CS222 Homework 5

Network Flow

Exercises for Algorithm Design and Analysis by Li Jiang, 2018 Autumn Semester

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1 Problem 1

1.1 Question

Figure 1 shows a flow network on which an s-t flow has been computed. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge. (Edges without boxed numbers have no flow being sent on them.)

(a) What is the value of this flow? Is this a maximum (s,t) flow in this graph?

(b) Find a minimum s-t cut in the flow network pictured in Figure 1, and also say what its capacity is.

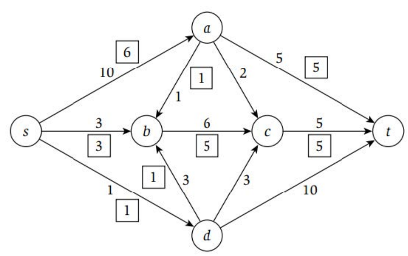

图 1: 1

1.2 Answer

(a) The value of this flow is 10. This is not the maximum flow, because s-a-c-b-d-t is a augmenting-path.

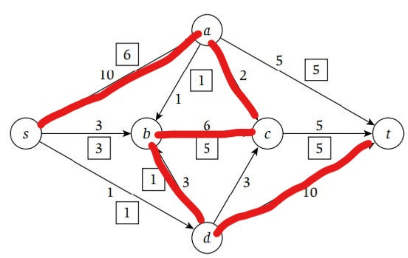

图 2: 2

(b) According to Maximum Flow, Minimum Cut Theorem, The maximum flow between vertices v_i and v_j in a graph G is exactly the weight of the smallest set of edges to disconnect G with v_i and v_j in different components. After adding the augmenting-path in a, the flow is as below:

There is no other augmenting-path, so the minimum cut is ({s, a, b, c}, {d, t}), the capacity is 11.

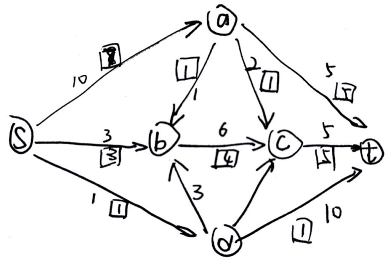

图 3: 3

2 Problem 2

2.1 Question

Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

(a) Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have f(e) = c_e)

(b) Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e; and let (A;B) be a minimum s-t cut with respect to these capacities fce : e 2 Eg. Now suppose we add 1 to every capacity; then (A;B) is still a minimum s-t cut with respect to these new capacities $\{1 + c_e : e \in E\}$.

2.2 Answer

(a) False. Counterexample:

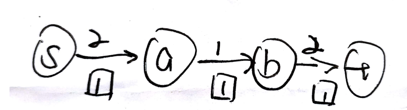

图 4: 4

(b) False. Counterexample:

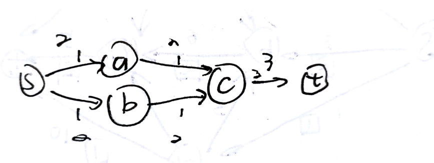

图 5: 5

A=s, B=G-A, (A,B) is the minimum cut of this flow. When every capacity add 1, capacity of A is 4, but the capacity of t is 3, (A,B) is not the minimum cut of the new flow.