

CS222 Homework 3

Exercises for Algorithm Design and Analysis by Li Jiang, 2018 Autumn Semester

1. Assume that all elements of $A[1..n]$ are distinct and sorted, and x is in A . Each element of A is equally likely to be in any position in the array. Please give the average case analysis of Algorithm BinarySearch ($n = 2^k$, $k \in \mathbb{N}$). The exact expression for the average number of comparisons $T(n)$ is required.
2. Given an integer array $A[1..n]$ and two integers $lower \leq upper$, design an algorithm using divide-and-conquer method to count the number of ranges (i, j) ($1 \leq i \leq j \leq n$) satisfying

$$lower \leq \sum_{k=i}^j A[k] \leq upper.$$

Example:

Given $A = [1, -1, 2]$, $lower = 1$, $upper = 2$, return 4.

The resulting four ranges are $(1, 1)$, $(3, 3)$, $(2, 3)$ and $(1, 3)$.

- (a) Complete the implementation using pseudo code.
 - (b) Write a recurrence for the running time of the algorithm.
3. Consider an n -node complete binary tree T , where $n = 2^d - 1$ for some d . Each node v of T is labeled with a real number x_v . You may assume that the real numbers labeling the nodes are all distinct. A node v of T is a local minimum if the label x_v is less than the label x_w for all nodes w that are joined to v by an edge.

You are given such a complete binary tree T , but the labeling is only specified in the following implicit way: for each node v , you can determine the value x_v by probing the node v . Show how to find a local minimum of T using only $O(\log n)$ probes to the nodes of T .

4. We've mentioned fast matrix multiplication theory, Principal components and Singular Value Decomposition in the class(05DivideAndConquerII.pdf, page 16-25). Now survey and read the corresponding paper about DNN weight matrix approximation and acceleration(e.g. low-rank factorization). Is there any other method to accomplish the same goal. Describe your thinking and ideas. Write an essay about one page.