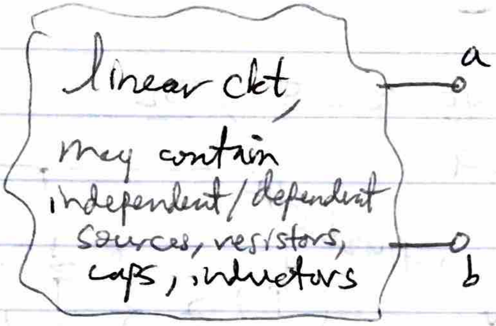
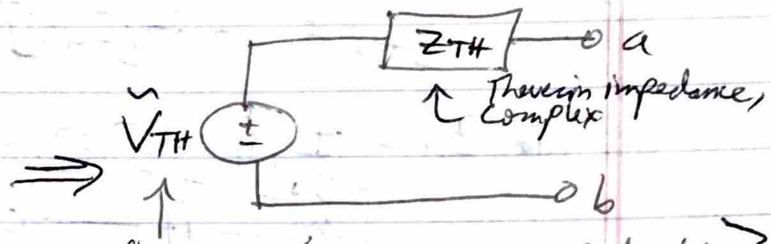
Thevenin & Norton Equivalent Ckt Models in the Phasor/Frequency Domain ⇒ Same Story!

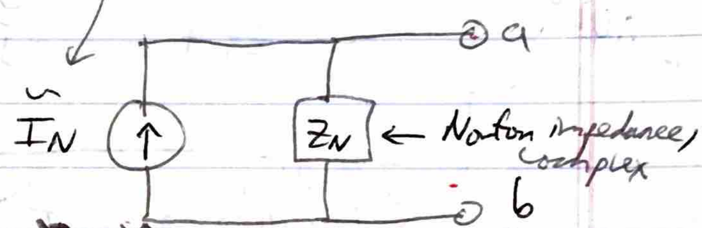


⇒



Phasor

⟨Thevenin equivalent ckt⟩

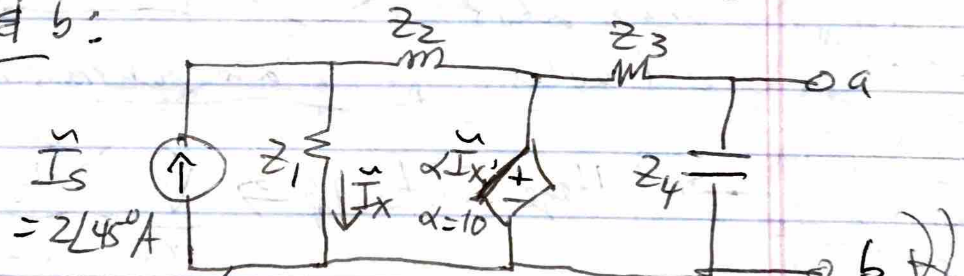


$\tilde{I}_N = \dfrac{\tilde{V}_{TH}}{Z_{TH}}$, $Z_N = Z_{TH}$ ⟨Norton equivalent ckt⟩

Let's work an example

⇒ AP 9.11

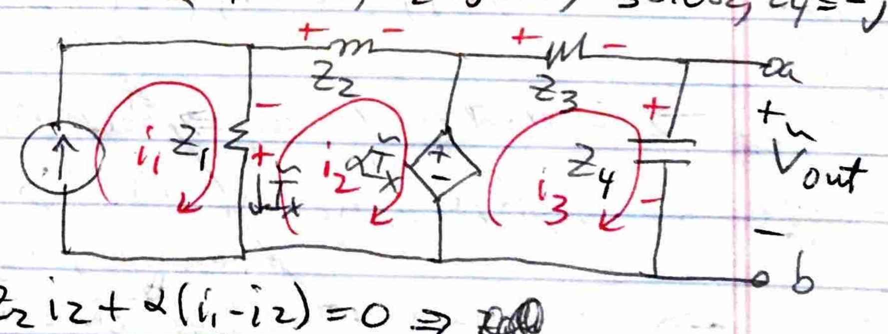
Determine phasor Thevenin & Norton equivalent ckt models wrt nodes a & b:



First, find $\tilde{V}_{TH}$
⇒ determine open-ckt voltage $\tilde{V}_{out}$

⟨$Z_1 = 20\Omega$, $Z_2 = j10\Omega$, $Z_3 = 10\Omega$, $Z_4 = -j10\Omega$⟩

⇒ I'll use mesh analysis to determine $\tilde{V}_{out} = \tilde{V}_{TH}$



$\underbrace{i_1 = \tilde{I}_s}_{\text{solved}}$, $\tilde{I}_x = i_1 - i_2$

KVL, mesh 2: $Z_1(i_2 - i_1) + Z_2 i_2 + \alpha(\underbrace{i_1 - i_2}_{\tilde{I}_x}) = 0 \Rightarrow (\alpha - Z_1)i_1 + (Z_1 + Z_2 - \alpha)i_2 = 0$

$i_1 = \tilde{I}_s$, so $(Z_1 + Z_2 - \alpha)i_2 = (Z_1 - \alpha)\tilde{I}_s$ (1)

KVL, mesh 3: $-\alpha(\underbrace{i_1 - i_2}_{\tilde{I}_x = i_1 - i_2 = \tilde{I}_s - i_2}) + Z_3 i_3 + Z_4 i_3 = 0 \Rightarrow \alpha i_2 + (Z_3 + Z_4)i_3 = \alpha\tilde{I}_s$ (2)

⇒ Can now solve for i_2 w/ (1) ⇒ $i_2 = \dfrac{(Z_1 - \alpha)\tilde{I}_s}{Z_1 + Z_2 - \alpha}$ → (cont'd)

$$i_2 = \frac{(20-10)(2\angle 45^\circ)}{20+j10-10} = \frac{10\,(2\angle 45^\circ)}{10+j10} = \frac{20\angle 45^\circ}{14.1421\angle 45^\circ}$$

$\Rightarrow \underline{i_2 = 1.4142\angle 0^\circ = 1.4142 = 1.4142 + j0} \Rightarrow$ Now, use (2) to solve for i_3:

$$i_3 = \frac{\alpha \check{I}_S - \alpha i_2}{Z_3 + Z_4} = \frac{10(2\angle 45^\circ) - (10)(1.4142)}{10 - j10} = \frac{20\angle 45^\circ - 14.1422}{14.1421\angle -45^\circ}$$

$$= \frac{\cancel{14.1422} + j\,14.1422 - \cancel{14.1422}}{14.1421\angle -45^\circ} = \frac{14.1422\angle 90^\circ}{14.1421\angle -45^\circ}$$

$\cong 1\angle 135^\circ \Rightarrow \underline{i_3 = 1\angle 135^\circ = -0.7071 + j\,0.7071}$ ckt is solved!

Now, compute $\check{V}_{out} = \check{V}_{TH} \Rightarrow \check{V}_{out} = Z_4 i_3 = (-j10)(1\angle 135^\circ)$

$\Rightarrow \check{V}_{out} = (10\angle -90^\circ)(1\angle 135^\circ) = 10\angle 45^\circ \Rightarrow \boxed{\check{V}_{TH} \overset{set}{=} \check{V}_{out} = 10\angle 45^\circ\ V \cong 7.0711 + j\,7.0711\ V}$

(*) Before computing $Z_{out} = Z_{TH}$, a few more words on impedance: $Z = R \pm jX$ [Ω]

$\Rightarrow Re\{Z\} = R$ [Ω], $Im\{Z\} = X$ [Ω] $\Rightarrow$ the real part of an impedance is called resistance, and the imaginary part of an impedance is called reactance

$\Rightarrow$ if reactance < 0, then impedance is said to be capacitive & if reactance > 0, then impedance is said to be inductive.

$\Rightarrow$ if reactance $= 0$, the impedance is said to be purely real or purely resistive

$\Rightarrow$ An analogy to $G = 1/R$ in phasor domain $\Rightarrow Y = 1/Z$ [$\mho$] is called admittance

$\Rightarrow Y = G \pm jB \Rightarrow Re\{Y\} = G$ [$\mho$] is called conductance & $Im\{Y\} = B$ [$\mho$] is called susceptance

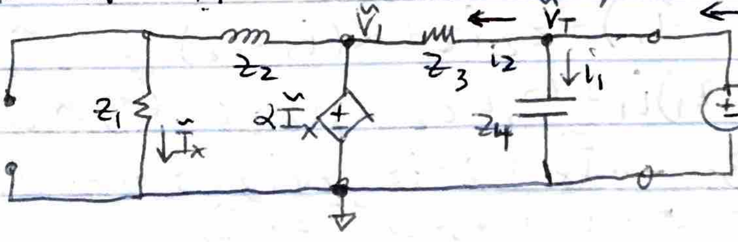
$\Rightarrow$ if susceptance > 0, then admittance is said to be capacitive & if susceptance < 0, then admittance is said to be inductive — if susceptance $= 0$, then admittance is said to be purely real or purely conductive

$\Rightarrow$ Finally, note that $Z (= 1/Y)$ and $Y (= 1/Z)$ are not phasors

Now, compute $Z_{out} = Z_{TH} \Rightarrow$ deactivate all independent sources $\Rightarrow \check{I}_S$ is replaced w/ open ckt

deactivated ckt contains dependent source, so attach a test source

$Z_{out} = \check{V}_T / \check{I}_T$



I'll use nodal analysis.

$\Rightarrow \check{V}_1 = \alpha \check{I}_X$

(cont'd)

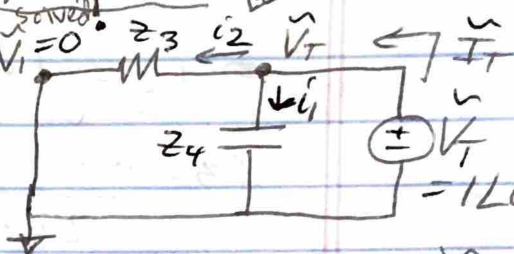
$$\Rightarrow \check{I}_x = \frac{\check{V}_1}{Z_1+Z_2} = \underbrace{Y_x}_{\sim} \check{V}_1 = Y_x \alpha \check{I}_x \quad (\check{V}_1 \to \alpha \check{I}_x)$$

⟨admittance → $Y_x = \frac{1}{Z_1+Z_2}$⟩ $\Rightarrow \check{I}_x = Y_x \alpha \check{I}_x$

$\Rightarrow$ Only valid sol'n is $\check{I}_x = 0$

$\Rightarrow$ So, $\check{I}_x = 0$ & $\check{V}_1 = \alpha \check{I}_x = 0 \Rightarrow$ the effective ckt is solved $\Rightarrow$ (since $\check{V}_1 = 0$)

KCL @ $\check{V}_T$ in the effective ckt: $\rightarrow$



$\Rightarrow \check{I}_T = i_1 + i_2$ (1) (admittances)

$\Rightarrow i_1 = Y_4 \check{V}_T$, $i_2 = Y_3 \check{V}_T \Rightarrow$ Back to (1): $\check{I}_T = Y_4 \check{V}_T + Y_3 \check{V}_T$
$(= 1/Z_4)$ $\quad (= 1/Z_3)$

$\Rightarrow \check{I}_T = (Y_4 + Y_3)\check{V}_T$

⟨$Y_3 = 1/Z_3 = 100m\mho$, $Y_4 = \frac{1}{Z_4} = +j100m\mho$⟩

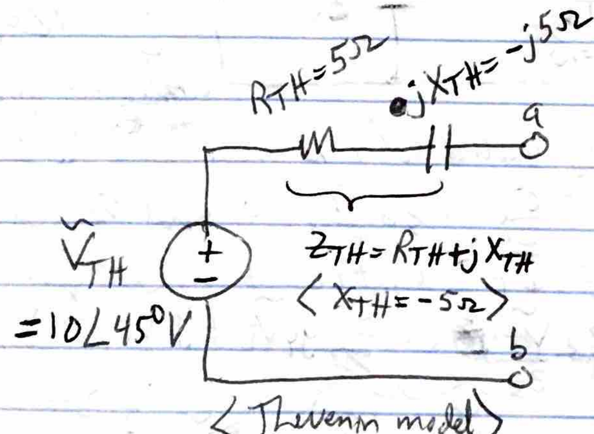
$$\Rightarrow Z_{out} = \check{V}_T/\check{I}_T = \frac{1}{Y_3+Y_4} = \frac{1}{100m + j100m} = \frac{1}{141.4214m\angle 45°}$$

$$= \frac{1\angle 0°}{141.4214m\angle 45°} = 7.0711\angle -45°\ \Omega\ \text{(ohms)} = 5 - j5\Omega$$

$\Rightarrow \boxed{Z_{TH} \overset{set}{=} Z_{out} = 7.0711\angle -45°\Omega = 5 - j5\Omega} \leftarrow$ capacitive. why? Because $Im\{Z_{TH}\} < 0$

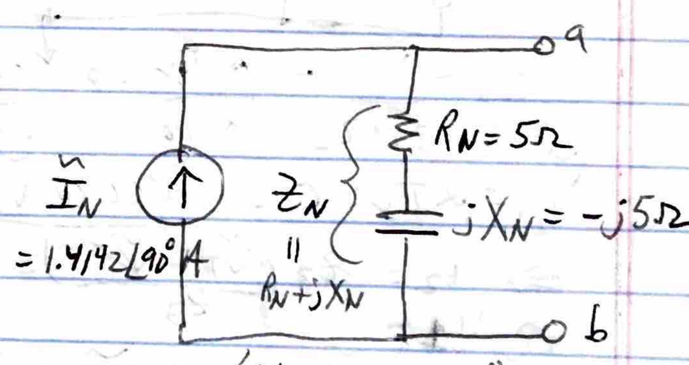
Now, Compute $\check{I}_N = \frac{\check{V}_{TH}}{Z_{TH}} = \frac{10\angle 45° V}{7.07\angle -45°\Omega} = 1.4142\angle 90° A$ &

$Z_N = Z_{TH} = 7.07\angle -45° = 5 - j5\Omega \Rightarrow$ we can now draw the models:



⟨Thevenin model⟩

Note that $Im\{Z_{TH}\} < 0$, so Z_{TH} is capacitive



⟨Norton model⟩

Note that $Im\{Z_N\} = X_N = -5\Omega < 0$, so Z_N is capacitive

Complex Power (see Sections 10.4 & 10.5 for more details) Chapter 10

In the phasor/frequency domain, the complex power S is defined as

$$S = P + jQ,$$

where P is the real power & Q is the reactive power; units?

real power is sometimes referred to as average power

$S \Rightarrow$ complex power $\Rightarrow$ units of volt-amps [VA] avg./real power is being delivered

$P \Rightarrow$ average or real power $\Rightarrow$ units of watts [W] $\Rightarrow$ (if $P<0 \Rightarrow$ delivered / else if $P>0 \Rightarrow$ dissipated/absorbed)

$Q \Rightarrow$ reactive power $\Rightarrow$ units of volt-amps reactive [VAR] $\Rightarrow$ (if $Q<0 \Rightarrow$ reactive power is being delivered / else if $Q>0 \Rightarrow$ dissipated/absorbed)

How to compute S? Note that apparent power is $|S|$: $|S| = \sqrt{P^2+Q^2}$ = magnitude of S

#1 $\quad S = \frac{1}{2}\check{V}\check{I}^* \left(= \frac{\check{V}}{\sqrt{2}}\frac{\check{I}^*}{\sqrt{2}} = \check{V}_{rms}\check{I}^*_{rms} \right)$

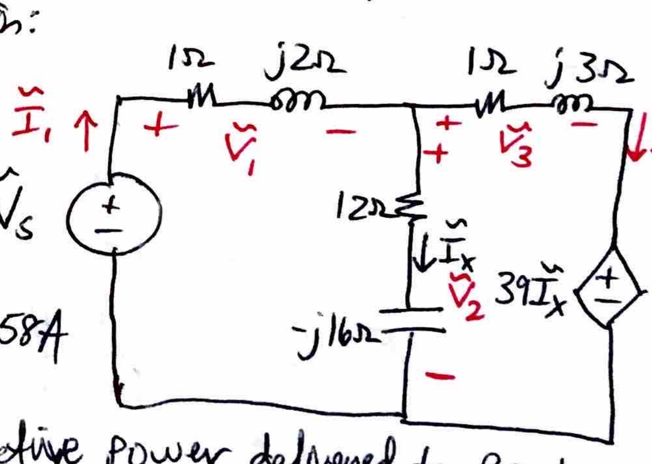
#2 $\quad S = \frac{1}{2}Z|\check{I}|^2 \left(= Z\frac{\check{I}}{\sqrt{2}}\frac{\check{I}^*}{\sqrt{2}} = \underbrace{Z\check{I}_{rms}}_{\check{V}_{rms}}\check{I}^*_{rms} = Z|\check{I}_{rms}|^2 \right)$

#3 $\quad S = \frac{1}{2}\frac{|\check{V}|^2}{Z^*} \left(= \frac{\check{V}}{\sqrt{2}}\left(\frac{\check{V}/\sqrt{2}}{Z}\right)^* = \check{V}_{rms}\underbrace{\left(\frac{\check{V}_{rms}}{Z}\right)^*}_{\check{I}^*_{rms}} = \frac{|\check{V}_{rms}|^2}{Z^*} \right)$

Let's work through an example to show how to do power checks in the phasor/frequency domain:

Example 10.7

Given: $\check{V}_s = 150\angle 0°\,V$, $\check{V}_1 = 78 - j104\,V$, $\check{V}_2 = 72 + j104\,V$, $\check{V}_3 = 150 - j130\,V$, $\check{I}_1 = -26 - j52\,A$, $\check{I}_x = -2 + j6\,A$, $\check{I}_2 = -24 - j58\,A$



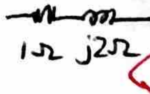
(a) Calculate the total average & reactive power delivered to each impedance.

(cont'd) →

(example, cont'd)

(a) The complex power delivered to the $1 + j2\,\Omega$ impedance is

37.2



$$\rightarrow S_1 = \frac{1}{2}\tilde{V}_1 \tilde{I}_1^{*} = \frac{1}{2}\underbrace{(78 - j104)}_{\tilde{V}_1}\underbrace{(-26 + j52)}_{\tilde{I}_1^{*}} = 1690 + j\,3380\ \text{VA} = P_1 + jQ_1$$

⟨note that passive sign convention is being adhered to here⟩

⇒ Thus the impedence is dissipating/absorbing an average/real power of $\underbrace{P_1 = 1690\,\text{W}}_{1\Omega \text{ resistor}}$ & a reactive power of $\underbrace{3380\ \text{VAR}}_{j2\Omega \text{ inductor}}$

(the sign of $\text{Im}\{S_1\} = Q_1$ is positive, so the 3380 VAR term is being dissipated/absorbed by the $j2\,\Omega$ inductor)

inductors always have $Q > 0$

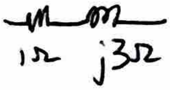
$$\rightarrow S_2 = \frac{1}{2}\tilde{V}_2 \tilde{I}_x^{*} = \frac{1}{2}\underbrace{(72 + j104)}_{\tilde{V}_2}\underbrace{(-2 - j6)}_{\tilde{I}_x^{*}} = 240 - j\,320\ \text{VA} = P_2 + jQ_2$$

⟨consistent w/ passive sign convention⟩

⇒ Therefore the impedance in the vertical branch is dissipating/absorbing an average/real power of $\underbrace{P_2 = 240\,\text{W}}_{12\Omega \text{ resistor}}$ & is delivering/supplying $\underbrace{320\ \text{VAR}}_{-j16\Omega \text{ cap}}$

(the sign of $\text{Im}\{S_2\} = Q_2$ is negative, so the 320 VAR term is being delivered/supplied by the $-j16\,\Omega$ capacitor)

caps always have $Q < 0$



$$\rightarrow S_3 = \frac{1}{2}\tilde{V}_3 \tilde{I}_2^{*} = \frac{1}{2}\underbrace{(150 - j130)}_{\tilde{V}_3}\underbrace{(-24 + j58)}_{\tilde{I}_2^{*}} = 1970 + j\,5910\ \text{VA} = P_3 + jQ_3$$

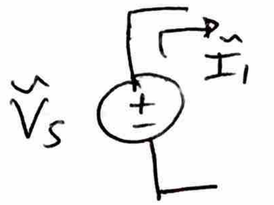
⇒ This impedance is dissipating/absorbing $\underbrace{1970\,\text{W} = 1.97\,\text{kW}}_{1\Omega \text{ resistor}}$ of average/real power & dissipating/absorbing a reactive power of $\underbrace{5910\,\text{VAR} = 5.91\,\text{kVAR}}_{j3\Omega \text{ inductor}}$

(cont'd)

(example, cont'd)

(b) Determine the complex power terms for the sources:

independent source, $\check{V}_s$ $\rightarrow$ $S_s = ?$ $\Rightarrow$

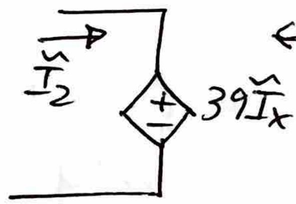


$\Leftarrow$ here, to be consistent with passive sign convention, we'll compute S_s as
$$S_s = -\frac{1}{2}\check{V}_s\check{I}_1^*$$

$$\Rightarrow S_s = -\frac{1}{2}\check{V}_s\check{I}_1^* = -\frac{1}{2}\underbrace{(150)}_{\check{V}_s}\underbrace{(-26+j52)}_{\check{I}_1^*} = 1950 - j3900\,\text{VA}$$
$$= P_s + jQ_s$$

$\Rightarrow$ $Re\{S_s\} = P_s > 0$, so $\check{V}_s$ is absorbing ("dissipating") 1950 W = 1.95 kW of real/avg. power, & since $Im\{S_s\} = Q_s < 0$, $\check{V}_s$ is delivering/supplying 3900 VAR = 3.9 kVAR of reactive power.

dependent source, $39\check{I}_x$ $\rightarrow$ $S_x = ?$ $\Rightarrow$



$\Leftarrow$ here, to be consistent with passive sign convention, S_x is computed as
$$S_x = \frac{1}{2}39\check{I}_x\check{I}_2^*$$

$$\Rightarrow S_x = \frac{1}{2}(39\check{I}_x)\check{I}_2^* = \frac{1}{2}\underbrace{(39)(-2+j6)}_{39\check{I}_x}\underbrace{(-24+j58)}_{\check{I}_2^*}$$

$$\Rightarrow S_x = -5850 - j5070\,\text{VA} = P_x + jQ_x$$

$\Rightarrow$ $Re\{S_x\} = P_x < 0$, so $39\check{I}_x$ source is delivering/supplying an average/real power of 5850 W = 5.85 kW, & since $Im\{S_x\} = Q_x < 0$, the $39\check{I}_x$ source is delivering/supplying a reactive power of 5070 VAR = 5.07 kVAR.

(c) Verify that the average/real power delivered equals the average/real power dissipated, and the magnetizing reactive power delivered equals the magnitizing reactive power absorbed/"dissipated"

(cont'd) $\rightarrow$

(example, cont'd)

(c) → The total average/real power being dissipated/absorbed is: 37.4 ✓

$$P_{dis} = P_1 + P_2 + P_3 + P_S = 5850W = 5.85kW$$

→ The total average/real power being delivered is:

$$P_{del} = |P_X| = 5850W = 5.85kW$$

⇒ Thus, $P_{dis} = P_{del} = 5.85kW$ ✓

⇒ For phasor ckts, we are not done with a power check until we also verify that $Q_{dis} = Q_{del}$

→ Magnetizing reactive power is being absorbed/"dissipated" by the inductors in the two horizontal branches in the ckt:

$$Q_{dis} = Q_1 + Q_3 = 9290VAR = 9.29kVAR$$

→ Magnetizing reactive power is being delivered by the independent voltage source, the capacitor, & the dependent voltage source:

$$Q_{del} = |Q_S| + |Q_2| + |Q_X| = 9290VAR = 9.29kVAR$$

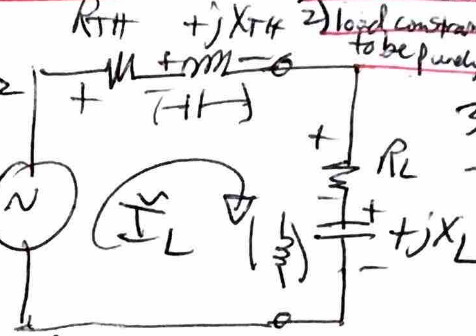
⇒ Thus $Q_{dis} = Q_{del} = 9.29kVAR$ ✓

⇒ The power check is now complete. ✓

→ How to accomplish max power transfer in phasor domain? Consider 2 cases: 1) No load constraints 2) load constrained to be purely resistive

37.5

$$P_{R_L} = \frac{1}{2} R_L |\tilde{I}_L|^2 = \left(\tfrac{1}{2}R_L\right)\frac{|\tilde{V}_{TH}|^2}{(R_{TH}+R_L)^2+(X_{TH}+X_L)^2}$$



$$\tilde{I}_L = ?\quad \tilde{I}_L = \frac{\tilde{V}_{TH}}{R_{TH}+jX_{TH}+R_L+jX_L}$$

$$\Rightarrow \tilde{I}_L = \frac{\tilde{V}_{TH}}{(R_{TH}+R_L)+j(X_{TH}+X_L)}$$

⇓ $X_{TH} > 0$ for inductive Z_{TH}
$X_{TH} < 0$ for capacitive Z_{TH}
<same story for X_L>

$$|\tilde{I}_L| = \frac{|\tilde{V}_{TH}|}{\sqrt{(R_{TH}+R_L)^2+(X_{TH}+X_L)^2}} \Rightarrow |\tilde{I}_L|^2 = \frac{|\tilde{V}_{TH}|^2}{(R_{TH}+R_L)^2+(X_{TH}+X_L)^2}$$

(I) Want to maximize P_{R_L} w/ no constraints on the load: ⇒ two load variables (not just one as before)

$$\Rightarrow \frac{\partial P_{R_L}}{\partial R_L} \stackrel{set}{=} 0 \text{ (i)} \quad \& \quad \frac{\partial P_{R_L}}{\partial X_L} \stackrel{set}{=} 0 \text{ (ii)}$$

⇒ (ii) To maximize P_{R_L} due the X_L term which only appears in the denominator of P_{R_L}, we should set $X_L = -X_{TH}$

⇒ (ii) So, to get $\frac{\partial P_{R_L}}{\partial X_L} = 0$, set $\boxed{X_L \stackrel{set}{=} -X_{TH} \text{ (1)}}$ ⇐ i.e., if $X_{TH} < 0$ (i.e., capacitive), set X_L to ~~a inductance~~ ~~reactance~~ ~~of~~ an inductor w/ reactance of $X_L = |X_{TH}| = -X_{TH}$

(*) What about (i) $\frac{\partial P_{R_L}}{\partial R_L} = 0$? ⇐ we'll now assume that $X_L = -X_{TH}$:

$$\Rightarrow \text{So, } P_{R_L} = \frac{1}{2}R_L\left(\frac{|\tilde{V}_{TH}|^2}{(R_{TH}+R_L)^2}\right)$$

⇐ this is essentially the same form of P_{R_L} that we had before...

⇒ we already know that P_{R_L} above is maximized when $\boxed{R_L \stackrel{set}{=} R_{TH} \text{ (2)}}$.

⇒ What about the scenario where we have a constraint on the load, e.g., Ze of load is purely real?

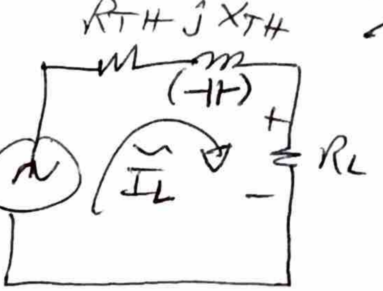
$$\& \; P_{R_L,max} = \frac{\frac{1}{2}R_{TH}|\tilde{V}_{TH}|^2}{(R_{TH}+R_{TH})^2} = \frac{\frac{1}{2}R_{TH}|\tilde{V}_{TH}|^2}{4R_{TH}^2}$$

$$\Rightarrow \boxed{P_{R_L,max} = \frac{|\tilde{V}_{TH}|^2}{8R_{TH}}}$$

⇐ <no constraints max avg power delivered to load> ⇐ if we use $\sqrt{2}\tilde{V}_{TH rms} = \tilde{V}_{TH}$, then $P_{R_L,max} = \frac{|\tilde{V}_{TH rms}|^2}{4R_{TH}}$

37.6

(II) Want to maximize P_{R_L} subject to the constraint that the load is purely resistive:



$$\tilde{I}_L = \frac{\tilde{V}_{TH}}{R_{TH}+R_L+jX_{TH}}, \quad P_{R_L} = \frac{1}{2}R_L|\tilde{I}_L|^2$$

$$\Rightarrow P_{R_L} = \left(\frac{1}{2}R_L\right)\frac{|\tilde{V}_{TH}|^2}{(R_{TH}+R_L)^2+X_{TH}^2} \Leftarrow \text{to maximize, set } \frac{dP_{R_L}}{dR_L}=0$$

$$\frac{dP_{R_L}}{dR_L} = \left(\frac{1}{2}|\tilde{V}_{TH}|^2\right)\frac{\left\{\left[(R_{TH}+R_L)^2+X_{TH}^2\right]\frac{dR_L}{dR_L} - R_L\frac{d}{dR_L}\left[(R_{TH}+R_L)^2+X_{TH}^2\right]\right\}}{\left[(R_{TH}+R_L)^2+X_{TH}^2\right]^2}$$

$\Rightarrow \frac{dP_{R_L}}{dR_L}=0$ when numerator$\left\{\frac{dP_{R_L}}{dR_L}\right\}=0 \Rightarrow \frac{d}{dR_L}\left[(R_{TH}+R_L)^2+X_{TH}^2\right] = \ldots$

$\ldots = \frac{d}{dR_L}\left[R_{TH}^2+2R_{TH}R_L+R_L^2+X_{TH}^2\right] = 2R_L+2R_{TH} \Rightarrow$ So, the num$\left\{\frac{dP_{R_L}}{dR_L}\right\}$ is equal to zero when $(R_{TH}+R_L)^2+X_{TH}^2 - R_L\left[2R_L+2R_{TH}\right]=0$

$$\Rightarrow R_{TH}^2+\cancel{2R_{TH}R_L}+R_L^2+X_{TH}^2 = 2R_L^2+\cancel{2R_{TH}R_L} \Rightarrow \underbrace{2R_L^2-R_L^2}_{R_L^2} = R_{TH}^2+X_{TH}^2$$

$\Rightarrow R_L = \pm\sqrt{R_{TH}^2+X_{TH}^2} \Leftarrow R_L = -\sqrt{R_{TH}^2+X_{TH}^2}$ is infeasible since R_L cannot be negative, so

$$\boxed{R_L = \sqrt{R_{TH}^2+X_{TH}^2} = |Z_{TH}| \quad (3)}$$ ⟸ yields $P_{R_L,max}$ when the load is constrained to be purely resistive

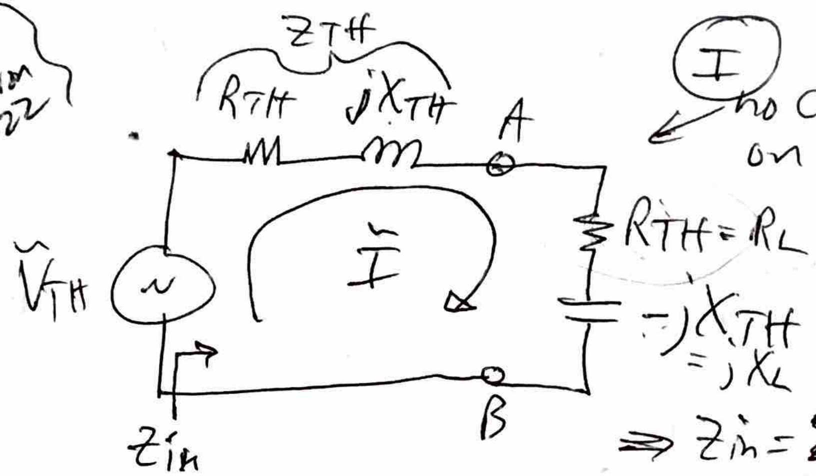
$$\& \; P_{R_L,max} = \frac{1}{2}R_L|\tilde{I}_L|^2\Big|_{R_L=\sqrt{R_{TH}^2+X_{TH}^2}} \Rightarrow \boxed{\frac{\frac{1}{2}\sqrt{R_{TH}^2+X_{TH}^2}\,|\tilde{V}_{TH}|^2}{\left(R_{TH}+\sqrt{R_{TH}^2+X_{TH}^2}\right)^2+X_{TH}^2} = P_{R_L,max}}$$

purely real constraint max avg power delivered to load

$\Rightarrow$ if we use $\sqrt{2}\tilde{V}_{TH,rms} = \tilde{V}_{TH}$,

then $$P_{R_L,max} = \frac{\sqrt{R_{TH}^2+X_{TH}^2}\,|\tilde{V}_{TH\,rms}|^2}{\left(R_{TH}+\sqrt{R_{TH}^2+X_{TH}^2}\right)^2+X_{TH}^2}$$

{in-class discussion Sp. 2022}



(I) no constraints on $Z_L \Rightarrow Z_L = Z_{TH}^*$

37.7

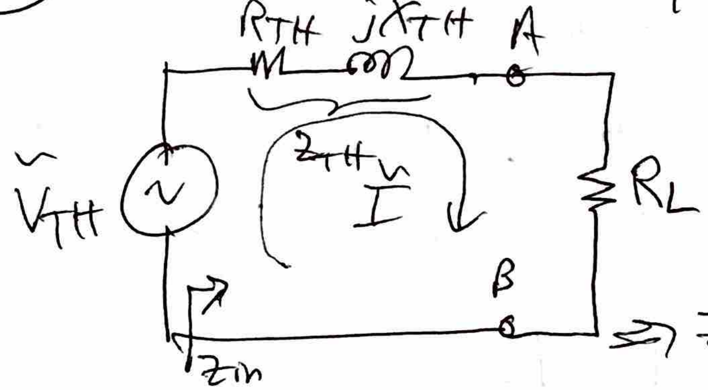
(37.7)

$\tilde{I} = \dfrac{\tilde{V}_{TH}}{Z_{in}} = A\angle\theta^\circ$

$\Rightarrow Z_{in} = 2R_{TH}$

$\Rightarrow$ avg/real power dissipated by Z_L = ? $P_{R_L} = \frac{1}{2} R_L |\tilde{I}|^2 = \frac{1}{2} R_{TH} A^2$ ($R_L = R_{TH}$)

(II) Z_L constrained to be purely real $\Rightarrow Z_L = R_L$:



$R_L = |Z_{TH}|$ ← real-valued

$\tilde{I} = \dfrac{\tilde{V}_{TH}}{Z_{in}} = B\angle\phi$

$\Rightarrow Z_{in} = R_{TH} + \underbrace{R_L}_{= |Z_{TH}|} + jX_{TH}$

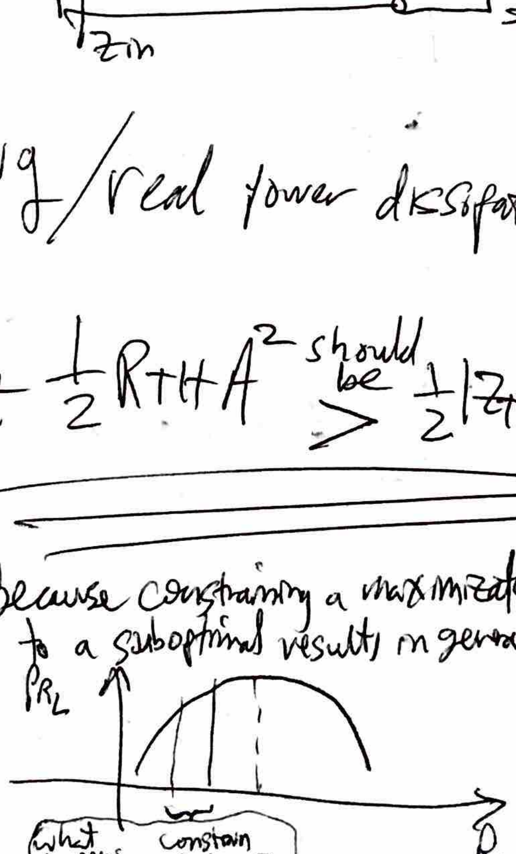
$\Rightarrow$ avg/real power dissipated by R_L = ? $P_{R_L} = \frac{1}{2} R_L |\tilde{I}|^2$ ($R_L = |Z_{TH}|$, $|\tilde{I}|^2 = B^2$)

$\Rightarrow P_{R_L} = \frac{1}{2} |Z_{TH}| B^2$

Note that $\frac{1}{2} R_{TH} A^2$ should be $> \frac{1}{2} |Z_{TH}| B^2$

$\Downarrow$

why? Because constraining a maximization problem leads to a suboptimal results in general.



⟨more hints on back!⟩ →

Prob 7

for parts (c) & (d) — 37.8

Wrong answers ~~formats~~ that you want to avoid:

Note: these are not the correct answers for part (a)!!

Suppose that you get $\check{V}_{TH} = 5\angle 52^\circ$ V & $Z_{TH} = 4.2 + j2.3\,\Omega \doteq 4.79\angle 28.71^\circ\,\Omega$ for part (a); now, for part (b), what kind of mistakes can we make?

part (b) Find Z_L to maximize avg. power delivery to load (no constraints!) $\Rightarrow$ set $Z_L = Z_{TH}^*$

So, $Z_L = Z_{TH}^* = \{4.2 + j2.3\}^* = 4.2 - j2.3\,\Omega$,

compare

WRONG! $\Rightarrow$ not ~~$Z_L = Z_{TH} = 4.2 + j2.3$~~! $\Leftarrow$ WRONG!

part (c) Find R_L (real-valued constraint!) to maximize avg. power delivery to load:

WRONG! $\Rightarrow$ ~~$R_L = \text{Re}\{Z_{TH}\} = 4.2\,\Omega$~~ $\Leftarrow$ WRONG!

Correct approach $\Rightarrow$ $R_L = |Z_{TH}| = |4.2 + j2.3| = \sqrt{4.2^2 + 2.3^2} \doteq 4.79\,\Omega$
$= |4.79\angle 28.71^\circ|$ $\left(\sqrt{17.64 + 5.29} = \sqrt{22.93}\right)$

Correct way!

part (d) P_{max} for parts (b) & (c)?

WRONG! $\Rightarrow$ ~~$P_{max} = \dfrac{V_{TH}^2 = 5V}{4R_L}$~~ $\Leftarrow$ WRONG!

Correct way? $\Rightarrow$ use procedure covered on previous page!

Also, wrong! $\Rightarrow$ ~~No discussion!~~ $\Leftarrow$ WRONG!

Correct way? $\Rightarrow$ discuss! $\Rightarrow$ $P_{max\,part(b)}$ should be $> P_{max\,part(c)}$

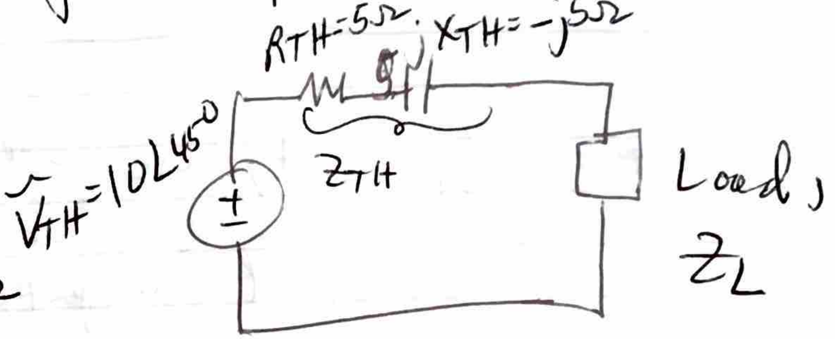
Summary

Max Power delivery in the phasor domain:

Take the following example Thevenin model:

$Z_{TH} = \underbrace{5 - j5\,\Omega}_{\text{capacitive}}$

$= \underbrace{7.07\angle -45^\circ\,\Omega}_{\text{capacitive}}$



(I) → Suppose we want to determine Z_L for max real/average power transfer to the load.

⇒ If there are no constraints on Z_L, then set $\underline{Z_L = Z_{TH}^*}$ ← i.e., if $Z_{TH} = R_{TH} + jX_{TH}$ is capacitive with $X_{TH} < 0$, then attach load $Z_L = R_{TH} - jX_{TH}$ inductive & vice versa

⇒ In this example, we'd set $Z_L = Z_{TH}^*$

⇒ $Z_L = (5 - j5)^* = 5 + j5\,\Omega$ ⇒ $Z_L = 5 + j5\,\Omega$ for max real power transfer, no Z_L constraints.

(II) → Now, suppose we want to maximize real/average power transfer to Z_L under the constraint that Z_L be purely real. How to design Z_L in this case?

⇒ Here, we'd set $\underline{Z_L = |Z_{TH}|}$ ← i.e., attach load resistor R_L equal to the magnitude of $Z_{TH} = \sqrt{R_{TH}^2 + X_{TH}^2}$

⇒ In this case, $Z_L = R_L = |5 - j5| = |7.07e^{-j45^\circ}|$

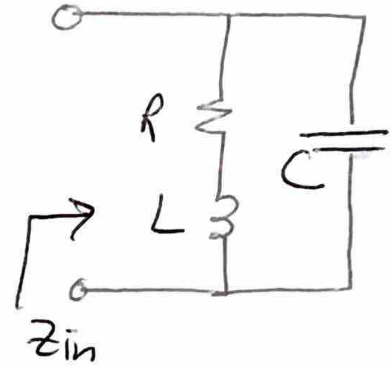
(A) ⇒ $Z_L = R_L = 7.07\,\Omega$ for max power transfer under the constraint that Z_L be purely real

See chpt. 10 for more on phasor-domain power calculations

Purely-real impedance

★ **Example:**

Ⓣ Dredo 36-37



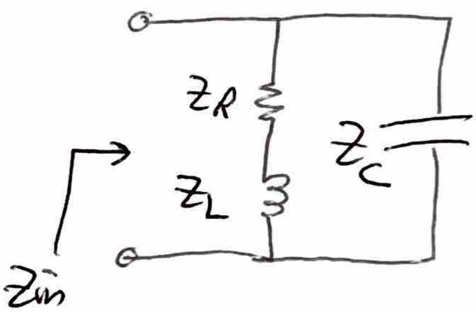
$R = 500\Omega$, $L = 50mH$ ← don't use these for part (a)!

(a) Derive an expression for C such that the impedance Zin is purely real (or purely resistive) at a frequency of $f = 1kHz$.

(b) Evaluate expression of part (a) for given R, L values, & determine Zin.

(c) Sketch Zin vs. f & highlight the result at f = 1kHz on sketch.

Sol'n (a) Consider phasor-transformed circuit →



$$Z_{in} = (Z_R + Z_L) // Z_C, \quad Z_R = R, \quad Z_L = j\omega L, \quad Z_C = \frac{1}{j\omega C}$$

$\Rightarrow Z_R + Z_L = R + j\omega L$ & so

$$Z_{in} = (Z_R + Z_L)//Z_C = \frac{(Z_R+Z_L)(Z_C)}{Z_R + Z_L + Z_C} = \left[\frac{(R+j\omega L)\left(\frac{1}{j\omega C}\right)}{R + j\omega L + \frac{1}{j\omega C}}\right]\frac{(j\omega C)}{(j\omega C)}$$

$$= \frac{R + j\omega L}{1 + \underbrace{(j\omega L)(j\omega C)}_{=-\omega^2 LC} + j\omega RC} = \frac{R + j\omega L}{(1-\omega^2 LC) + j\omega RC}$$

← we want Zin purely real; let's multiply by complex conjugate of denom. to get all j terms into numerator.

$$\Rightarrow Z_{in} = \frac{(R+j\omega L)}{(1-\omega^2 LC + j\omega RC)} \frac{(1-\omega^2 LC - j\omega RC)}{(1-\omega^2 LC - j\omega RC)} = \frac{R(1-\omega^2 LC) + \omega^2 RLC - j\omega R^2 C + j\omega L(1-\omega^2 LC)}{(1-\omega^2 LC)^2 + (\omega RC)^2}$$

$$\Rightarrow Z_{in} = \frac{R - \cancel{\omega^2 RLC} + \cancel{\omega^2 RLC} + j\left[\omega L - \omega R^2 C - \omega^3 L^2 C\right]}{(1-\omega^2 LC)^2 + (\omega RC)^2}$$

$$\Rightarrow Z_{in} = \frac{R + j\omega\left[L - R^2 C - \omega^2 L^2 C\right]}{(1-\omega^2 LC)^2 + (\omega RC)^2} \Rightarrow$$

we want $Im\{Z_{in}\} = 0$, so set

$$Im\{Z_{in}\} = \frac{\omega\left[L - R^2 C - \omega^2 L^2 C\right]}{(1-\omega^2 LC)^2 + (\omega RC)^2} = 0$$

(Cont'd) →

(2) redo 36-37

(a) (cont'd) $\text{Im}\{Z_{in}\}$ will equal zero if $\omega[L - R^2C - \omega^2L^2C]$ equals zero.

$$\Rightarrow \cancel{\omega}[L - R^2C - \omega^2L^2C] \overset{\text{set}}{=} 0 \Rightarrow L = C[R^2 + \omega^2L^2]$$

$$\Rightarrow \boxed{C = \frac{L}{R^2 + \omega^2L^2}}$$

← this equation gives the value of C needed to make Z_{in} purely resistive (i.e., $\text{Im}\{Z_{in}\} = 0$)

(b) $$C = \frac{L}{R^2 + \omega^2L^2} = \frac{50m}{(500)^2 + [(2\pi)(1k)]^2(50m)^2} \Rightarrow \boxed{C = 143.39nF}$$

$$\& \quad Z_{in} = \frac{R + j\omega[\underbrace{L - R^2C - \omega^2L^2C}_{\to 0}]}{(1 - \omega^2LC)^2 + (\omega RC)^2} = \frac{R}{(1 - \omega^2LC)^2 + (\omega RC)^2}$$

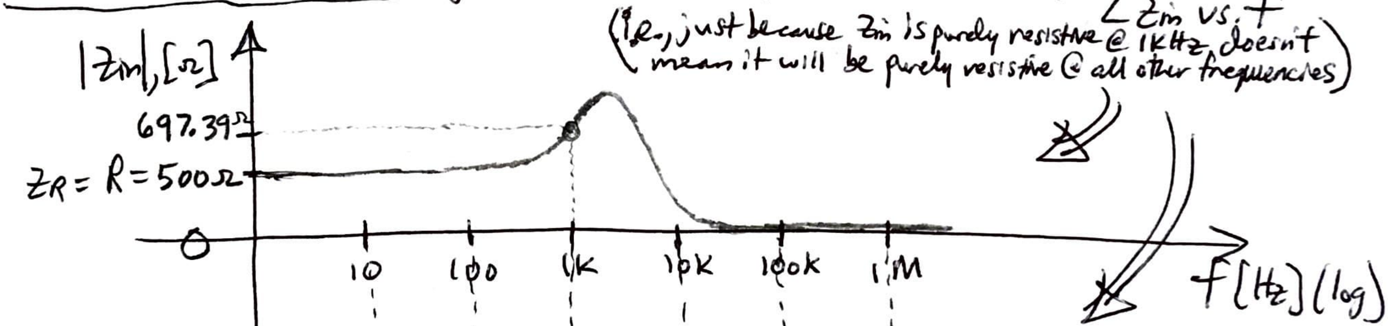
$$= \frac{500}{[1 - (2\pi 1k)^2(50m)(143.39n)]^2 + [(2\pi 1k)(500)(143.39n)]^2}$$

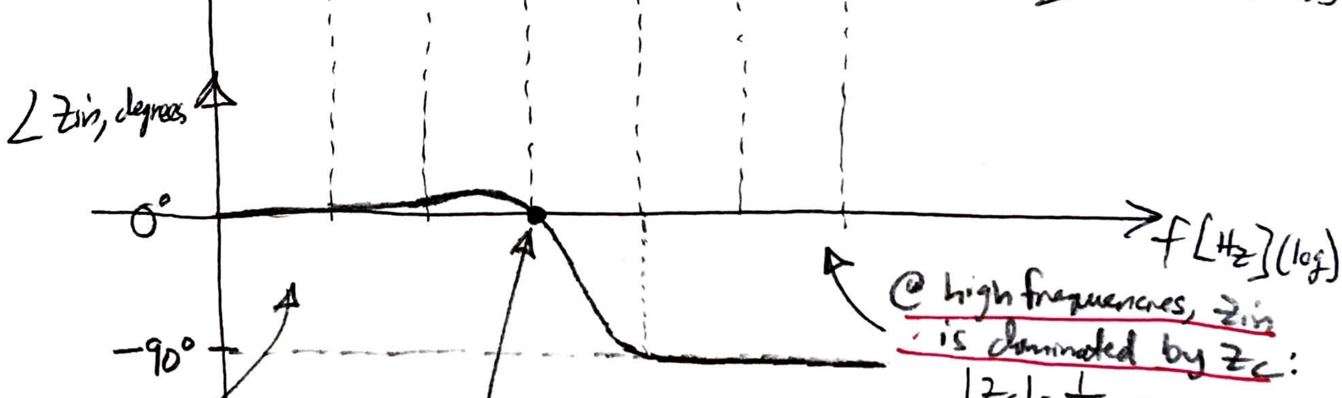
$$\Rightarrow \boxed{Z_{in} = 697.39\Omega}$$

← Note that (1) Z_{in} is purely resistive ($\text{Im}\{Z_{in}\} = 0$) @ 1kHz & (2) $Z_{in} > R = 500\Omega$ @ $f = 1kHz$.

(c) Sketch Z_{in} vs. frequency ← Z_{in} is complex, in general, so plot $|Z_{in}|$ vs. f & $\angle Z_{in}$ vs. f

(i.e., just because Z_{in} is purely resistive @ 1kHz, doesn't mean it will be purely resistive @ all other frequencies)





@ low frequencies, Z_{in} is dominated by Z_R; $|Z_L|$ is small & $|Z_C|$ is large compared to Z_R.

$\angle Z_{in} = 0°$ @ $f = 1kHz$ ⇒ Z_{in} is purely real @ $f = 1kHz$.

@ high frequencies, Z_{in} is dominated by Z_C: $|Z_C| = \frac{1}{\omega C} \to 0$ as $\omega \to \infty$

<see Zin_example.asc LTSpice demo>