This exam is to be completed *individually*. Students must be logged in to Zoom with their cameras on and be *in camera viewsight for the entire exam* in order to be eligible for any credit, partial or otherwise. Further, I expect that your solutions, *whether correct or incorrect*, be unique to you and to not look like anyone else's solutions – I *will* be checking all exams for evidence of cheating!

Work out your solutions to the exam and upload them as ONE PDF FILE to the Canvas submission portal for the Final Exam by the due date and time listed on Canvas. **No late submissions will be accepted**.

Scan your solutions and create a single PDF file using any method you prefer (e.g., with your phone, scanner, etc.), however, BEFORE you upload your exam solutions as ONE PDF FILE to Canvas, double-check your ONE PDF FILE to make sure that you are uploading the correct file and verify that your file is easy to read, the pages are in the proper order, and all pages are oriented upright. You will only have one upload attempt, so **if you upload the incorrect file or I cannot make any sense out of it, you will receive zero credit, no exceptions**.

Box your solutions so that they are easy for me to find. Any plots/sketches you produce should contain key numerical values and be drawn with enough clarity so that one has a reasonable chance of attempting to understand your work and what the key takeaways are. **Expect zero credit for not showing your work**.

This exam has 4 problems. All problems are weighted equally.

Students may not discuss the contents of this exam with anyone else except the instructor for this course. By "discuss," this also includes, e.g., emailing your classmate and writing "I figured it out! Give me a virtual high five!" – I don't want you to discuss *any* aspect of this exam with anyone else, either another student in the class or otherwise, except for me, *period*.

This exam is open notes and you may refer to the course Canvas page; **looking online for help otherwise is not allowed**. The use of a calculator is, *of course*, allowed, ***but the use of a circuit simulator, such as LTSpice, is not allowed***.

Students who have received any type of unauthorized assistance or who have copied answers from another student, *whether the answers are correct or incorrect*, will receive an automatic F grade in the course.

Good luck!

1. Consider the circuit shown in Figure 1 below.

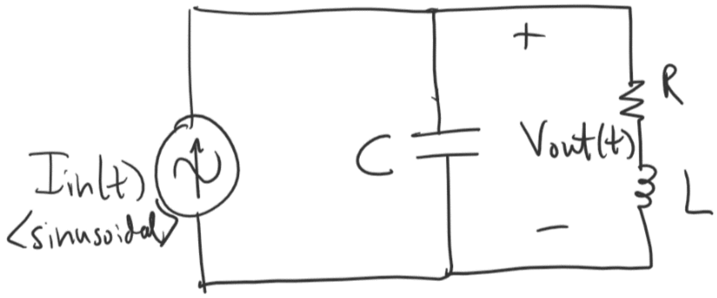

Figure 1. Circuit for Problem 1.

For an input current of the form $\mathrm{I_{in}}(t) = \mathrm{A}\cos(2\pi f_0 t),$ the output voltage will be of the form $\mathrm{V_{out}}(t) = \mathrm{B}\cos(2\pi f_0 t + \theta)$.

If **R = 5 Ω**, **L = 1 mH**, $\boldsymbol{f_0}$ **= 60 Hz [i.e., $\boldsymbol{\omega_0 = 2\pi(60)}$ rps]**, and **θ = 3°**, determine the numerical value of capacitor C.

Note that you are being asked to compute the (*positive-valued*) capacitance (*with units of F*), not an impedance. Note that the phase angle θ is positive-valued. The numerical values of A and B are not given and are not needed to solve this problem.

Show your work, and list your numerical answer in proper engineering notation format with appropriate units and two decimal places of precision.

Do not show your work in this exam booklet

Do not show your work in this exam booklet

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2. Consider the circuit shown in Figure 2 below, where V_{in} is the input voltage, V_{out} is the output voltage, and the component values are **R = 12 kΩ**, **L = 7 mH**, and **C = 12 nF**.

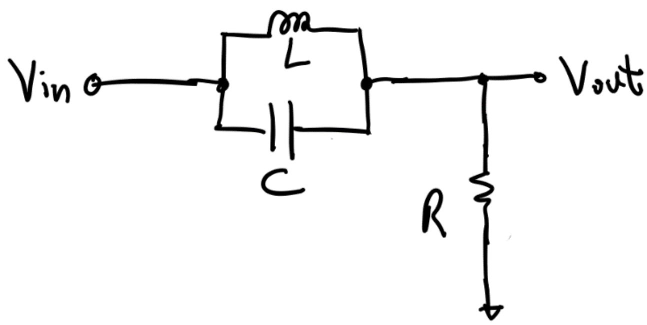

Figure 2. Circuit for Problem 2.

The circuit of Figure 2 has transfer function $H(s)$ of the form

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{\left(s/\omega_0\right)^2+1}{\left(s/\omega_0\right)^2+\frac{1}{Q}\left(s/\omega_0\right)+1}.$$

Note that the transfer function above is given in proper Bode approximation form.

Determine the numerical value of the (*unitless*) quality factor Q. Do not represent your numerical Q value in dB; just list your numerical Q value as a unitless quantity.

To solve for the numerical quality factor Q, you should first analyze the circuit of Figure 2 above, derive its transfer function $H(s)$, put the transfer function into proper Bode approximation form, and then from there you should be able to derive an expression for Q in terms of the circuit components R and/or L and/or C. Finally, you should then be able to compute the numerical quality factor Q. Note that Q should be positive-valued.

Show your work, and make it really easy for me to find your (*unitless*) numerical quality factor Q value represented <u>with two decimal places of precision</u>.

<u>*Do not show your work in this exam booklet*</u>

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3. Consider the circuit shown in Figure 3 below, constructed with ideal opamps.

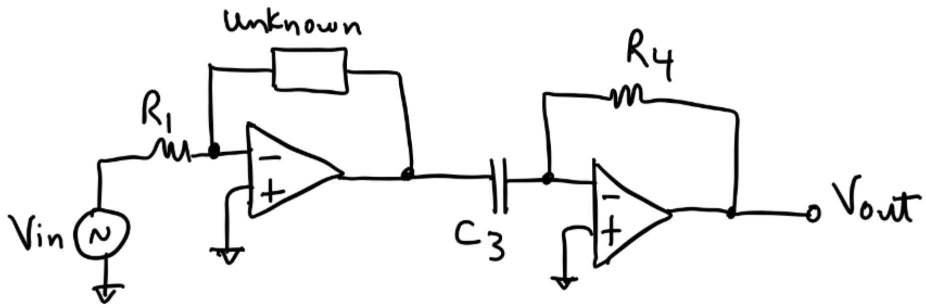

Figure 3. Circuit for Problem 3.

Here, **R_1 = 5 kΩ**, **C_3 = 800 nF**, and **R_4 = 15 kΩ**. Additionally, the circuit of Figure 3 contains an unknown element (*R*, *L*, *or C*), which you are to determine below.

<u>The following measured data were obtained from the circuit of Figure 3:</u>

- For $V_{in}(t) = 2\cos(\omega_1 t + 20^\circ)$ V at ω_1 = 5 krps, $V_{out}(t) = 12\cos(\omega_1 t + 200^\circ)$ V.
- For $V_{in}(t) = 3\cos(\omega_2 t + 15^\circ)$ V at ω_2 = 10 krps, $V_{out}(t) = 72\cos(\omega_2 t + 195^\circ)$ V.

Determine **i)** whether the unknown component is a resistor, capacitor, or an inductor, and **ii)** compute the numerical value of the unknown component.

<u>**To be very clear:**</u> if you determine that the unknown component is a resistor, then state the (*positive-valued*) numerical resistance in ohms; else if you determine that the unknown component is a capacitor, then state the (*positive-valued*) numerical capacitance in farads; else if you determine that the unknown component is an inductor, then state the (*positive-valued*) numerical inductance in henrys; *I am **not** looking for an impedance here*.

You should list just one numerical component value (*R*, *L*, *or C*) as your answer please. Make it really easy for me to determine which kind of component (*R*, *L*, *or C*) you think the unknown component is, and make it really easy for me to find your numerical component value.

List your numerical answer in proper engineering notation format with appropriate units and two decimal places of precision. *Show your work, and briefly explain how you determined what the unknown component (R, L, or C) is*.

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4. Consider the circuit shown below in Figure 4, constructed with an ideal voltage source of 5 V, an ideal switch, **an inductor L = 200 nH**, and a resistor R.

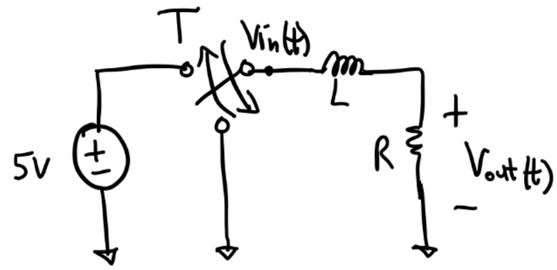


Figure 4. Circuit for Problem 4.

The ideal switch in the circuit above is toggled every T = 5 ns to create the voltage input $V_{in}(t)$ data shown in the top panel of Figure 5 below. The corresponding output voltage $V_{out}(t)$ data is shown in the bottom panel of Figure 5 below.

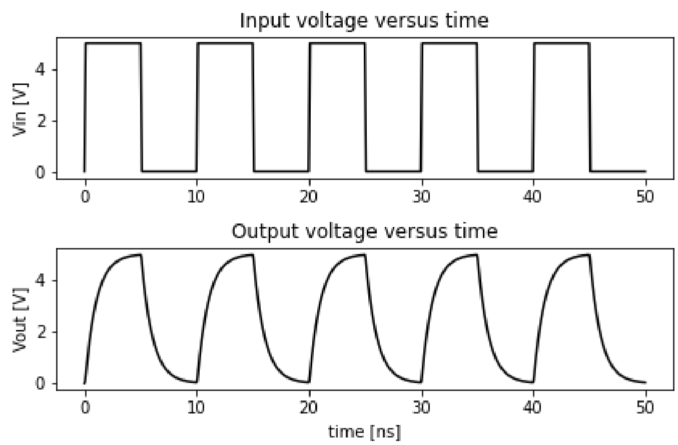


Figure 5. Input and output voltage data for Problem 4. *The horizontal axis is time in ns.*

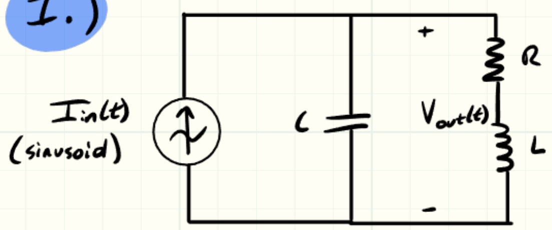
As shown in the bottom panel of Figure 5 above, the output voltage settles out to *approximately* **5 V** just before the input voltage is switched to 0 V, and the output voltage settles out to *approximately* **0 V** just before the input voltage is switched to 5 V.

Determine a plausible and reasonable numerical **lower-bound** for the resistor R value (*in units of* Ω), such that if you were to use a resistor value **smaller** than your stated **lower-bound**, the output would not achieve the same settling performance described above. *Just as your numerical answer will be incorrect if it is **too small**, a numerical answer which is arbitrarily **too large** will also be considered incorrect; there actually is one correct answer to this problem. Thus I suggest to use sound reasoning to arrive at your result.*

Show your work, and briefly and coherently explain your solution procedure. List your numerical **lower-bound** resistance value in proper engineering notation format with appropriate units and two decimal places of precision.

Austin Riha | Final Exam | Problem 1

1.)



$I_{in}(t) = A\cos(2\pi f_0 t)$

$V_0(t) = B\cos(2\pi f_0 t + \theta)$

$R = 5\Omega \quad L = 1mH$

$f_0 = 60Hz \quad \theta = 3°$

$\omega_0 = 2\pi f_0 = 2\pi(60Hz)$

$\omega_0 = 376.99 rps$

$Z_{RL} = Z_R + Z_L$

$Z_{RL} = 5\Omega + j(376.99)(1mH)$

$Z_{RL} = 5 + 0.376j$

$Z_C = \frac{1}{j\omega C}$

$Z_C = -j\frac{1}{(376.99)C} \qquad Y = \frac{1}{Z}$

$\frac{1}{Z_{eq}} = \frac{1}{Z_{RL}} + \frac{1}{Z_C}$

$\frac{1}{Z_{eq}} = \frac{1}{5 + 0.376j} + \frac{1}{376.99Cj}$

$Y_{RL} = 0.198 - 0.015j$

$Y_C = 376.99Cj$

$Y_{eq} = Y_{RL} + Y_C$

$Y_{eq} = (0.198 - 0.015j) + (376.99Cj)$

$\tan(\theta) = \frac{Imag}{Real} \rightarrow \tan(-3°)$

$\tan(-3°) = \frac{376.99C - 0.015}{0.198}$

$-0.0524 = \frac{376.99C - 0.015}{0.198}$

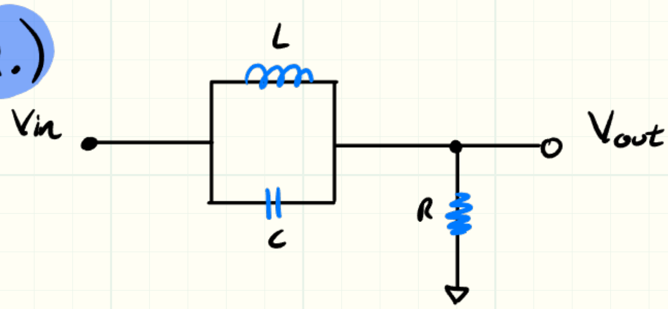
$-0.01 = 376.99C - 0.015$

$0.005 = 376.99C$

$\boxed{C = 13.262\mu F}$

Austin Riha | Final Exam | Problem 2

2.)



$R = 12k\Omega \quad L = 7mH$

$C = 12nF$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\left(\frac{s}{\omega_0}\right)^2 + 1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + 1}$$

$$Z_R = R \quad Z_L = sL \quad Z_C = \frac{1}{sC}$$

$$Z_{LC} = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{\cancel{s}L \cdot \frac{1}{\cancel{s}C}}{sL + \frac{1}{sC}} = \frac{L/C}{sL + 1/sC} \cdot \frac{sC}{sC}$$

$$Z_{LC} = \frac{\frac{Ls\cancel{C}}{\cancel{C}}}{s^2LC + 1} = \frac{Ls}{s^2LC+1}$$

$$V_{out} = V_{in}\left(\frac{Z_R}{Z_{LC} + Z_R}\right)$$

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{R}{Z_{LC} + R}$$

$$H(s) = \frac{R}{\left(\frac{Ls}{s^2LC+1}\right) + R} \cdot \frac{s^2LC+1}{s^2LC+1}$$

$$H(s) = \frac{Rs^2LC + R}{Ls + Rs^2LC + R} \cdot \frac{1}{RLC}$$

$$H(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC}$$

$$\omega_0^2 = \frac{1}{LC} \; ; \; \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\left[\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + 1\right]\omega_0^2$$

$$\frac{s^2\cancel{\omega_0^2}}{\cancel{\omega_0^2}} + \frac{s\omega_0^{\cancel{2}}}{Q\cancel{\omega_0}} + \omega_0^2$$

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2$$

$$\cancel{s}\frac{\omega_0}{Q} = \frac{\cancel{s}}{RC}$$

$$\frac{Q}{\omega_0} = RC$$

$$Q = \omega_0 \cdot RC$$

$$Q = \frac{1}{\sqrt{LC}} \cdot RC$$

$$Q = \frac{RC}{\sqrt{LC}}$$

$$Q = R\sqrt{\frac{C^2}{LC}}$$

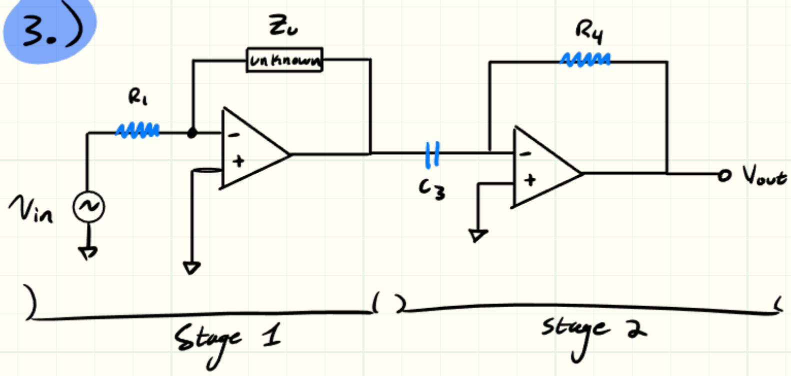
$$Q = R\sqrt{C/L}$$

$$Q = (12k)\sqrt{\frac{(12\times10^{-9})}{0.007H}}$$

$$\boxed{Q = 15.711}$$

Austin Riha | Final Exam | Problem 3

3.)



Find missing Component

$$H(s) = A_1 \cdot A_2$$

$$A_1 = -\frac{Z_U}{R_1}$$

$$A_2 = -\frac{R_4}{Z_{C_3}} = -j\omega R_4 C_3$$

$$H(j\omega) = \left(-\frac{Z_u}{R_1}\right)(-j\omega R_4 C_3)$$

$$H(j\omega) = \left(\frac{R_4 C_3}{R_1}\right) j\omega \cdot Z_u$$

for ω_1 phase shift is 180°

180° = - Real

plug into $H(j\omega)$

$$H(j\omega) = \frac{R_4 C_3}{R_1} \cdot j\omega\, [R]$$

$H(j\omega) = 90°$ (positive imag.)

$$H(j\omega) = \frac{R_4 C_3}{R_1} \cdot \cancel{j\omega}\left(\frac{1}{\cancel{j\omega} C}\right)$$

$$H(j\omega) = \frac{R_4 C_3}{R_1} \cdot \frac{1}{C}$$

$H(j\omega) = 0°$ (Positive Real)

$$H(j\omega) = \left(\frac{R_4 C_3}{R_1}\right) \cdot j\omega \cdot (j\omega L)$$

$$H(j\omega) = \left(\frac{R_4 C_3}{R_1}\right) \cdot \omega^2 (j)^2 L$$

$$H(j\omega) = \left(\frac{L R_4 C_3}{R_1}\right) \cdot -\omega^2$$

$H(j\omega) = 180°$ ✓

(Negative Real)

$\boxed{\text{The unknown part is an inductor}}$

$$H(j\omega) = \omega^2 L \left(\frac{R_4 C_3}{R_1}\right)$$

$$H(j\omega) = \omega^2 L \left(\frac{15000 \cdot 800 \times 10^{-9}}{5000}\right)$$

$$H(j\omega) = \omega^2 L (0.0000024)$$

$$\omega_1 = 5000 \text{ rps} \; ; \; H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{12V}{2V} = 6V$$

$$6V = (5000)^2 (L)(0.0000024)$$

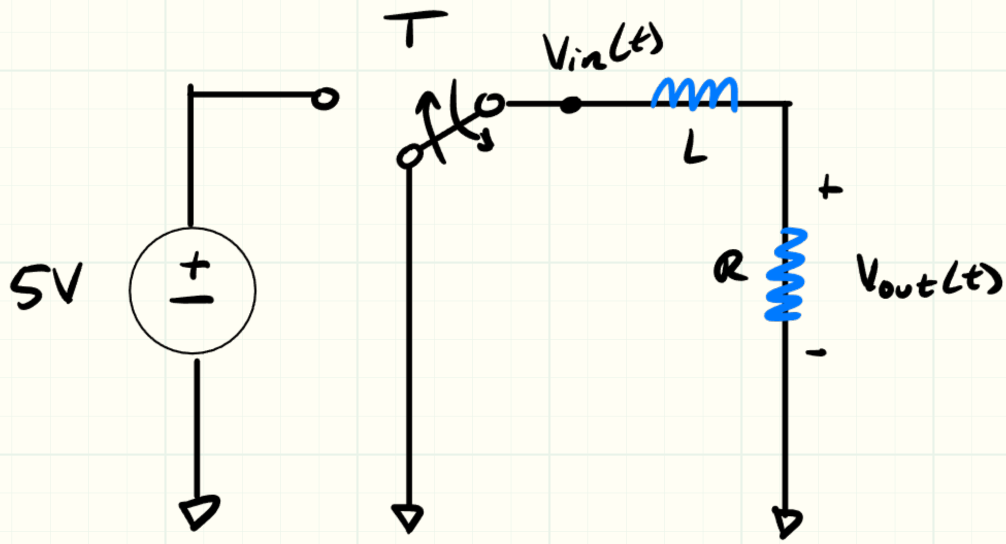
$$6V = 60L$$

$$L = 6/60 = 0.1H$$

$$\boxed{L = 0.1\,H}$$

Austin Riha | Final Exam | Problem 4

4.)



$\tau = \frac{L}{R}$

$T = 5ns$

$5\tau \leq T$

$5\frac{L}{R} \leq 5 \times 10^{-9} s$

$\frac{L}{R} \leq 1 \times 10^{-9}$

$R = \frac{L}{1 \times 10^{-9}}$

$R = \frac{200 \cancel{\times 10^{-9}}}{1 \cancel{\times 10^{-9}}}$

$\boxed{R = 200\Omega}$