<New for Sp. 2021>

Bode approximation example

Suppose transfer function $H(s) = \dfrac{2000\,(s/500)}{s + 200}$ ⇐ produce Bode approximation of frequency response $H(j\omega)$

SOL'N

First, we need to write $H(s)$ in proper Bode approximation form:

$$H(s) = \frac{2000(s/500)}{s+200} = \frac{\cancel{2000}^{\,10}(s/500)}{\cancel{200}(s/200+1)} = \frac{10\,(s/500)}{(1+s/200)}$$

K, gain term (the 10); zero at DC, with 0dB crossing of 500rps (the $s/500$); pole at 200 rps (the $1+s/200$).

Note that we could also factor the 500 out of the zero at DC term & work with $H(s) = \dfrac{(1/50)(s/1)}{(1+s/200)}$, but $K \ll 1$ in this case...

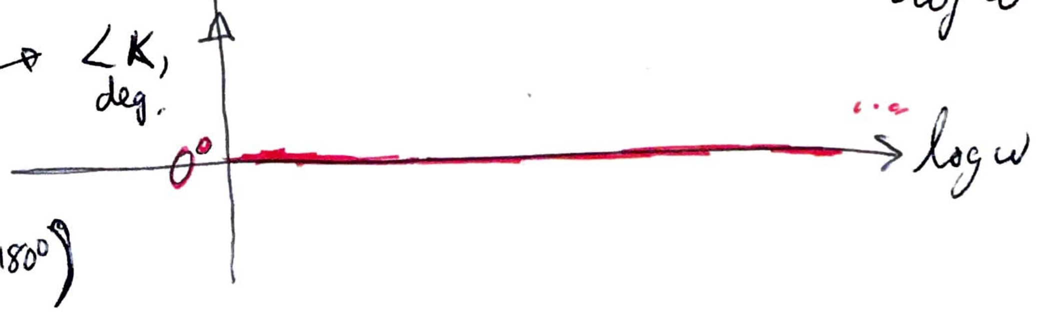
... OR, we could "fold" the $K=10$ term into the zero at DC & work with $H(s) = \dfrac{(s/50)}{(1+s/200)}$; we have options!

Now, $H(s) = \dfrac{10\,(s/500)}{(1+s/200)}$ ← 3 terms; we will plot out approximations of the Bode responses of these 3 terms first; then combine as a final step

(*) K, gain term

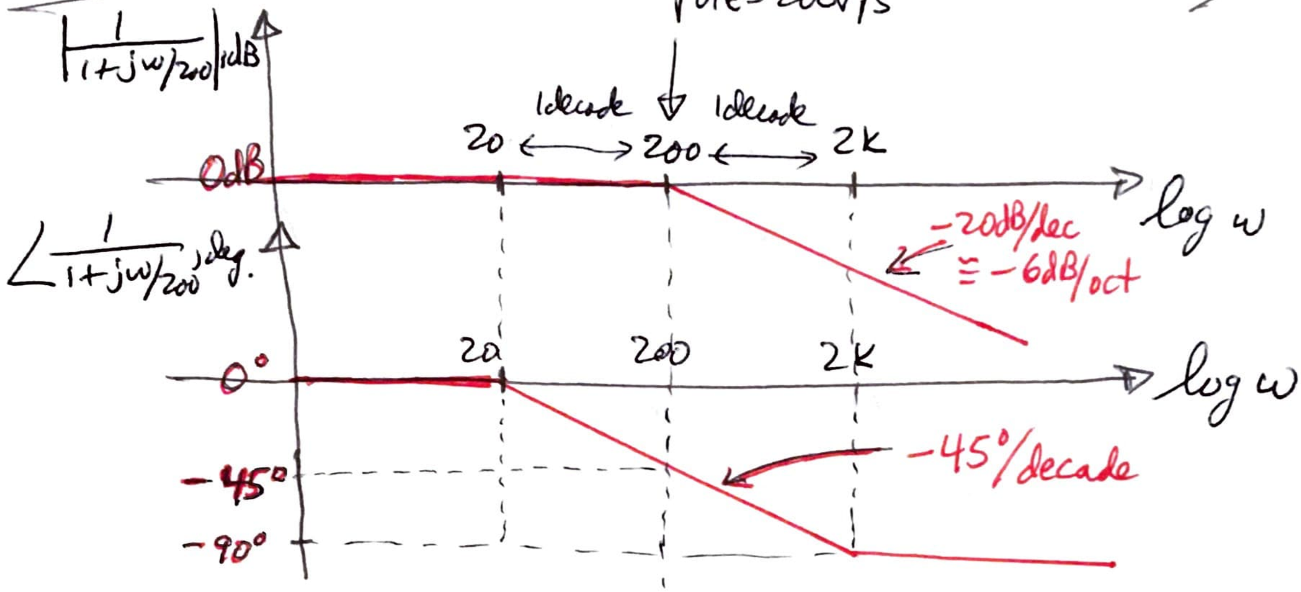
$20\log_{10}(K) = 20\text{dB}$ (with $K = 10$)

$K > 0$, so $0°$ phase shift. (if $K<0$, then $\pm 180°$ phase shift)

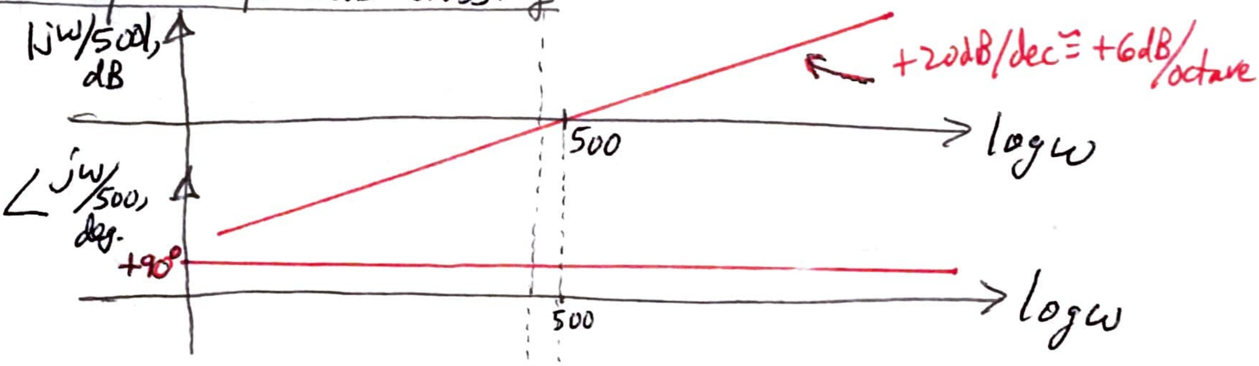


(cont'd)

(*) pole @ 200rps term



(+) zero @ DC w/ 500rps 0dB crossing

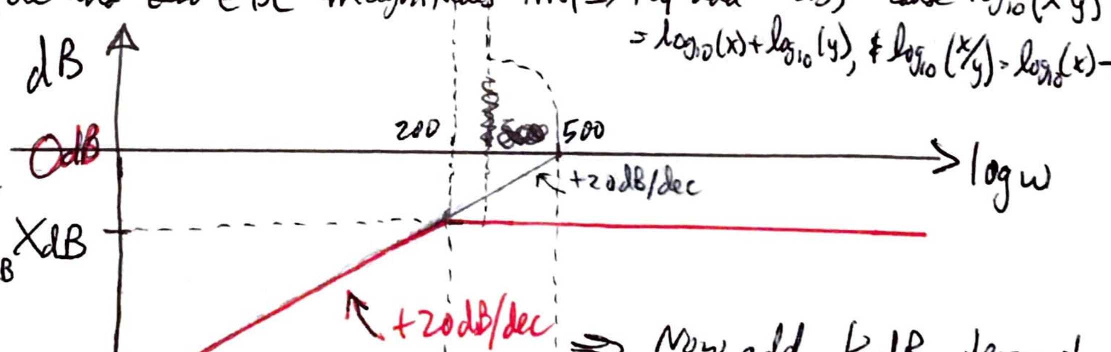


→ Let's combine pole and zero @ DC magnitudes first ⇒ they add in dB, because $\log_{10}(xy) = \log_{10}(x)+\log_{10}(y)$, & $\log_{10}(x/y) = \log_{10}(x)-\log_{10}(y)$

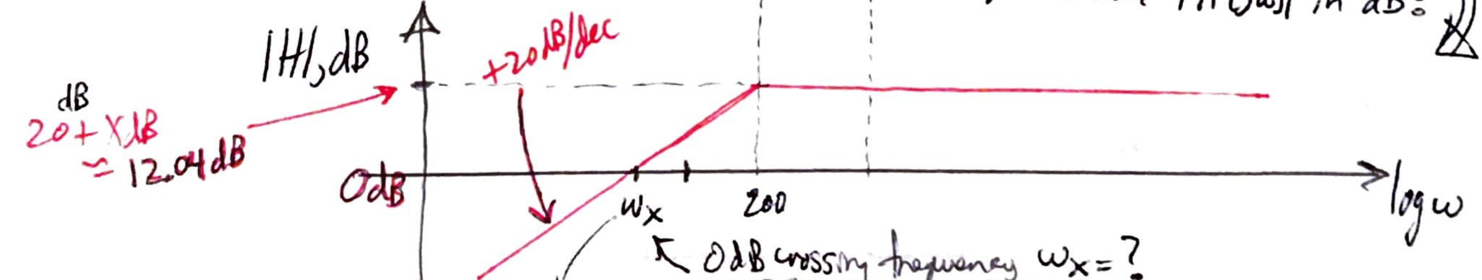
\# of decades between 200 & 500: $\log_{10}\left(\frac{500}{200}\right) \cong 0.398$ dec.

$X\text{dB} + \frac{20\text{dB}}{\text{dec}}(0.398\text{dec}) = 0\text{dB}$

$\Rightarrow X\text{dB} \simeq -7.96\text{dB}$



⇒ Now add KdB term to get overall $|H(j\omega)|$ in dB:



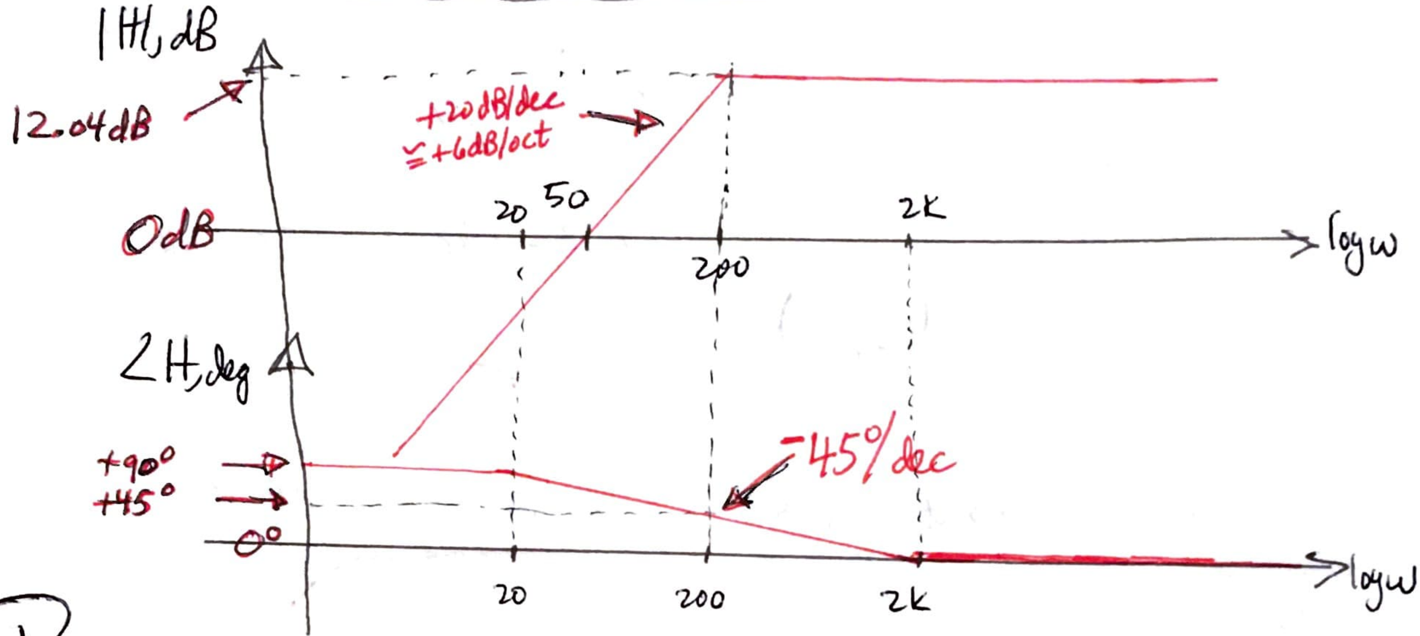
↖ 0dB crossing frequency ω_x = ?

$\Rightarrow 0\text{dB} + \frac{20\text{dB}}{\text{dec}}\left(\#\text{ decades between } \omega_x \text{ \& } 200\right) = 12.04\text{dB} \Rightarrow \#\text{ decades} = \frac{12.04}{20} \cong 0.602 \text{ decades}$

$\Rightarrow \log_{10}\left(200/\omega_x\right) = 0.602 \Rightarrow \omega_x = 200/10^{0.602} \simeq 200/4 = 50\text{rps}$

(cont'd)

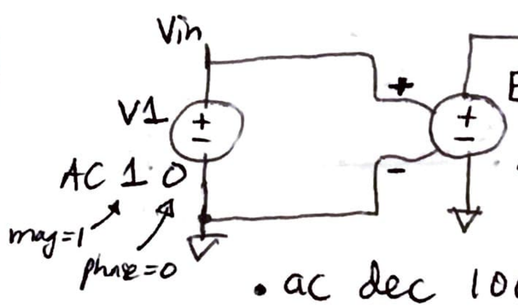
→ the overall approximate phase response for $H(j\omega)$ is obtained by shifting the pole phase up by 90° (due to the phase of the zero at DC); the K term is positive, so has 0° phase contribution; here is the total Bode approximation of $H(j\omega)$, for completeness:



#1

→ Can confirm approximation w/ high-accuracy LTSpice simulation; if you have the circuit, you can simulate that; if you have $H(s)$, you can use the "Laplace" VCVS (E source); e.g.,

.asc file example



put this in "value" field of the E source

Laplace(10*(s/500)/(1+s/200))

.ac dec 100 1 1k ← LTSpice works in Hz, not rps; 200rps ≅ 31.83Hz, 50 rps ≅ 7.96 Hz

<see screenshots of .asc & Bode plot>

→ Errors in the approximation relative to the high-accuracy result:

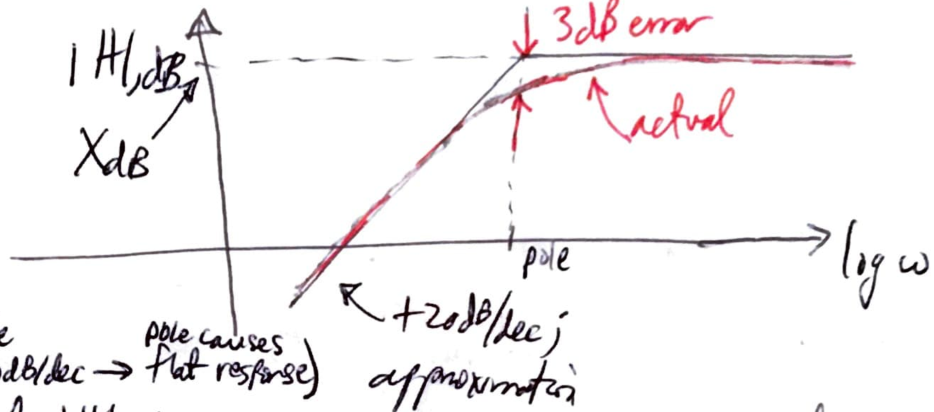
⇒ Actual phase response has more curvature

⇒ The magnitude response has a 3dB error at the pole frequency

2→

More on "3dB" errors at pole/zero frequencies:

(*) "3dB" errors in the approximation at pole frequencies ⇒

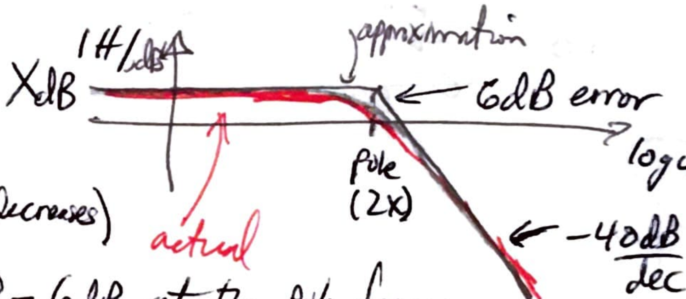


why is this a pole?
poles cause magnitude response to decrease after the pole, relative to the preceding trend. (i.e., +20dB/dec → pole causes flat response)

⇒ here, the actual |H|,dB is XdB − 3dB at the pole frequency

⇒ if the trend (preceding) (after) the pole is $\left(\begin{smallmatrix}+\\-\end{smallmatrix}\frac{20dB}{dec}\right)n$, n integer, $n \geq 1$,

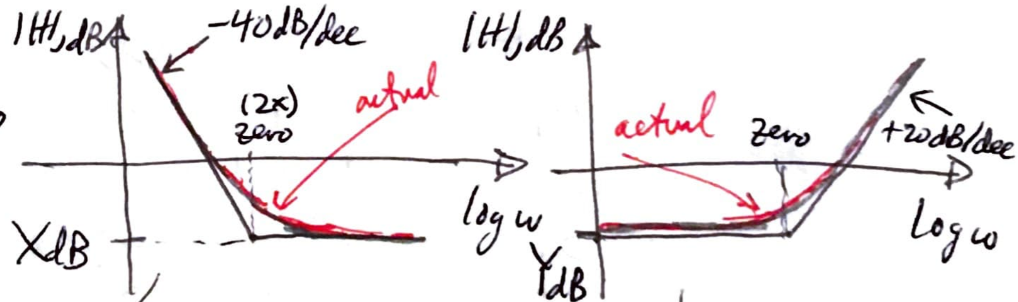
then the error of the approximation relative to the actual will be −n 3dB at the pole.



⇒ Another pole scenario ⇒
(we know it's a pole because |H| decreases)

Here, the actual |H|,dB is XdB − 6dB at the pole frequency.

(*) "3dB" errors in the approximation at zero frequencies; two example scenarios



here, the ~~actual~~ actual |H|,dB is XdB + 6dB at the zero frequency

here, the actual |H|,dB is YdB + 3dB at the zero frequency.

basic idea: if the trend (preceding) (after) the zero is $\left(\begin{smallmatrix}-\\+\end{smallmatrix}\frac{20dB}{dec}\right)n$, n integer, $n \geq 1$,

then the error of the approximation relative to the actual will be +n 3dB at the zero.

→ p

#2

→ Can also confirm Bode approximation w/ high-accuracy Matlab simulation.

option (a)

⇒ can write $H(s)$ in the following form:

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

numerator polynomial in s, order m

denominator polynomial in s, order n

⇒ store coefficients for the two polynomials in two vectors, say num & den:

```
num=[bm  bm-1 .... b1  b0];
den=[an  an-1 .... a1  a0];
```

⇒ create a transfer function object, say H, using tf:

```
H=tf(num, den);
```

⇒ have Matlab plot out Bode response:

```
bode(H)
```

⇒ can optionally tell Matlab to plot Bode response over some specific range or rad/sec values; e.g.,

```
wmin=10;  wmax=1e4;
bode(H,{wmin, wmax})
```

Option (b)

⇒ create transfer function object `s=tf('s')`

⇒ then specify transfer function directly as expression in s; e.g.,

```
H=(s+1)/(s^2+3*s+1);
```

⇒ then plot Bode response with `bode(H)` (or `bode(H,{wmin, wmax})`, as above)

→

→ For the example we are working on here (starting on p. 46), we have:

$$H(s) = \left[\frac{10\left(s/\cancel{500}\right)}{\left(1+s/200\right)}\right]\frac{(\cancel{500})}{(500)} = \frac{10s}{2.5s+500} = \frac{10s+0}{2.5s+500}$$

⇒ using option (a) on previous page, we have, in Matlab:

```
num=[10 0]; den=[2.5 500];
H=tf(num,den);
bode(H)
```

⇒ <if ran in class, note the very good agreement between Bode approximation & actual Bode response in Matlab>

⇒ See help bode in Matlab for more details

<Note that help <function> is very useful for your future courses/career, too!>

→ Another Matlab bode example: plot Bode response of

$$H(s) = \frac{s^2 + 0.1s + 7.5}{s^4 + 0.12s^3 + 9s^2}$$

⇒ In Matlab, do:

```
H=tf([1 0.1 7.5],[1 0.12 9 0 0]);
bode(H)
```

s?, $j\omega$?, $H(s)$?, phasors? ⟨week 14 discussion⟩ Sp. 2021 ELEN 2425

→ 

$V_{out}(s) = V_{in}(s)\,H(s)$

Laplace transform of $v_{in}(t)$:

$V_{in}(s) = \mathcal{L}\{v_{in}(t)\}$

Laplace transform of ckt/system impulse response $h(t)$:

$H(s) = \mathcal{L}\{h(t)\}$

⟨system poles & zeros in the complex s-plane⟩

more on $h(t)$ in Signals & Systems

Laplace transform of $v_{out}(t)$:

$V_{out}(s) = \mathcal{L}\{v_{out}(t)\}$

→ Setting $s = j\omega$ yields the Fourier transform of the signal of interest:

frequency response

$$V_{in}(j\omega) = \mathcal{F}\{v_{in}(t)\}, \quad H(j\omega) = \mathcal{F}\{h(t)\}, \quad V_{out}(j\omega) = \mathcal{F}\{v_{out}(t)\} = V_{in}(j\omega)\,H(j\omega)$$

⟨system frequency response⟩

→ In the specific case of sinusoidal signals, the Fourier transform of the time-domain signal can be viewed as a phasor, e.g.,

$V_{in}(j\omega) = \tilde{V}_{in}$, $V_{out}(j\omega) = \tilde{V}_{out}$, where

$$\tilde{V}_{out} = \tilde{V}_{in}\,H(j\omega)$$

☆ Note that the connection between Fourier transforms & phasors of sinusoidal signals can be somewhat nuanced, but this is essentially the main idea. (For sinusoids, the phasor transform & Fourier transform are essentially two different ways of representing the same information.)