

KCL @ Vy:  $i_1 = i_4 + i_2 + i_3$

$$Y_{R_1}(V_{in} - V_y) = Y_{C_1}(V_y - V_{out}) + Y_{R_2}V_y + Y_{C_3}(V_y - V_x)$$

$$\Rightarrow V_y = \frac{Y_{R_1}V_{in} + Y_{C_4}V_{out}}{Y_{R_1} + Y_{R_2} + Y_{C_4} + Y_{C_3}} \quad (1)$$

KCL @ Vx:  $i_3 = i_5 \Rightarrow Y_{C_3}(V_y - V_x) = Y_{R_5}(V_x - V_{out})$

$$\Rightarrow V_y = -\frac{Y_{R_5}}{Y_{C_3}}V_{out} \quad (2) \quad \text{Set RHSs of (1) \& (2) equal, eliminate } V_y.$$

$$\Rightarrow Y_{R_1}V_{in} + Y_{C_4}V_{out} = -\left(\frac{Y_{R_5}}{Y_{C_3}}\right)[Y_{R_1} + Y_{R_2} + Y_{C_4} + Y_{C_3}]V_{out}$$

admittance  $= Y_Z$

$$Y_{R_1} = \frac{1}{R_1}, \quad Y_{R_2} = \frac{1}{R_2}, \quad Y_{C_3} = \frac{1}{ZC_3}, \quad Y_{C_4} = \frac{1}{ZC_4}, \quad Y_{R_5} = \frac{1}{R_5} = G_5$$

$$\Rightarrow ZG_1C_3V_{in} = -[ZC_3C_4 + ZG_5(C_3 + C_4) + (G_1 + G_2)G_5]V_{out}$$

$$\begin{aligned}
 H(\omega) &= \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{-\omega G_1 C_3}{\omega^2 C_3 C_4 + \omega G_5 (C_3 + C_4) + G_5 (G_1 + G_2)} \quad (\text{cancel } C_3 \text{ out of den}) \\
 &= \frac{-\omega G_1 C_3}{\omega^2 C_3 C_4 + \frac{\omega G_5 (C_3 + C_4)}{C_3 C_4} + \frac{G_5 (G_1 + G_2)}{C_3 C_4}} \\
 &= \frac{-\omega \left( \frac{+}{R_1 C_4} \right)}{\omega^2 + \omega \left( \frac{C_3 + C_4}{R_5 C_3 C_4} \right) + \frac{(1/R_1 + 1/R_2)}{R_5 C_3 C_4}}
 \end{aligned}$$

$$H(\omega) = \frac{-\omega_x \omega}{\omega^2 + 2 \zeta \omega_0 \omega + \omega_0^2} \quad \omega_x = \omega_0$$

$$= \frac{-\omega \omega_0 \omega}{\omega^2 + \frac{\omega_0}{Q} \omega + \omega_0^2}$$

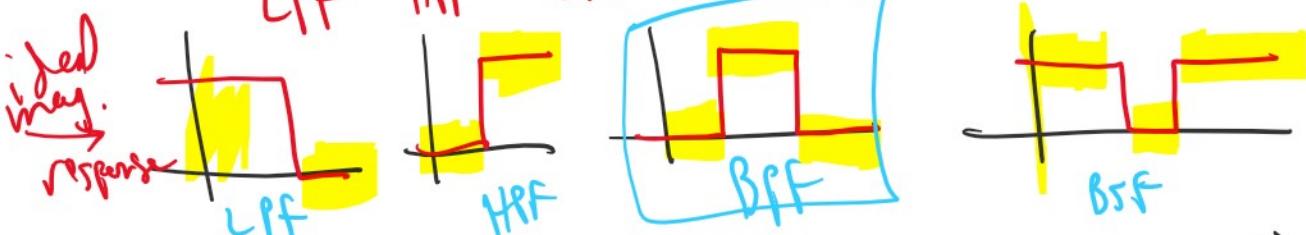
$\omega_0$  = natural freq.  
(resonant)

$\zeta$  = damping coefficient  
factor

$Q = \frac{1}{2\zeta}$  = quality factor

What kind of filter will this be?

LPF → HF → BPF → BSF (at there are others)



Consider the 2 extremes:  $\omega \rightarrow 0$  (DC) &  $\omega \rightarrow \infty$  (VHF)

$$\lim_{\omega \rightarrow 0} H(\omega) = \frac{0}{\omega_0^2} = 0 \quad \parallel \quad \lim_{\omega \rightarrow \infty} \frac{-\omega \omega_0 \omega}{\omega^2 + \frac{\omega_0}{Q} \omega + \omega_0^2}$$

