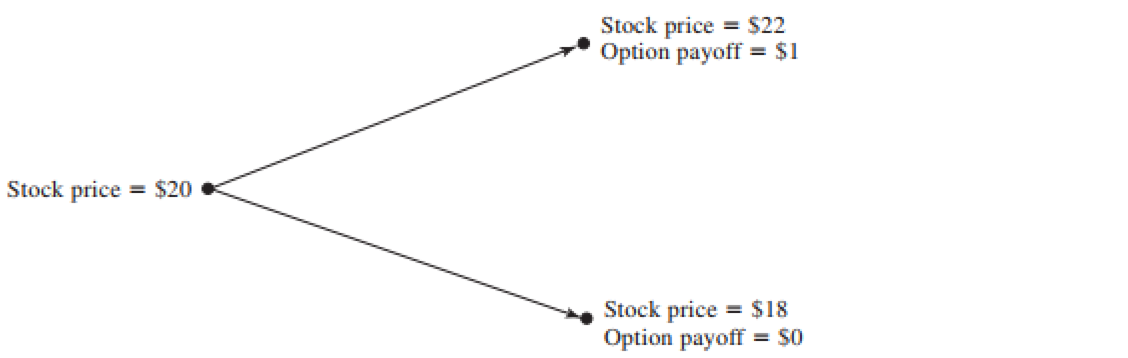
Risk-Neutral Valuation #Question 13.26

- A stock price is currently $50.
- It is known that at the end of 6 months it will be either $60 or $42.
- The risk-free rate of interest with continuous compounding is 12% per annum.
- Calculate the value of a 6-month European call option on the stock
- with an exercise price of $48.
- Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answers.

Risk-Neutral Valuation #Question 13.31

- Footnote 1 of this chapter shows that the correct discount rate to use for the real-world expected payoff in the case of the call option considered in Figure 13.1 is 55.96%. (Real-world expected return on the stock is 10%)
- Show that if the option is a put rather than a call the discount rate is -70.4%.
- Explain why the two real-world discount rates are so different.

Figure 13.1 Stock price movements for numerical example in Section 13.1.



- Footnote 1: "Since we know the correct value of the option is 0.545, we can deduce that the correct real-world discount rate is 55.96%. This is because 0.545 = 0.6266e^(-0.5596*3/12)."

More-than-1-Step Binomial Tree

More-than-1-Step Binomial Tree #Question 13.11

- A stock price is currently $100.
- Over each of the next two 6-month periods it is expected to go up by 10% or down by 10%.
- The risk-free interest rate is 8% per annum with continuous compounding.
- What is the value of a 1-year European call option with a strike price of $100?

More-than-1-Step Binomial Tree #Question 13.12

- For the situation considered in Problem 13.11,
- what is the value of a 1-year European put option with a strike price of $100?
- Verify that the European call and European put prices satisfy put–call parity.

More-than-1-Step Binomial Tree #Question 13.17

- A stock price is currently $50.
- Over each of the next two 3-month periods it is expected to go up by 6% or down by 5%.
- The risk-free interest rate is 5% per annum with continuous compounding.
- What is the value of a 6-month European call option with a strike price of $51?

Volatility and Binomial Tree

Volatility and Binomial Tree #Question 13.20

- Calculate u, d, and p when a binomial tree is constructed to value an option on a foreign currency.
- The tree step size is 1 month,
- the domestic interest rate is 5% per annum, the foreign interest rate is 8% per annum,
- and the volatility is 12% per annum.

Volatility and Binomial Tree #Question 13.24

- The current price of a non-dividend-paying biotech stock is $140 with a volatility of 25%.
- The risk-free rate is 4%. For a 3-month time step:
- (a) What is the percentage up movement?
- (b) What is the percentage down movement?
- (c) What is the probability of an up movement in a risk-neutral world?
- (d) What is the probability of a down movement in a risk-neutral world?
- Use a two-step tree to value a 6-month European call option and a 6-month European put option.
- In both cases the strike price is $150.

Volatility and Binomial Tree #Question 13.29

- Consider a European call option on a non-dividend-paying stock
- where the stock price is \$40, the strike price is \$40, the risk-free rate is 4% per annum,
- the volatility is 30% per annum, and the time to maturity is 6 months.
- (a) Calculate u, d, and p for a two-step tree.
- (b) Value the option using a two-step tree.
- (c) Verify that DerivaGem gives the same answer.
- (d) Use DerivaGem to value the option with 5, 50, 100, and 500 time steps.

Binomial Tree for Valuing Exotic Options

Binomial Tree for Valuing Exotic Options #Question 13.19

- A stock price is currently $25.
- It is known that at the end of 2 months it will be either $23 or $27.
- The risk-free interest rate is 10% per annum with continuous compounding.
- Suppose S_T is the stock price at the end of 2 months.
- What is the value of a derivative that pays off S_T^2 at this time?

Binomial Tree for Valuing Exotic Options #Question 13.18

- For the situation considered in Problem 13.17,
- what is the value of a 6-month European put option with a strike price of $51?
- Verify that the European call and European put prices satisfy put–call parity.
- If the put option were American, would it ever be optimal to exercise it early at any of the nodes on the tree?

Binomial Tree for Valuing Exotic Options #Question 13.22

- A stock index is currently 1,500.
- Its volatility is 18%.
- The risk-free rate is 4% per annum (continuously compounded) for all maturities and
- the dividend yield on the index is 2.5%.
- Calculate values for u, d, and p when a 6-month time step is used.
- What is the value a 12-month American put option with a strike price of 1,480
- given by a two-step binomial tree.

Binomial Tree for Valuing Exotic Options #Question 13.23

- The futures price of a commodity is \$90.
- Use a three-step tree to value
- (a) a 9-month American call option with strike price \$93 and
- (b) a 9-month American put option with strike price \$93.
- The volatility is 28% and
- the risk-free rate (all maturities) is 3% with continuous compounding.

Binomial Tree for Valuing Exotic Options #Question 13.27

- A stock price is currently $40.
- Over each of the next two 3-month periods it is expected to go up by 10% or down by 10%.
- The risk-free interest rate is 12% per annum with continuous compounding.
- (a) What is the value of a 6-month European put option with a strike price of $42?
- (b) What is the value of a 6-month American put option with a strike price of $42?

Binomial Tree for Valuing Exotic Options #Question 13.28

- Using a “trial-and-error” approach,
- estimate how high the strike price has to be in Problem 13.27 for it to be optimal to exercise the option immediately.

Binomial Tree for Valuing Exotic Options #Question 13.30

- Repeat Problem 13.29 for an American put option on a futures contract.
- The strike price and the futures price are $50,
- the risk-free rate is 10%, the time to maturity is 6 months, and the volatility is 40% per annum.

Dynamic Delta Hedging

Dynamic Delta Hedging #Question 13.21

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- The volatility of a non-dividend-paying stock whose price is $78, is 30%.
- The risk-free rate is 3% per annum (continuously compounded) for all maturities.
- Calculate values for u, d, and p when a 2-month time step is used.
- What is the value a 4-month European call option with a strike price of $80 given by a two-step binomial tree.
- Suppose a trader sells 1,000 options (10 contracts).
- What position in the stock is necessary to hedge the trader's position at the time of the trade?

Dynamic Delta Hedging #Question 13.13

- Consider a situation where stock price movements during the life of a European option are governed by a two-step binomial tree.
- Explain why it is not possible to set up a position in the stock and the option that remains riskless for the whole of the life of the option.

Dynamic Delta Hedging #Question 13.25

- In Problem 13.14, suppose a trader sells 10,000 European call options and the two-step tree describes the behavior of the stock.
- How many shares of the stock are needed to hedge the 6-month European call for the first and second 3-month period?
- For the second time period, consider both the case where the stock price moves up during the first period and the case where it moves down during the first period.