

CS222 Homework 2

Greedy Algorithms

Deadline: 2018-10-7 Sunday 24:00

Exercises for Algorithm Design and Analysis by Li Jiang, 2018 Autumn Semester

1. You are consulting for a trucking company that does a large amount of business shipping packages between New York and Boston. The volume is high enough that they have to send a number of trucks each day between the two locations. Trucks have a fixed limit W on the maximum amount of weight they are allowed to carry. Boxes arrive at the New York station one by one, and each package i has a weight w_i . The trucking station is quite small, so at most one truck can be at the station at any time. Company policy requires that boxes are shipped in the order they arrive; otherwise, a customer might get upset upon seeing a box that arrived after his make it to Boston faster. At the moment, the company is using a simple greedy algorithm for packing: they pack boxes in the order they arrive, and whenever the next box does not fit, they send the truck on its way.

But they wonder if they might be using too many trucks, and they want your opinion on whether the situation can be improved. Here is how they are thinking. Maybe one could decrease the number of trucks needed by sometimes sending off a truck that was less full, and in this way allow the next few trucks to be better packed.

Prove that, for a given set of boxes with specified weights, the greedy algorithm currently in use actually minimizes the number of trucks that are needed. Your proof should follow the type of analysis we used for the Interval Scheduling Problem: it should establish the optimality of this greedy packing algorithm by identifying a measure under which it "stays ahead" of all other solutions.

2. Let's consider a long, quiet country road with houses scattered very sparsely along it. (We can picture the road as a long line segment, with an eastern endpoint and a western endpoint.) Further, let's suppose that despite the bucolic setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road, so that every house is within four miles of one of the base stations.

Give an efficient algorithm that achieves this goal, using as few base stations as possible.

3. Let $G = (V, E)$ be an (undirected) graph with costs $c_e \geq 0$ on the edges $e \in E$. Assume you are given a minimum-cost spanning tree T in G . Now assume that a new edge is added to G , connecting two nodes $v, w \in V$ with cost c .
 - (a) Give an efficient algorithm to test if T remains the minimum-cost spanning tree with the new edge added to G (but not to the tree T). Make your algorithm run in time $O(|E|)$. Can you do it in $O(|V|)$ time? Please note any assumptions you make about what data structure is used to represent the tree T and the graph G .
 - (b) Suppose T is no longer the minimum-cost spanning tree. Give a linear-time algorithm (time $O(|E|)$) to update the tree T to the new minimum-cost spanning tree.

4. Given a list of n natural numbers d_1, d_2, \dots, d_n , show how to decide in polynomial time whether there exists an undirected graph $G = (V, E)$ whose node degrees are precisely the numbers d_1, d_2, \dots, d_n . (That is, if $V = v_1, v_2, \dots, v_n$, then the degree of v_i should be exactly d_i .) G should not contain multiple edges between the same pair of nodes, or "loop" edges with both endpoints equal to the same node.

5. As mentioned in the class, greedy algorithm is also used for neural network pruning. Please read related papers and write an essay about two pages to show your thinking. You'd better create a new LaTeX project to write this essay. Here are some papers that you can refer to and you can also find related papers by yourself.
 1. Yoon et al. "Combined Group and Exclusive Sparsity for Deep Neural Networks". ICML2017
 2. Liu et al. "Learning efficient convolutional networks through network slimming". ICCV2017
 3. He et al. "Channel Pruning for Accelerating Very Deep Neural Networks". ICCV 2017
 4. Sun et al. "meProp: Sparsified Back Propagation for Accelerated Deep Learning with Reduced Overfitting". ICML2017