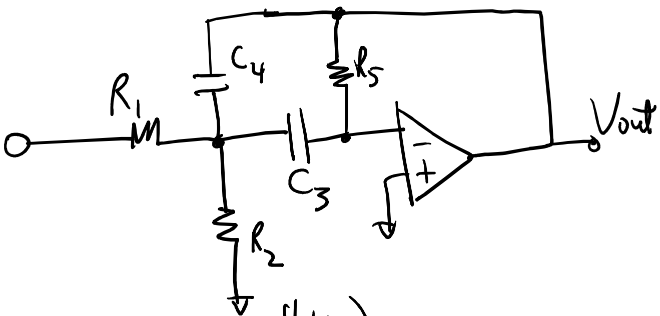
Bode Example

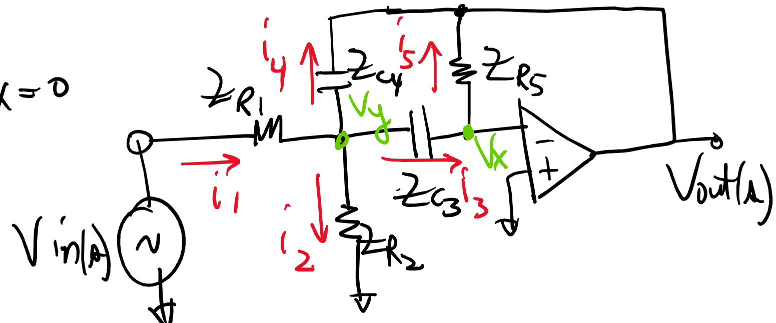


Derive TF (transfer function) $H(s) = V_{out}(s)/V_{in}(s)$, &

Sketch Bode approximation of frequency response $H(j\omega)$

Sol'n V_X is driven to ground by negative feedback $\Rightarrow V_X = 0$

$\Rightarrow V_X$ is a virtual ground.



KCL @ V_y: $i_1 = i_2 + i_3 + i_4$

$$\Rightarrow Y_{R_1}(V_{in} - V_y) = Y_{R_2} V_y + Y_{C_3}(V_y - \cancelto{0}{V_x}) + Y_{C_4}(V_y - V_{out})$$

$$\Rightarrow V_y = \frac{Y_{R_1} V_{in} + Y_{C_4} V_{out}}{Y_{R_1} + Y_{R_2} + Y_{C_3} + Y_{C_4}} \quad (1)$$

KCL @ V_X: $i_3 = i_5 \Rightarrow Y_{C_3}(V_y - \cancelto{0}{V_x}) = Y_{R_5}(\cancelto{0}{V_x} - V_{out})$

$$\Rightarrow V_y = -\frac{Y_{R_5}}{Y_{C_3}} V_{out} \quad (2)$$

$\Rightarrow$ Now set the RHSs of (1) & (2) equal to eliminate V_y

$$\Rightarrow \frac{Y_{R_1} V_{in} + Y_{C_4} V_{out}}{Y_{R_1} + Y_{R_2} + Y_{C_3} + Y_{C_4}} \overset{\text{set}}{=} -\frac{Y_{R_5}}{Y_{C_3}} V_{out}$$

$\rightarrow$ do some algebra … & get $\frac{V_{out}(s)}{V_{in}(s)} = H(s)$

$$\Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-s\left(\frac{1}{R_1 C_4}\right)}{s^2 + s\left(\frac{C_3 + C_4}{\ldots}\right) + \left(\frac{G_1 + G_2}{\ldots}\right)} \quad (3)$$

$$V_{in}(s) \qquad \overline{s^2 + s\left(\frac{C_3+C_4}{R_5C_3C_4}\right) + \left(\frac{G_1+G_2}{R_5C_3C_4}\right)} \quad (5)$$

This $H(s)$ has the form

<Recall that $Q = 1/2\zeta$>

$$H(s) = \frac{-\alpha\omega_0 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \quad \text{OR} \quad \frac{-\alpha\omega_0 s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

<where $\alpha\omega_0 = \frac{1}{R_1C_4}$>

compare

$\Rightarrow$ Note that $H(s)$ is not yet in proper Bode approximation form. (more soon.)

What kind of filter is this?

(*) $\lim_{s\to 0} H(s) = 0$ ← attenuates low frequencies, so this can't be a LPF or BSF

(*) $\lim_{s\to\infty} H(s) = 1/s = 0$ ← attenuates high frequencies as well, so this can't be a HPF, either

→ This must be a BPF

Now, let's put $H(s)$ into proper Bode form:

$$H(s) = \frac{-\alpha\omega_0 s}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \overset{\text{OR}}{=} \frac{-\alpha\omega_0 s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Factor ω_0^2 out of denominator:

$$\Rightarrow H(s) = \frac{-\left(\alpha\,\cancel{\omega_0}/\omega_0^{\cancel{2}}\right)s}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} = \frac{-\left[s\big/\left(\omega_0/\alpha\right)\right]}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

Suppose $R_1 = 49k\Omega$, $R_2 = 1k\Omega$, $R_5 = 98k\Omega$, $C_3 = C_4 = 427.3763\,pF$

From (3) above,

$$\rightarrow \omega_0 = \sqrt{\frac{G_1+G_2}{R_5 C_3 C_4}} \simeq \underbrace{238.7611\,krps}_{\omega_0} \left(\simeq 2\pi\,(\underbrace{38kHz}_{f_0})\right)$$

$$\rightarrow \alpha\omega_0 = \frac{1}{R_1 C_4} \Rightarrow \alpha = \frac{\left(1/R_1 C_4\right)}{\omega_0} = G_1\sqrt{\frac{R_5 C_3}{C_4(G_1+G_2)}}$$

$$\rightarrow \underbrace{\omega_0/\alpha}_{} = \frac{\sqrt{\frac{G_1+G_2}{R_5 C_3 \cancel{C_4}}}}{G_1\sqrt{\frac{R_5 C_3}{\cancel{C_4}(G_1+G_2)}}} = \frac{R_1(G_1+G_2)}{R_5 C_3} = \frac{1+R_1/R_2}{R_5 C_3}$$

0dB crossing frequency of zero @ DC term

$$\Rightarrow \omega_0/\alpha \simeq 1.1938\,Mrps \left(\simeq 2\pi\,(190kHz)\right)$$

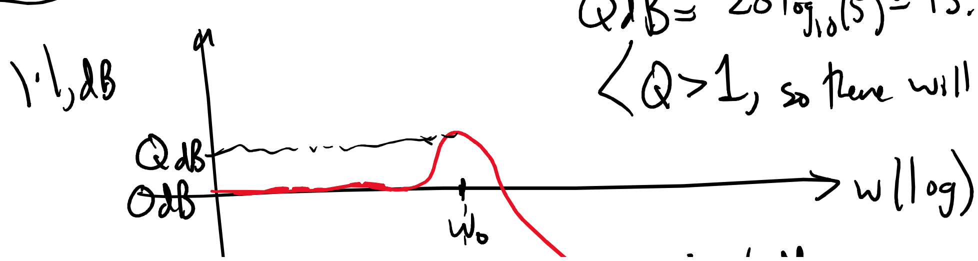
$$\rightarrow Q = \frac{1}{2\zeta} = \frac{\sqrt{\frac{(G_1+G_2)}{R_5 C_3 C_4}}}{\left(\frac{C_3+C_4}{R_5 C_3 C_4}\right)} = \frac{\sqrt{(G_1+G_2)R_5 C_3 C_4}}{C_3+C_4}$$

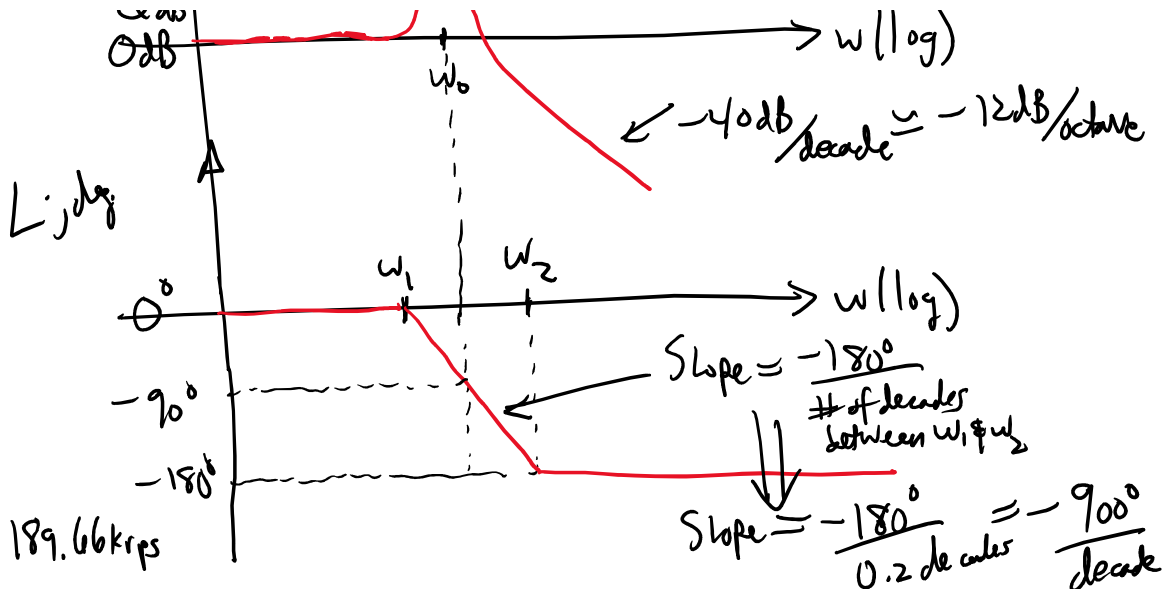
$$\Rightarrow \underbrace{Q = 5 \;\;\&\;\; \zeta = \frac{1}{2Q} = 0.1}_{\text{underdamped!}}$$

→ let's sketch the underdamped denominator term first:

$Q\,dB = 20\log_{10}(5) \simeq 13.98dB$

⟨$Q > 1$, so there will be gain peaking⟩



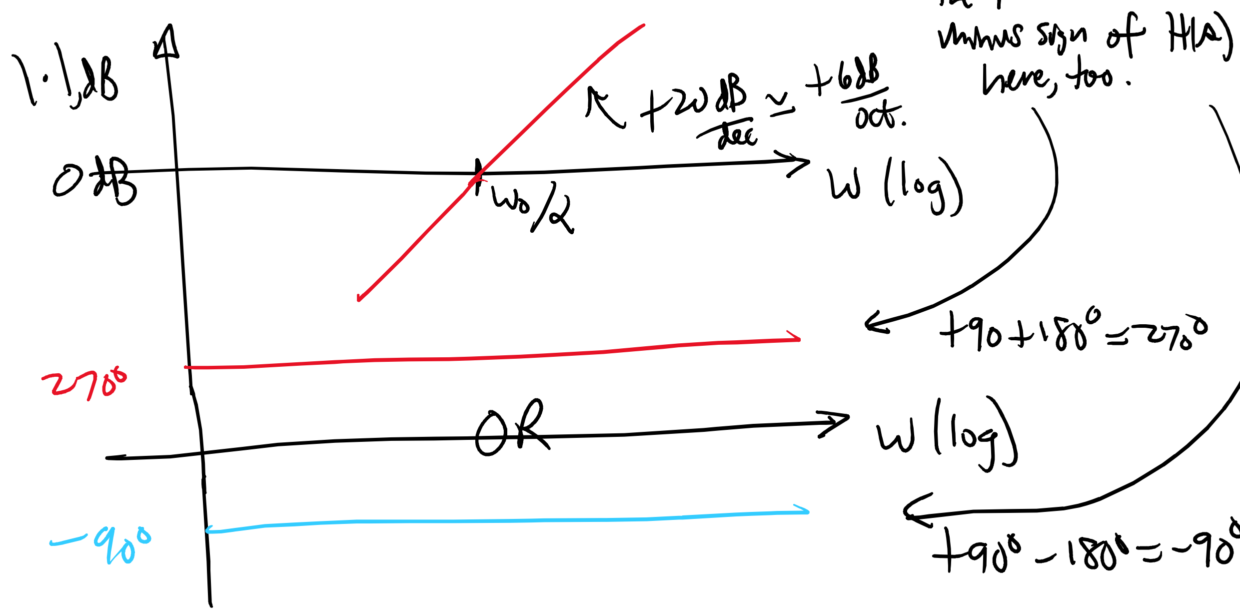


$\omega_1 = \omega_0 10^{-\zeta} = 189.66$ krps

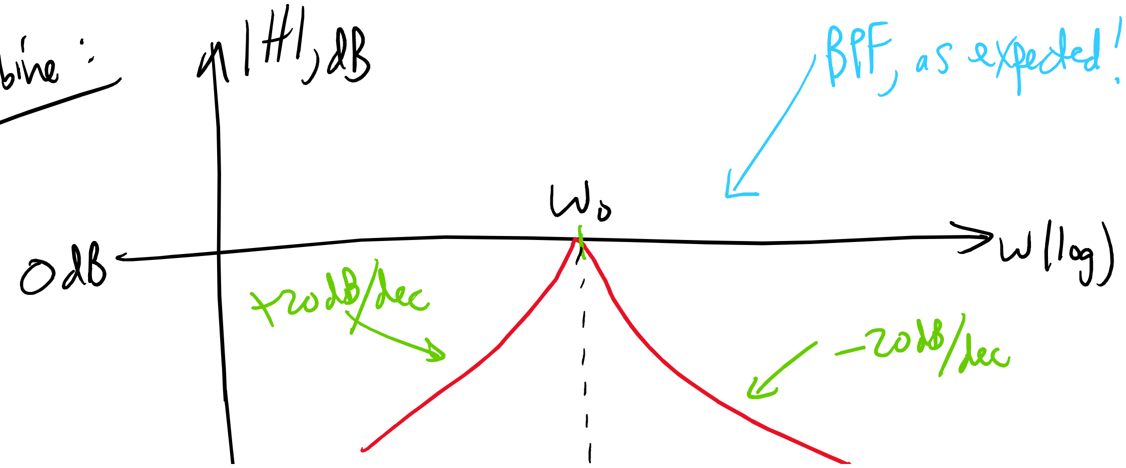
$\omega_2 = \omega_0 10^{+\zeta} = 300.58$ krps

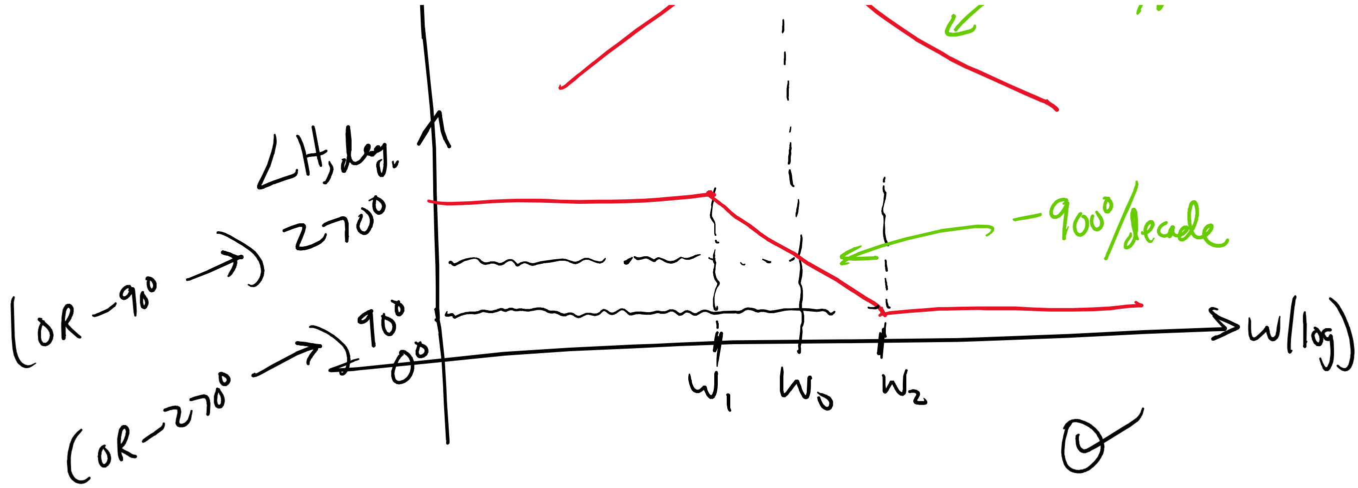
<Note that ζ is not less than 0.02>

$\Rightarrow$ Now let's sketch the zero @ DC term: $\Leftarrow$ I'll also include the phase due to the minus sign of H(s) here, too.



Now, combine:




∠H, deg.
270°
(OR −90°
90°
0°
(OR −270°
−900°/decade
w(log)
w1
w0
w2