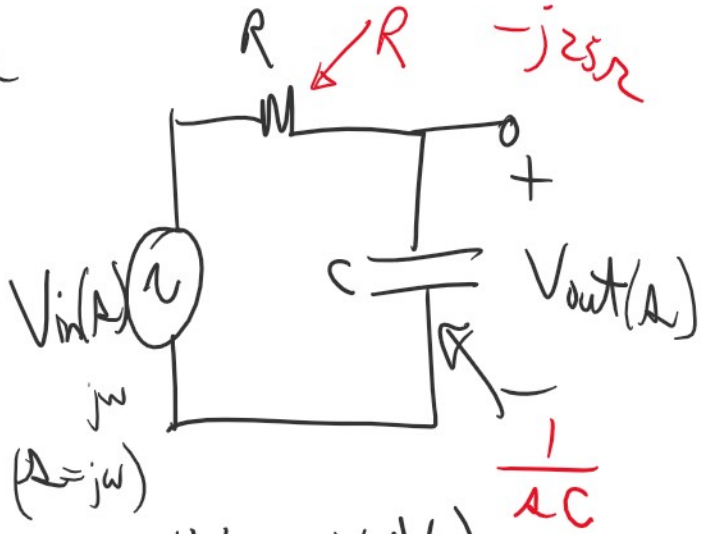
Friday, October 26, 2018 9:59 AM

Consider the frequency response of an RC filter:



transfer function: how much of the input "transfers" to the output

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

pole/zero view

frequency response $\Rightarrow$ $H(jw) = \frac{V_{out}(jw)}{V_{in}(jw)}$

what type of filter? LPF, HPF, BPF

$$\Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{sC}}{\frac{1}{sC}+R} = \frac{1}{1+sRC} = \frac{1}{1+s/w_p}$$

TF

proper Bode approx form

$w_p = 1/RC$

$$\Rightarrow \text{Freq. response } H(jw) = \frac{V_{out}(jw)}{V_{in}(jw)} = \frac{1}{1+jwRC}$$

Suppose $V_{in}(t) = 2\cos(10t+20^\circ) + 10\cos(100kt-40^\circ)$ V

$w_1 = 10$ rps $\quad w_2 = 100$ krps

$\angle 2$ $\quad \angle 0^\circ$

$V_{out}(t) = 2|H(jw)|\ldots$

$$V_{out}(t) = 2\,|H(j\omega_1)|\cos(\omega_1 t + 20^\circ + \angle H(j\omega_1)) + \dots$$

$0 < x \le 1$, $x \ge 1$

≈ 0

$\approx 100m$

$\approx -130^\circ$

$$\dots + 10\,|H(j\omega_2)|\cos(\omega_2 t - 40^\circ + \angle H(j\omega_2))$$

$$|H(j\omega)| = \left|\frac{1}{1+j\omega RC}\right| = \frac{|1|}{|1+j\omega RC|} \approx \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\angle H(j\omega) = \angle\frac{1}{1+j\omega RC} \approx \underbrace{\angle(1)}_{\angle(1\angle 0^\circ)} - \angle(1+j\omega RC)$$

$1\angle 0^\circ$

$$\approx 0^\circ - \tan^{-1}\left(\frac{\omega RC}{1}\right) \approx -\tan^{-1}(\omega RC)$$

$R \approx 1k\Omega$, $C \approx 1\mu F$

$$|H(j\omega_1)| = \frac{1}{\sqrt{1+[(10)(1k)(1\mu)]^2}} \approx 999.95m\,V/V$$

$(\approx 1\,V/V) \leftarrow 0dB$

$$\angle H(j\omega_1) \approx -\tan^{-1}((10)(1k)(1\mu)) \approx -572.94\,m\,deg.$$

$(\approx 0^\circ)$

$|H(\omega_1)|$ $\quad$ 1 $\quad$ $R \approx 1k\Omega$

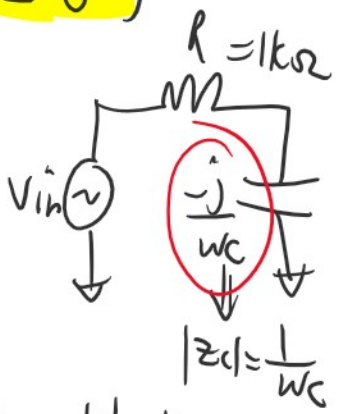
$$|H(j\omega_2)| \simeq \frac{1}{\sqrt{1+[(100k)(1k)(1\mu)]^2}}$$

$$\simeq 9.9995\ mV/V \quad (\simeq 10\ mV/V)$$

$$\angle H(j\omega_2) \simeq -\tan^{-1}((100k)(1k)(1\mu))$$

$$\simeq -89.4271^\circ \ (\simeq -90^\circ)$$

$(\simeq 0^\circ)$



Bode response of an RC filter