CS222 Homework 1

Stable Matching and Algorithm Analysis

Exercises for Algorithm Design and Analysis by Li Jiang, 2018 Autumn Semester

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1 True. Consider there are $(m, w1)$ and $(m1, w)$ in S, because m is prefer w and w is prefer m, S is not stable.

2 as below

表 1: women

$women$	1^{st}	2^{nd}	3^{rd}
w	m''	$m'(fake)$	$m(fake)$
w'	m	m'	m''
w''	m	m'	m''

表 2: men

men	1^{st}	2^{nd}	3^{rd}
m	w	w'	w''
m'	w	w''	w'
m''	w'	w	w''

表 3: result

	$fake$	$real$
w	m''	m
w'	m	m''
w''	m'	m'

3 The problem is similar to stable marriage problem, and we can use Gale-Shapley algorithm to solve the problem. For wires, their perferance is the distance to source. Input wires need switching early and output wires need the data stream to be switched to it late.

Proof: There is A valid switching of the data streams.
1. Suppose, for sake of contradiction, that $Iutput_1$ is not matched upon termination of GS algorithm.
2. Then exist $Onput_1$, is not matched upon termination.
3. $Onput_1$ should be never 'proposed' to because it doesn't be matched.
4. But, $Iutput_1$ is proposed to every output, since it ends up unmatched.

4 $n^{log_2 n} = 2^{log_2 n^2}$
Result:$g1 < g5 < g4 < g3 < g2 < g7 < g6$

5 a. False. $g(n) = 1, f(n) = 2$, then $log(g(n)) = 0 < logf(n)$
b. Fasle. $g(n) = log(n), f(n) = 2log(n)$, then $c * 2^{g(n)} = cn < 2^{f(n)} = cn^2$
c. True. $f(n) \leq cg(n) \rightarrow f(n)^2 \leq (cg(n))^2$

6 a. $(n-1)*(n-1)*(j-i+1)$ and $1 < j-i+1 < n \rightarrow f(n) = n^3$
b. for $i < \frac{n}{4}$, $j > \frac{3n}{4}$, $t > (\frac{3n}{4} - \frac{n}{4}) * \frac{n}{4} * \frac{n}{4} = \frac{n^3}{32} \rightarrow \Omega(n^3)$
c. $g(n) = n^2$, the unnecessary steps is computing B(i,j) through $\sum_i^j A[k]$ rather than using the relationship between $B(i,j)\&B(i,j-1)$
matlab code

```
for i=1:n
   B(i,i+1)=A(i)+A(i+1);
end
for k=2:n
   for i=1:n-k
      j=i+k;
      B(i,j)=B(i,j-1)+A(j);
   end
end
```