CS222 Homework 1

Stable Matching and Algorithm Analysis

Exercises for Algorithm Design and Analysis by Li Jiang, 2018 Autumn Semester

- 潘佳萌

- 516030910510

1 Because the greedy algorithm locally optimal solution, for the problem, we need to prove the solution is global optimal solution. Assume there are n boxes which is $b_1, b_2, ..., b_n$, and b_i's weight is w_i. For each truck, $\sum w \leq W$. $\forall i \leq j$, b_i is earlier than b_j. Because the use of the greedy algorithm, other algorithm will use more trucks than N. For k-1 trucks, if GA fits b_i boxes, and other algorithm fits b_j boxes where $i < j$, but in GA the kth truck should fit as more as possible, the GA is the best algorithm.

2 Greedy algorithm. From eastern side, if there is a house 4 miles to this side, then put a station 4-mile west, then remove all the house 4 miles to the station. Iterate this process until all houses are removes.
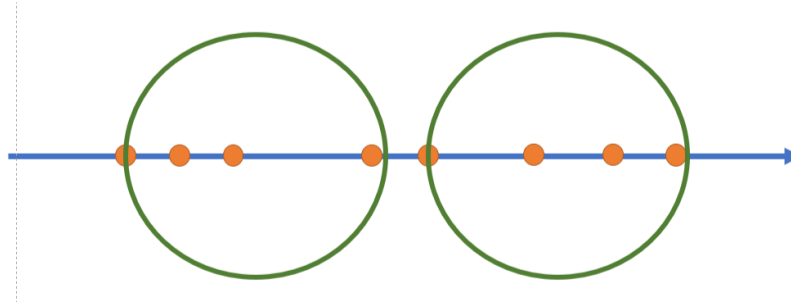

图 1: base station

3 (a) Represent T in a adjacency list, find all the edge connecting to v、w, this step is O(|V|), and compare them to (v,w), if they are greater than (v,w), then T is still the minimum spanning tree.

   (b) Find all the edge in cycle $v \rightarrow w \rightarrow w$, and remove the greatest one.

4 We can use the Havel-Hakimi Algorithm to solve this problem. Assume the degree sequence is S, $S = d_1, d_2, d_3, ..., d_n$. $d_i \geq d_{i+1}$

   a if any $d_i \geq n$ then fail;

   b if there is an odd number of odd degrees then fail;

   c if there is a $d_i \leq 0$ then fail;

   d if all $d_i = 0$ then report success;

   e reorder S into non-increasing order;

   f let $k = d_1$;

   g remove d_1 from S;

   h subtract 1 from the first k terms remaining of the new sequence;

   i go to step c;

Example:

图 2: S = 4,3,3,3,1
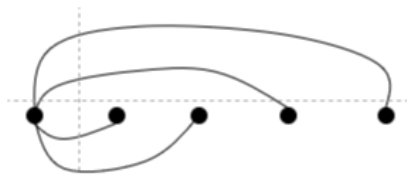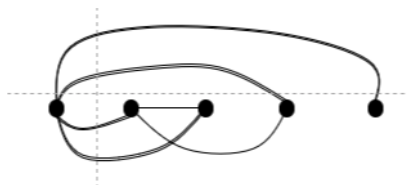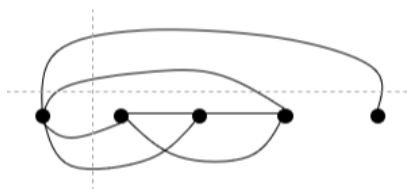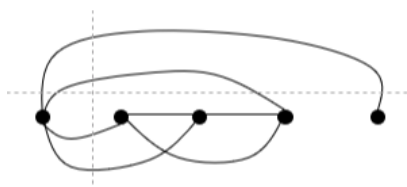
图 3: S = 2,2,2,0

图 4: S = 1,1,0

图 5: S = 0,0

图 6: Report Success