

- 1) Consider the circuit shown in Figure 1 below.

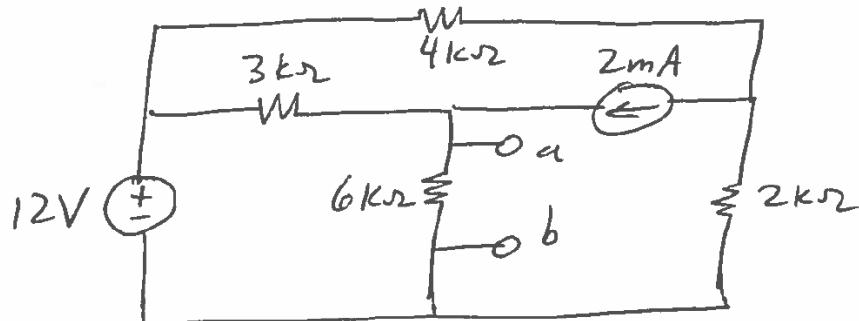
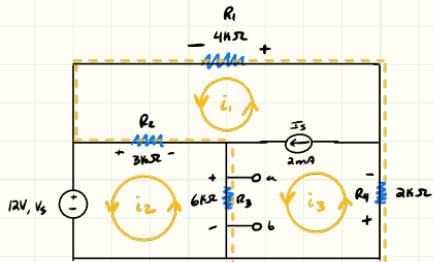


Figure 1. Circuit for Problem 1.

- a) Find the Thevenin and Norton equivalent circuit models with respect to nodes a and b. **Clearly draw the Thevenin and Norton models with numerical component values and the nodes of interest labeled on the model schematics.** *Show your work.* List your numerical model parameters with appropriate units and in proper engineering notation format with two decimal places of precision.
- b) Verify your by-hand answers for the Thevenin and Norton model parameters in LTSpice. **Write out the answers you got from LTSpice** and attach any supporting evidence that you actually simulated the circuit to verify your work. The answers you write out for this part should be the same as the results you provided in part (a); if they don't match **by both sign and magnitude**, expect zero credit. Also, expect zero credit 1) if your by-hand and simulated results agree but the results are incorrect, 2) if you do not perform the LTSpice verification altogether, 3) if you only do the LTSpice verification but not the by-hand solution, or 4) if you do not label key node voltages and/or use properly-oriented 0 V voltage source ammeters for measuring key branch currents in LTSpice.

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$$I_S = i_3 - i_1 \quad (1)$$

KVL at mesh 1 & 3

$$R_4 i_3 + R_1 i_1 + R_2 (i_1 - i_2) + R_3 (i_3 - i_2) = 0$$

$$R_4 i_3 + R_1 i_1 + R_2 i_1 - R_2 i_2 + R_3 i_3 - R_3 i_2 = 0$$

$$(R_1 + R_2) i_1 + (-R_2 - R_3) i_2 + (R_3 + R_4) i_3 = 0 \quad (2)$$

KVL at  $i_2$

$$-R_3 (i_3 - i_2) - R_2 (i_1 - i_2) + V_s = 0$$

$$R_3 (i_3 - i_2) + R_2 (i_1 - i_2) = V_s$$

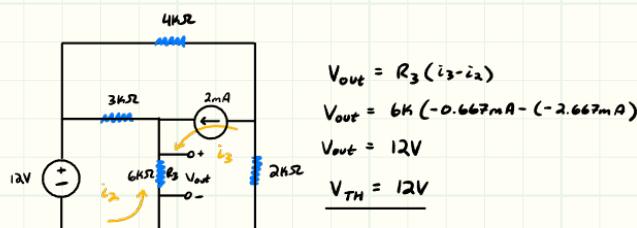
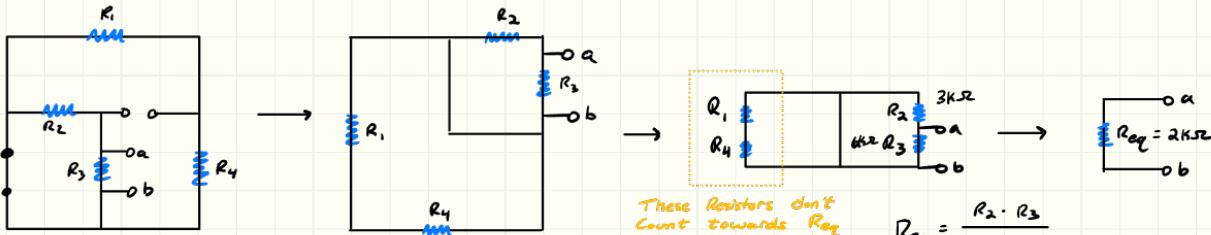
$$R_3 i_3 - R_3 i_2 + R_2 i_1 - R_2 i_2 = V_s$$

$$(R_2) i_1 + (-R_2 - R_3) i_2 + (R_3) i_3 = V_s \quad (3)$$

$$\begin{bmatrix} I_S \\ V_s \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ (R_2) & (-R_2 - R_3) & (R_3) \\ (R_3) & (-R_2 - R_3) & (R_2 + R_3) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 3k & -9k & 6k \\ 7k & -9k & 8k \end{bmatrix} \begin{bmatrix} 0.002A \\ 12V \\ 0 \end{bmatrix}$$

$$i_1 = -2.667mA \quad i_2 = -2.667mA \quad i_3 = -0.667mA$$

Finding  $R_{TH}$ ; Deactivate CKT.



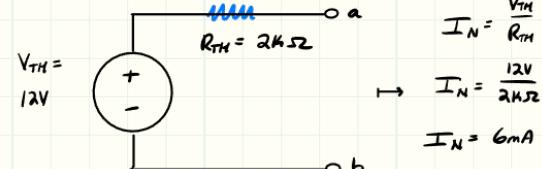
These Resistors don't Count towards  $R_{eq}$  because the Voltage Potential across them is zero

$$R_{eq} = \frac{R_2 \cdot R_3}{R_2 + R_3}$$

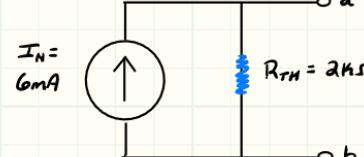
$$R_{eq} = \frac{(3k\Omega \cdot 6k\Omega)}{3k\Omega + 6k\Omega}$$

$$R_{eq} = R_{TH} = 2k\Omega$$

$$R_{TH} = 2k\Omega$$



Thevenin

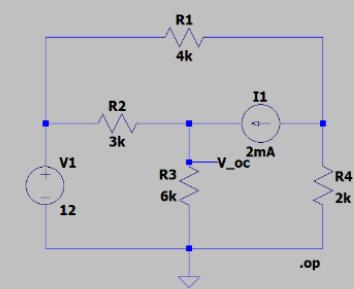


Norton

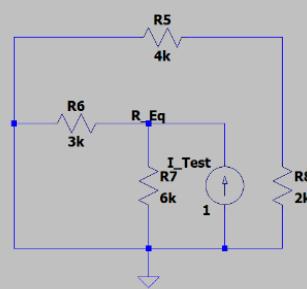
#### --- Operating Point ---

```
V(n001): 12      voltage
V(v_oc): 12      voltage
V(n002): 1.33333 voltage
V(r_eq): 2000    voltage
V(n003): 0       voltage
V(n005): -12     voltage
V(n004): 0       voltage
V(n006): -12     voltage
I(I1): 0.002    device_current
I(I_test): 1     device_current
I(R2): 0       device_current
I(R3): -0.002   device_current
I(R4): 0.000666667 device_current
I(R1): -0.002666667 device_current
I(R6): 0.6666667 device_current
I(R7): 0.3333333 device_current
I(R5): 0       device_current
I(R8): 0       device_current
I(R10): 0.004   device_current
I(R9): 0       device_current
I(R12): 0       device_current
I(R11): 0.002   device_current
I(V1): -0.002666667 device_current
I(V_test): -0.006  device_current
I(I_n): 0.006   device_current
```

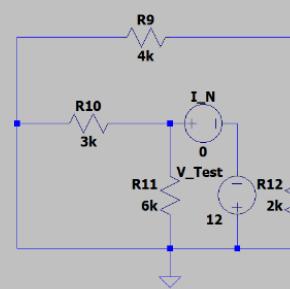
#### -- Finding $V_{th}$ --



#### -- Finding $R_{eq}$ --



#### -- Finding $I_{Norton}$ --



- 2) Consider the circuit shown in Figure 2 below.

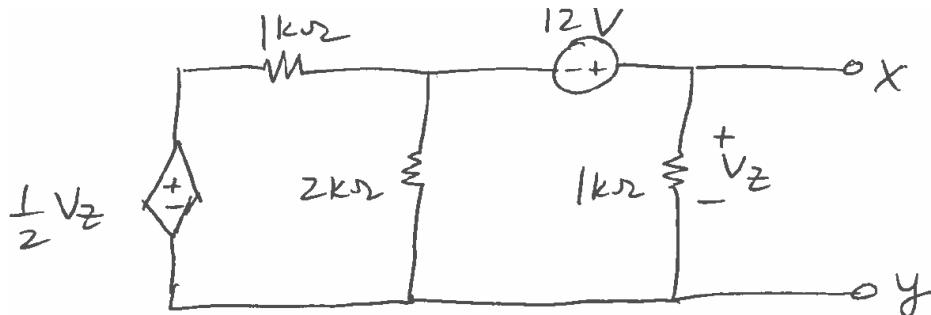
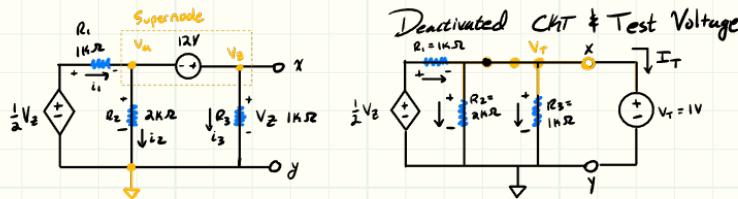


Figure 2. Circuit for Problem 2.

- a) Find the Thevenin and Norton equivalent circuit models with respect to nodes x and y. **Clearly draw the Thevenin and Norton models with numerical component values and the nodes of interest labeled on the model schematics. Show your work.** List your numerical model parameters with appropriate units and in proper engineering notation format with two decimal places of precision.
- b) Verify your by-hand answers for the Thevenin and Norton model parameters in LTSpice. **Write out the answers you got from LTSpice** and attach any supporting evidence that you actually simulated the circuit to verify your work. The answers you write out for this part should be the same as the results you provided in part (a); if they don't match **by both sign and magnitude**, expect zero credit. Also, expect zero credit 1) if your by-hand and simulated results agree but the results are incorrect, 2) if you do not perform the LTSpice verification altogether, 3) if you only do the LTSpice verification but not the by-hand solution, or 4) if you do not label key node voltages and/or use properly-oriented 0 V voltage source ammeters for measuring key branch currents in LTSpice.

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KCL @ Supernode

$$i_1 = i_2 + i_3$$

$$G_1(\frac{1}{2}V_z - V_a) = G_{12}(V_a - 0) + G_{13}(V_z - 0)$$

$$\frac{1}{2}G_1V_z - G_1V_a = G_{12}V_a + G_{13}V_z$$

$$0 = G_{12}V_a + G_{13}V_z + G_1V_a - \frac{1}{2}G_1V_z$$

$$0 = (G_{12} + G_{13})V_a + (-\frac{1}{2}G_1 + G_{13})V_z \quad (1)$$

$$V_s = V_z - V_a$$

$$12V = -V_a + V_z \quad (2)$$

$$\begin{cases} G_1 = \frac{1}{1000} \text{ S} \\ G_{12} = \frac{1}{2000} \text{ S} \\ G_{13} = \frac{1}{1000} \text{ S} \end{cases}$$

$$I_T = 2 \text{ mA}$$

$$R_{TH} = \frac{V_r}{I_T} = \frac{1V}{2 \text{ mA}}$$

$$R_{TH} = 500 \Omega$$

$$\begin{bmatrix} 0 \\ 12V \end{bmatrix} = \begin{bmatrix} (G_{12} + G_{13}) & (-\frac{1}{2}G_1 + G_{13}) \\ -1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_z \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_z \end{bmatrix} = \begin{bmatrix} 3/2000 & 1/2000 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 12V \end{bmatrix}$$

$$V_a = -3V \quad I_N = \frac{V_{TH}}{R_{TH}} = \frac{9V}{500 \Omega}$$

$$V_z = 9V \quad I_N = 18 \text{ mA}$$

$$V_{TH} = 9V \quad I_N = 18 \text{ mA}$$

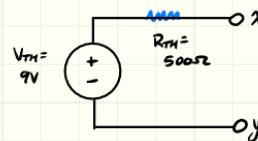
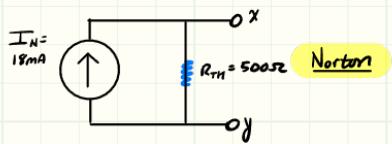
KCL at V<sub>r</sub>

$$G_1(\frac{1}{2}V_z - V_r) - G_{12}(V_r - 0) - G_{13}(V_r - 0) - I_T = 0$$

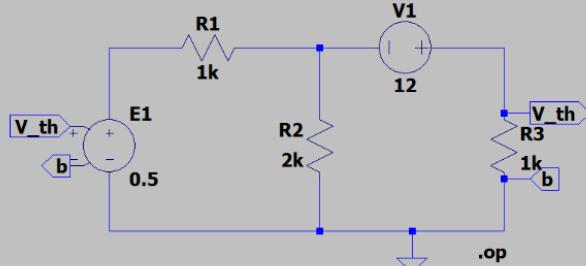
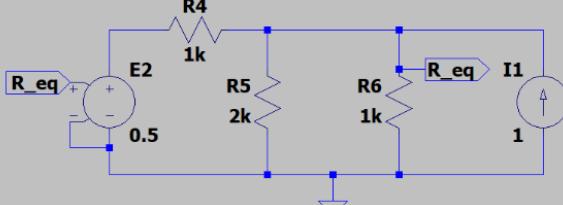
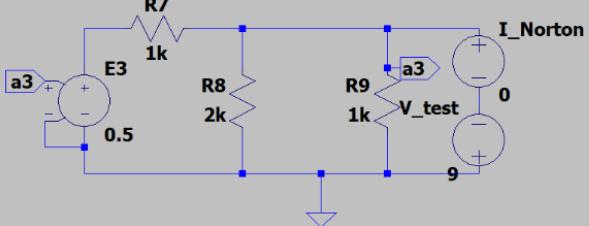
$$\frac{1}{2}G_1V_z - G_1V_r - G_{12}V_r - G_{13}V_r - I_T = 0$$

$$\frac{1}{2}G_1V_z + (-G_1 - G_{12} - G_{13})V_r = I_T$$

$$\frac{1}{2}\left(\frac{1}{1000 \Omega}\right)(9V) + \left(-\frac{1}{1000 \Omega} - \frac{1}{2000 \Omega} - \frac{1}{1000 \Omega}\right)(1V) = I_T$$

TheveninNorton**--- Operating Point ---**

V(n001):	4.5	voltage
V(v_th):	9	voltage
V(n002):	-3	voltage
V(n003):	250	voltage
V(r_eq):	500	voltage
V(n004):	-4.5	voltage
V(a3):	-9	voltage
V(n005):	-9	voltage
I(I1):	1	device_current
I(R1):	0.0075	device_current
I(R2):	0.0015	device_current
I(R3):	0.009	device_current
I(R4):	0.25	device_current
I(R5):	-0.25	device_current
I(R6):	-0.5	device_current
I(R7):	0.0045	device_current
I(R8):	0.0045	device_current
I(R9):	0.009	device_current
I(E1):	-0.0075	device_current
I(E2):	0.25	device_current
I(E3):	-0.0045	device_current
I(V1):	-0.009	device_current
I(I_norton):	0.018	device_current
I(V_test):	-0.018	device_current

**-- Finding V<sub>th</sub> --****-- Finding R<sub>th</sub> --****-- Finding I<sub>norton</sub> --**

- 3) The variable resistor,  $R_L$ , in the circuit shown in Figure 3 is adjusted for maximum power transfer to  $R_L$ . **For the parts below, include appropriate units with your numerical answers in proper engineering notation format.** Keep at least two decimal places of precision. *Show your work.*

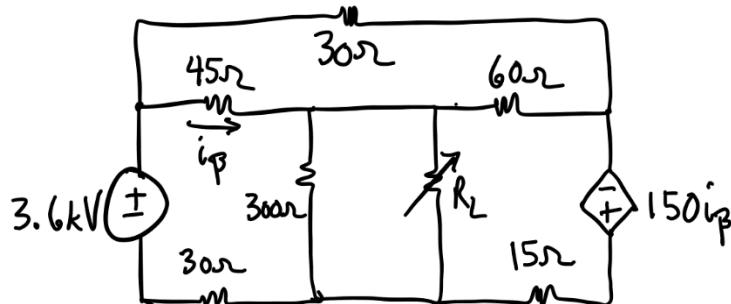
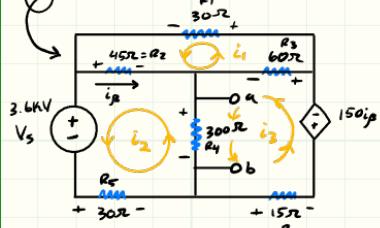
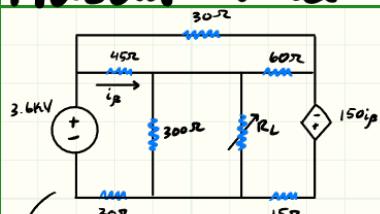


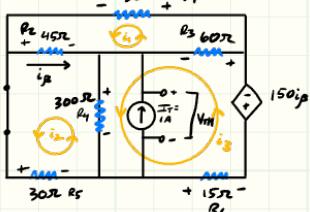
Figure 3. Circuit for Problem 3.

- Find the numerical value of  $R_L$ .
- Find the maximum power transferred to  $R_L$ .
- Verify your results for parts (a) and (b) in LTSpice. **Write out the answers you got from LTSpice** and attach any supporting evidence that you actually simulated the circuit to verify your work. The answers you write out for this part should be the same as the results you provided in part (a); if they don't match **by both sign and magnitude**, expect zero credit. Also, expect zero credit 1) if your by-hand and simulated results agree but the results are incorrect, 2) if you do not perform the LTSpice verification altogether, 3) if you only do the LTSpice verification but not the by-hand solution, or 4) if you do not label key node voltages and/or use properly-oriented 0 V voltage source ammeters for measuring key branch currents in LTSpice.

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Deactivate Ckt1 and Find Req



$$b.) P_L = \frac{(V_{TH})^2}{4R_L} = \frac{(360V)^2}{4(90)} = 360W$$

$$P_L = 360W$$

$$i_B = i_1 - i_2 \rightarrow 150(i_1 - i_2)$$

KVL @ i<sub>1</sub>

$$\begin{aligned} R_2(i_1 - i_2) - R_3(i_3 - i_1) + R_4i_4 &= 0 \\ R_2i_1 - R_2i_2 - R_2i_3 + R_2i_1 + R_4i_4 &= 0 \\ (R_1 + R_2 + R_3)i_1 + (-R_2)i_2 + (-R_3)i_3 &= 0 \quad (1) \end{aligned}$$

KVL @ i<sub>2</sub>

$$V_s + R_5(i_2) - R_4(i_2 - i_1) - R_2(i_1 - i_2) = 0$$

$$-V_s = R_5i_2 - R_4i_2 + R_4i_1 - R_2i_1 + R_2i_2$$

$$-V_s = (-R_2)i_1 + (R_2 + R_4 + R_5)i_2 + (-R_4)i_3 \quad (2)$$

KVL @ i<sub>3</sub>

$$R_6(i_3) + [150(i_1 - i_2)] + R_3(i_3 - i_1) + R_4(i_3 - i_2) = 0$$

$$R_6i_3 + 150i_1 - 150i_2 + R_3i_3 - R_3i_1 + R_4i_3 - R_4i_2 = 0$$

$$(150 - R_3)i_1 + (-150 - R_4)i_2 + (R_3 + R_4 + R_6)i_3 = 0 \quad (3)$$

$$\begin{bmatrix} (R_1 + R_2 + R_3) & (-R_2) & (-R_3) \\ (-R_2) & (R_5 + R_4 + R_6) & (-R_4) \\ (150 - R_3) & (-150 - R_4) & (R_3 + R_4 + R_6) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -V_s \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 135\Omega & -45\Omega & -60\Omega \\ -45\Omega & 375\Omega & -300\Omega \\ 90\Omega & -450\Omega & 375\Omega \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3.6kV \\ 0 \end{bmatrix}$$

$$i_1 = 78A \quad i_2 = 99.6A \quad i_3 = 100.8A$$

$$i_B = i_1 - i_2 = \boxed{78A - 99.6A} = -21.6A$$

$$V_{TH} = R_4(i_3 - i_2)$$

$$V_{TH} = 300\Omega(100.8 - 99.6)$$

$$\underline{V_{TH} = 360V}$$

$$i_{SC} = i_2 - i_3$$

from i<sub>1</sub>

$$-45i_1 - 60i_2 + 135i_3 = 0$$

from i<sub>2</sub>

$$-3600 + 45i_1 - 45i_3 + 30i_2 = 0$$

$$70i_1 - 45i_3 = 3600$$

from i<sub>3</sub>

$$75i_2 - 60i_3 - 150(i_1 - i_3) = 0$$

$$-150i_1 + 75i_2 + 90i_3 = 0$$

$$\begin{bmatrix} 75 & 0 & -45 \\ -150 & 75 & 90 \\ -45 & -60 & 135 \end{bmatrix}^{-1} \begin{bmatrix} 3600 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

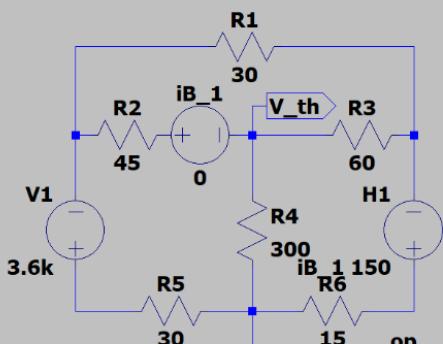
$$i_1 = 92A \quad i_{SC} = 96 - 92 = 4A$$

$$i_2 = 96A \quad R_{TH} = \frac{V_{TH}}{i_{SC}} = \frac{360V}{4A}$$

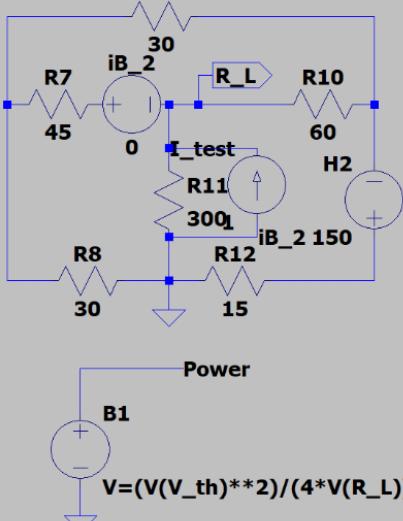
$$R_{TH} = 90\Omega$$

$$\boxed{R_L = 90\Omega}$$

-- Finding V<sub>th</sub> --



-- Finding R<sub>L</sub> --



$$Power$$

$$V = (V(V_{th})^2) / (4 * V(R_L))$$

--- Operating Point ---

V(n002):	1728	voltage
V(n001):	-612	voltage
V(p001):	360	voltage
V(v_th):	360	voltage
V(n003):	2988	voltage
V(n004):	-1512	voltage
V(p002):	90	voltage
V(n005):	57	voltage
V(n006):	92	voltage
V(r_l):	90	voltage
V(n007):	-18	voltage
V(power):	360	voltage
I(B1):	0	device_current
I(H1):	100.8	device_current
I(H2):	1.2	device_current
I(I_test):	1	device_current
I(R1):	78	device_current
I(R2):	21.6	device_current
I(R3):	22.8	device_current
I(R4):	1.2	device_current
I(R5):	-99.6	device_current
I(R6):	-100.8	device_current
I(R7):	0.733333	device_current
I(R8):	-1.9	device_current
I(R10):	0.0333333	device_current
I(R9):	1.16667	device_current
I(R11):	0.3	device_current
I(R12):	-1.2	device_current
I(V1):	-99.6	device_current
I(Ib_1):	-21.6	device_current
I(Ib_2):	-0.733333	device_current

- 4) Current  $i$  in Figure 4 below can be written as  $i = i_{90V} + i_{40V}$ , where  $i_{90V}$  and  $i_{40V}$  are the components of  $i$  due to the 90 V and 40 V voltage sources, respectively.

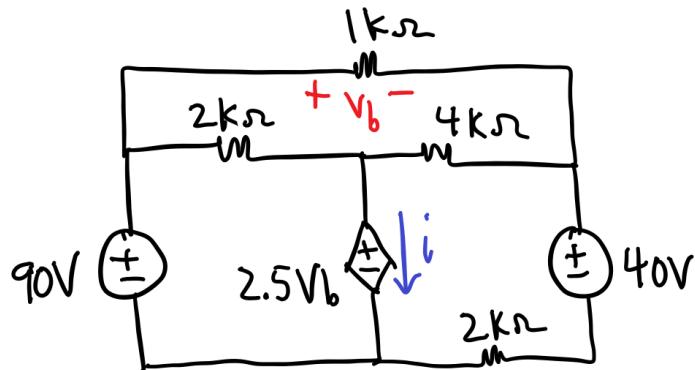
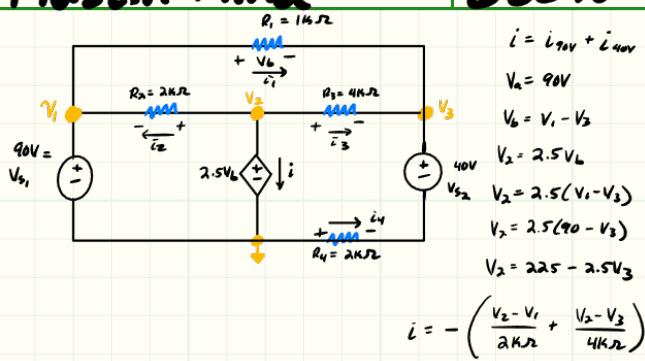


Figure 4. Circuit for Problem 4.

For the parts below, include appropriate units with your numerical answers in proper engineering notation format and with at least two decimal places of precision. Keep at least two decimal places of precision. *Show your work.*

- Determine the numerical value of  $i_{90V}$ .
- Determine the numerical value of  $i_{40V}$ .
- Using the principle of superposition and the results of parts (a) and (b), determine the numerical value of  $i$ .
- Verify your results for parts (a), (b), and (c) in LTSpice. **Write out the answers you got from LTSpice** and attach any supporting evidence that you actually simulated the circuit to verify your work. The answers you write out for this part should be the same as the results you provided in part (a); if they don't match **by both sign and magnitude**, expect zero credit. Also, expect zero credit 1) if your by-hand and simulated results agree but the results are incorrect, 2) if you do not perform the LTSpice verification altogether, 3) if you only do the LTSpice verification but not the by-hand solution, or 4) if you do not label key node voltages and/or use properly-oriented 0 V voltage source ammeters for measuring key branch currents in LTSpice.



Superposition ( $i_{90V}$ ): short 40V source

$R_1 = 14.5\Omega$

$i_{90V} = \frac{V_1 - V_3}{2K\Omega}$

$\left( \frac{V_1 - V_3}{1K\Omega} \right) + \left( \frac{V_2 - V_3}{4K\Omega} \right) - \left( \frac{V_2 - 0}{2K\Omega} \right) = 0$

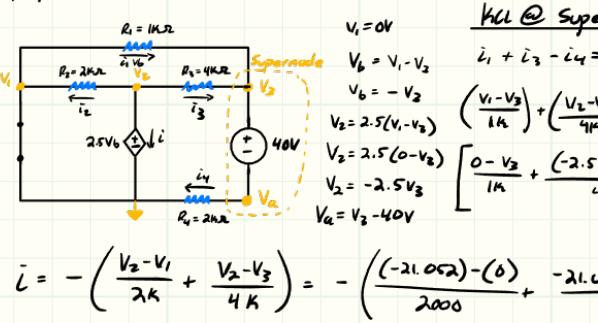
$\frac{(90V) - V_3}{1K\Omega} + \frac{(225 - 2.5V_3) - V_3}{4K\Omega} = \frac{V_3}{2K\Omega}$

$4(90) - 4V_3 + 225 - 3.5V_3 = 2V_3$

$360 + 225 = 9.5V_3$

$V_3 = 61.579V$

Superposition ( $i_{40V}$ ): short 90V.



KCL @ Supernode  $V_2 = 71.052V$

$i_1 + i_3 - i_4 = 0$

$\left( \frac{V_1 - V_3}{1K\Omega} \right) + \left( \frac{V_2 - V_3}{4K\Omega} \right) - \left( \frac{V_4 - 0}{2K\Omega} \right) = 0$

$\left[ \frac{0 - V_3}{1K\Omega} + \frac{(-2.5V_3) - V_3}{4K\Omega} - \frac{(V_4 - 40) - 0}{2K\Omega} \right] \cdot 4K = 0$

$4(-V_3) - 3.5V_3 - 2(V_3 - 40) = 0$

$-4V_3 - 3.5V_3 - 2V_3 + 80 = 0$

$80V = 9.5V_3$

$V_3 = 8.42V$

$V_2 = -2.5V_3$

$V_2 = -2.5(8.42)$

$V_2 = -21.052$

$i = i_{90V} + i_{40V}$

$i = (7.11mA) + (17.89mA)$

$i = 25mA$

