CS222 Homework 6

(NP and Computational Intractability)

Deadline: 2018-11-20 Tuesday 18:00

Exercises for Algorithm Design and Analysis by Li Jiang, 2018 Autumn Semester

1. For each of the two questions below, decide whether the answer is (i) "Yes," (ii) "No," or (iii) "Unknown, because it would resolve the question of whether P = NP." Give a brief explanation of your answer.
   (a) Let's define the decision version of the Interval Scheduling Problem from Chapter 4 as follows: Given a collection of intervals on a time-line, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k? Question: Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?
   (b) Question: Is it the case that Independent Set $\leq_P$ Interval Scheduling?

2. *PARTITION*: Given a finite set A and a size $s(a) \in \mathbb{Z}$ for each $a \in A$, is there a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)$?
   SUBSET SUM: Given a finite set A, a size $s(a) \in \mathbb{Z}$ for each $a \in A$ and an integer B, is there a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = B$?
   KNAPSACK: Given a finite set A, a size $s(a) \in \mathbb{Z}$ and a value $v(a) \in \mathbb{Z}$ for each $a \in A$ and integers B and K, is there a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) \leq B$ and $\sum_{a \in A'} v(a) \geq K$?
   (a) Prove *PARTITION* $\leq_p$ *SUBSET SUM*.
   (b) Prove *SUBSET SUM* $\leq_p$ *KNAPSACK*.

3. Since the 3-Dimensional Matching Problem is NP-complete, it is natural to expect that the corresponding 4-Dimensional Matching Problem is at least as hard. Let us define 4-Dimensional Matching as follows. Given sets W, X, Y, and Z, each of size n, and a collection C of ordered 4-tuples of the form w_i, x_j, y_k, z_l, do there exist n 4-tuples from C so that no two have an element in common?

   Prove that 4-Dimensional Matching is NP-complete.

4. You are given a directed graph G = (V, E) with weights w_e on its edges $e \in E$. The weights can be negative or positive. The *Zero-Weight-Cycle Problem* is to decide if there is a simple cycle in G so that the sum of the edge weights on this cycle is exactly 0.

   Prove that this problem is NP-complete.