Assignment07

May 15, 2019

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[Apply K-means algorithm to both image value and its spatial domain]

For a given input image (either gray or color), apply a K-means algorithm that is designed to take into consideration of both the image intensity and its spatial domain with varying parameters: the number of clusters and the trade-off between the intensity energy and the spatial energy.

The objective function is given by:

$\sum_k \sum_{x \in I(k)} [\|f(x) - m_k\|^2 + a * \|x - c_k\|^2]$

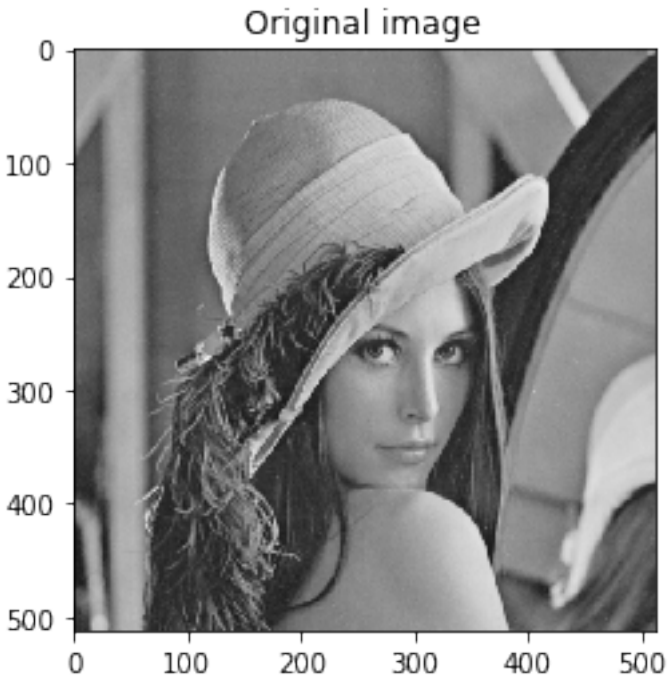
where $I(k)$ denotes the index set of x that belongs to cluster k, m_k denotes the centroid of image intensity for cluster k, c_k denotes the centroid of spatial location for cluster k, and a determines the importance between the image intensity and the spatial relation.

- Visualize the clustering results with varying k and a using the centroid color m_k for each cluster k.
- Visualize the energy curve for both the intensity energy and the spatial energy.

```
In [24]: import numpy as np
         import matplotlib.pyplot as plt
         from PIL import Image
```

```
In [27]: def rgb2gray(rgb):
             return np.dot(rgb[...,:3], [0.2989, 0.5870, 0.1140])
```

```
In [239]: # Read the image file
          #img = plt.imread('../../exercise/python/data/tiger.jpeg')
          #img_rows = img.shape[0]
          #img_cols = img.shape[1]

          img = Image.open('../../exercise/python/data/lena.png','r')
          img = img.convert('L') #makes it greyscale
          img_matrix = np.asarray(img.getdata(),dtype=np.float64).\
          reshape((img.size[1],img.size[0]))
          img_rows = img_matrix.shape[0]
          img_cols = img_matrix.shape[1]
          plt.imshow(img_matrix, cmap='gray', vmin=0, vmax=255)
          plt.title("Original image")
          plt.show()
```



In [240]:
```
# Define a K-means clustering function
def KMeans(img_matrix, img_rows, img_cols, K, alpha):
    # K = 5
    # alpha = 0.1

    centroid_row = np.random.randint(0, img_rows, (K, 1))
    centroid_col = np.random.randint(0, img_cols, (K, 1))
    centroid_value = np.random.randint(0, 255, (K, 1))
    centroid = np.hstack([centroid_row, centroid_col, centroid_value])
    centroid_list = []
    label_map = np.zeros((img_rows, img_cols))
    energy_list = []
    energy_list_spatial = []
    energy_list_intensity = []

    # K-Means clustering by L2 norm
    distance_list = []
    distance_list_spatial = []
    distance_list_intensity = []
    count_list = np.zeros((K))

    for iteration in range(0, 10):
        energy = 0
        energy_spatial = 0
        energy_intensity = 0
```

```
for i in range(0, img_rows):
    for j in range(0, img_cols):
        # Compare the distance with each centroid.
        for iterator in range(0, K):
            distance_list.append\
            ((img_matrix[i,j]-centroid[iterator,2])**2+\
             alpha*((i-centroid[iterator,0])**2+\
             (j-centroid[iterator,1])**2))
            distance_list_spatial.append\
            ((i-centroid[iterator,0])**2+\
             (j-centroid[iterator,1])**2)
            distance_list_intensity.append\
            ((img_matrix[i,j]-centroid[iterator,2])**2+\
             alpha*((i-centroid[iterator,0])**2))
        # Choose the label that has the minimum distance
        # and update the label.
        label_map[i][j] = np.argmin(distance_list)
        energy += (np.min(distance_list))
        energy_spatial += (np.min(distance_list_spatial))
        energy_intensity += (np.min(distance_list_intensity))
        # Re-initialize distance_list
        distance_list = []
        distance_list_spatial = []
        distance_list_intensity = []


# Update the energy list
energy_list.append(energy)
energy_list_spatial.append(energy_spatial)
energy_list_intensity.append(energy_intensity)
# Update centroids.
centroid.fill(0)
# Iterate through the image label map matrix.
for i in range(0, img_rows):
    for j in range(0, img_cols):
        centroid[int(label_map[i][j]),0] += i
        centroid[int(label_map[i][j]),1] += j
        centroid[int(label_map[i][j]),2] += img_matrix[i][j]
        count_list[int(label_map[i][j])] += 1

centroid[:,0] = np.true_divide(centroid[:,0], count_list)
centroid[:,1] = np.true_divide(centroid[:,1], count_list)
centroid[:,2] = np.true_divide(centroid[:,2], count_list)

# Reinitialize count_list
count_list.fill(0)

centroid_list.append(centroid)
```

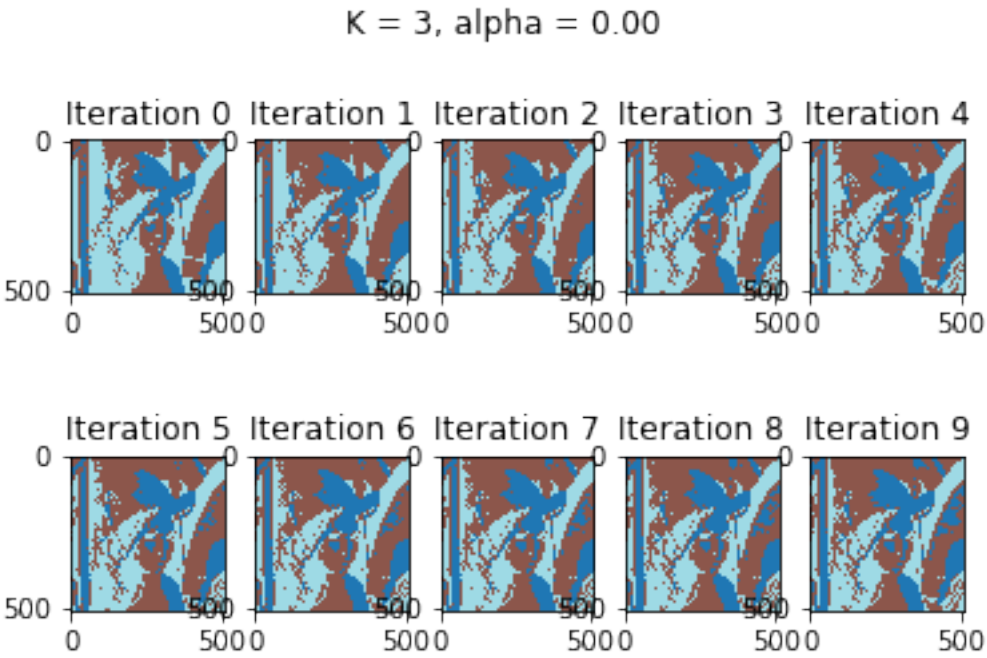
```
            print("%d th iteration" % iteration)



            # Visualize image
            plt_sub = plt.subplot(2, 5, iteration+1)
            plt_sub.title.set_text("Iteration %d" % iteration)
            plt_sub.imshow(label_map,cmap='tab20')
            # Visualize spatial centroids
            # plt_sub.scatter(centroid[:,0], centroid[:,1], c='b')
        plt.suptitle("K = %d, alpha = %.2f" % (K, alpha))
        plt.show()

        # Plot the total energy graph
        plt.title("Total Energy graph")
        plt.xlabel("Iteration")
        plt.ylabel("Energy")
        plt.plot(energy_list)
        plt.show()

        # Plot the spatial energy graph
        plt.title("Spatial Energy graph")
        plt.xlabel("Iteration")
        plt.ylabel("Energy")
        plt.plot(energy_list_spatial)
        plt.show()

        # Plot the intensity energy graph
        plt.title("Intensity Energy graph")
        plt.xlabel("Iteration")
        plt.ylabel("Energy")
        plt.plot(energy_list_intensity)
        plt.show()
```

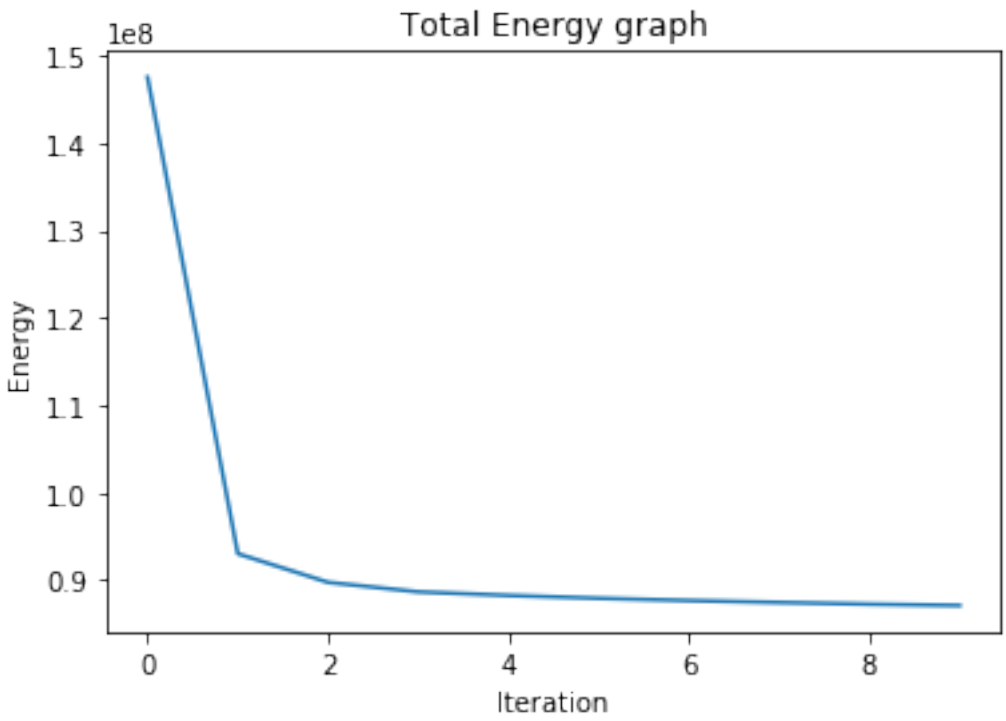
1 K = 3

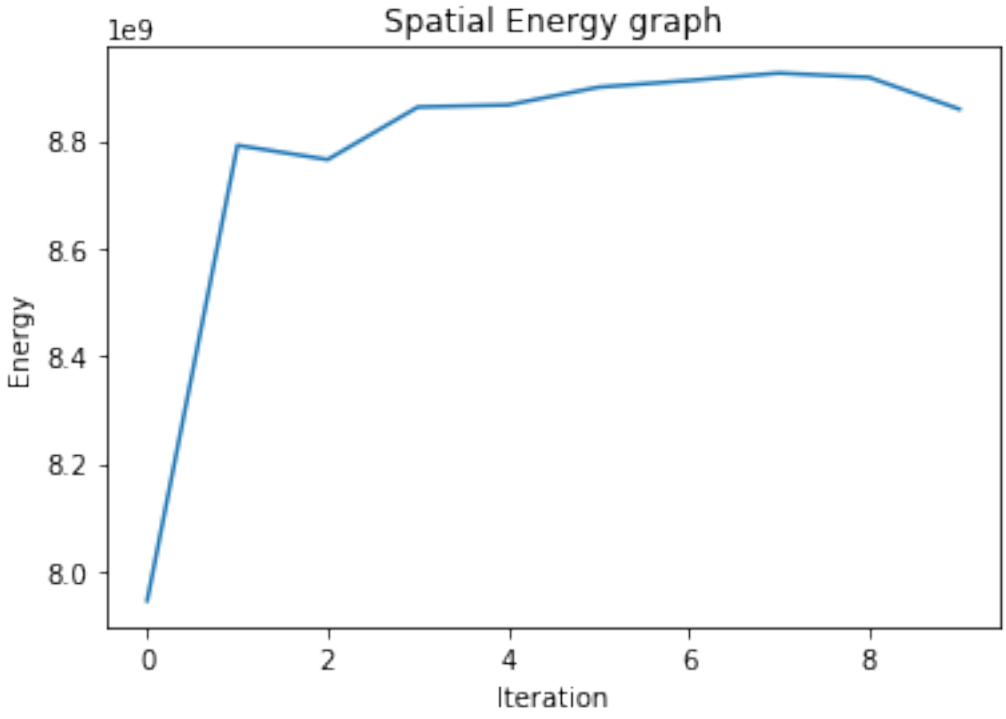
```
In [242]: K = 3
          KMeans(img_matrix, img_rows, img_cols, K, 0)
          KMeans(img_matrix, img_rows, img_cols, K, 0.01)
          KMeans(img_matrix, img_rows, img_cols, K, 0.03)
          KMeans(img_matrix, img_rows, img_cols, K, 0.05)
          KMeans(img_matrix, img_rows, img_cols, K, 0.07)
          KMeans(img_matrix, img_rows, img_cols, K, 0.08)
          KMeans(img_matrix, img_rows, img_cols, K, 0.10)
          KMeans(img_matrix, img_rows, img_cols, K, 0.25)
          KMeans(img_matrix, img_rows, img_cols, K, 0.50)
```

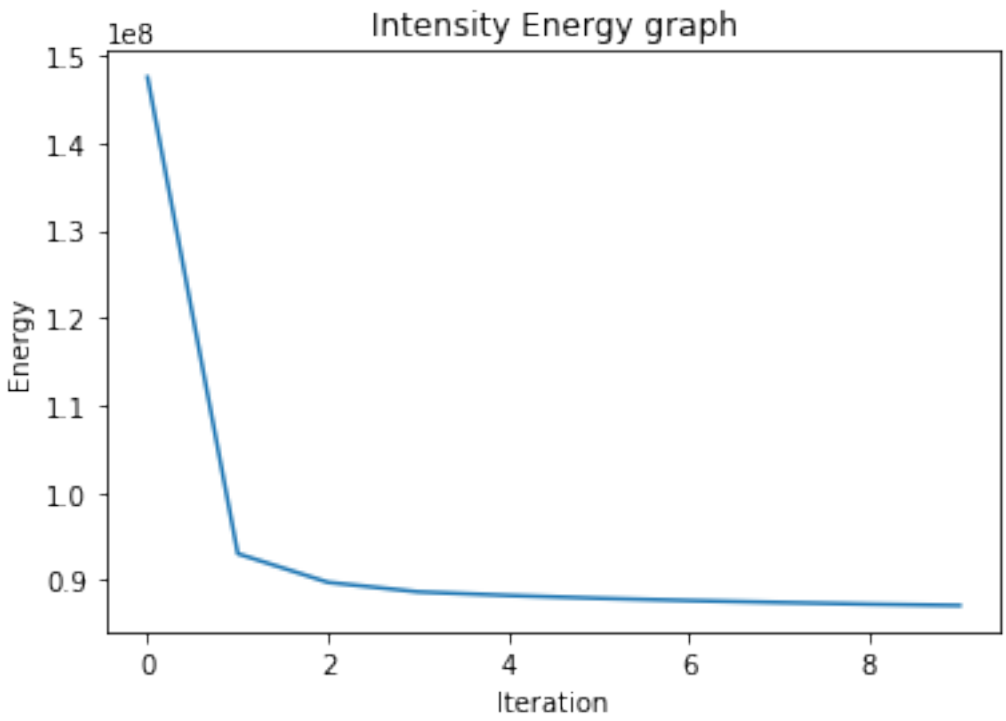
```
KMeans(img_matrix, img_rows, img_cols, K, 0.75)
KMeans(img_matrix, img_rows, img_cols, K, 1)
```

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

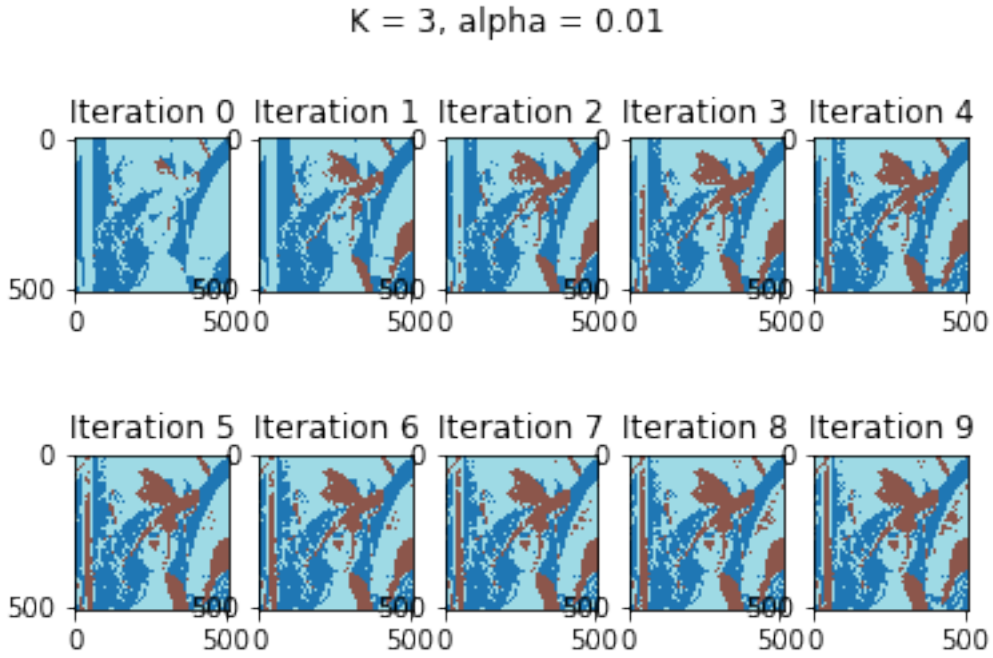


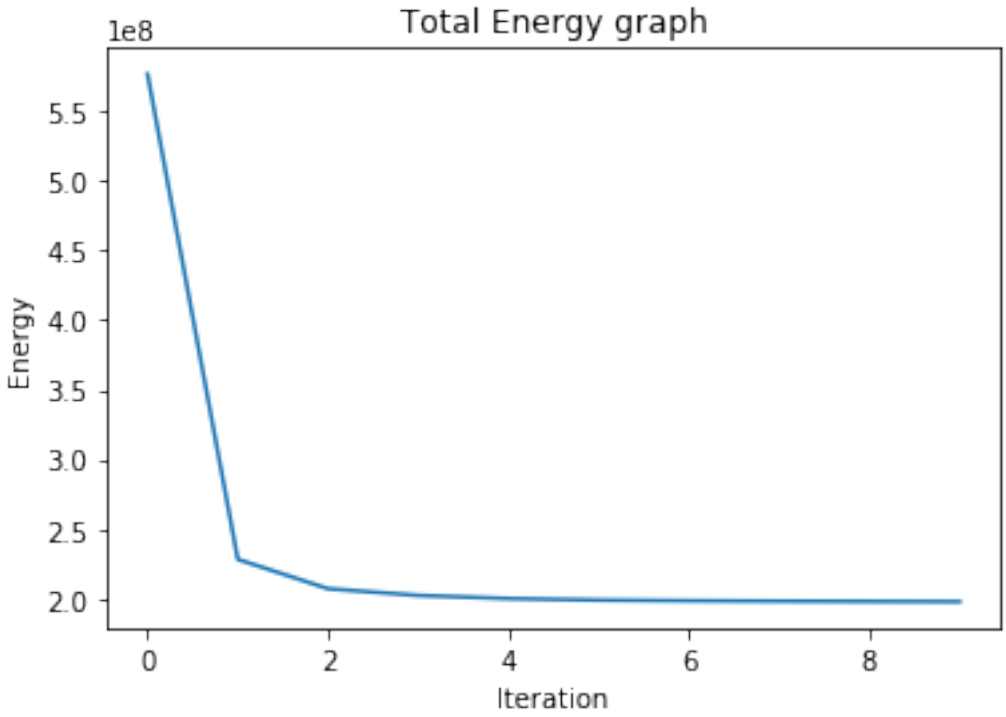

Total Energy graph
1e8
1.5
1.4
1.3
1.2
1.1
1.0
0.9
Energy
0
2
4
6
8
Iteration

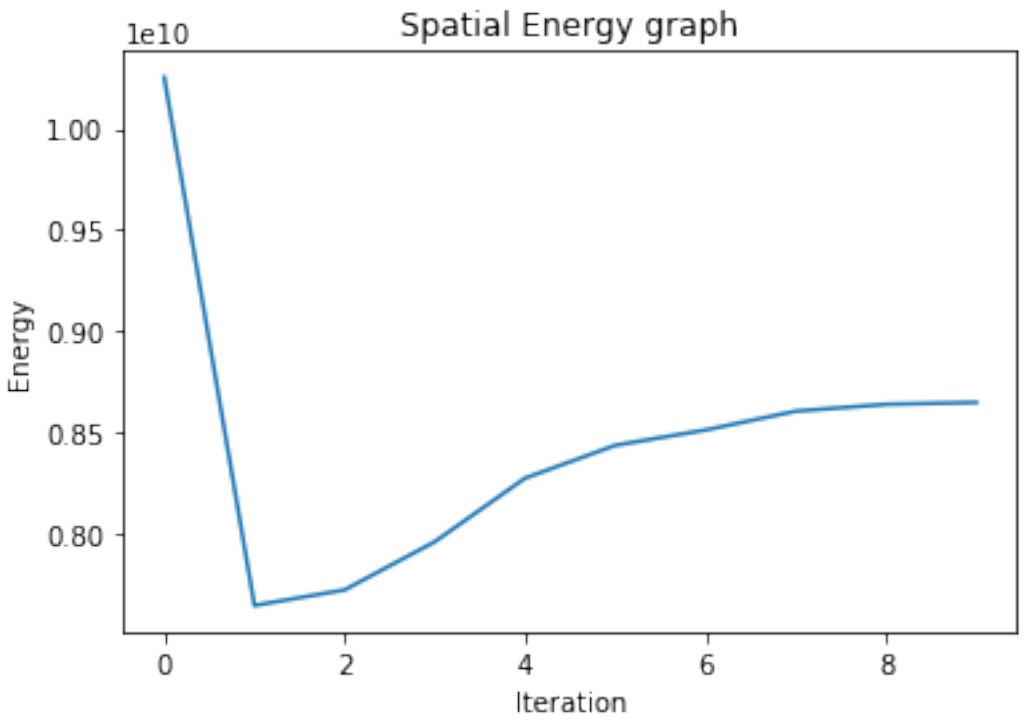

Spatial Energy graph
1e9
8.8
8.6
8.4
8.2
8.0
Energy
0
2
4
6
8
Iteration

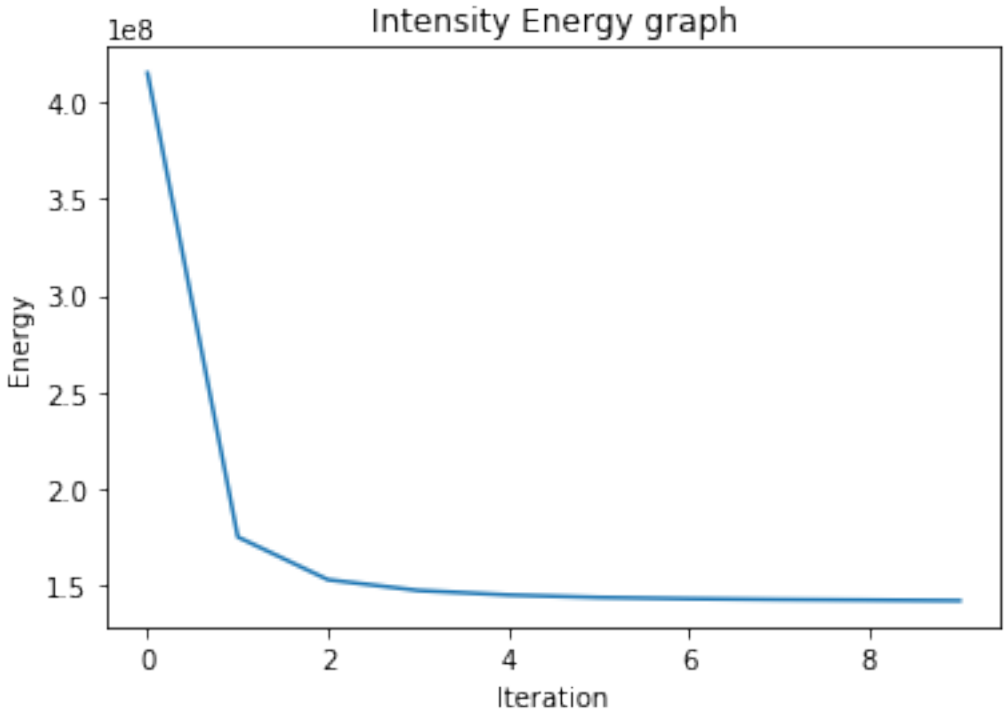



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

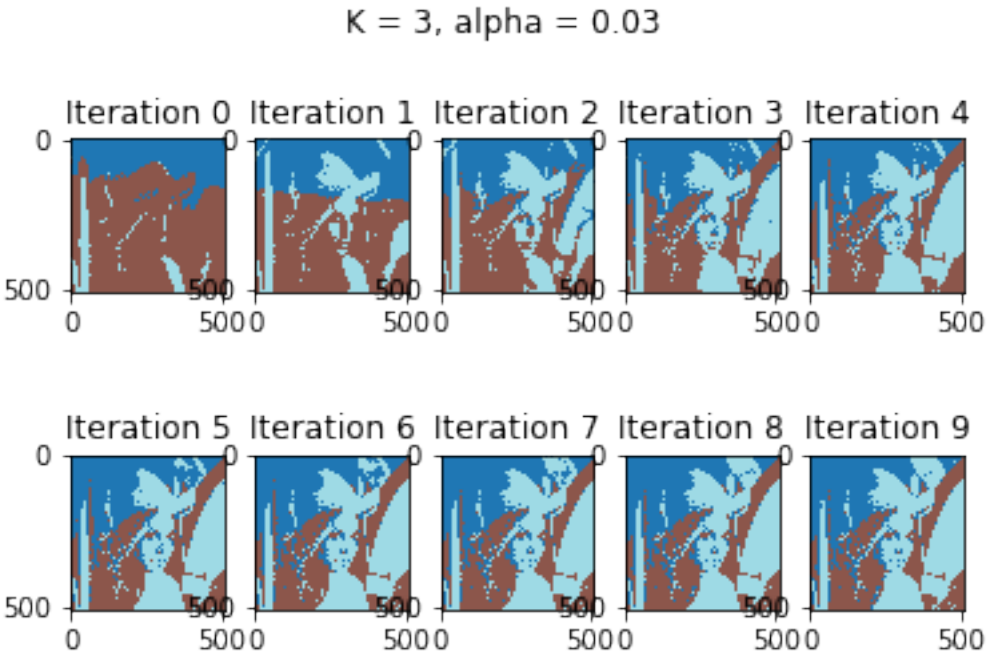

K = 3, alpha = 0.01
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

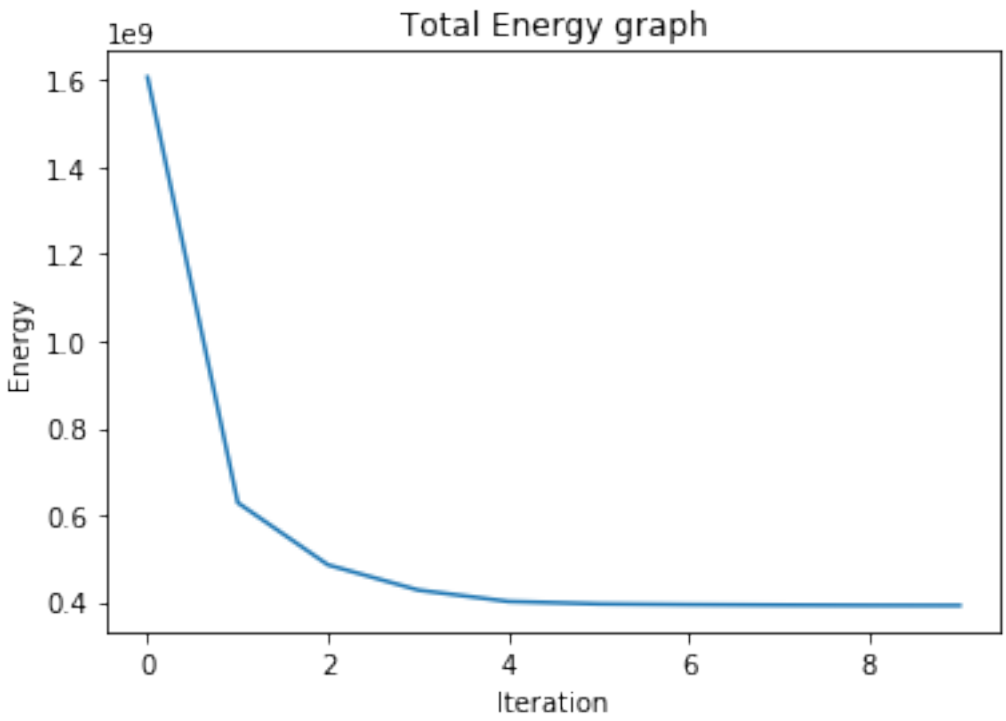

Total Energy graph
1e8
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

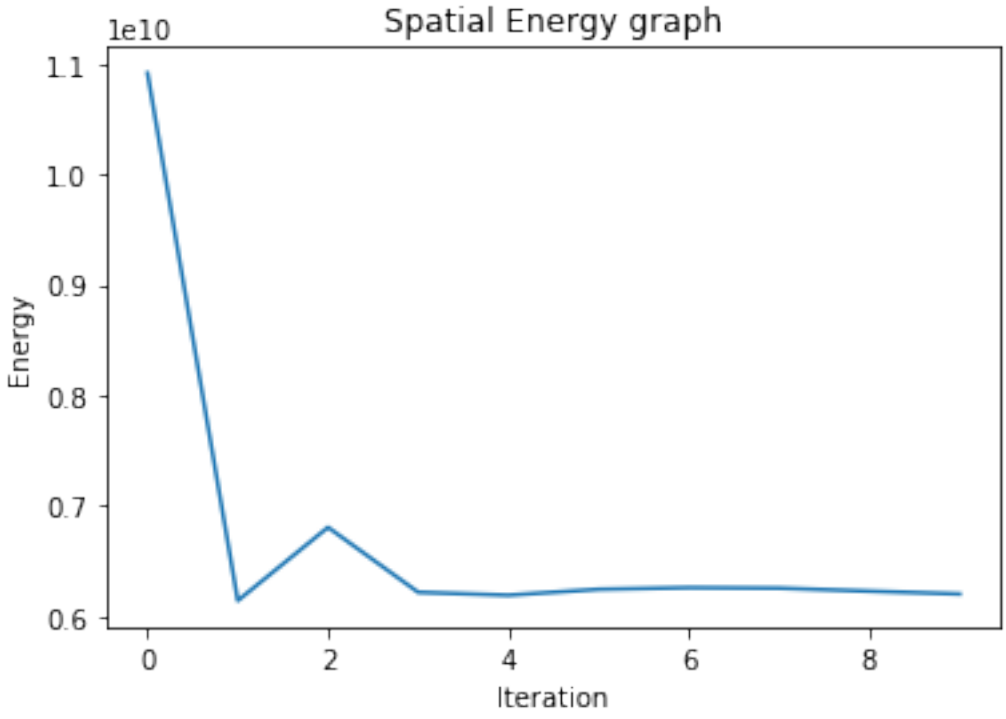

Spatial Energy graph
1e10
1.00
0.95
0.90
0.85
0.80
Energy
0
2
4
6
8
Iteration

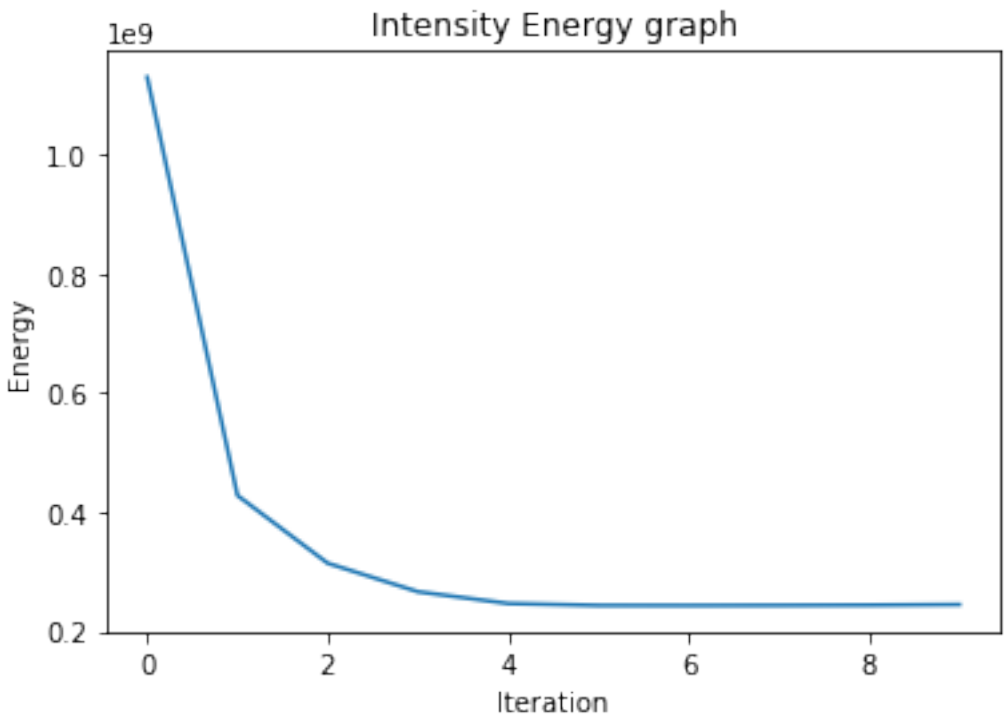

Intensity Energy graph
1e8
4.0
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

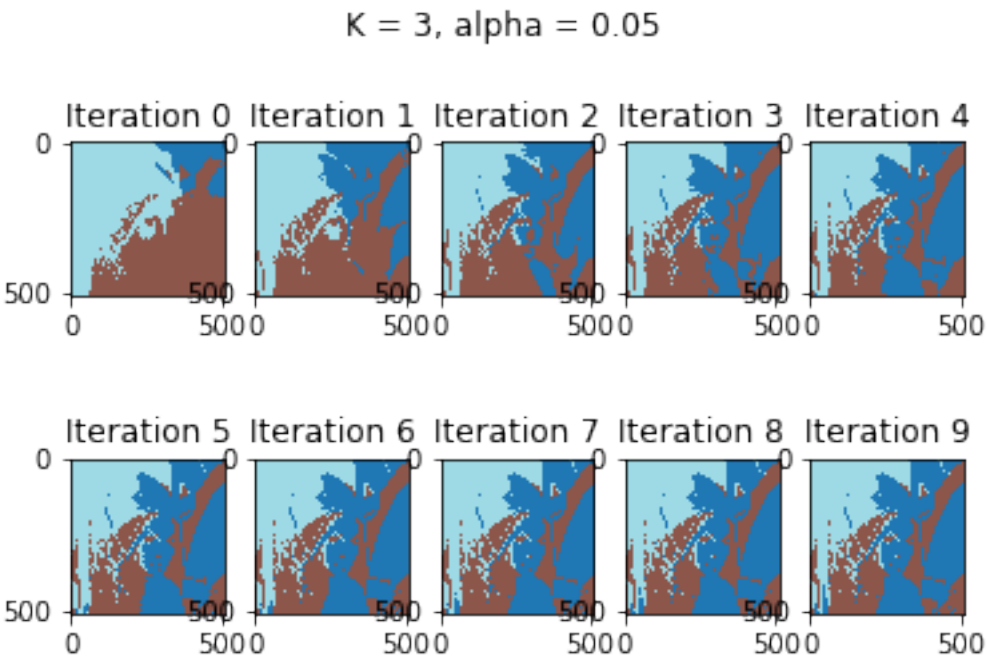


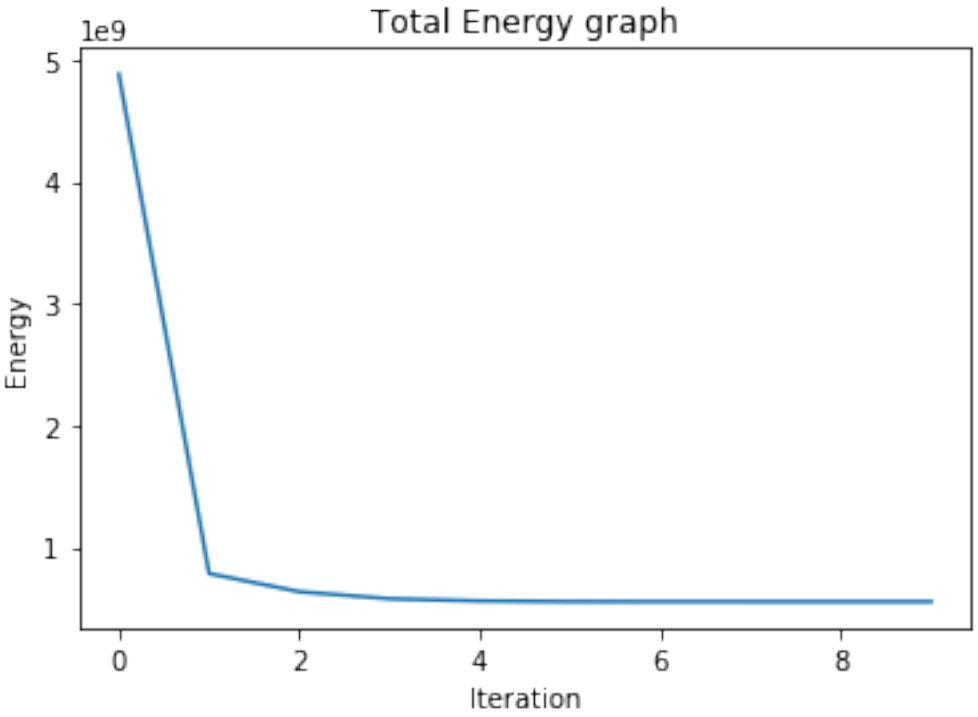

Total Energy graph
1e9
1.6
1.4
1.2
1.0
0.8
0.6
0.4
Energy
0
2
4
6
8
Iteration

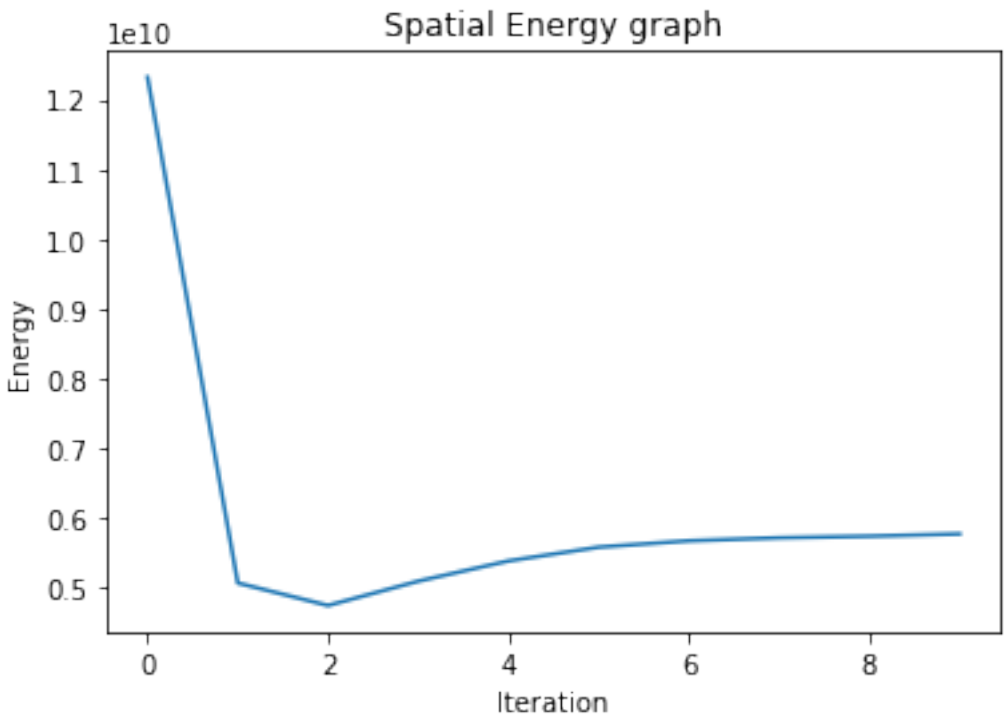

Spatial Energy graph
1e10
1.1
1.0
0.9
0.8
0.7
0.6
Energy
0
2
4
6
8
Iteration

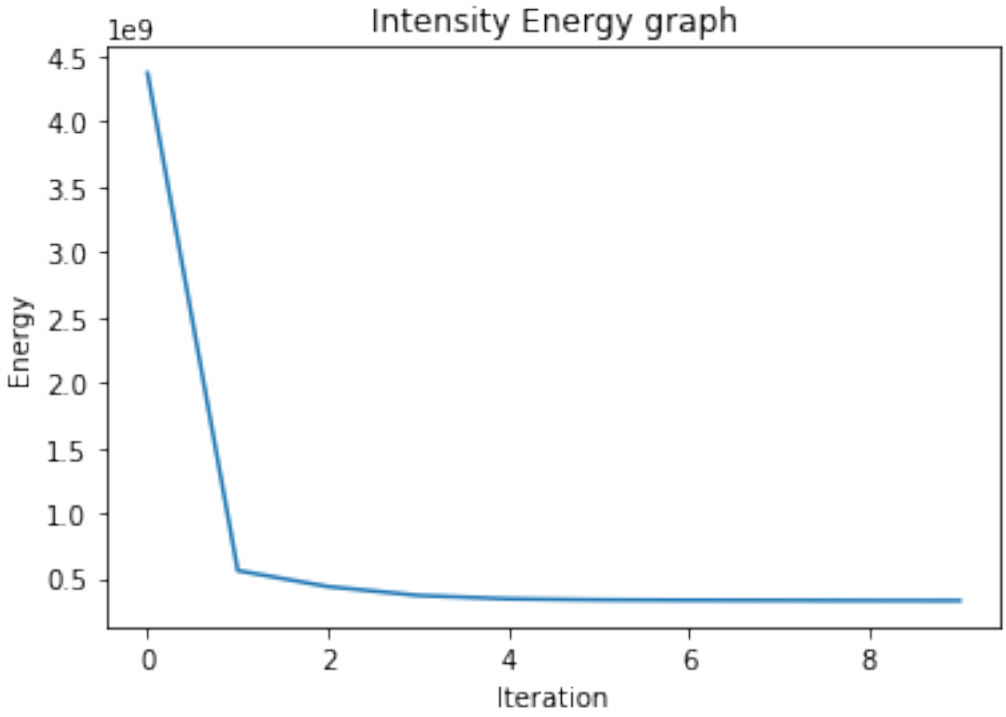



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

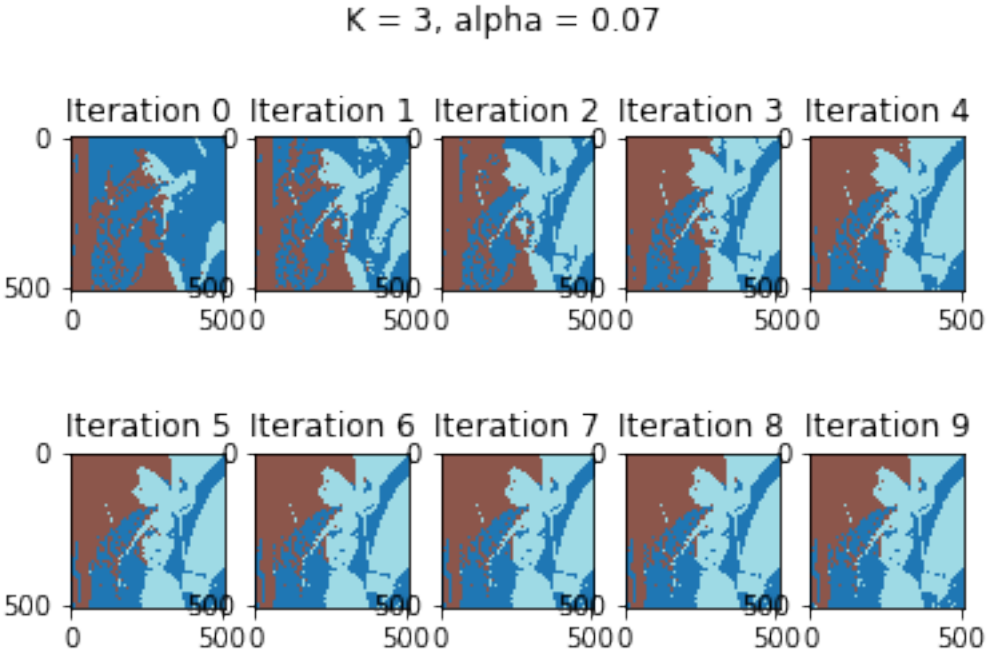

K = 3, alpha = 0.05
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

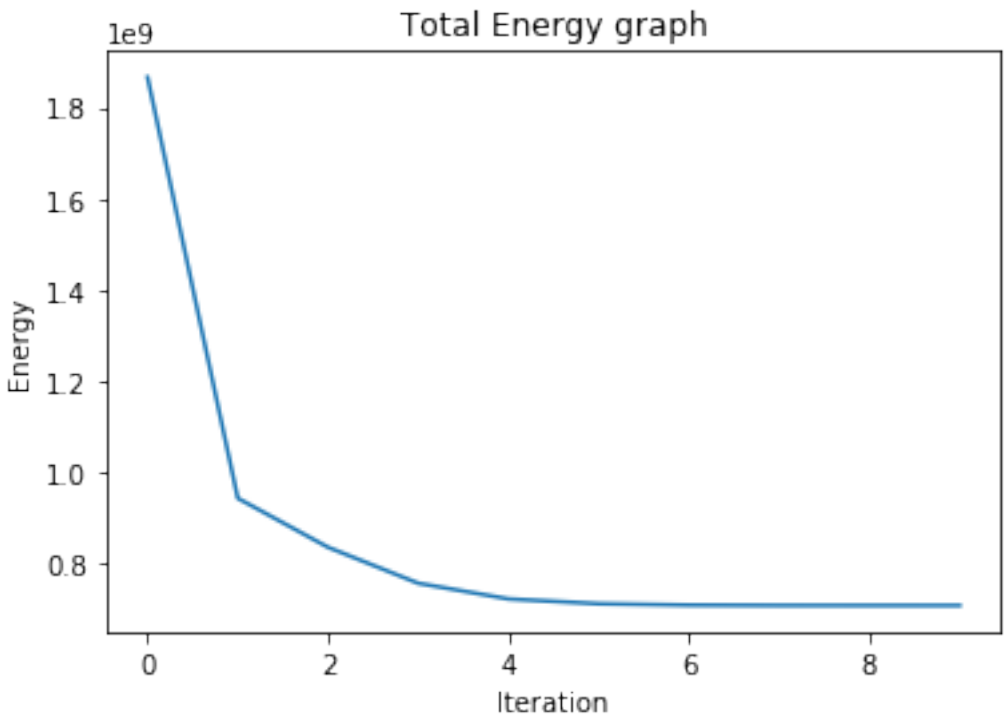

Total Energy graph
1e9
5
4
3
2
1
Energy
0
2
4
6
8
Iteration

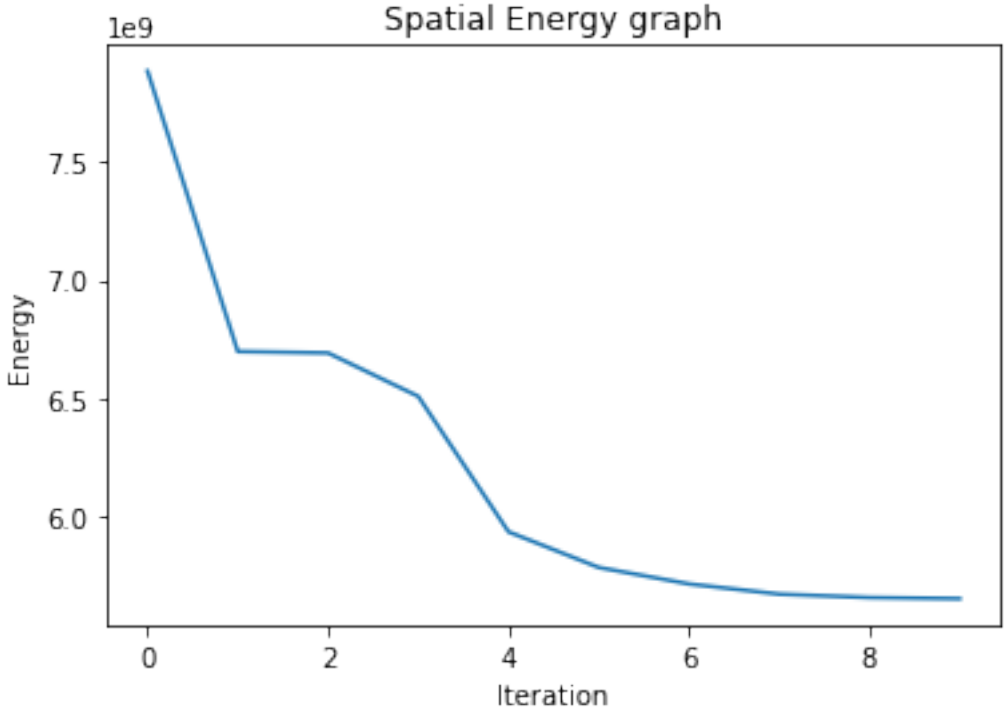

Spatial Energy graph
1e10
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

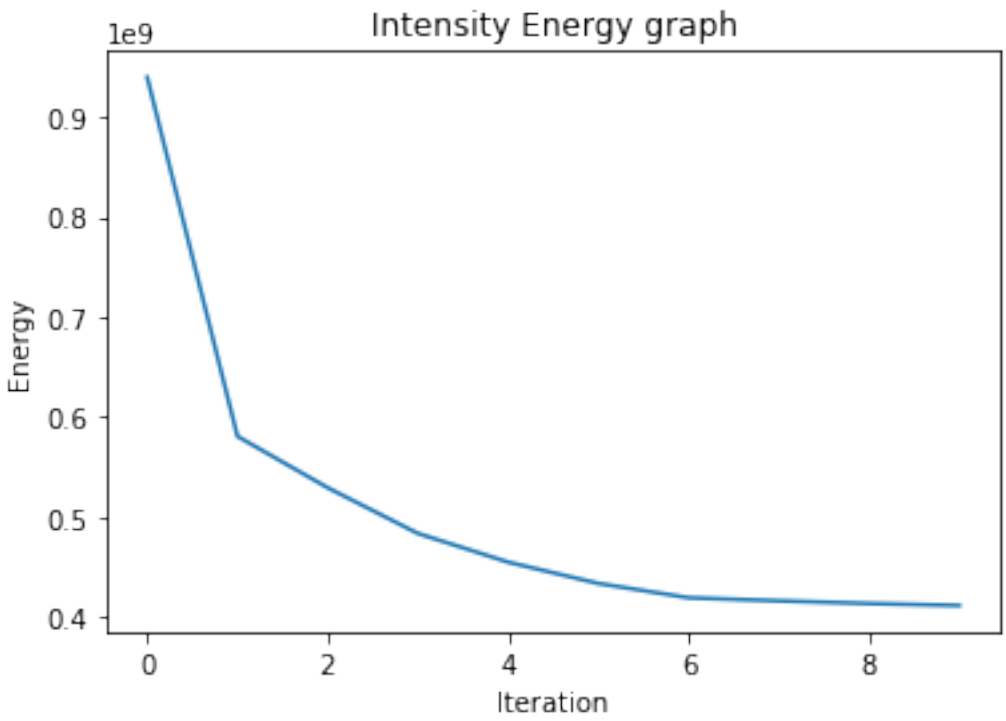

Intensity Energy graph
1e9
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

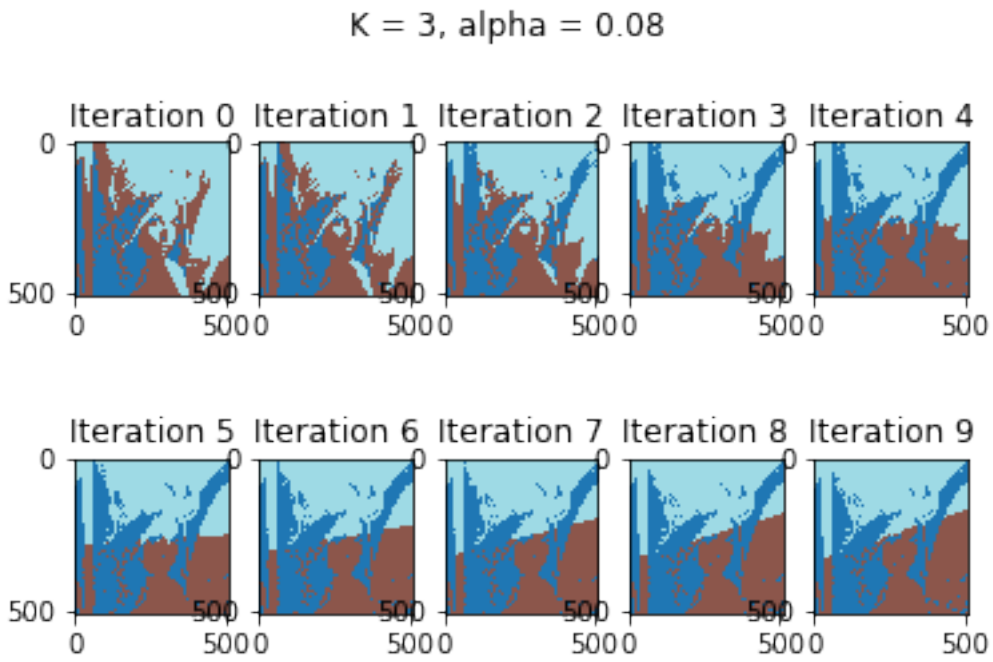


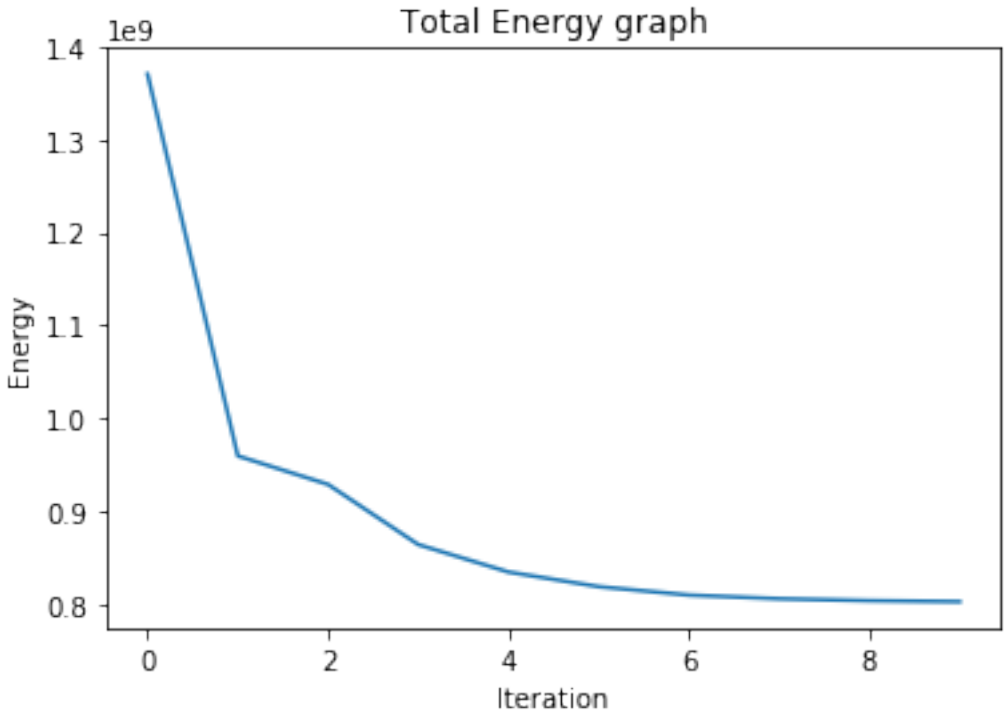

Total Energy graph
1e9
1.8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

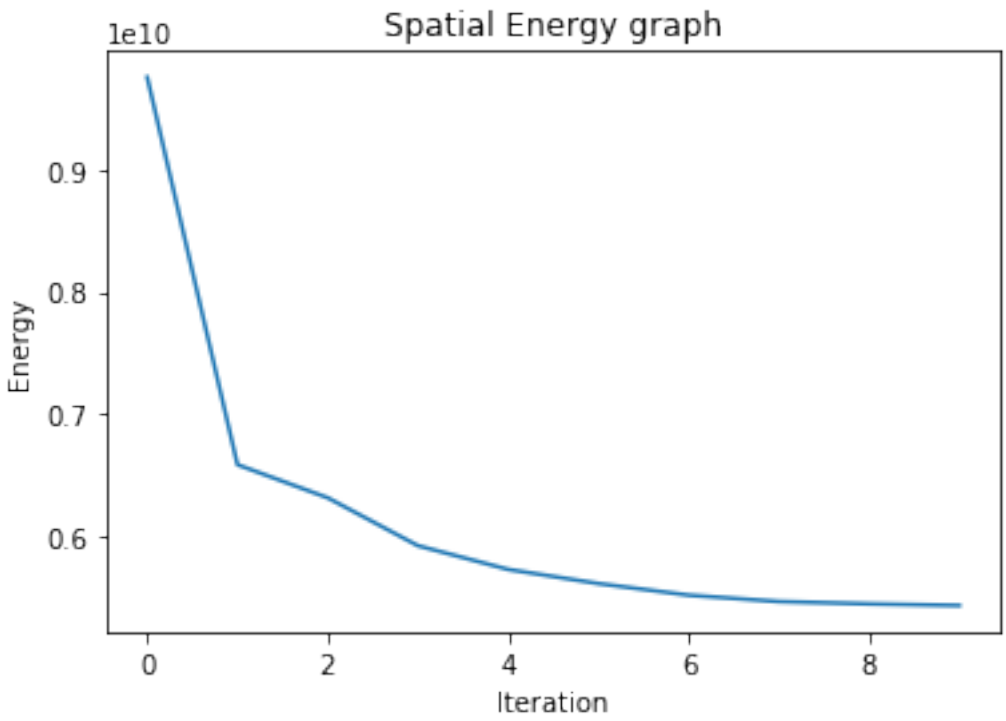

Spatial Energy graph
1e9
7.5
7.0
6.5
6.0
Energy
0
2
4
6
8
Iteration

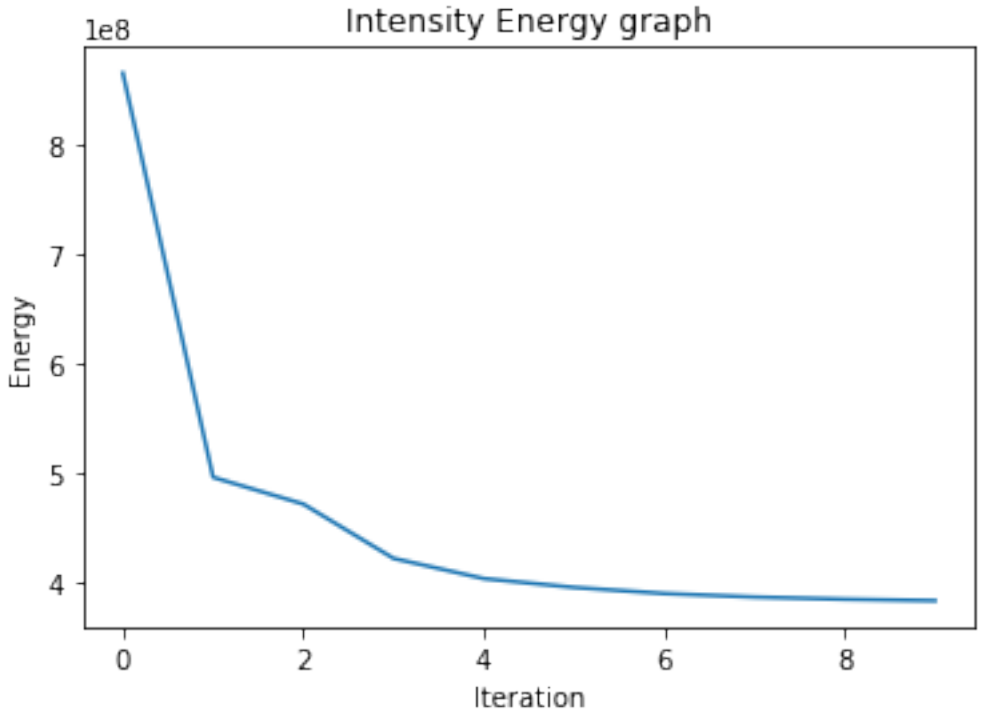



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

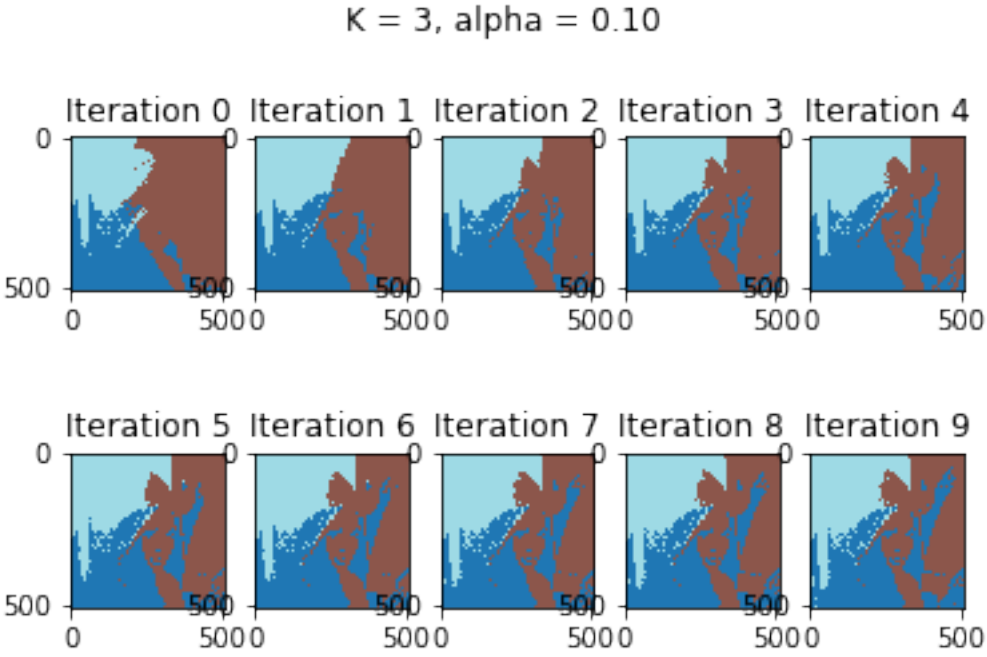

K = 3, alpha = 0.08
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

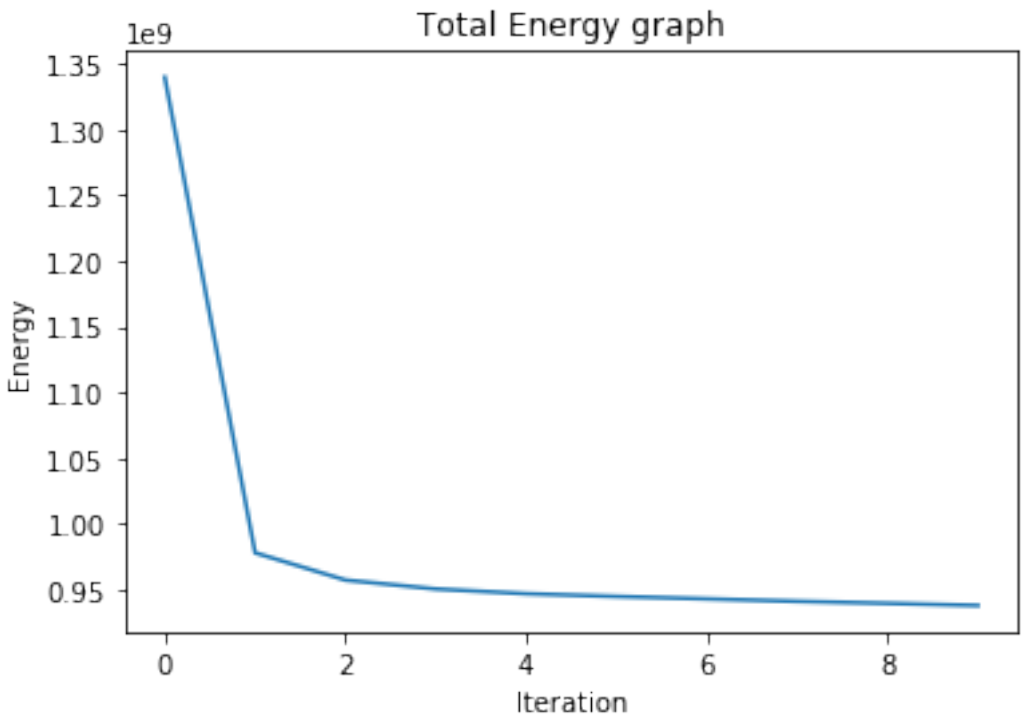

Total Energy graph
1e9
1.4
1.3
1.2
1.1
1.0
0.9
0.8
Energy
0
2
4
6
8
Iteration

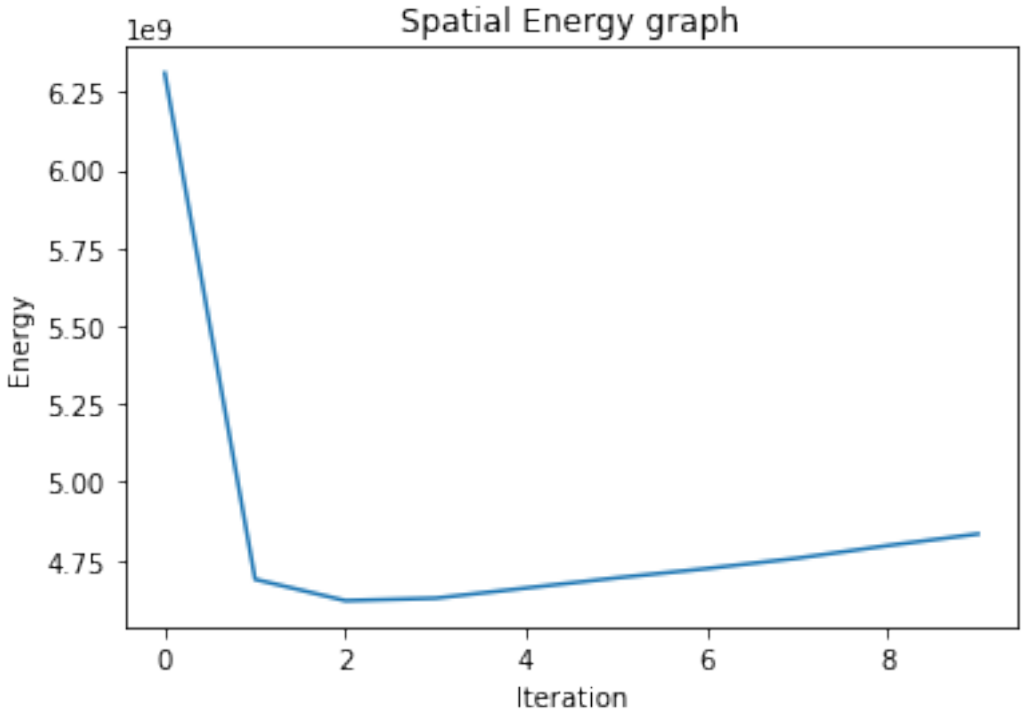

Spatial Energy graph
1e10
0.9
0.8
0.7
0.6
Energy
0
2
4
6
8
Iteration

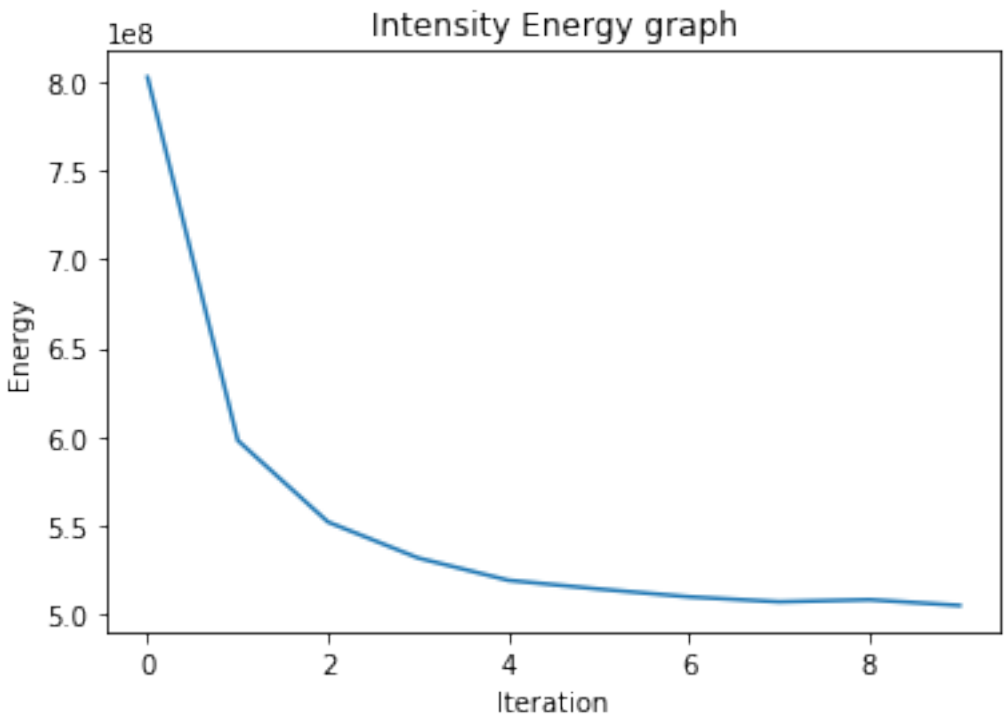

Intensity Energy graph
1e8
8
7
6
5
4
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

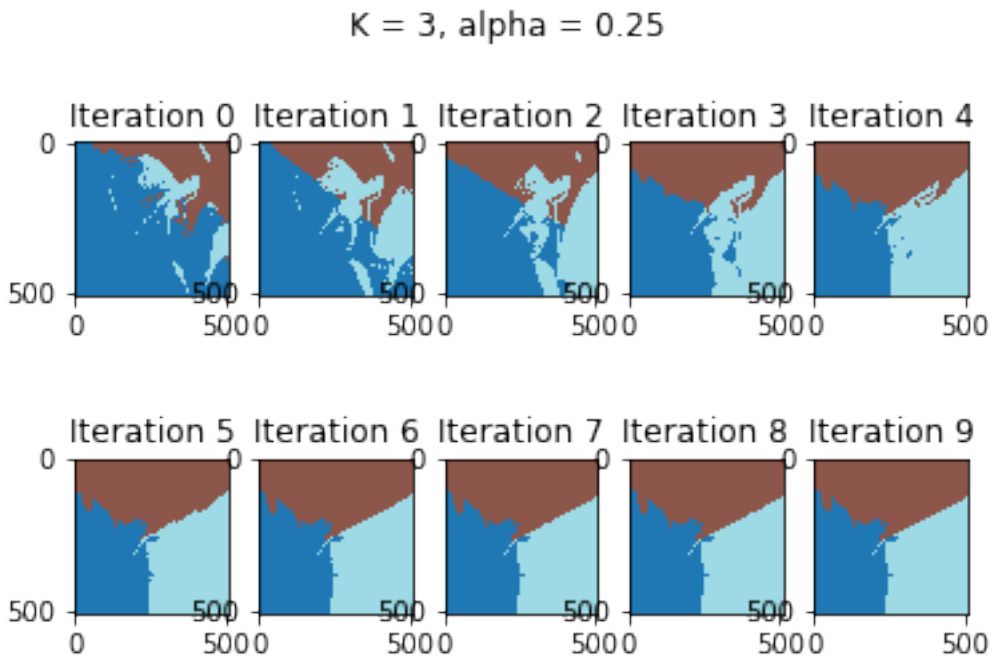


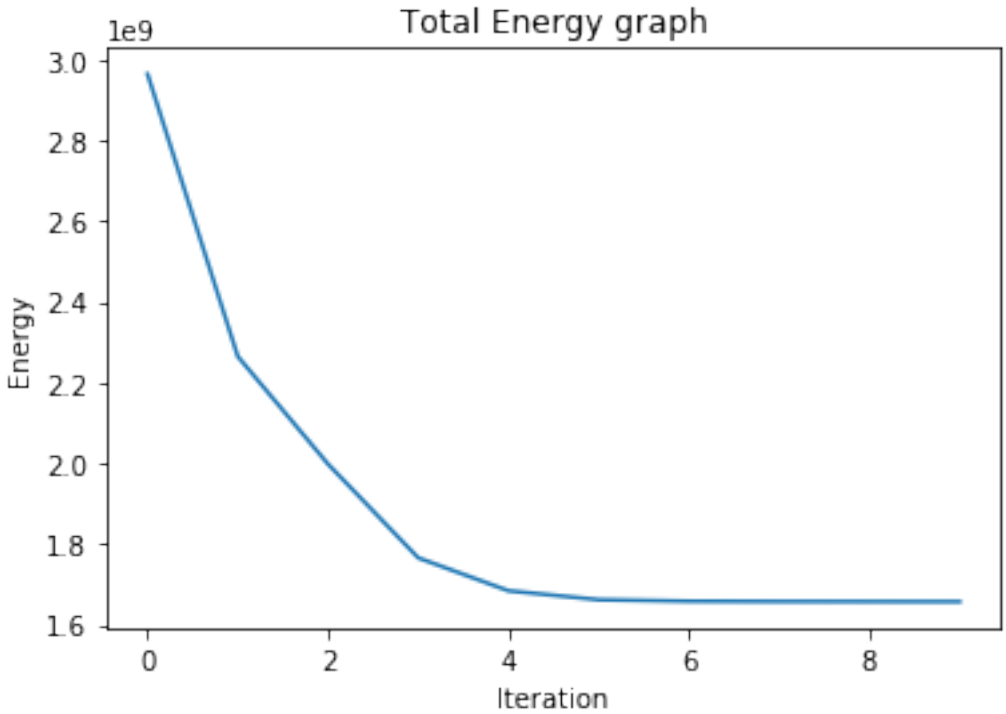

Total Energy graph
1e9
1.35
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
Energy
0
2
4
6
8
Iteration

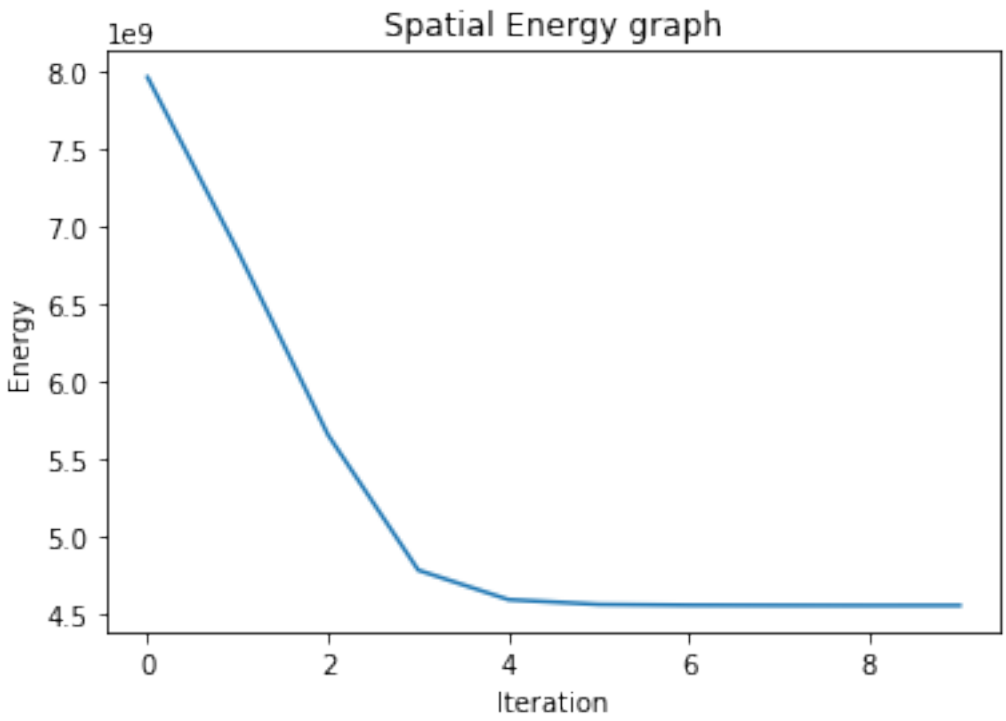

Spatial Energy graph
1e9
6.25
6.00
5.75
5.50
5.25
5.00
4.75
Energy
0
2
4
6
8
Iteration

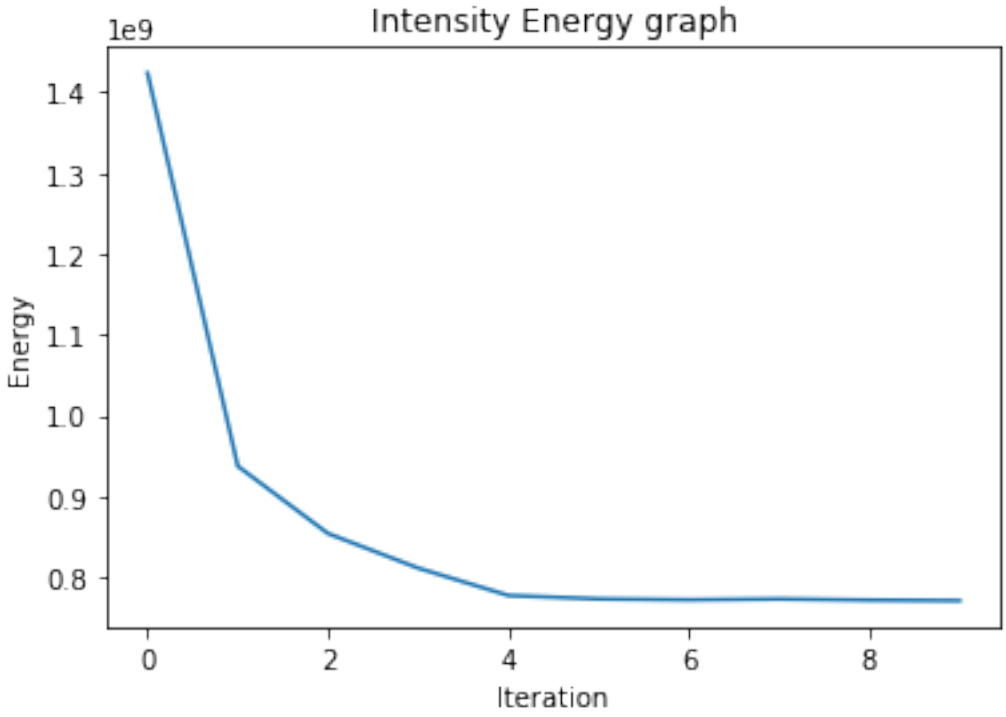



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

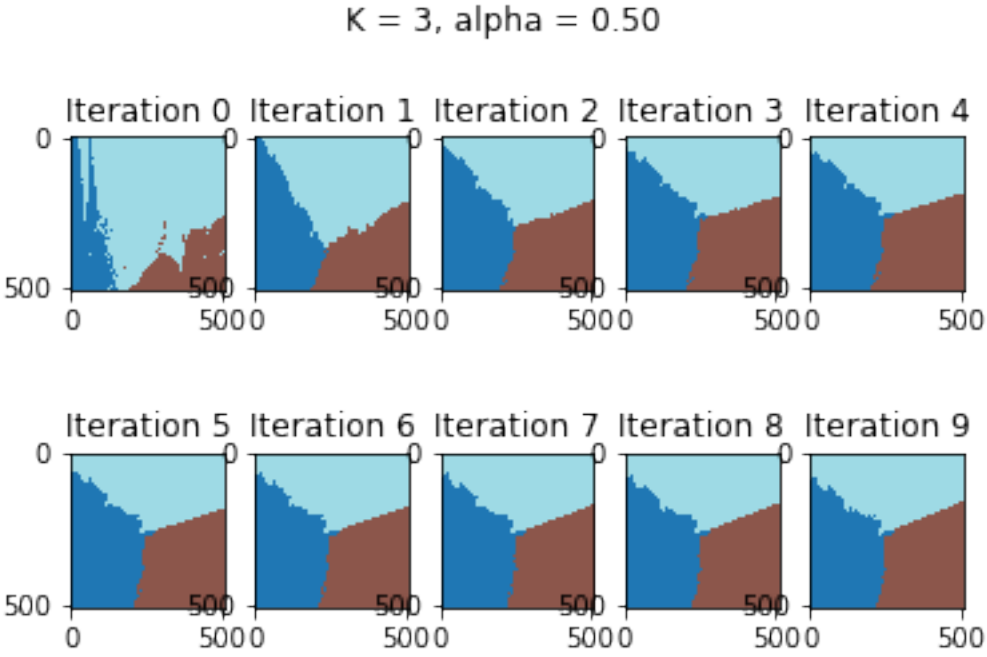

K = 3, alpha = 0.25
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

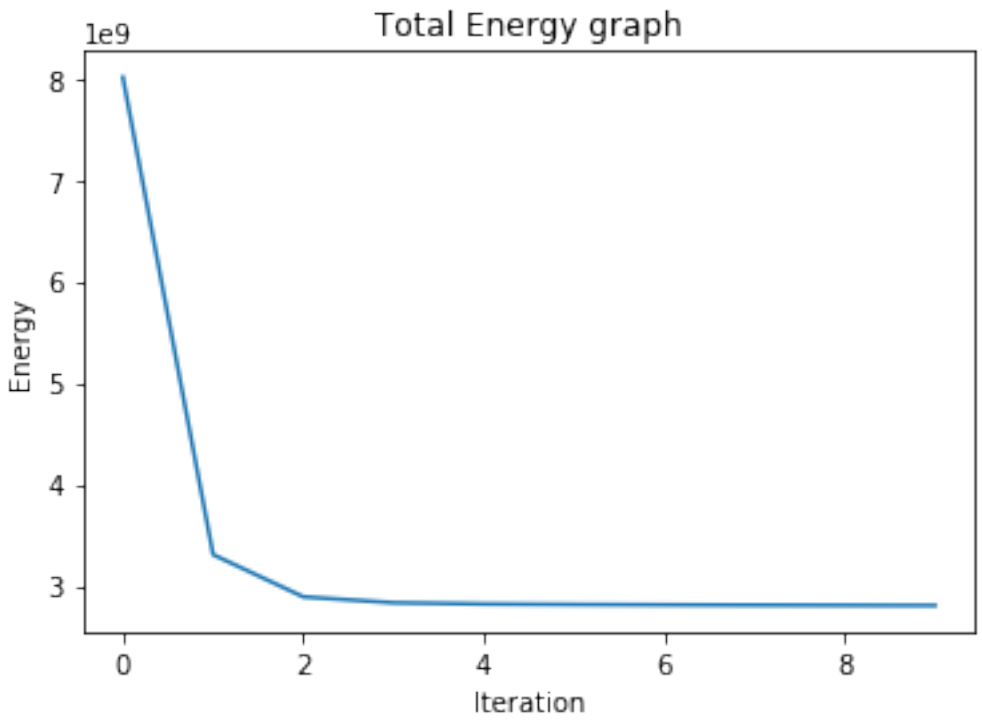

Total Energy graph
1e9
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
Energy
0
2
4
6
8
Iteration

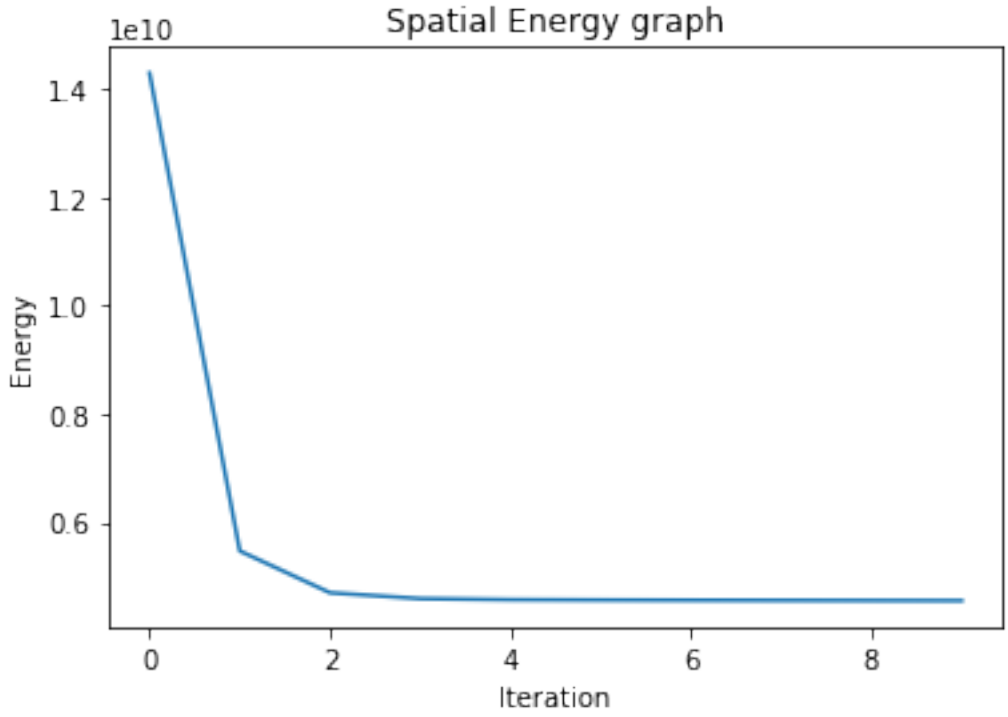

Spatial Energy graph
1e9
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
Energy
0
2
4
6
8
Iteration

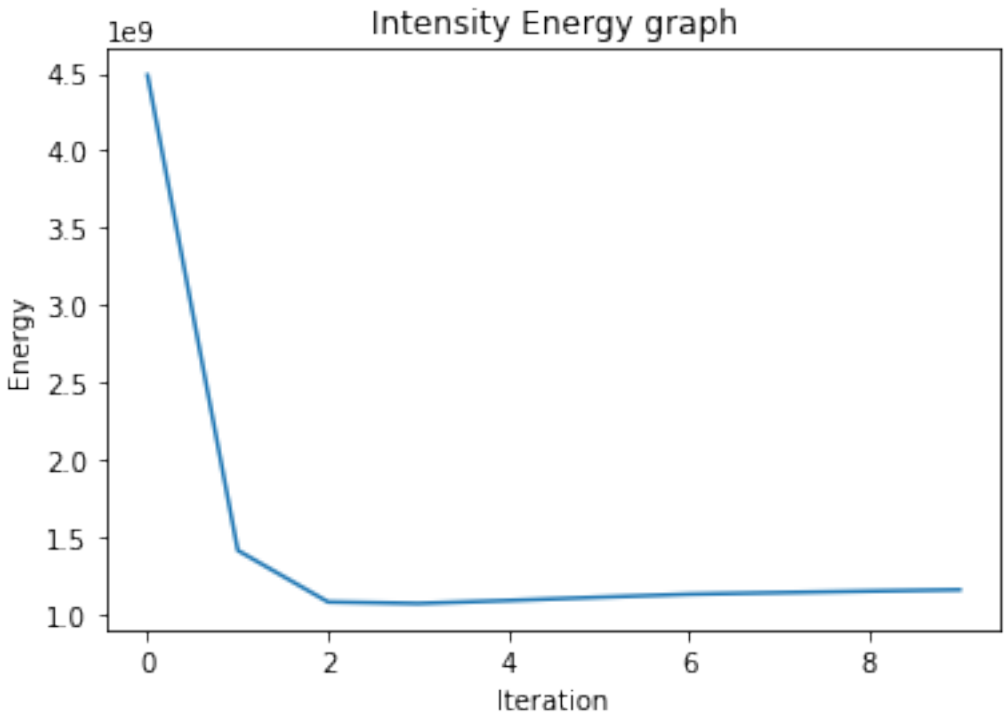

Intensity Energy graph
1e9
1.4
1.3
1.2
1.1
1.0
0.9
0.8
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

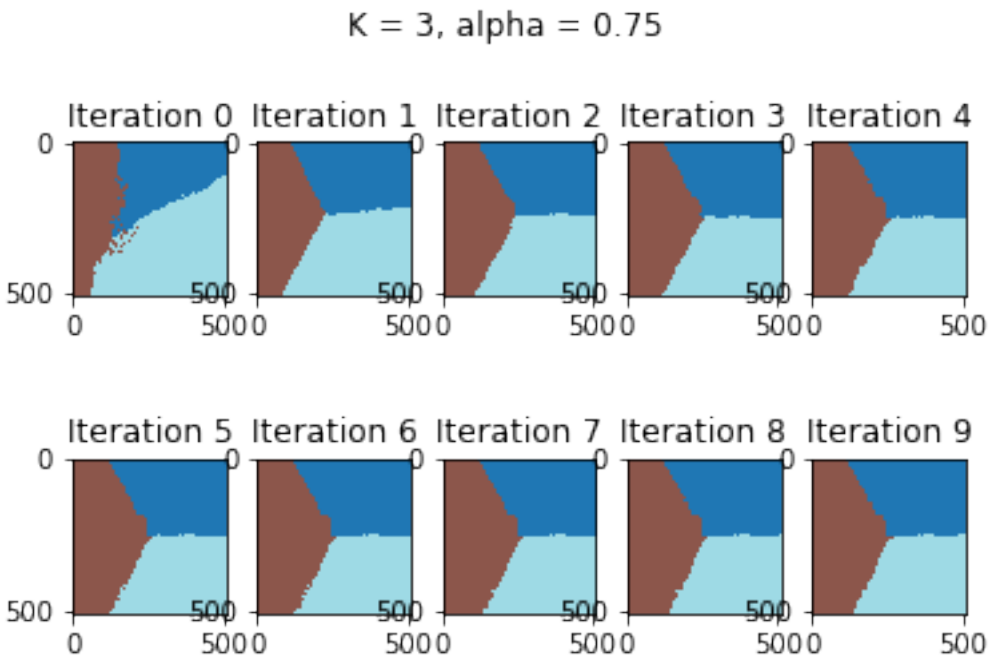


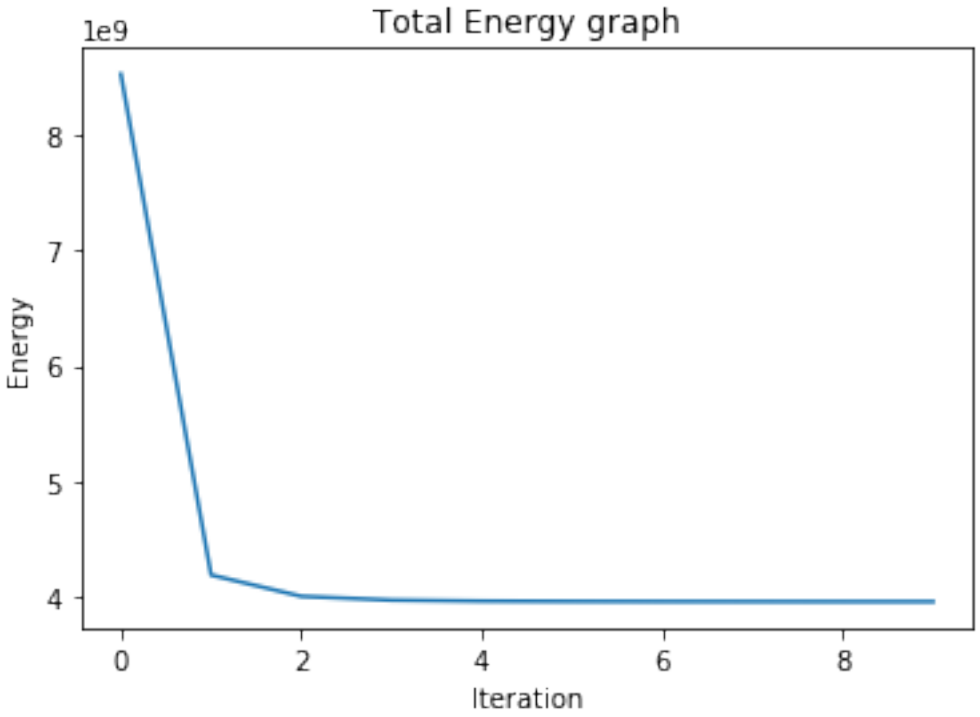

Total Energy graph
1e9
8
7
6
5
4
3
Energy
0
2
4
6
8
Iteration

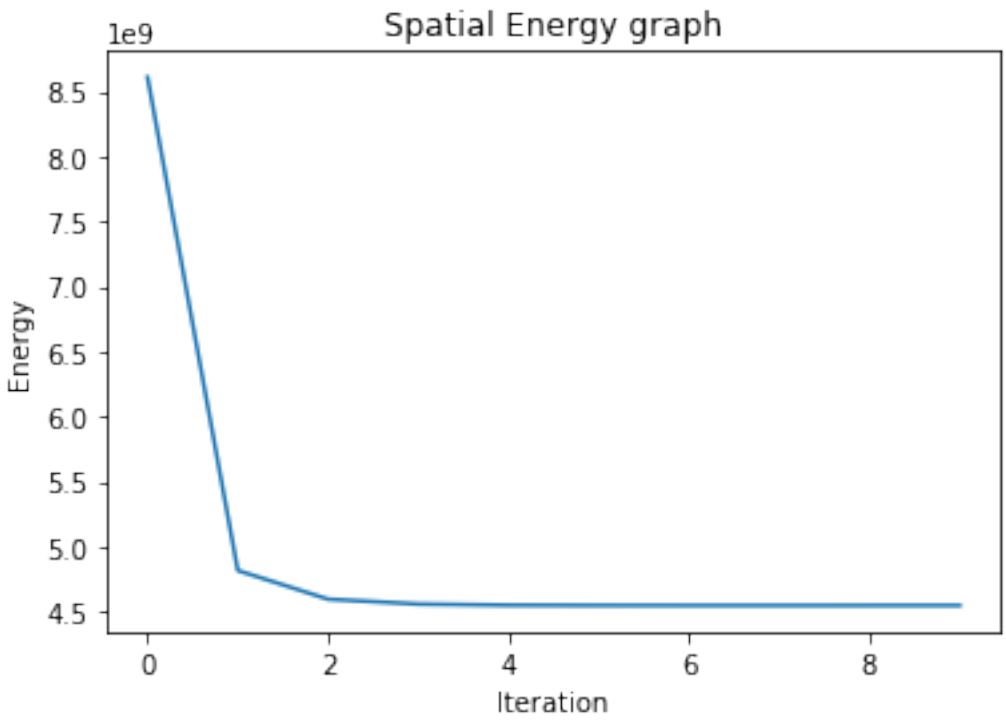

Spatial Energy graph
1e10
1.4
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

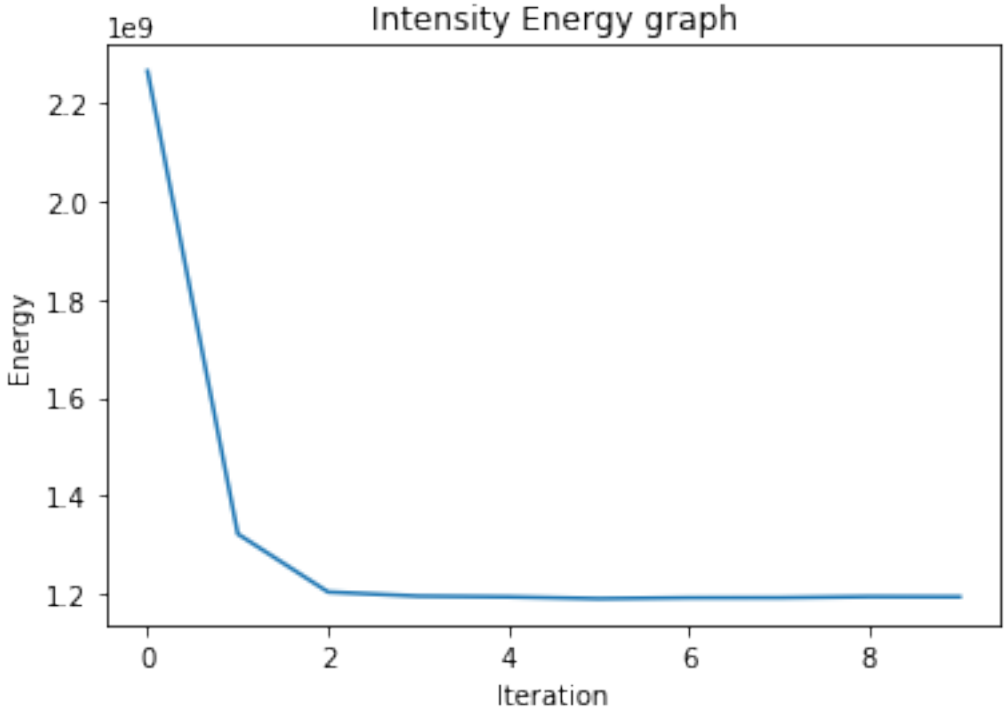



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

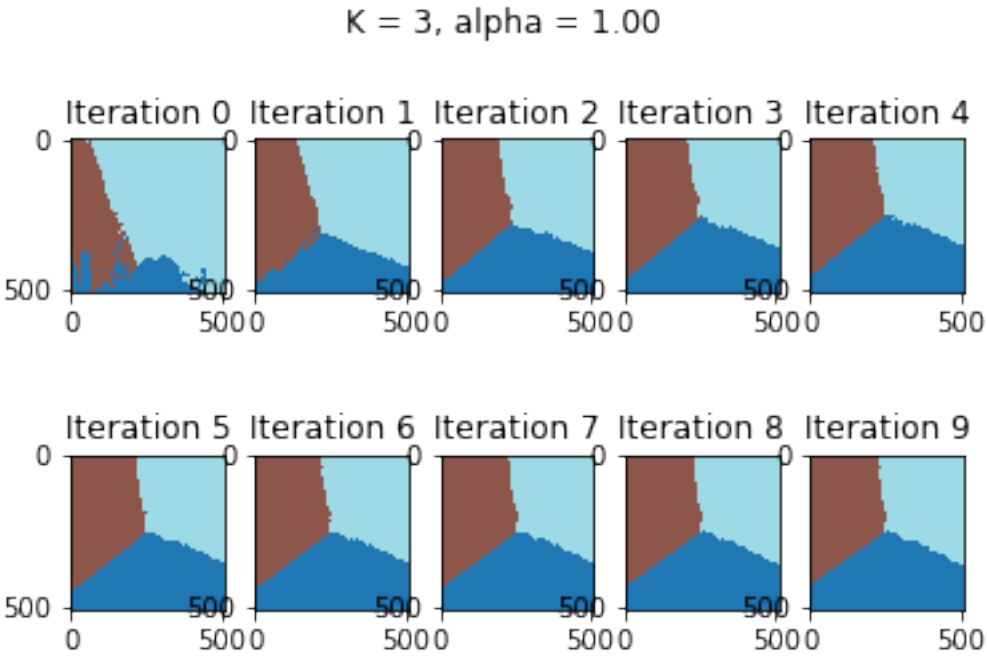

K = 3, alpha = 0.75
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

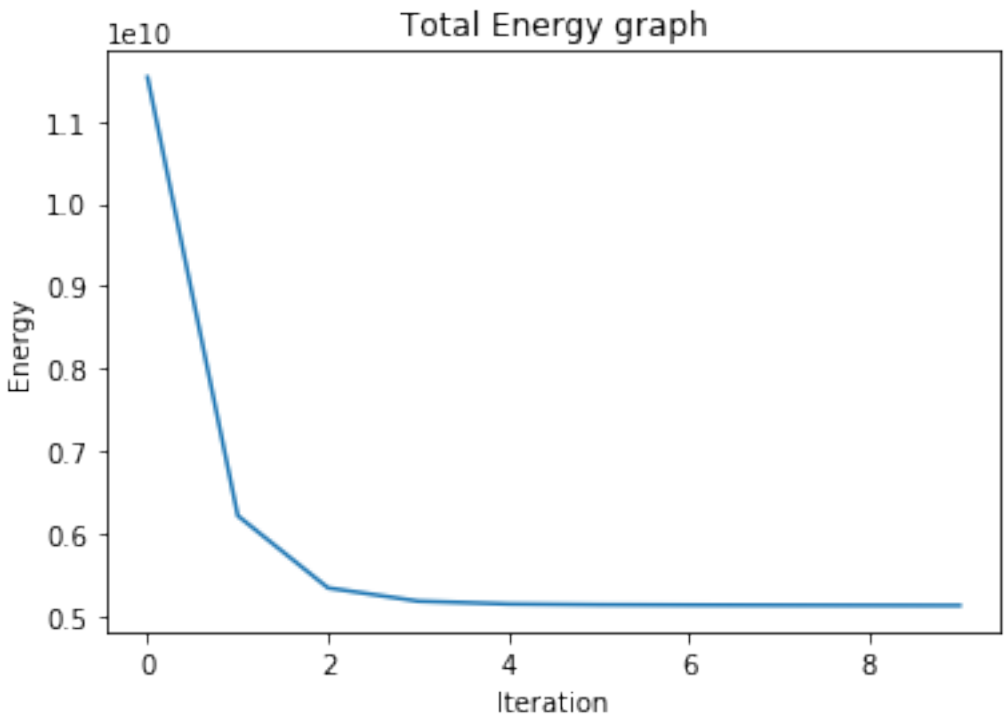

Total Energy graph
1e9
Energy
Iteration
8
7
6
5
4
0
2
4
6
8

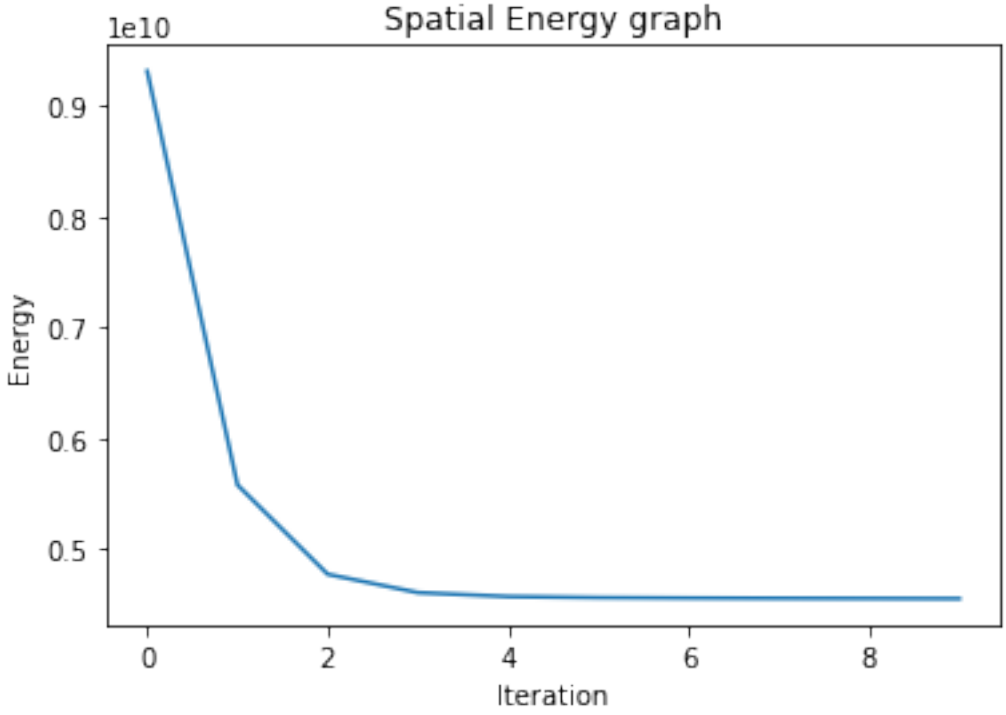

Spatial Energy graph
1e9
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
Energy
0
2
4
6
8
Iteration

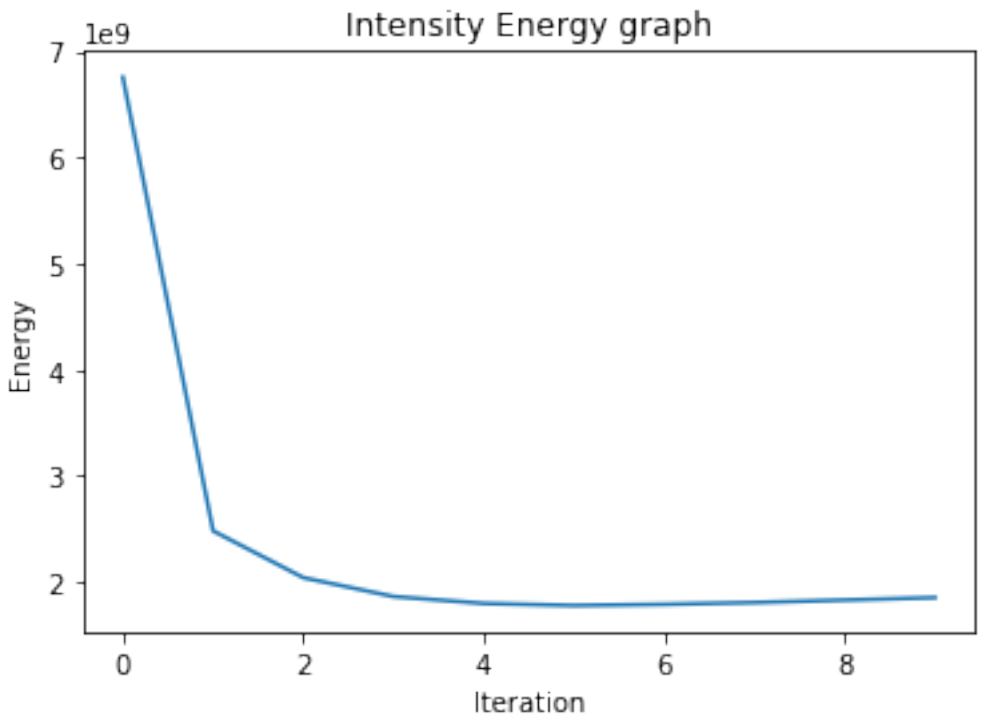

Intensity Energy graph
1e9
2.2
2.0
1.8
1.6
1.4
1.2
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```




Total Energy graph
1e10
1.1
1.0
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

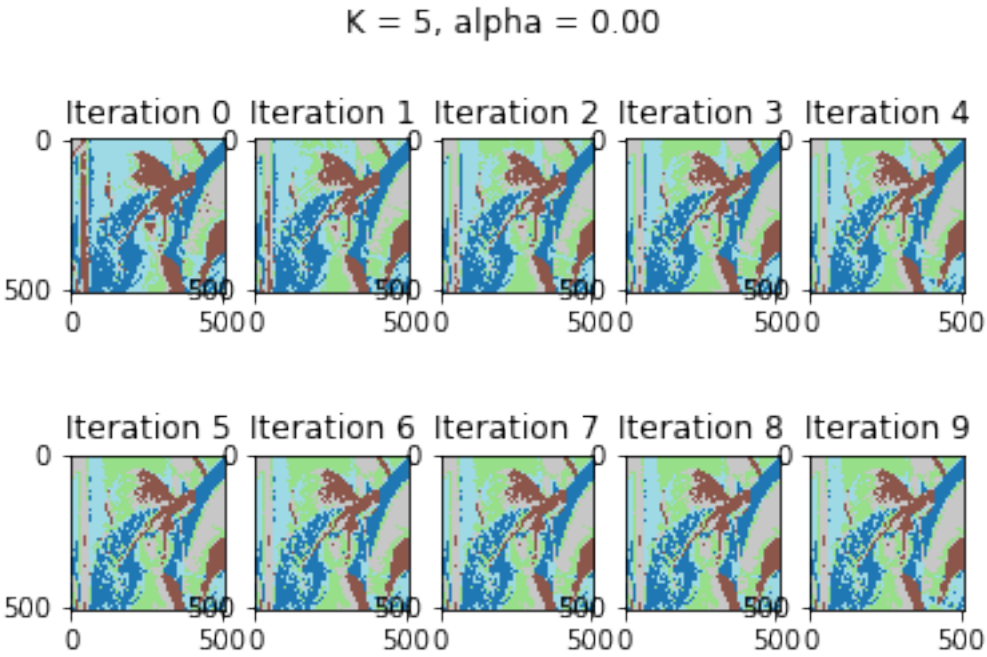

Spatial Energy graph
1e10
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

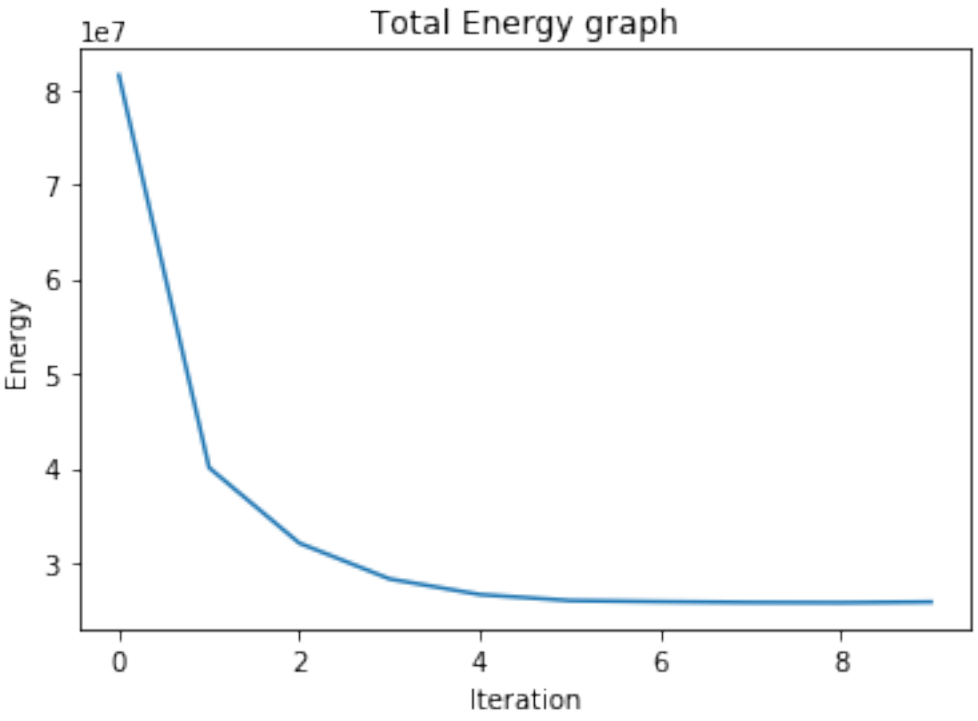


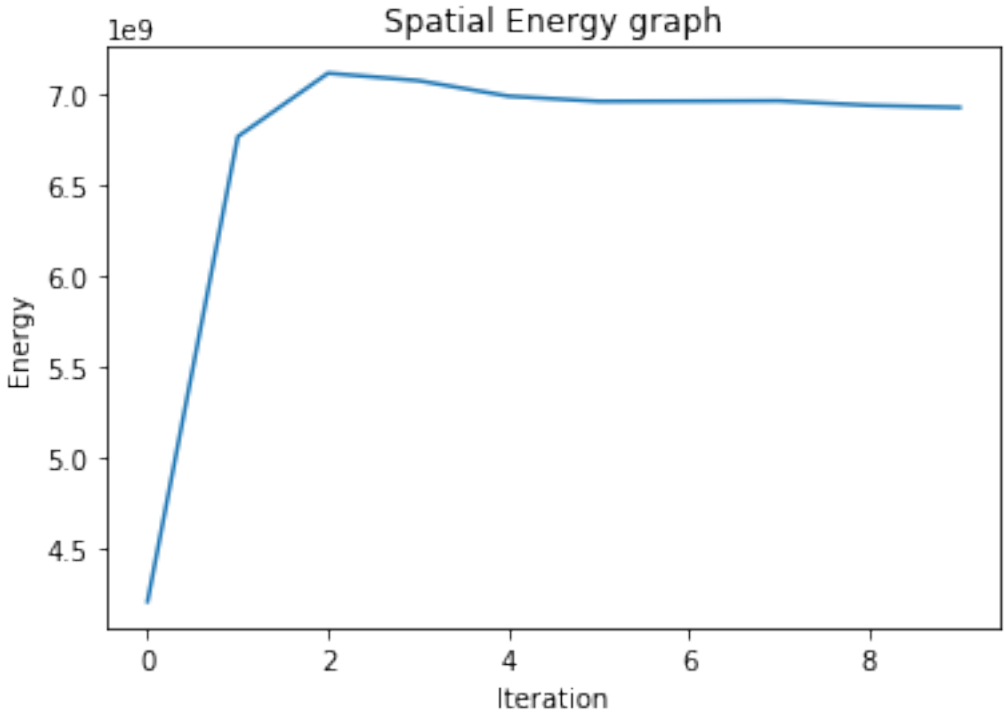
2 K = 5

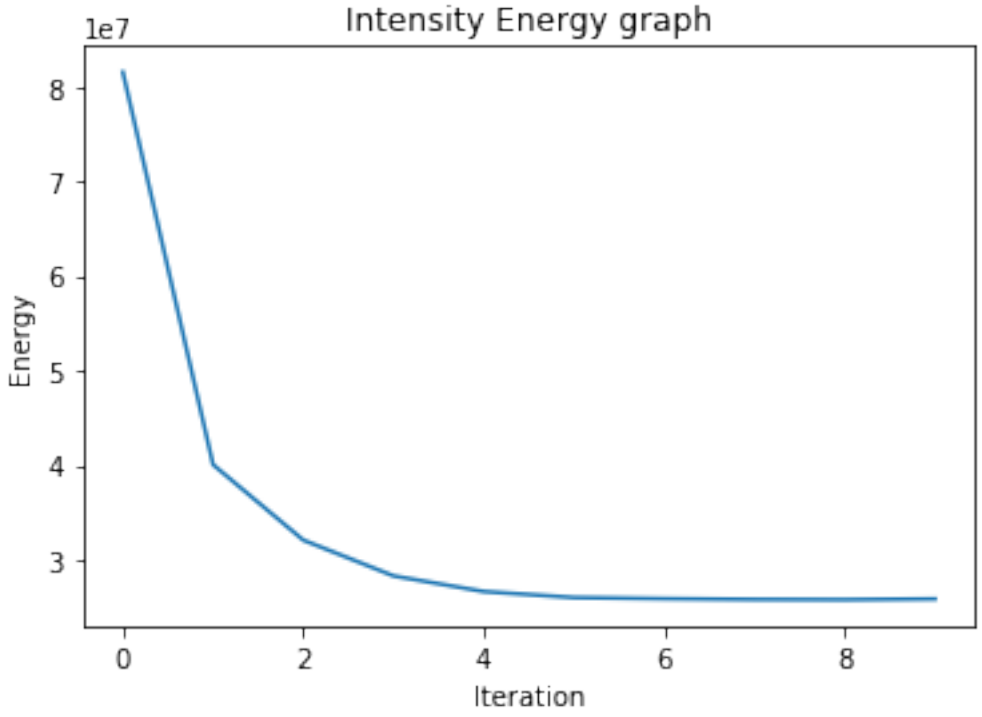
```
In [243]: K = 5
          KMeans(img_matrix, img_rows, img_cols, K, 0)
          KMeans(img_matrix, img_rows, img_cols, K, 0.01)
          KMeans(img_matrix, img_rows, img_cols, K, 0.03)
          KMeans(img_matrix, img_rows, img_cols, K, 0.05)
          KMeans(img_matrix, img_rows, img_cols, K, 0.07)
          KMeans(img_matrix, img_rows, img_cols, K, 0.08)
          KMeans(img_matrix, img_rows, img_cols, K, 0.10)
          KMeans(img_matrix, img_rows, img_cols, K, 0.25)
          KMeans(img_matrix, img_rows, img_cols, K, 0.50)
          KMeans(img_matrix, img_rows, img_cols, K, 0.75)
          KMeans(img_matrix, img_rows, img_cols, K, 1)

0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
```

```
8 th iteration
9 th iteration
```

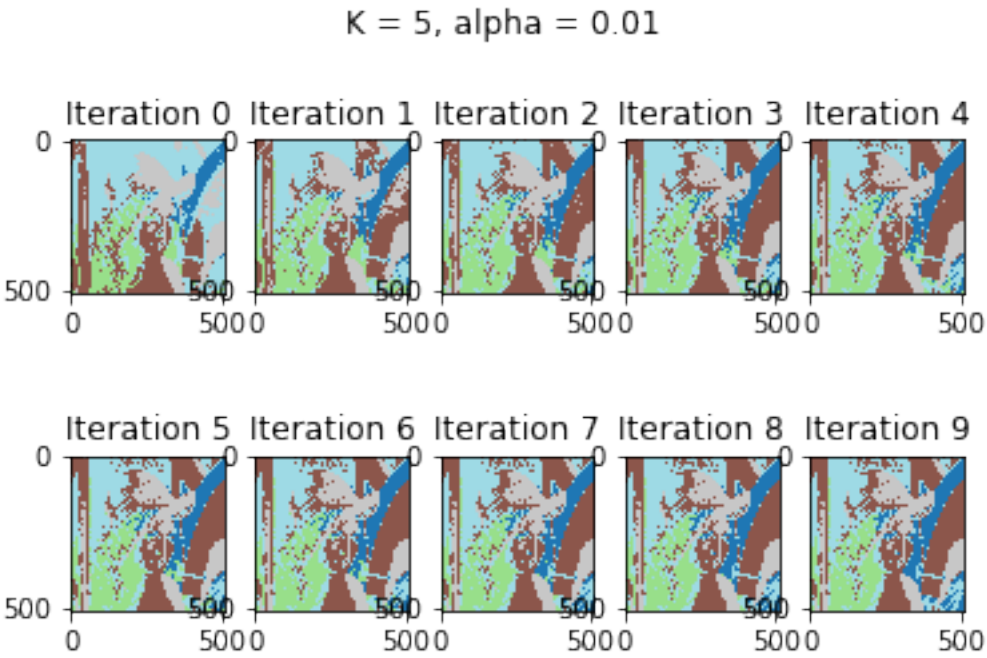


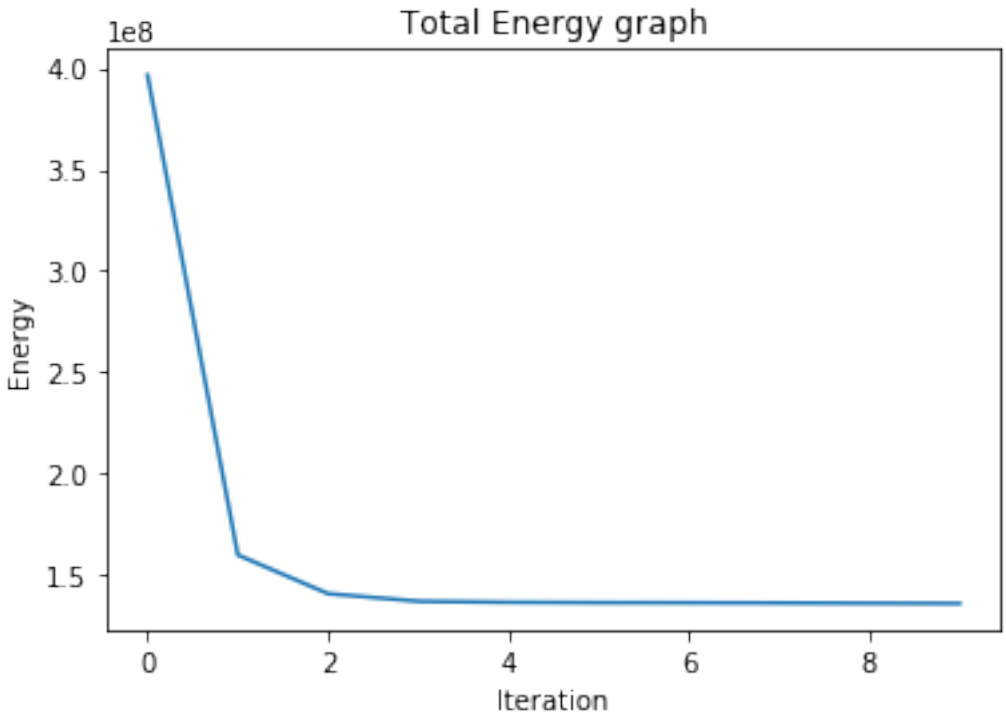


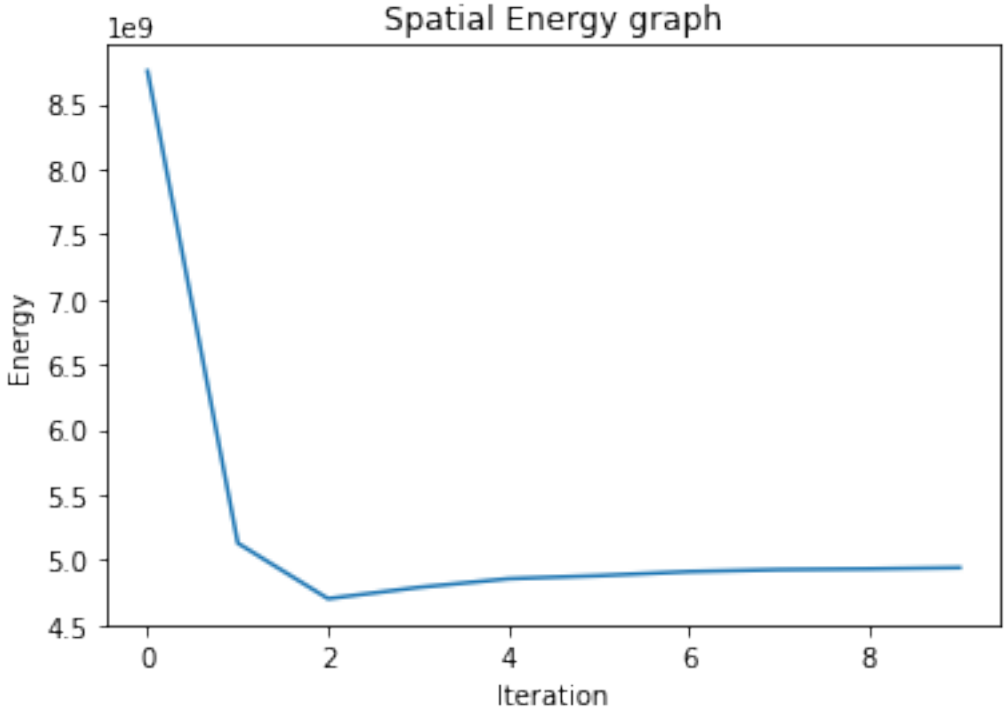

Spatial Energy graph
1e9
7.0
6.5
6.0
5.5
5.0
4.5
Energy
0
2
4
6
8
Iteration

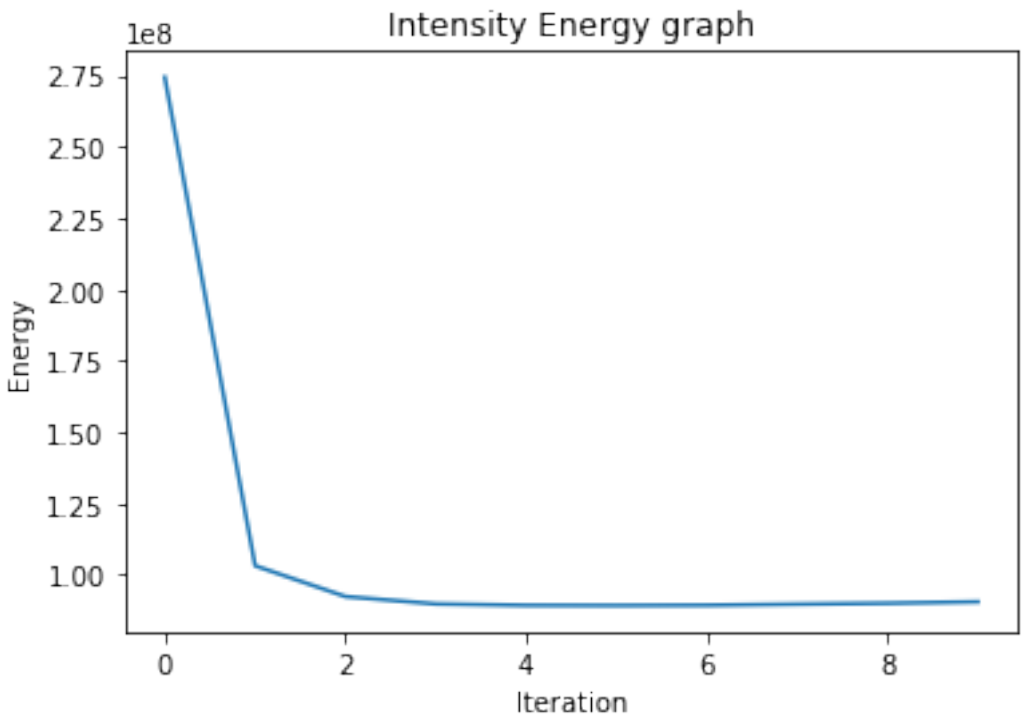

Intensity Energy graph
1e7
8
7
6
5
4
3
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

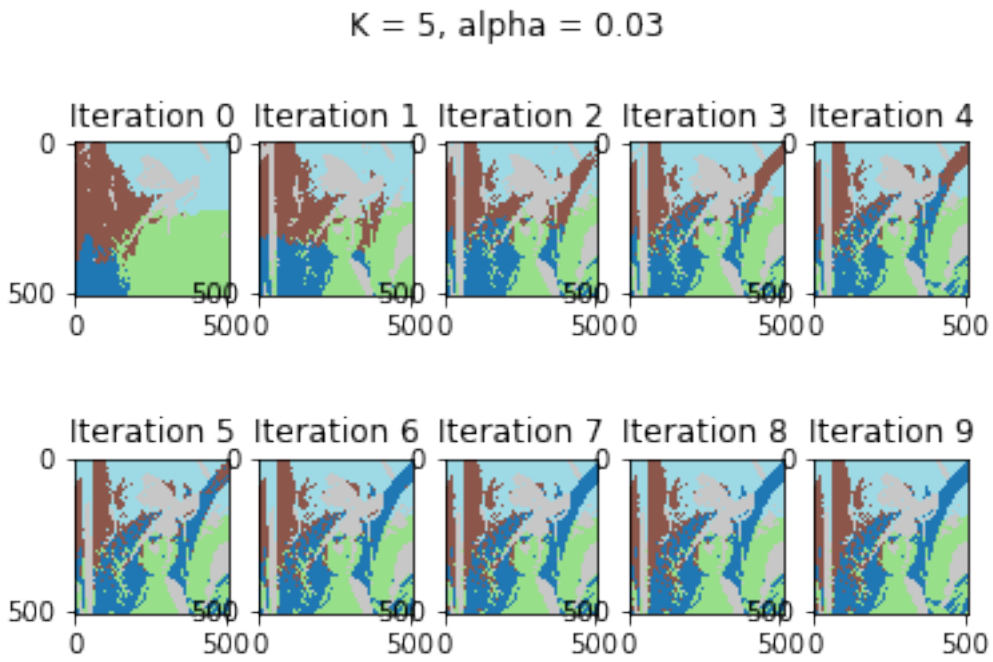


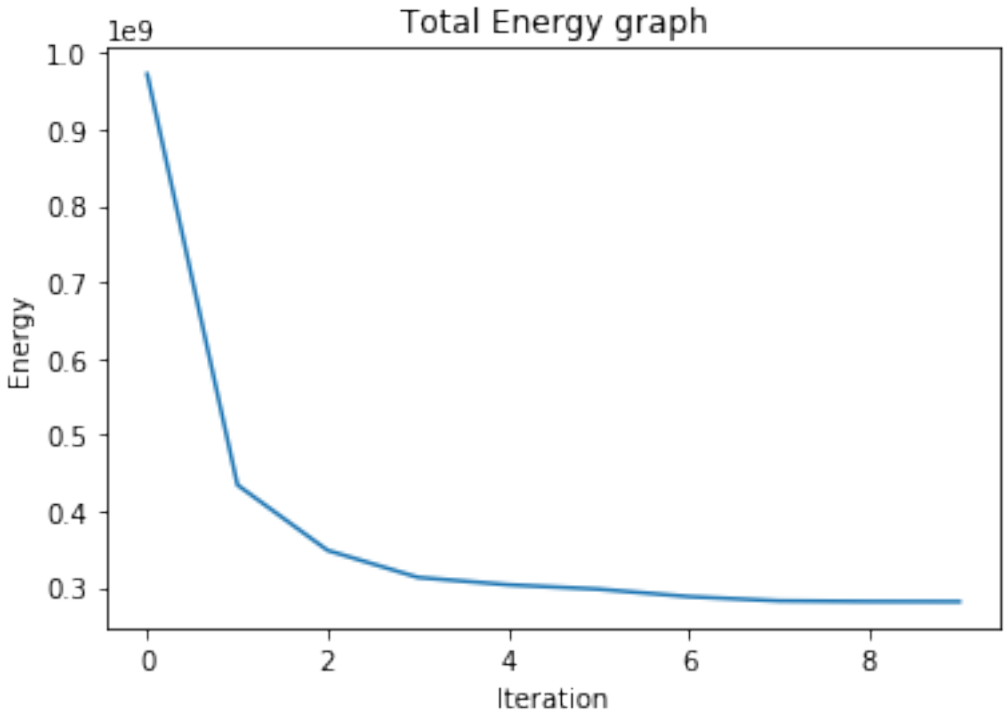

Total Energy graph
1e8
4.0
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

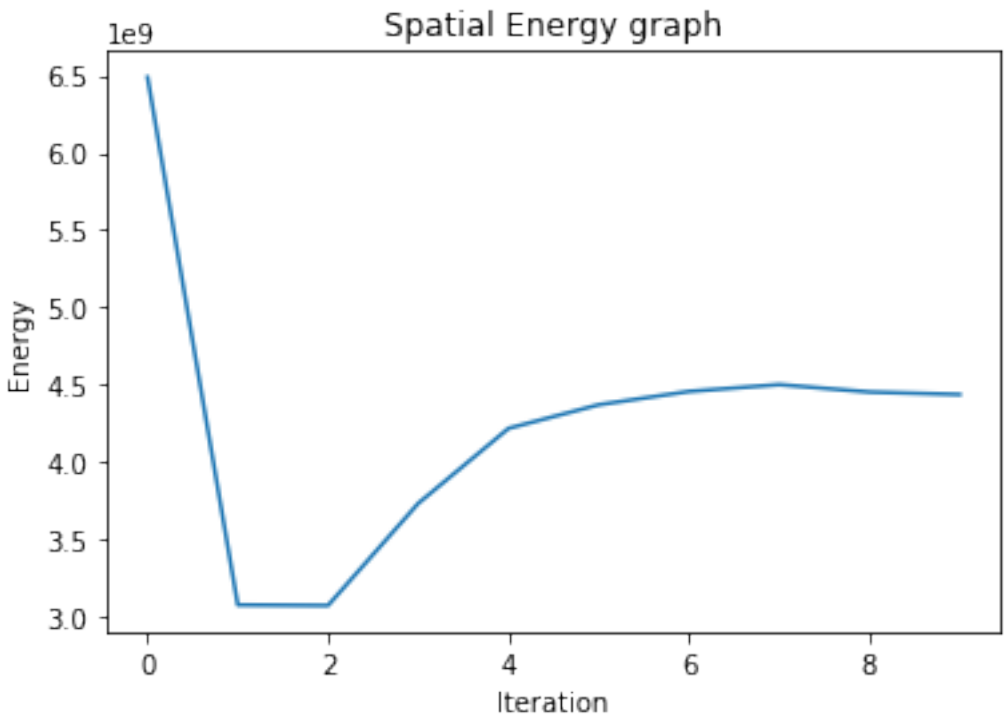

Spatial Energy graph
1e9
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
Energy
0
2
4
6
8
Iteration

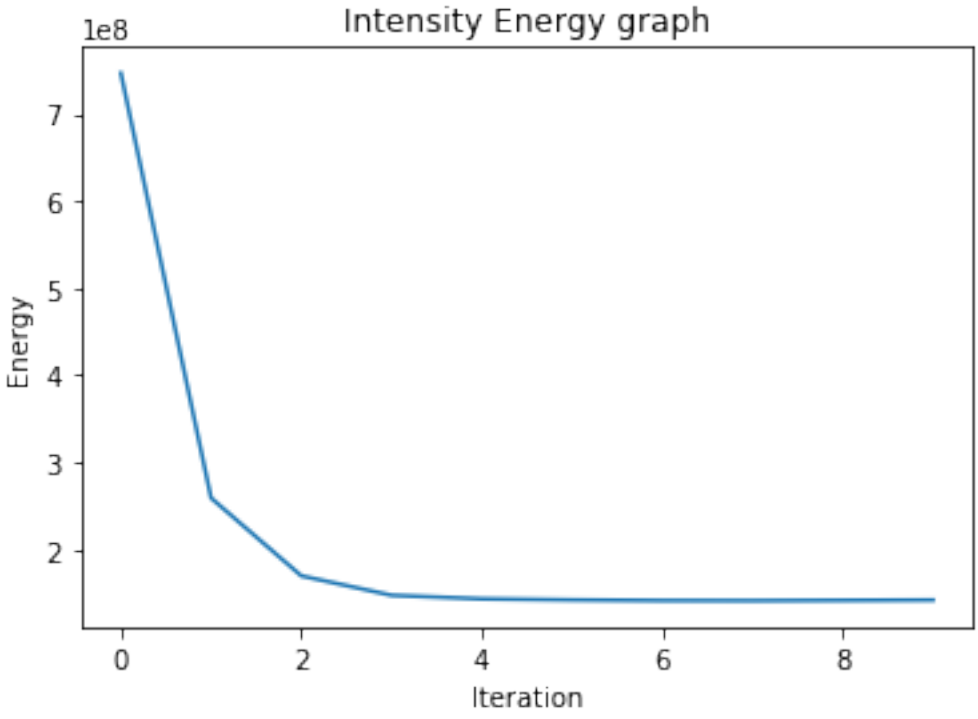



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

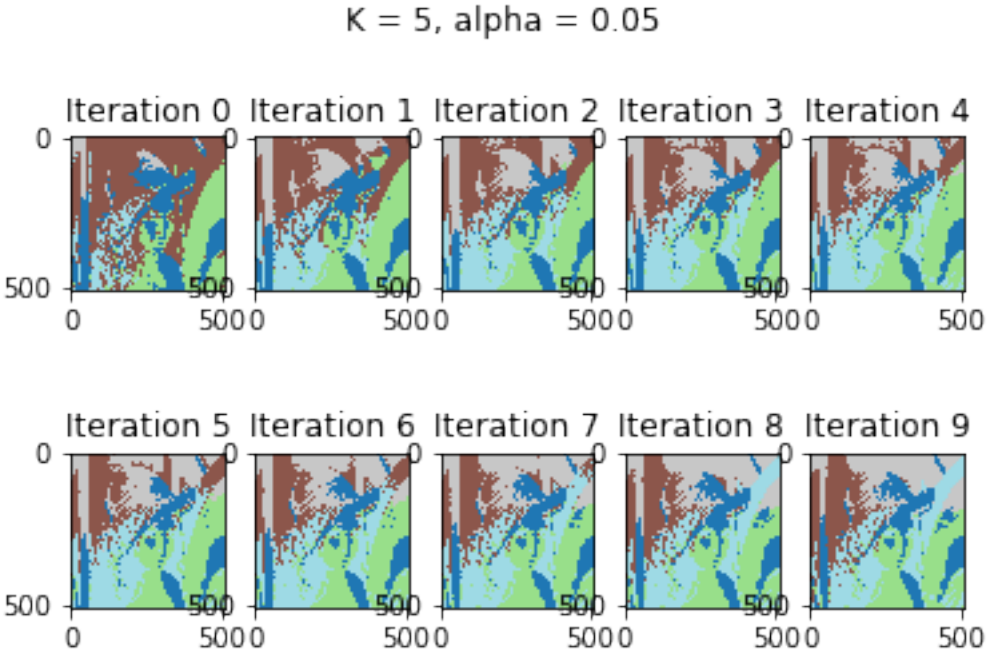

K = 5, alpha = 0.03
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

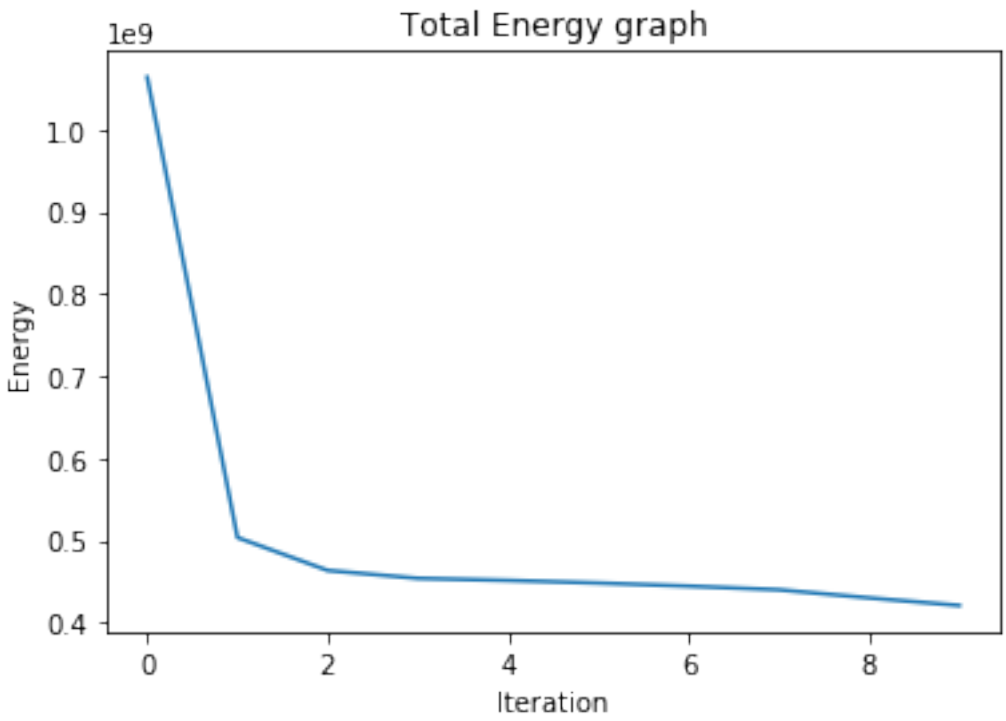

Total Energy graph
1e9
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
Energy
0
2
4
6
8
Iteration

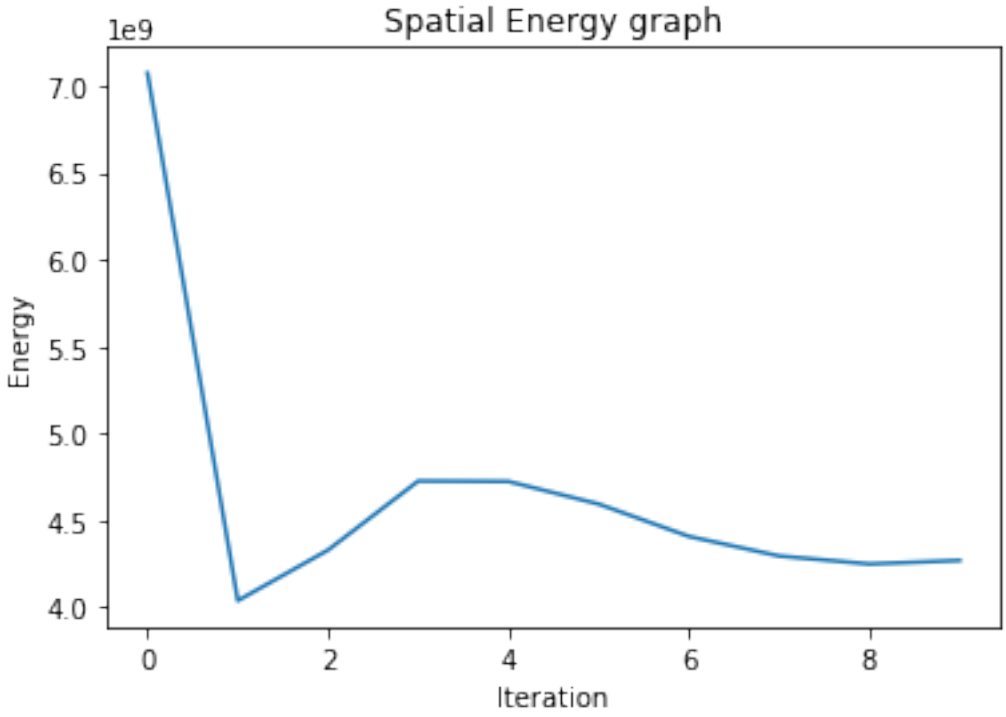

Spatial Energy graph
1e9
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
Energy
0
2
4
6
8
Iteration

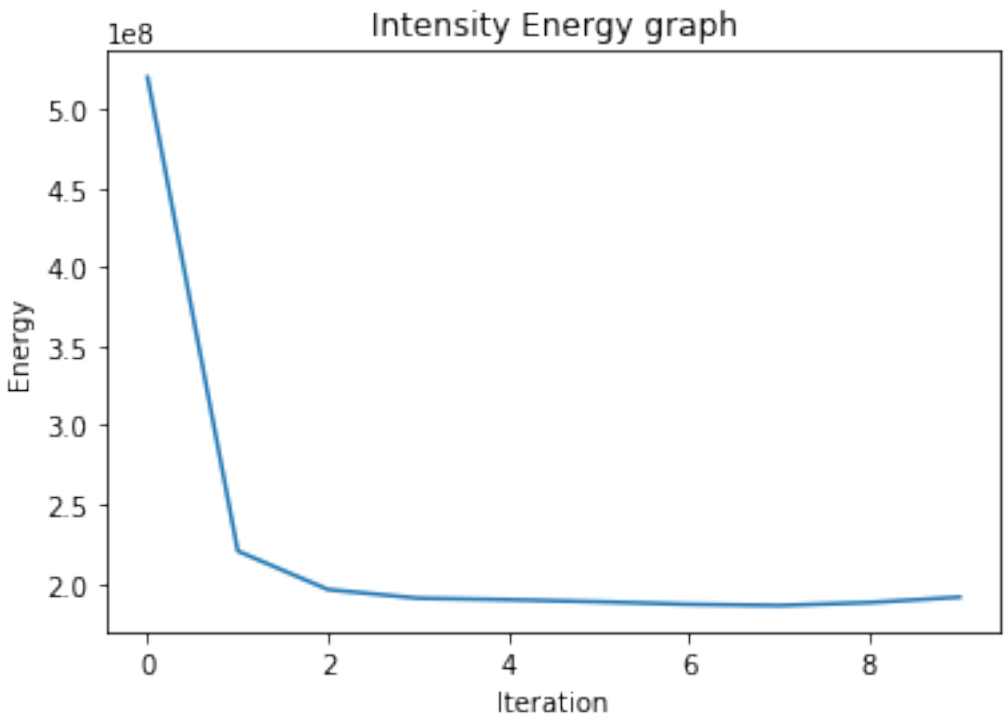

Intensity Energy graph
1e8
7
6
5
4
3
2
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

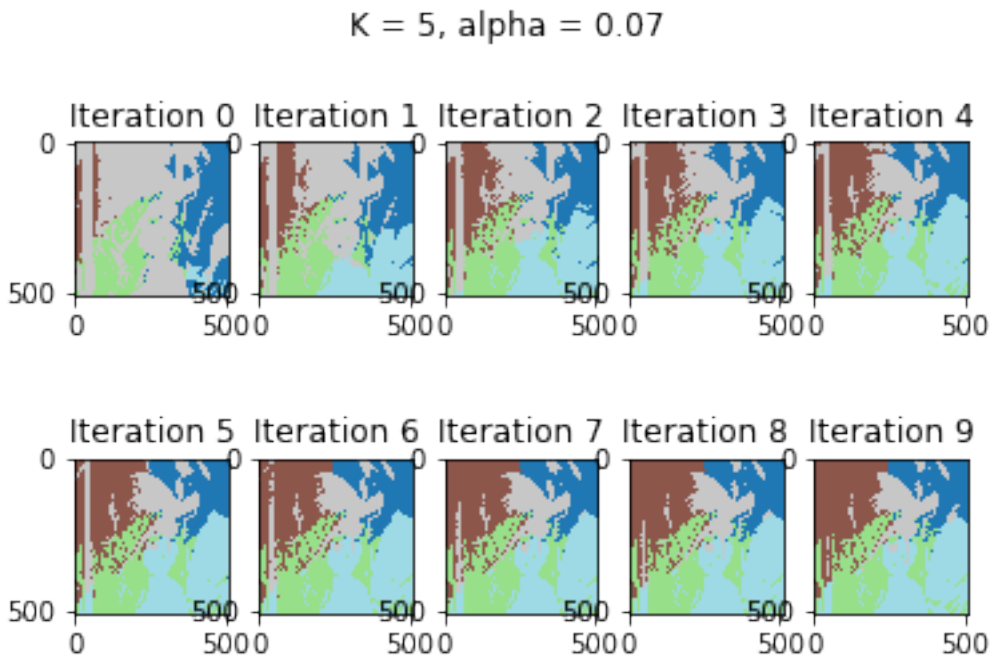


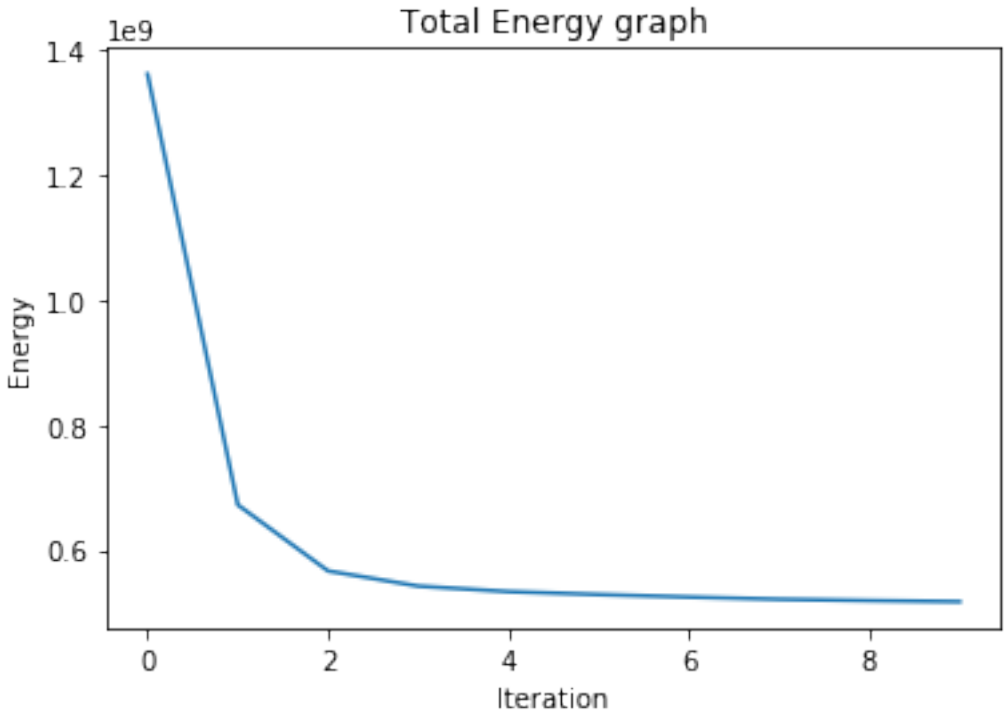

Total Energy graph
1e9
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration

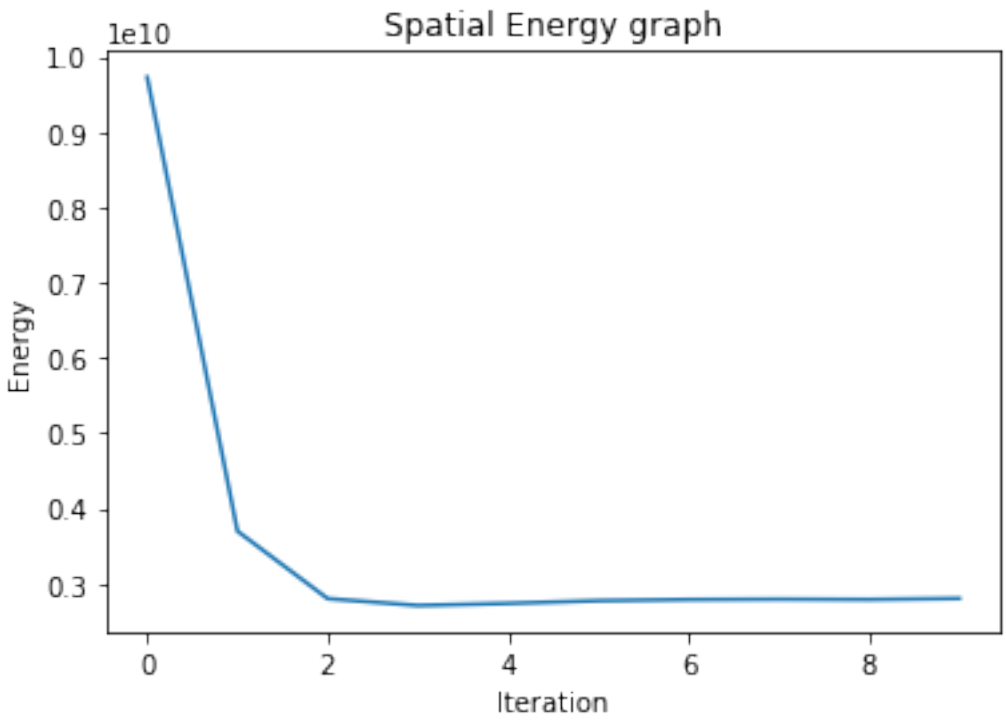

Spatial Energy graph
1e9
7.0
6.5
6.0
5.5
5.0
4.5
4.0
Energy
0
2
4
6
8
Iteration

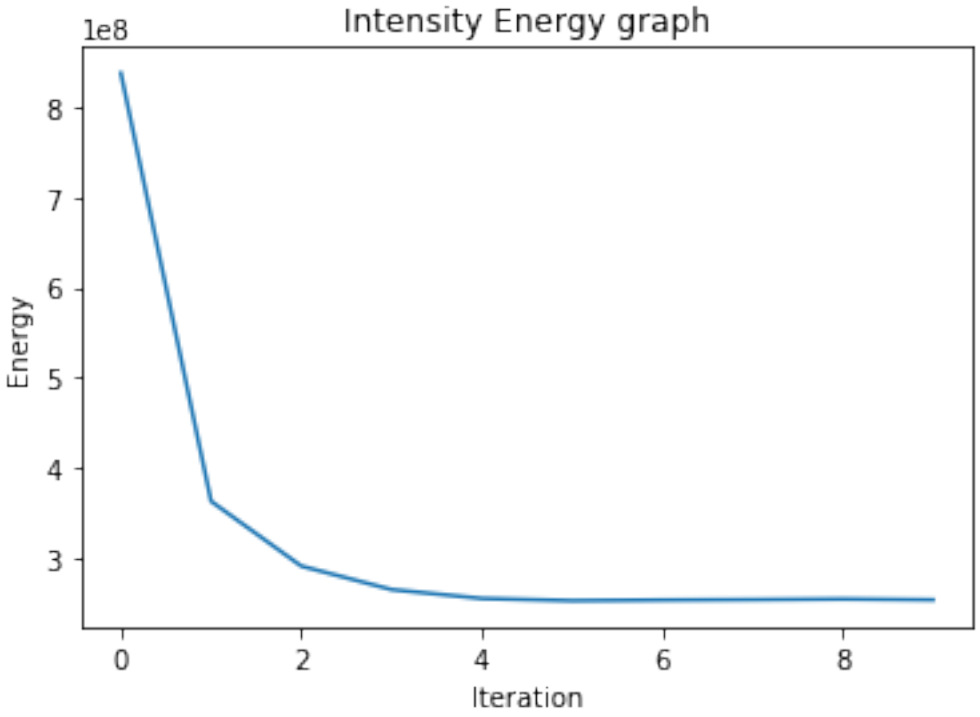



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

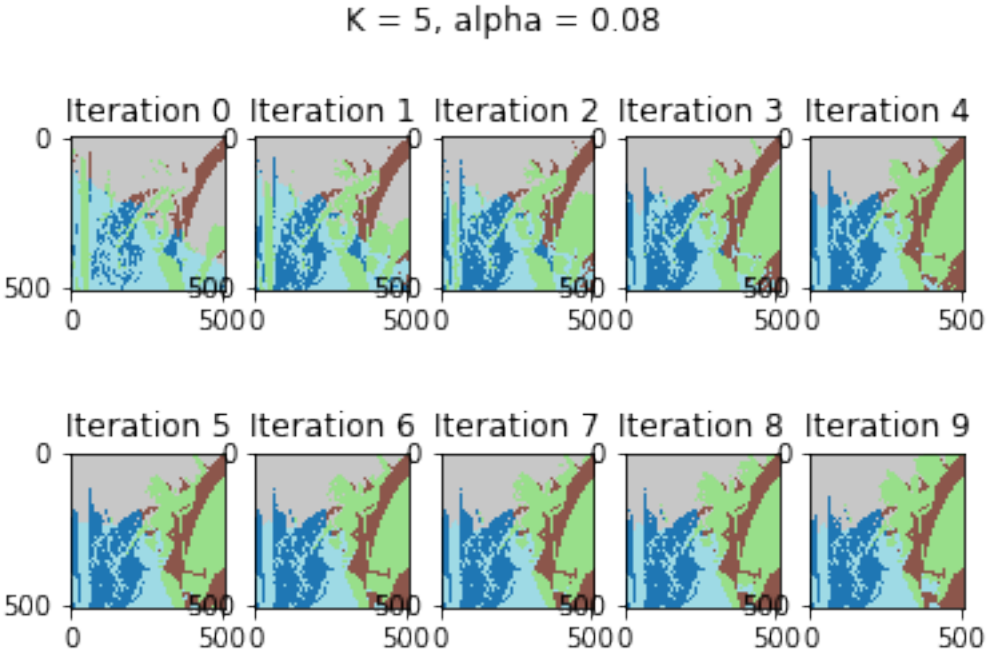

K = 5, alpha = 0.07
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

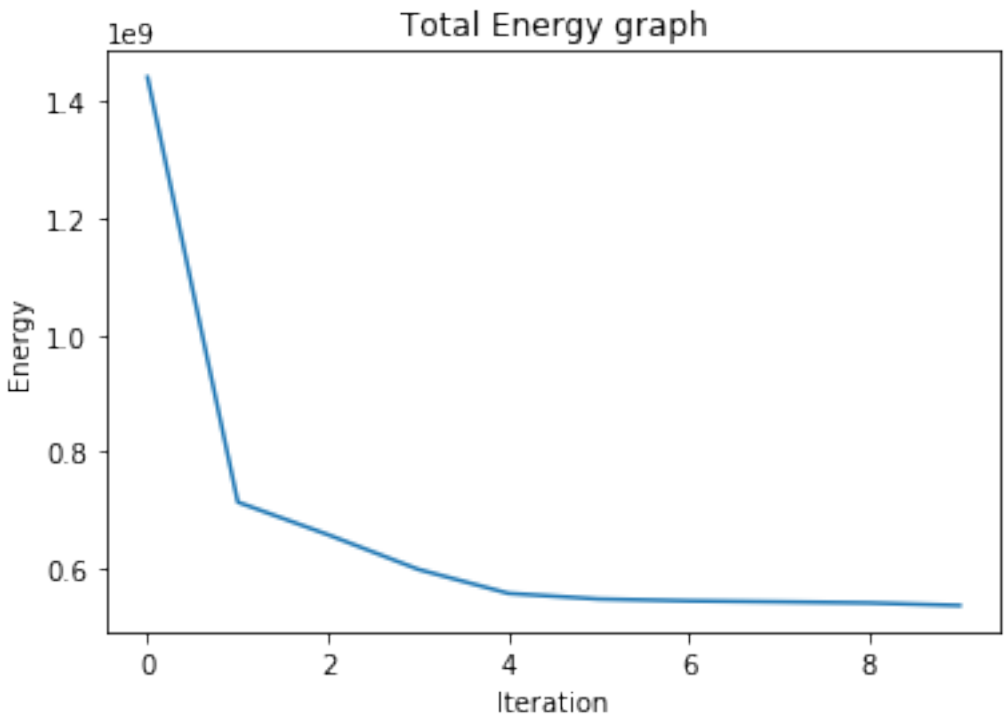

Total Energy graph
1e9
1.4
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

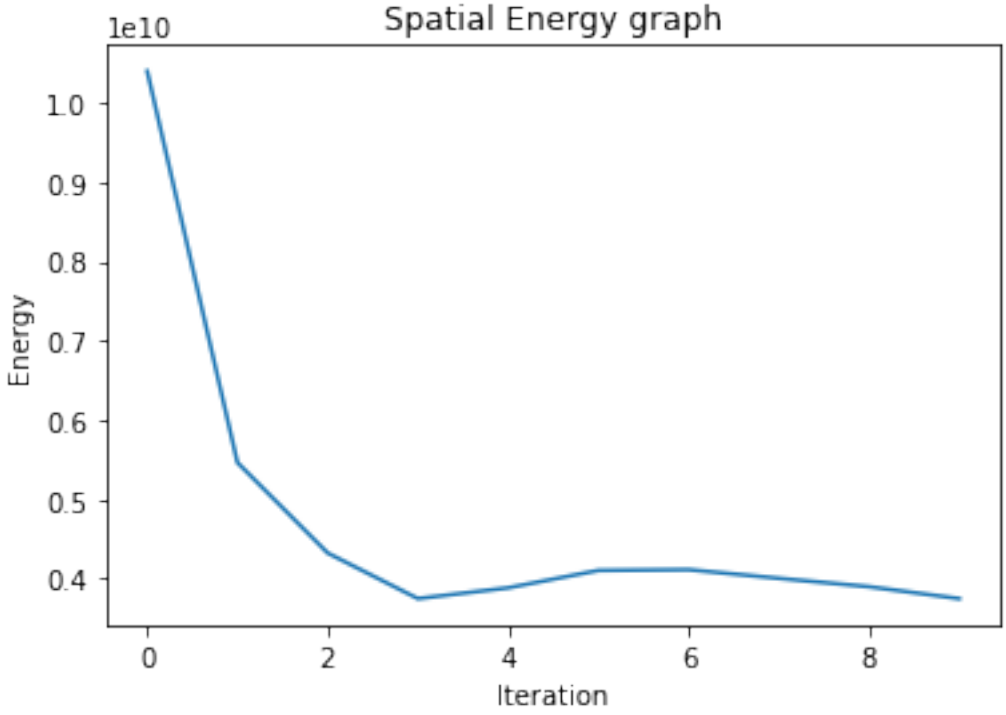

Spatial Energy graph
1e10
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
Energy
0
2
4
6
8
Iteration

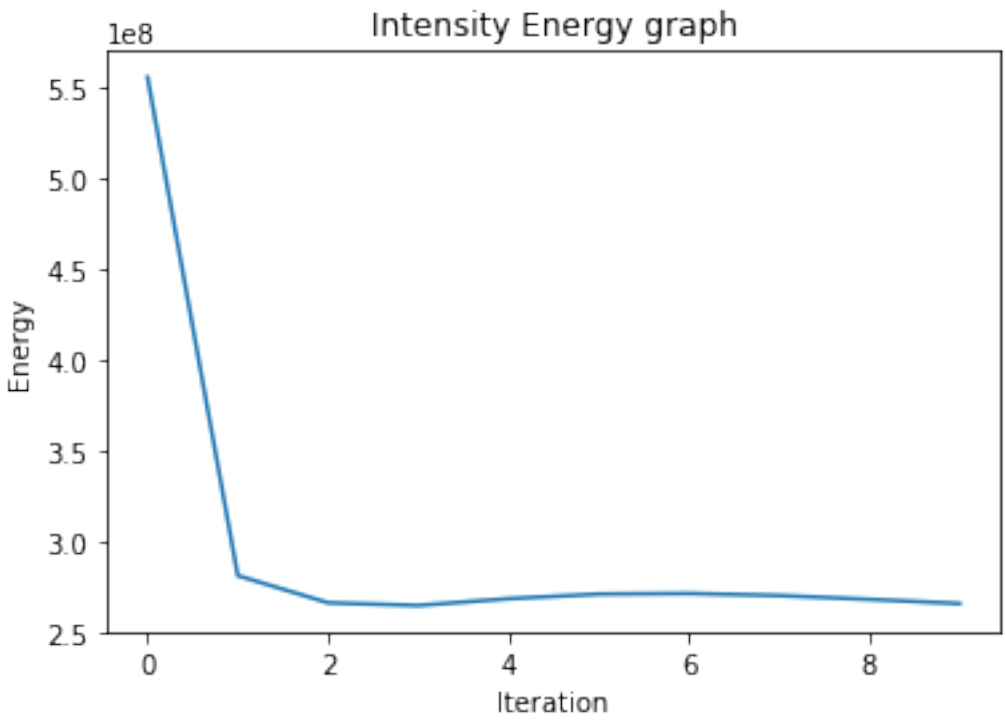

Intensity Energy graph
1e8
8
7
6
5
4
3
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

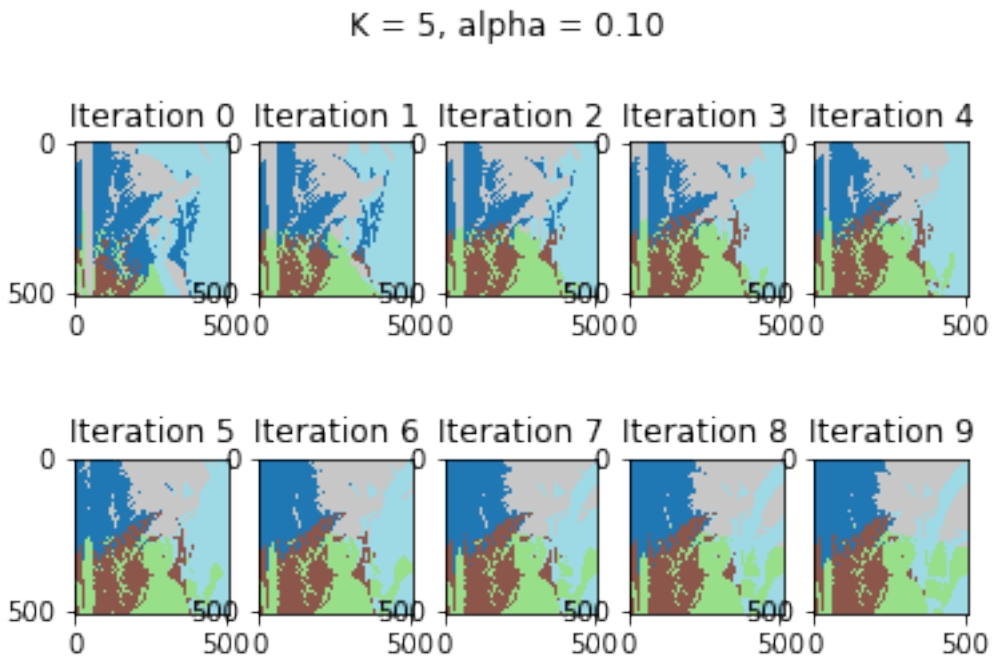


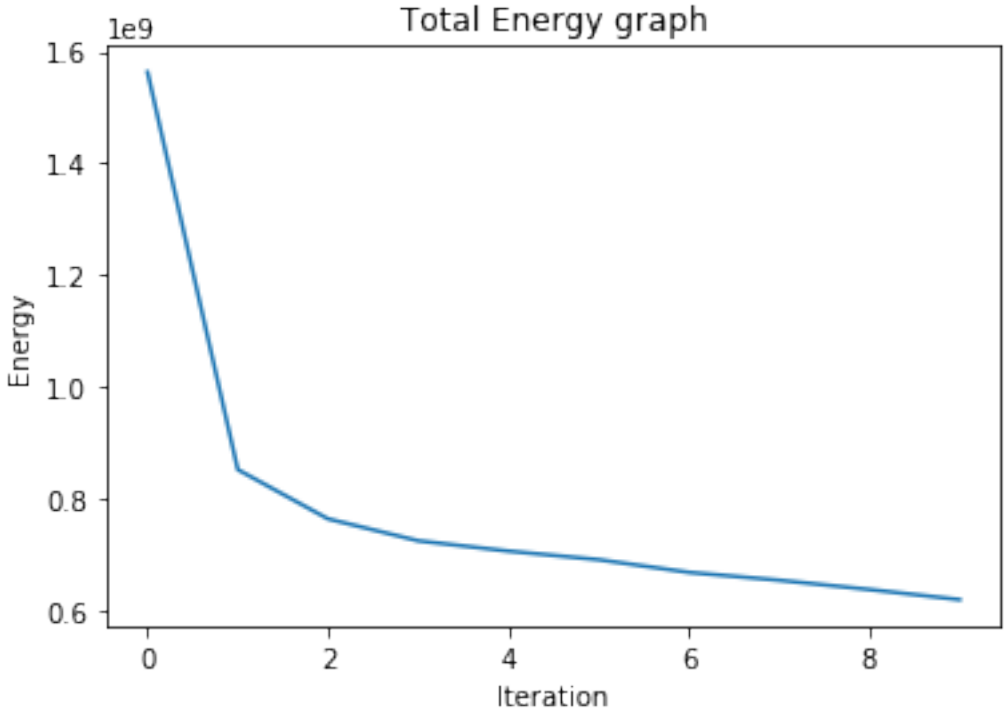

Total Energy graph
1e9
1.4
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

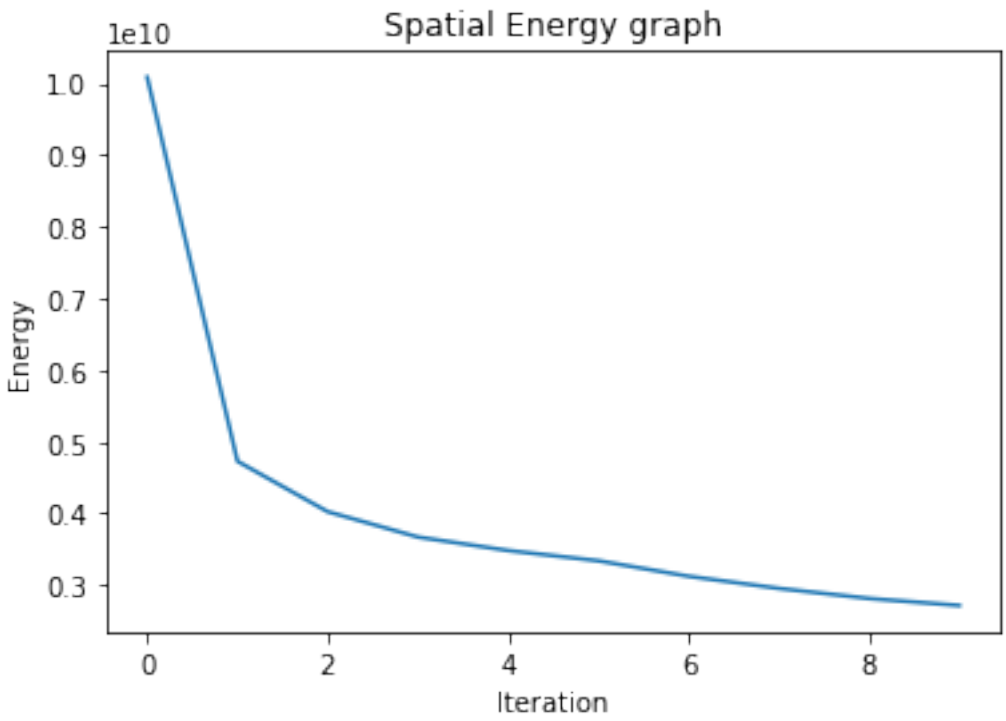

Spatial Energy graph
1e10
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration

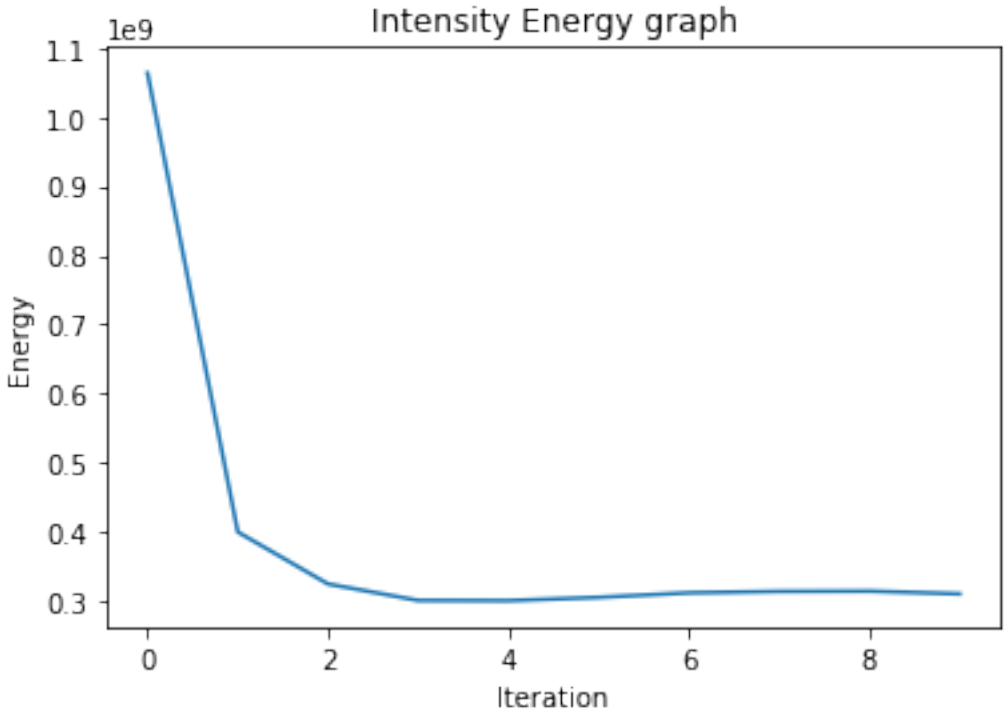



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

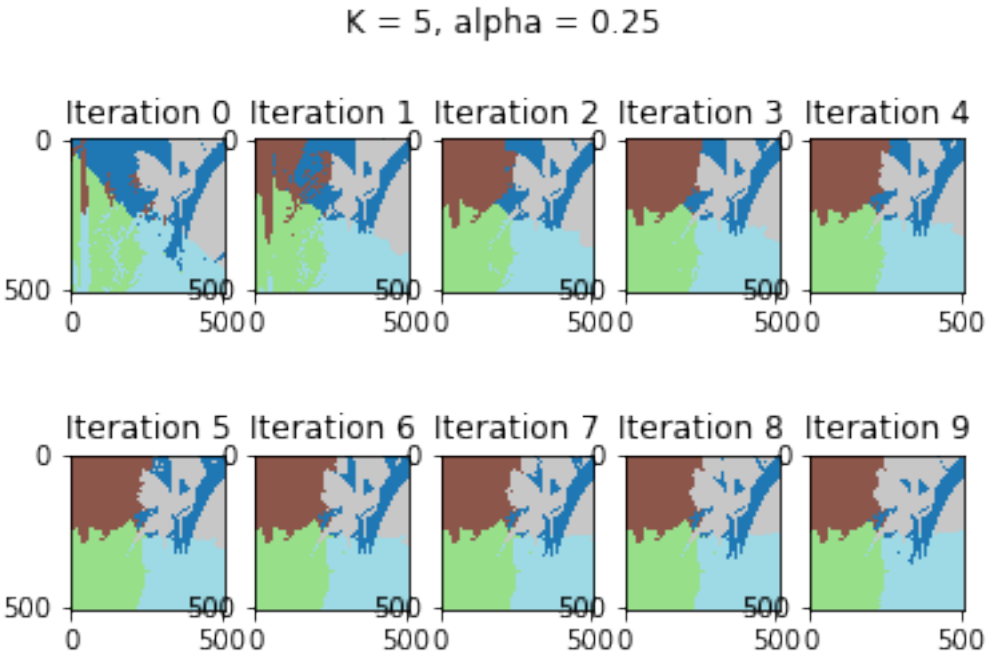

K = 5, alpha = 0.10
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

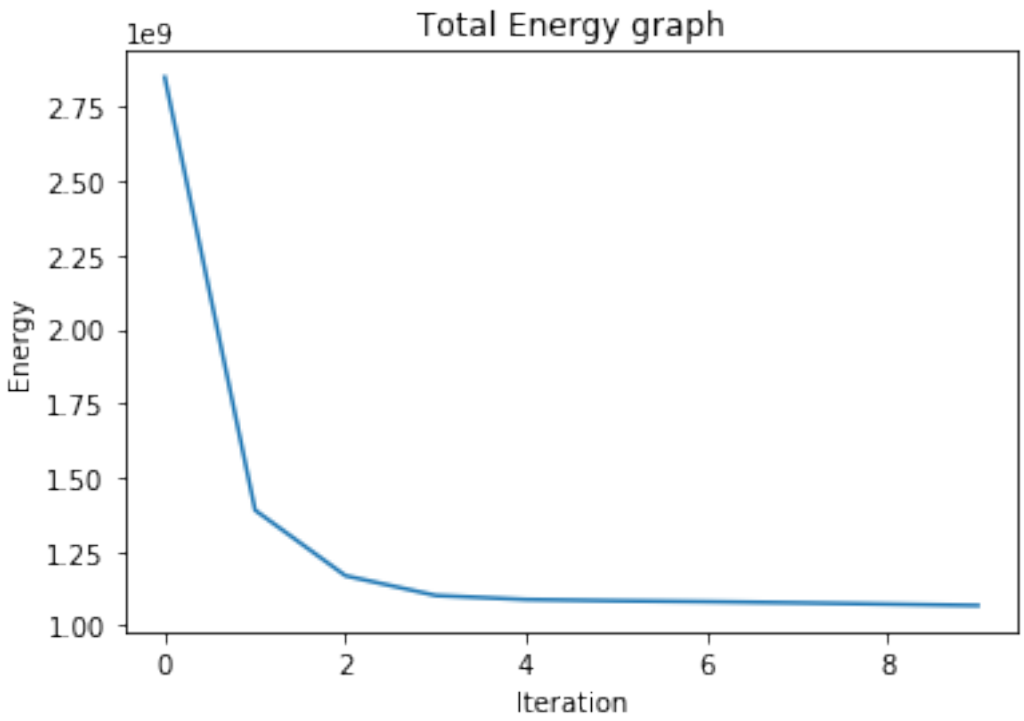

Total Energy graph
1e9
1.6
1.4
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

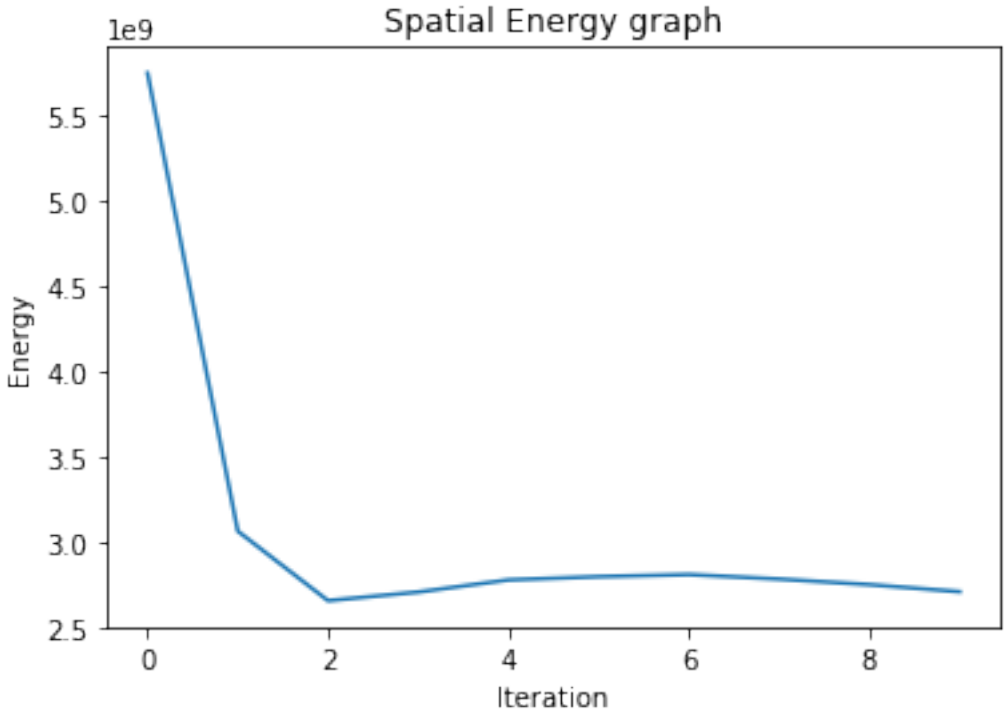

Spatial Energy graph
1e10
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
Energy
0
2
4
6
8
Iteration

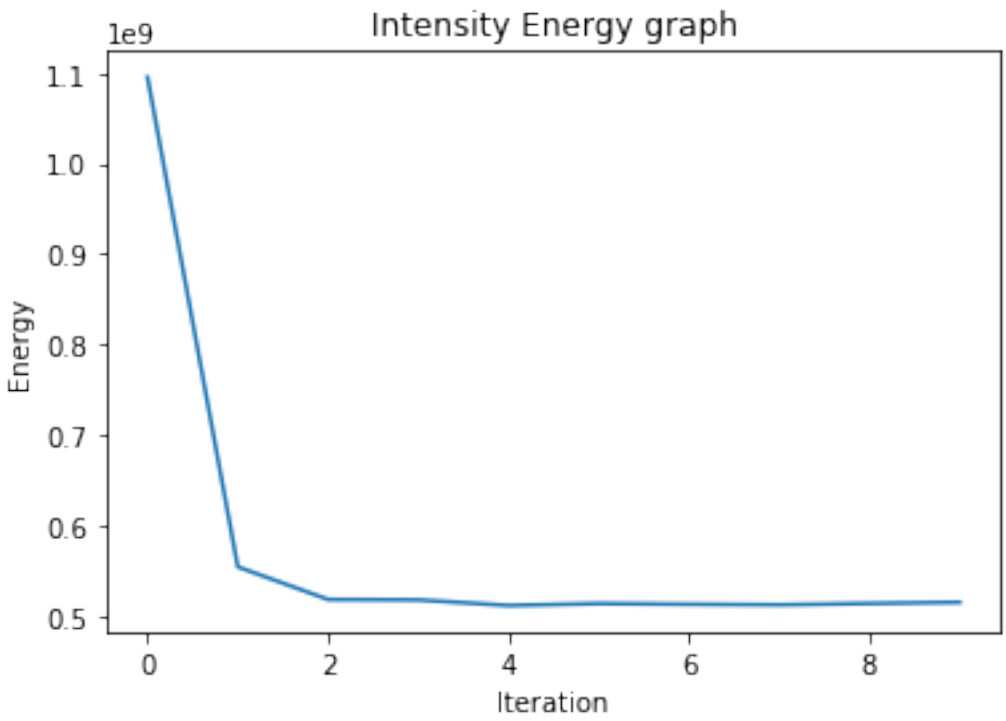

Intensity Energy graph
1e9
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

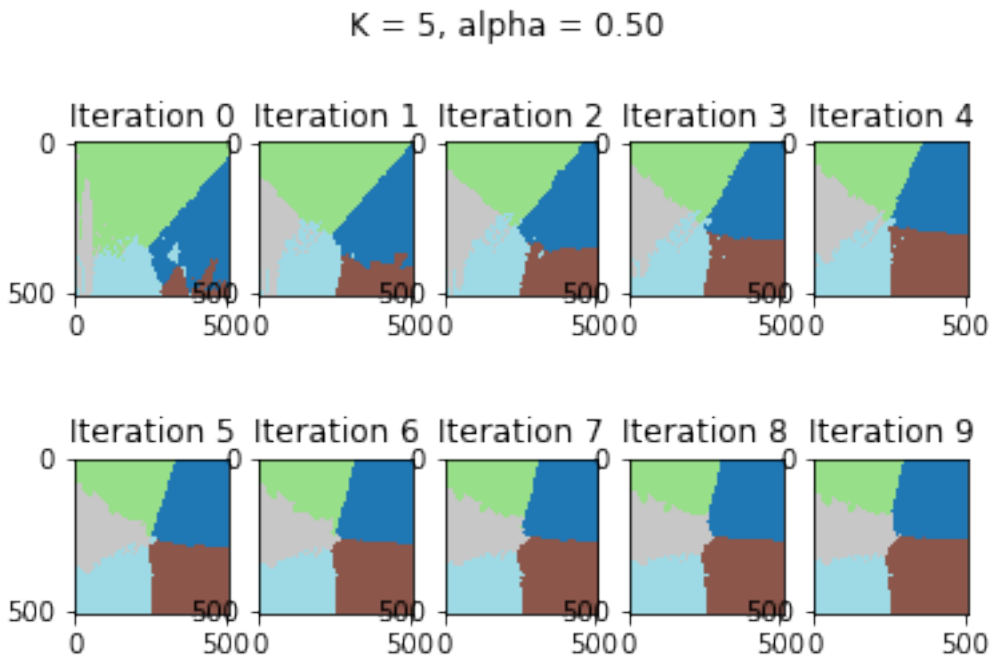


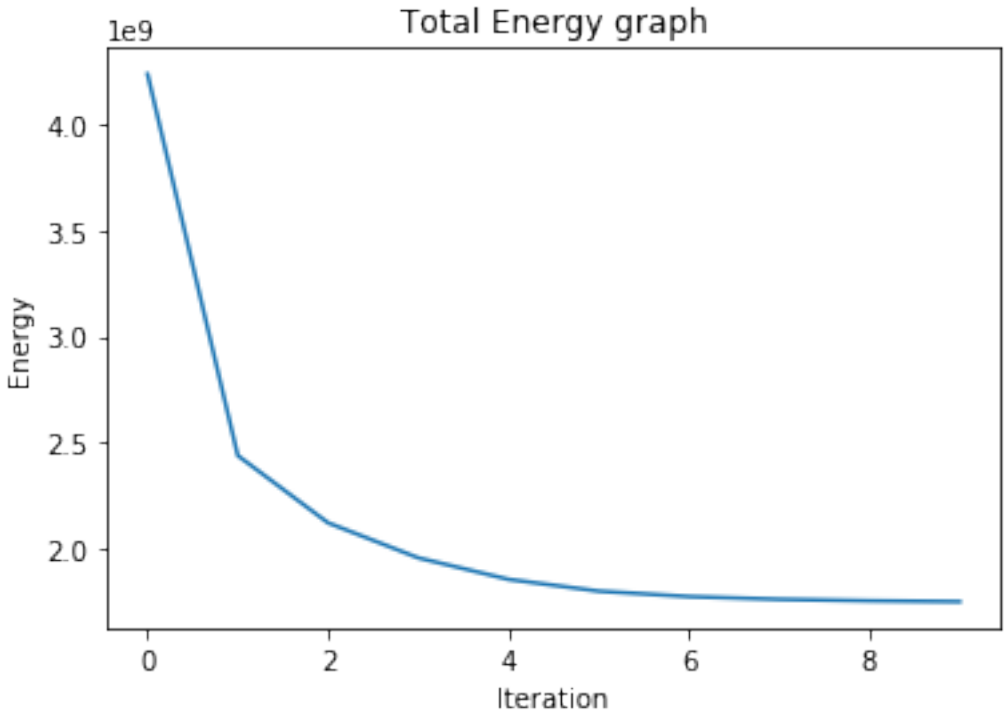

Total Energy graph
1e9
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
Energy
0
2
4
6
8
Iteration

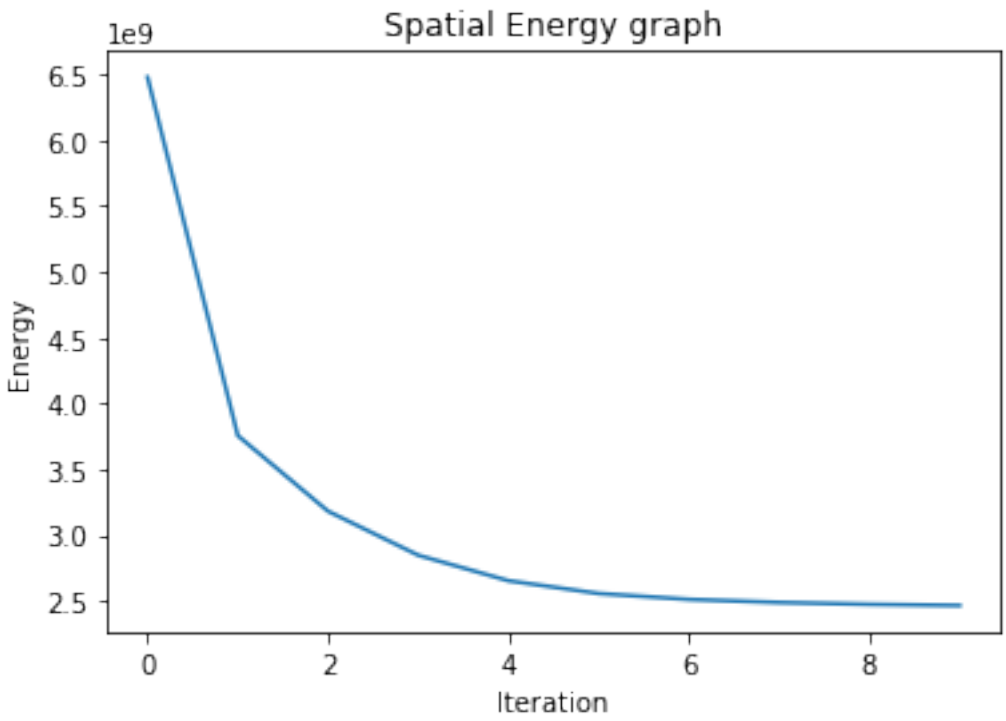

Spatial Energy graph
1e9
5.5
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

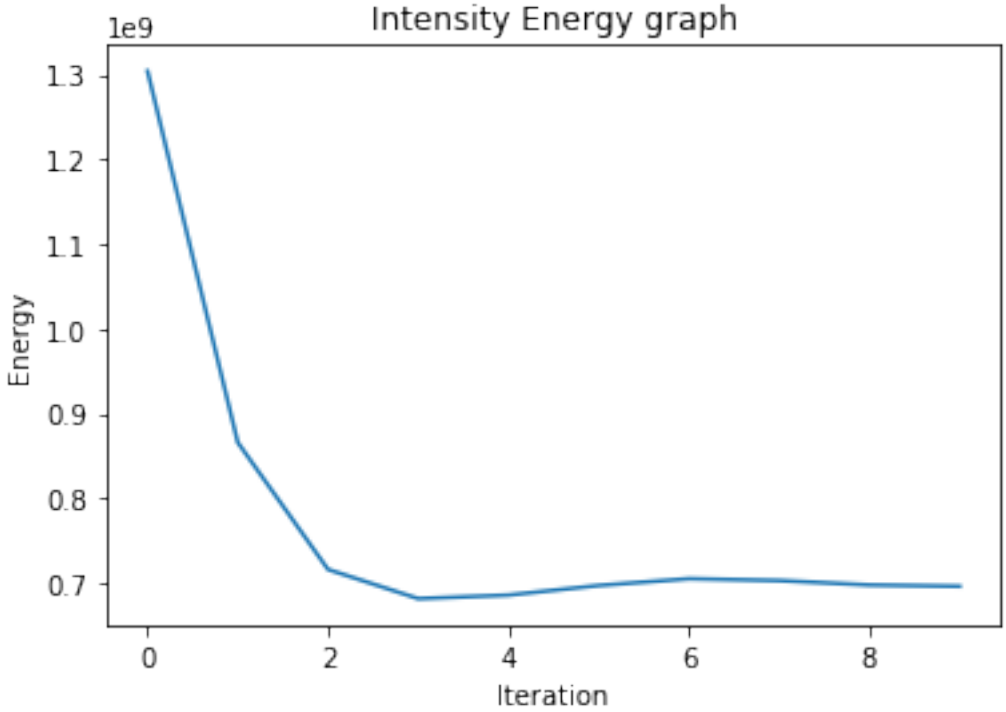



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

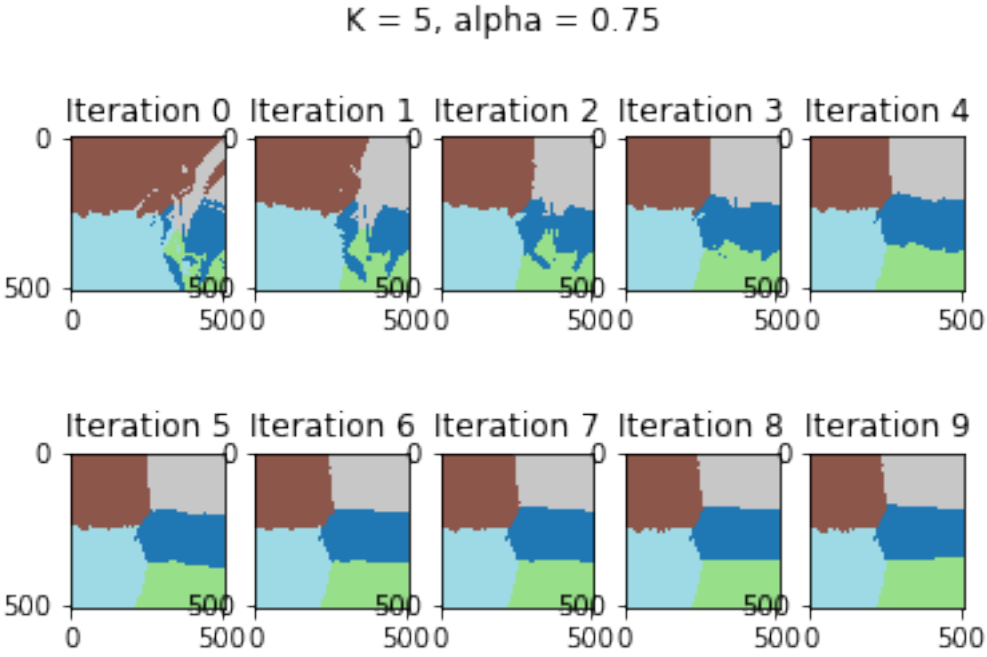

K = 5, alpha = 0.50
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

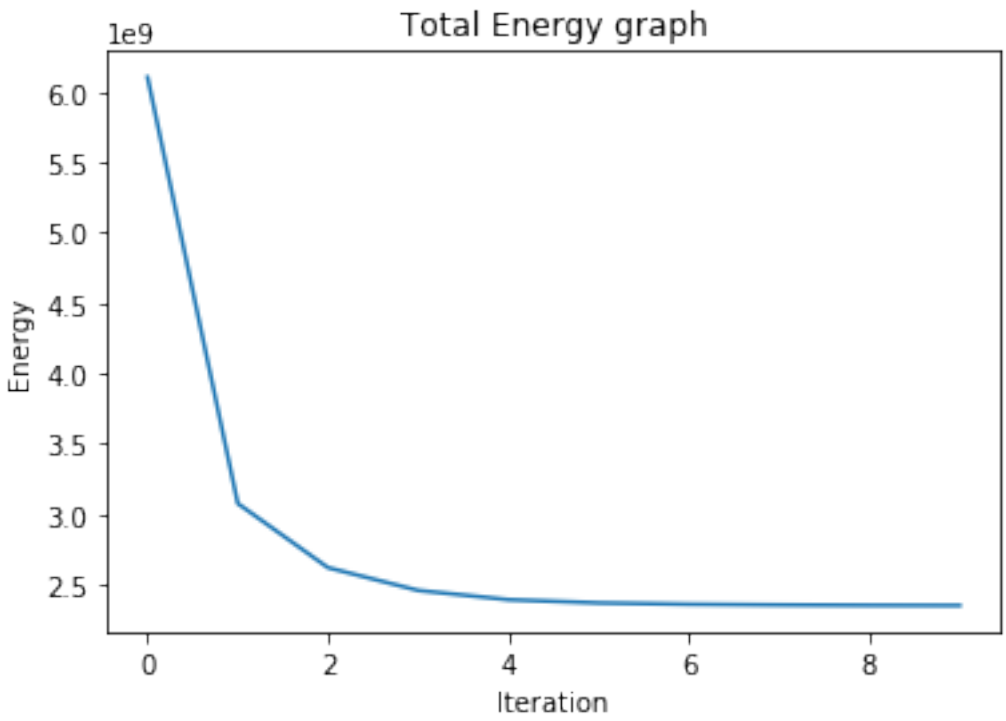

Total Energy graph
1e9
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

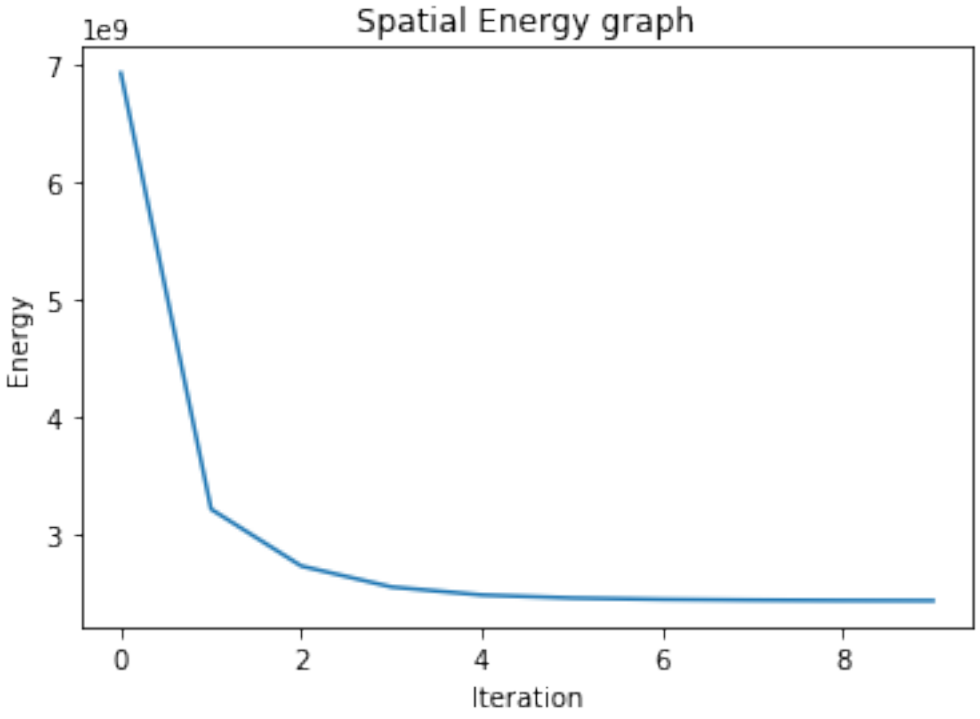

Spatial Energy graph
1e9
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

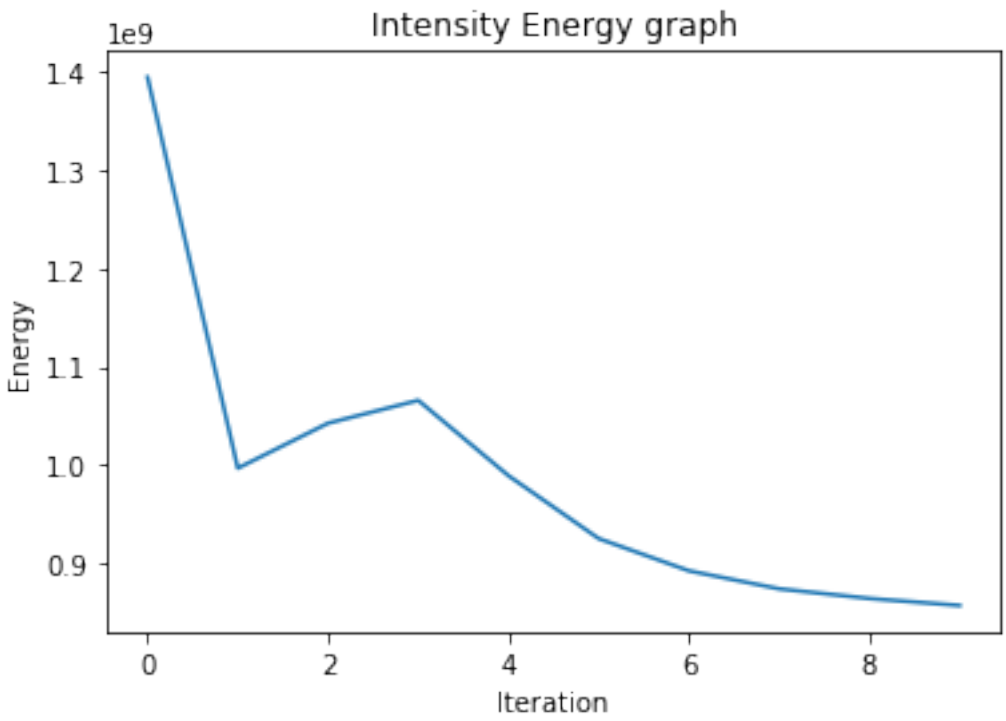

Intensity Energy graph
1e9
1.3
1.2
1.1
1.0
0.9
0.8
0.7
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

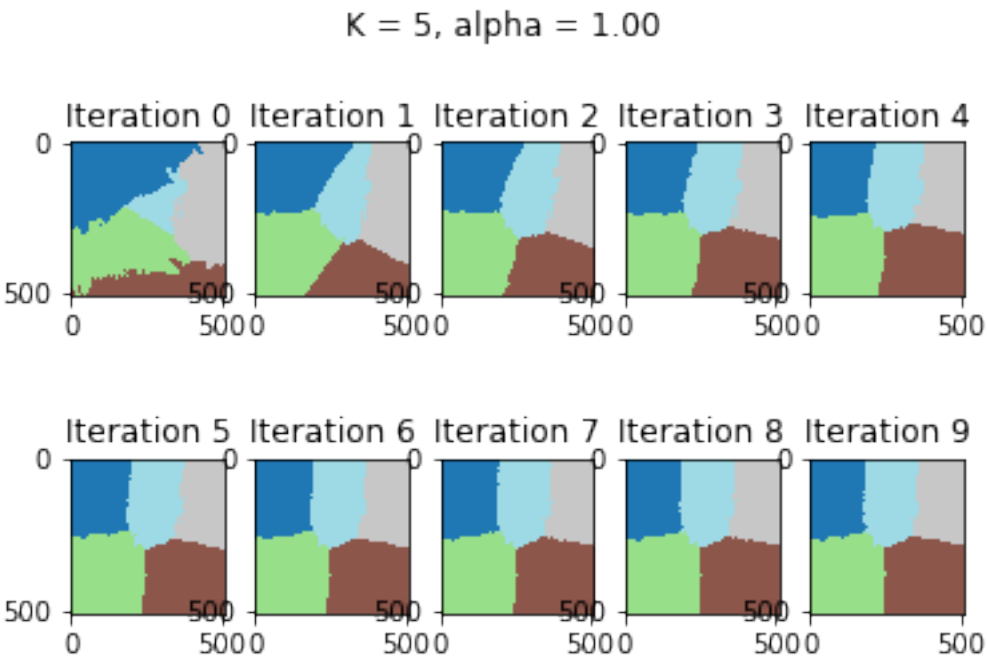


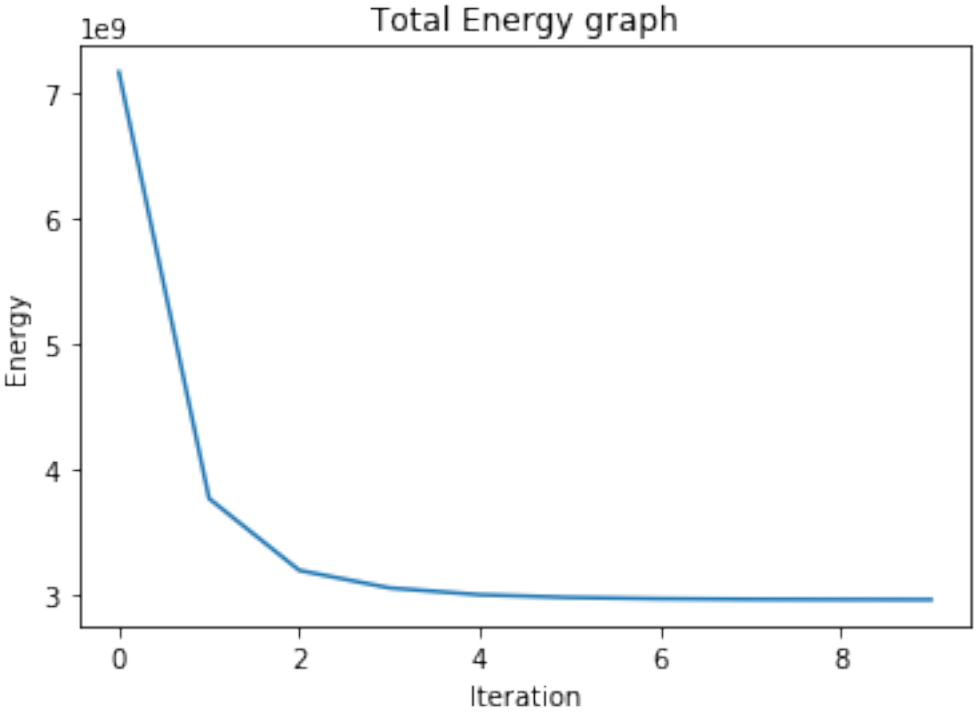

Total Energy graph
1e9
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

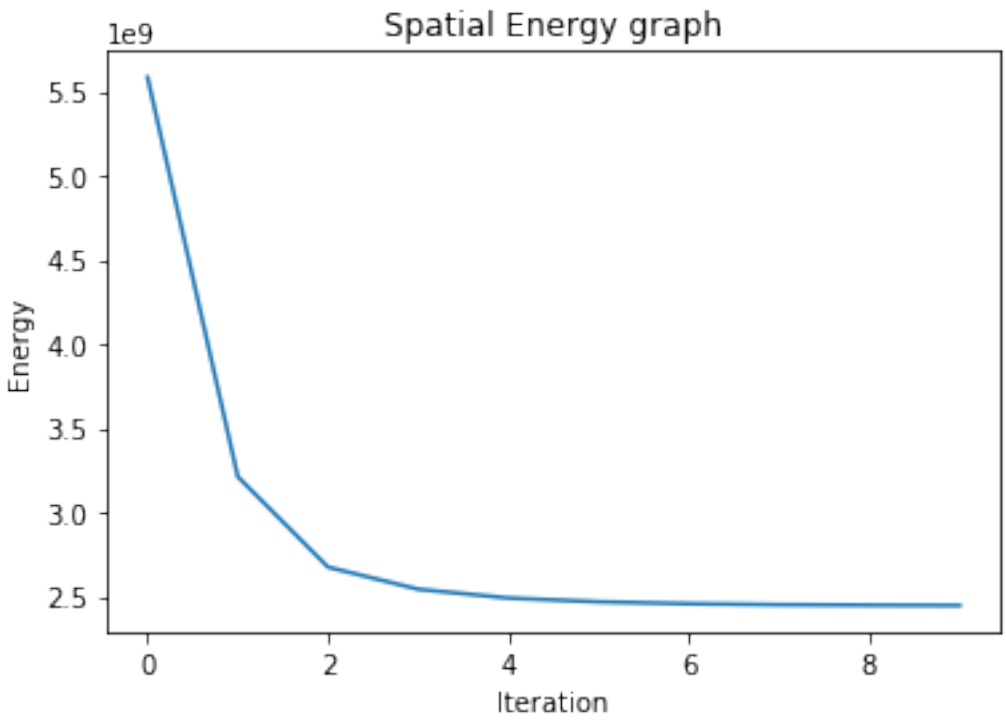

Spatial Energy graph
1e9
7
6
5
4
3
Energy
0
2
4
6
8
Iteration

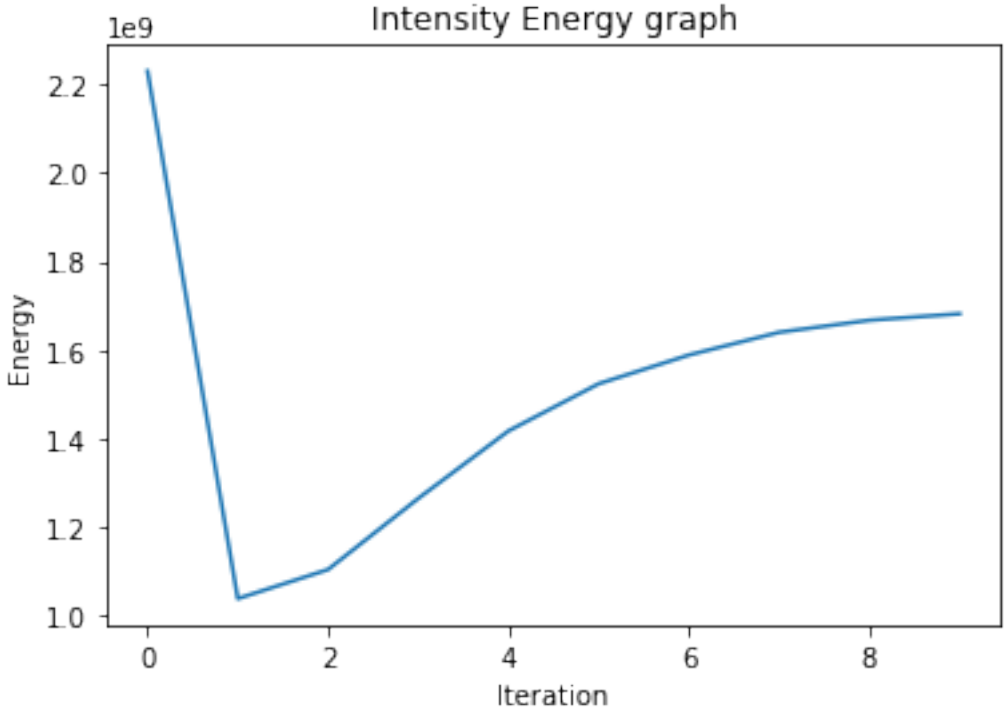



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```


K = 5, alpha = 1.00
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500


Total Energy graph
1e9
7
6
5
4
3
Energy
0
2
4
6
8
Iteration

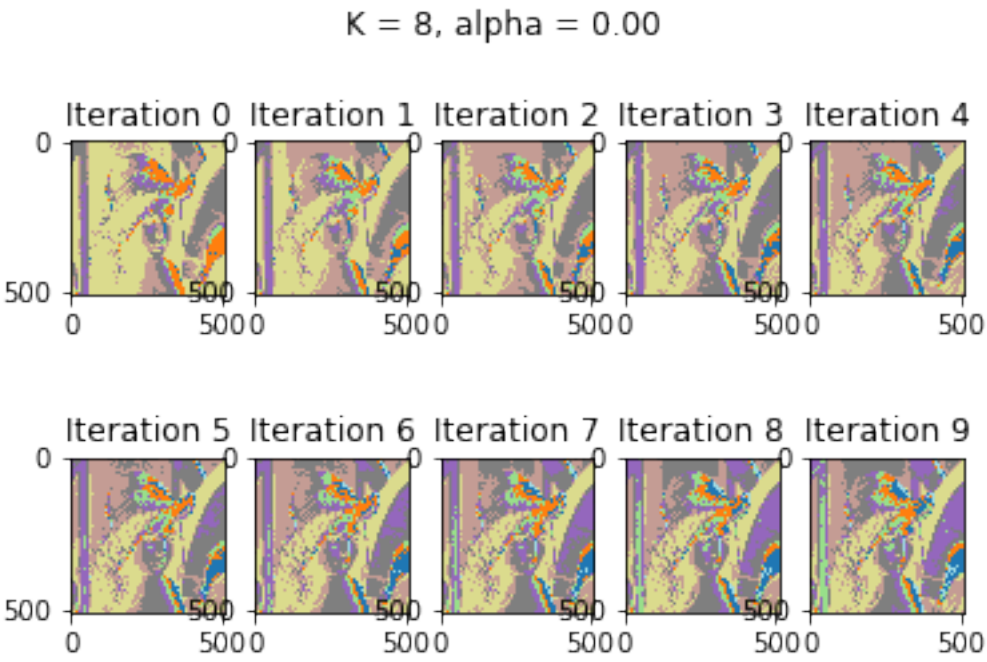

Spatial Energy graph
1e9
5.5
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

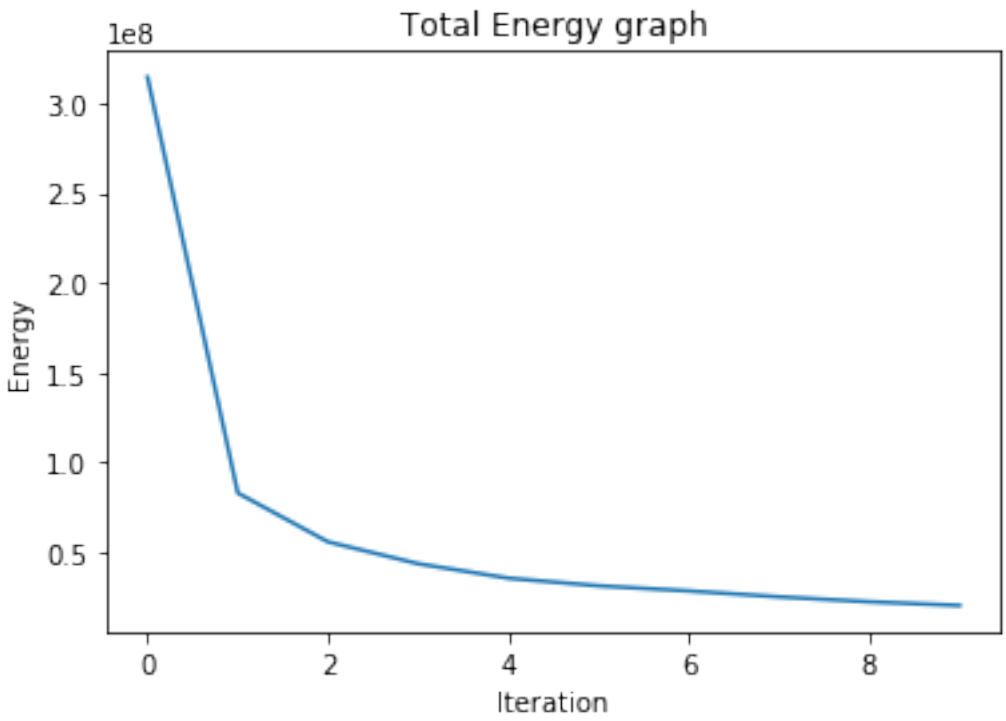

Intensity Energy graph
1e9
2.2
2.0
1.8
1.6
1.4
1.2
1.0
Energy
0
2
4
6
8
Iteration

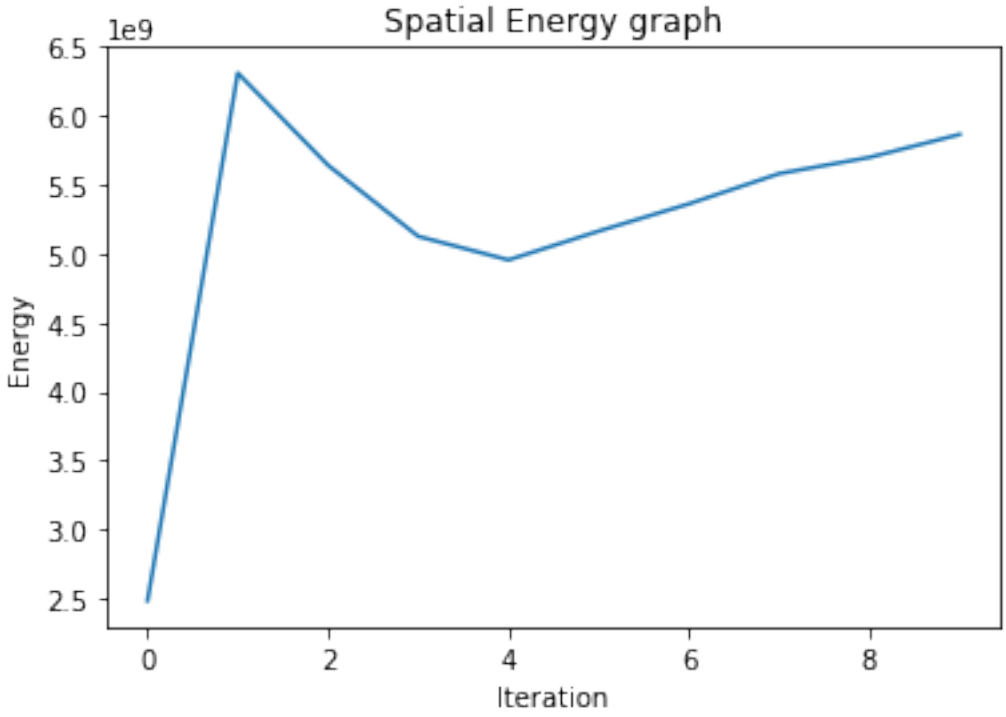
3 K = 8

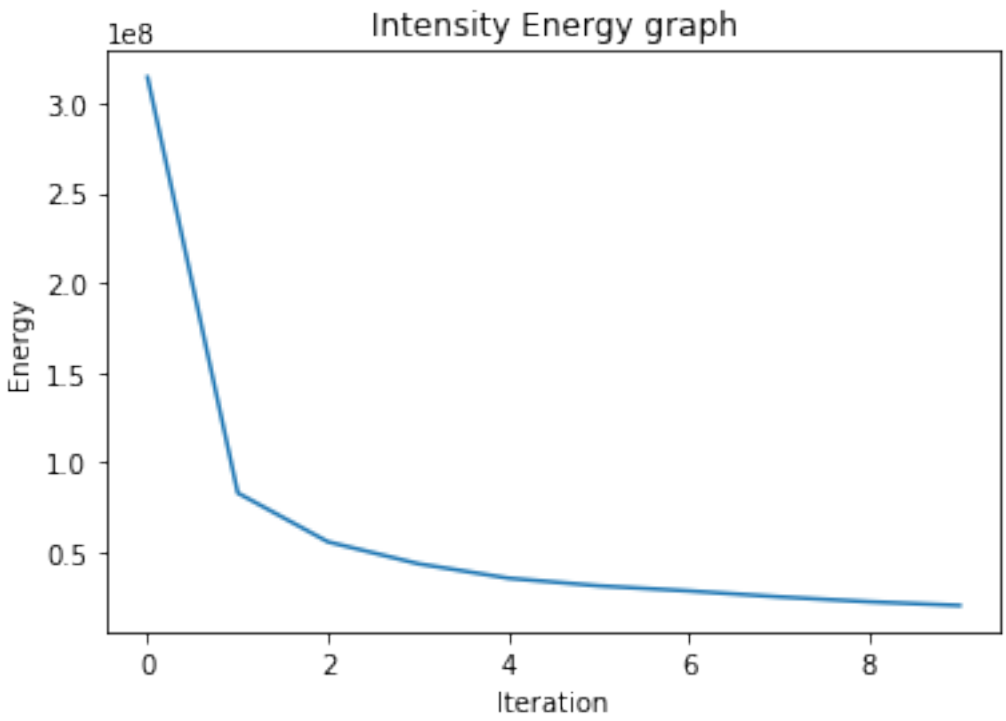
```
In [244]: K = 8
          KMeans(img_matrix, img_rows, img_cols, K, 0)
          KMeans(img_matrix, img_rows, img_cols, K, 0.01)
          KMeans(img_matrix, img_rows, img_cols, K, 0.03)
          KMeans(img_matrix, img_rows, img_cols, K, 0.05)
          KMeans(img_matrix, img_rows, img_cols, K, 0.07)
          KMeans(img_matrix, img_rows, img_cols, K, 0.08)
          KMeans(img_matrix, img_rows, img_cols, K, 0.10)
          KMeans(img_matrix, img_rows, img_cols, K, 0.25)
          KMeans(img_matrix, img_rows, img_cols, K, 0.50)
          KMeans(img_matrix, img_rows, img_cols, K, 0.75)
          KMeans(img_matrix, img_rows, img_cols, K, 1)
```

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

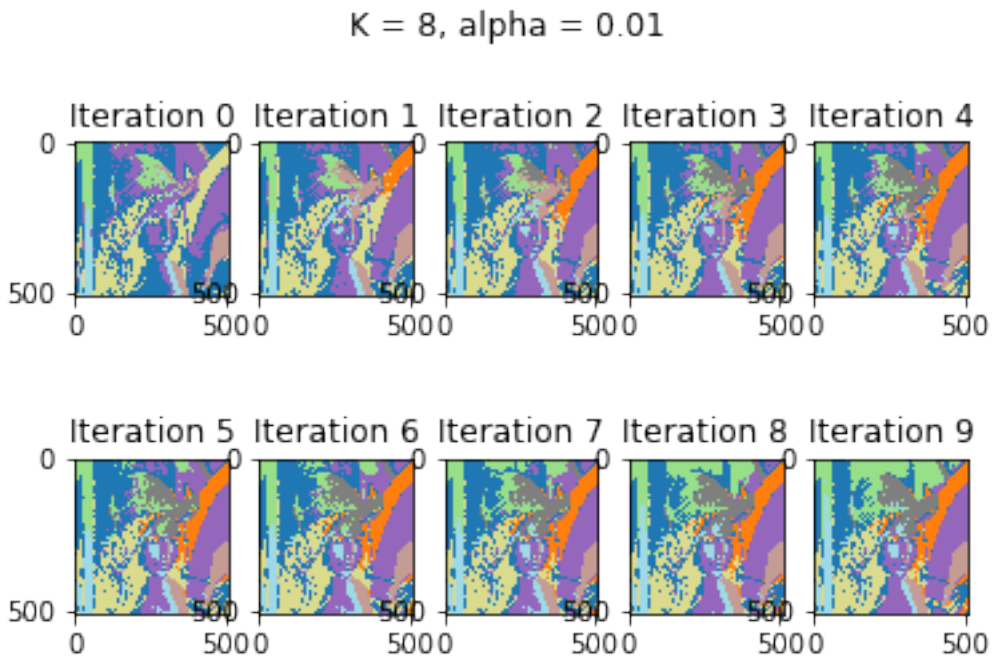


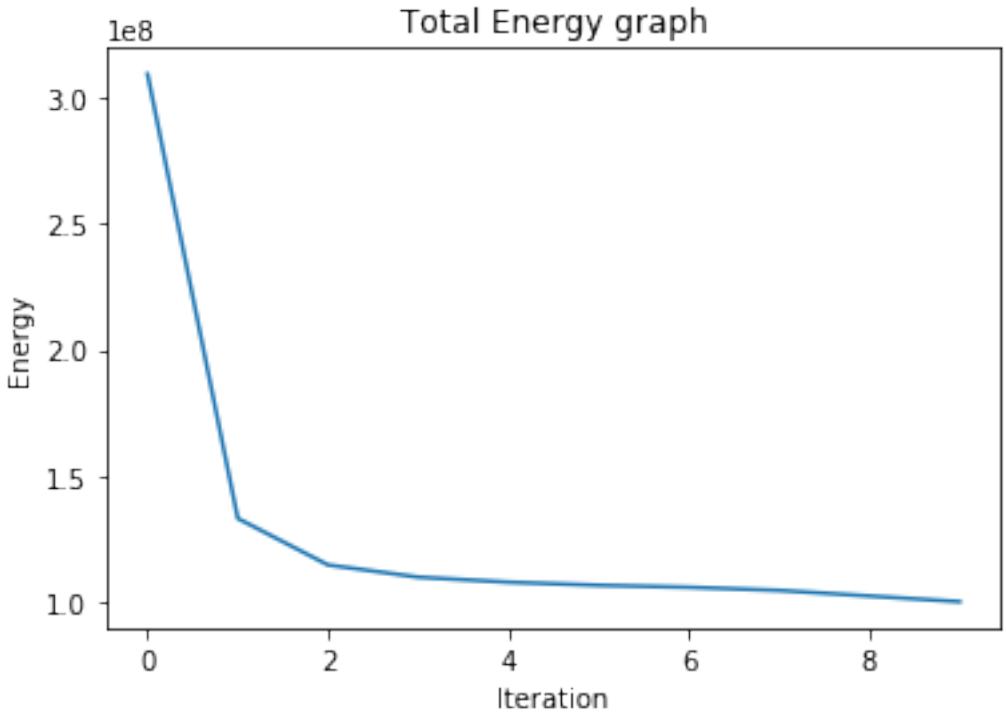

Total Energy graph
1e8
3.0
2.5
2.0
1.5
1.0
0.5
Energy
0
2
4
6
8
Iteration

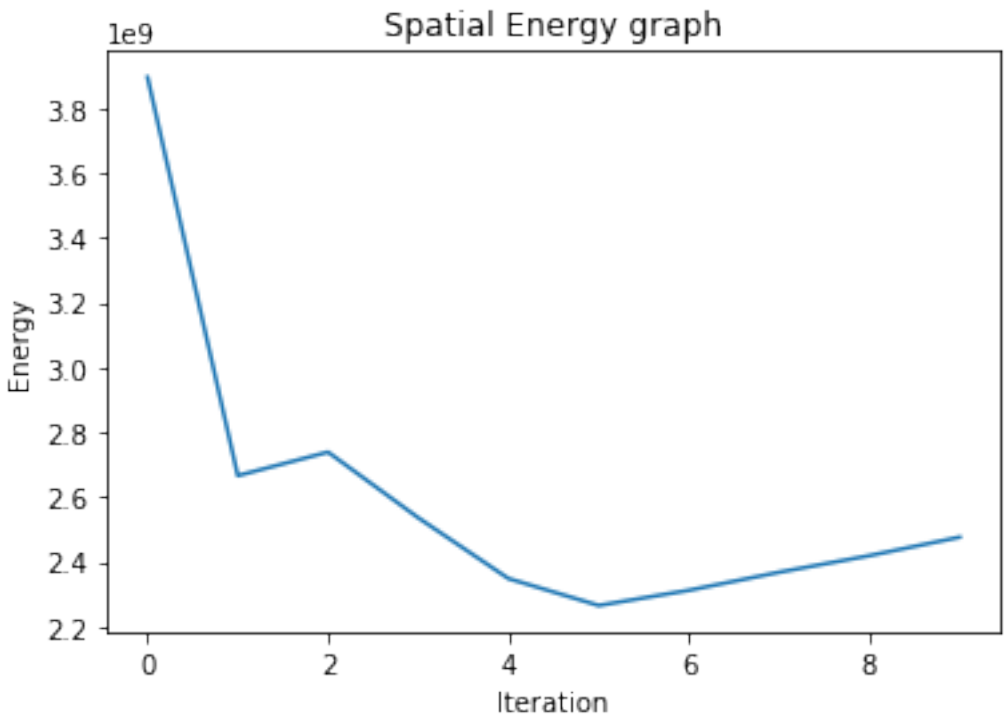

Spatial Energy graph
1e9
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

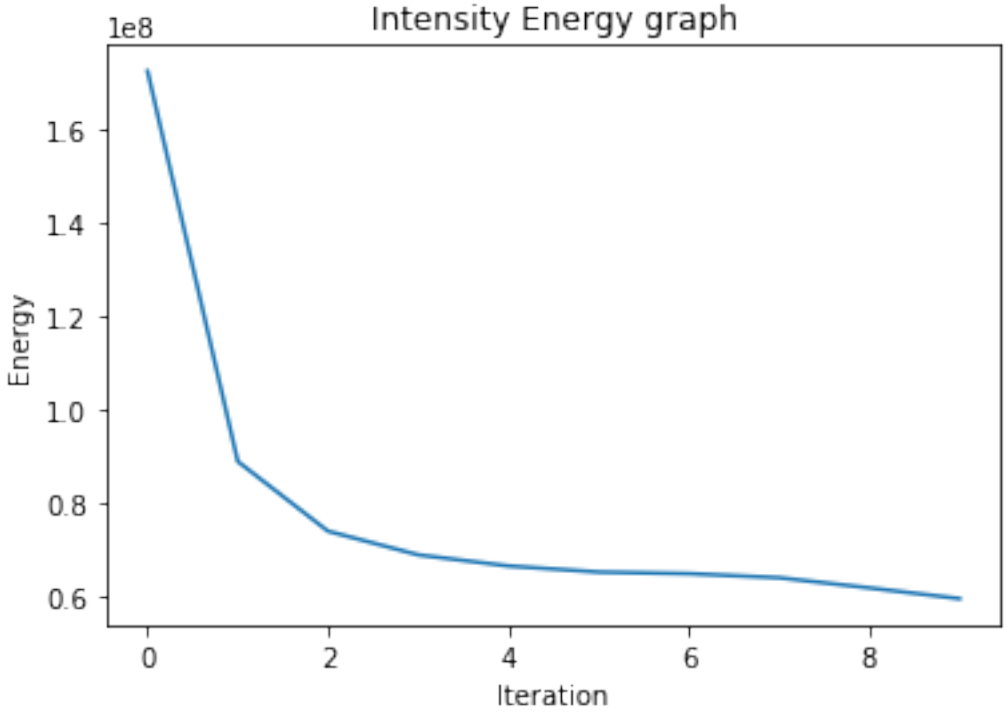



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```


K = 8, alpha = 0.01
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

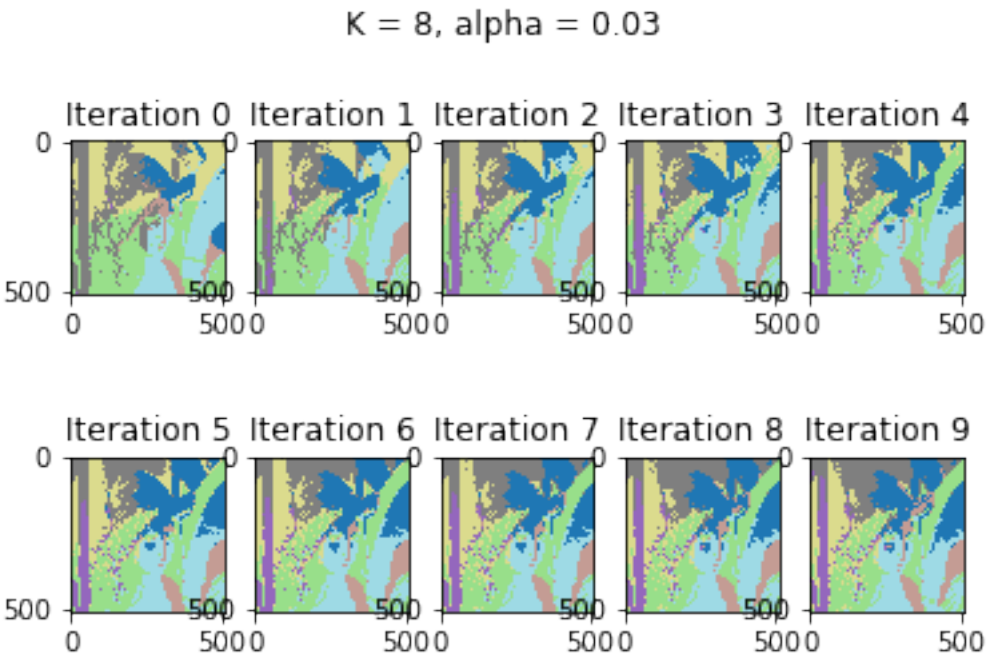

Total Energy graph
1e8
3.0
2.5
2.0
1.5
1.0
Energy
0
2
4
6
8
Iteration

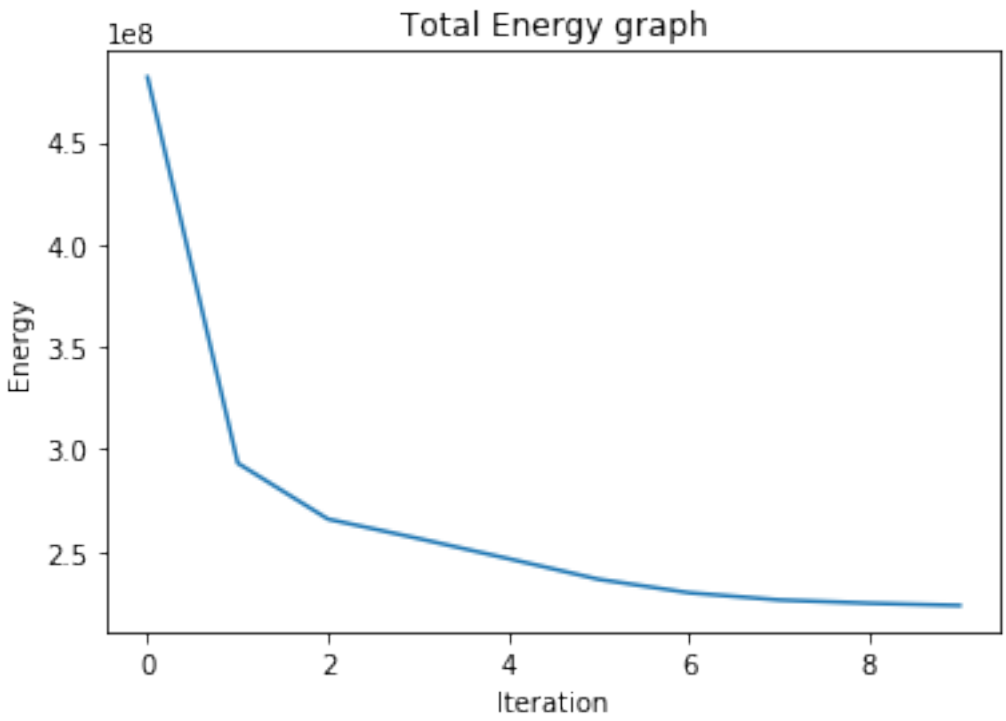

Spatial Energy graph
1e9
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
Energy
0
2
4
6
8
Iteration

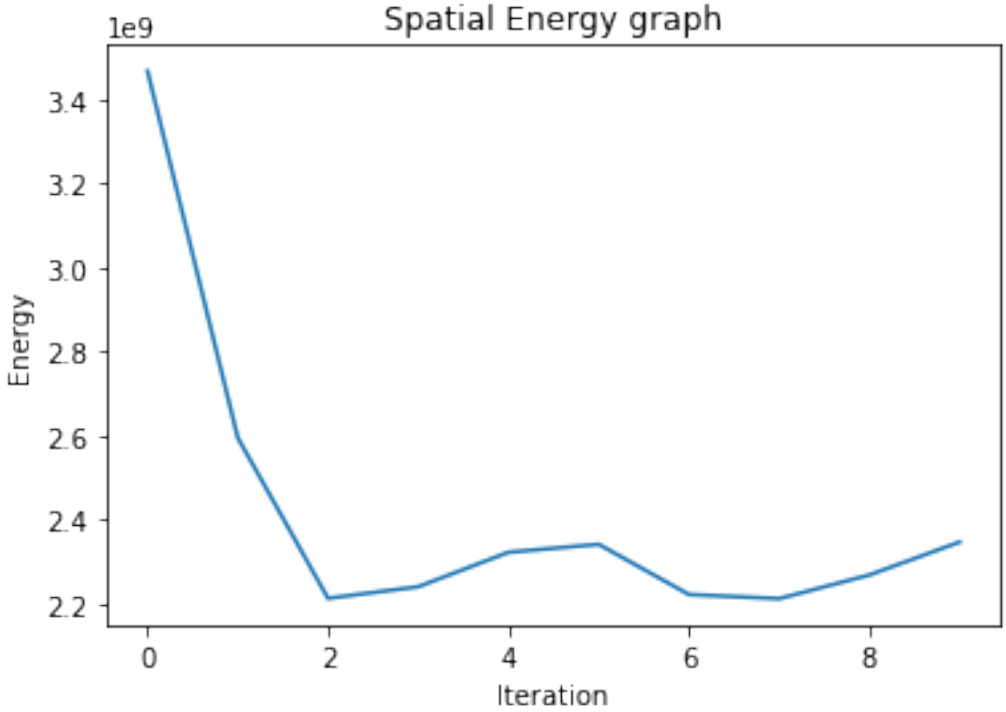

Intensity Energy graph
1e8
1.6
1.4
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

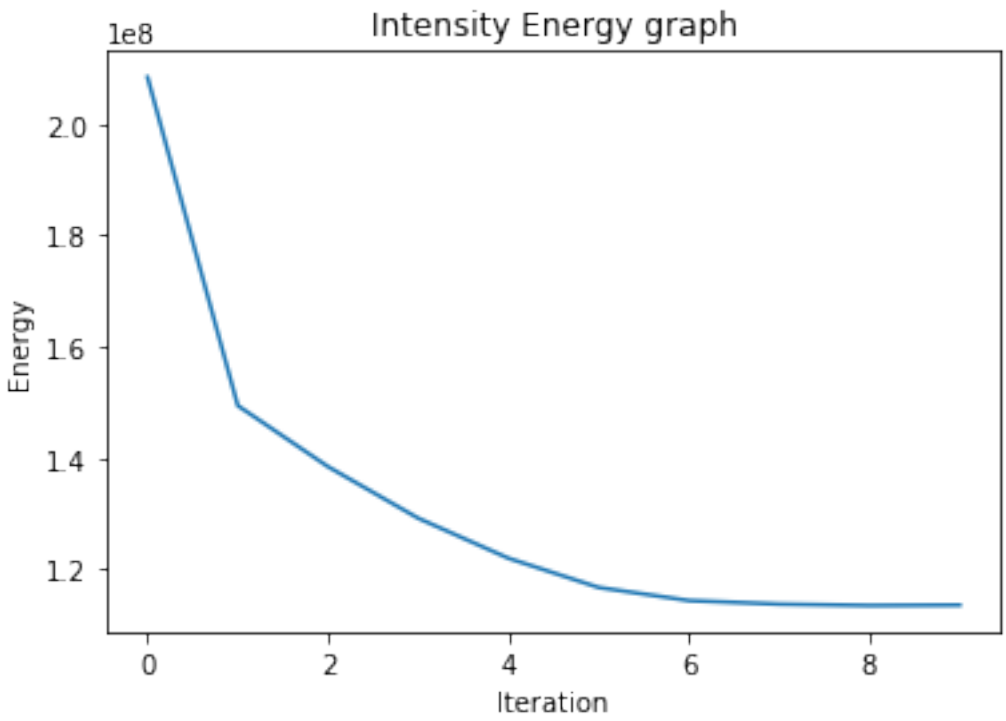
```
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:66: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:67: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:68: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:32: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:33: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:35: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:36: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:39: RuntimeWarning: overi
```

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

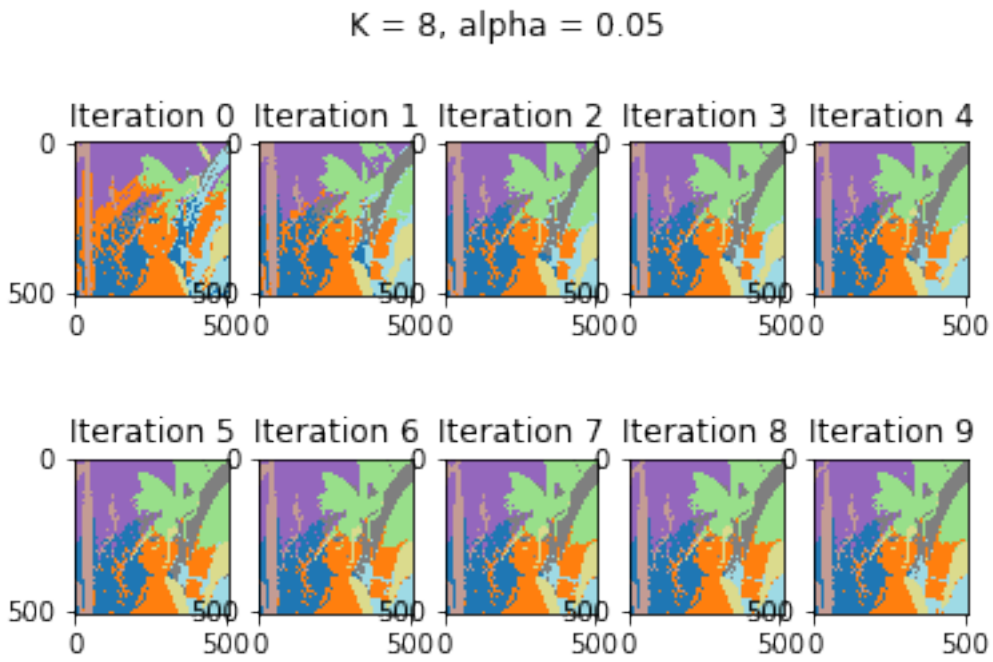


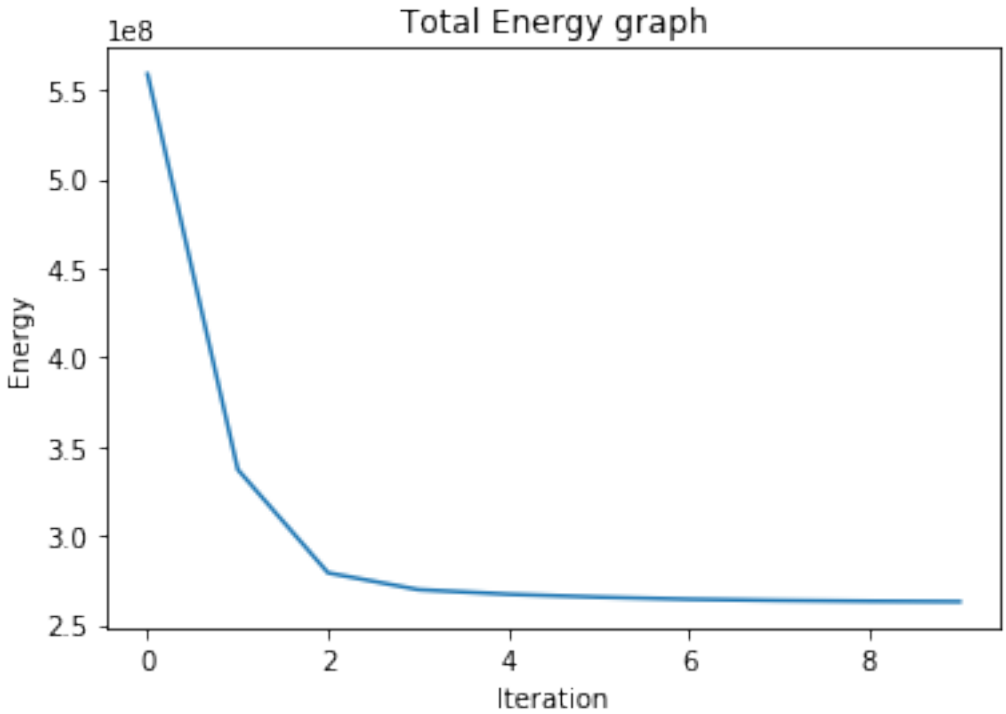

Total Energy graph
1e8
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

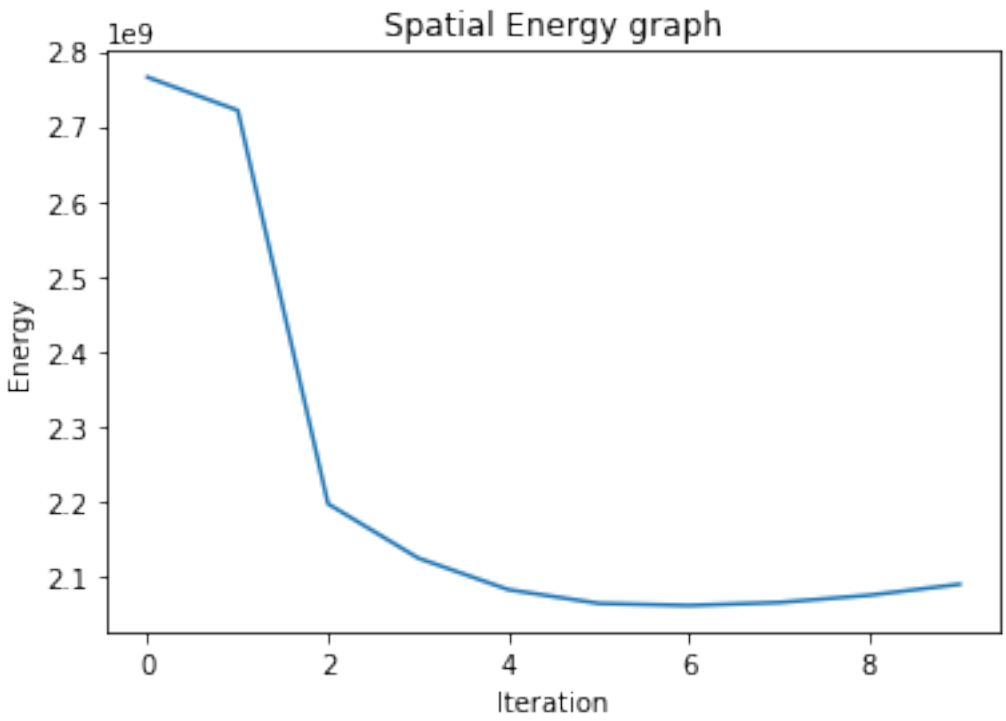

Spatial Energy graph
1e9
3.4
3.2
3.0
2.8
2.6
2.4
2.2
Energy
0
2
4
6
8
Iteration

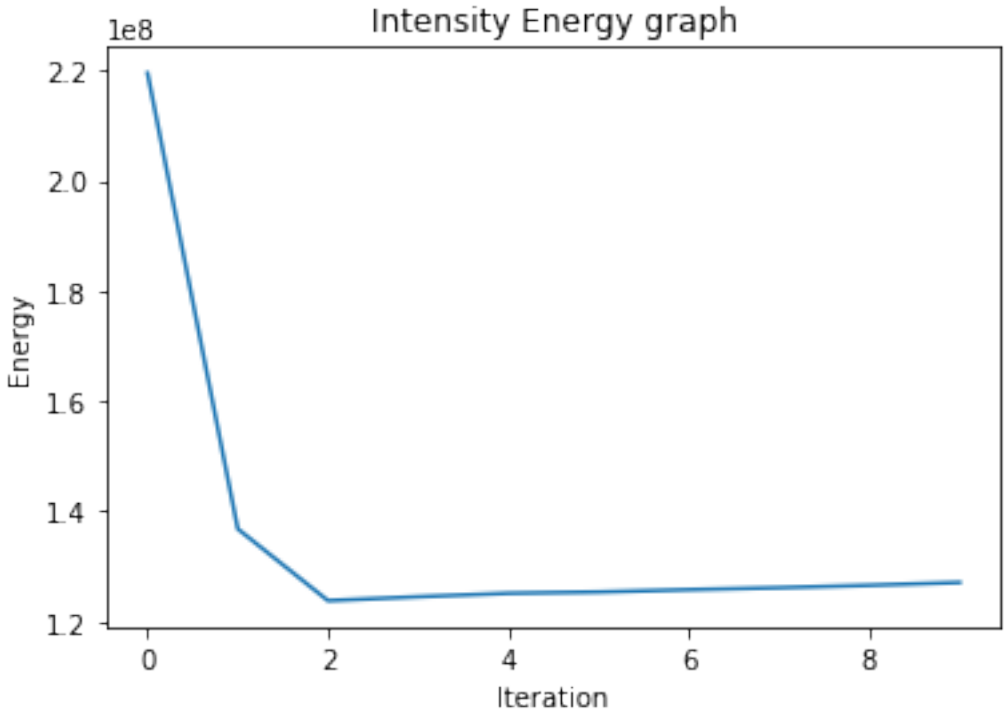



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

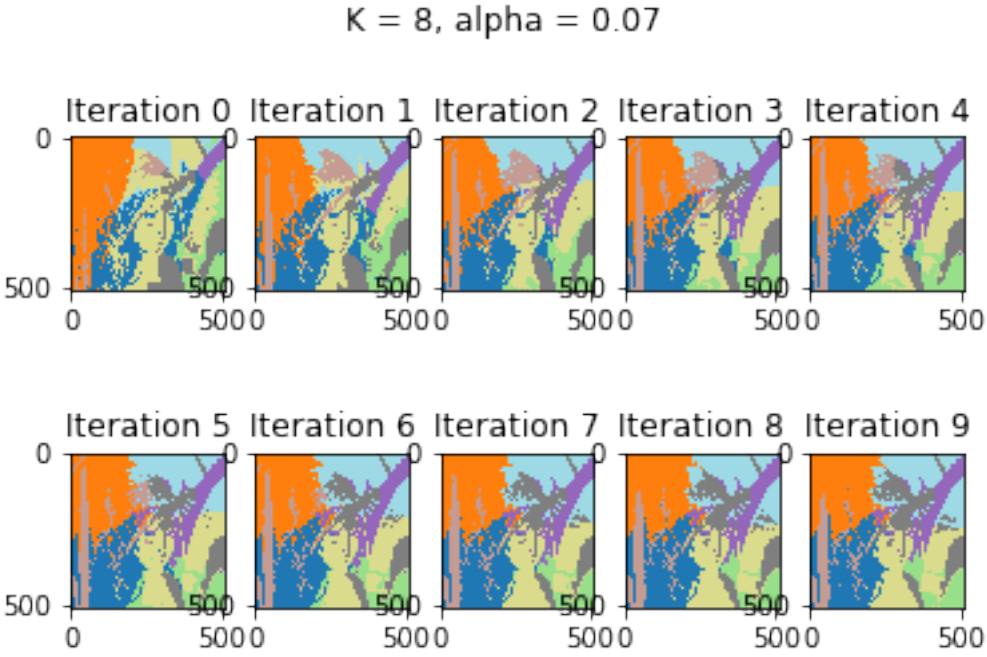

K = 8, alpha = 0.05
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

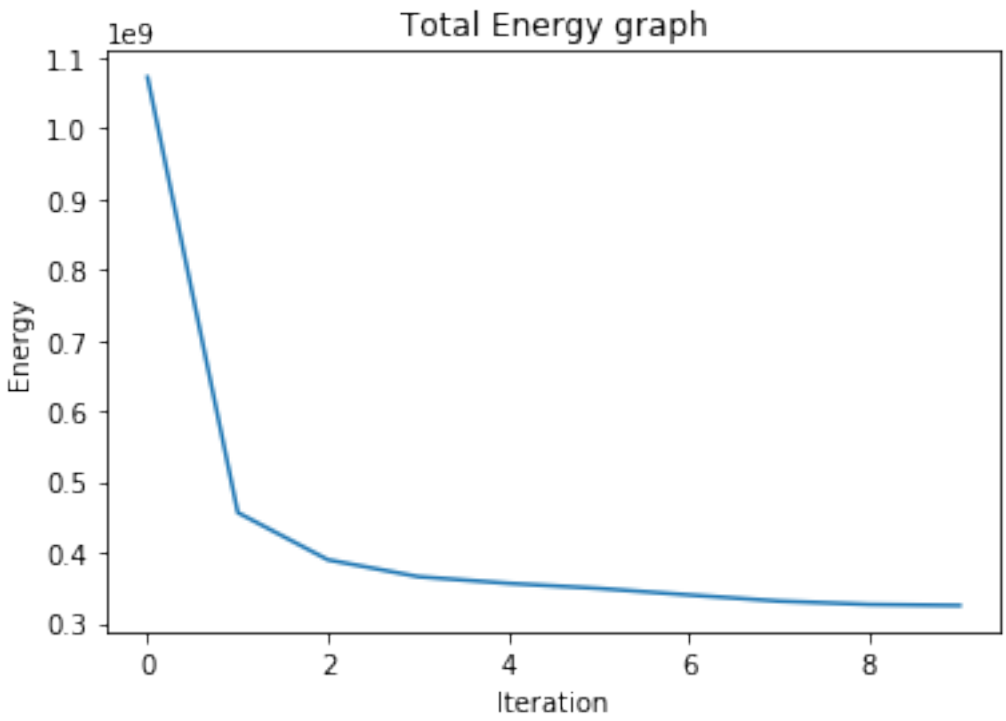

Total Energy graph
1e8
5.5
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

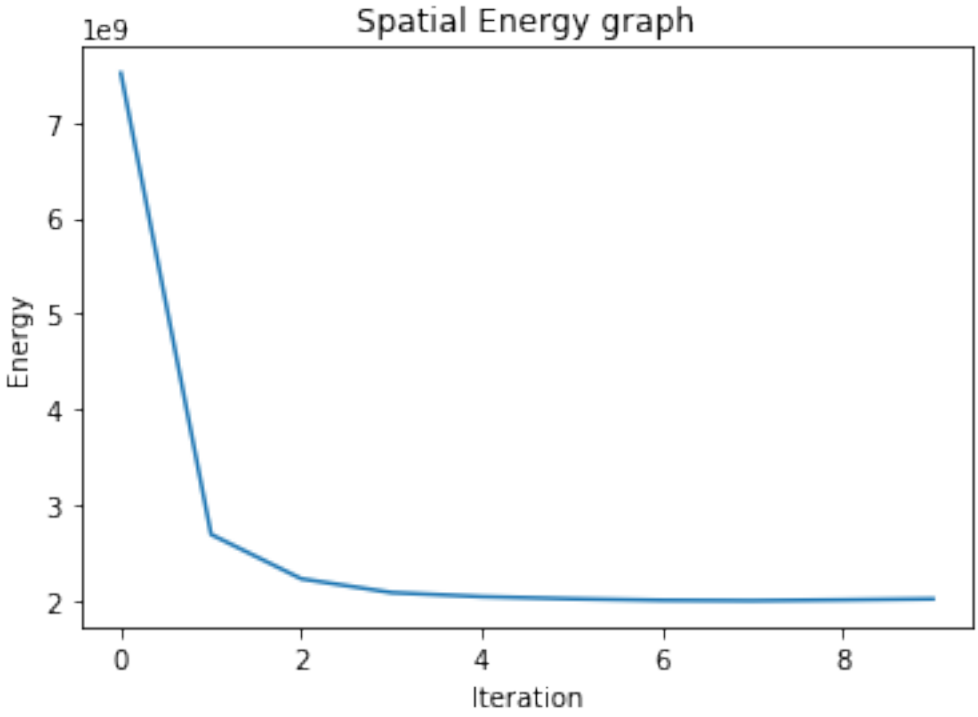

Spatial Energy graph
1e9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
Energy
0
2
4
6
8
Iteration

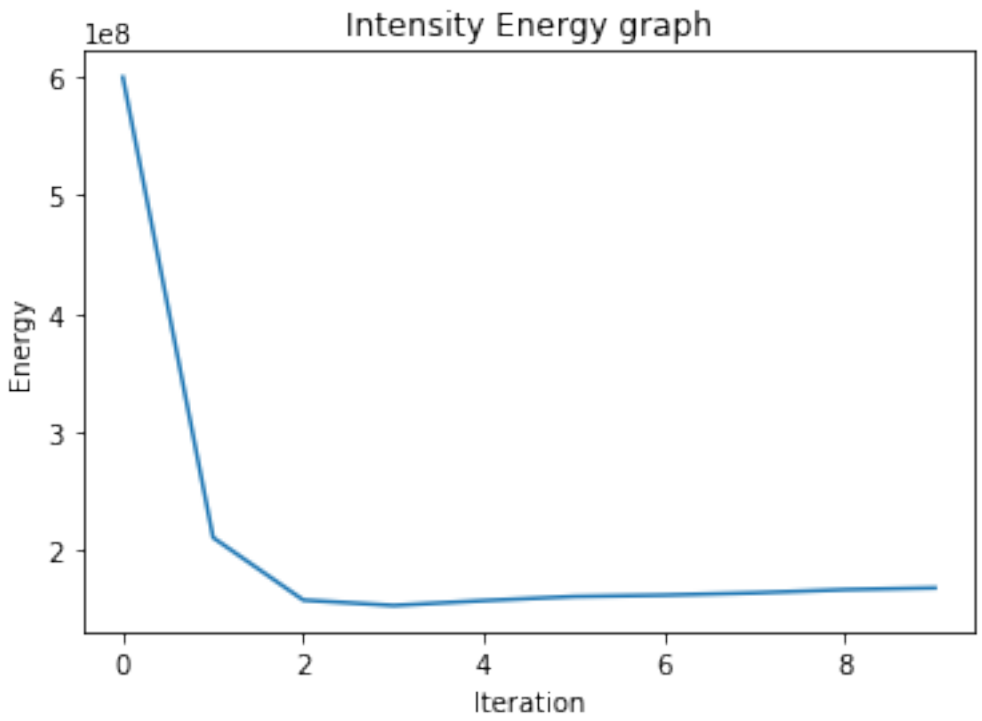

Intensity Energy graph
1e8
2.2
2.0
1.8
1.6
1.4
1.2
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

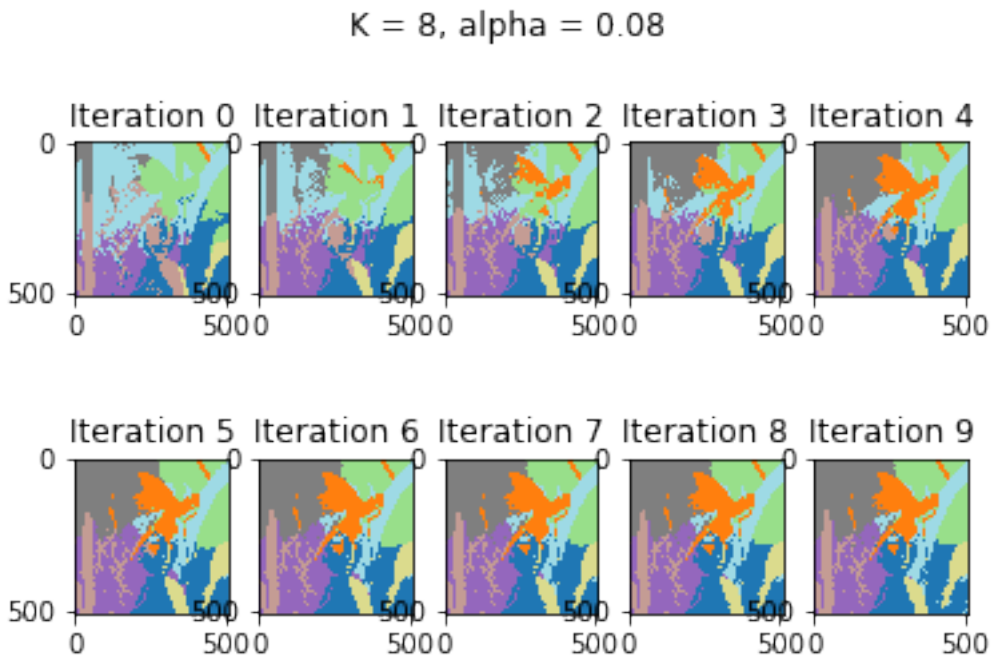


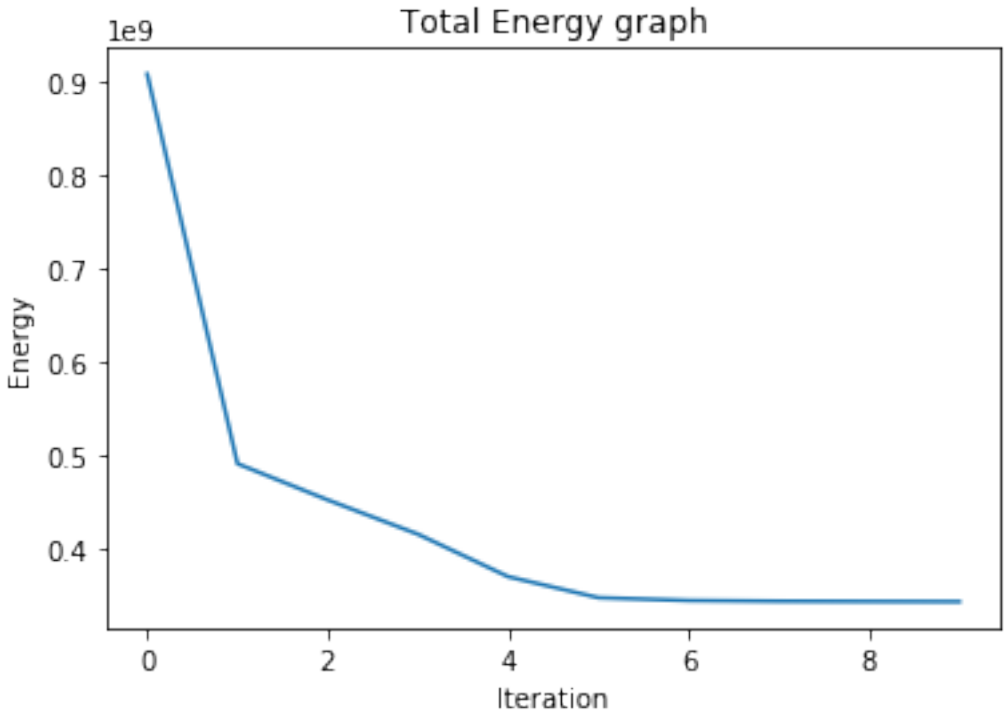

Total Energy graph
1e9
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
Energy
0
2
4
6
8
Iteration

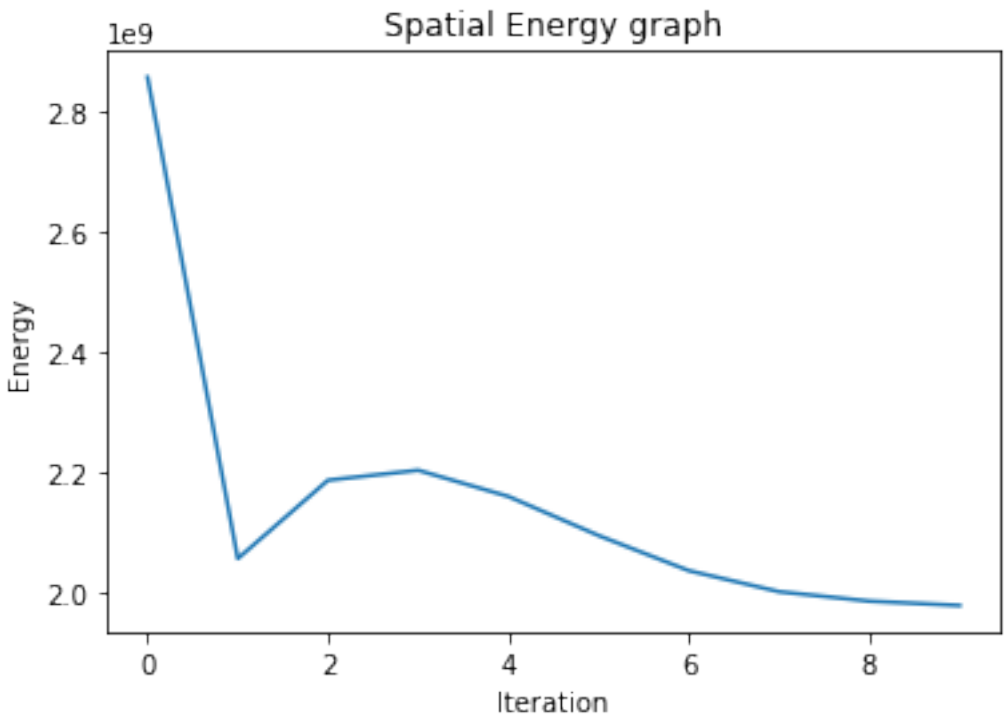

Spatial Energy graph
1e9
7
6
5
4
3
2
Energy
0
2
4
6
8
Iteration

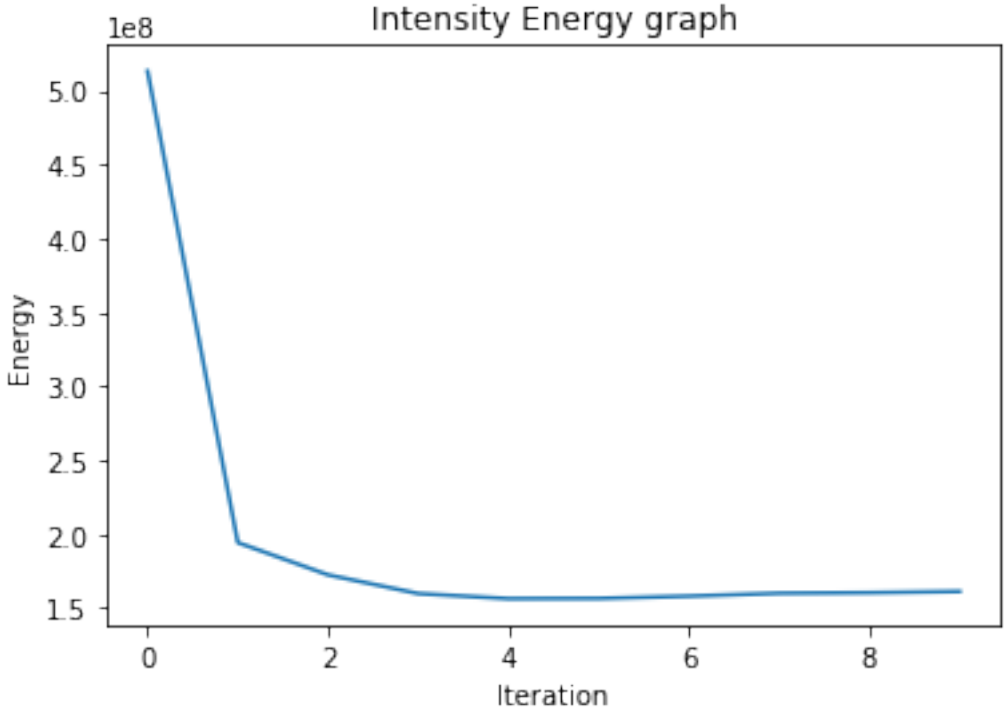



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

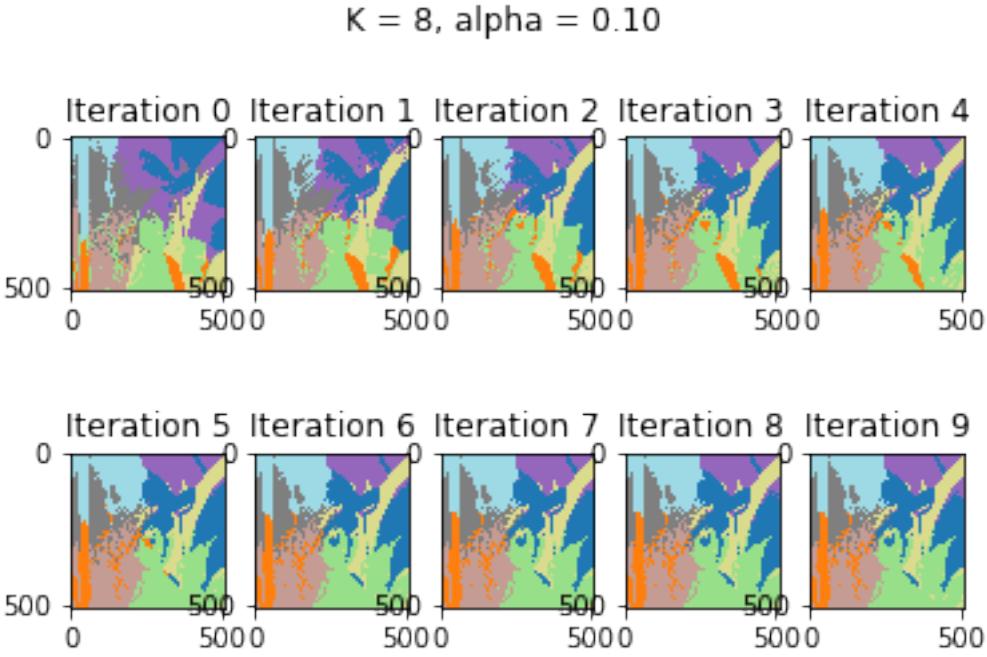

K = 8, alpha = 0.08
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

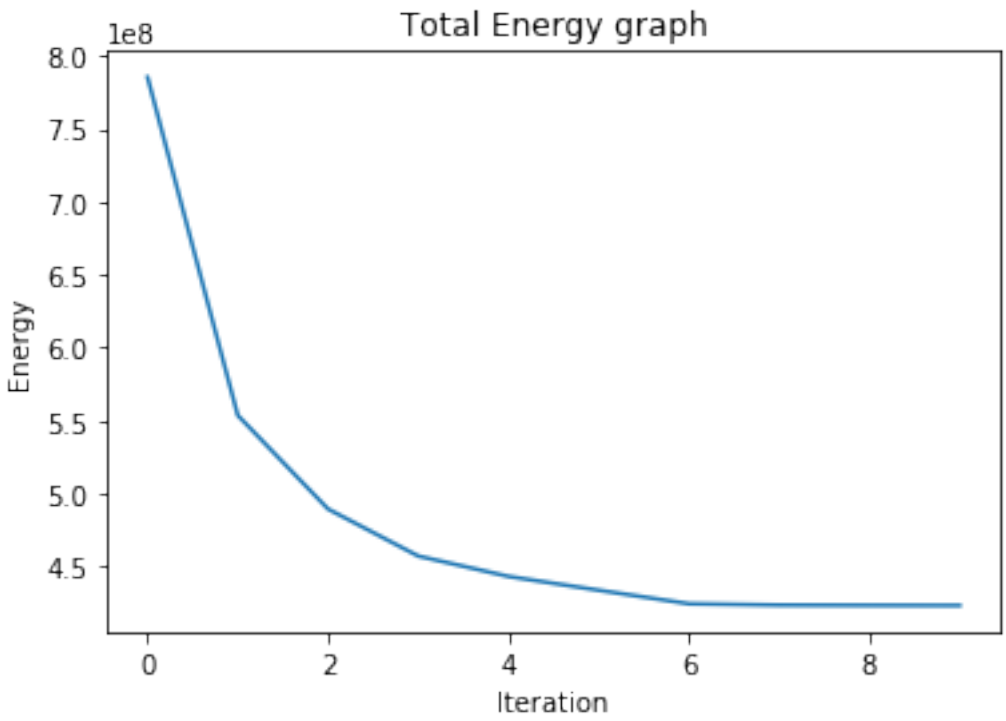

Total Energy graph
1e9
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration

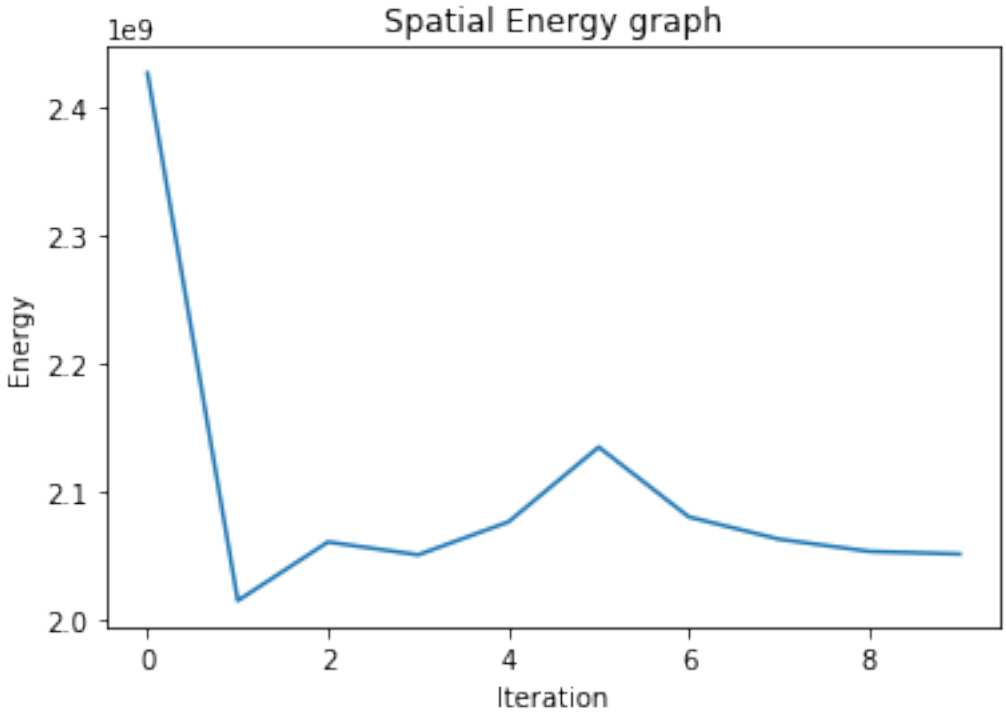

Spatial Energy graph
1e9
2.8
2.6
2.4
2.2
2.0
Energy
0
2
4
6
8
Iteration

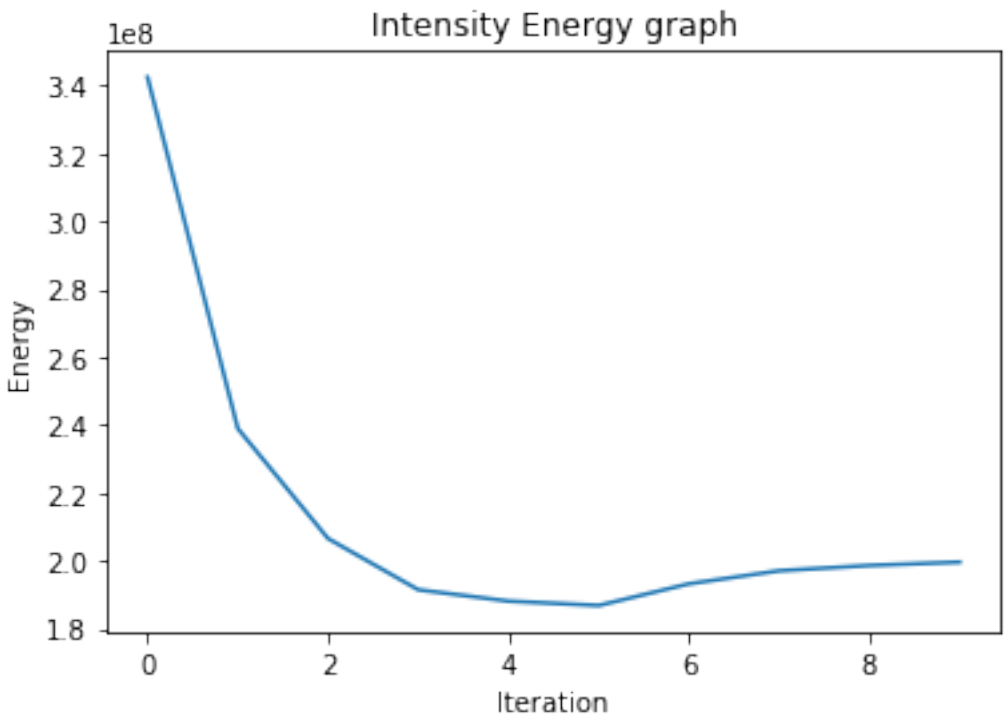

Intensity Energy graph
1e8
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

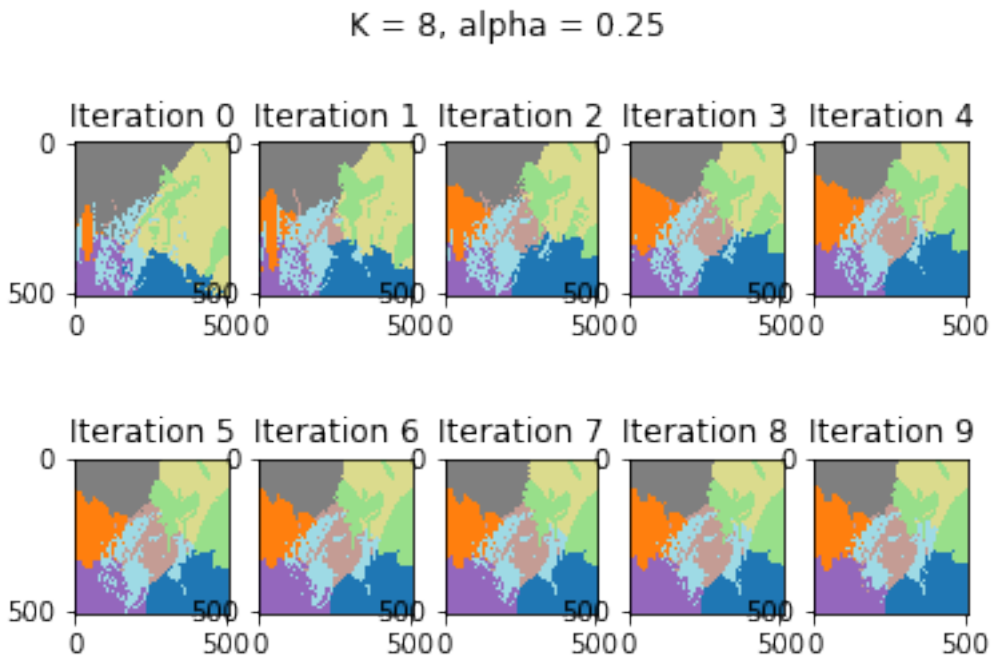


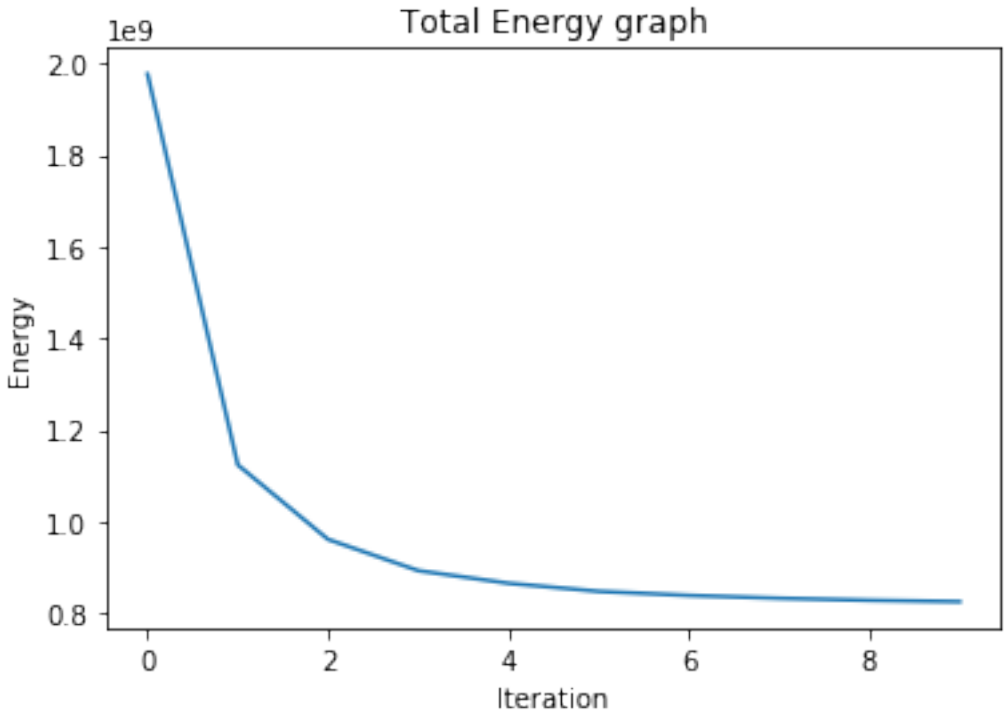

Total Energy graph
1e8
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
Energy
0
2
4
6
8
Iteration

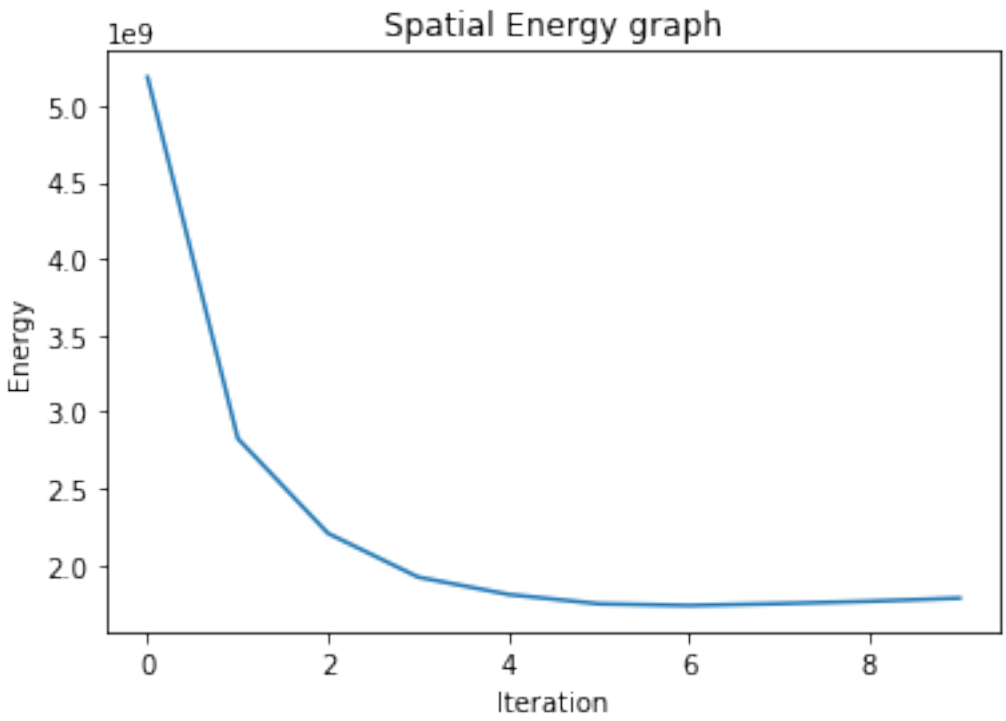

Spatial Energy graph
1e9
2.4
2.3
2.2
2.1
2.0
Energy
0
2
4
6
8
Iteration

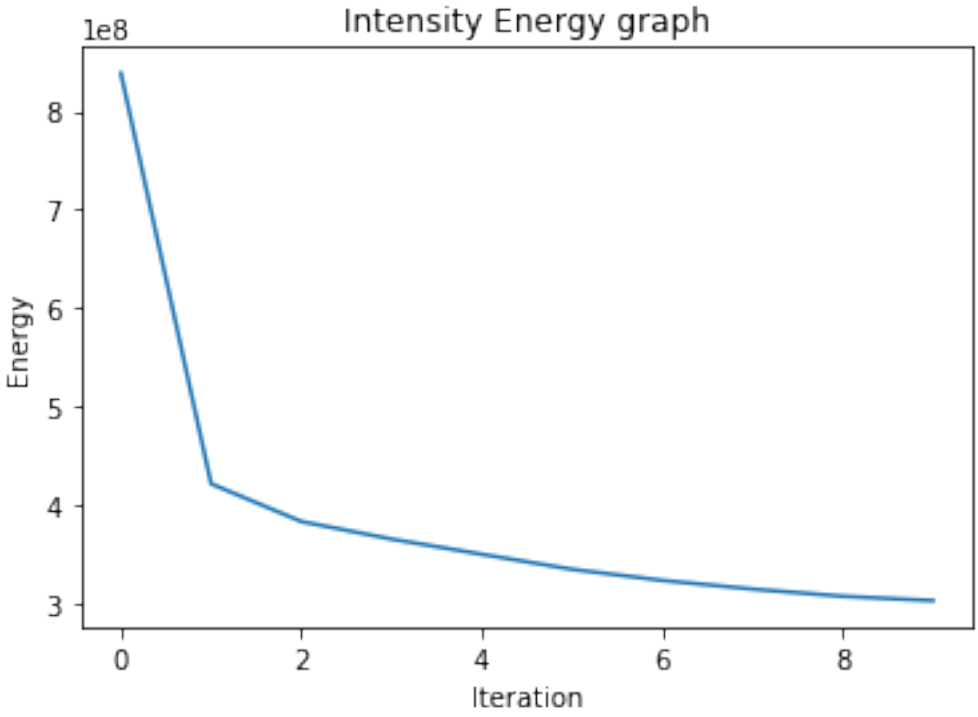



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

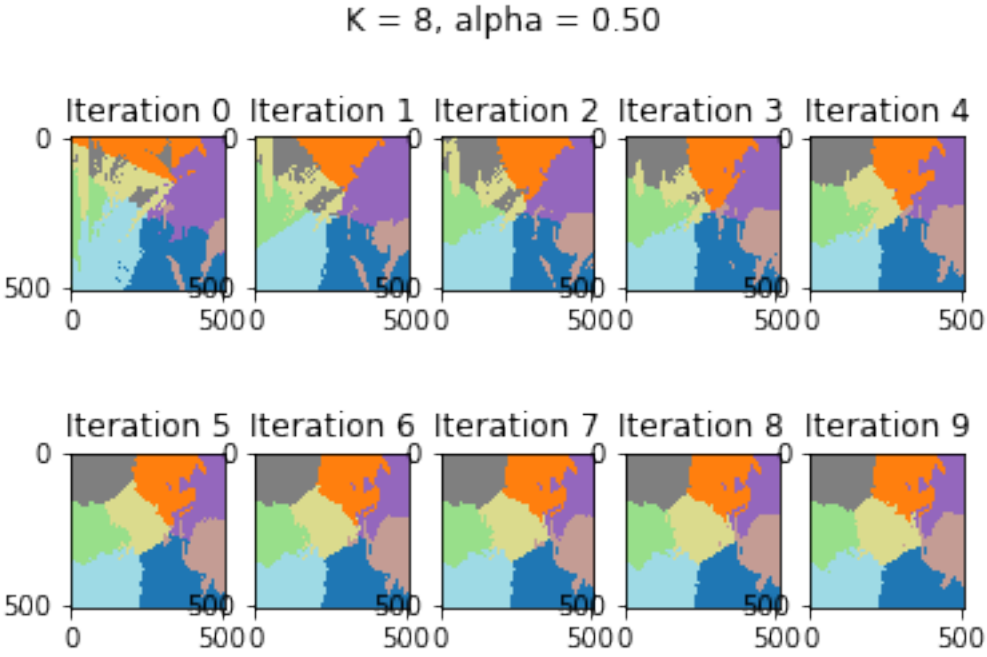

K = 8, alpha = 0.25
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

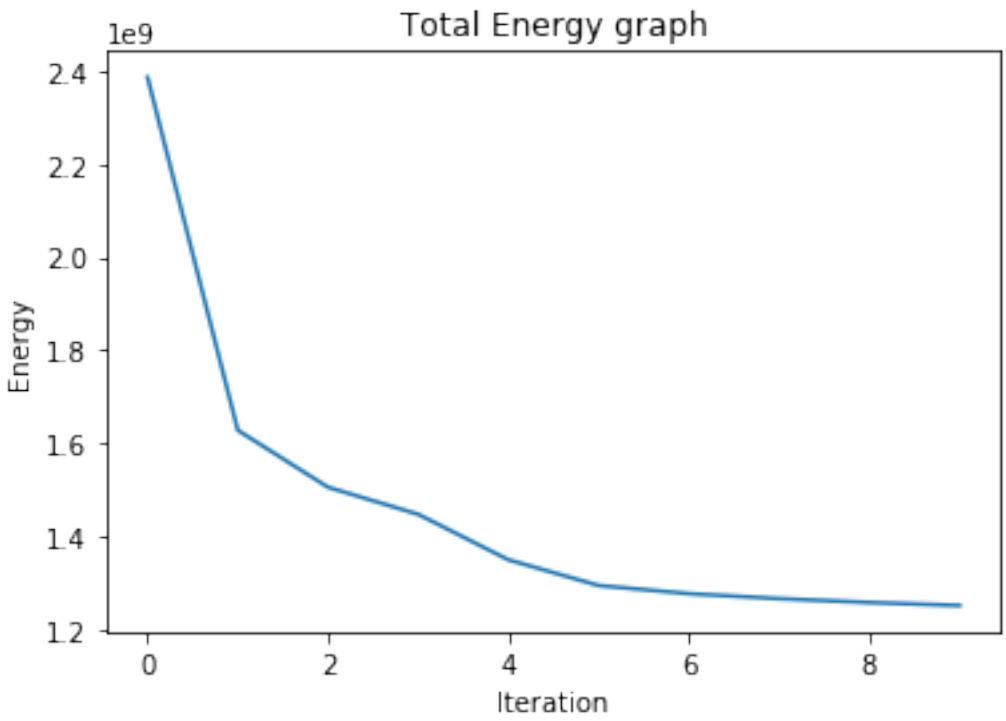

Total Energy graph
1e9
2.0
1.8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

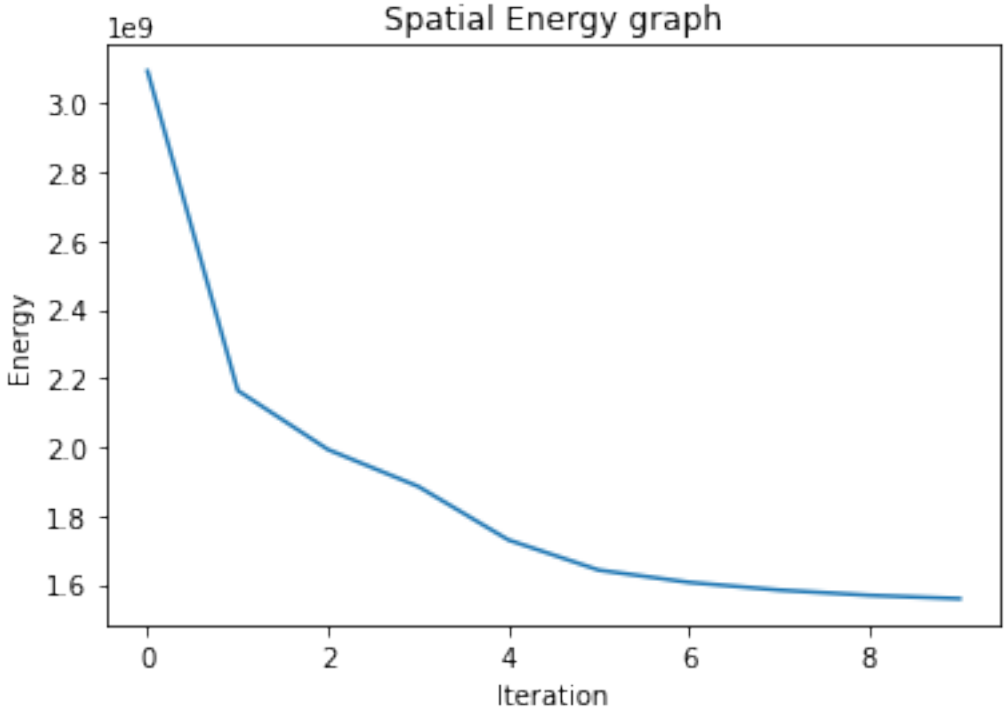

Spatial Energy graph
1e9
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

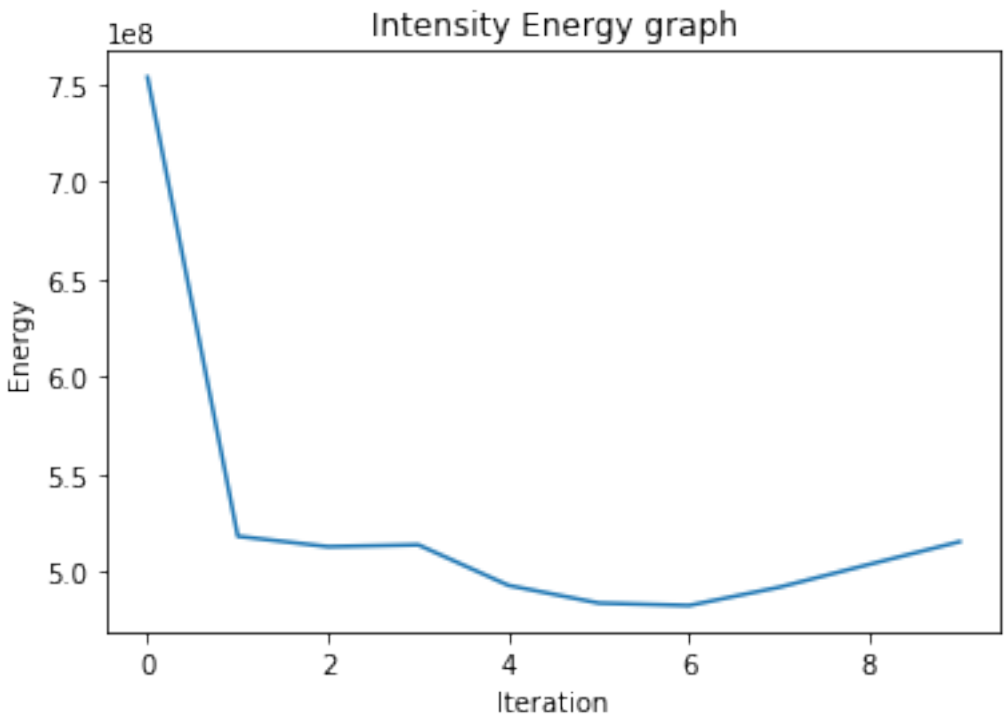

Intensity Energy graph
1e8
8
7
6
5
4
3
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

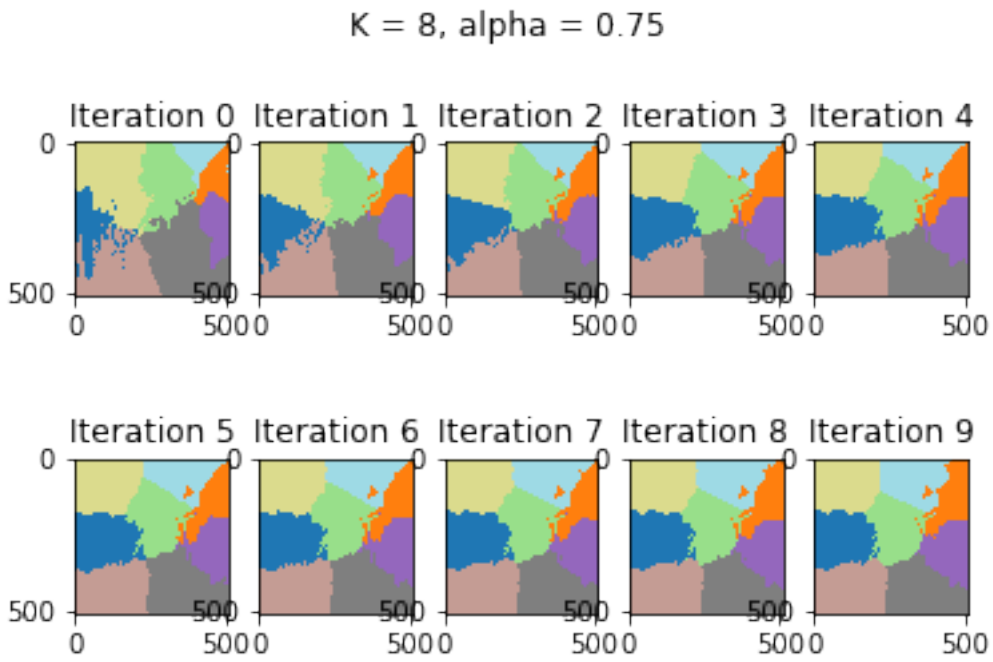


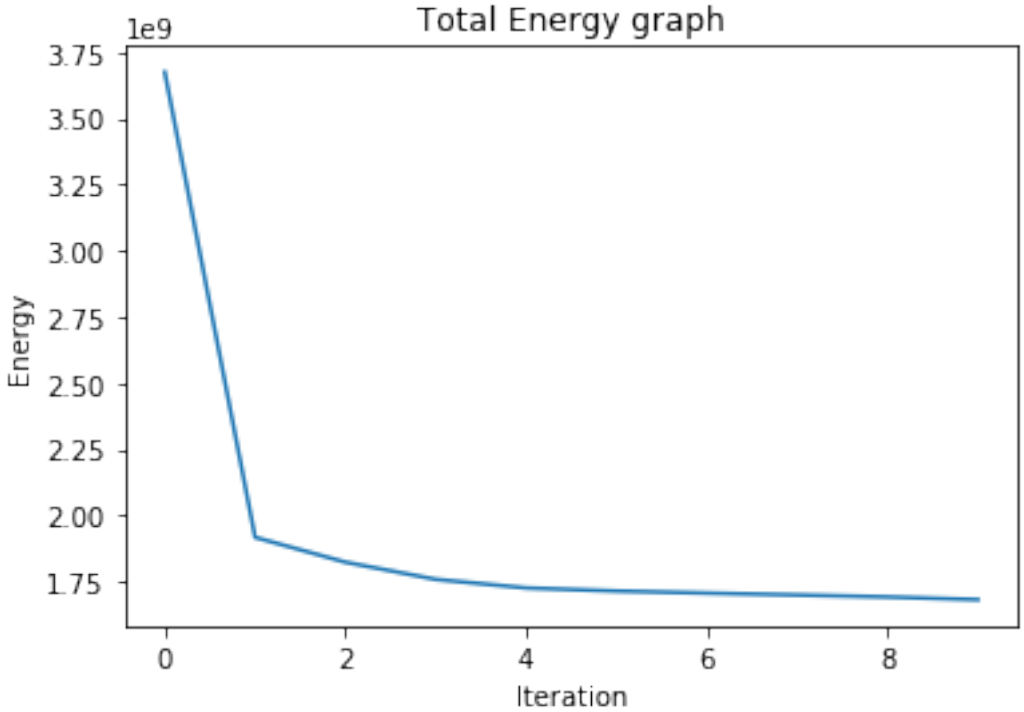

Total Energy graph
1e9
2.4
2.2
2.0
1.8
1.6
1.4
1.2
Energy
0
2
4
6
8
Iteration

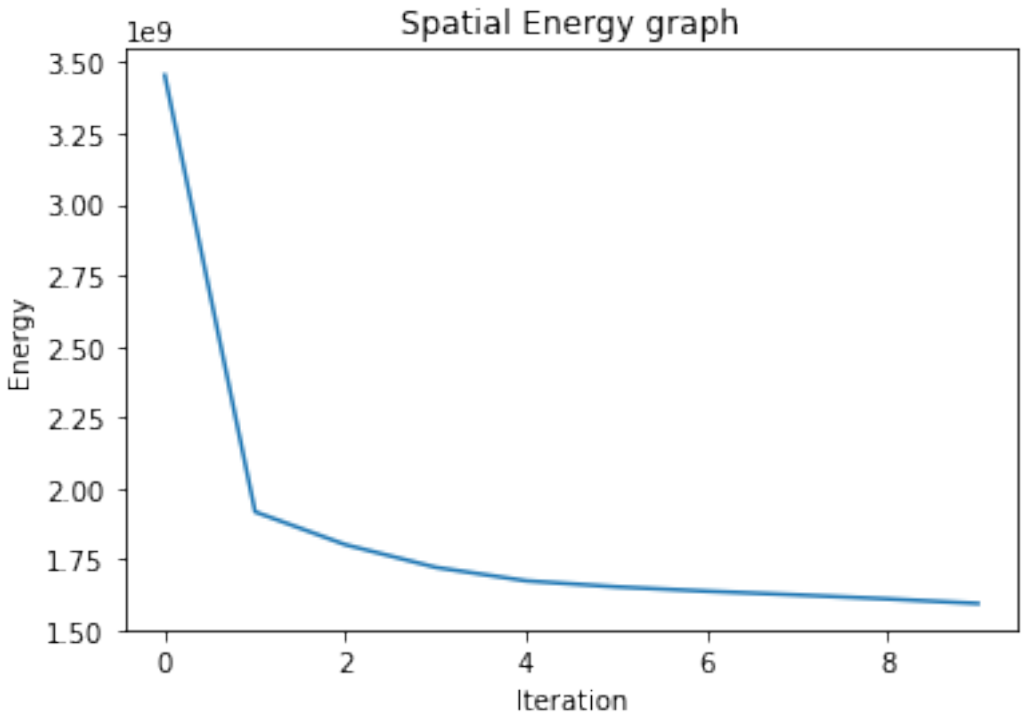

Spatial Energy graph
1e9
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
Energy
0
2
4
6
8
Iteration

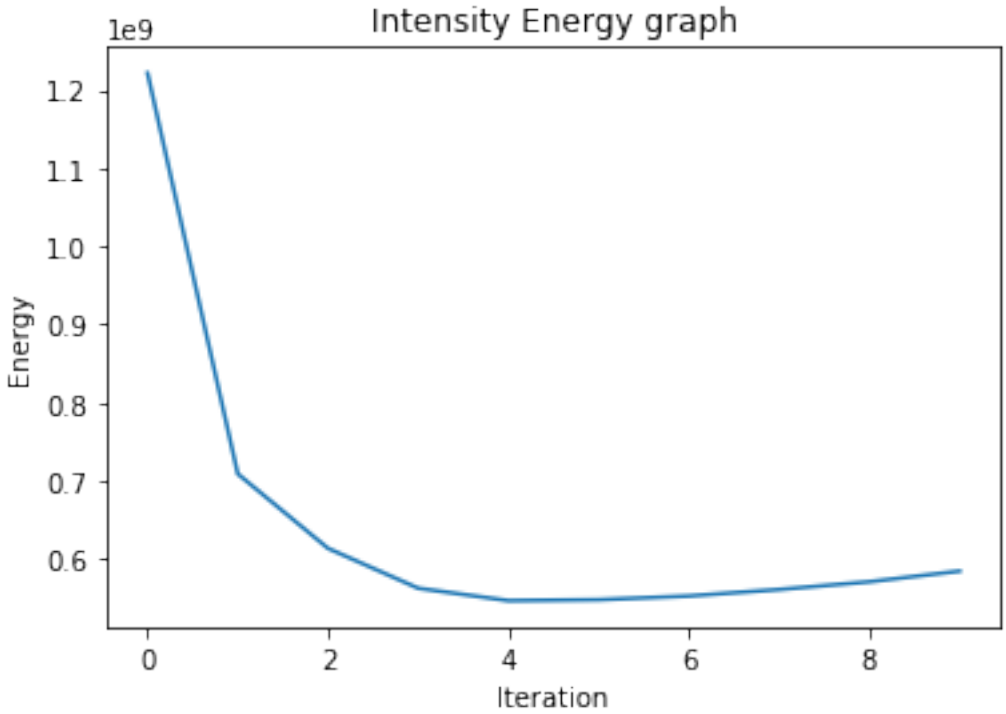



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

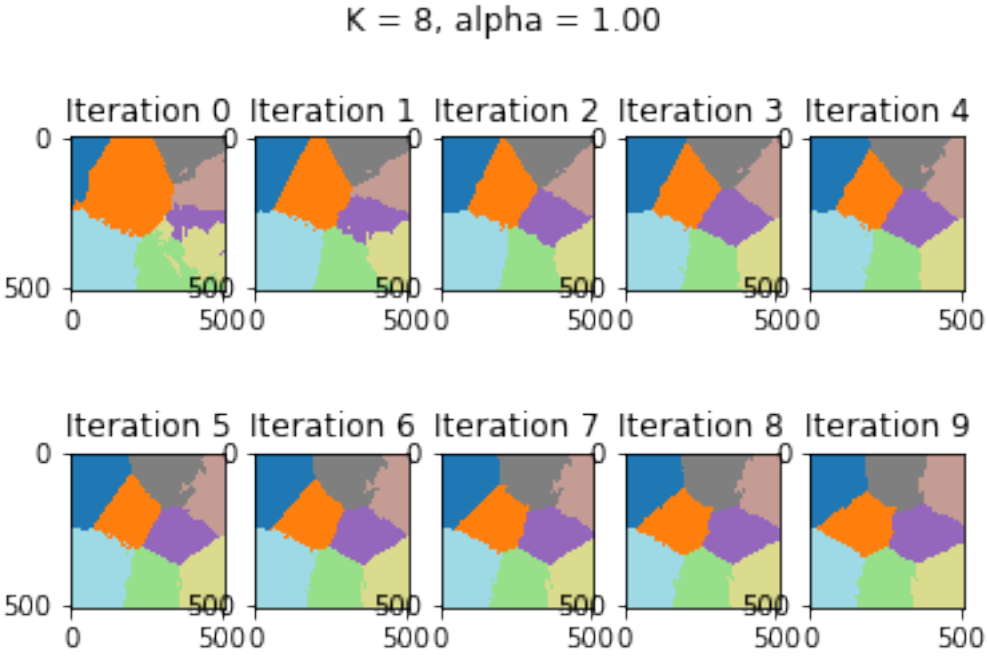

K = 8, alpha = 0.75
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

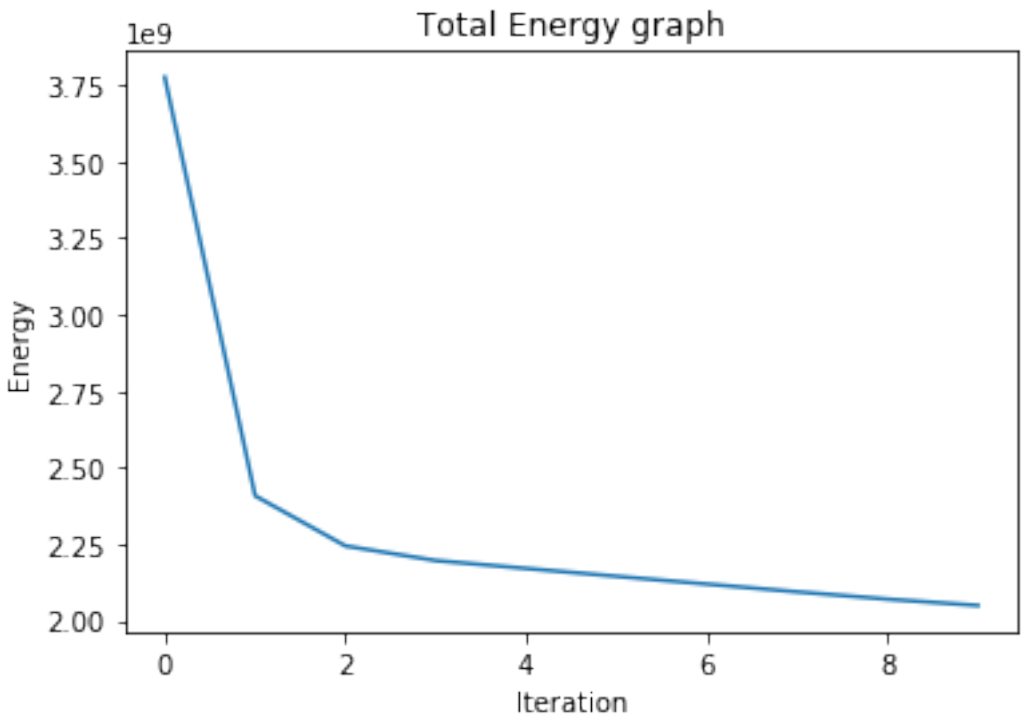

Total Energy graph
1e9
3.75
3.50
3.25
3.00
2.75
2.50
2.25
2.00
1.75
Energy
0
2
4
6
8
Iteration

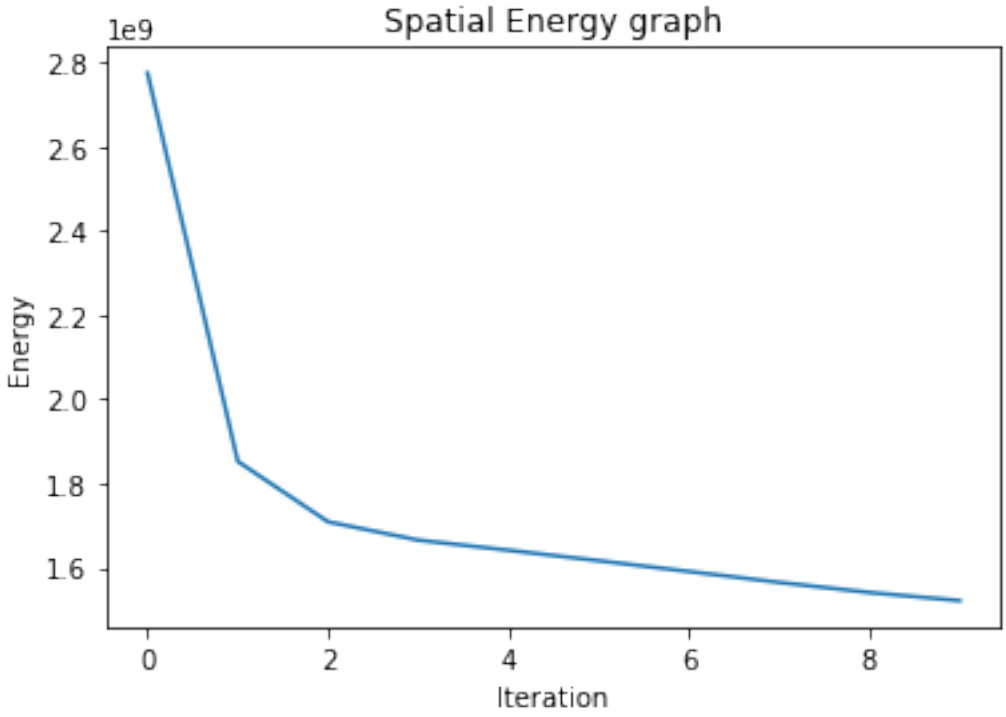

Spatial Energy graph
1e9
3.50
3.25
3.00
2.75
2.50
2.25
2.00
1.75
1.50
Energy
0
2
4
6
8
Iteration

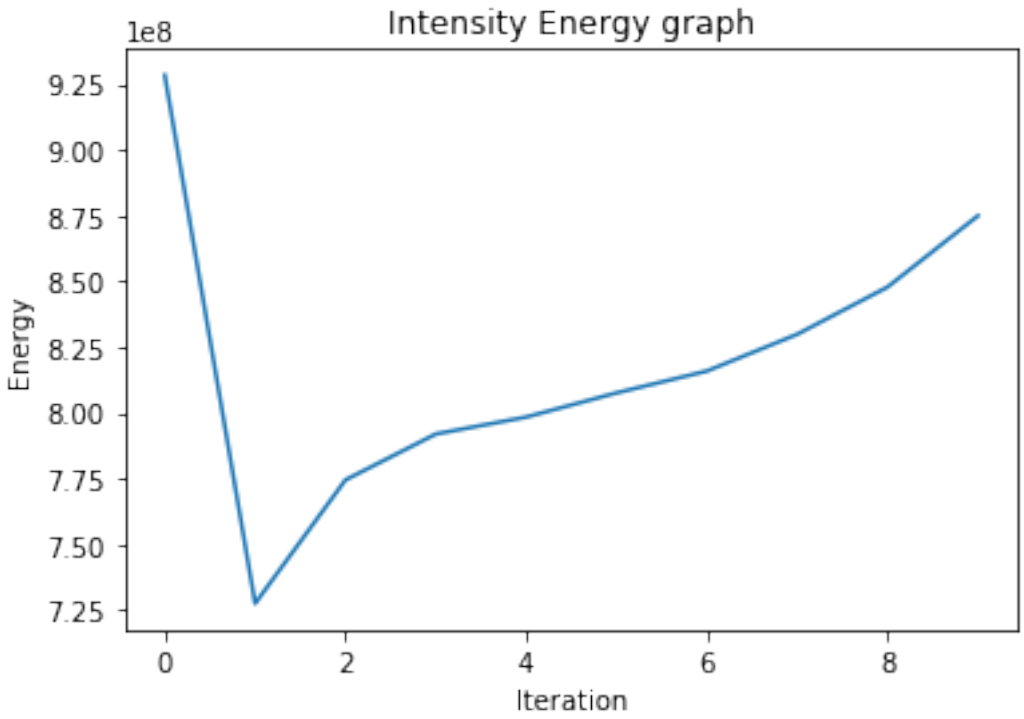

Intensity Energy graph
1e9
1.2
1.1
1.0
0.9
0.8
0.7
0.6
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```




Total Energy graph
1e9
3.75
3.50
3.25
3.00
2.75
2.50
2.25
2.00
Energy
0
2
4
6
8
Iteration


Spatial Energy graph
1e9
2.8
2.6
2.4
2.2
2.0
1.8
1.6
Energy
0
2
4
6
8
Iteration

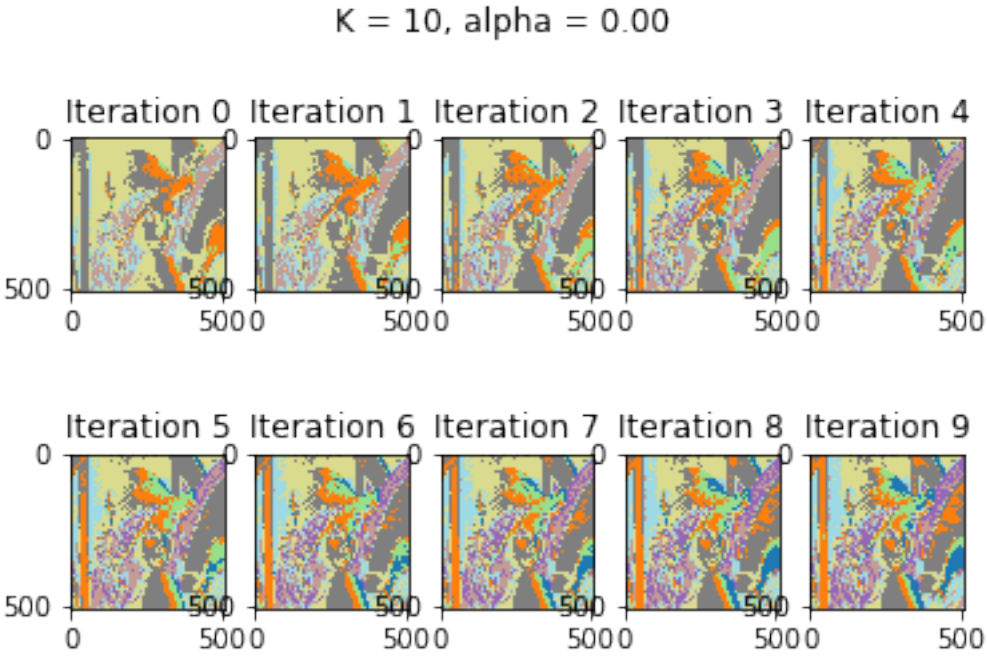


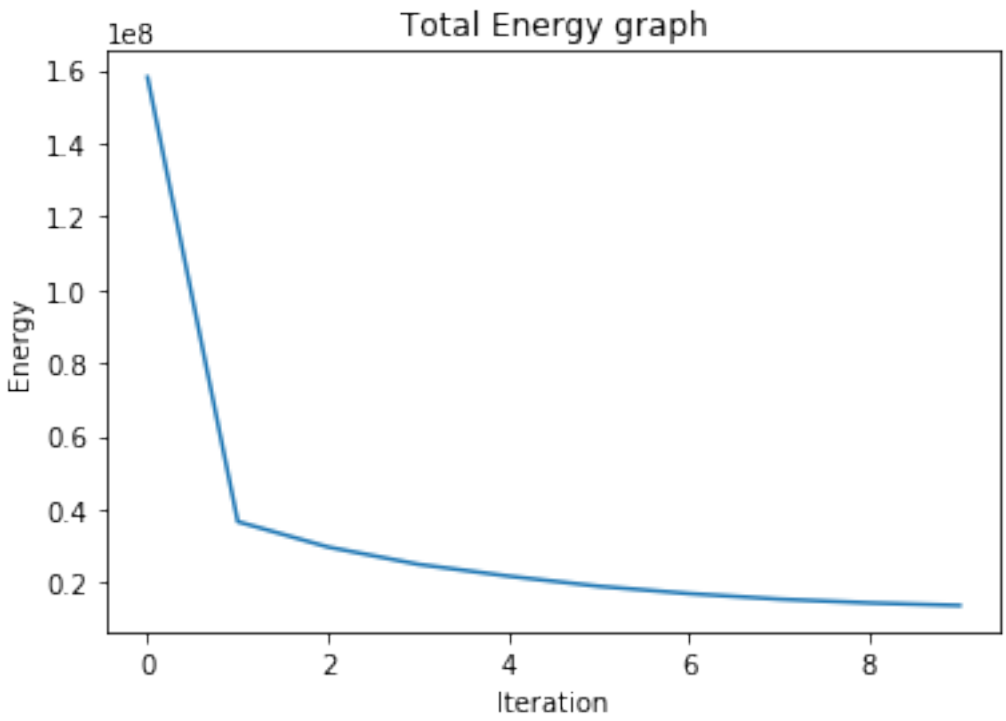
4 K = 10

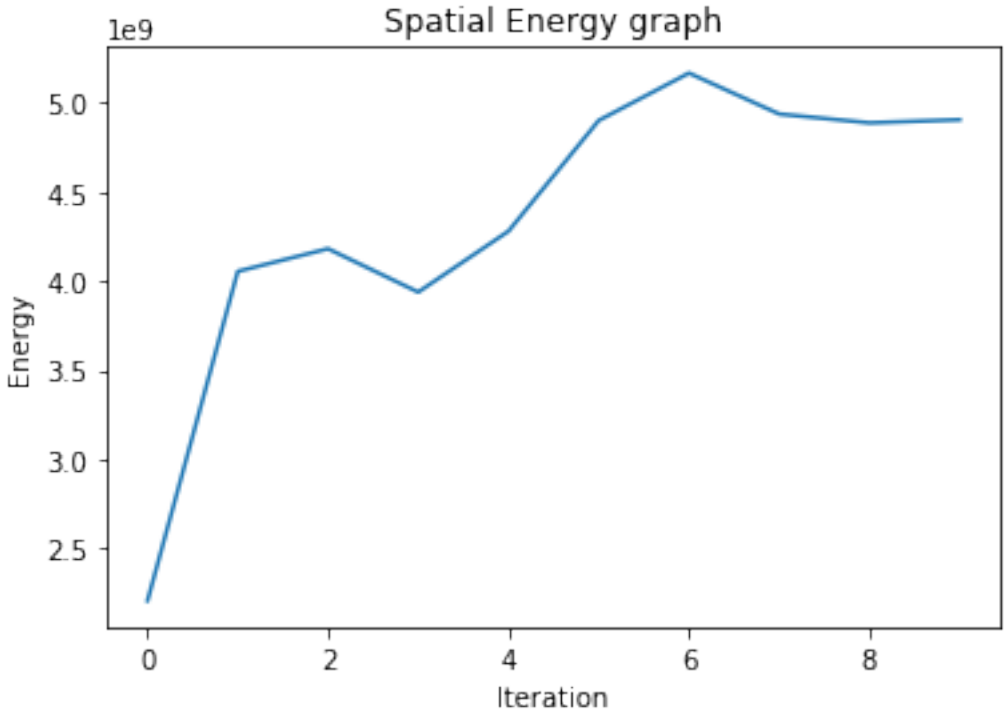
```
In [245]: K = 10
          KMeans(img_matrix, img_rows, img_cols, K, 0)
          KMeans(img_matrix, img_rows, img_cols, K, 0.01)
          KMeans(img_matrix, img_rows, img_cols, K, 0.03)
          KMeans(img_matrix, img_rows, img_cols, K, 0.05)
          KMeans(img_matrix, img_rows, img_cols, K, 0.07)
          KMeans(img_matrix, img_rows, img_cols, K, 0.08)
          KMeans(img_matrix, img_rows, img_cols, K, 0.10)
          KMeans(img_matrix, img_rows, img_cols, K, 0.25)
          KMeans(img_matrix, img_rows, img_cols, K, 0.50)
          KMeans(img_matrix, img_rows, img_cols, K, 0.75)
          KMeans(img_matrix, img_rows, img_cols, K, 1)

/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:66: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:67: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:68: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:32: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:33: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:35: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:36: RuntimeWarning: overi
```

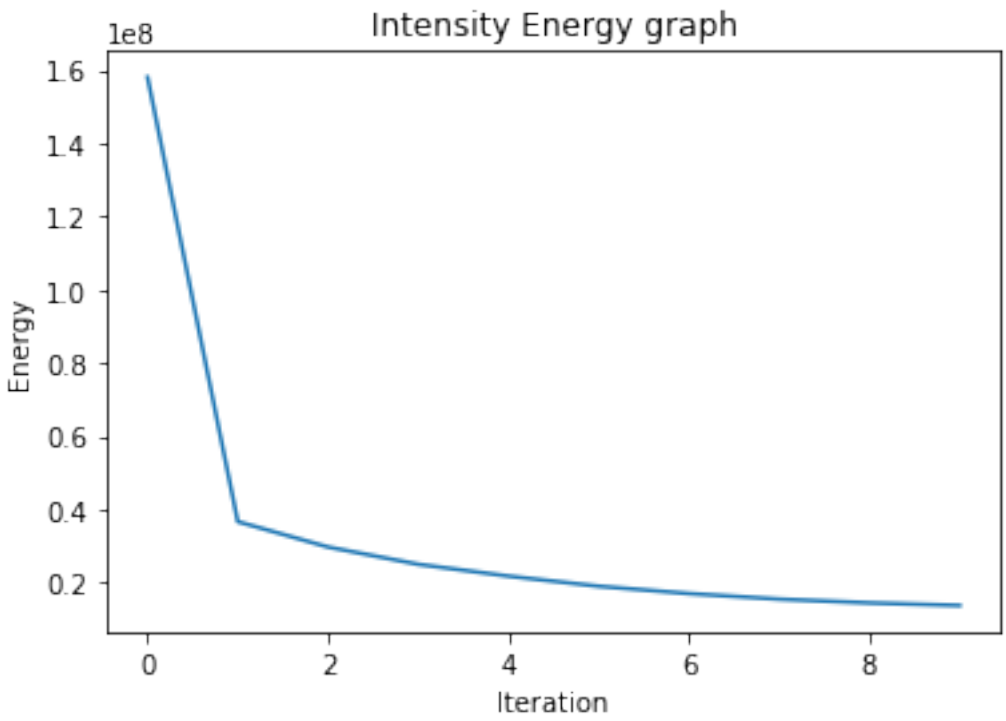
```
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:39: RuntimeWarning: over
```

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

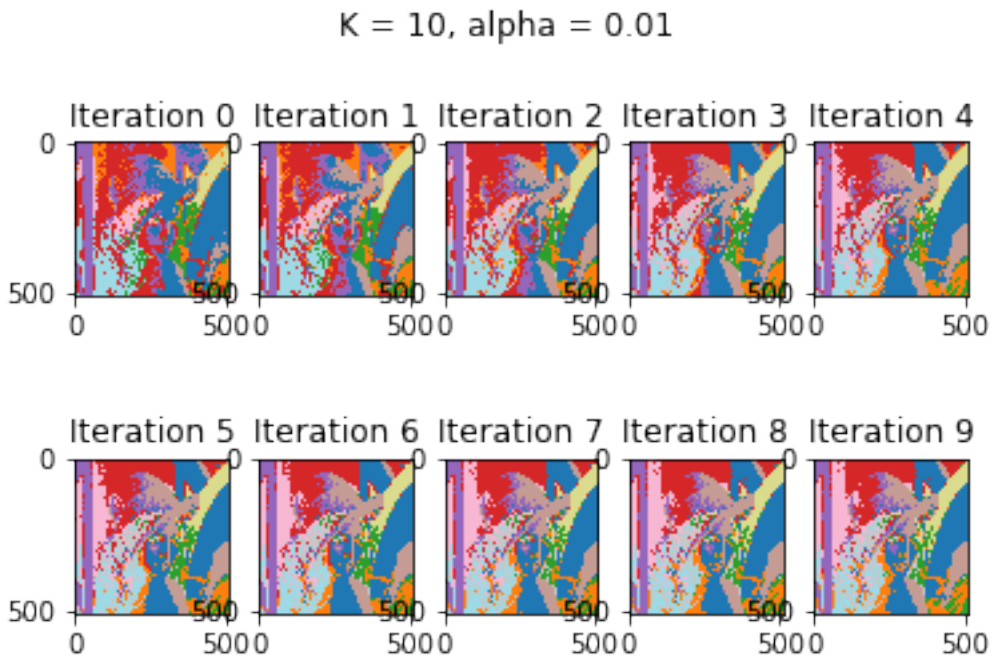


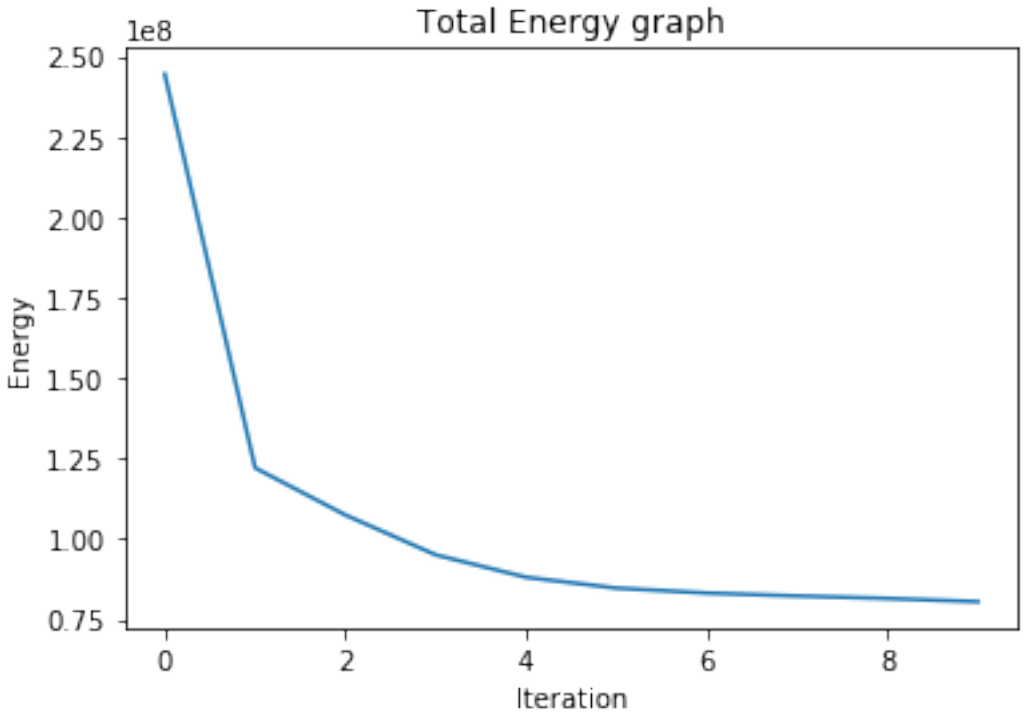

Total Energy graph
1e8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
Energy
0
2
4
6
8
Iteration

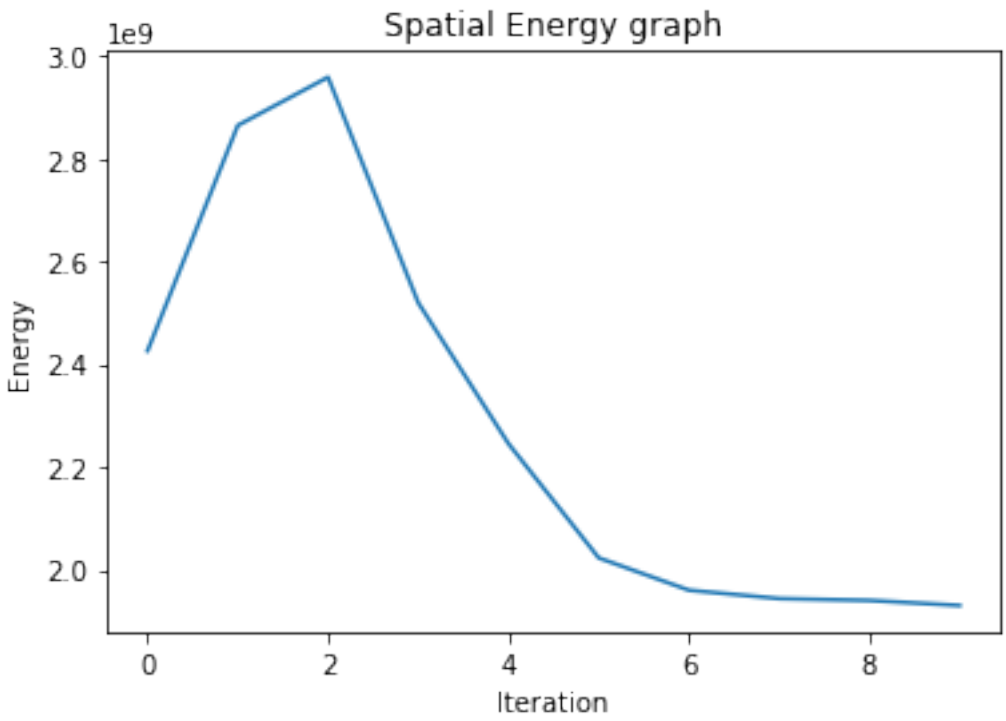

Spatial Energy graph
1e9
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

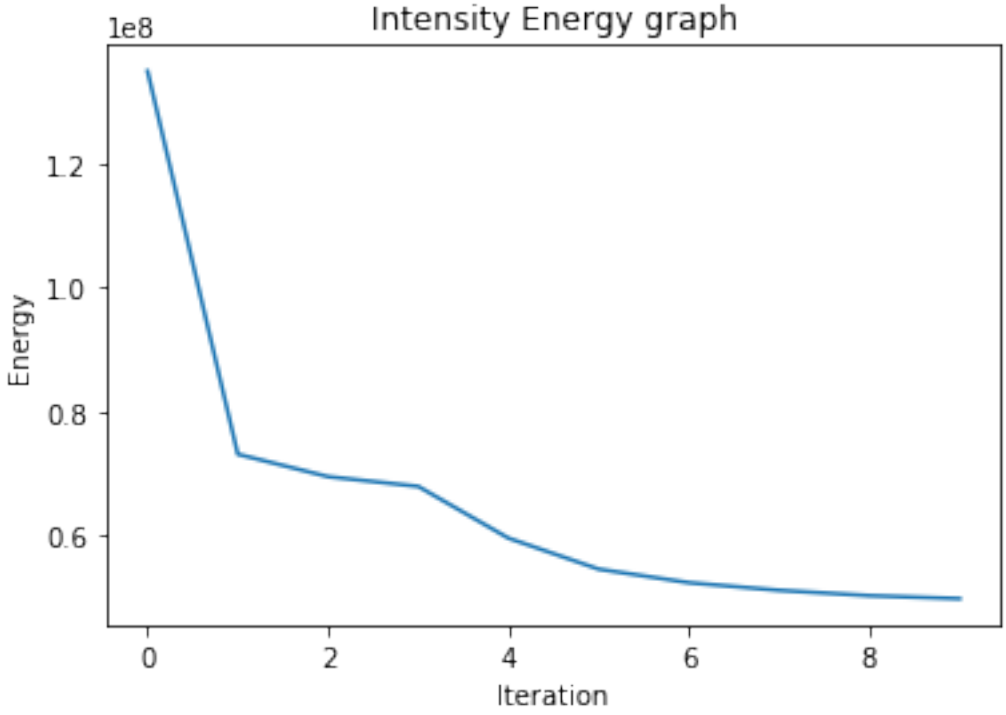



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

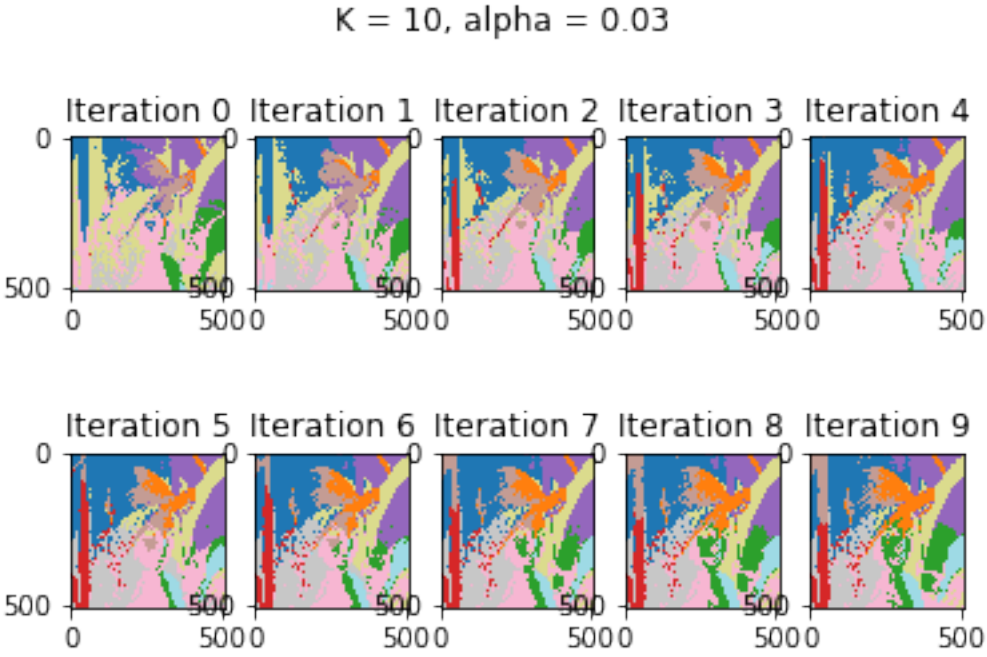

K = 10, alpha = 0.01
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

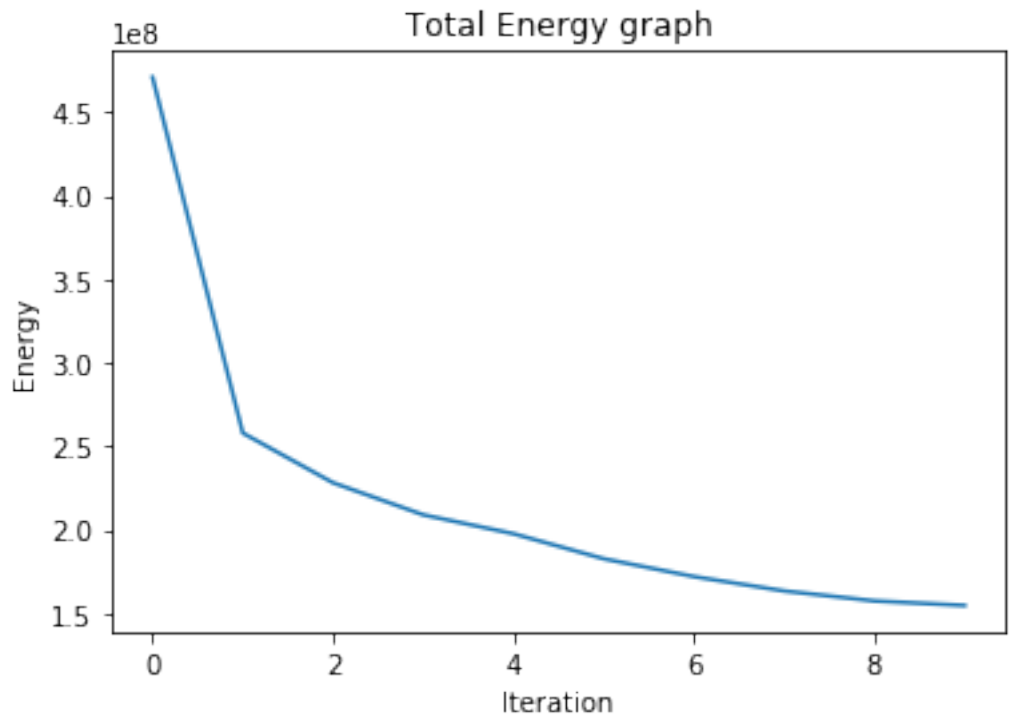

Total Energy graph
1e8
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
Energy
0
2
4
6
8
Iteration

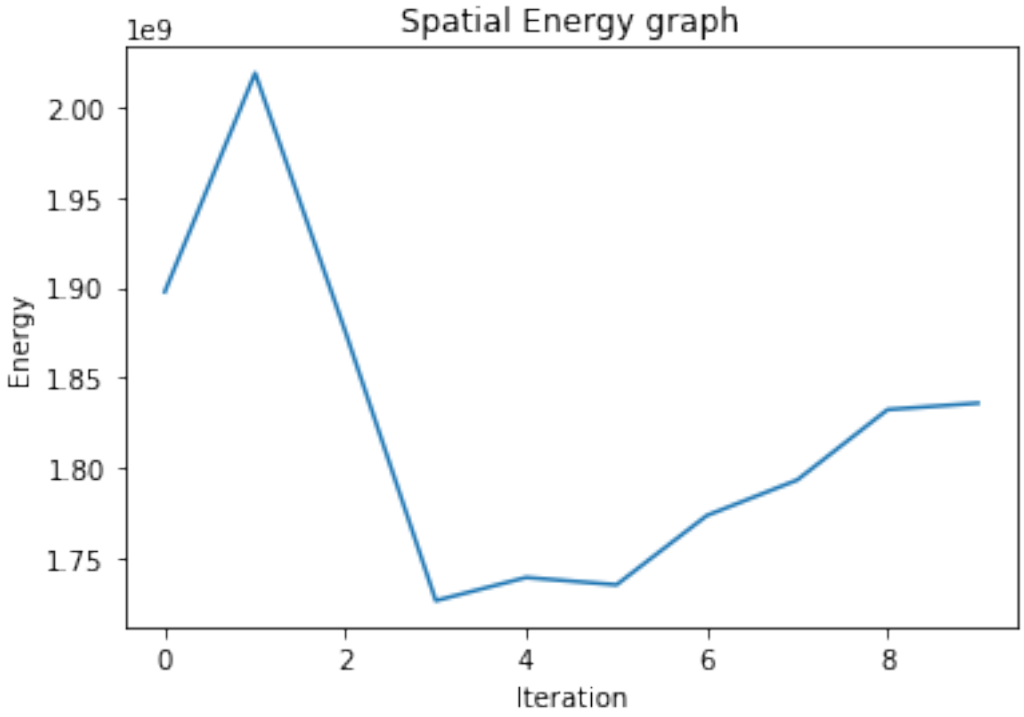

Spatial Energy graph
1e9
3.0
2.8
2.6
2.4
2.2
2.0
Energy
0
2
4
6
8
Iteration

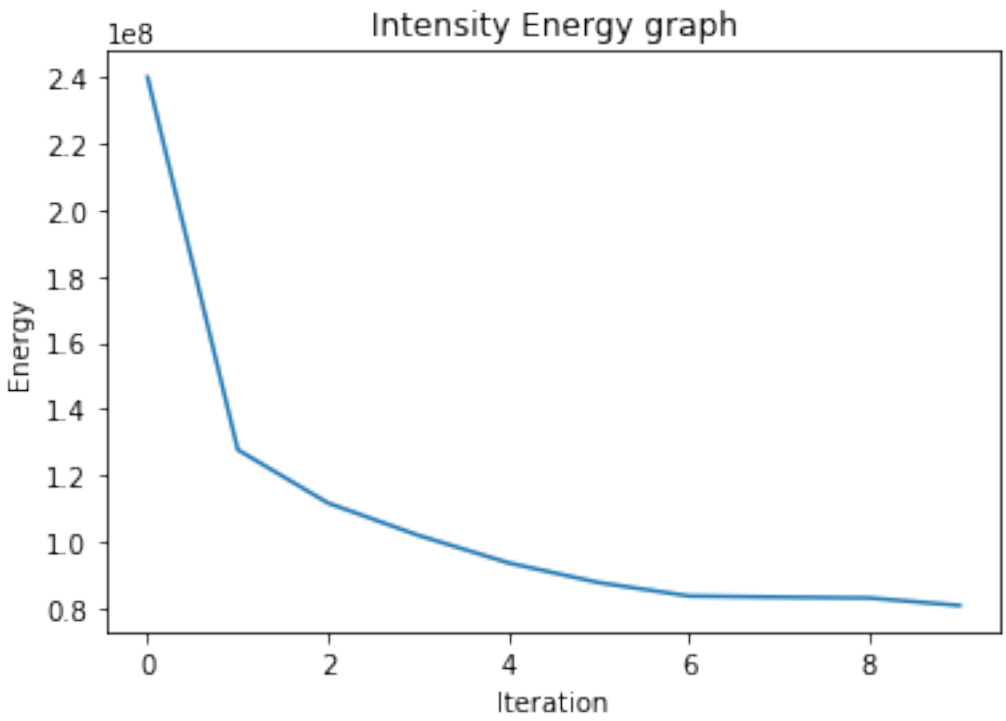

Intensity Energy graph
1e8
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

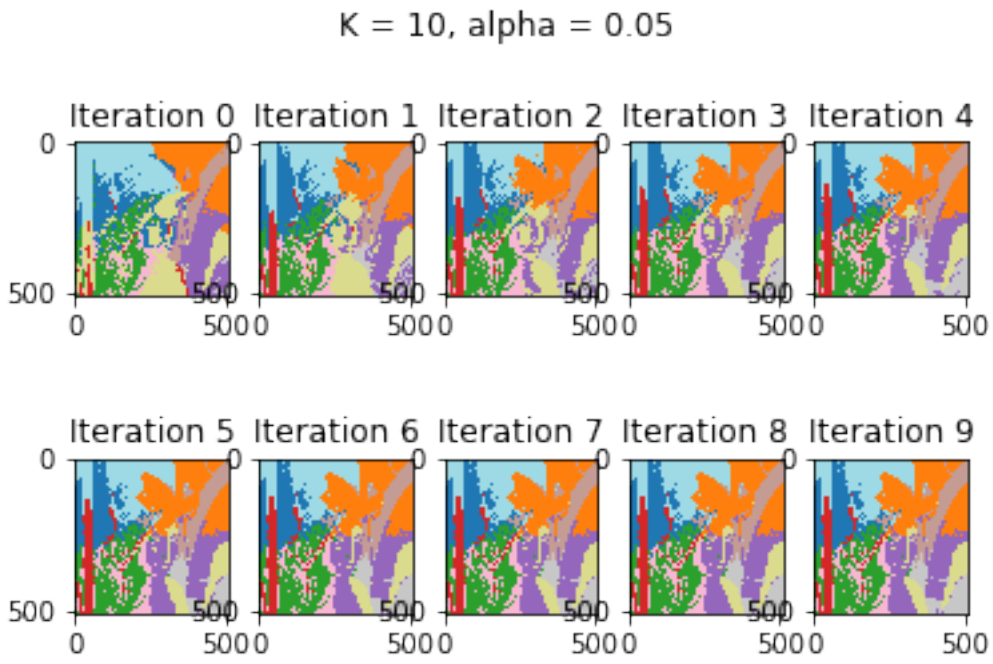


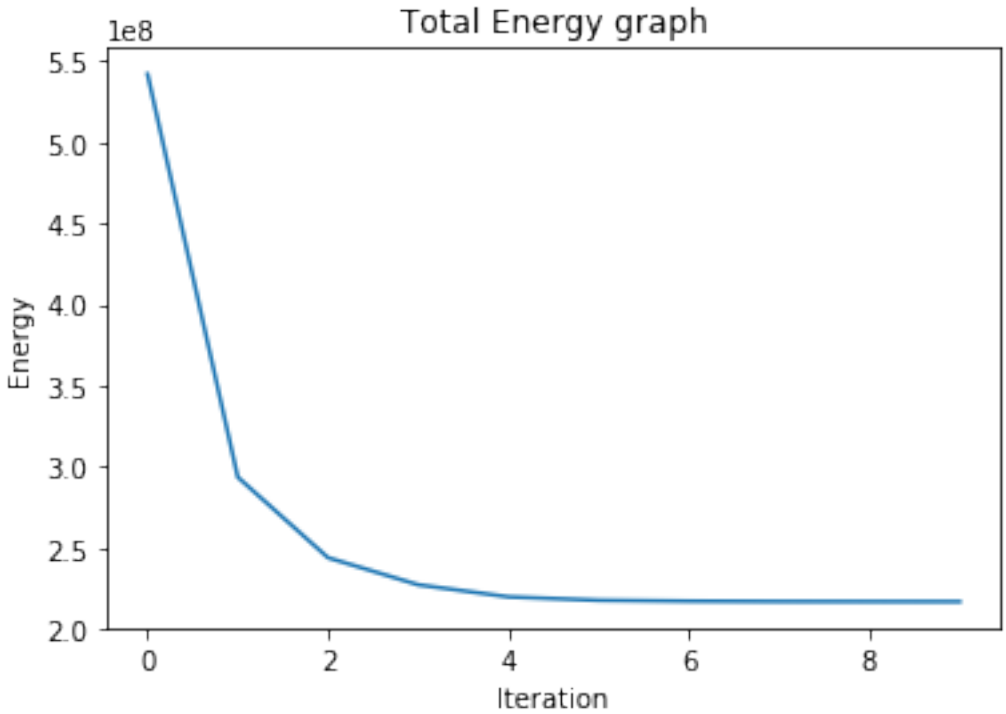

Total Energy graph
1e8
4.5
4.0
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

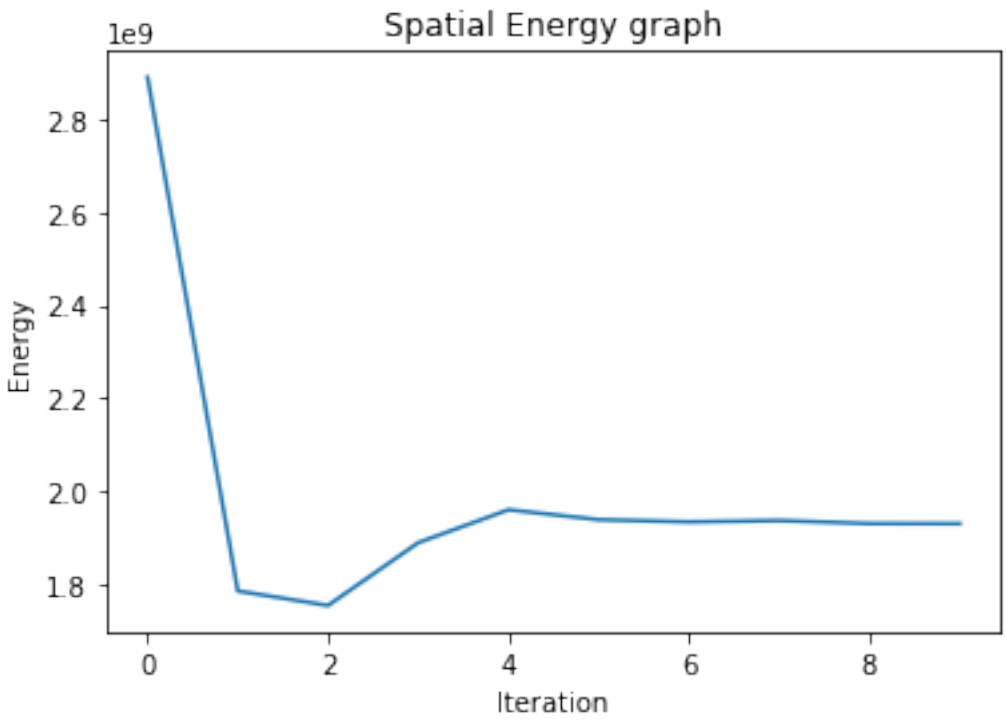

Spatial Energy graph
1e9
2.00
1.95
1.90
1.85
1.80
1.75
Energy
0
2
4
6
8
Iteration

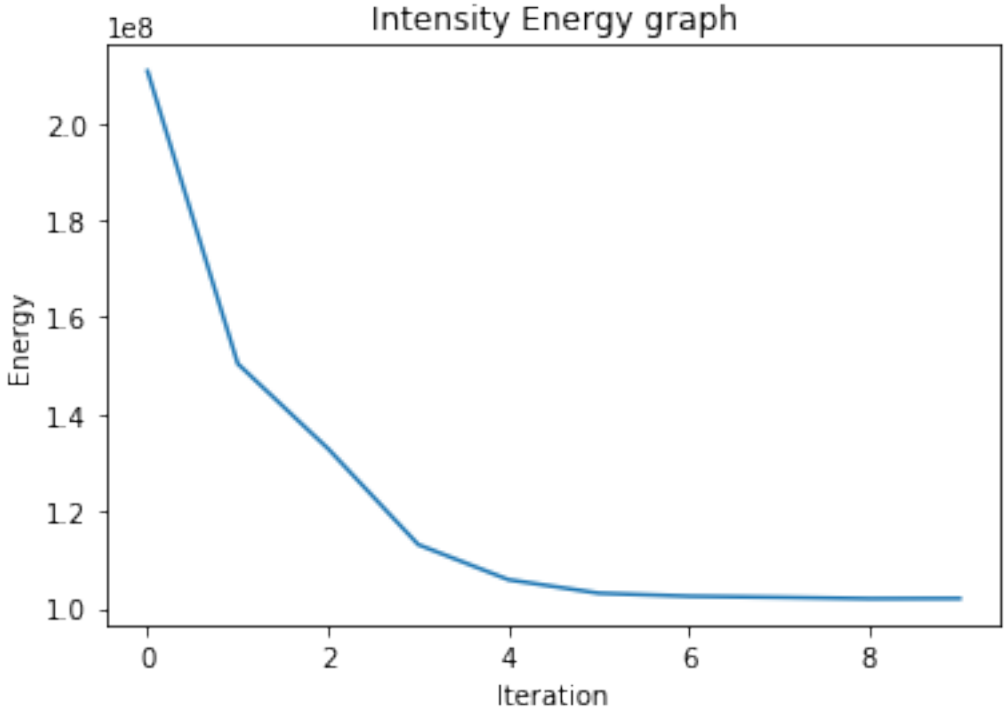



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

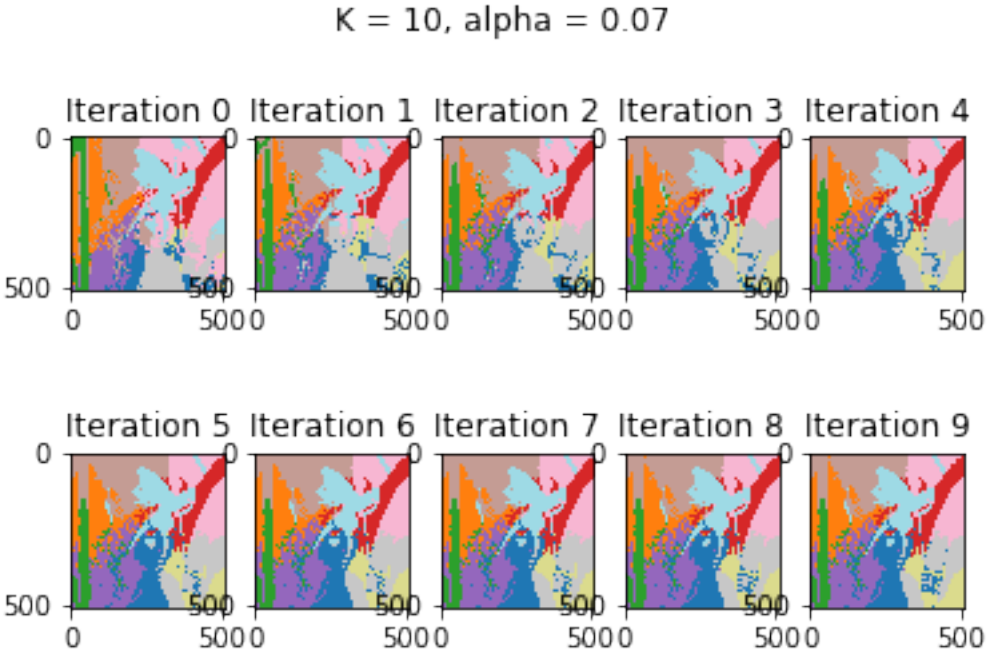

K = 10, alpha = 0.05
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

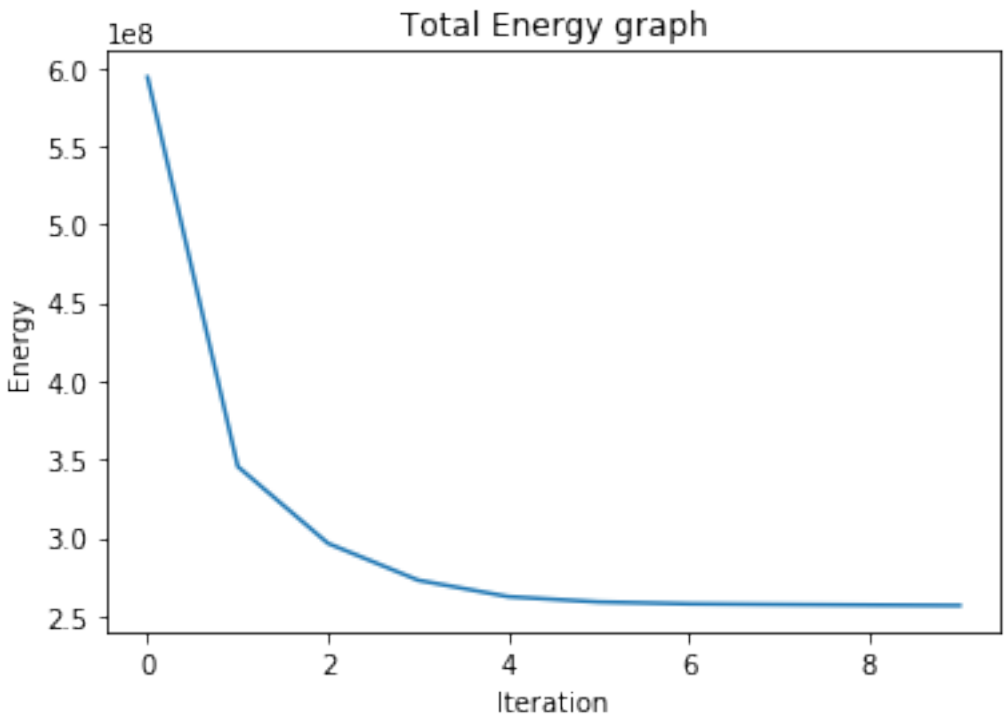

Total Energy graph
1e8
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

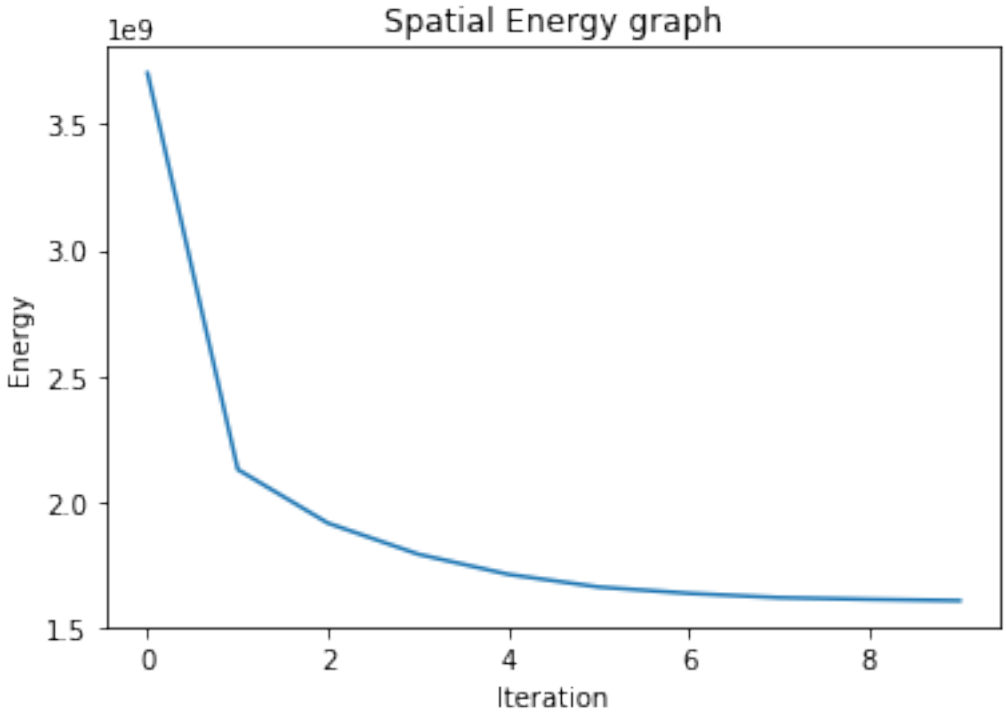

Spatial Energy graph
1e9
2.8
2.6
2.4
2.2
2.0
1.8
Energy
0
2
4
6
8
Iteration

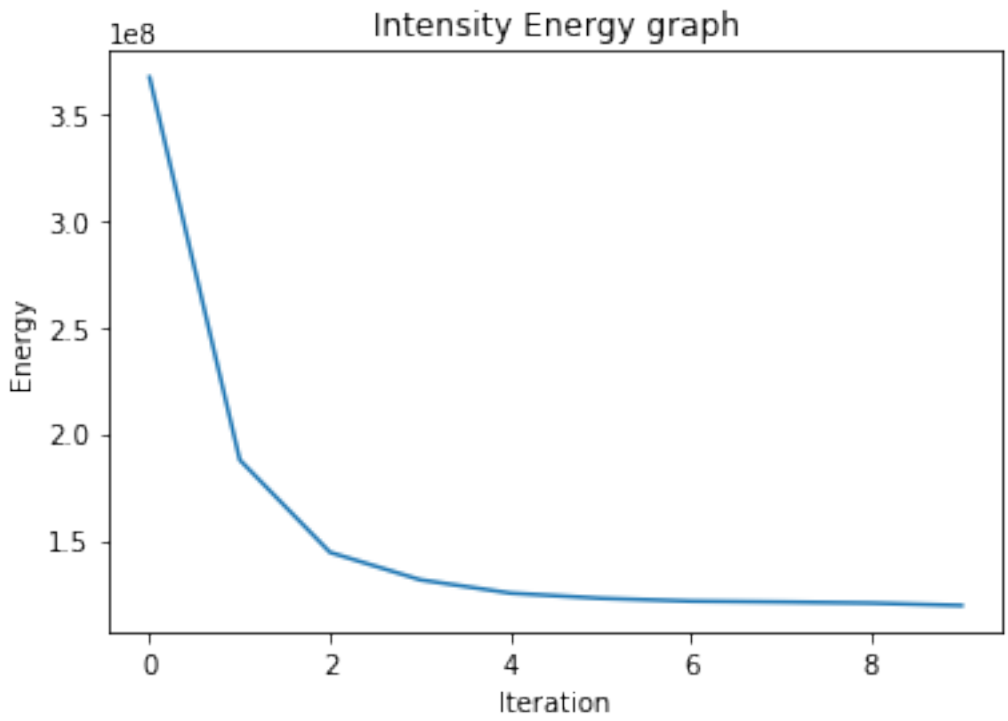

Intensity Energy graph
1e8
2.0
1.8
1.6
1.4
1.2
1.0
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

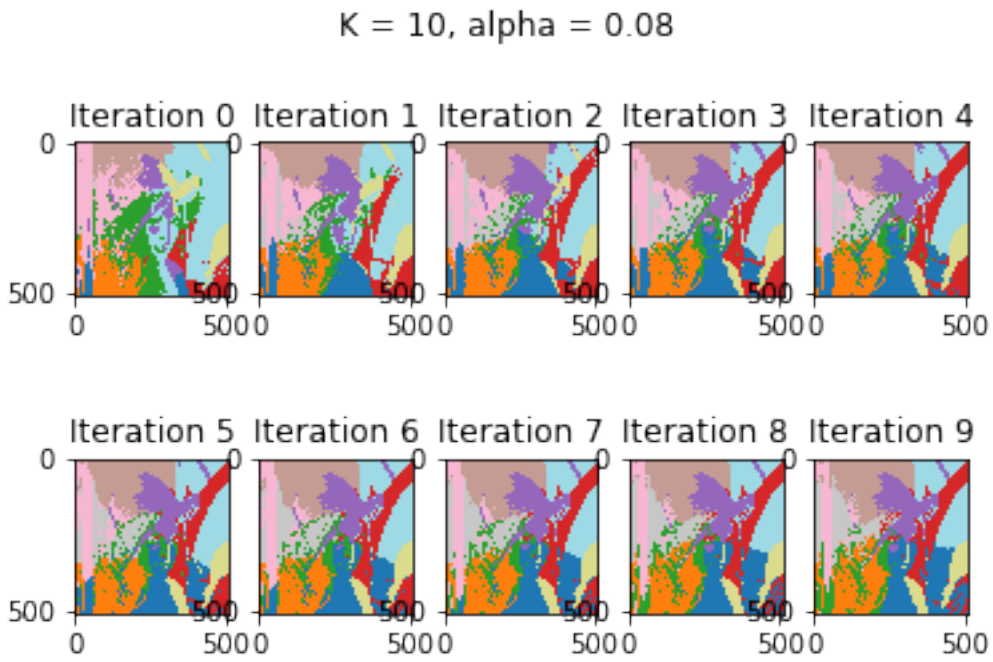


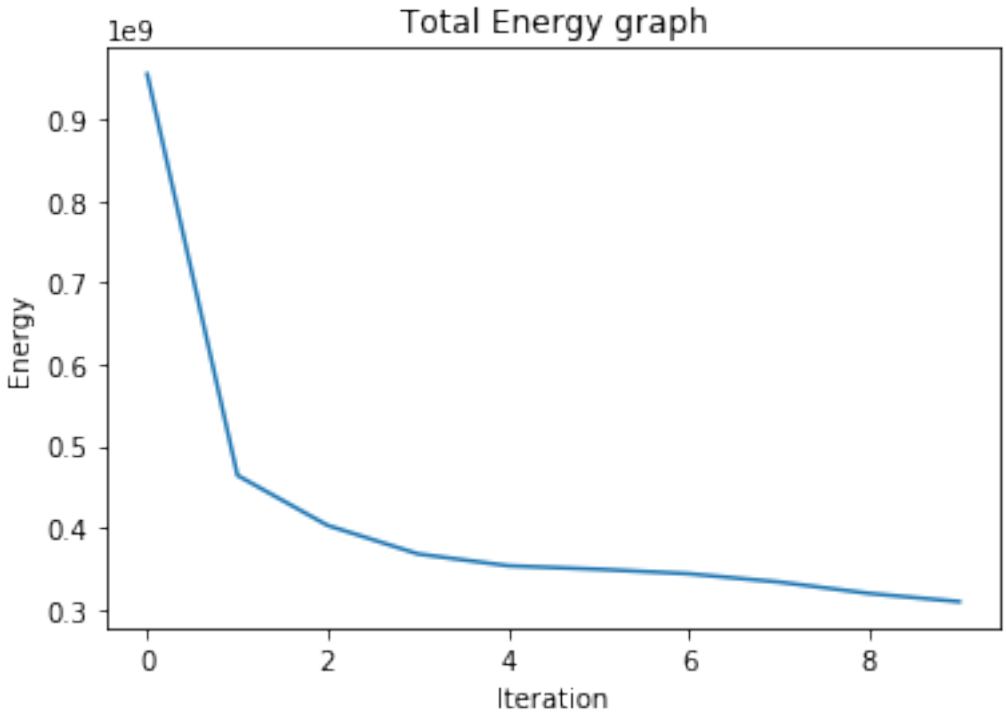

Total Energy graph
1e8
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

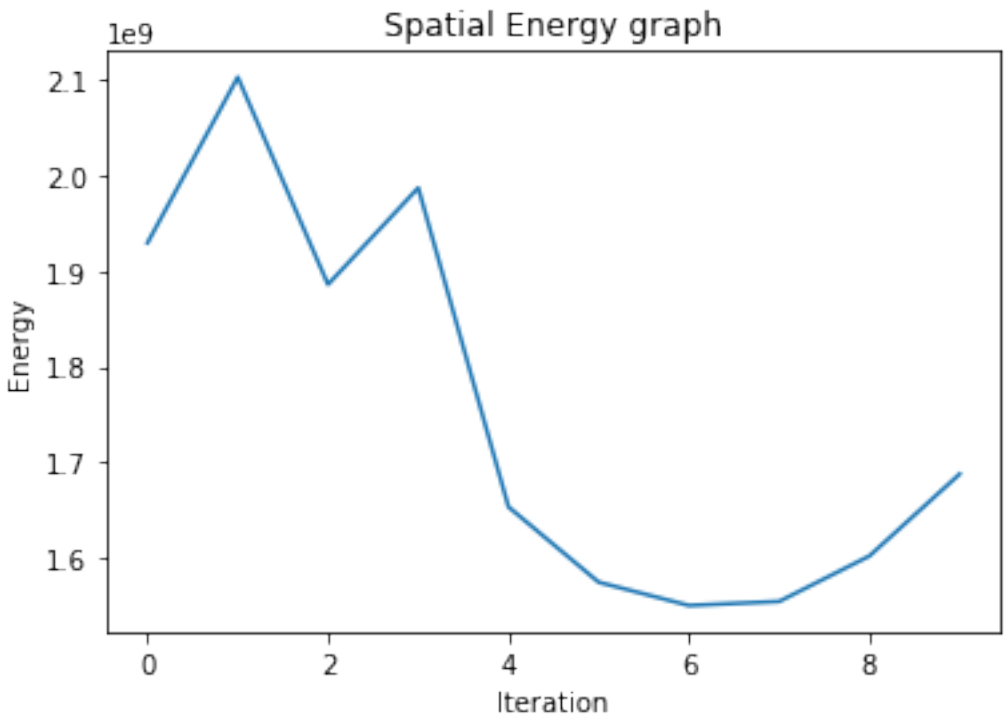

Spatial Energy graph
1e9
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

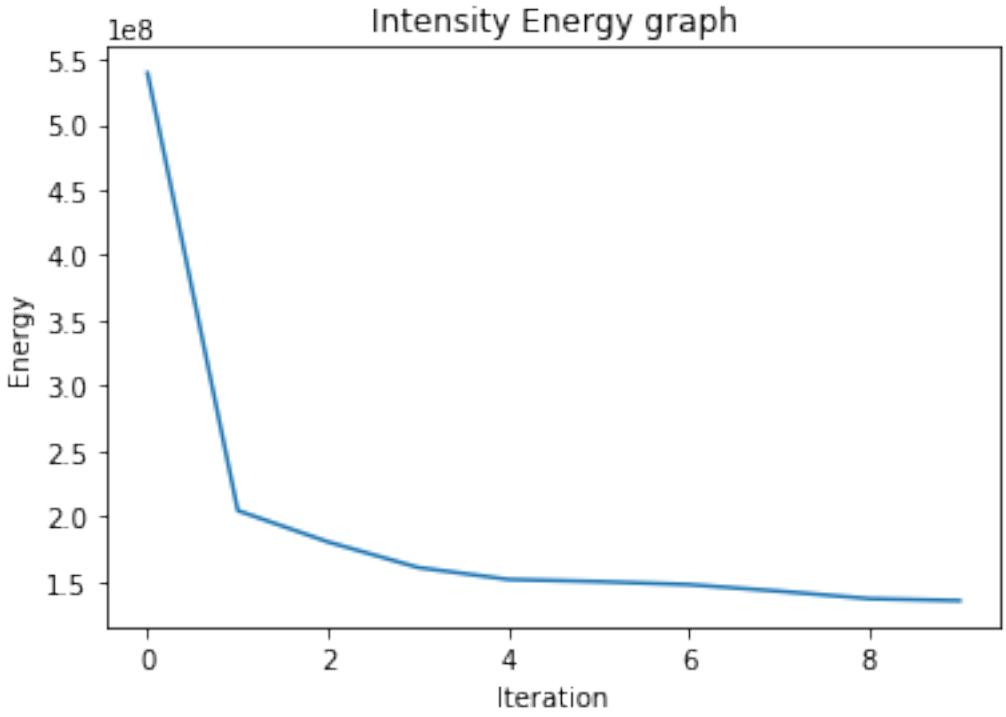



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

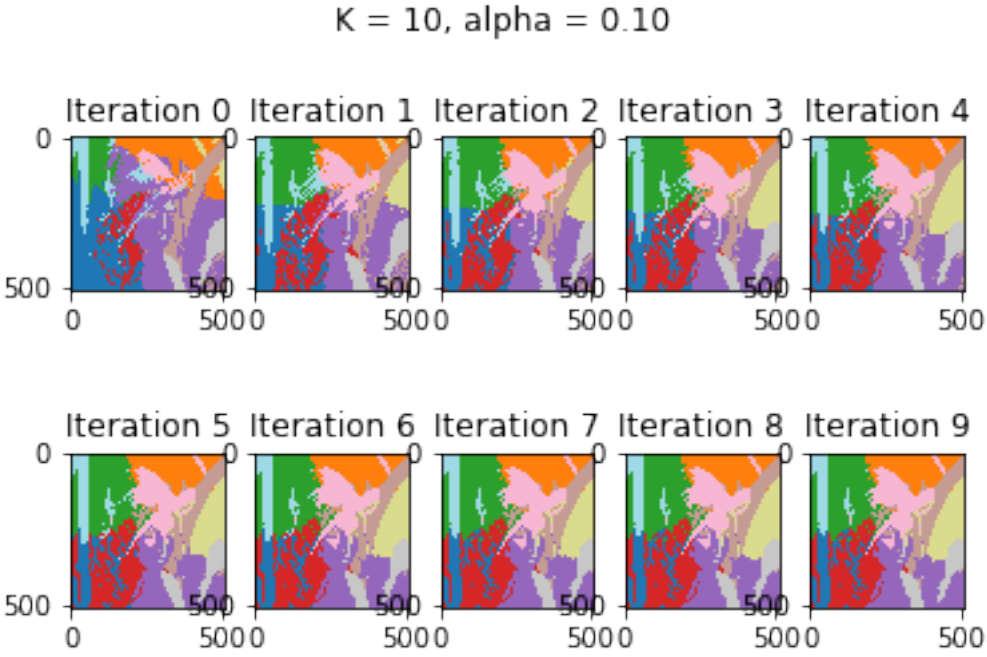

K = 10, alpha = 0.08
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

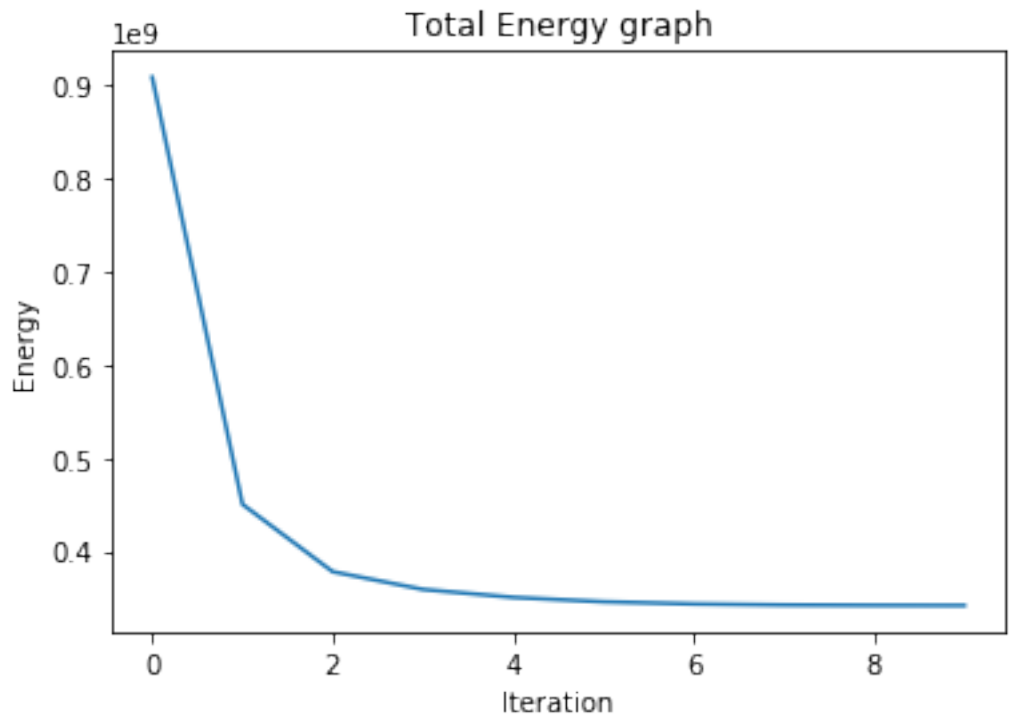

Total Energy graph
1e9
0.9
0.8
0.7
0.6
0.5
0.4
0.3
Energy
0
2
4
6
8
Iteration

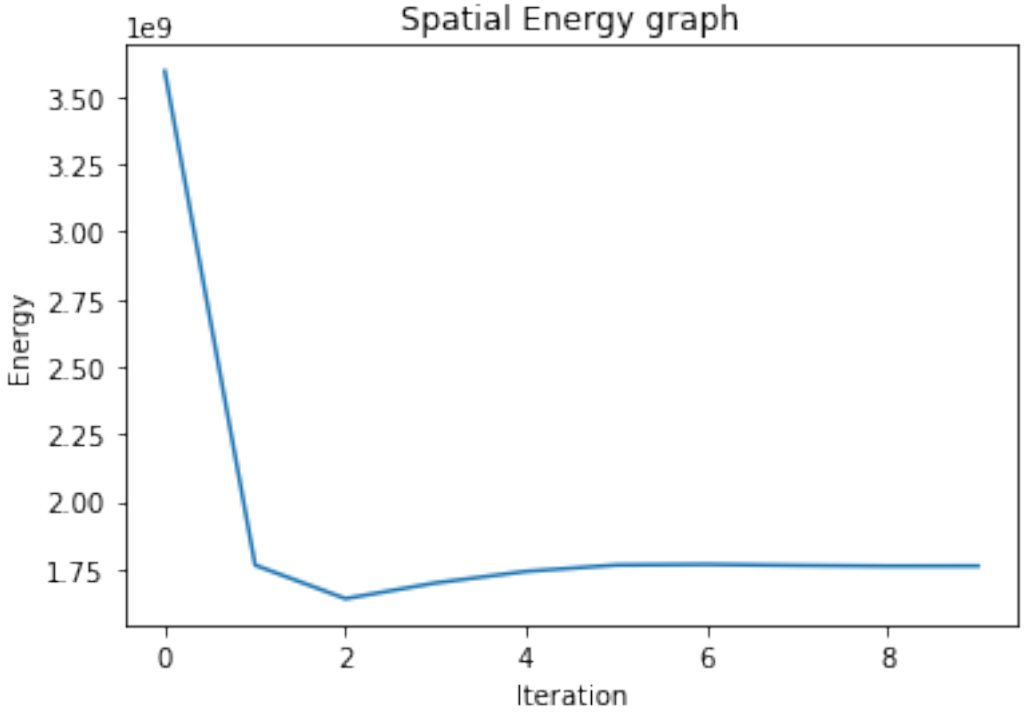

Spatial Energy graph
1e9
2.1
2.0
1.9
1.8
1.7
1.6
Energy
0
2
4
6
8
Iteration

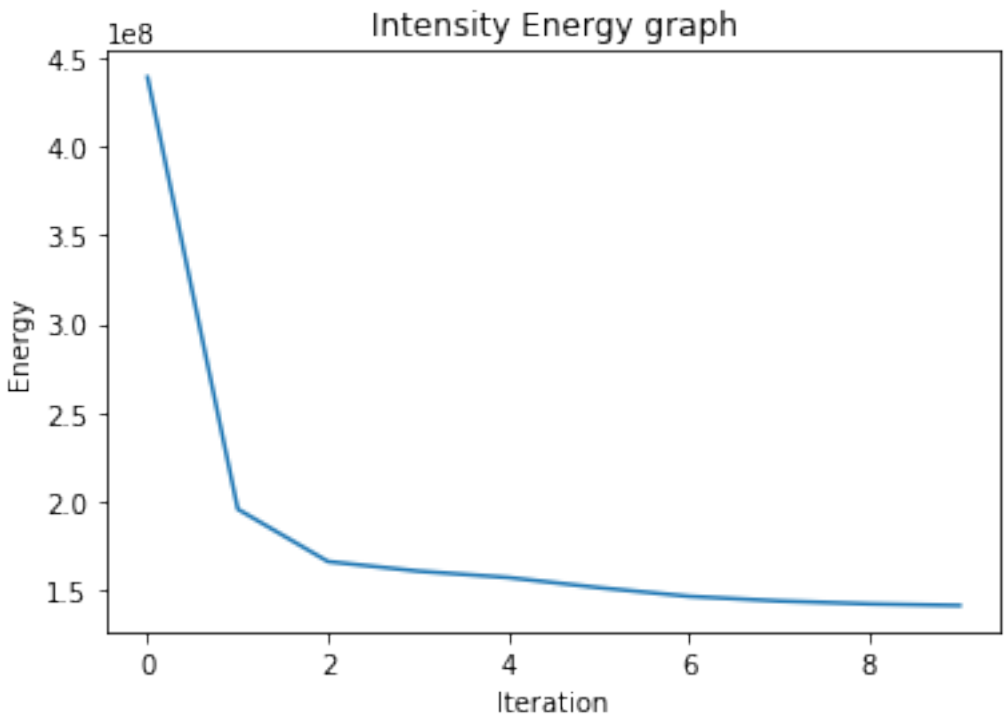

Intensity Energy graph
1e8
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

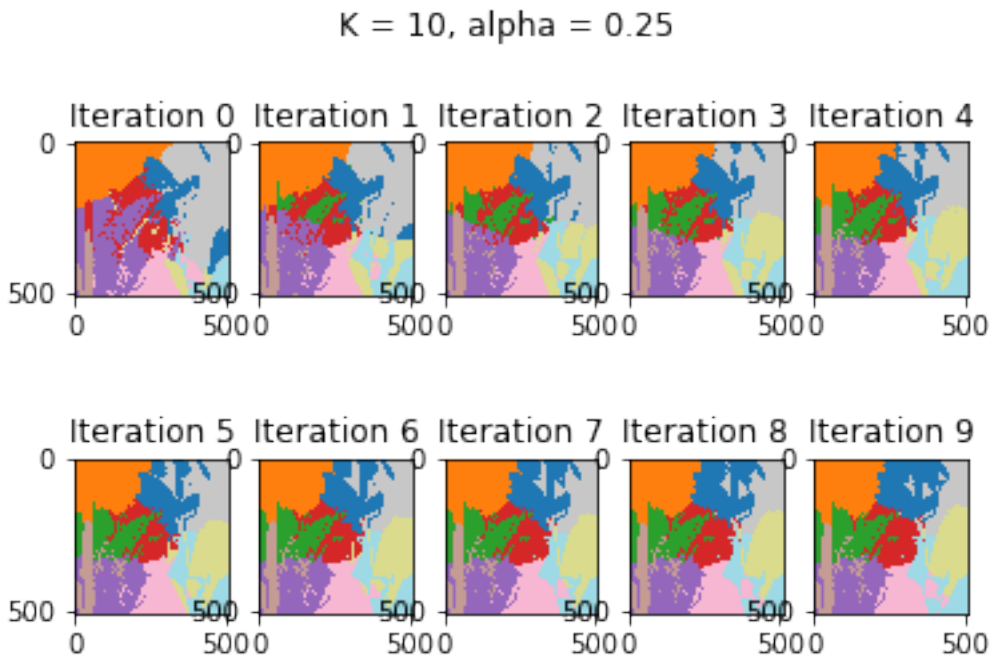


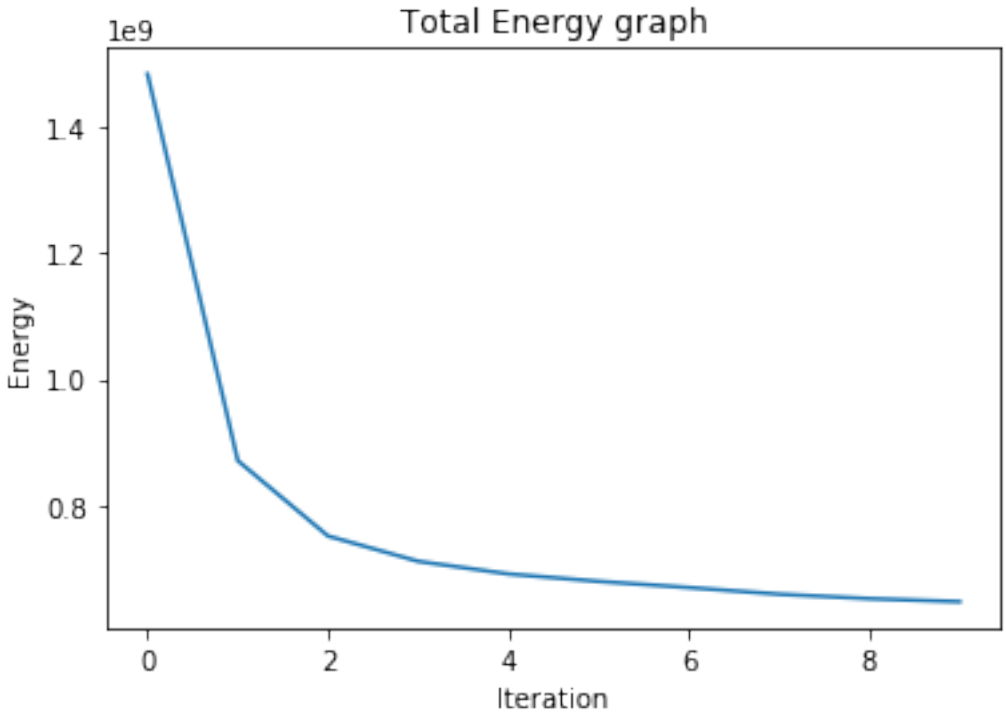

Total Energy graph
1e9
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration

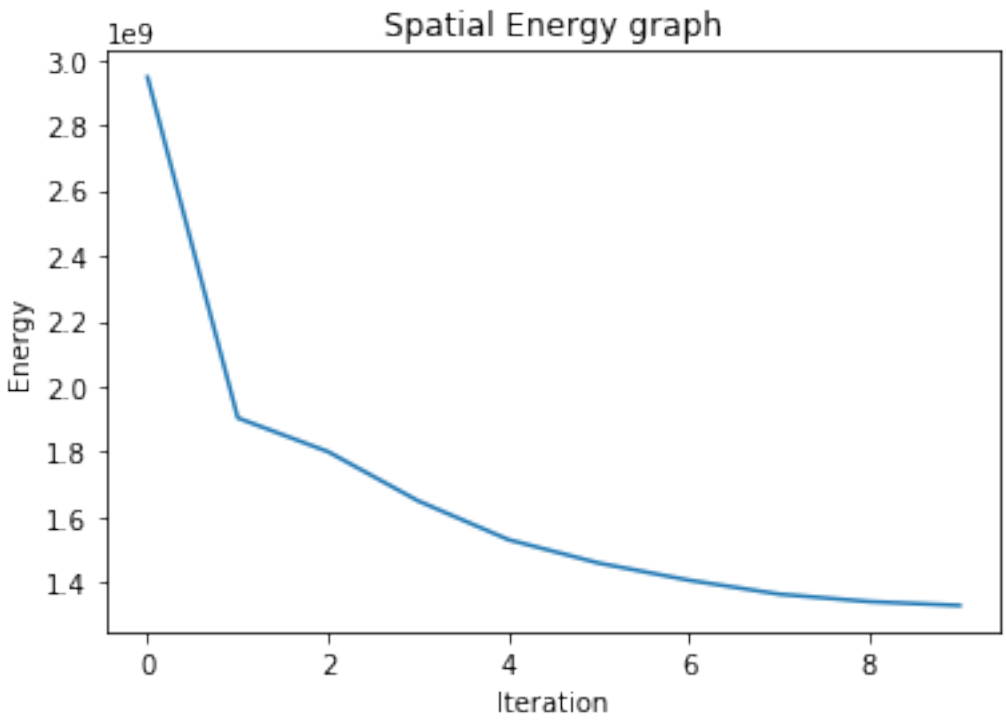

Spatial Energy graph
1e9
3.50
3.25
3.00
2.75
2.50
2.25
2.00
1.75
Energy
0
2
4
6
8
Iteration

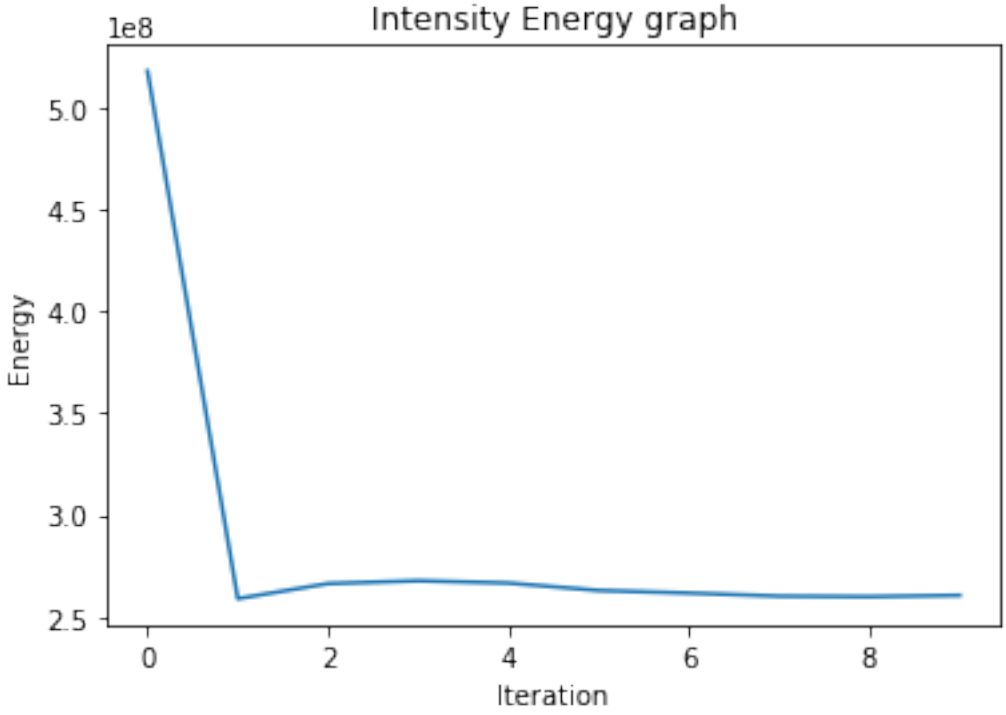



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

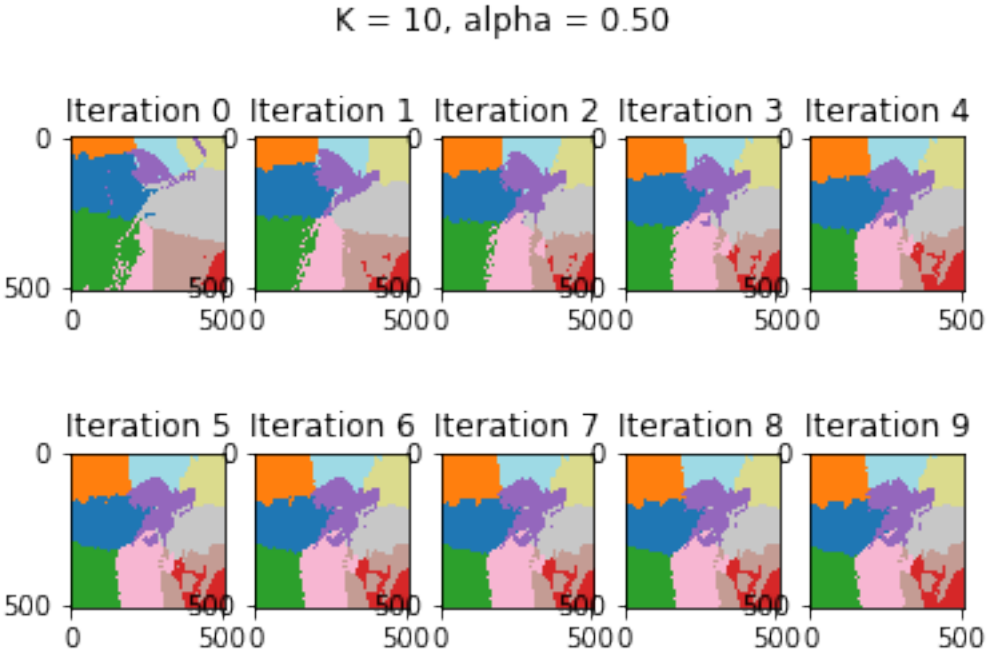

K = 10, alpha = 0.25
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

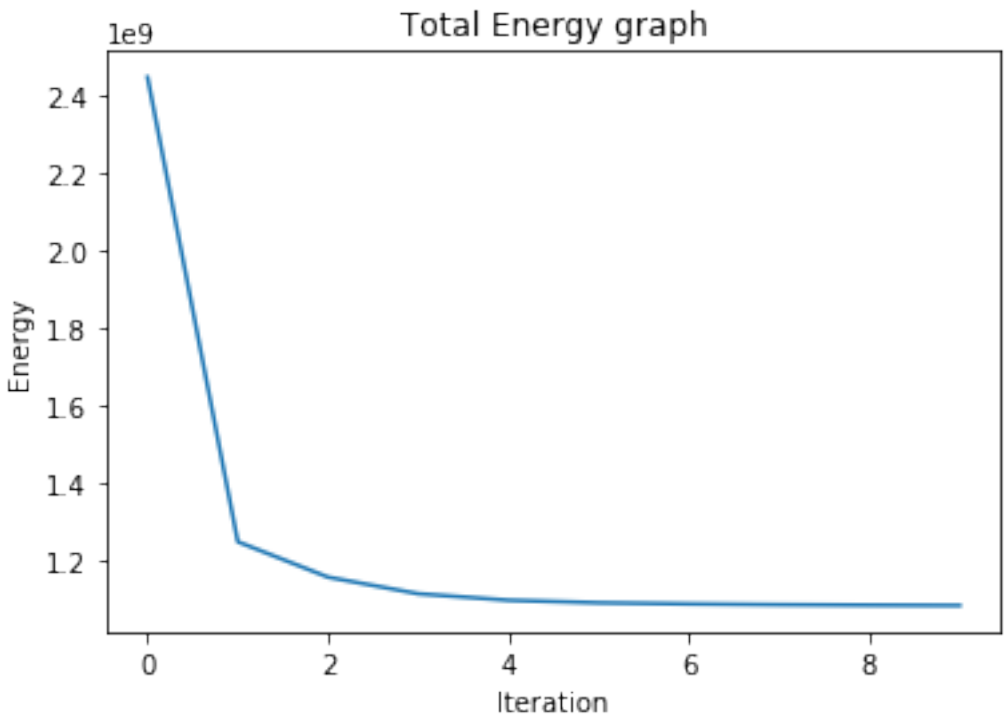

Total Energy graph
1e9
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

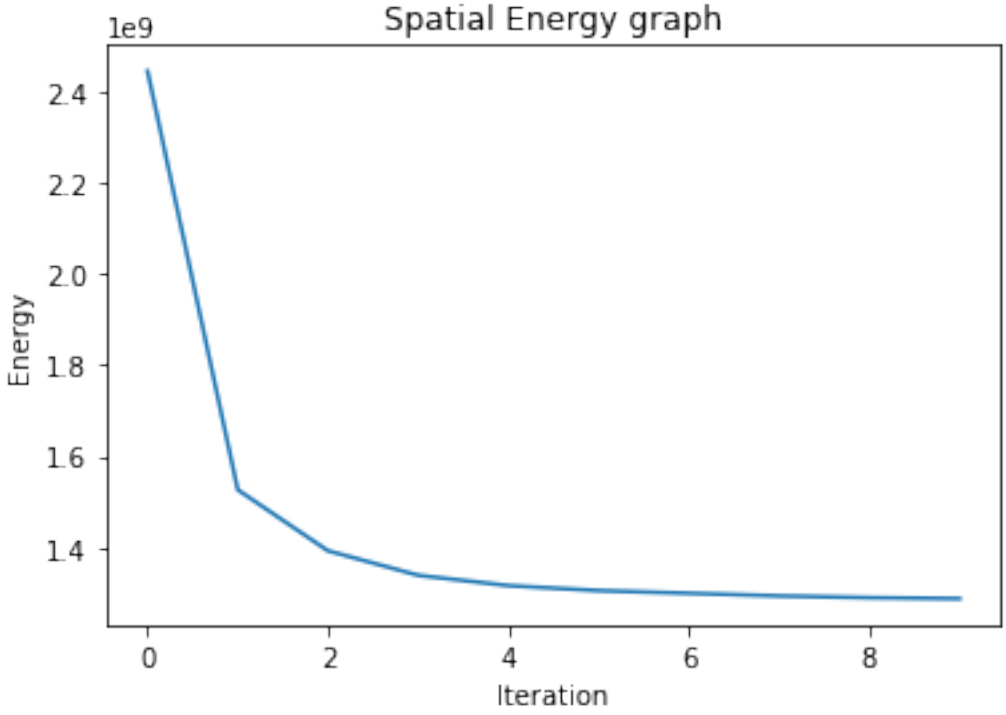

Spatial Energy graph
1e9
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
Energy
0
2
4
6
8
Iteration

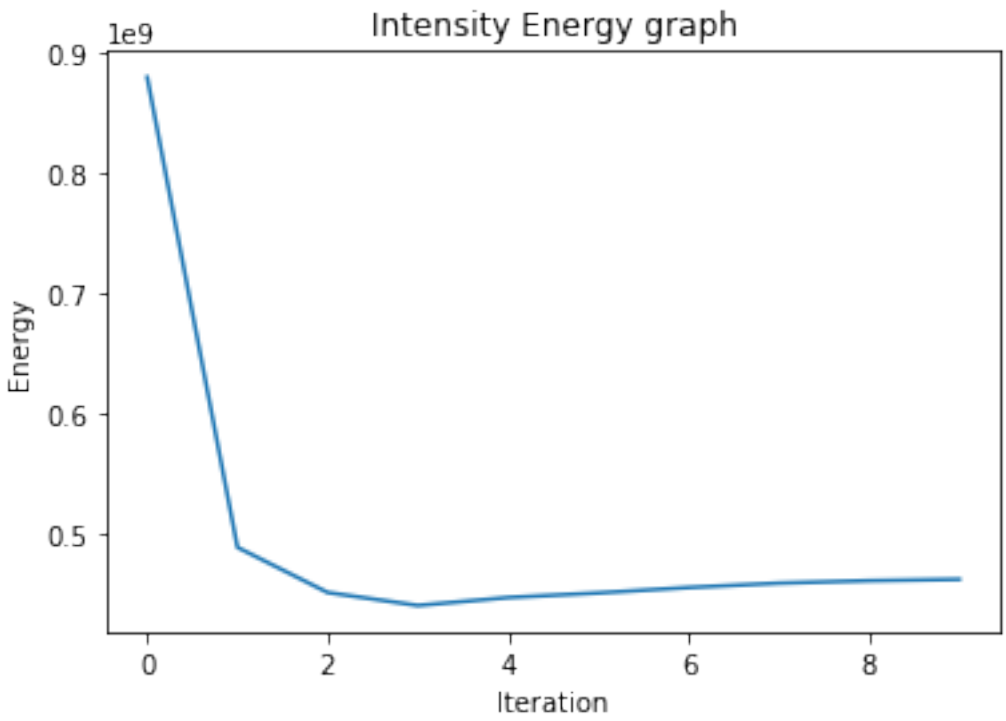

Intensity Energy graph
1e8
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

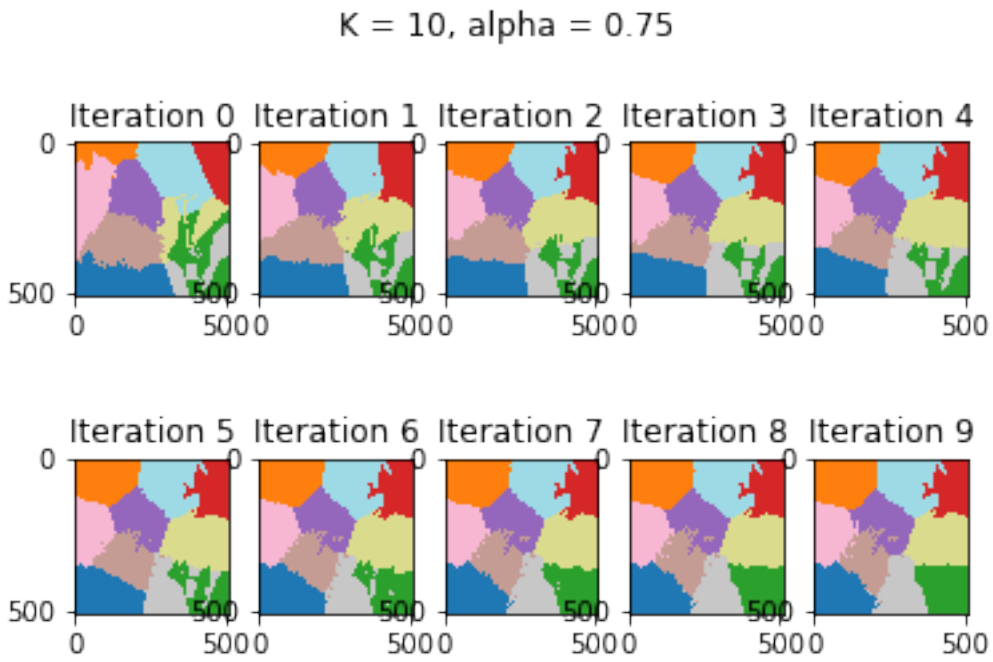


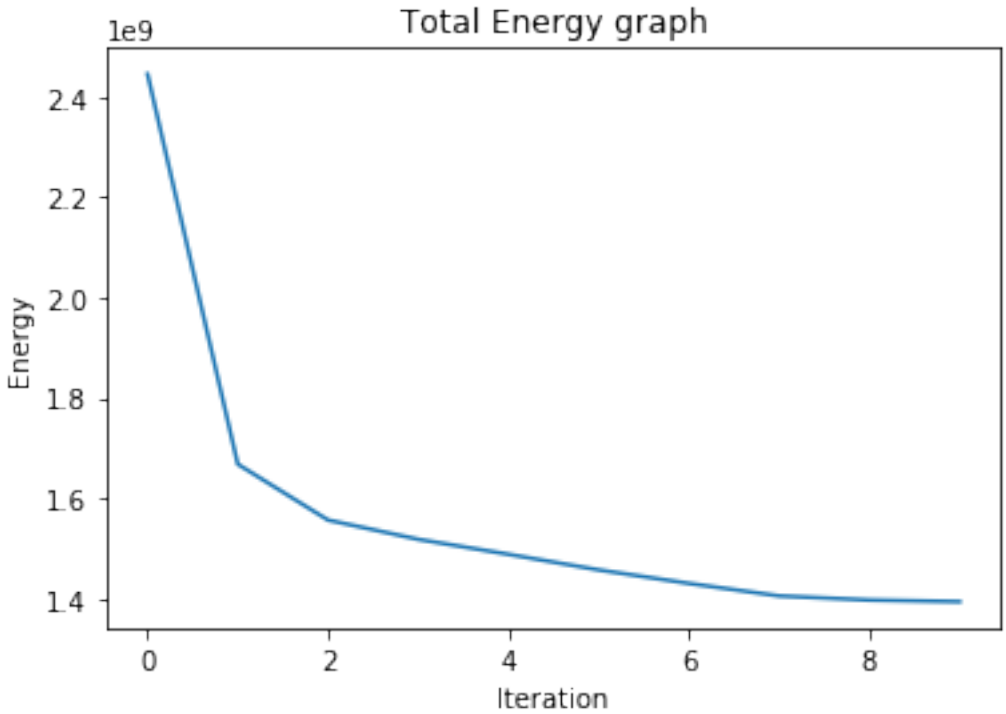

Total Energy graph
1e9
2.4
2.2
2.0
1.8
1.6
1.4
1.2
Energy
0
2
4
6
8
Iteration

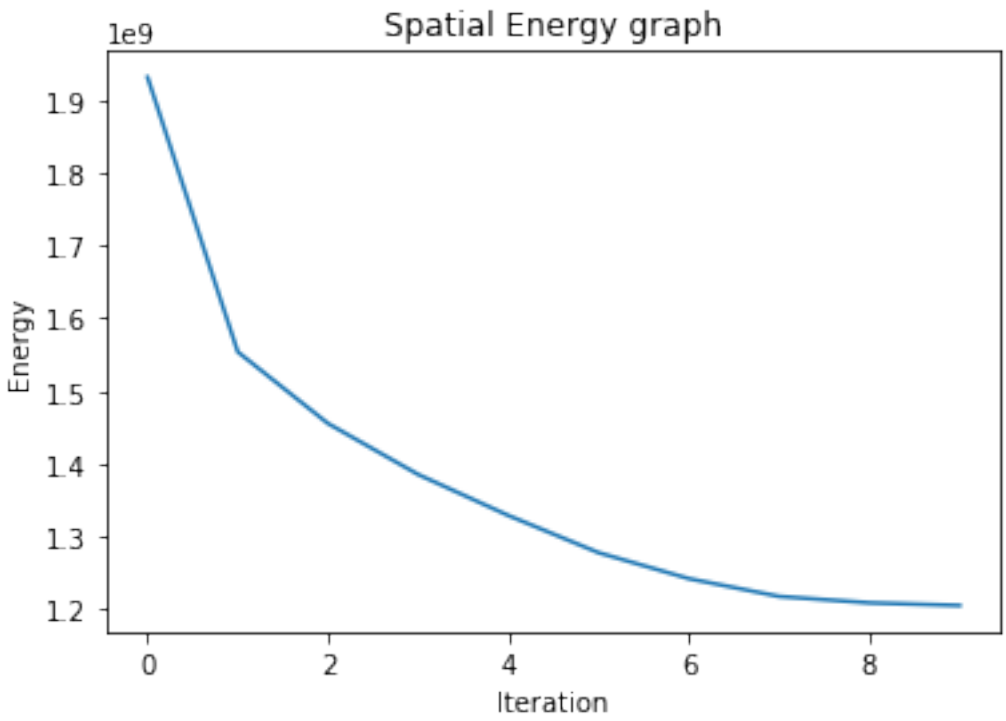

Spatial Energy graph
1e9
2.4
2.2
2.0
1.8
1.6
1.4
Energy
0
2
4
6
8
Iteration

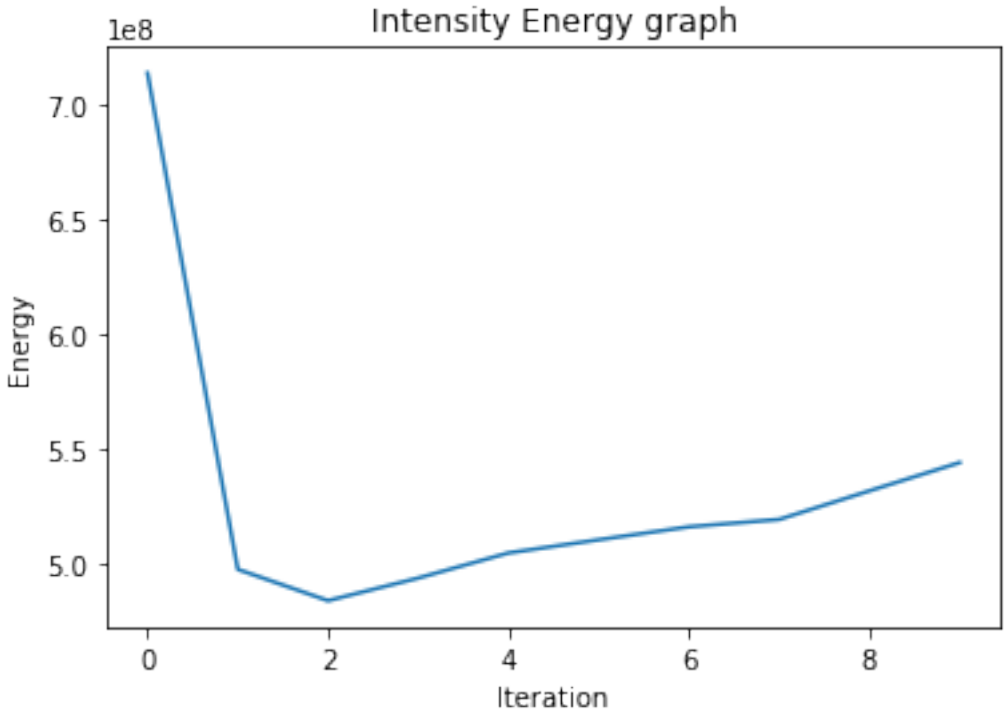



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

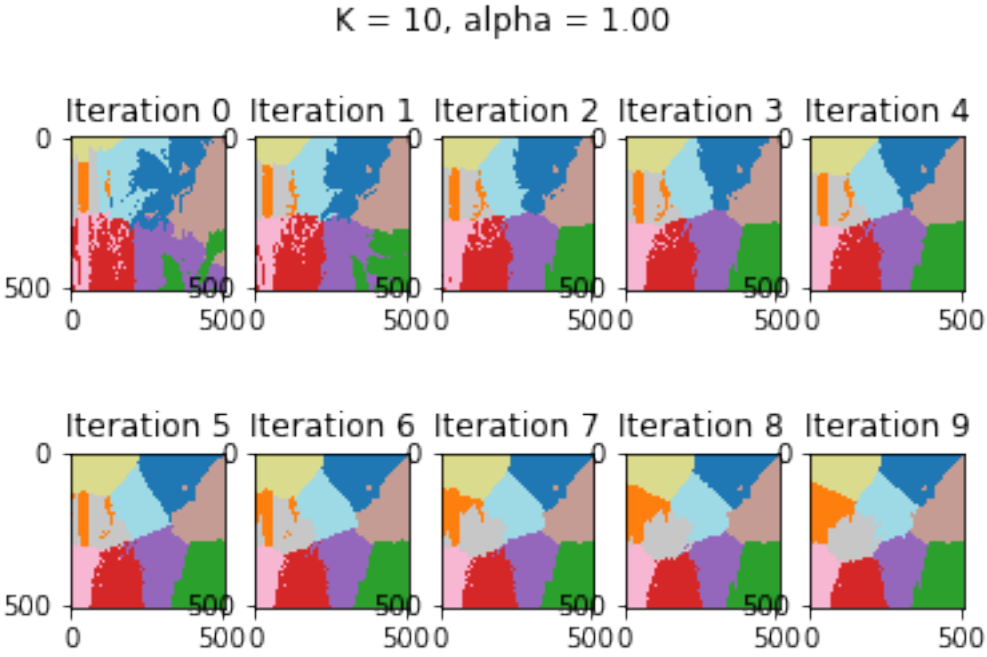

K = 10, alpha = 0.75
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

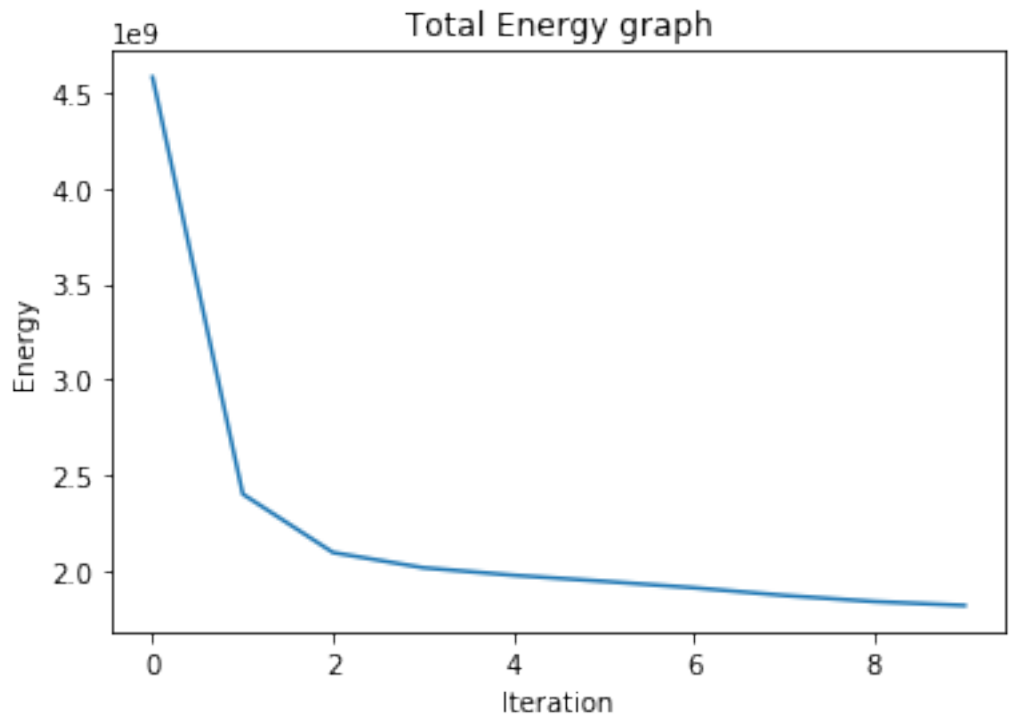

Total Energy graph
1e9
2.4
2.2
2.0
1.8
1.6
1.4
Energy
0
2
4
6
8
Iteration

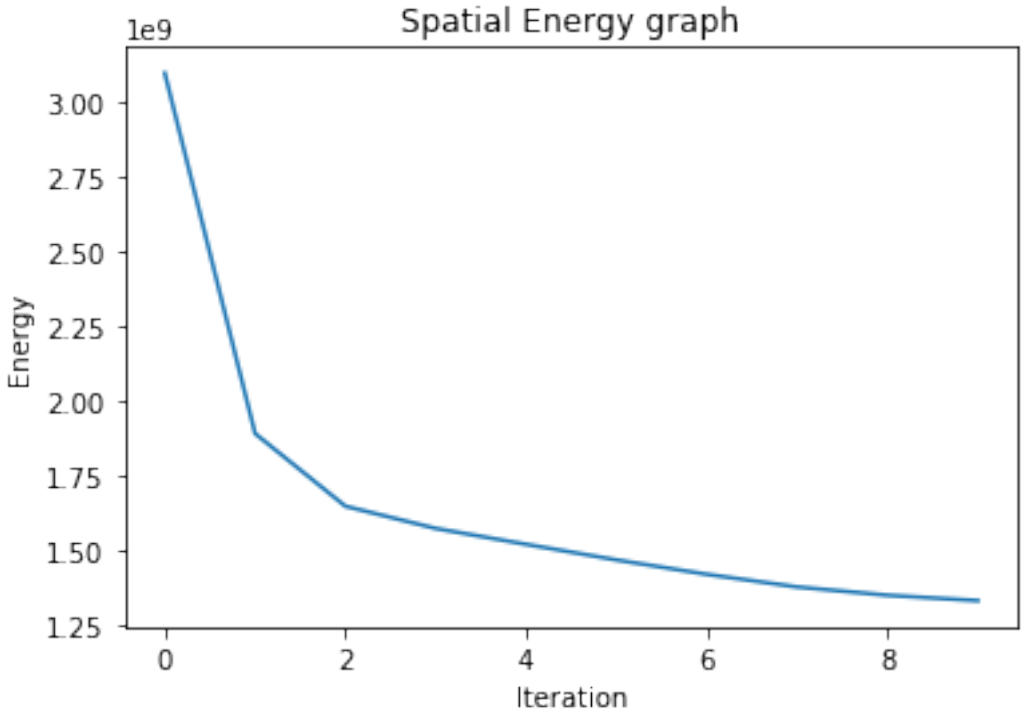

Spatial Energy graph
1e9
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
Energy
0
2
4
6
8
Iteration

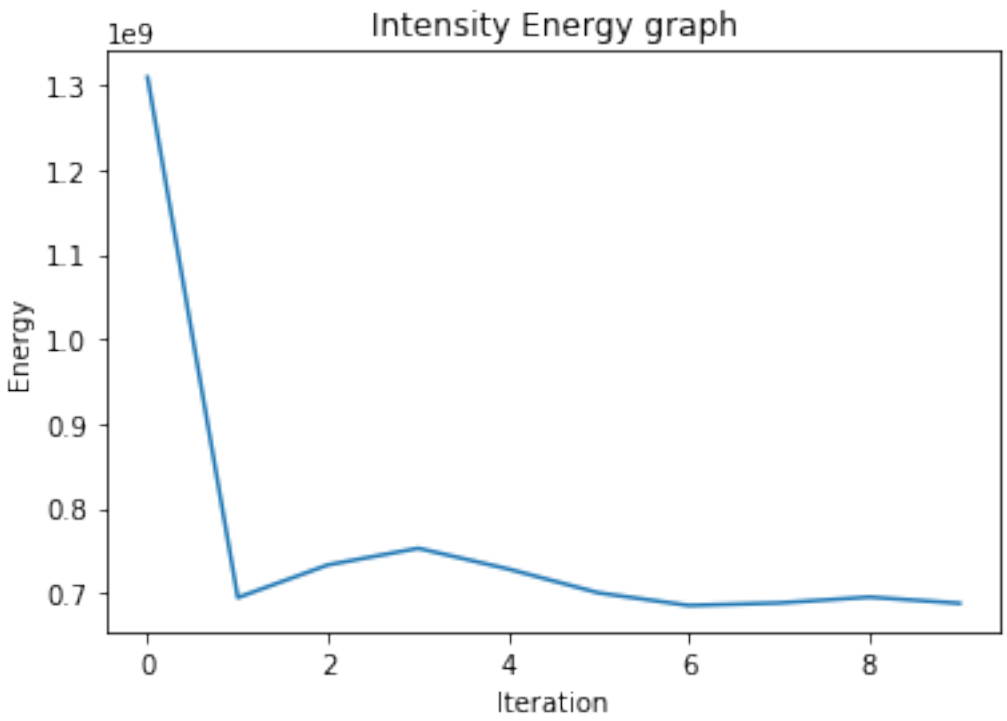

Intensity Energy graph
1e8
7.0
6.5
6.0
5.5
5.0
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```




Total Energy graph
1e9
4.5
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration


Spatial Energy graph
1e9
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
Energy
0
2
4
6
8
Iteration

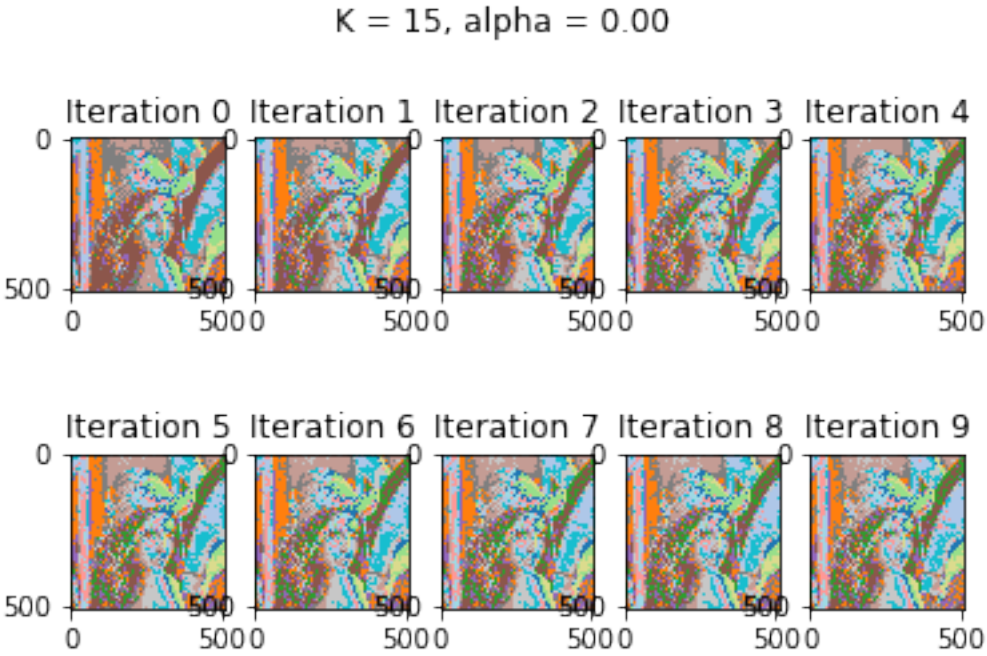


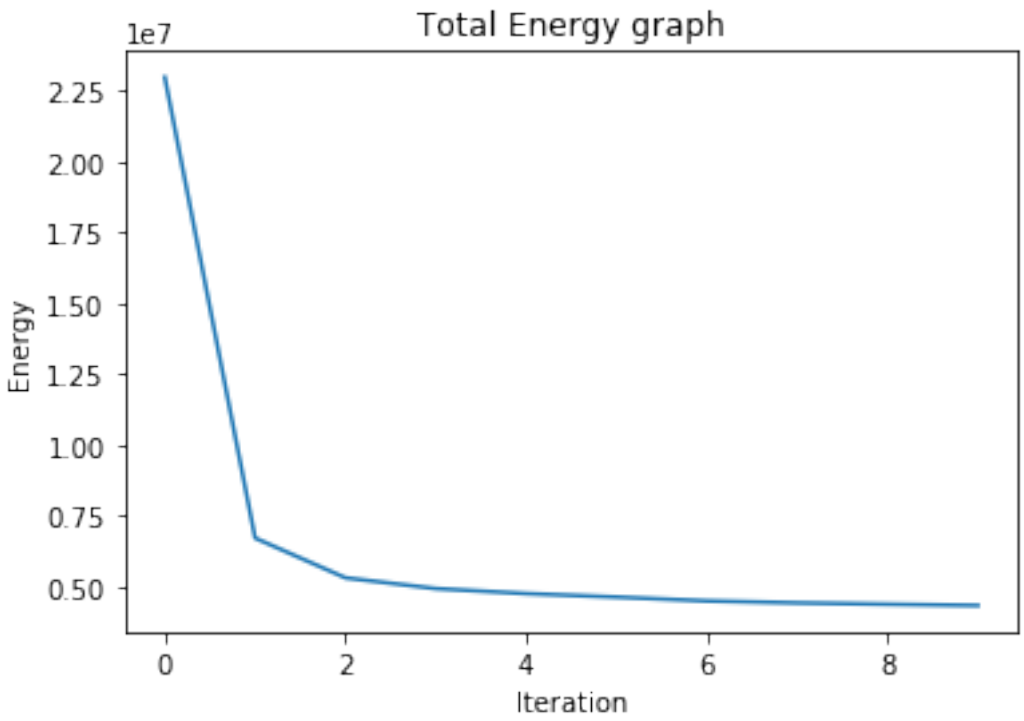
5 K = 15

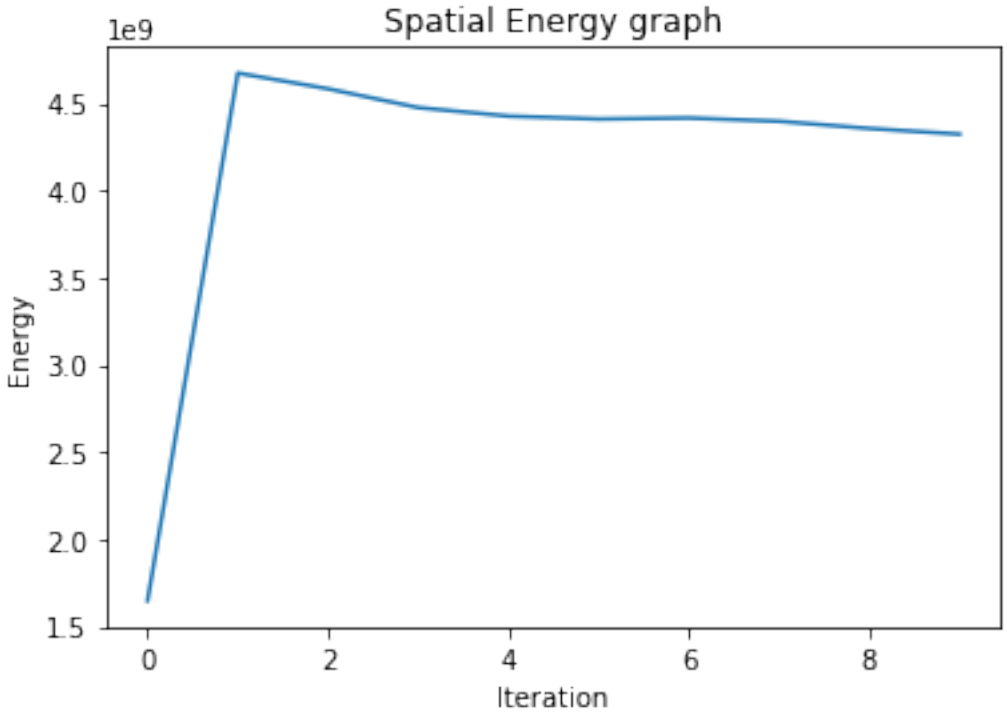
```
In [246]: K = 15
          KMeans(img_matrix, img_rows, img_cols, K, 0)
          KMeans(img_matrix, img_rows, img_cols, K, 0.01)
          KMeans(img_matrix, img_rows, img_cols, K, 0.03)
          KMeans(img_matrix, img_rows, img_cols, K, 0.05)
          KMeans(img_matrix, img_rows, img_cols, K, 0.07)
          KMeans(img_matrix, img_rows, img_cols, K, 0.08)
          KMeans(img_matrix, img_rows, img_cols, K, 0.10)
          KMeans(img_matrix, img_rows, img_cols, K, 0.25)
          KMeans(img_matrix, img_rows, img_cols, K, 0.50)
          KMeans(img_matrix, img_rows, img_cols, K, 0.75)
          KMeans(img_matrix, img_rows, img_cols, K, 1)
```

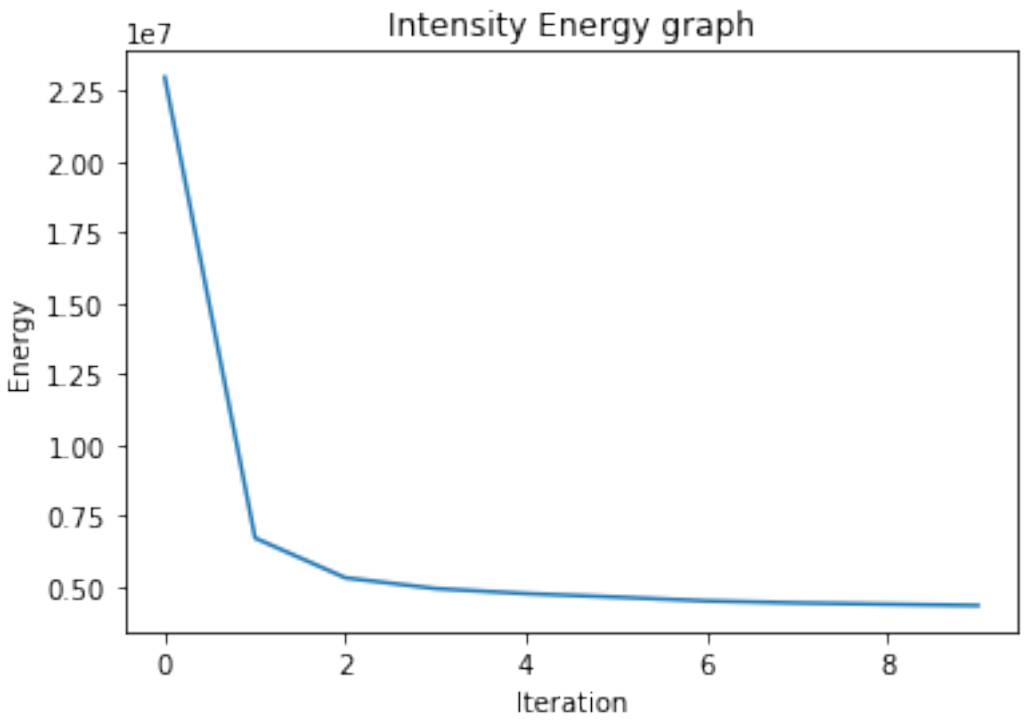
```
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:66: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:67: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:68: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:32: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:33: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:35: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:36: RuntimeWarning: overi
```

```
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:39: RuntimeWarning: over

0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

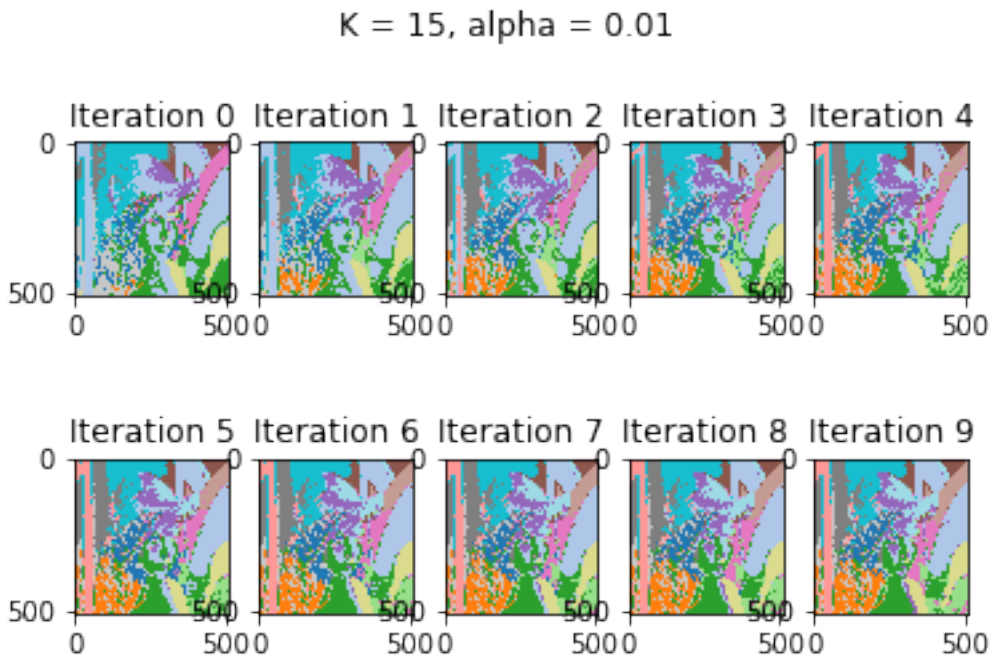


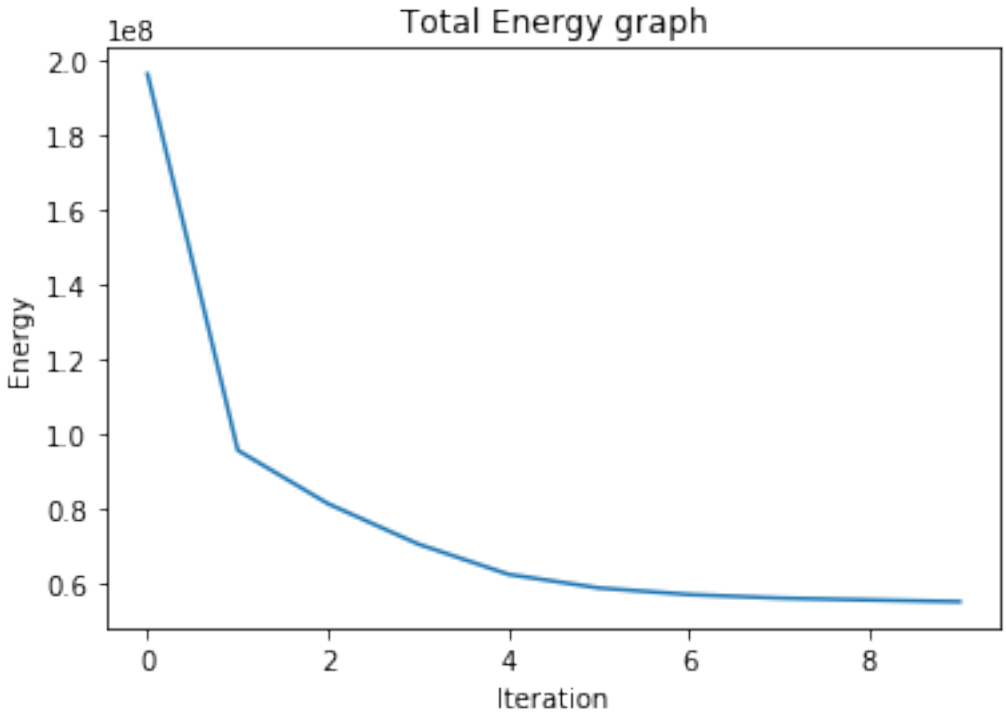

Total Energy graph
1e7
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
Energy
0
2
4
6
8
Iteration

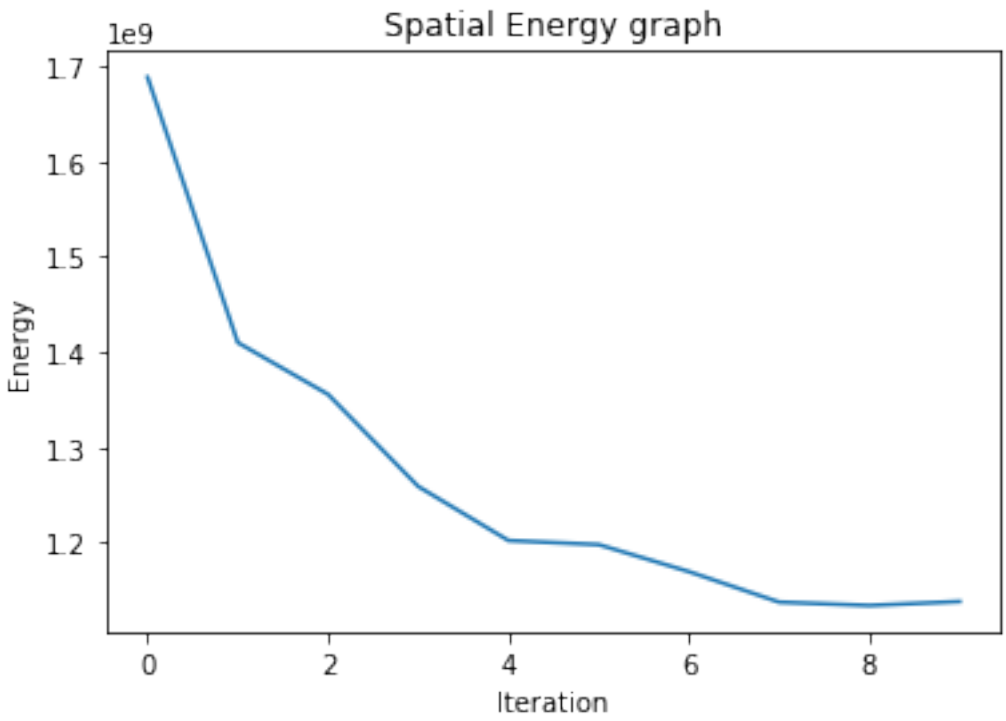

Spatial Energy graph
1e9
4.5
4.0
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

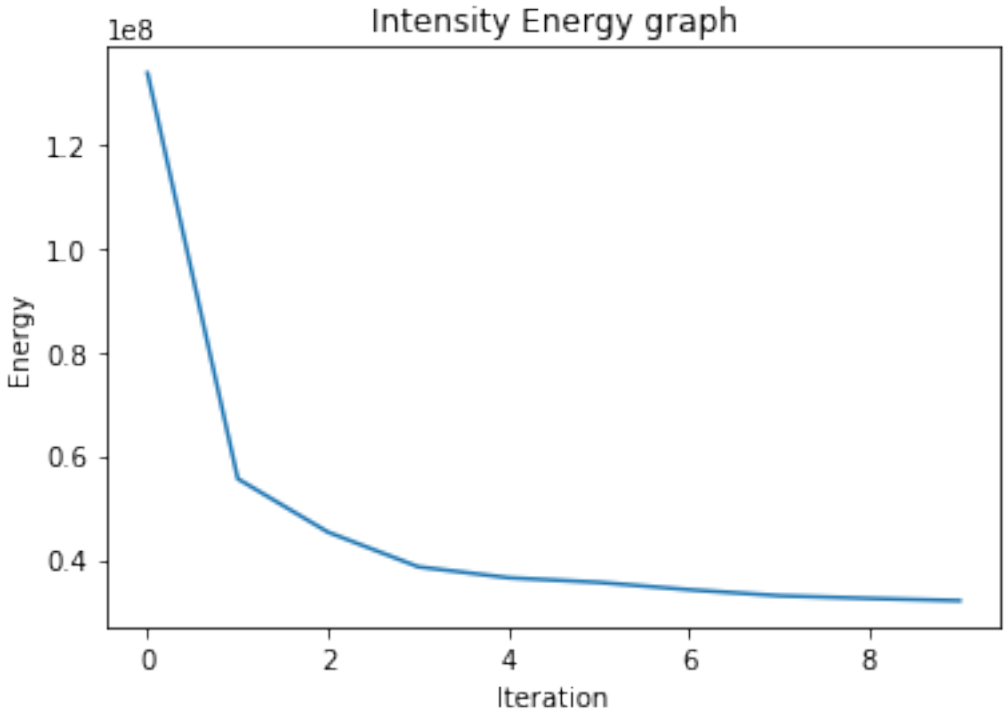



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

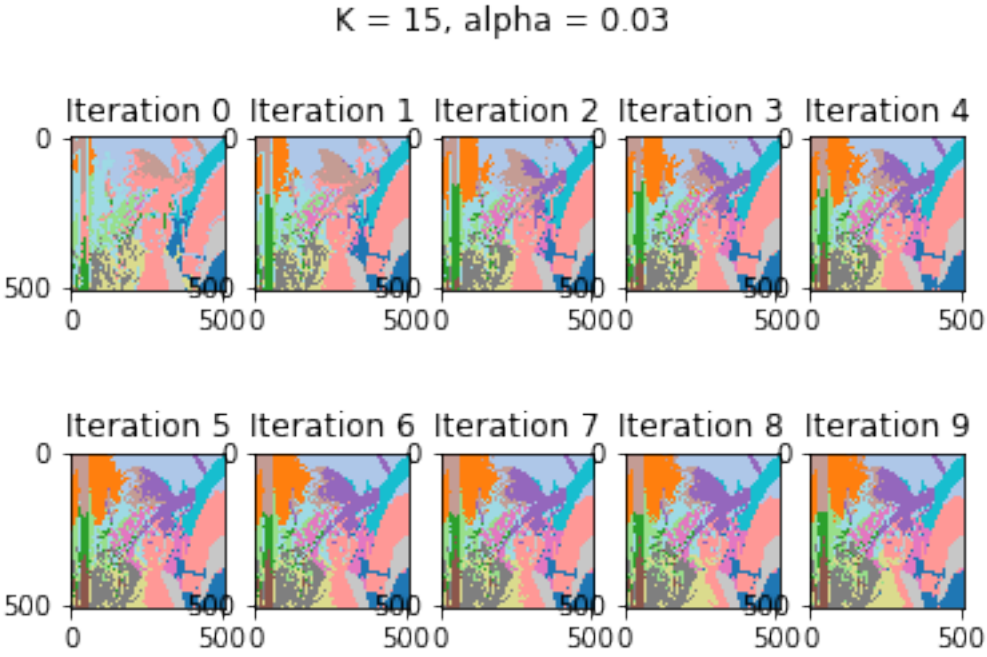

K = 15, alpha = 0.01
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

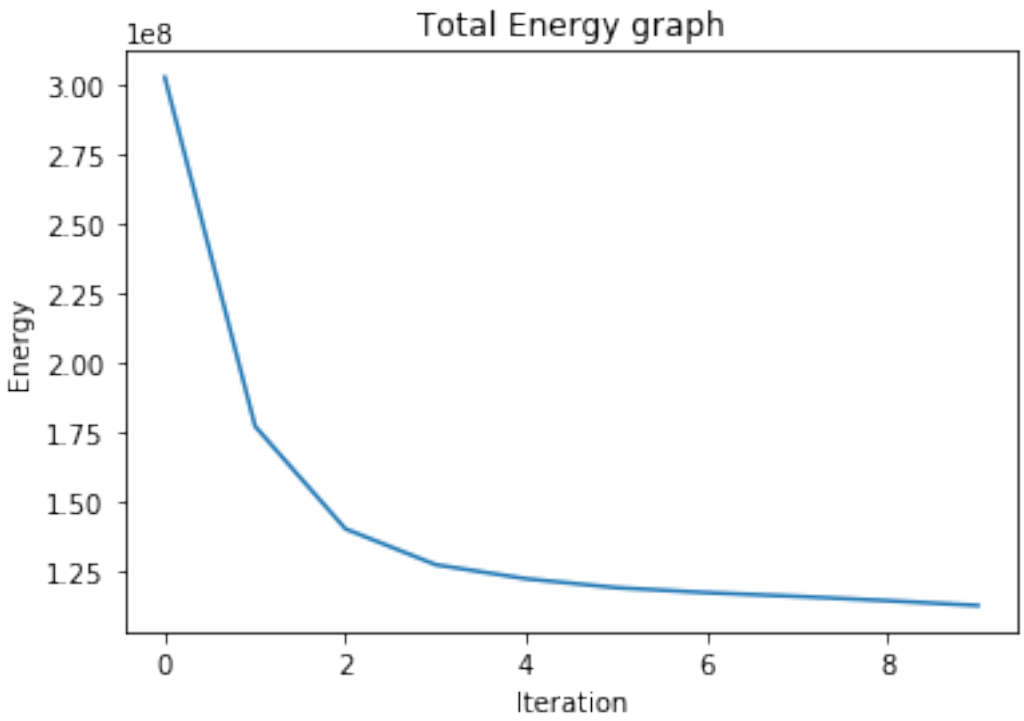

Total Energy graph
1e8
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

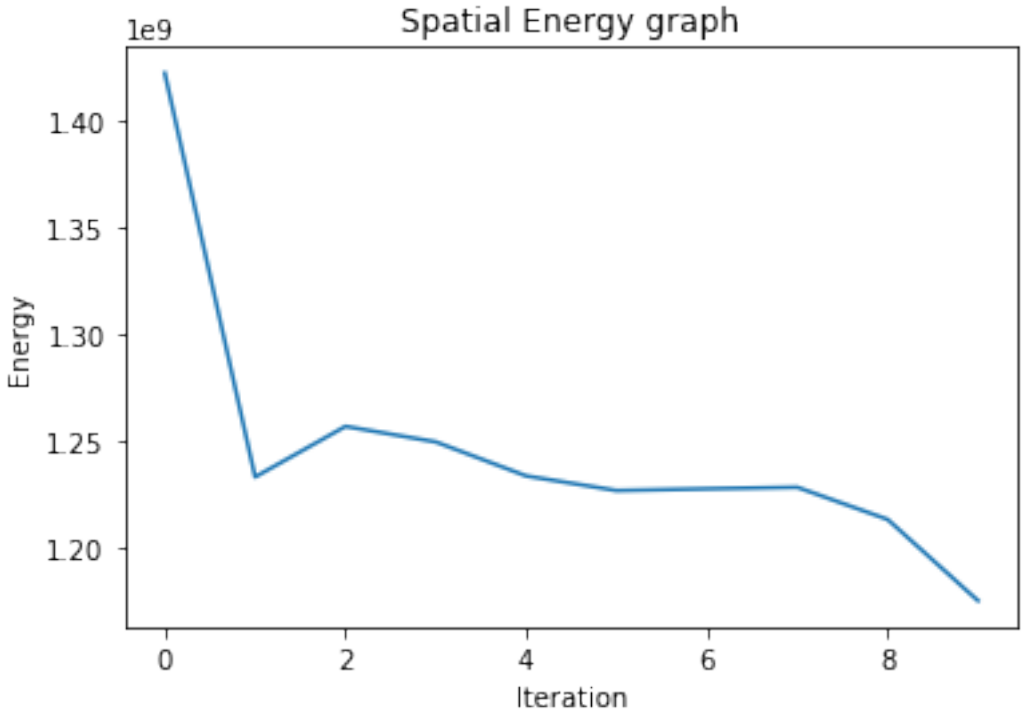

Spatial Energy graph
1e9
1.7
1.6
1.5
1.4
1.3
1.2
Energy
0
2
4
6
8
Iteration

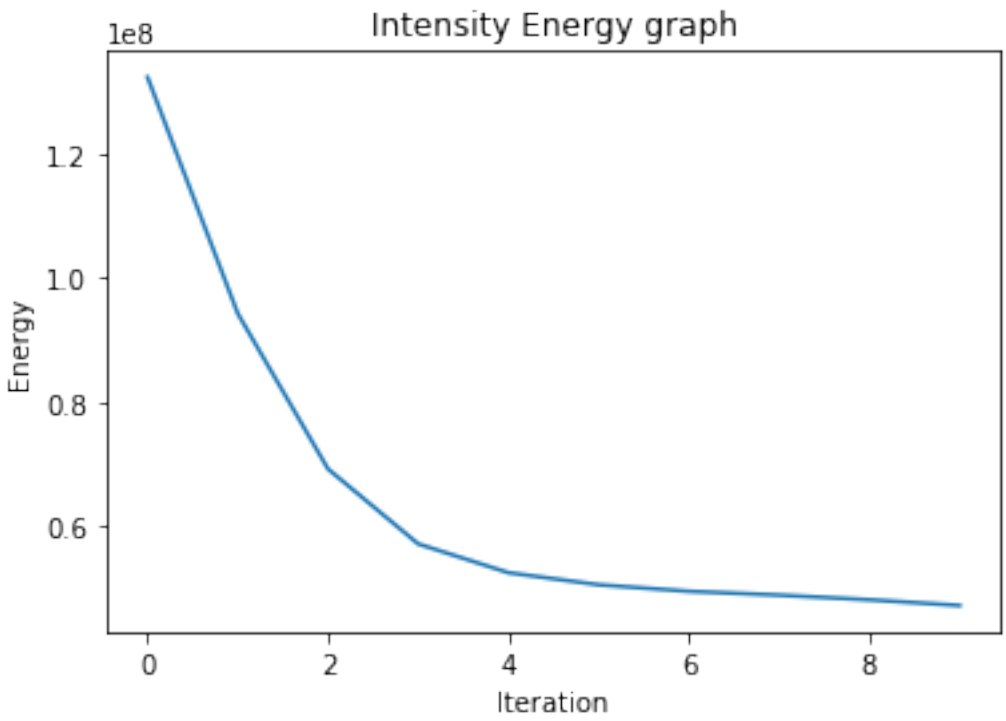

Intensity Energy graph
1e8
1.2
1.0
0.8
0.6
0.4
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

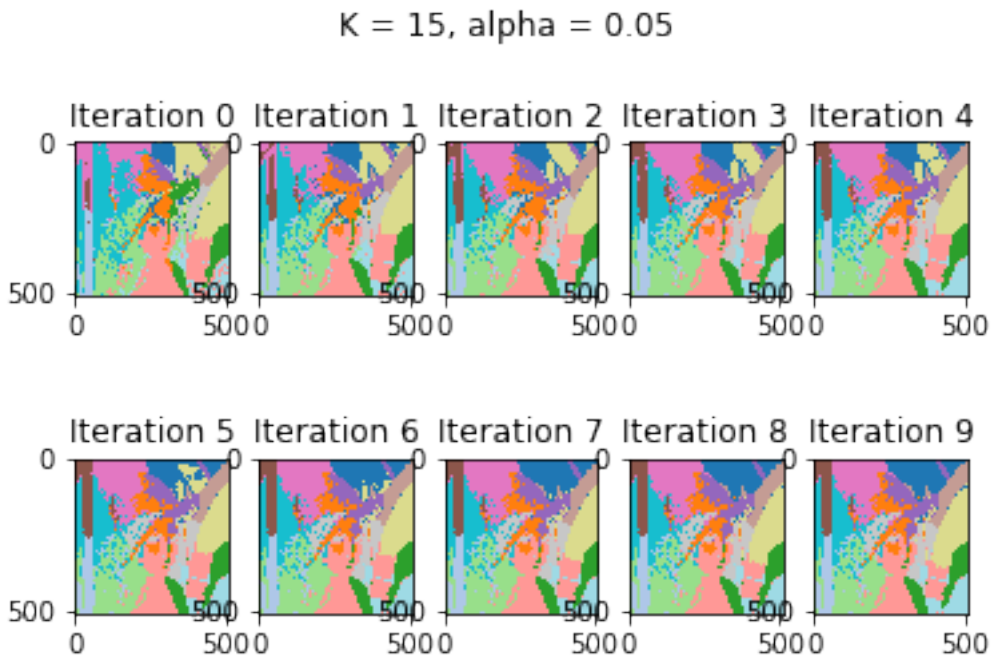


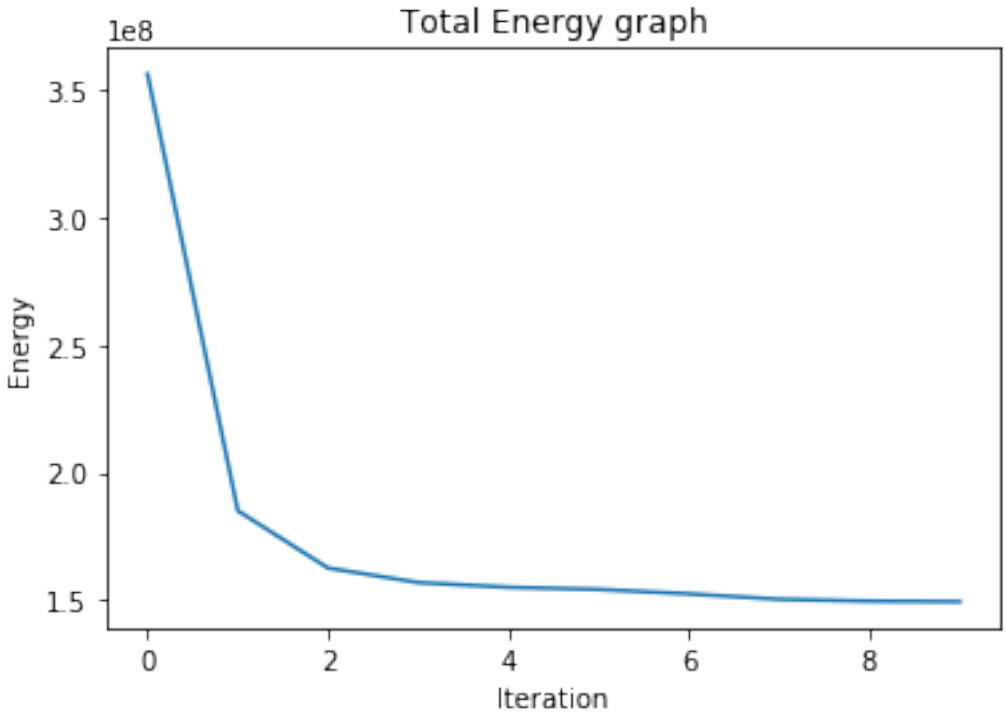

Total Energy graph
1e8
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
Energy
0
2
4
6
8
Iteration

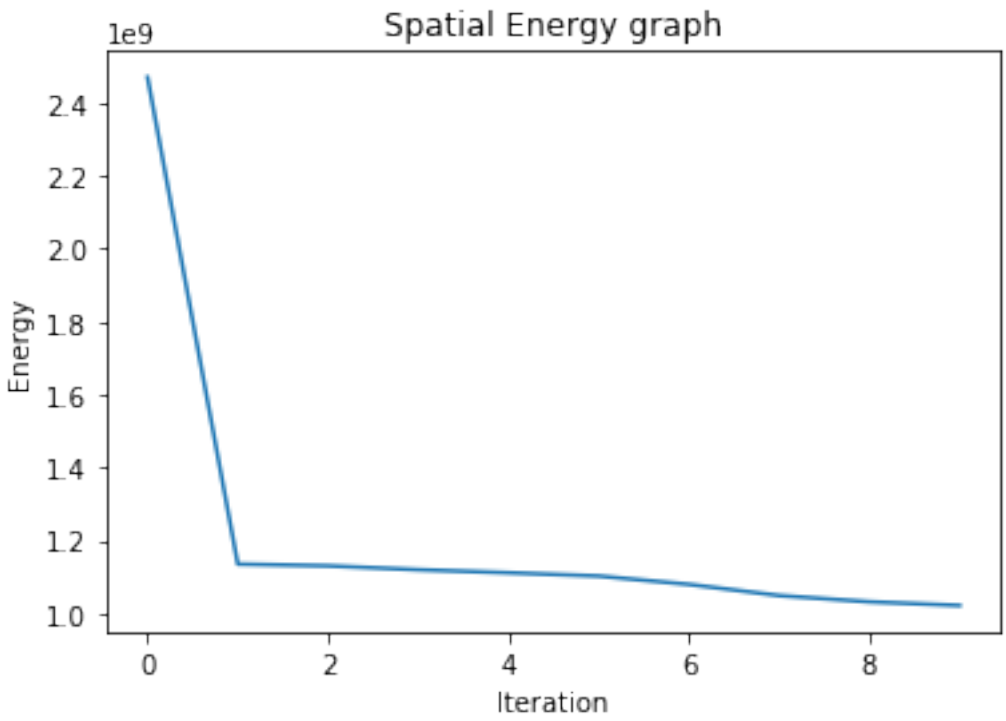

Spatial Energy graph
1e9
1.40
1.35
1.30
1.25
1.20
Energy
0
2
4
6
8
Iteration

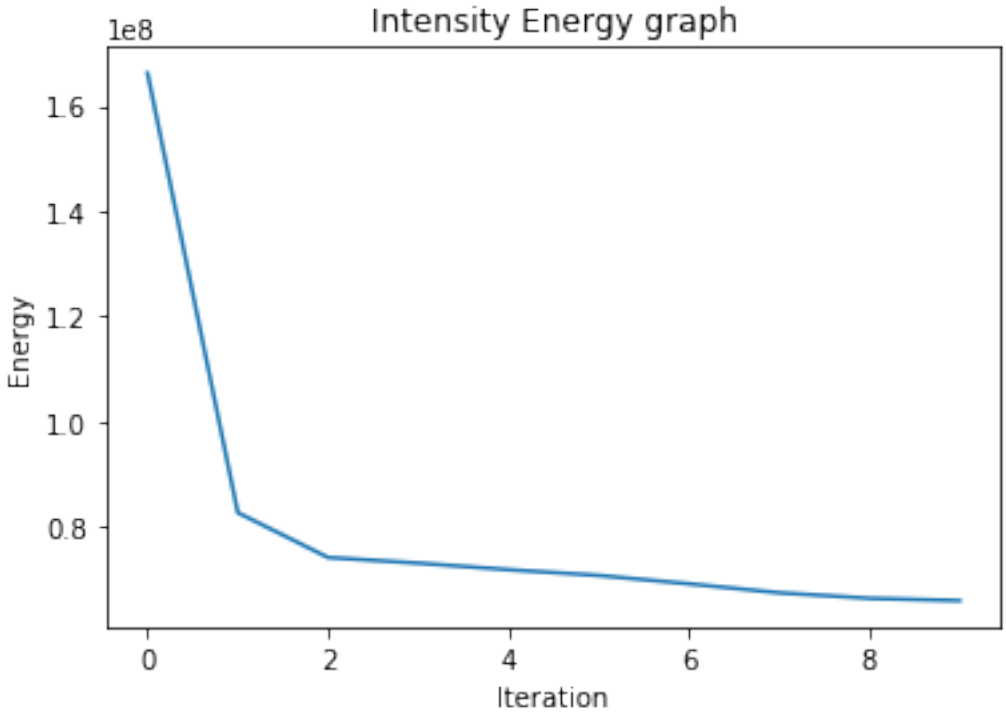



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

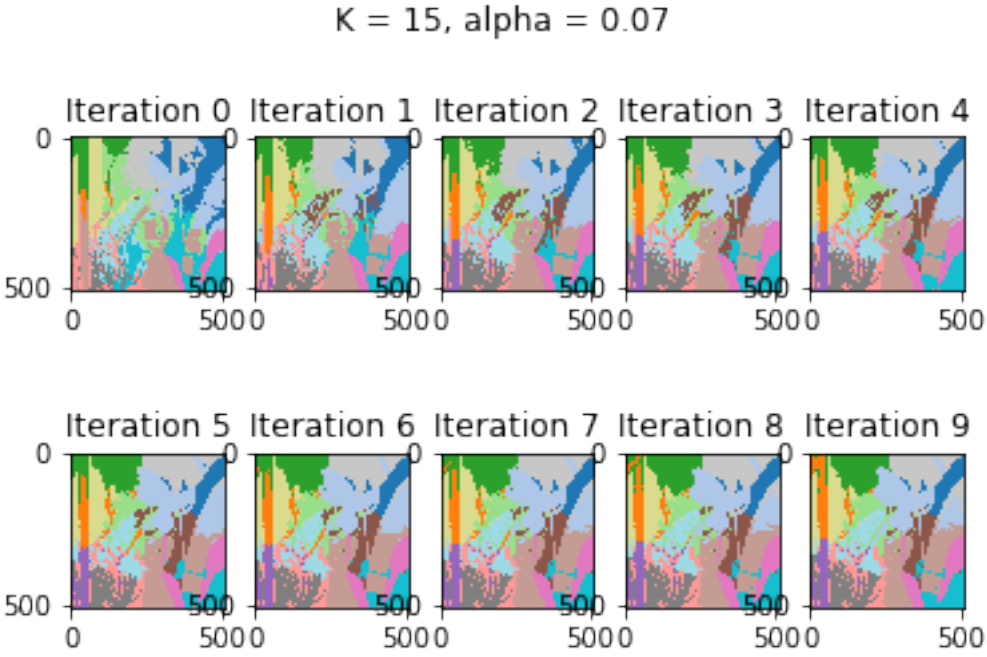

K = 15, alpha = 0.05
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

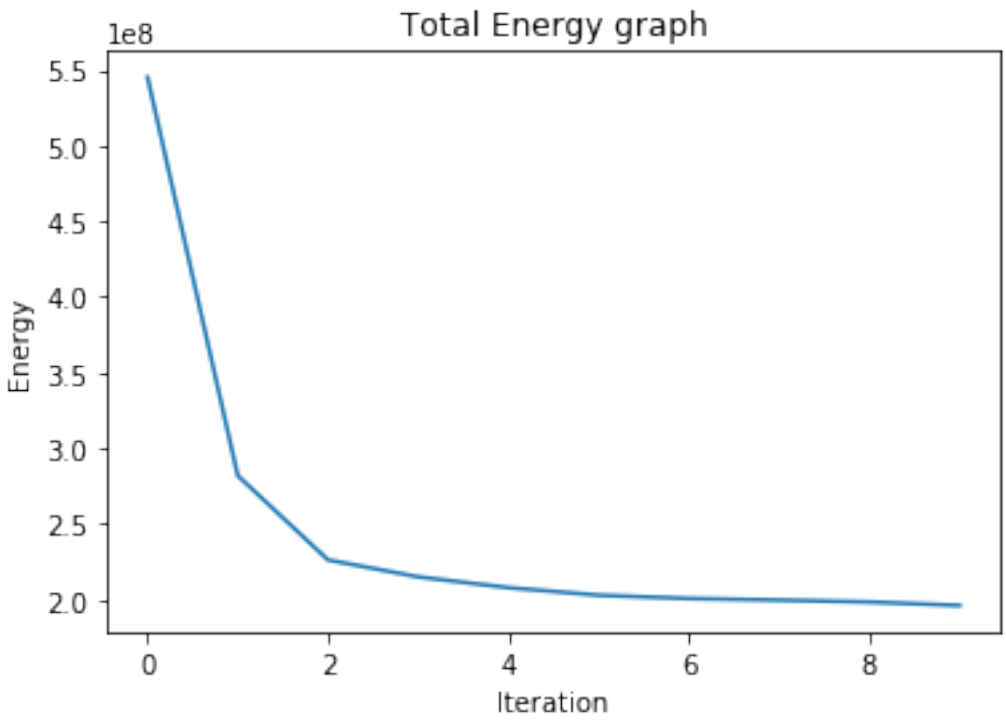

Total Energy graph
1e8
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

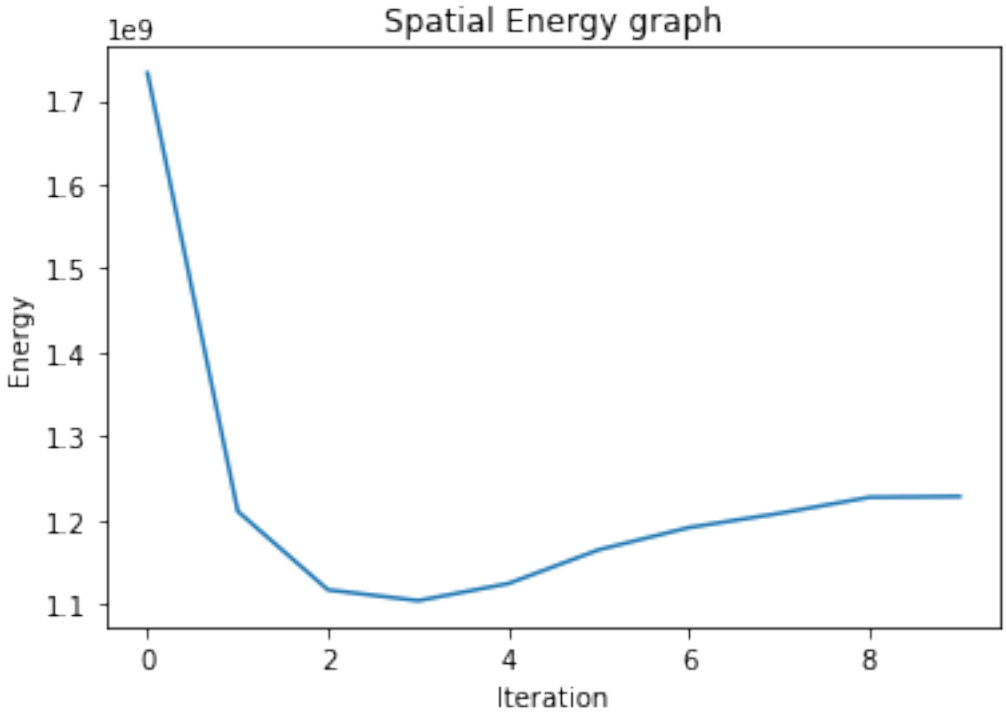

Spatial Energy graph
1e9
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
Energy
0
2
4
6
8
Iteration

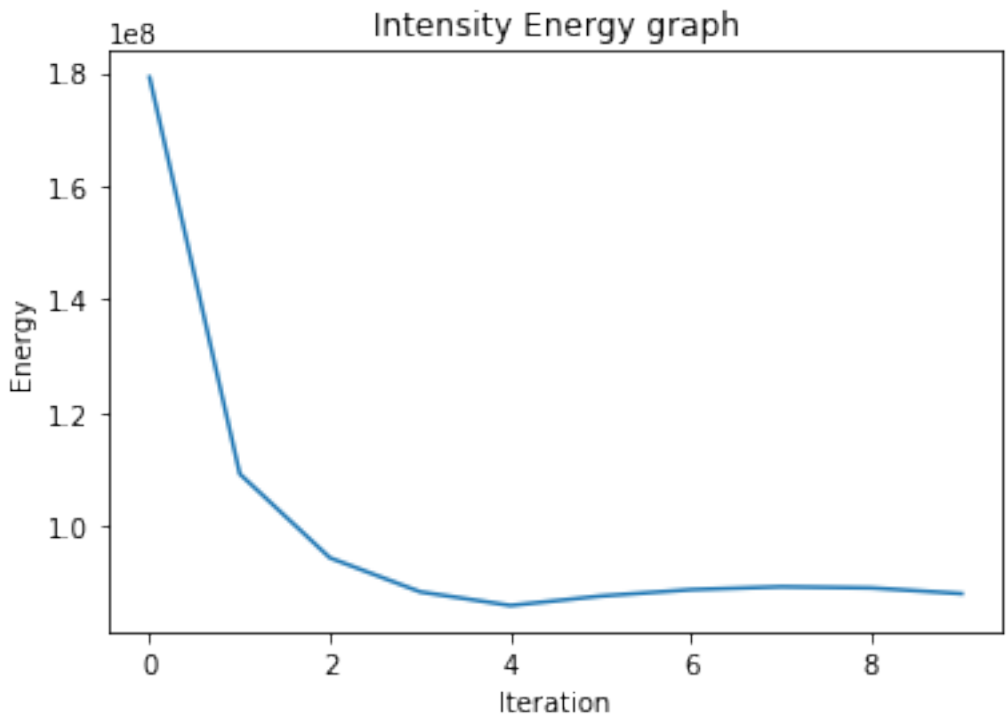

Intensity Energy graph
1e8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

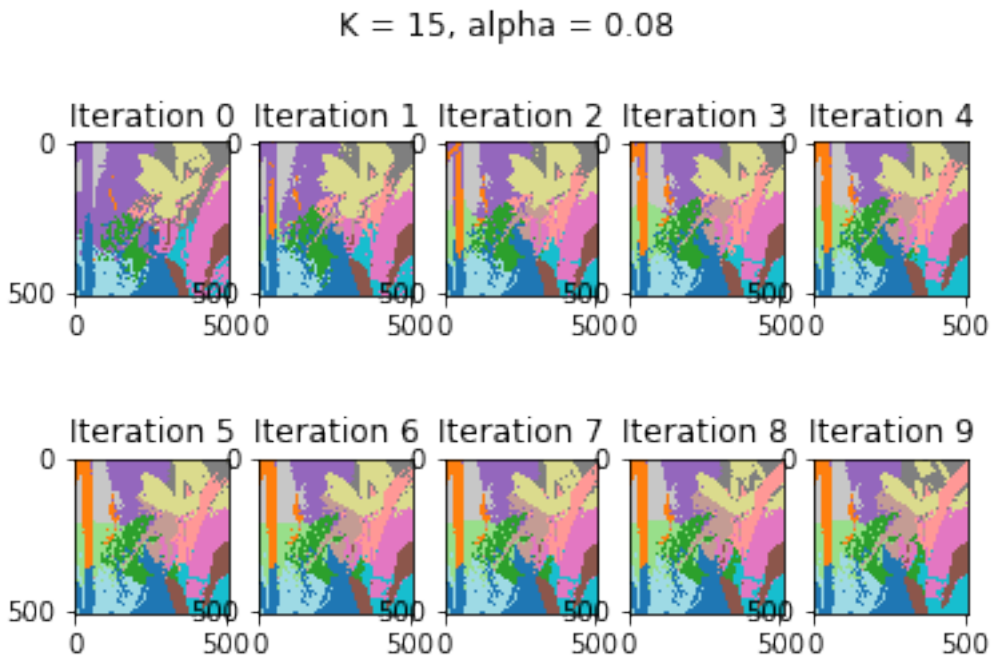


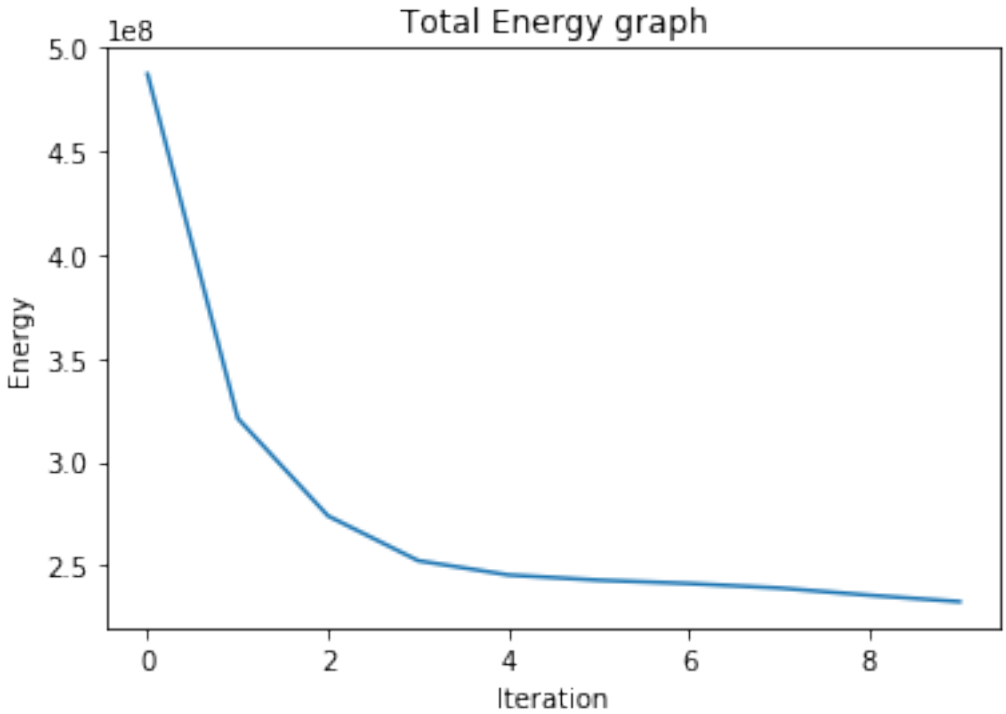

Total Energy graph
1e8
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

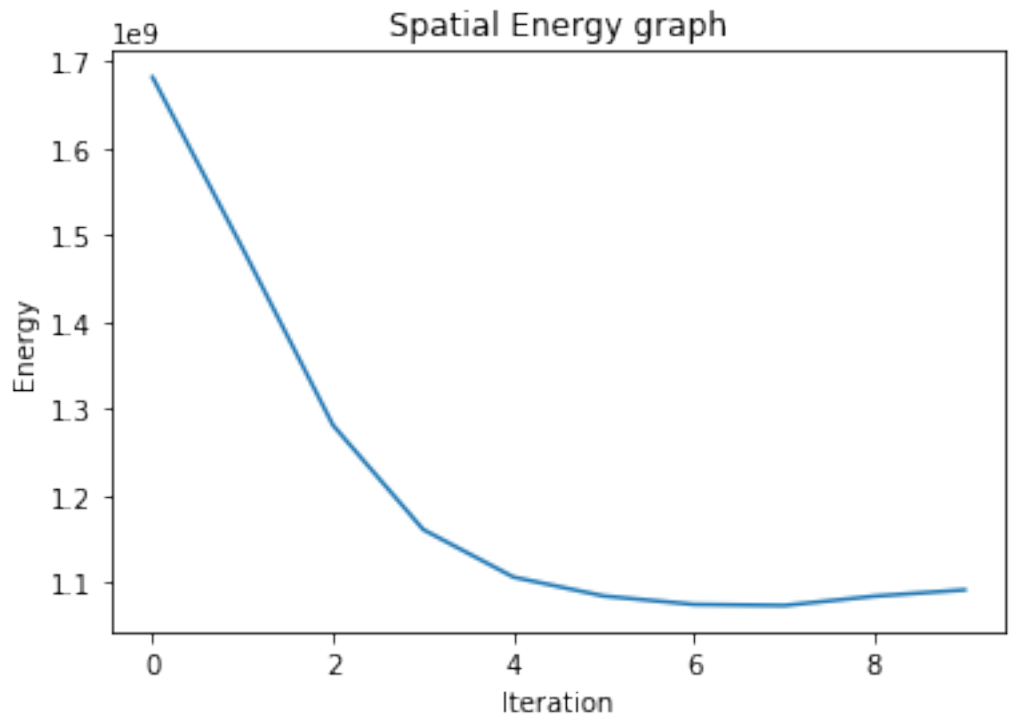

Spatial Energy graph
1e9
1.7
1.6
1.5
1.4
1.3
1.2
1.1
Energy
0
2
4
6
8
Iteration

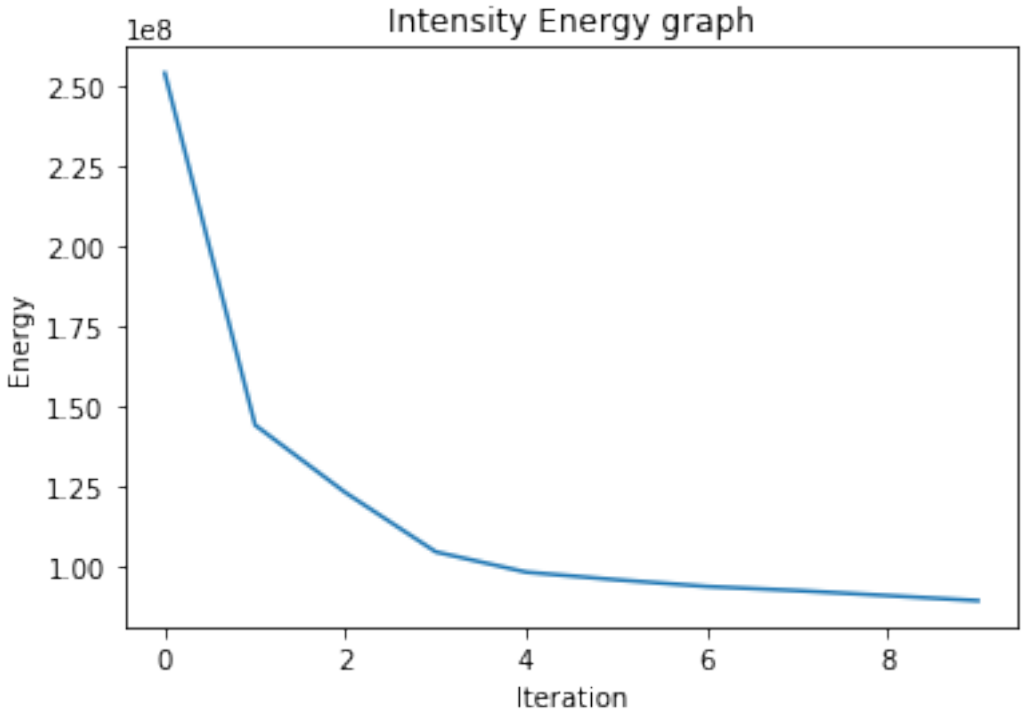



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

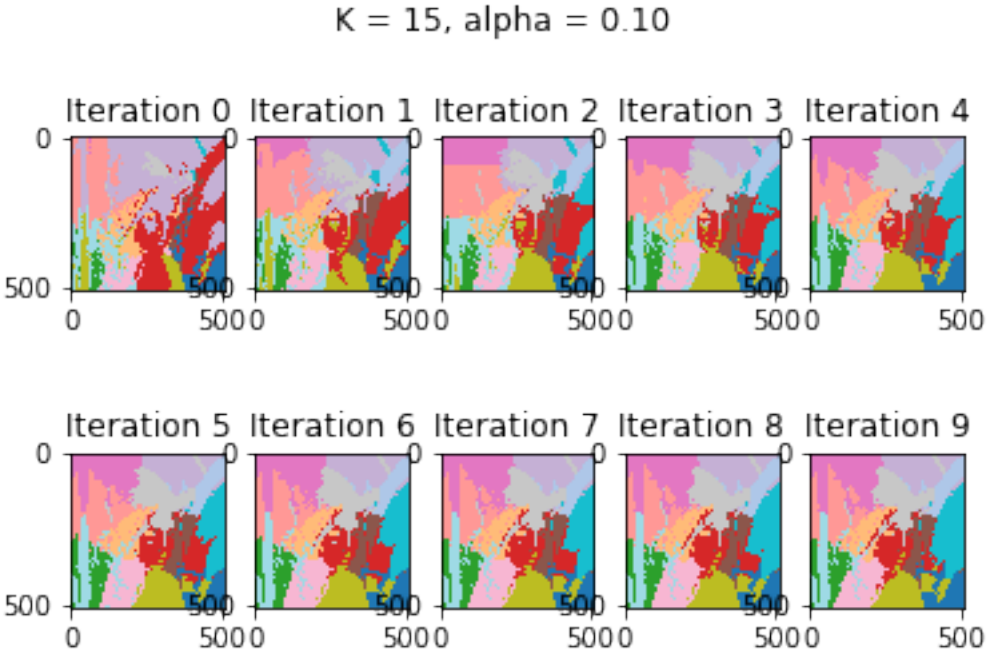

K = 15, alpha = 0.08
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

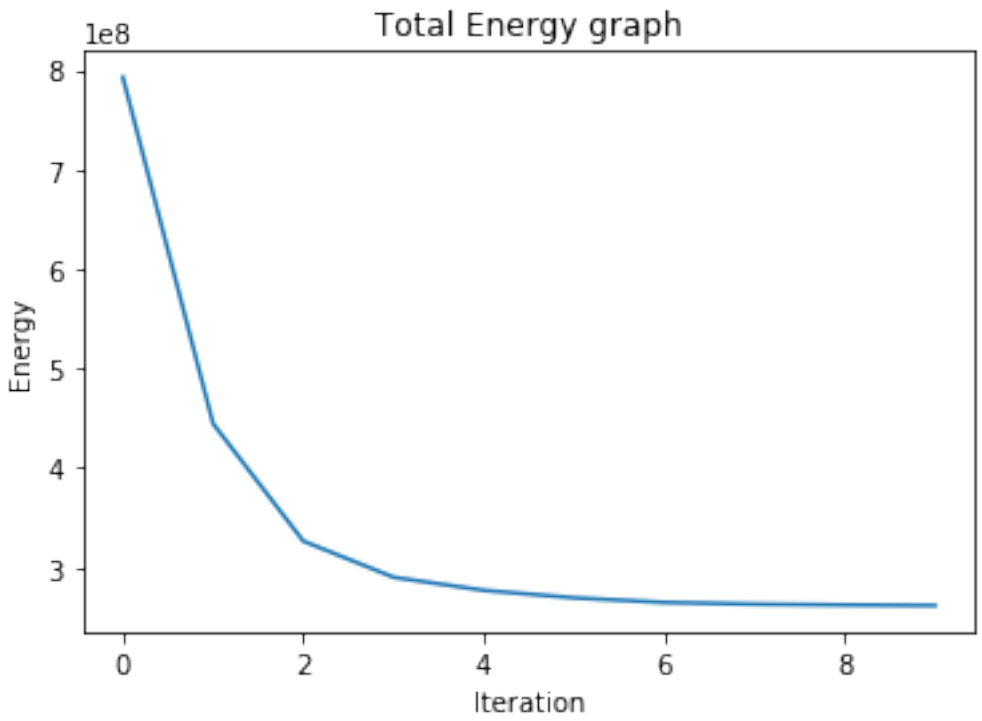

Total Energy graph
1e8
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

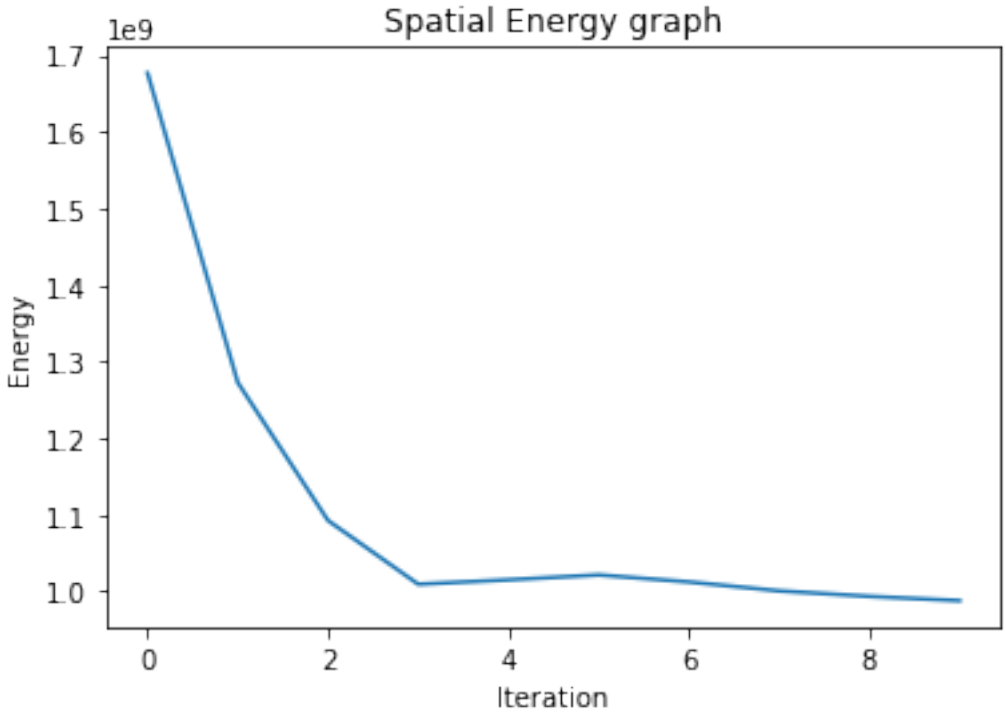

Spatial Energy graph
1e9
1.7
1.6
1.5
1.4
1.3
1.2
1.1
Energy
0
2
4
6
8
Iteration

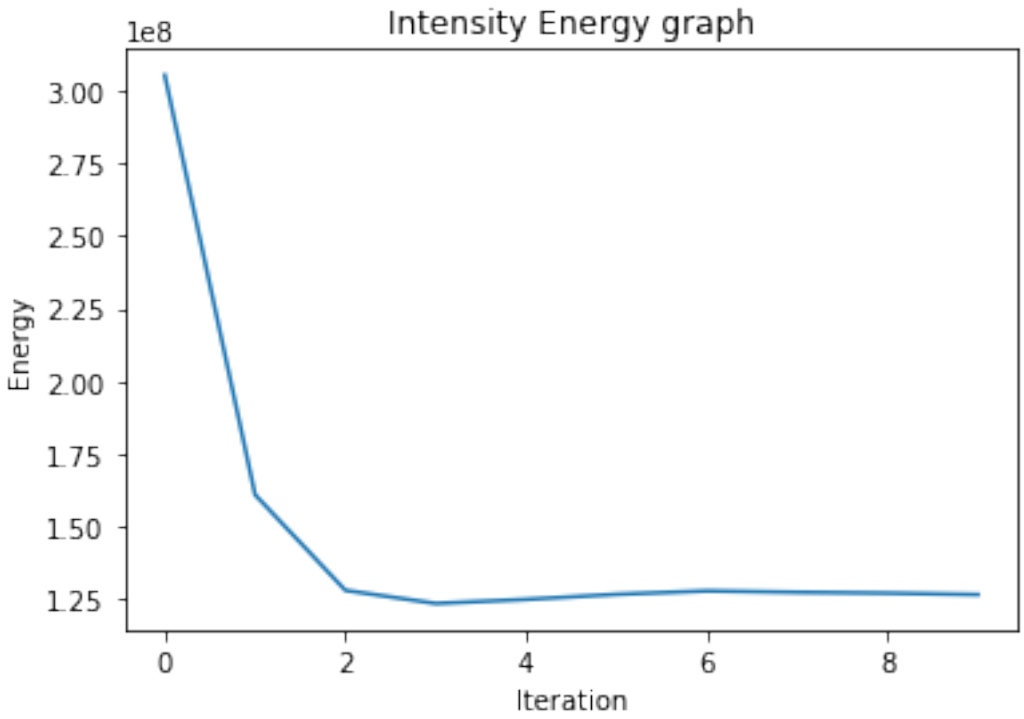

Intensity Energy graph
1e8
2.50
2.25
2.00
1.75
1.50
1.25
1.00
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

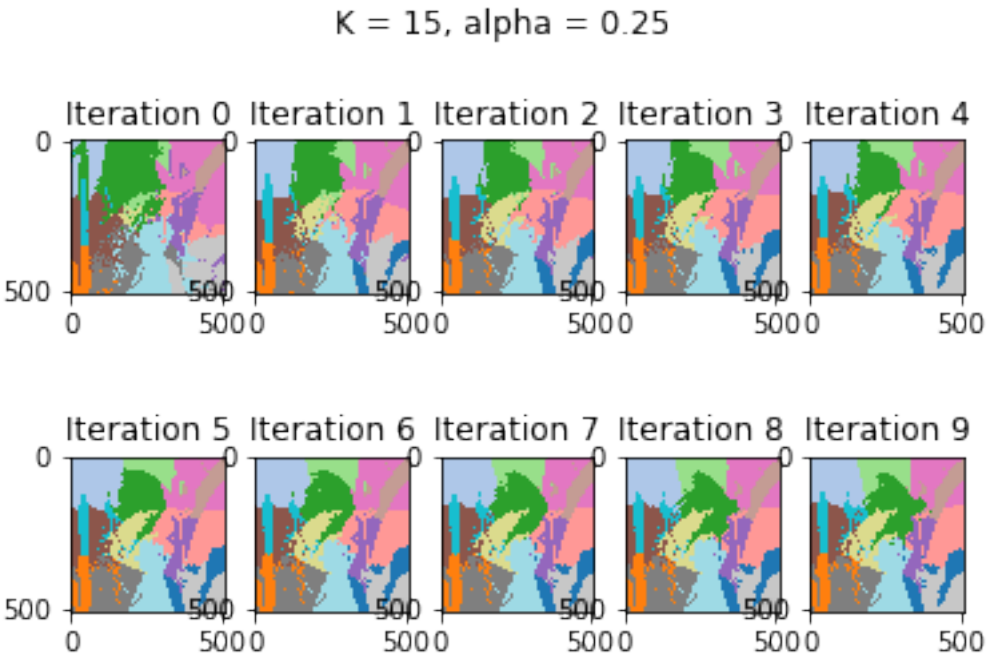


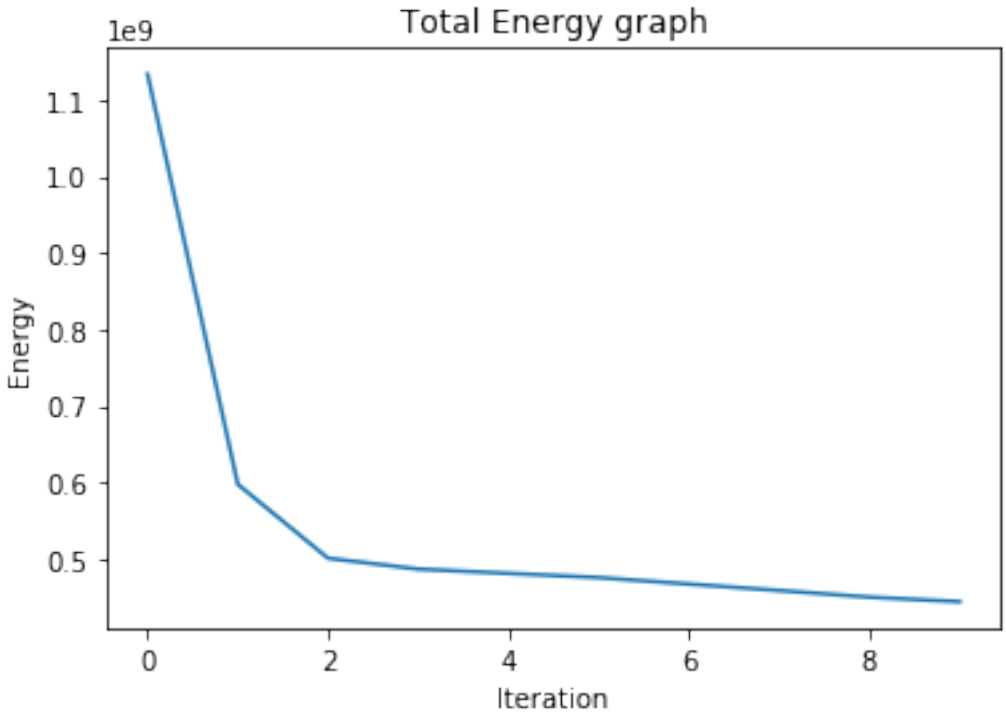

Total Energy graph
1e8
8
7
6
5
4
3
Energy
0
2
4
6
8
Iteration

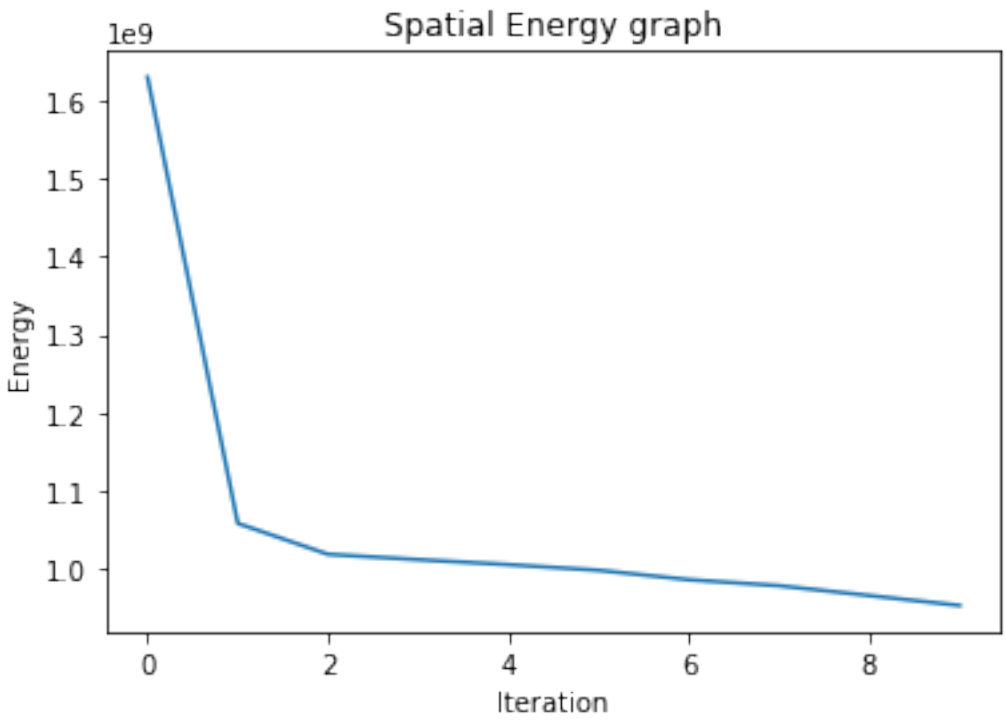

Spatial Energy graph
1e9
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
Energy
0
2
4
6
8
Iteration

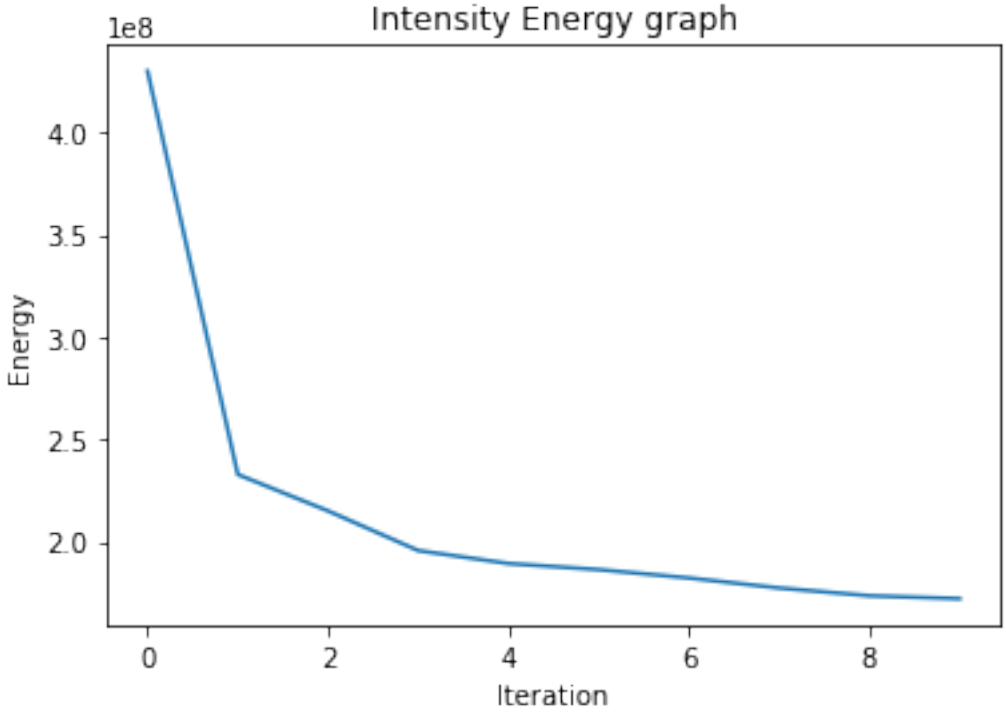



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

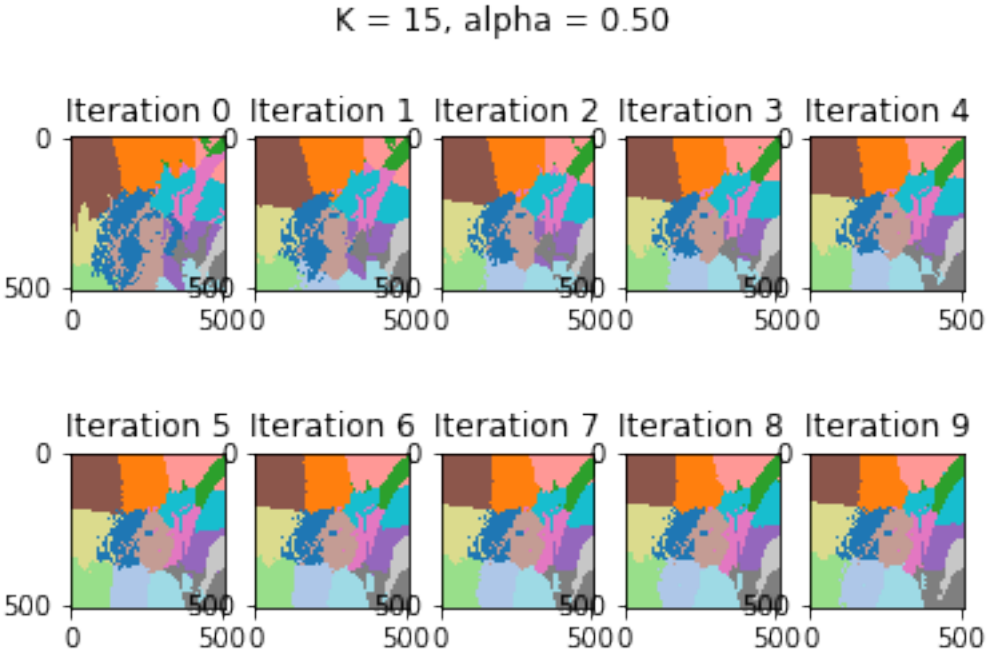

K = 15, alpha = 0.25
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

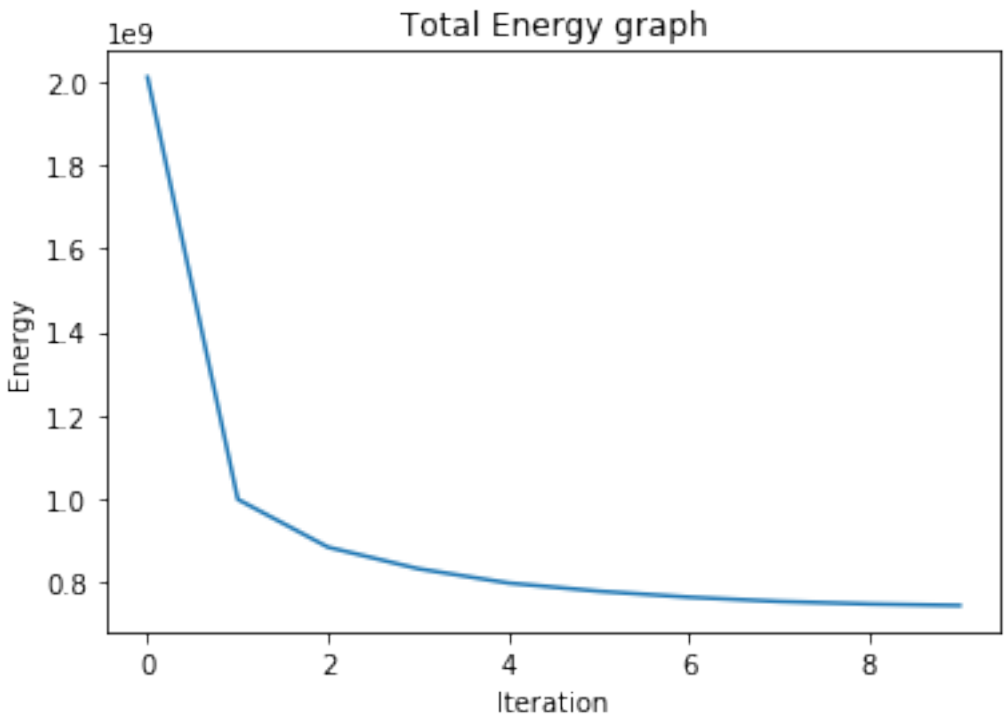

Total Energy graph
1e9
1.1
1.0
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

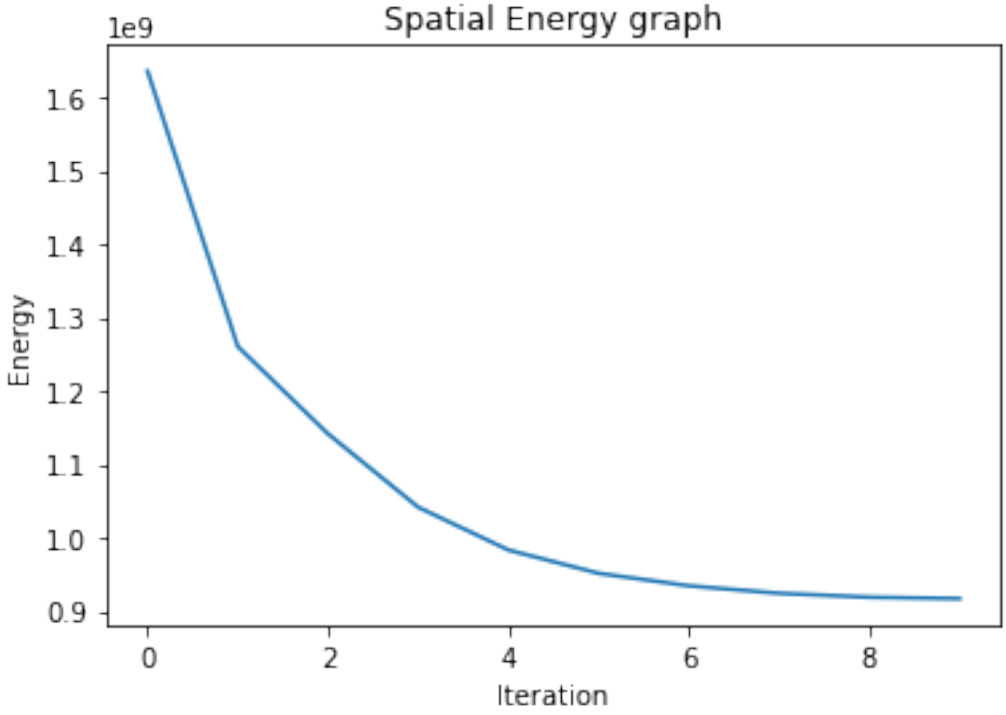

Spatial Energy graph
1e9
1.6
1.5
1.4
1.3
1.2
1.1
1.0
Energy
0
2
4
6
8
Iteration

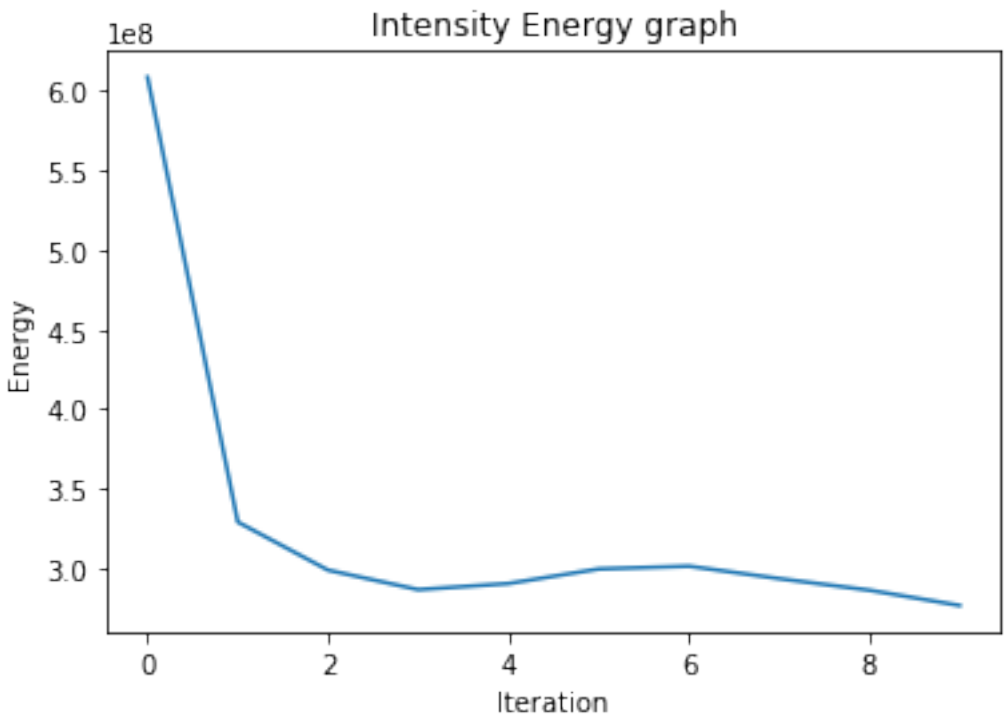

Intensity Energy graph
1e8
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

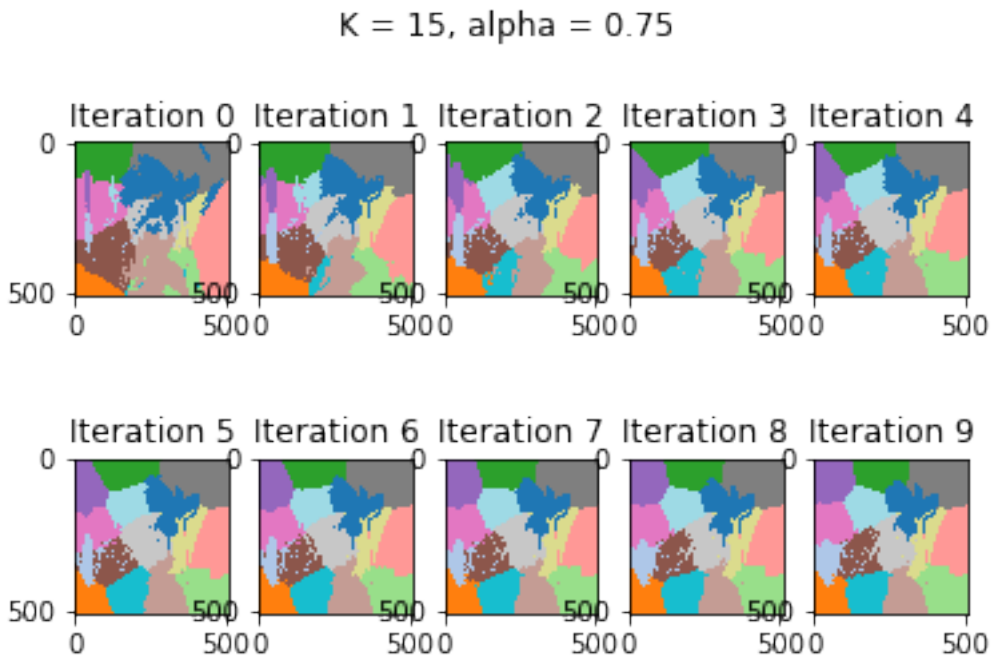


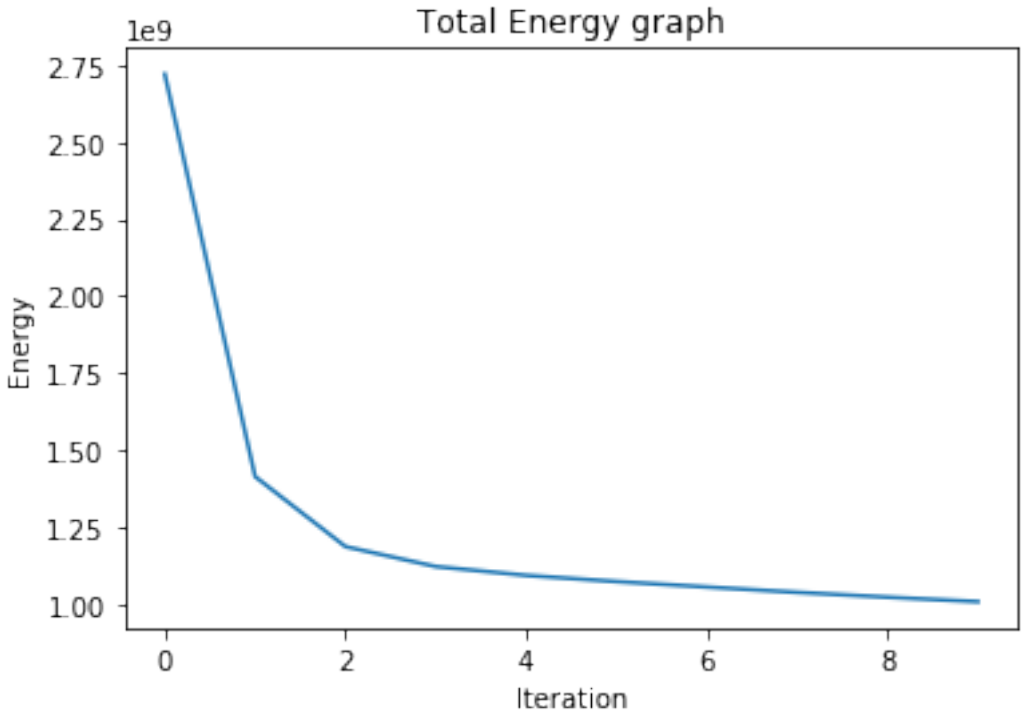

Total Energy graph
1e9
2.0
1.8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

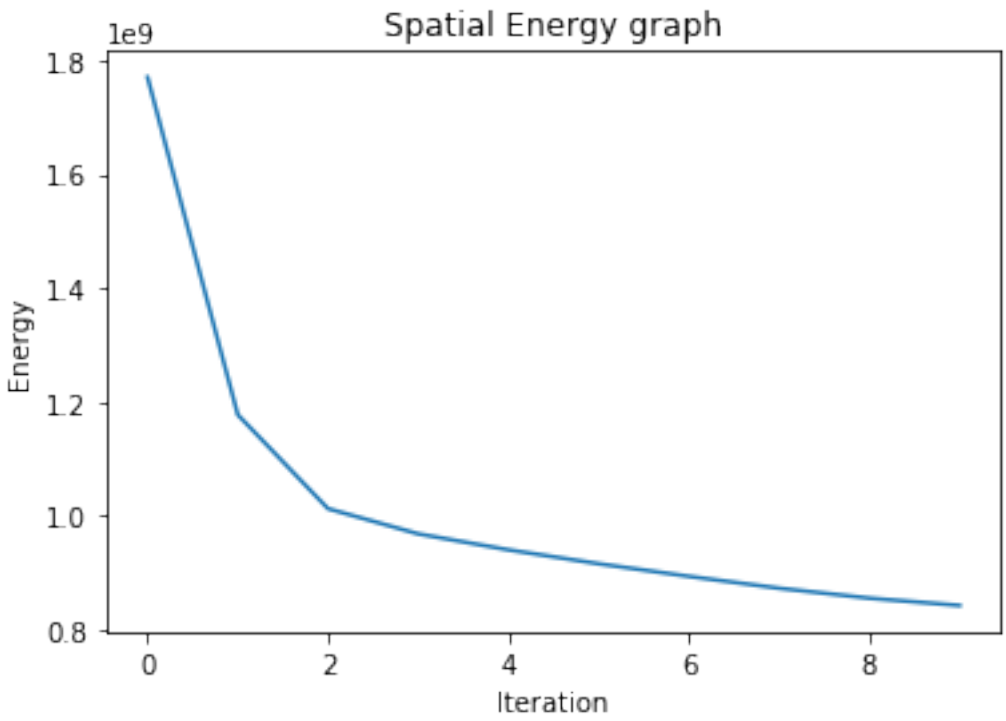

Spatial Energy graph
1e9
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
Energy
0
2
4
6
8
Iteration

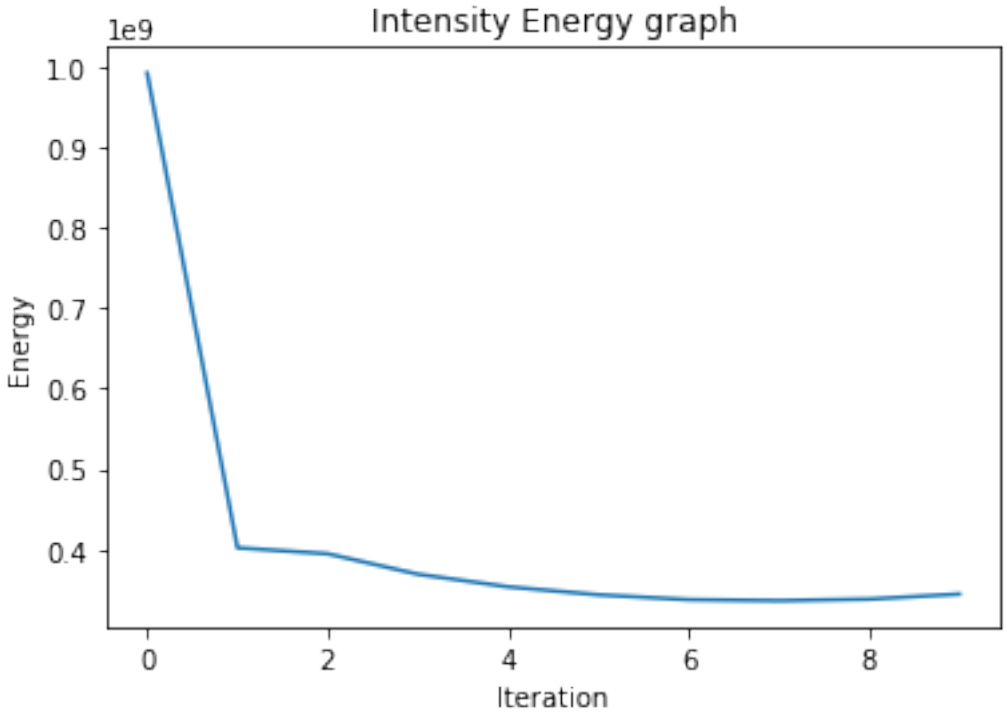



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

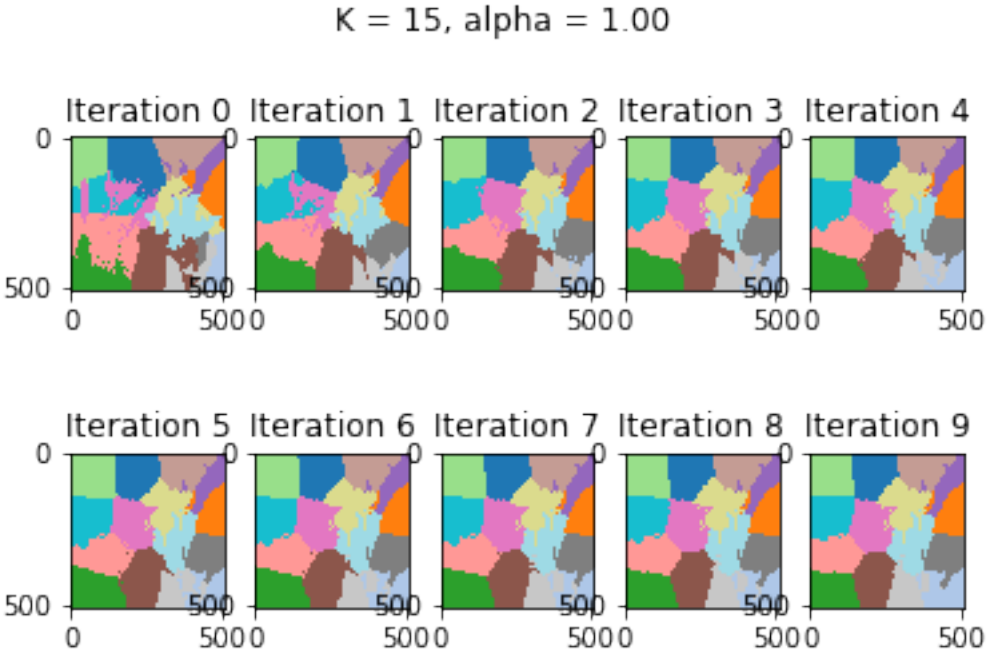

K = 15, alpha = 0.75
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

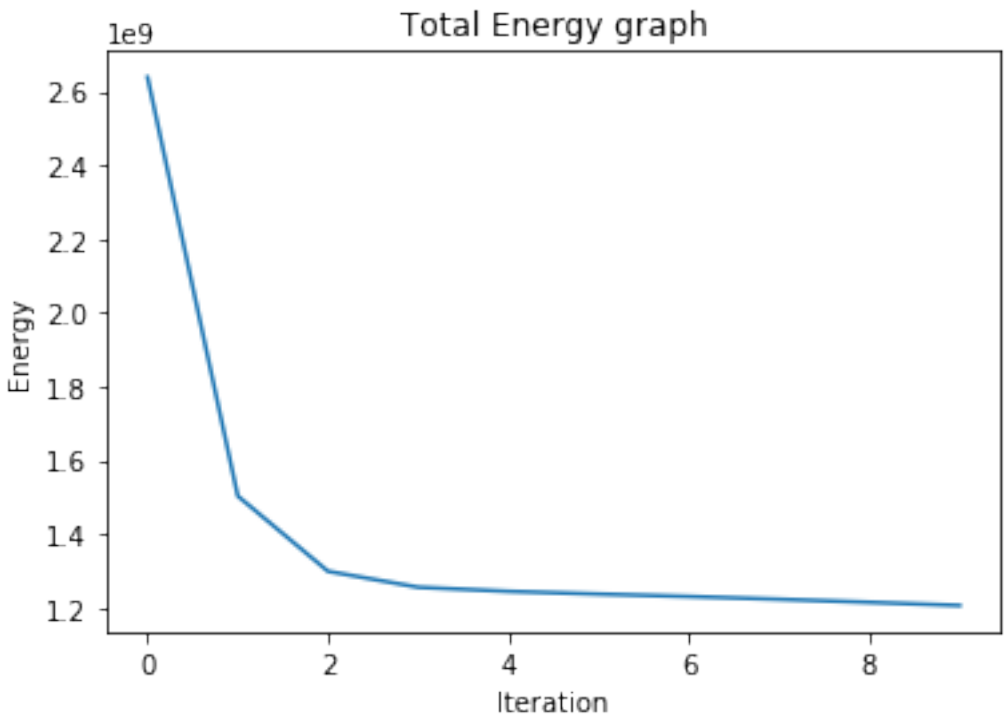

Total Energy graph
1e9
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
Energy
0
2
4
6
8
Iteration

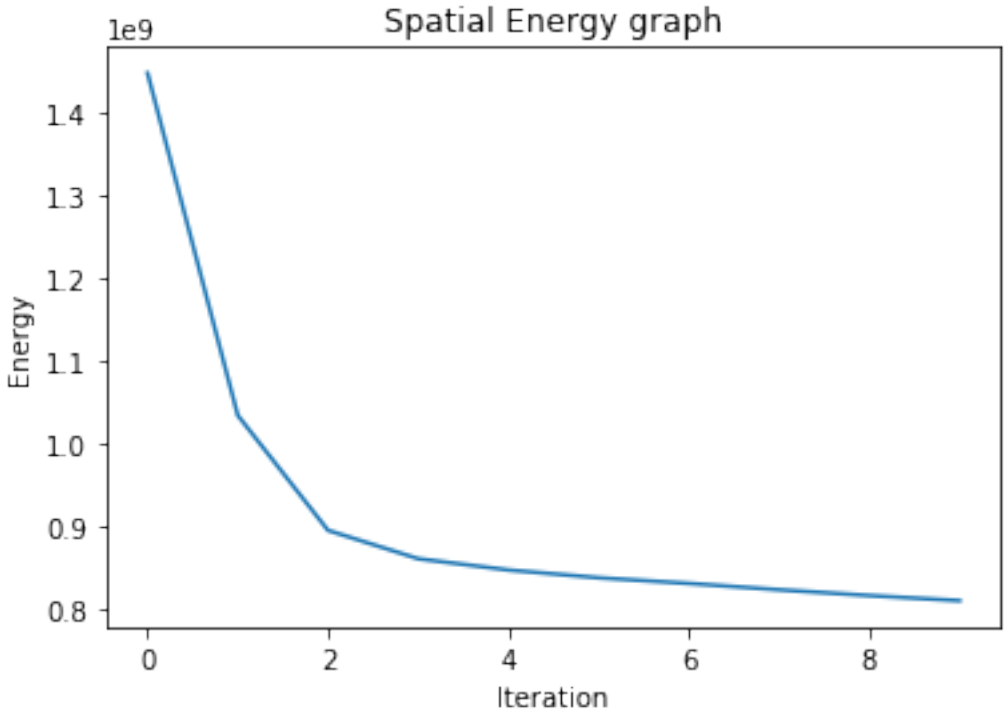

Spatial Energy graph
1e9
1.8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

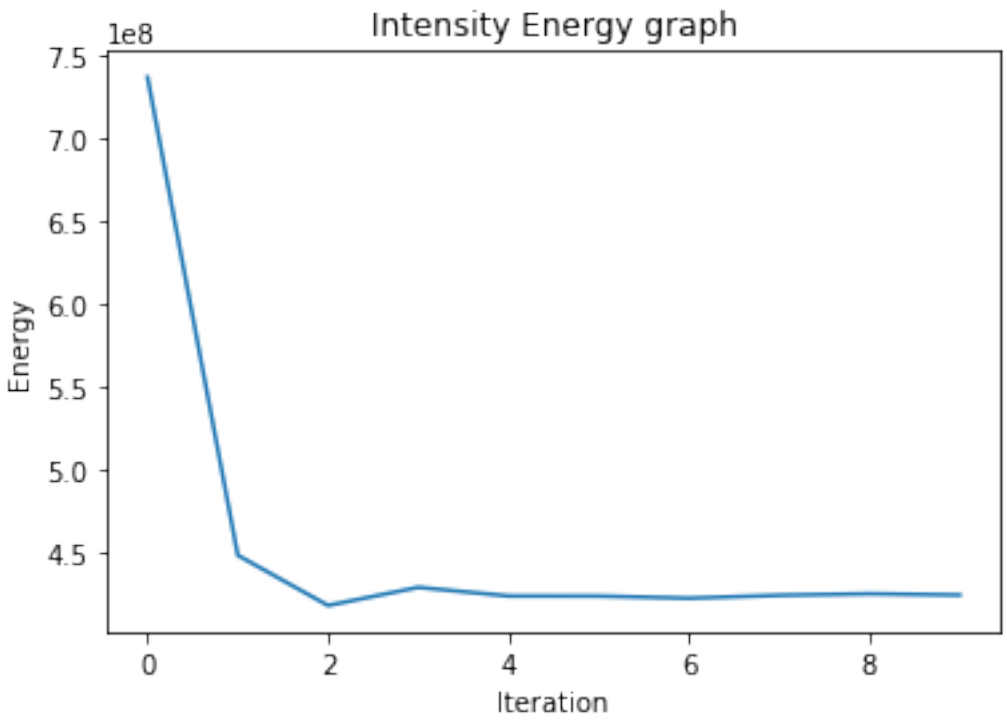

Intensity Energy graph
1e9
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```




Total Energy graph
1e9
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
Energy
0
2
4
6
8
Iteration


Spatial Energy graph
1e9
1.4
1.3
1.2
1.1
1.0
0.9
0.8
Energy
0
2
4
6
8
Iteration



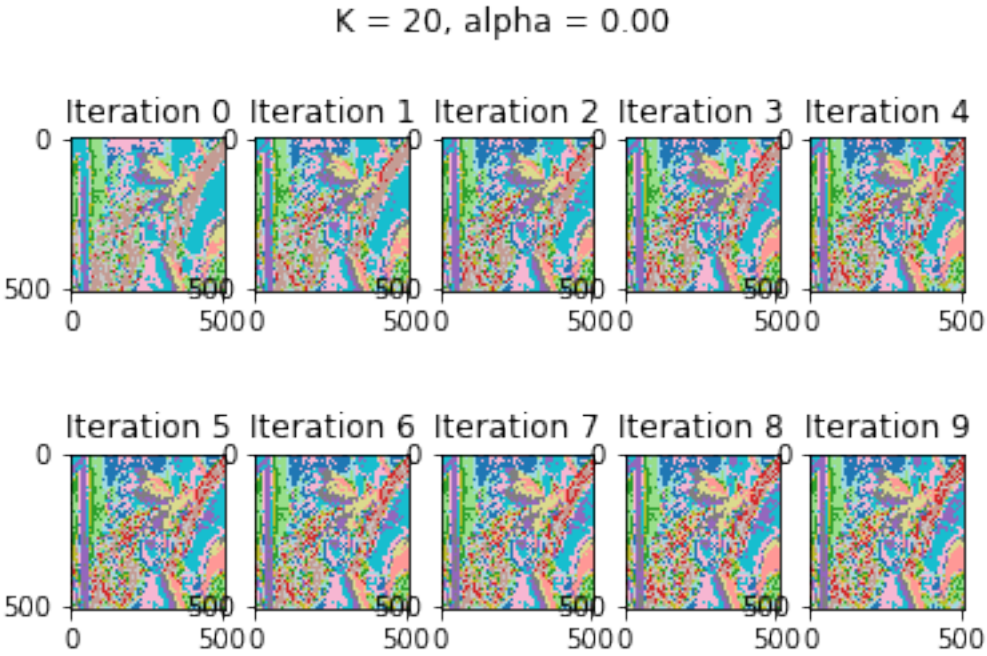
6 K = 20

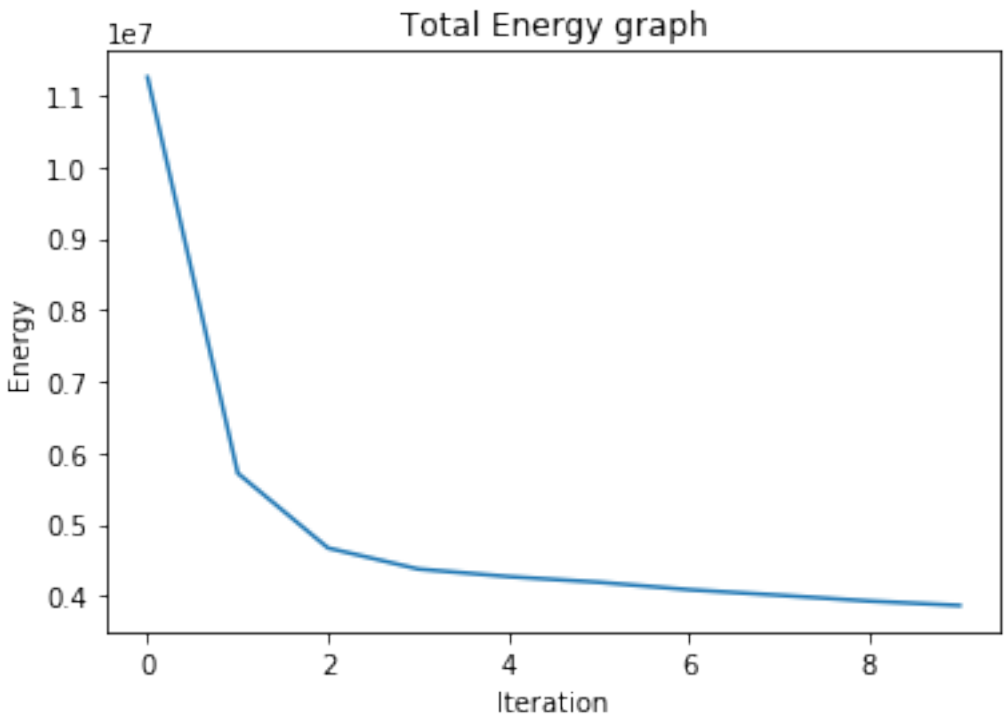
```
In [247]: K = 20
          KMeans(img_matrix, img_rows, img_cols, K, 0)
          KMeans(img_matrix, img_rows, img_cols, K, 0.01)
          KMeans(img_matrix, img_rows, img_cols, K, 0.03)
          KMeans(img_matrix, img_rows, img_cols, K, 0.05)
          KMeans(img_matrix, img_rows, img_cols, K, 0.07)
          KMeans(img_matrix, img_rows, img_cols, K, 0.08)
          KMeans(img_matrix, img_rows, img_cols, K, 0.10)
          KMeans(img_matrix, img_rows, img_cols, K, 0.25)
          KMeans(img_matrix, img_rows, img_cols, K, 0.50)
          KMeans(img_matrix, img_rows, img_cols, K, 0.75)
          KMeans(img_matrix, img_rows, img_cols, K, 1)
```

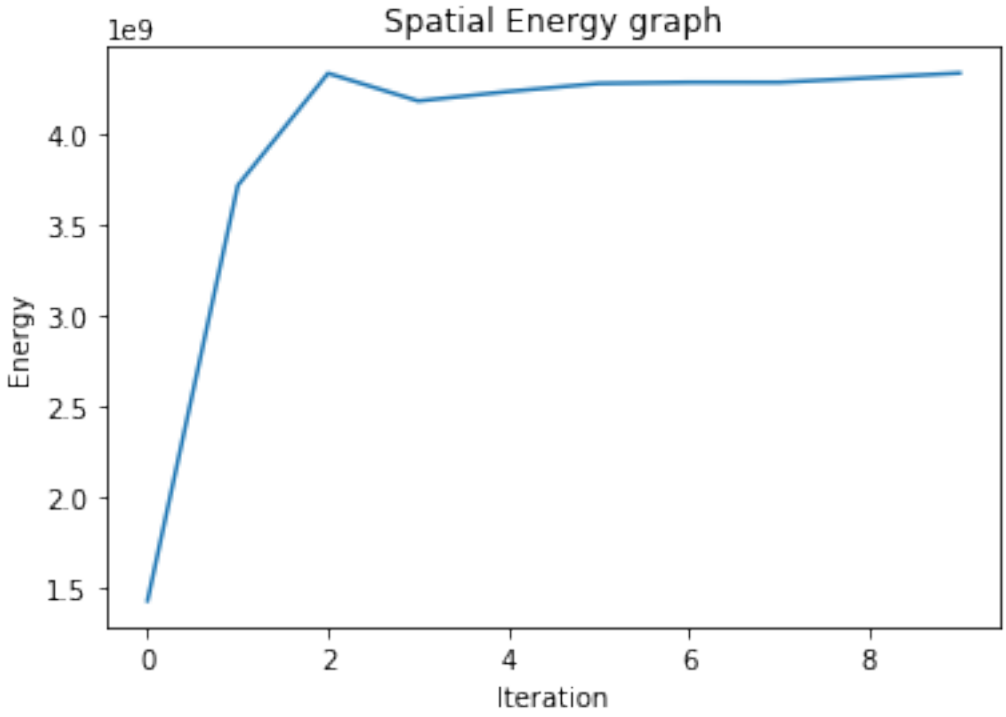
```
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:66: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:67: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:68: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:32: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:33: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:35: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:36: RuntimeWarning: overi
```

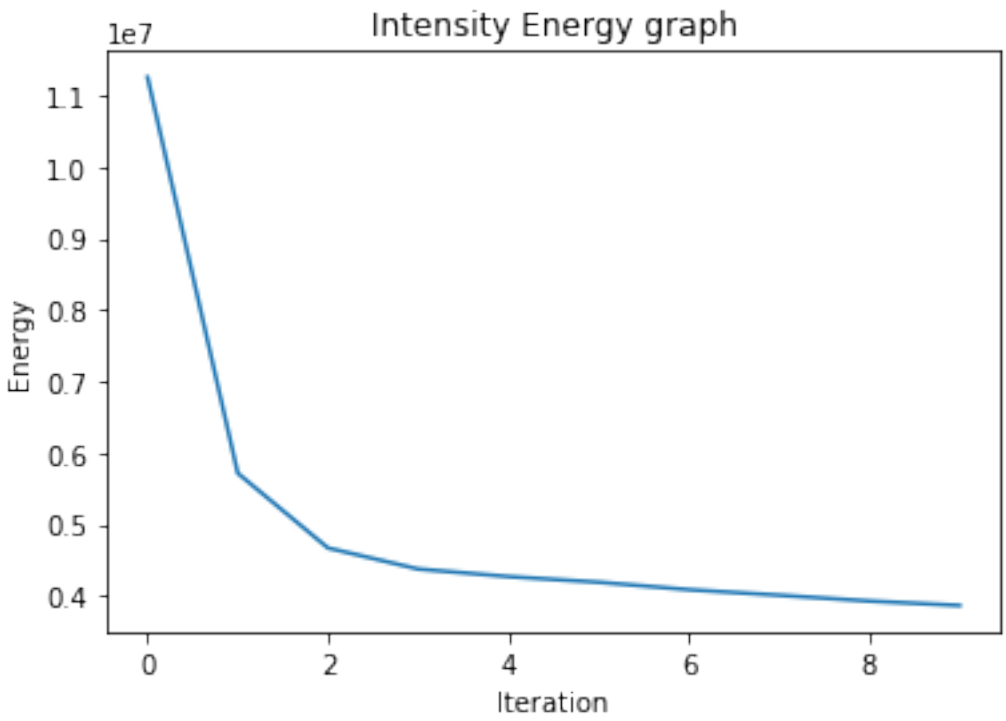
```
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:39: RuntimeWarning: over
```

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

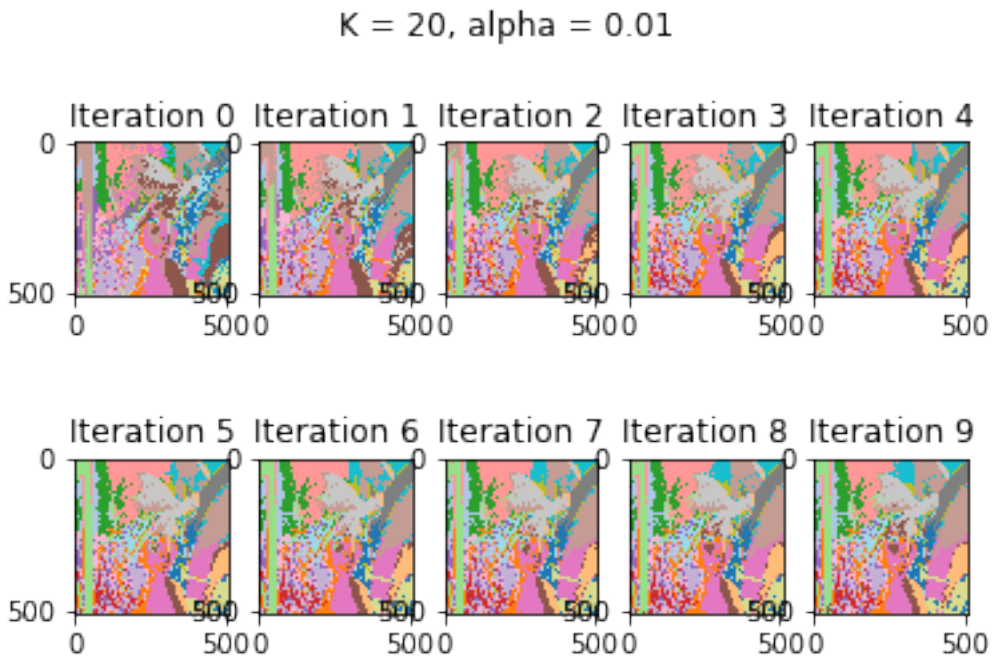


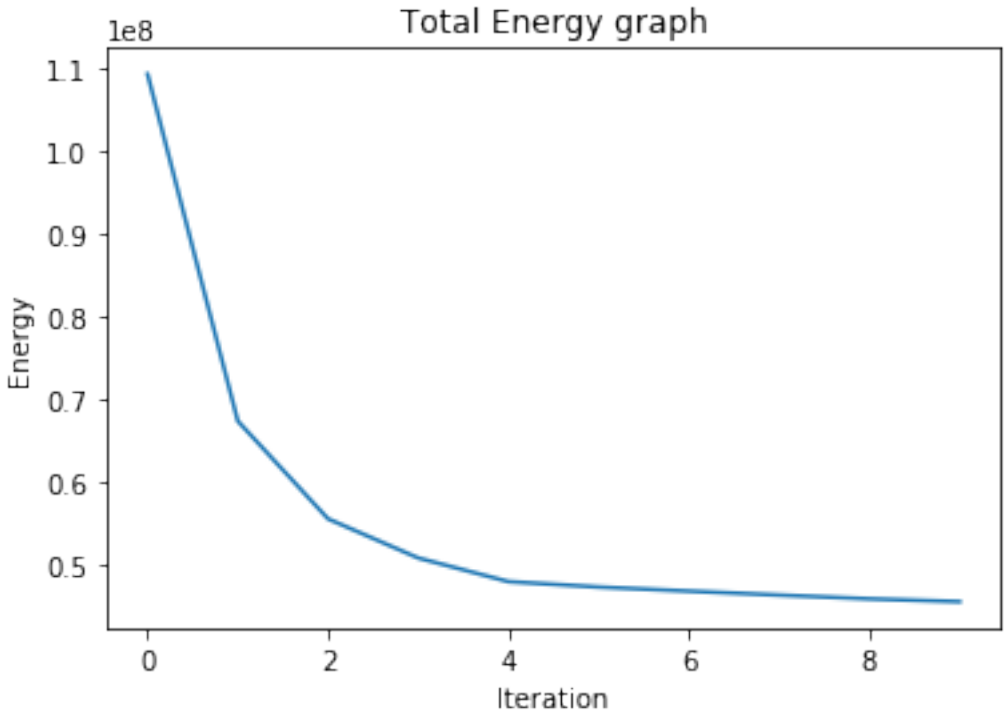

Total Energy graph
1e7
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration

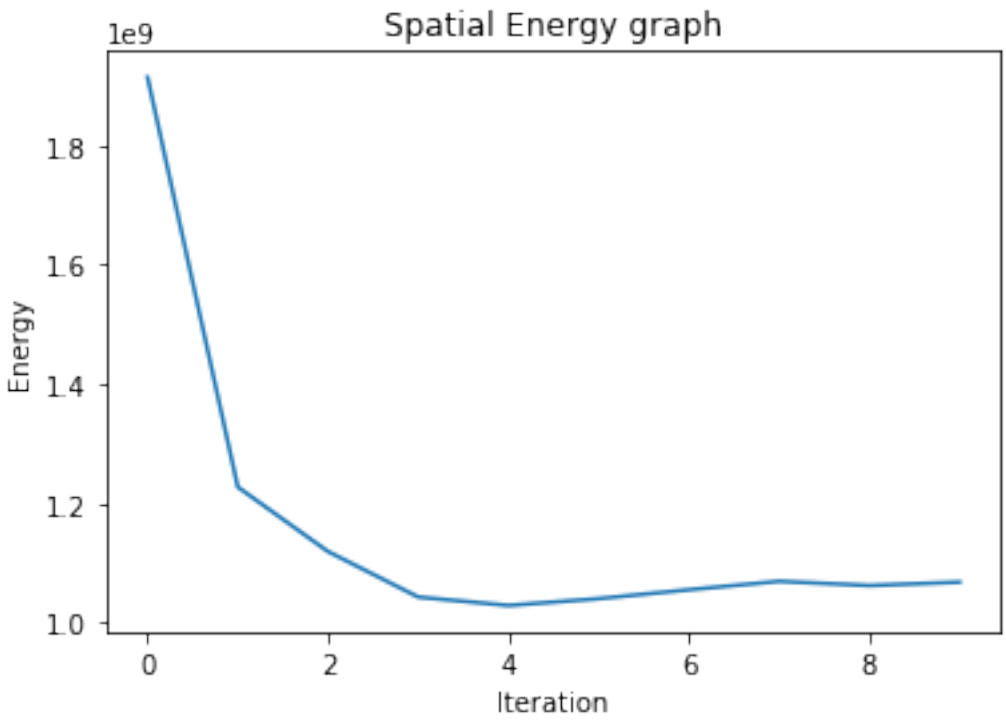

Spatial Energy graph
1e9
4.0
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

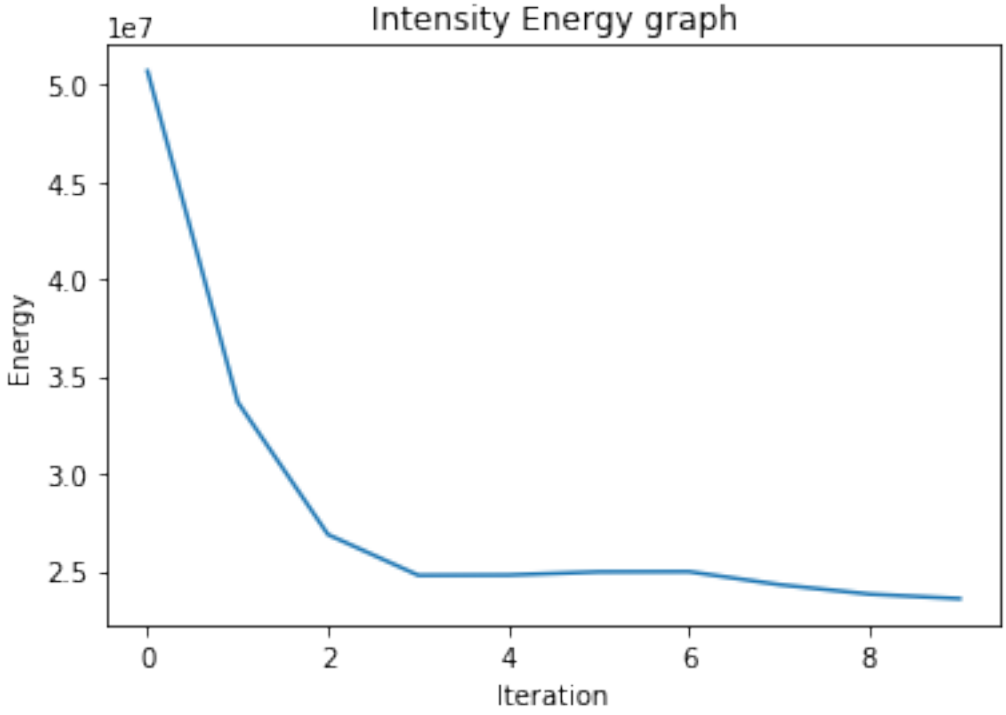



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

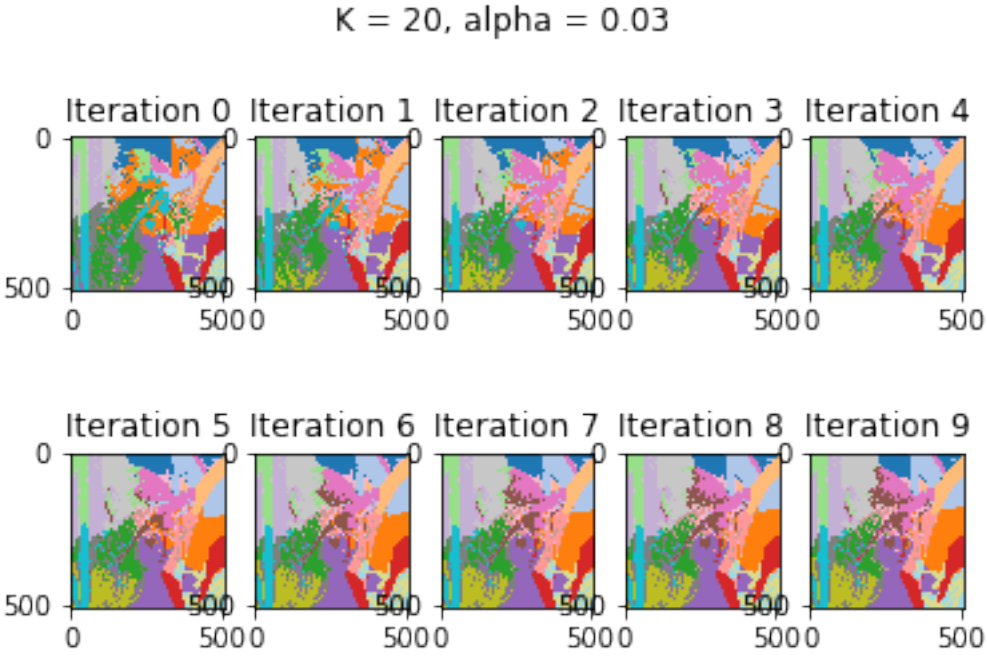

K = 20, alpha = 0.01
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

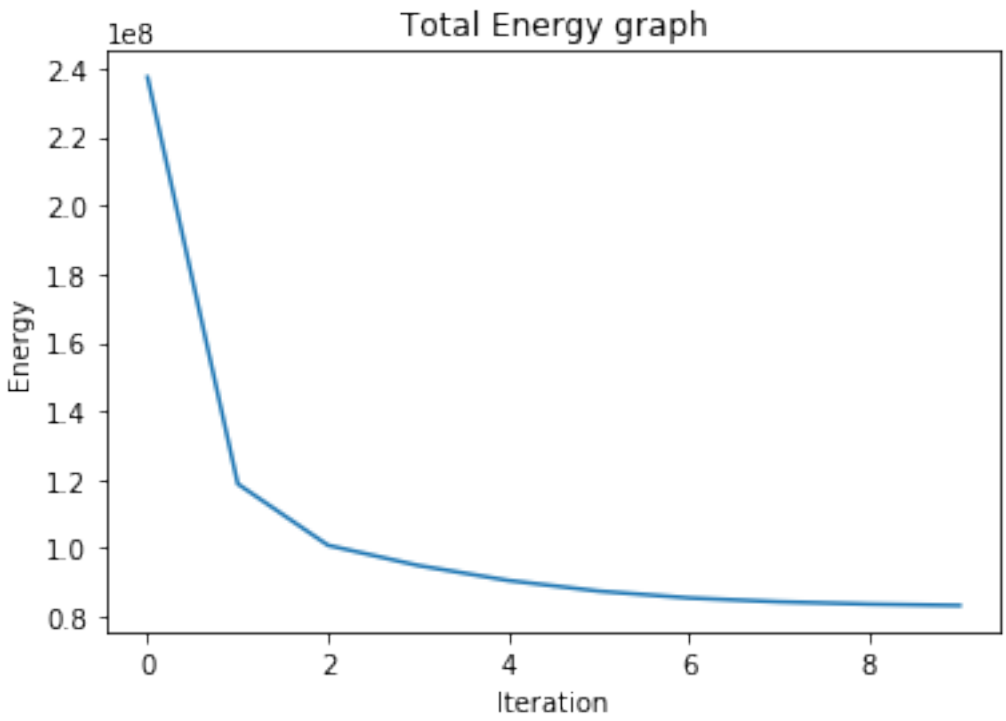

Total Energy graph
1e8
1.1
1.0
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

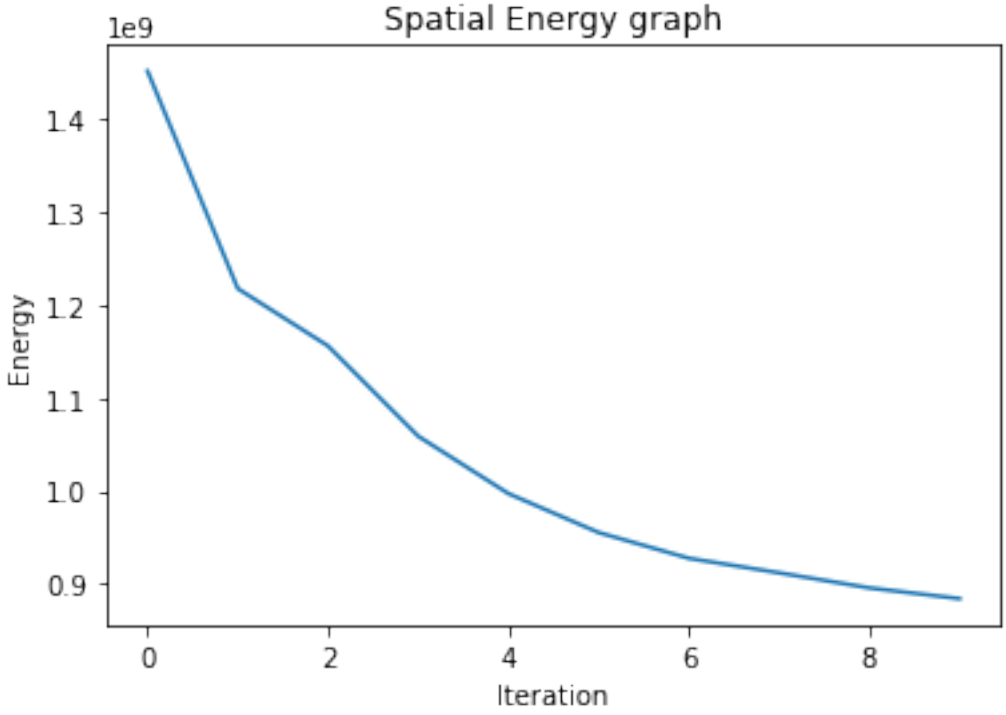

Spatial Energy graph
1e9
1.8
1.6
1.4
1.2
1.0
Energy
0
2
4
6
8
Iteration

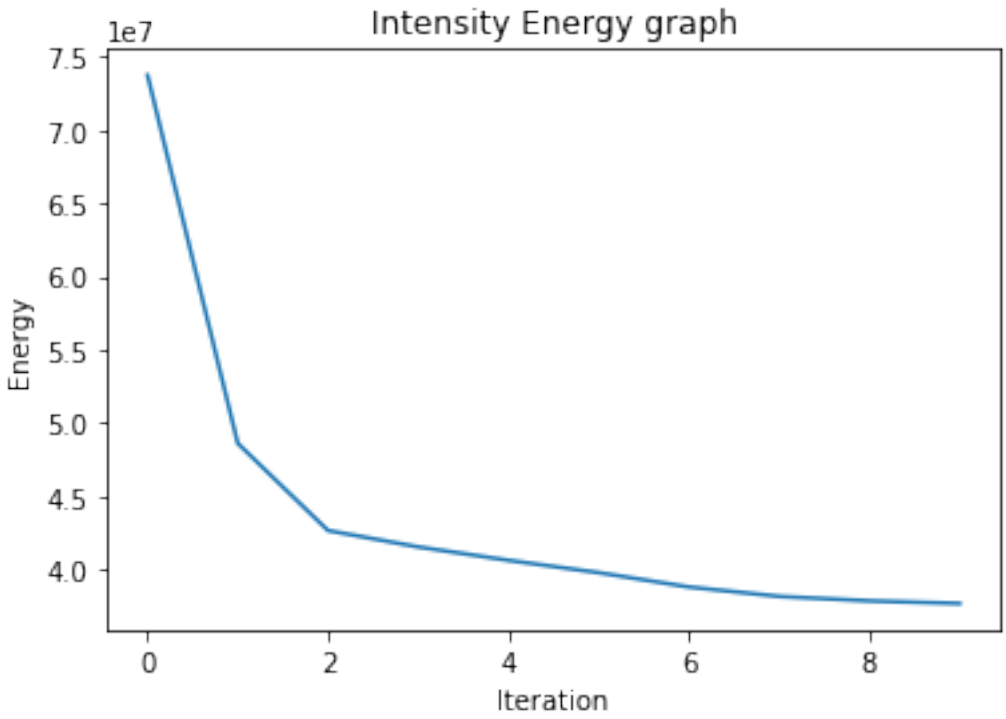

Intensity Energy graph
1e7
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

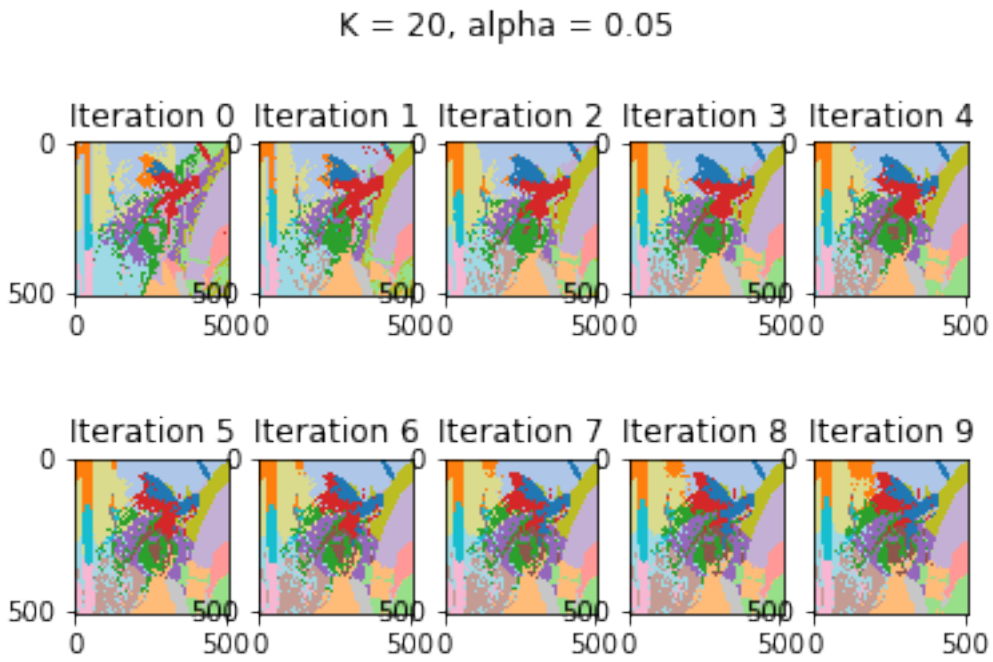


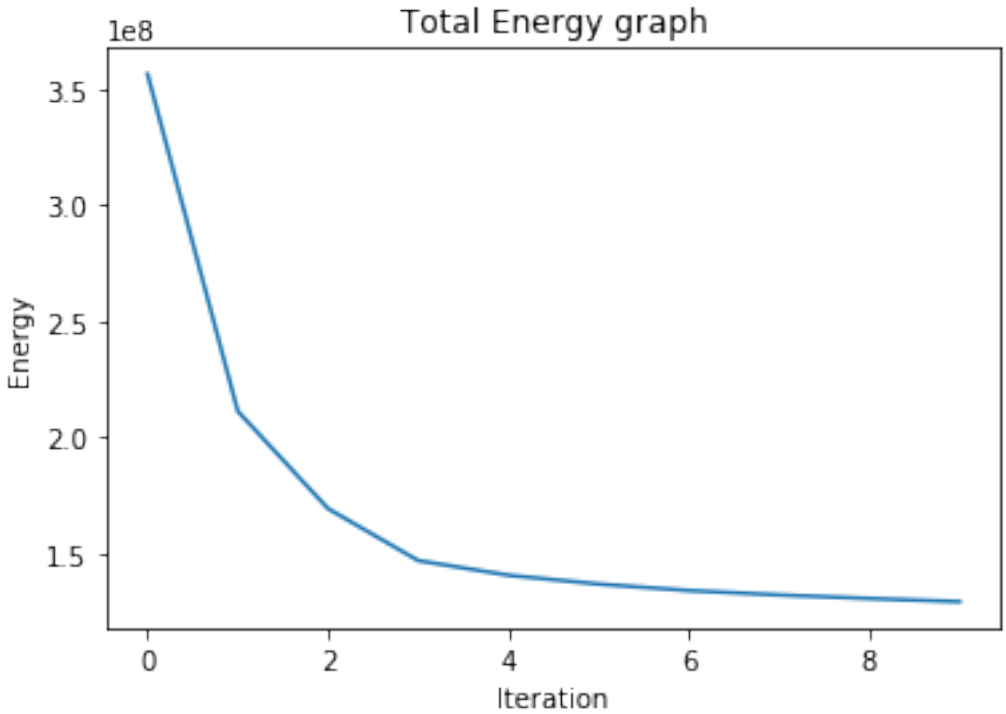

Total Energy graph
1e8
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

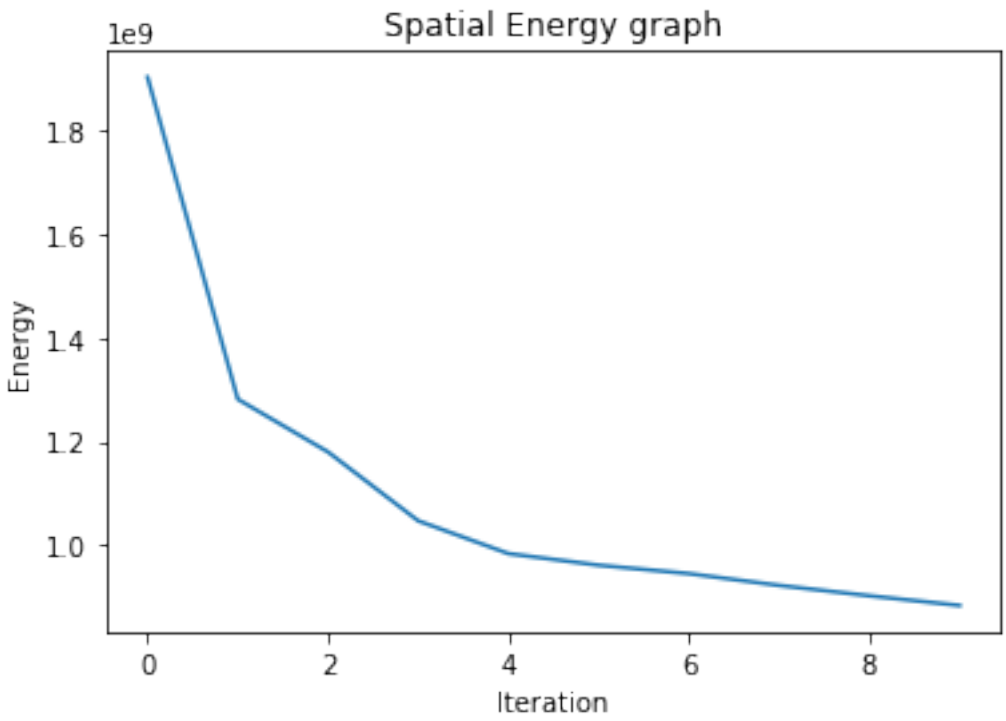

Spatial Energy graph
1e9
1.4
1.3
1.2
1.1
1.0
0.9
Energy
0
2
4
6
8
Iteration

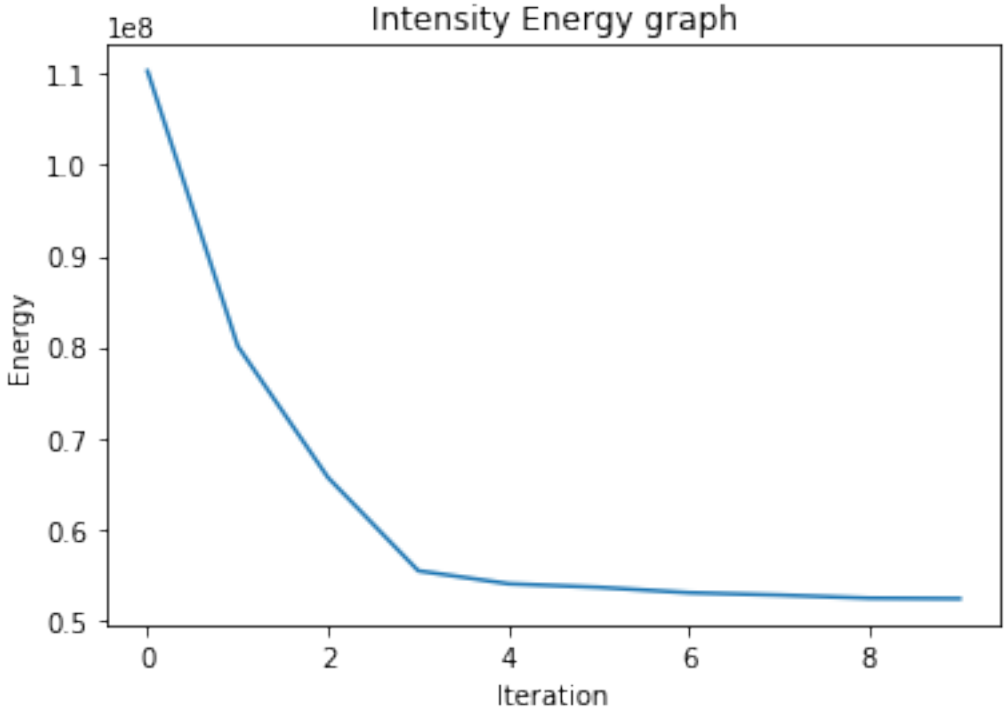



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

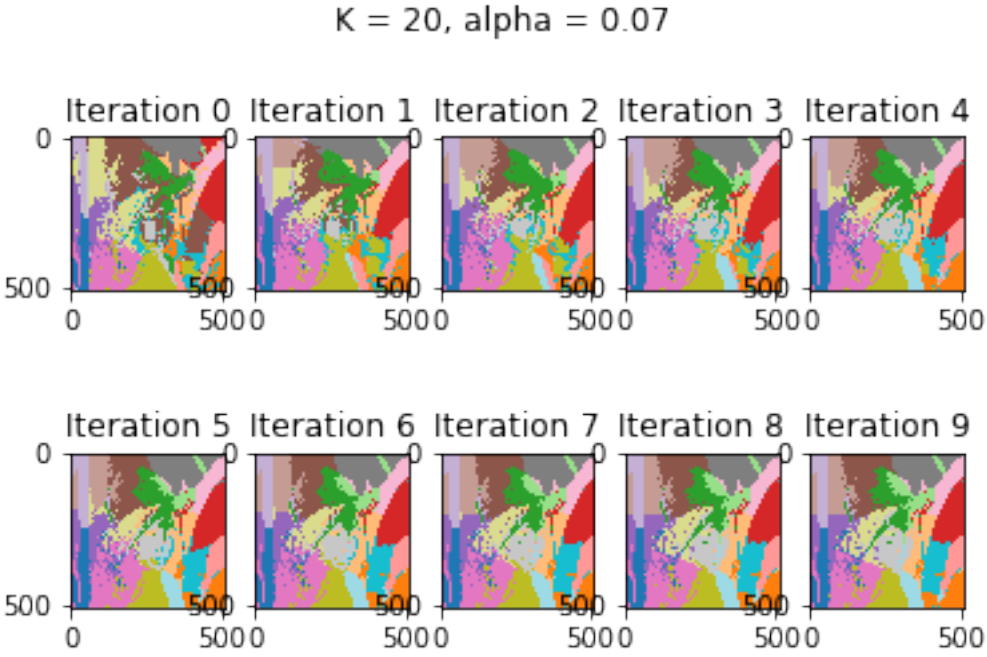

K = 20, alpha = 0.05
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

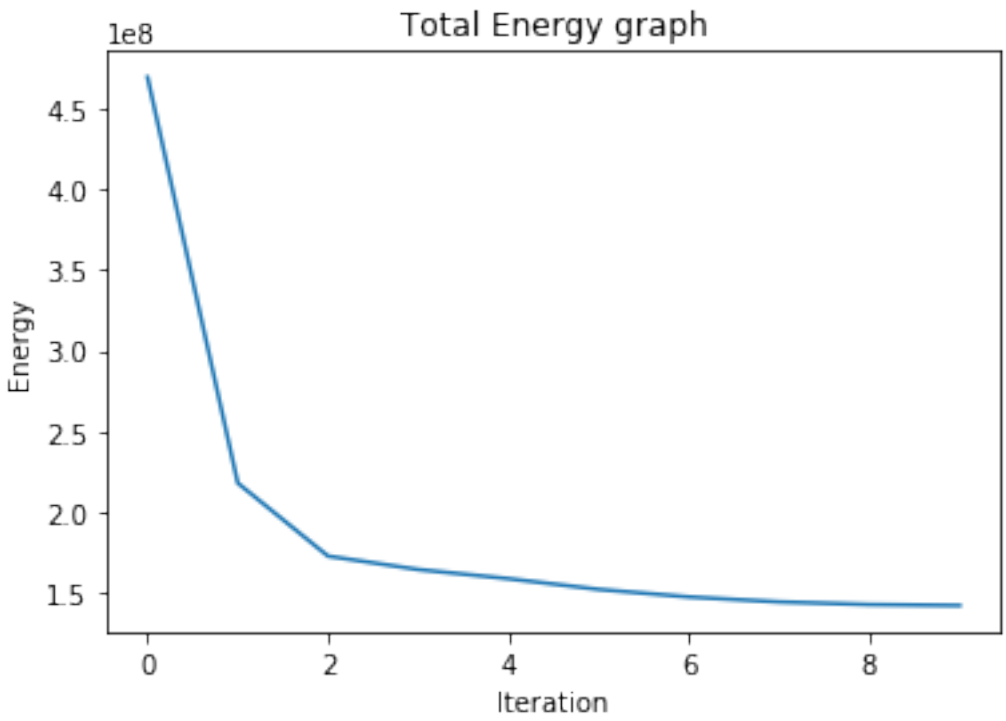

Total Energy graph
1e8
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

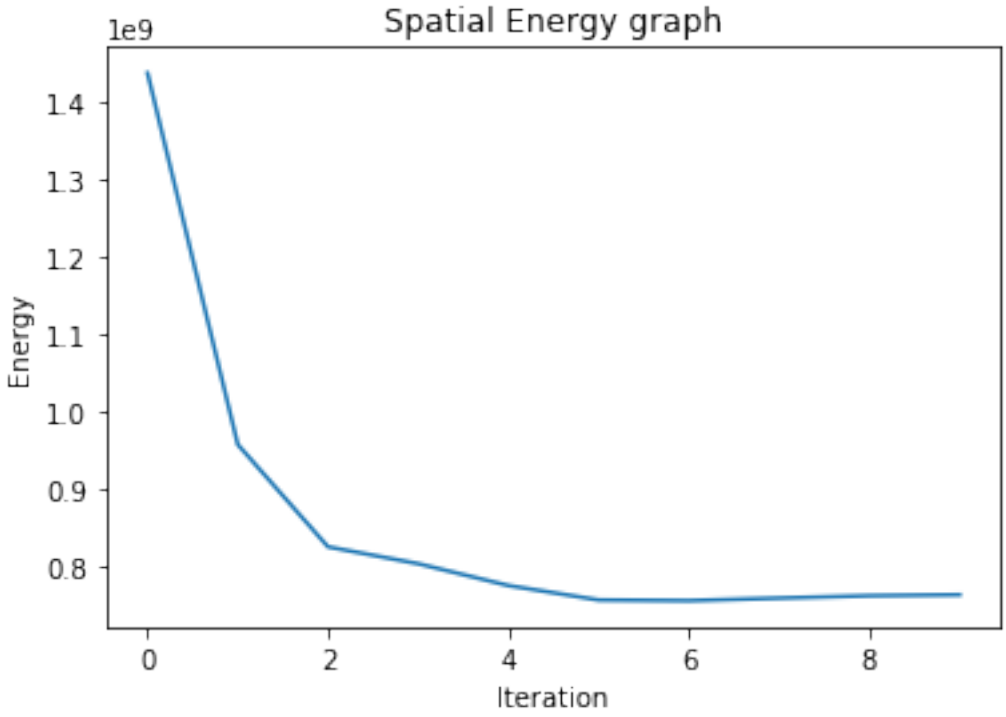

Spatial Energy graph
1e9
1.8
1.6
1.4
1.2
1.0
Energy
0
2
4
6
8
Iteration

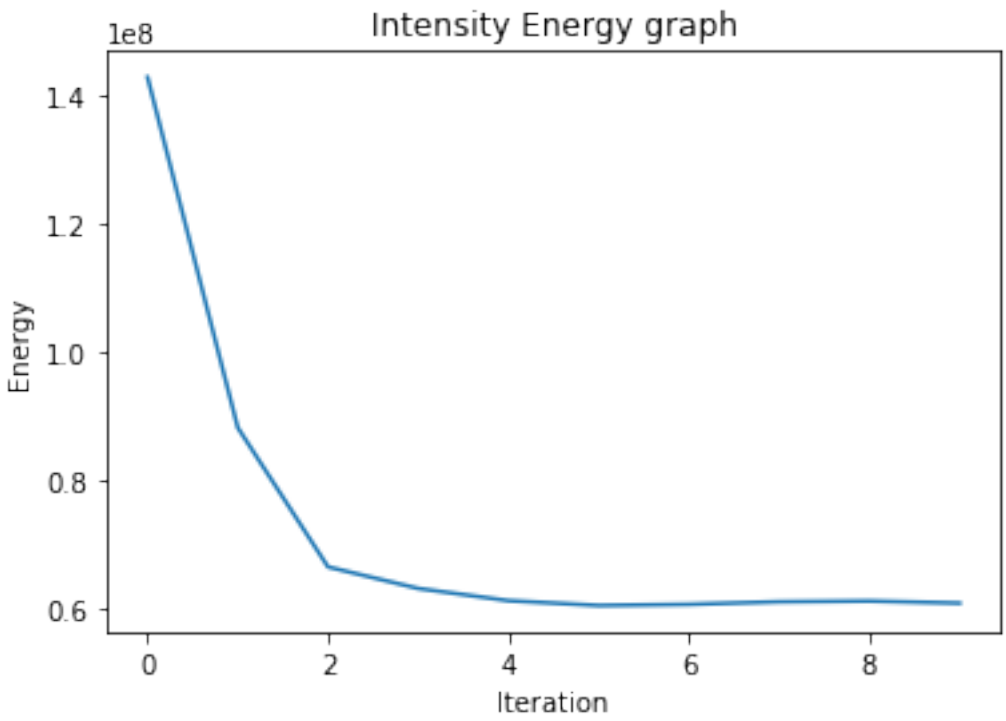

Intensity Energy graph
1e8
1.1
1.0
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

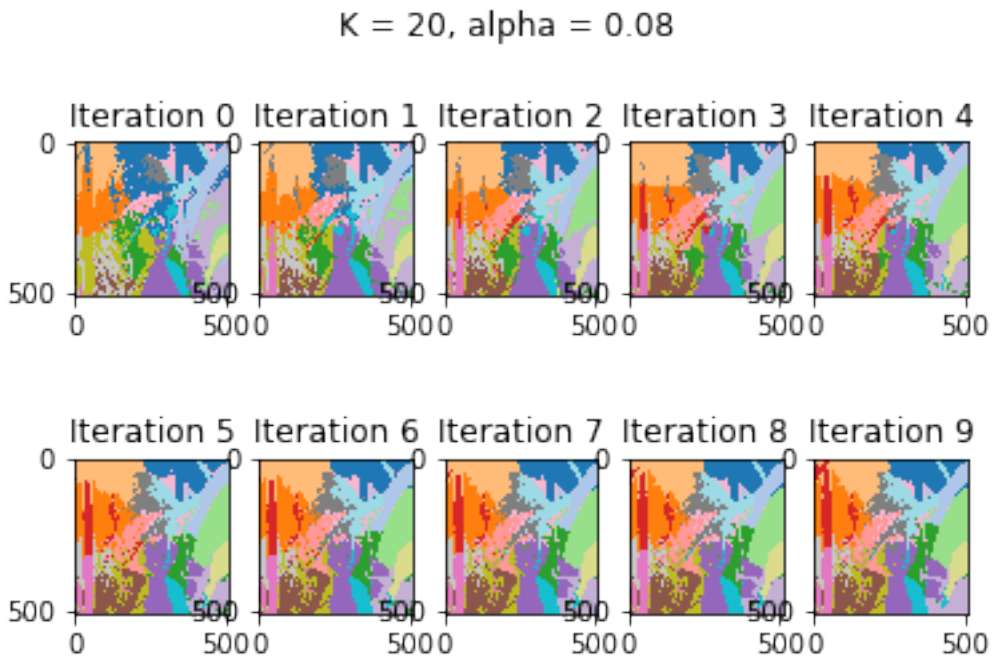


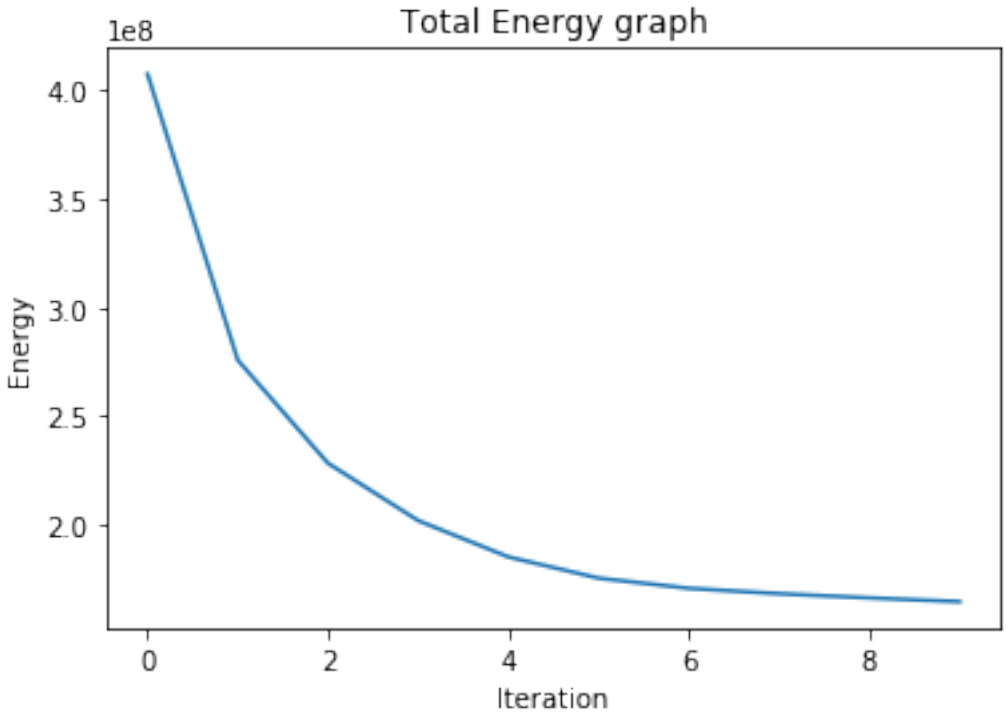

Total Energy graph
1e8
4.5
4.0
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

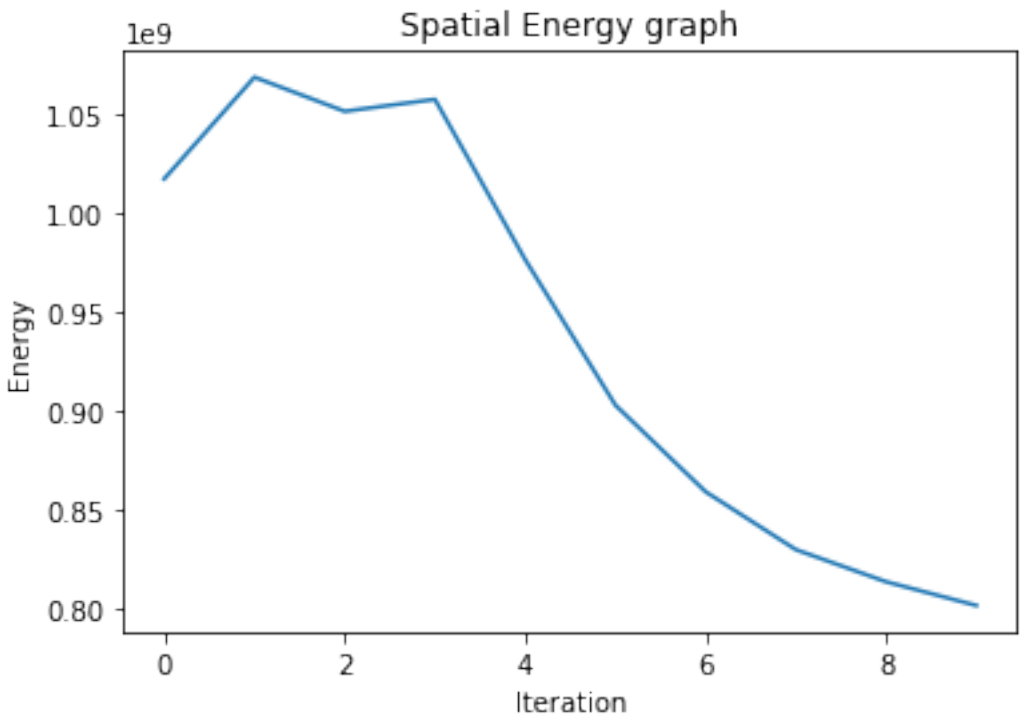

Spatial Energy graph
1e9
1.4
1.3
1.2
1.1
1.0
0.9
0.8
Energy
0
2
4
6
8
Iteration

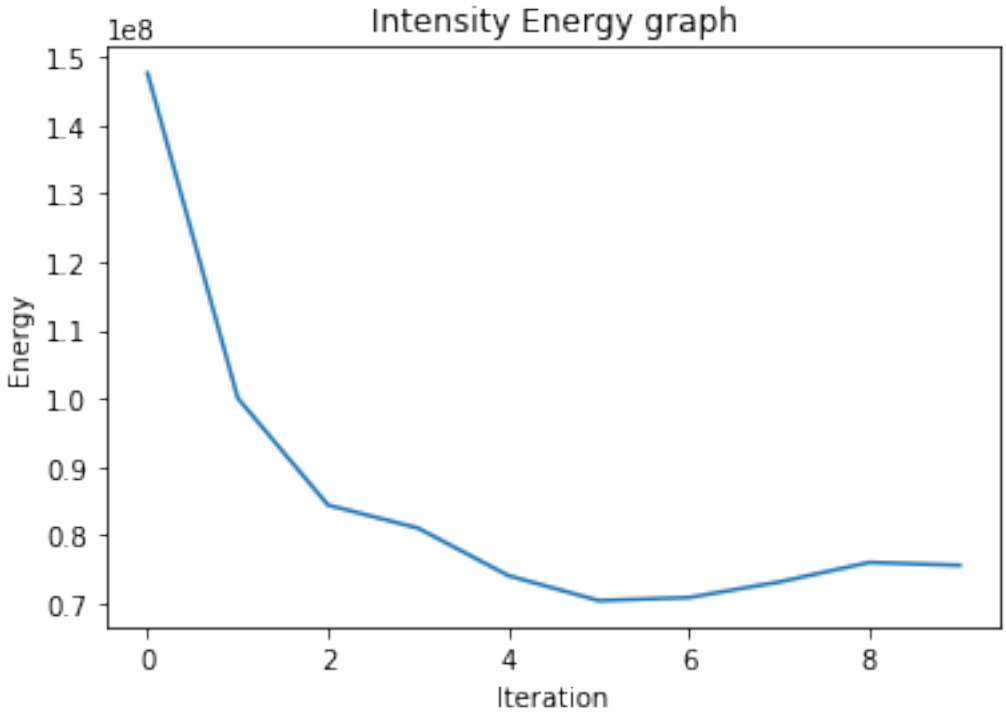



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

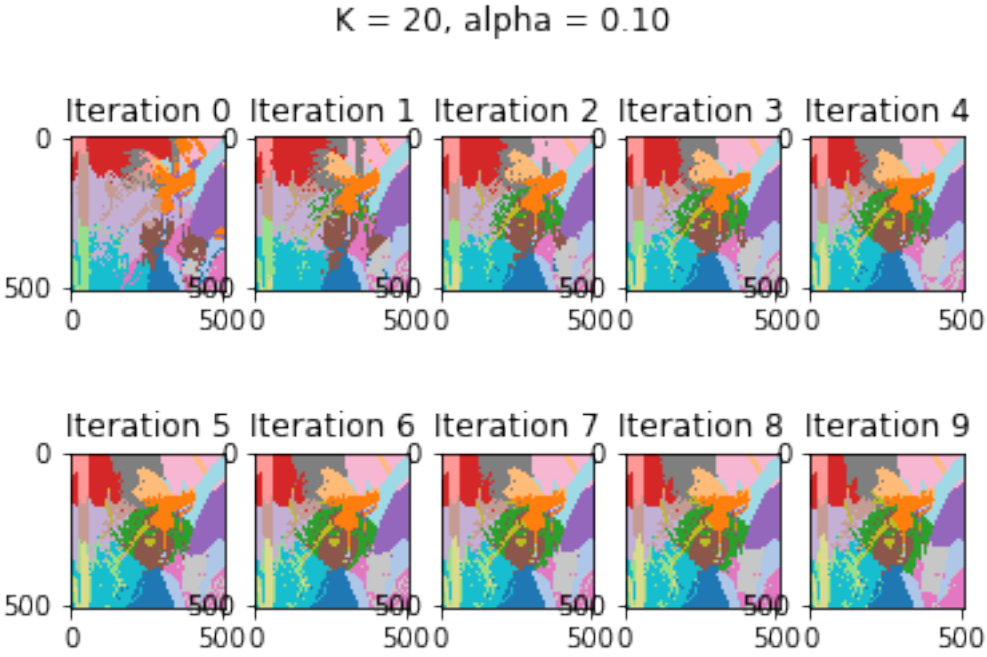

K = 20, alpha = 0.08
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

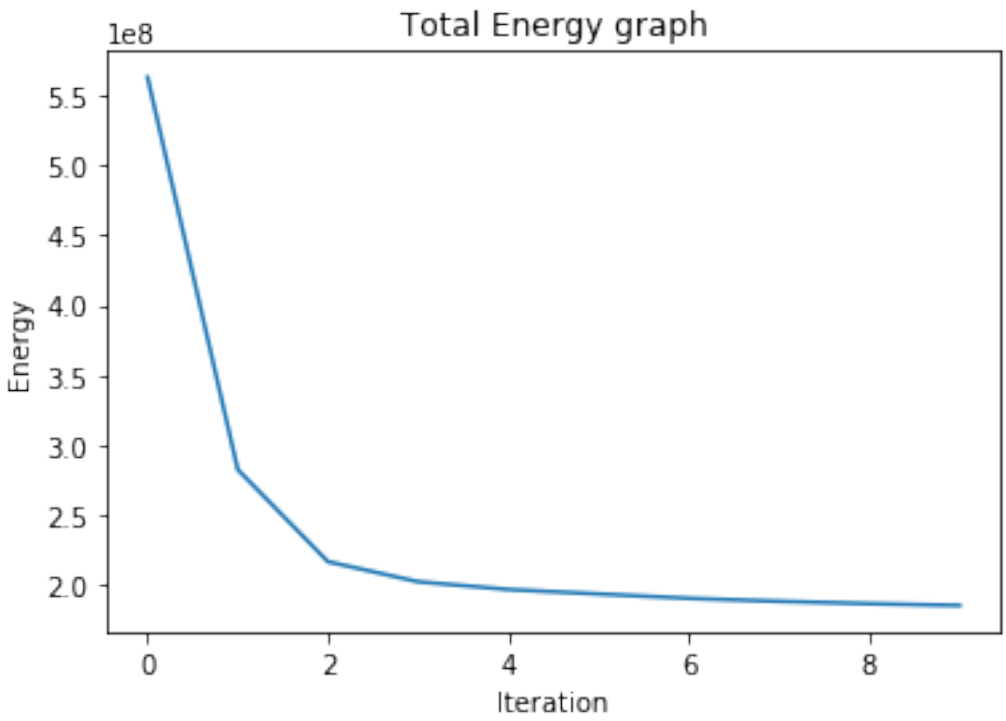

Total Energy graph
1e8
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

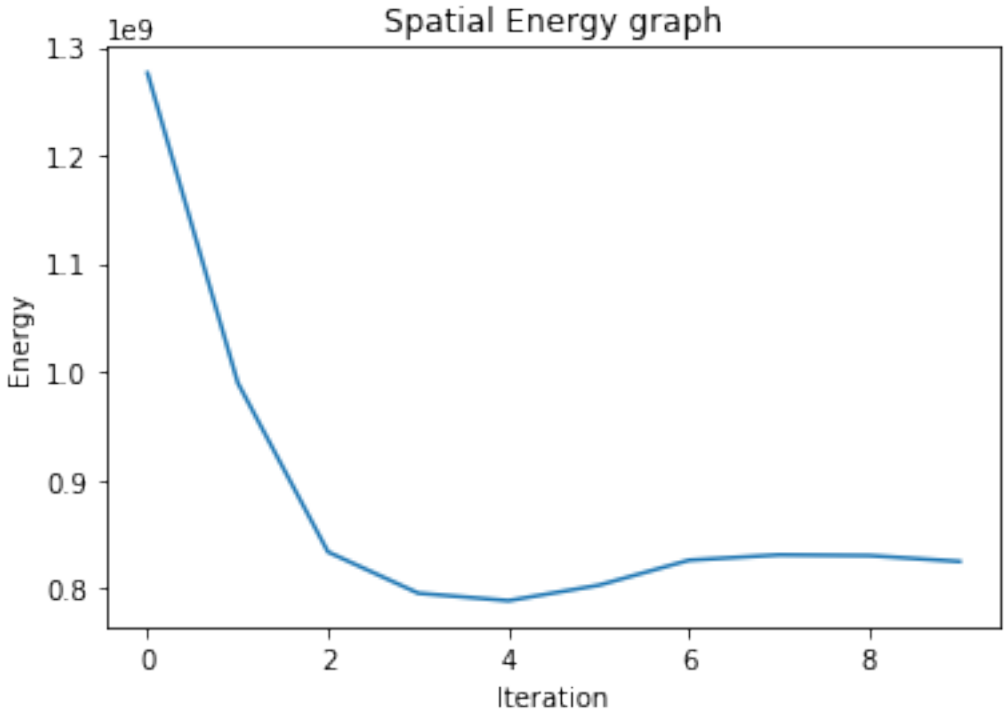

Spatial Energy graph
1e9
1.05
1.00
0.95
0.90
0.85
0.80
Energy
0
2
4
6
8
Iteration

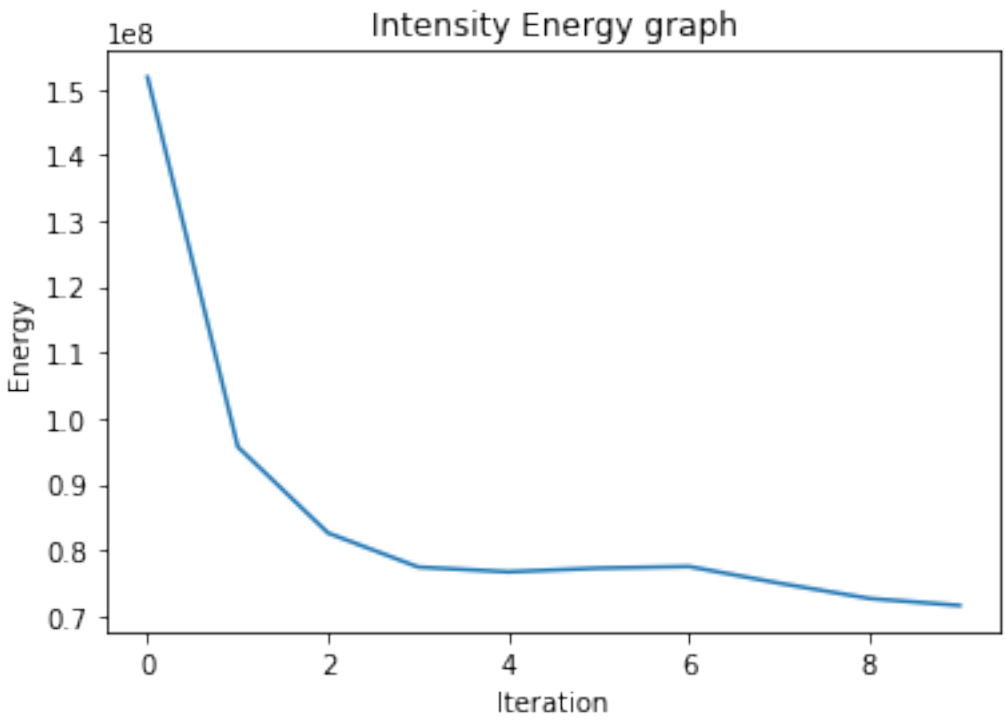

Intensity Energy graph
1e8
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

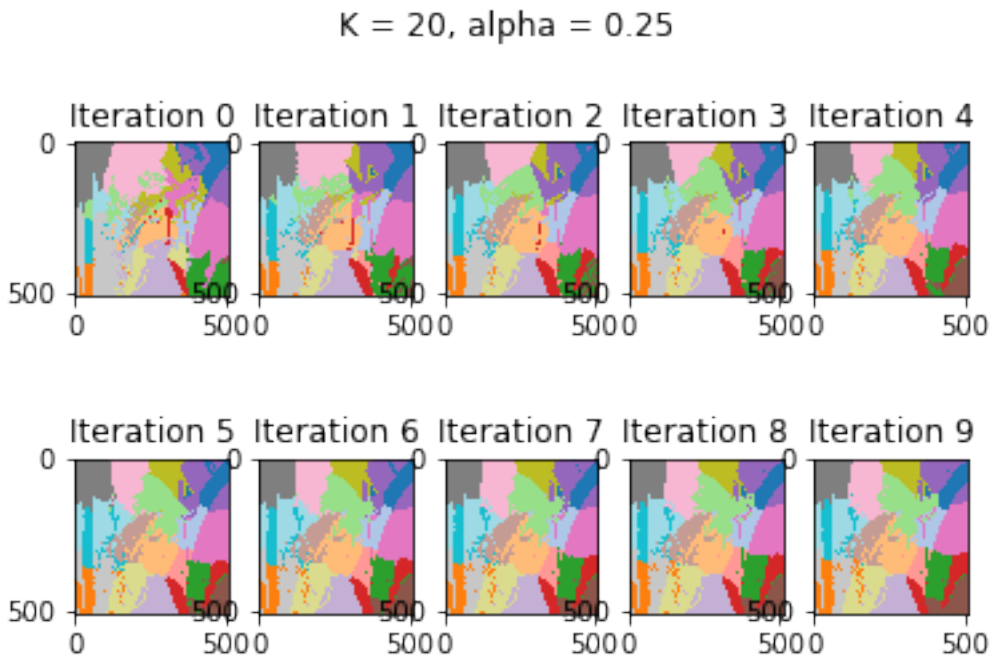


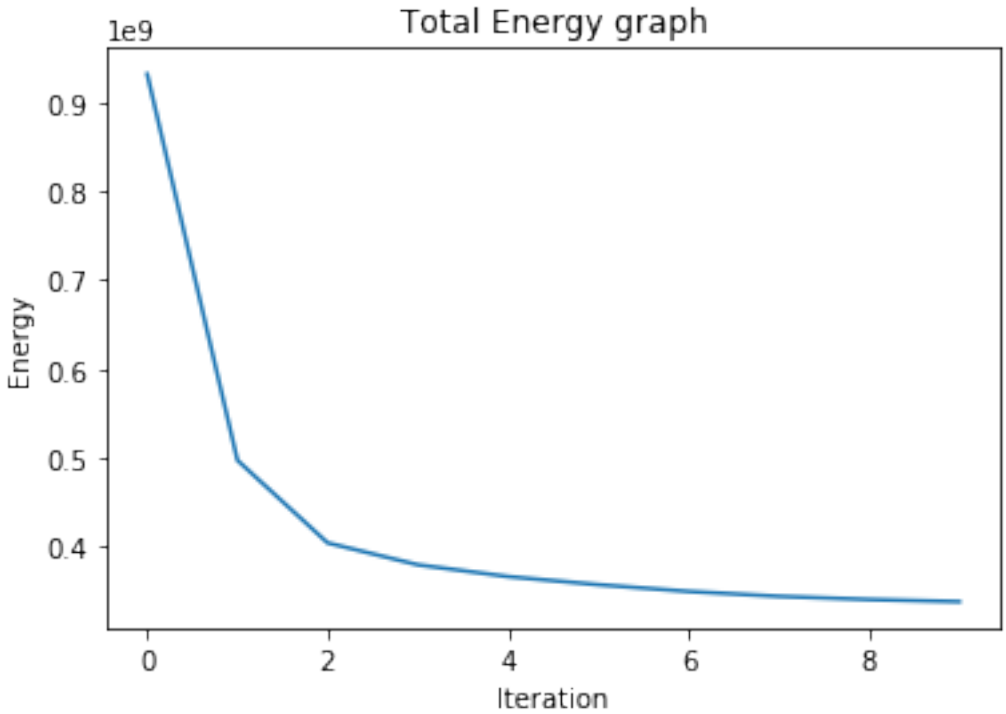

Total Energy graph
1e8
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

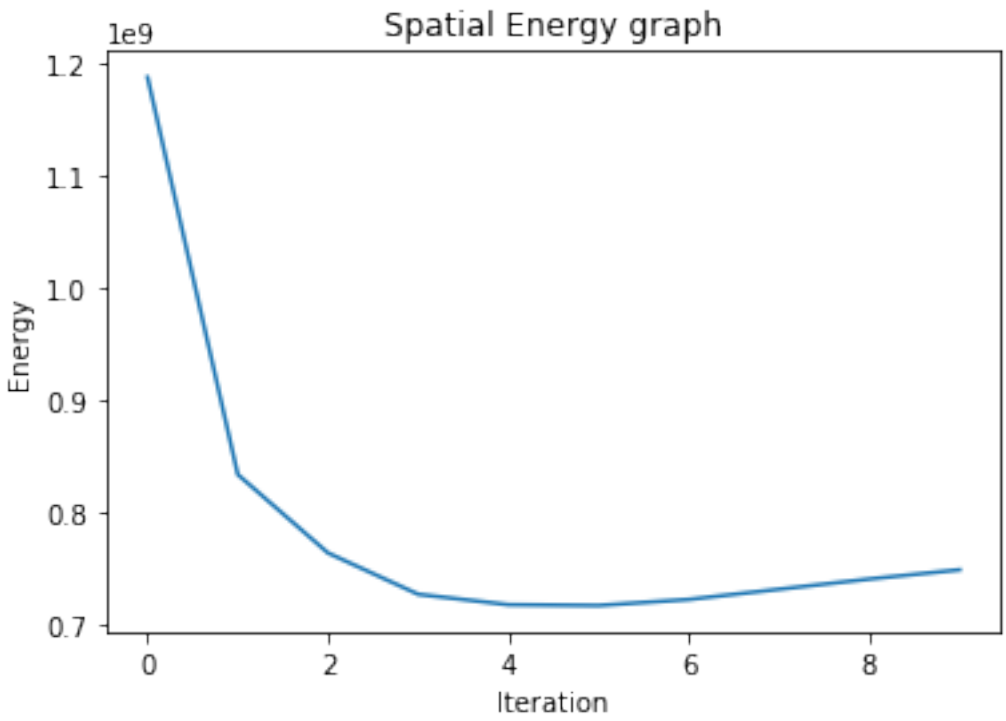

Spatial Energy graph
1e9
1.3
1.2
1.1
1.0
0.9
0.8
Energy
0
2
4
6
8
Iteration

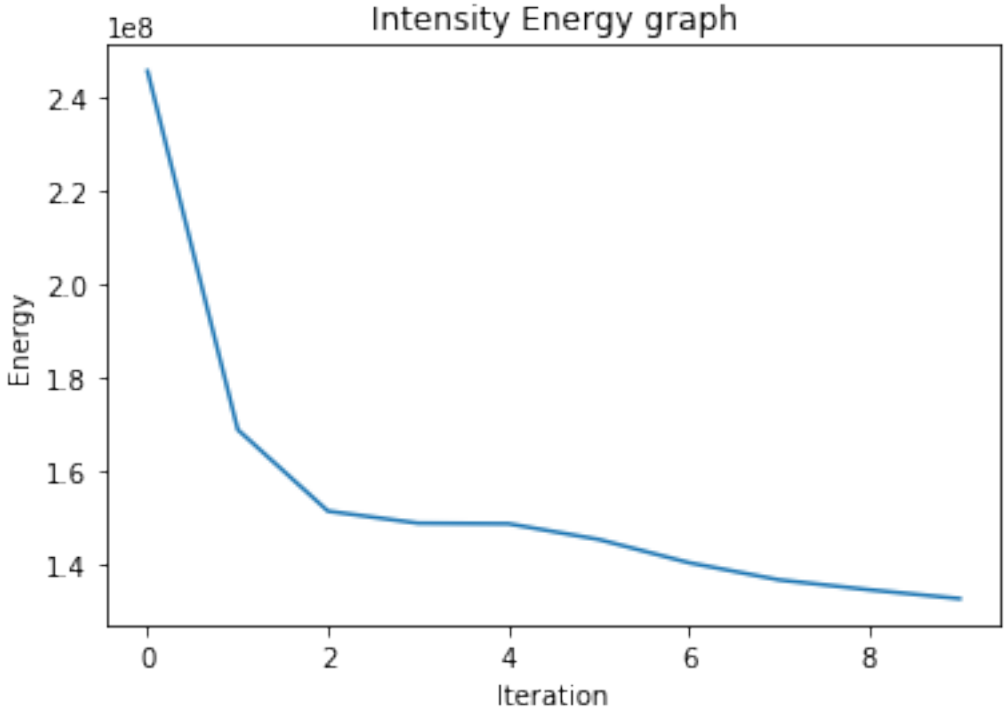



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

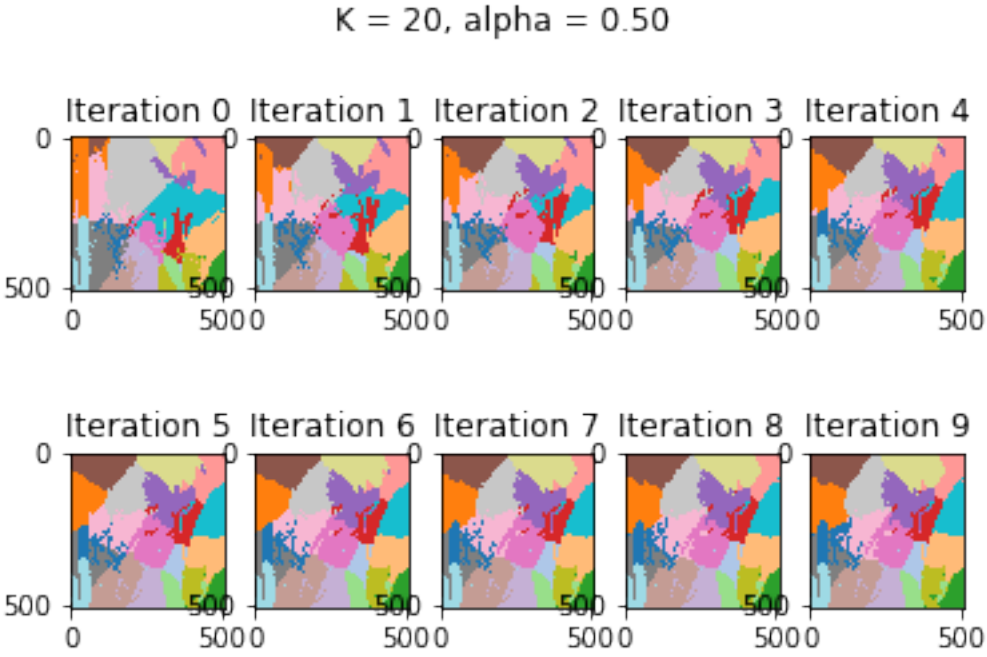

K = 20, alpha = 0.25
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

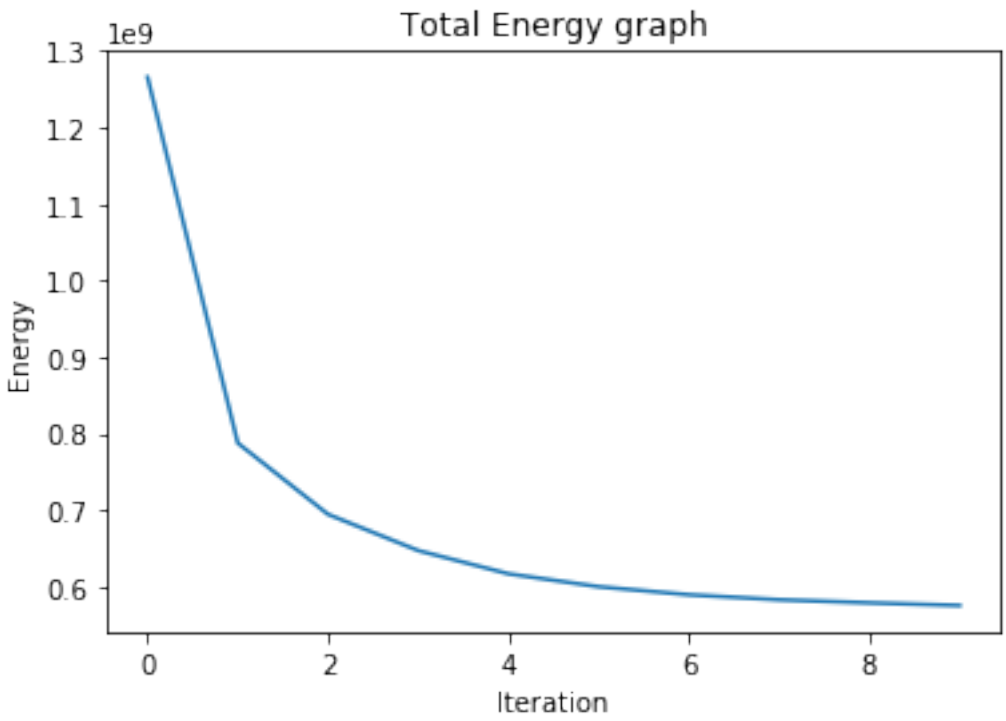

Total Energy graph
1e9
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration

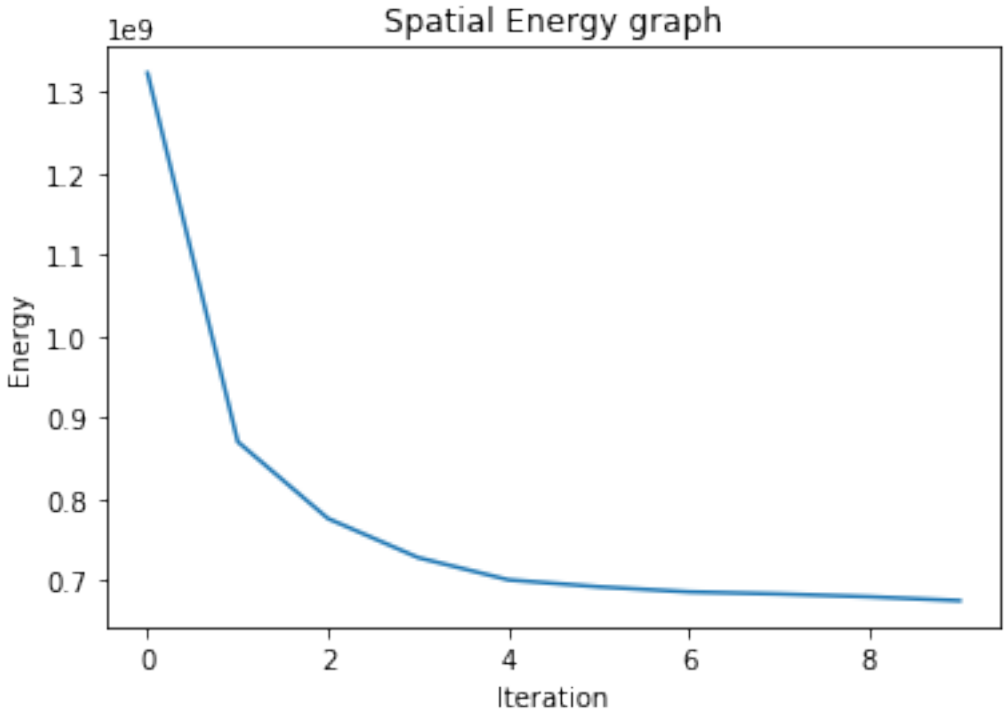

Spatial Energy graph
1e9
1.2
1.1
1.0
0.9
0.8
0.7
Energy
0
2
4
6
8
Iteration

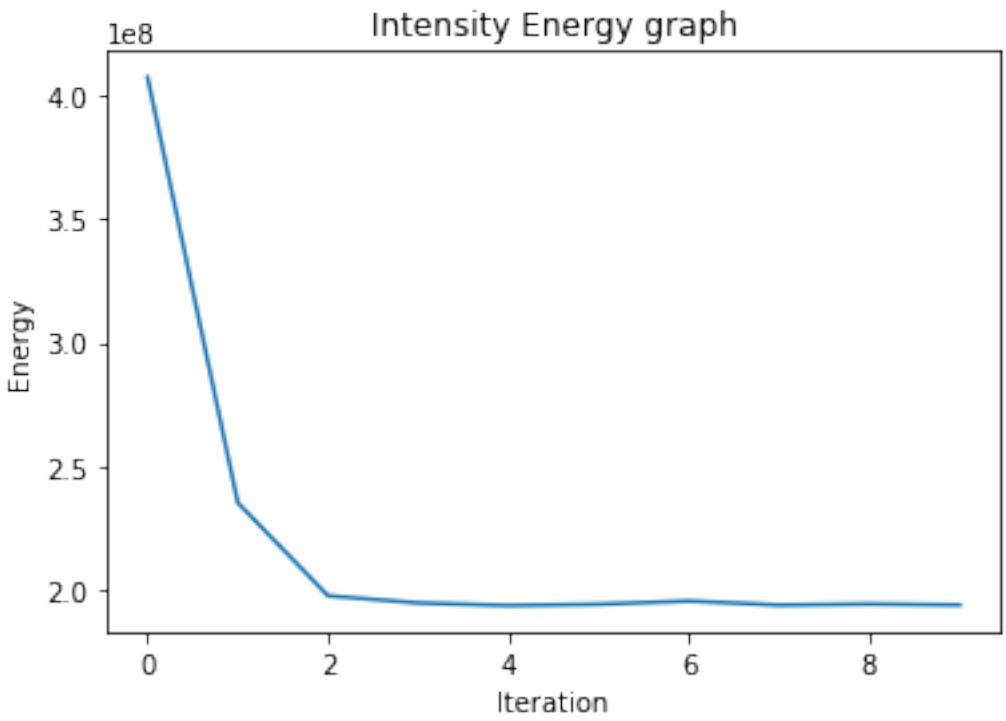

Intensity Energy graph
1e8
2.4
2.2
2.0
1.8
1.6
1.4
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

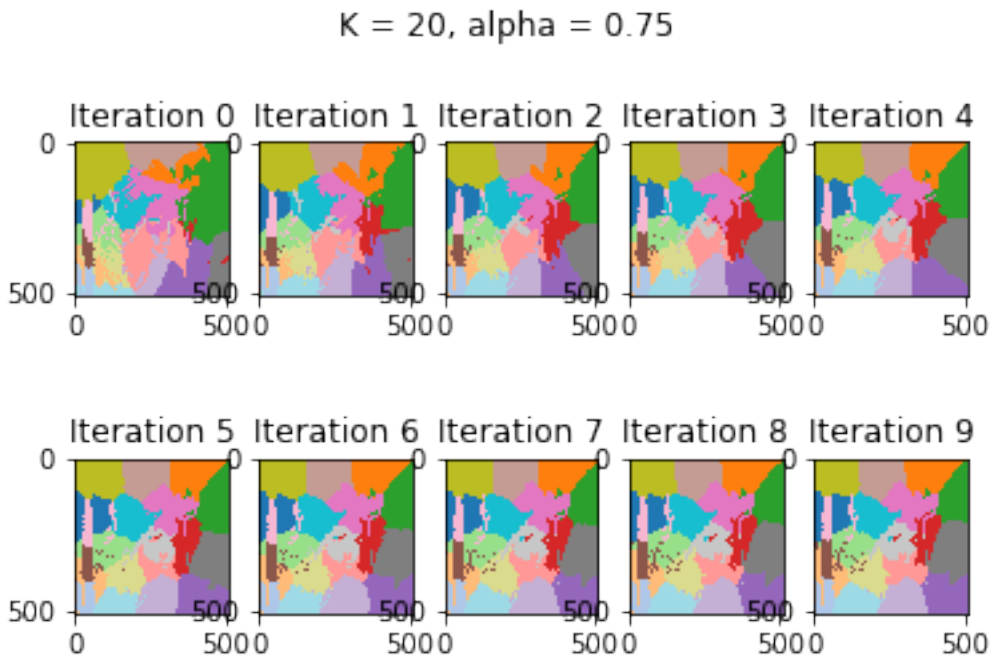


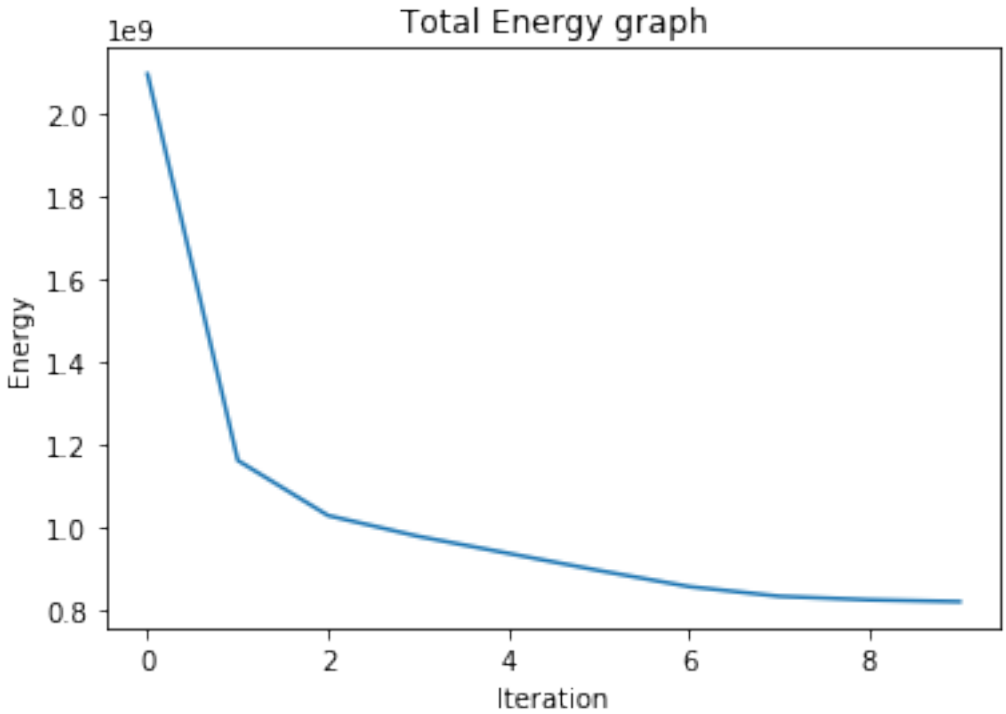

Total Energy graph
1e9
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
Energy
0
2
4
6
8
Iteration

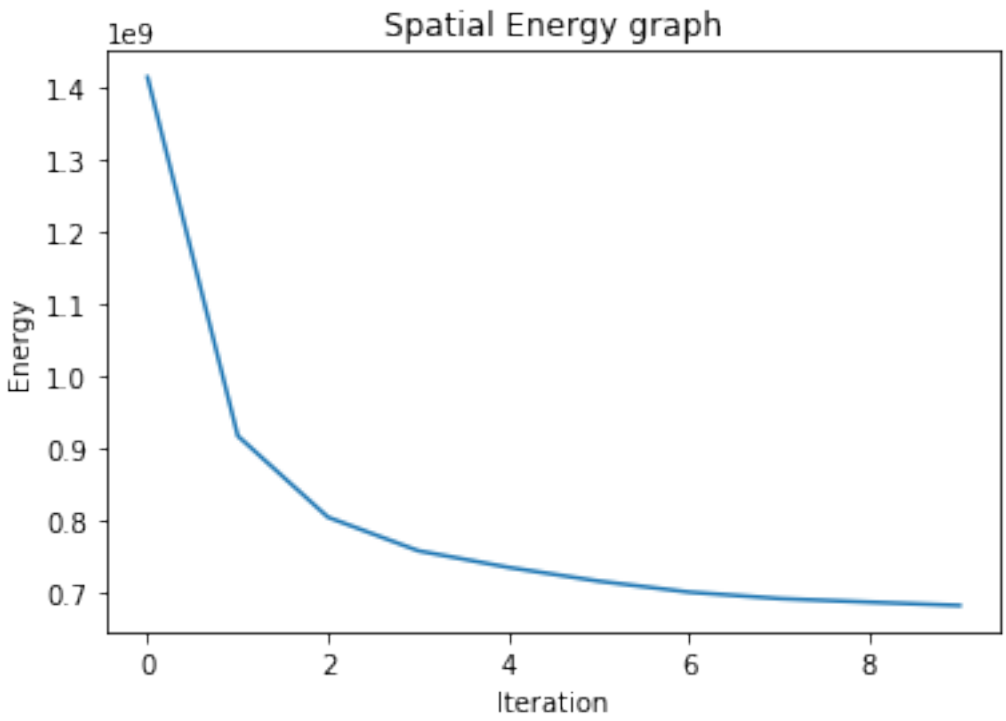

Spatial Energy graph
1e9
1.3
1.2
1.1
1.0
0.9
0.8
0.7
Energy
0
2
4
6
8
Iteration

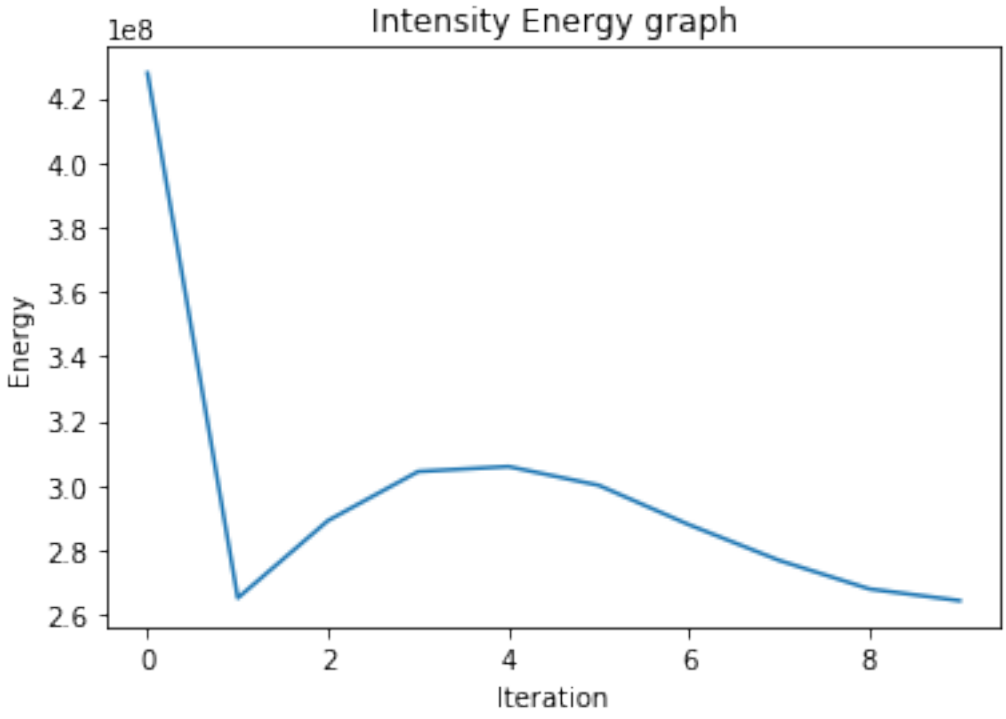



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

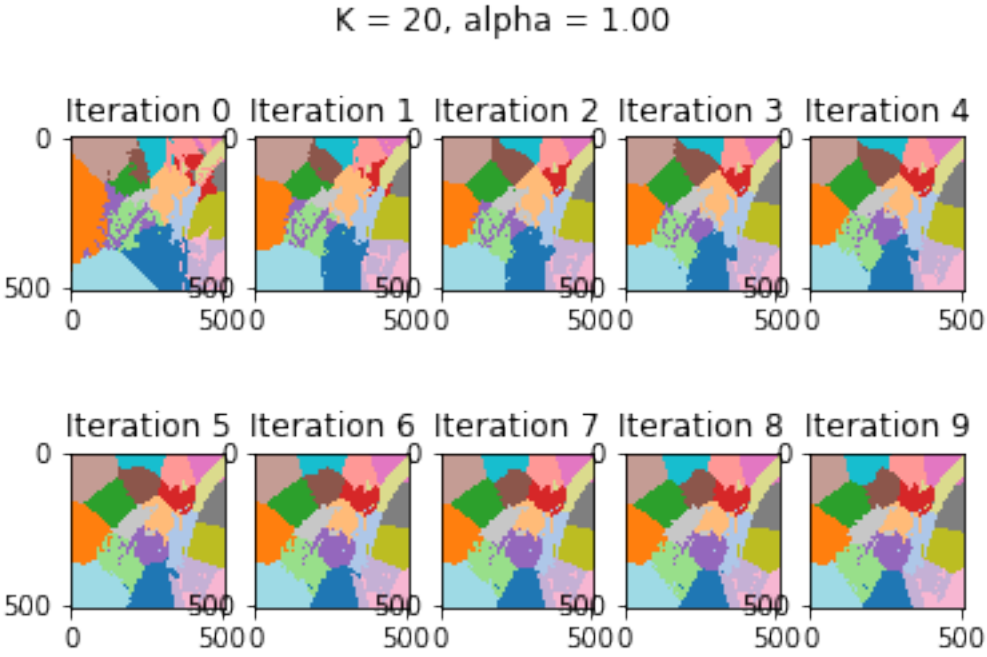

K = 20, alpha = 0.75
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

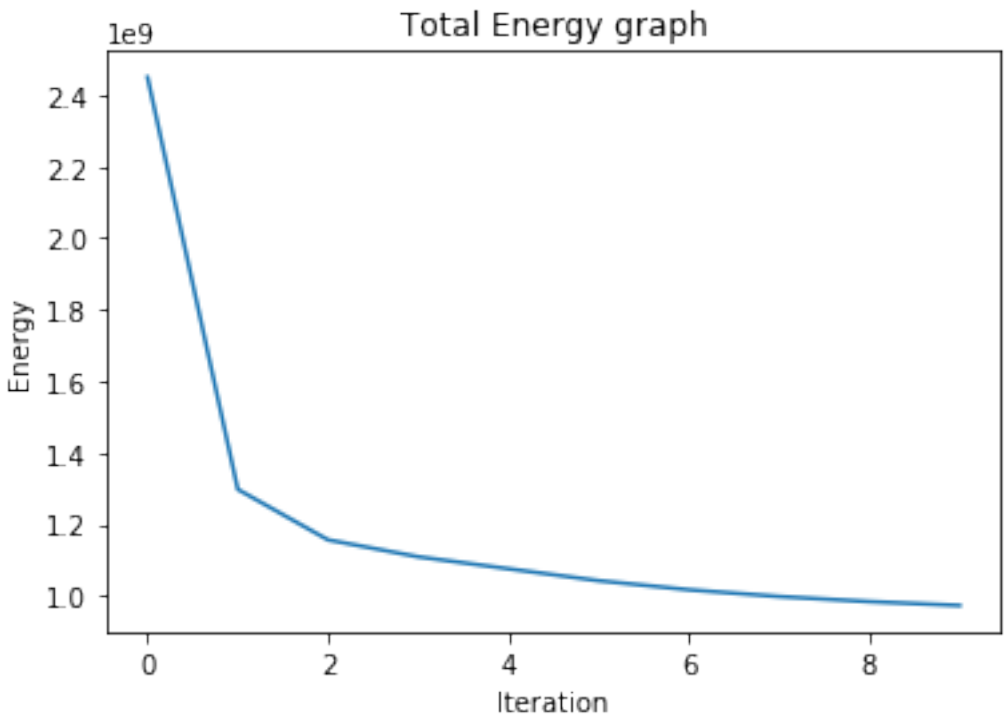

Total Energy graph
1e9
2.0
1.8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

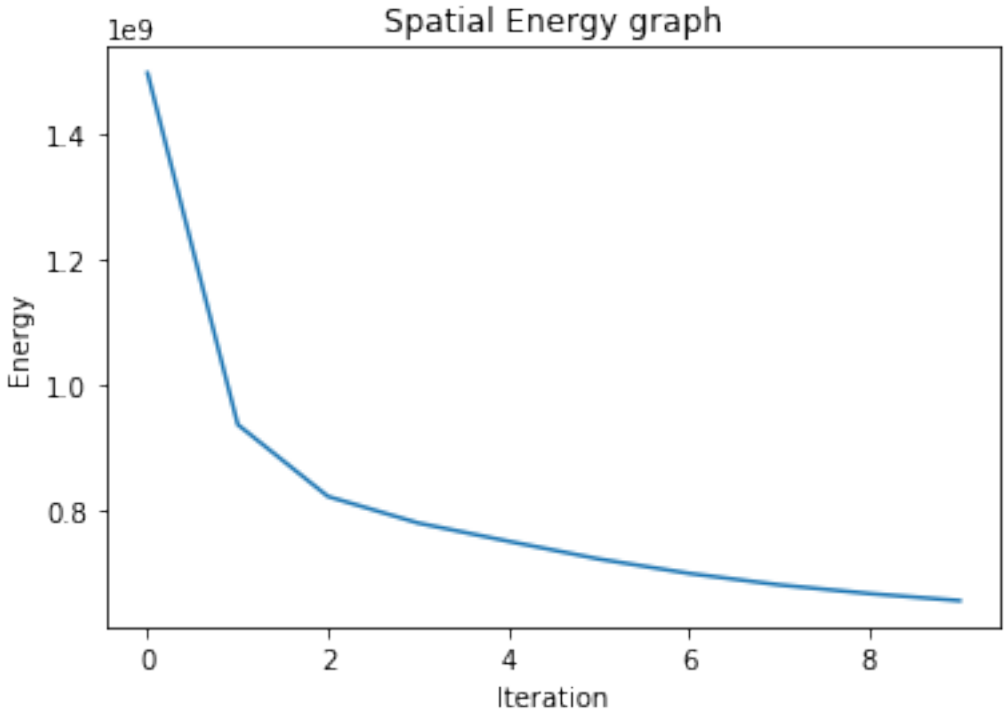

Spatial Energy graph
1e9
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
Energy
0
2
4
6
8
Iteration

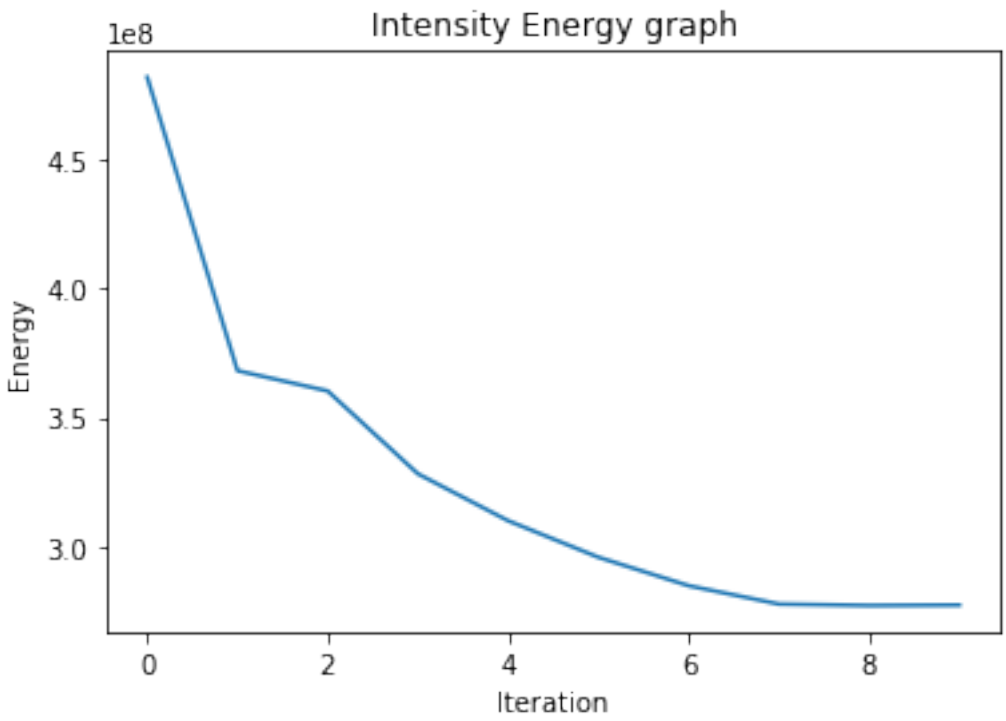

Intensity Energy graph
1e8
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```




Total Energy graph
1e9
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
Energy
0
2
4
6
8
Iteration


Spatial Energy graph
1e9
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

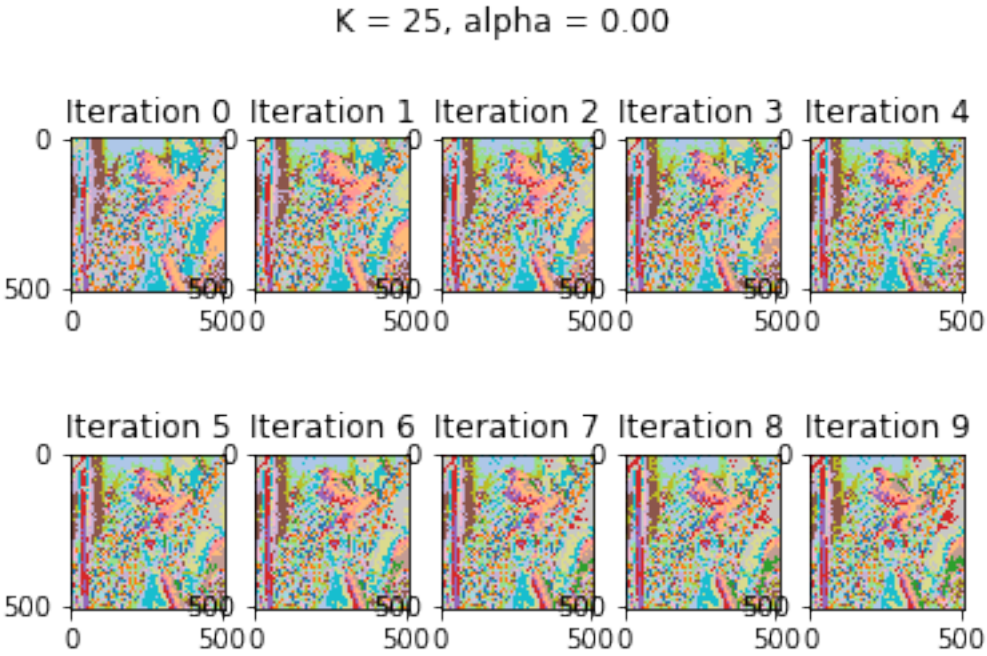


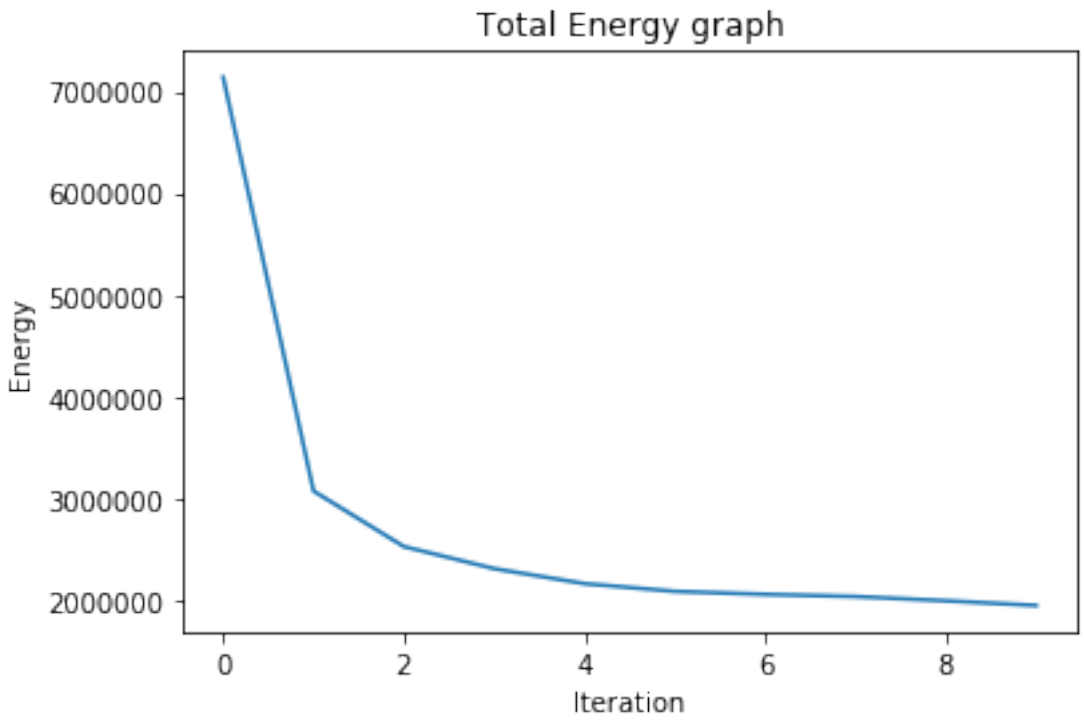
7 K = 25

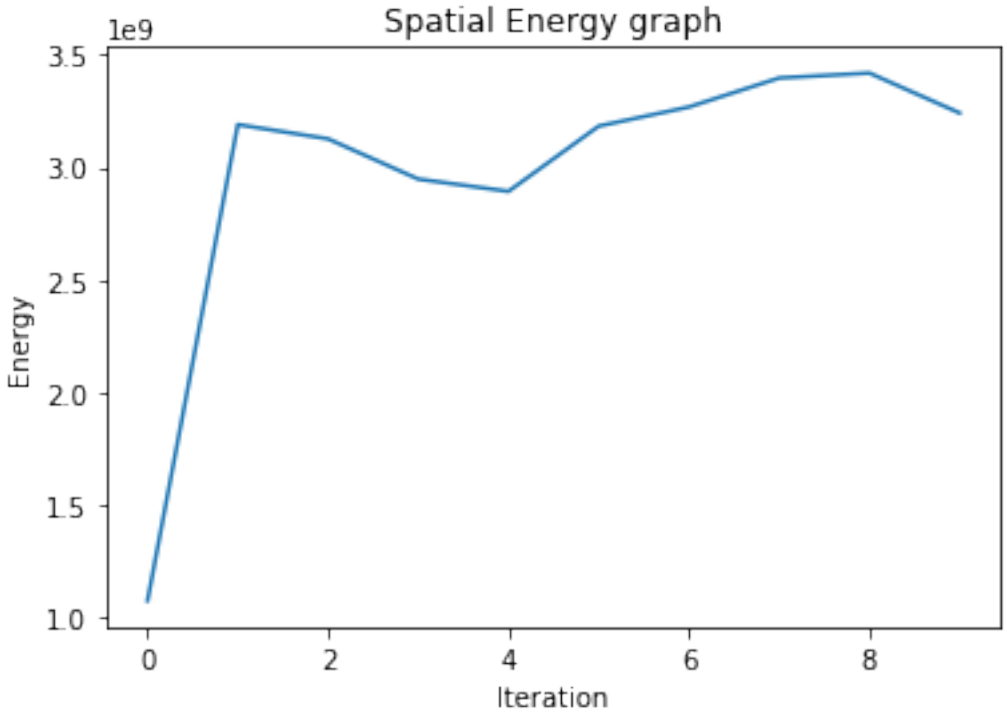
```
In [248]: K = 25
          KMeans(img_matrix, img_rows, img_cols, K, 0)
          KMeans(img_matrix, img_rows, img_cols, K, 0.01)
          KMeans(img_matrix, img_rows, img_cols, K, 0.03)
          KMeans(img_matrix, img_rows, img_cols, K, 0.05)
          KMeans(img_matrix, img_rows, img_cols, K, 0.07)
          KMeans(img_matrix, img_rows, img_cols, K, 0.08)
          KMeans(img_matrix, img_rows, img_cols, K, 0.10)
          KMeans(img_matrix, img_rows, img_cols, K, 0.25)
          KMeans(img_matrix, img_rows, img_cols, K, 0.50)
          KMeans(img_matrix, img_rows, img_cols, K, 0.75)
          KMeans(img_matrix, img_rows, img_cols, K, 1)

/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:66: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:67: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:68: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:32: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:33: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:35: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:36: RuntimeWarning: overi
```

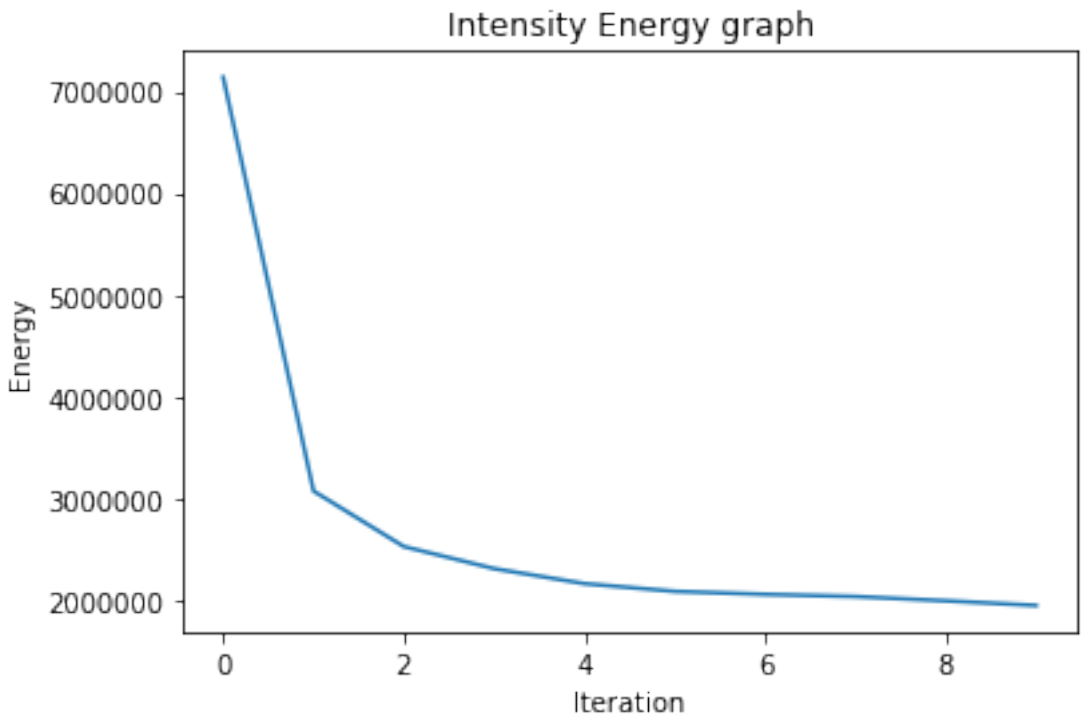
```
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:39: RuntimeWarning: over
```

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

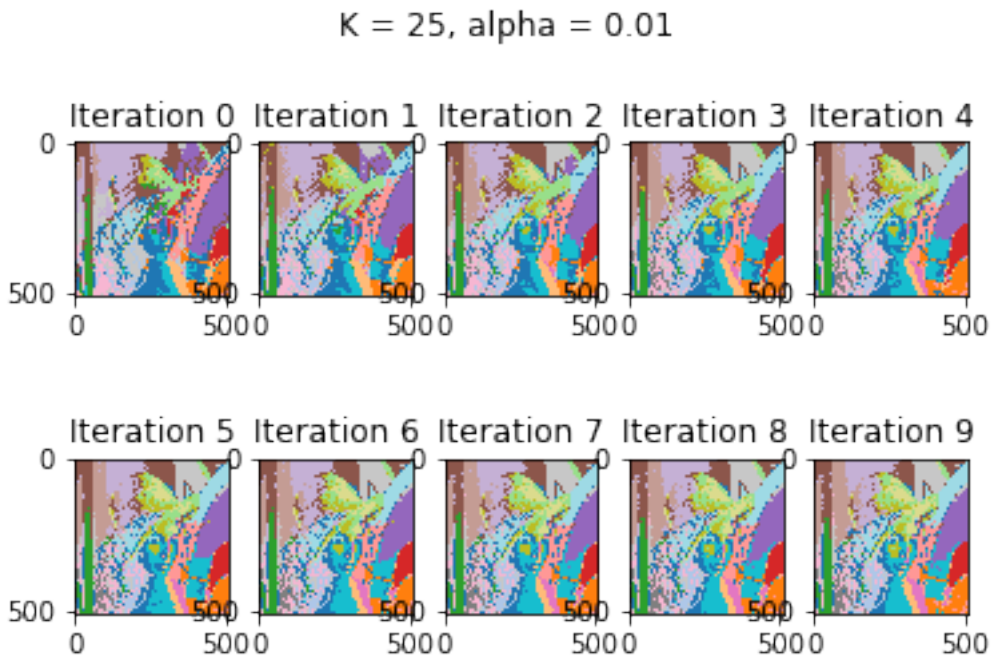


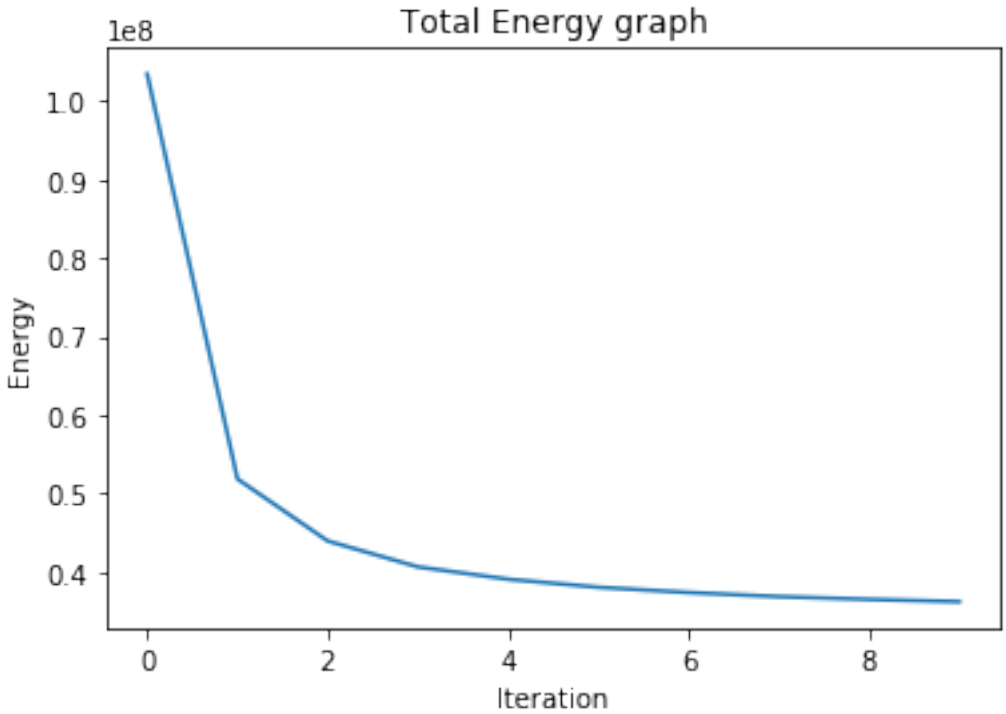

Total Energy graph
7000000
6000000
5000000
4000000
3000000
2000000
Energy
0
2
4
6
8
Iteration

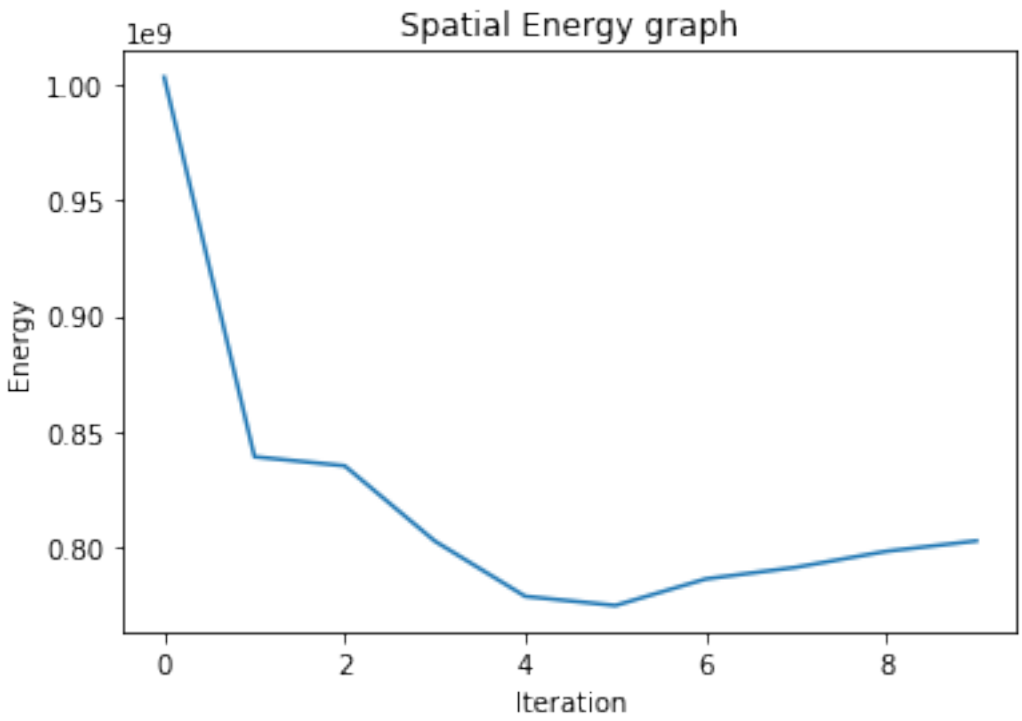

Spatial Energy graph
1e9
3.5
3.0
2.5
2.0
1.5
1.0
Energy
0
2
4
6
8
Iteration

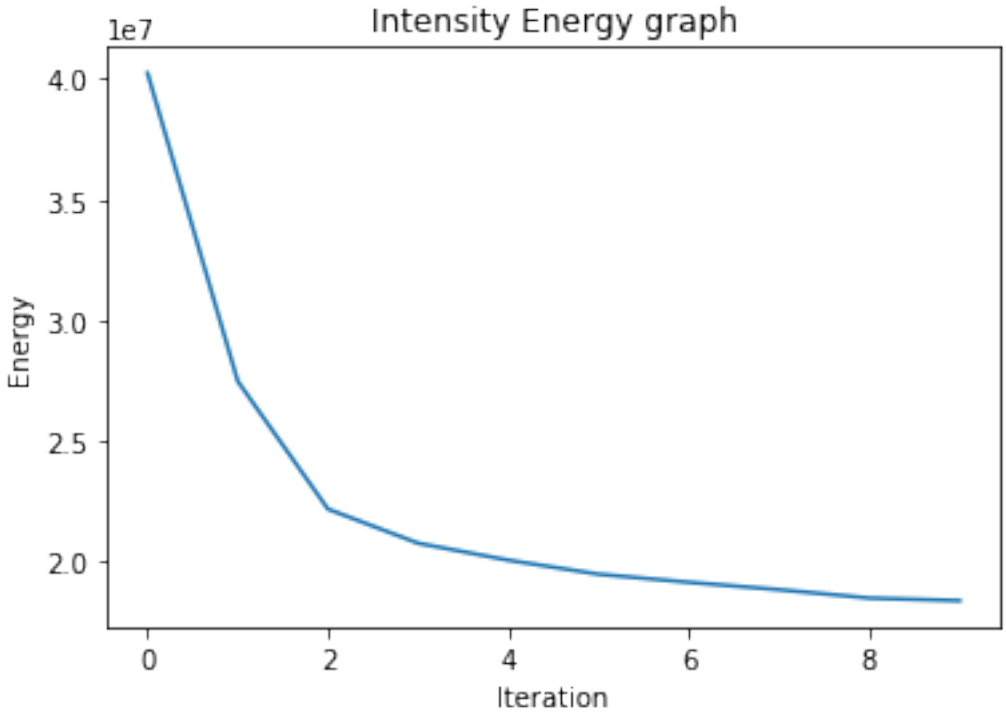



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

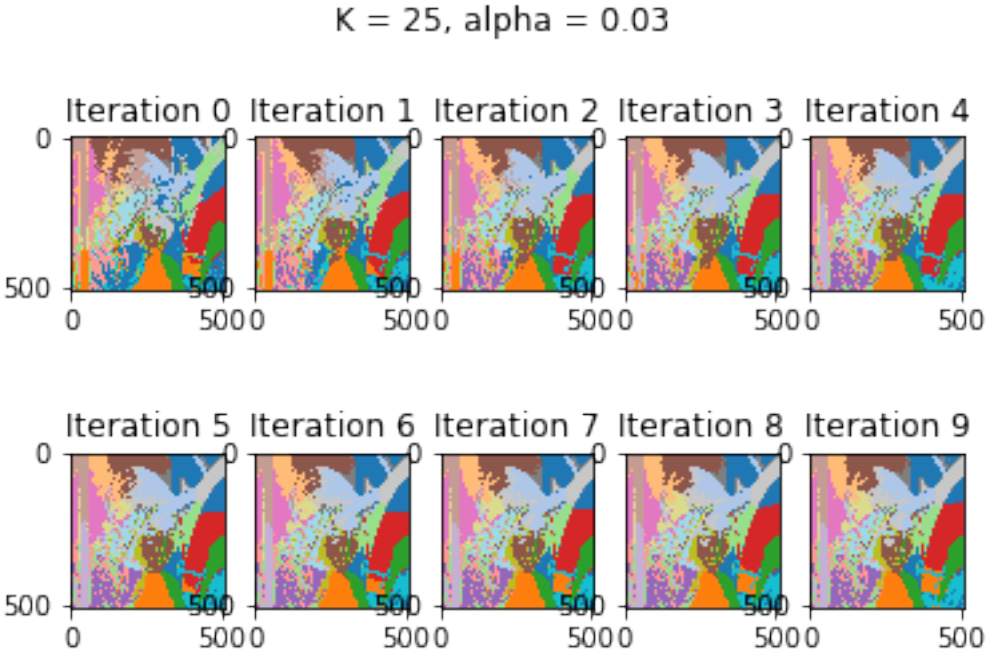

K = 25, alpha = 0.01
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

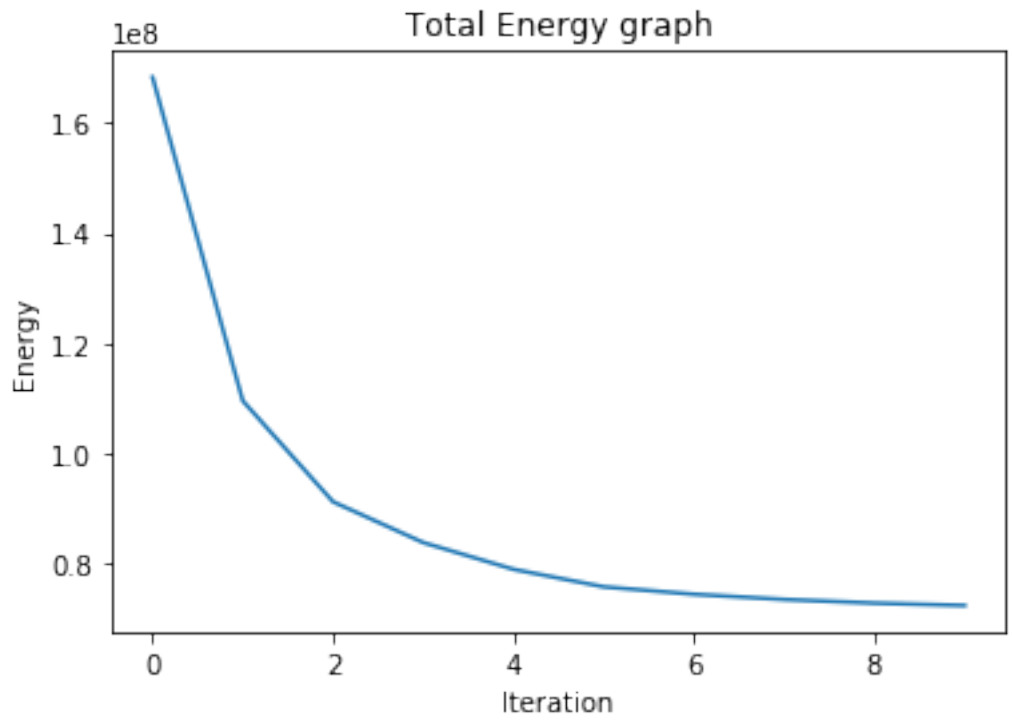

Total Energy graph
1e8
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration

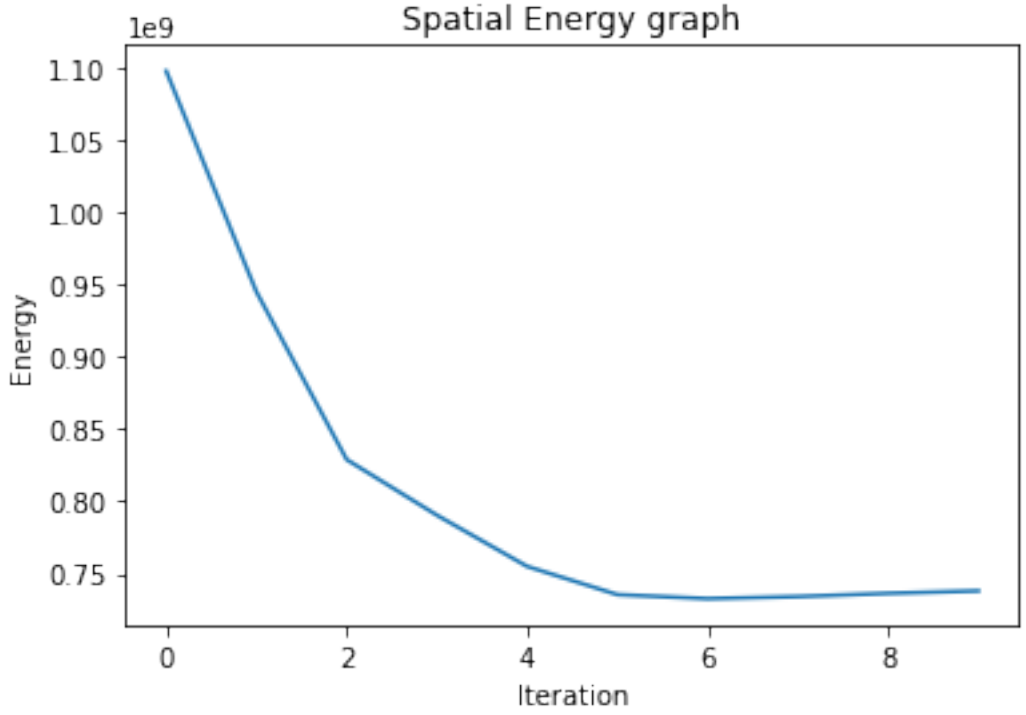

Spatial Energy graph
1e9
1.00
0.95
0.90
0.85
0.80
Energy
0
2
4
6
8
Iteration

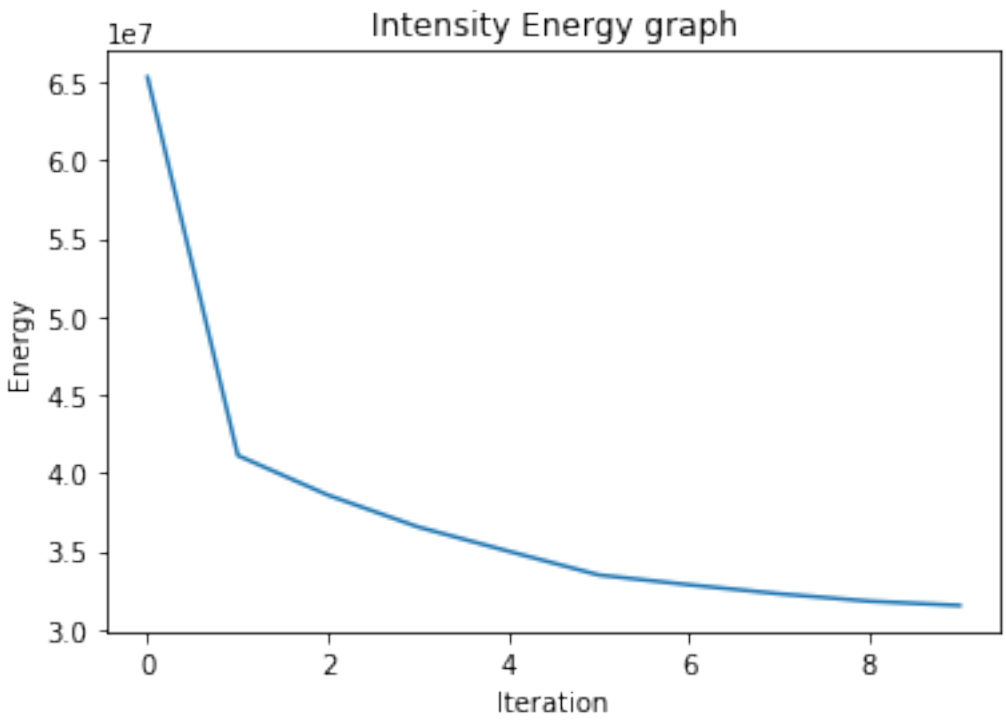

Intensity Energy graph
1e7
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

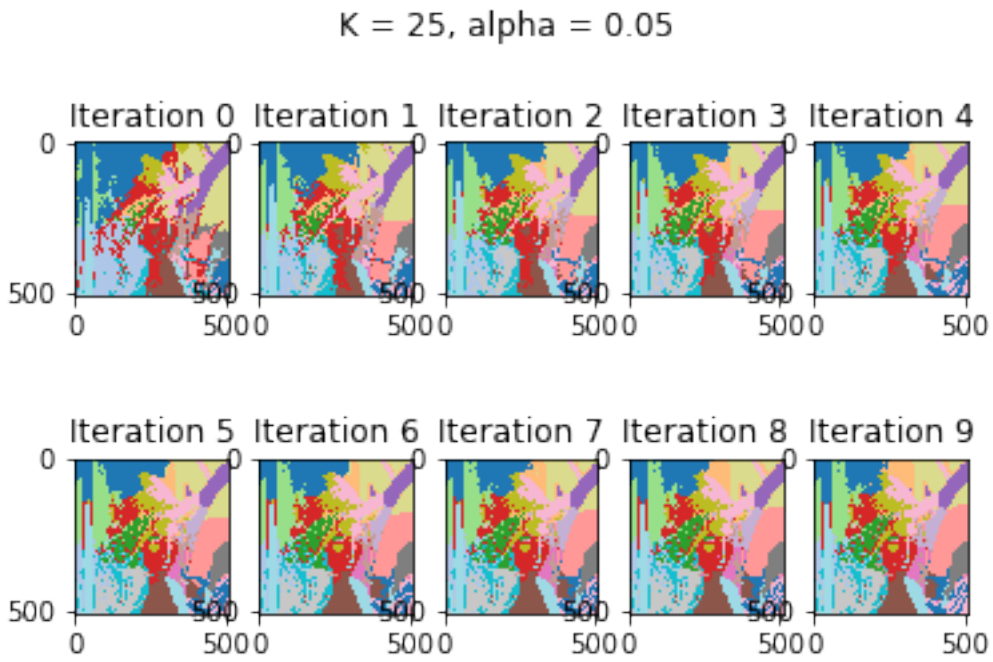


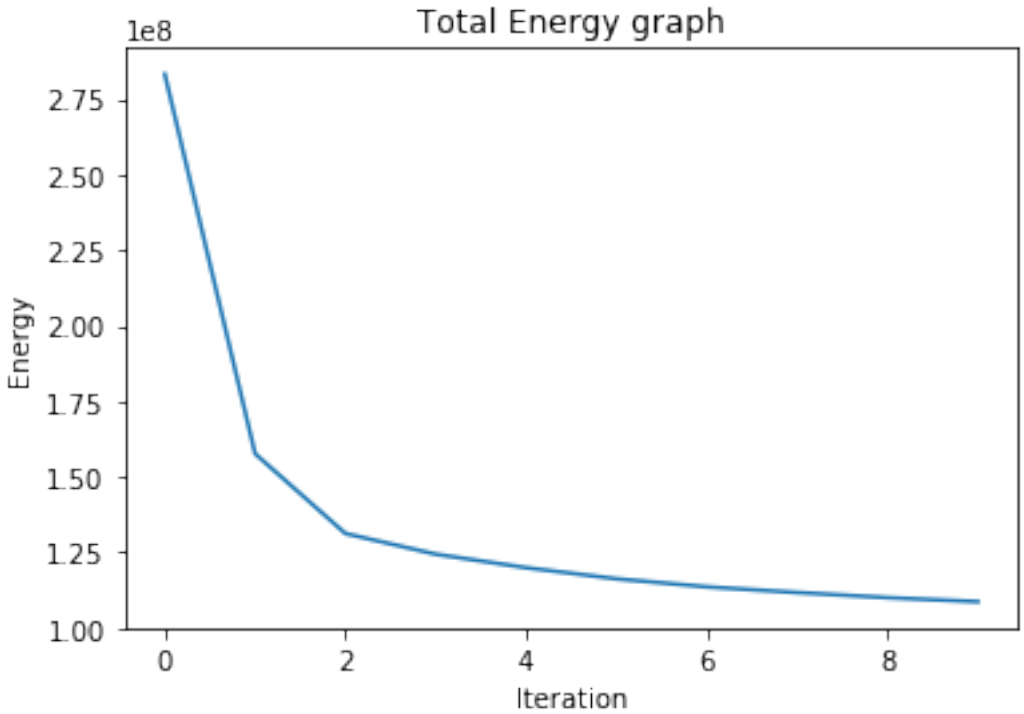

Total Energy graph
1e8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

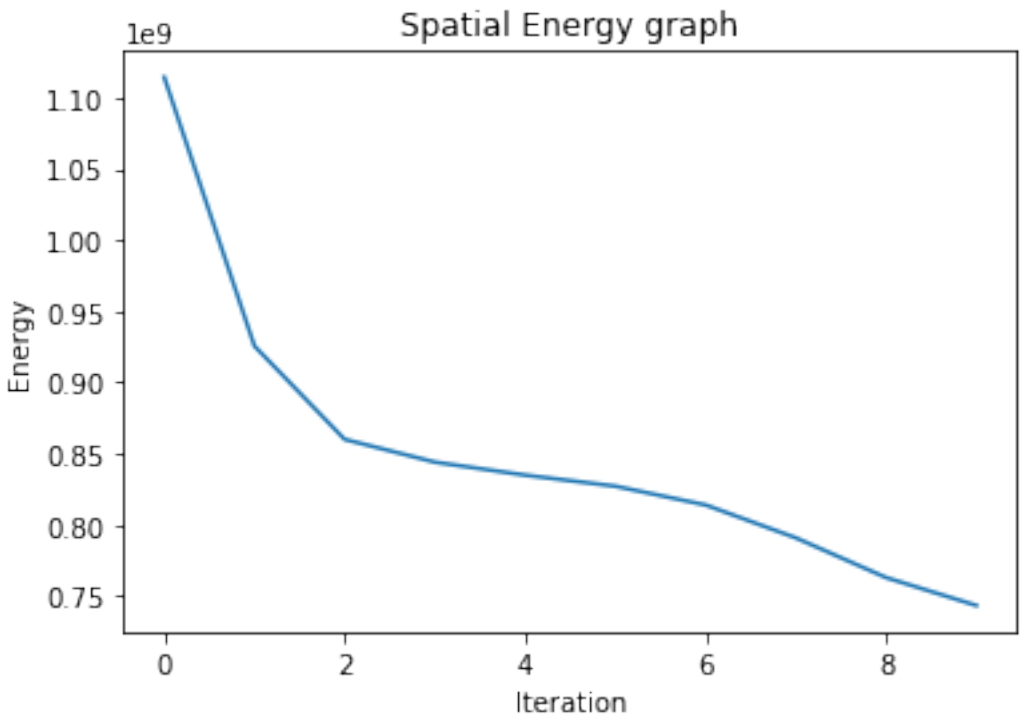

Spatial Energy graph
1e9
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
Energy
0
2
4
6
8
Iteration

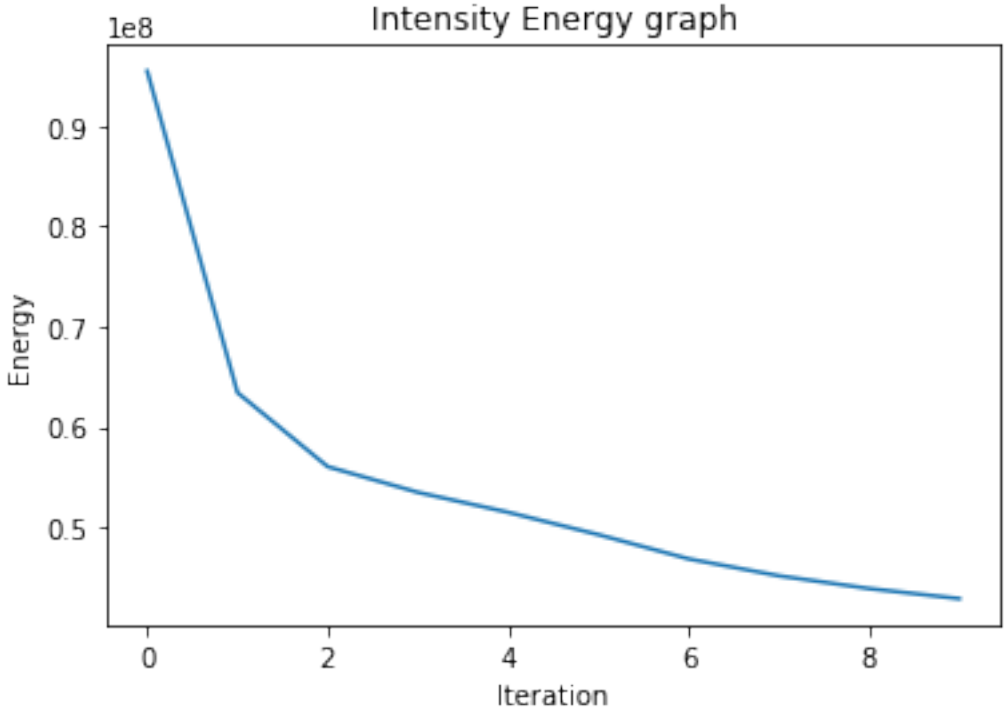



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

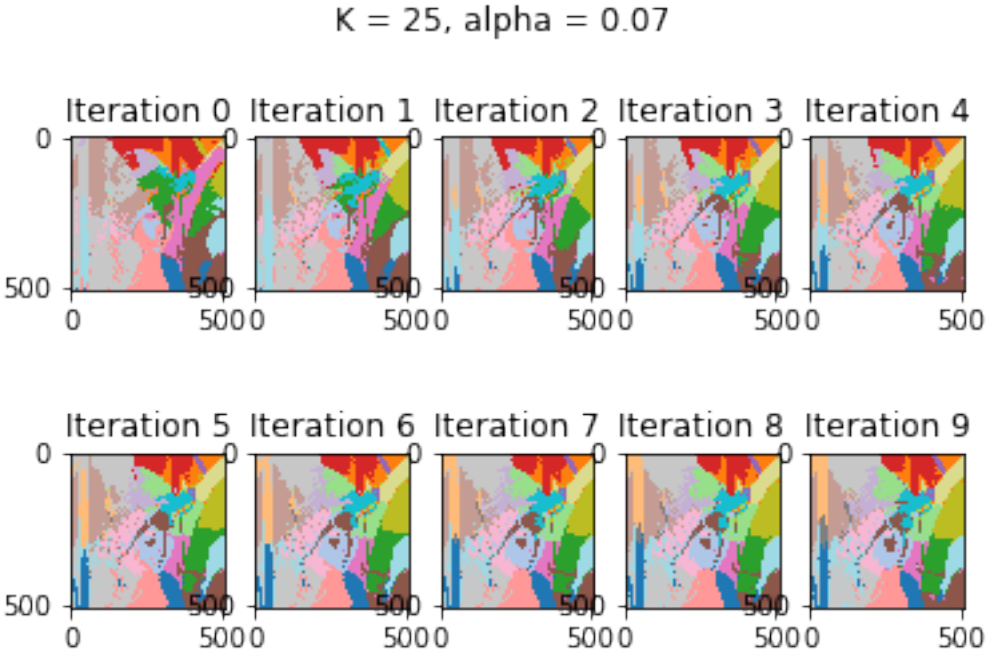

K = 25, alpha = 0.05
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

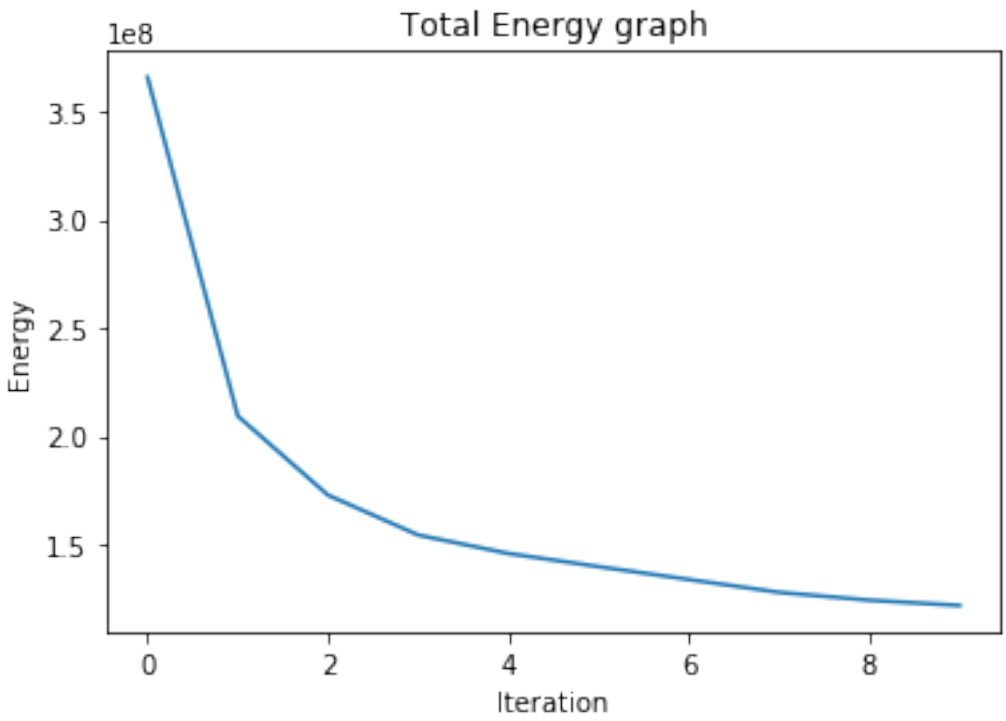

Total Energy graph
1e8
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
Energy
0
2
4
6
8
Iteration

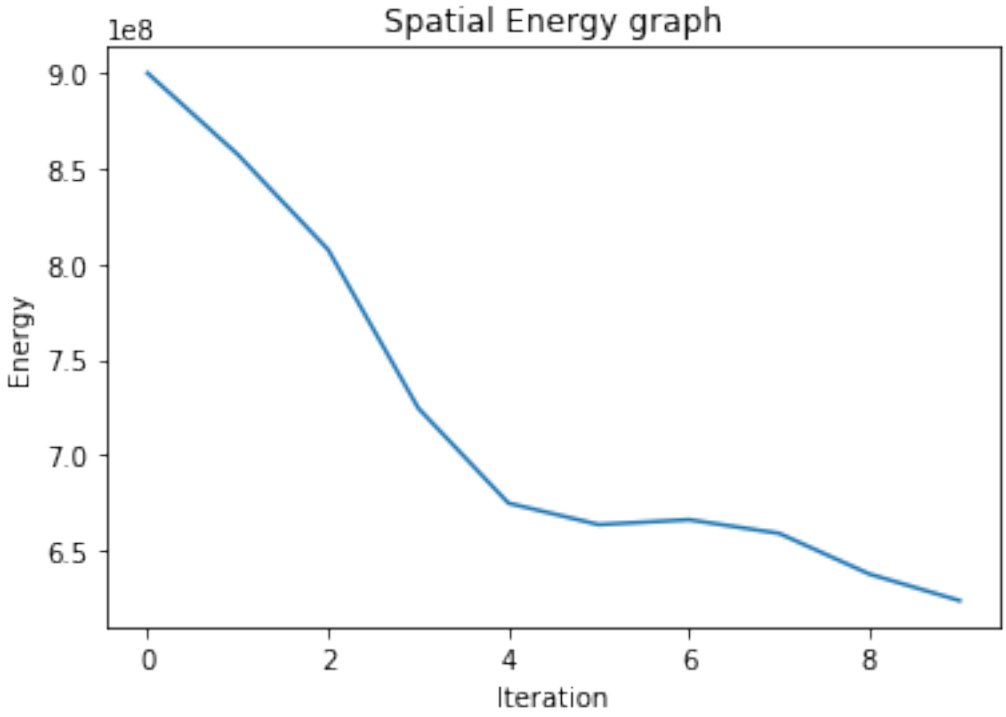

Spatial Energy graph
1e9
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
Energy
0
2
4
6
8
Iteration

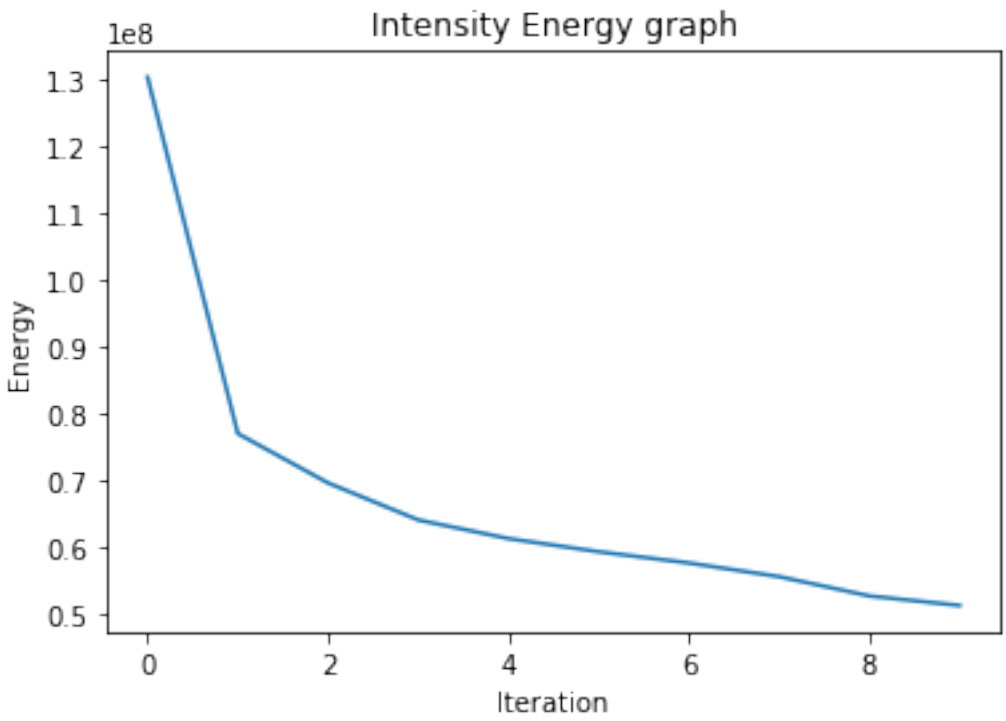

Intensity Energy graph
1e8
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

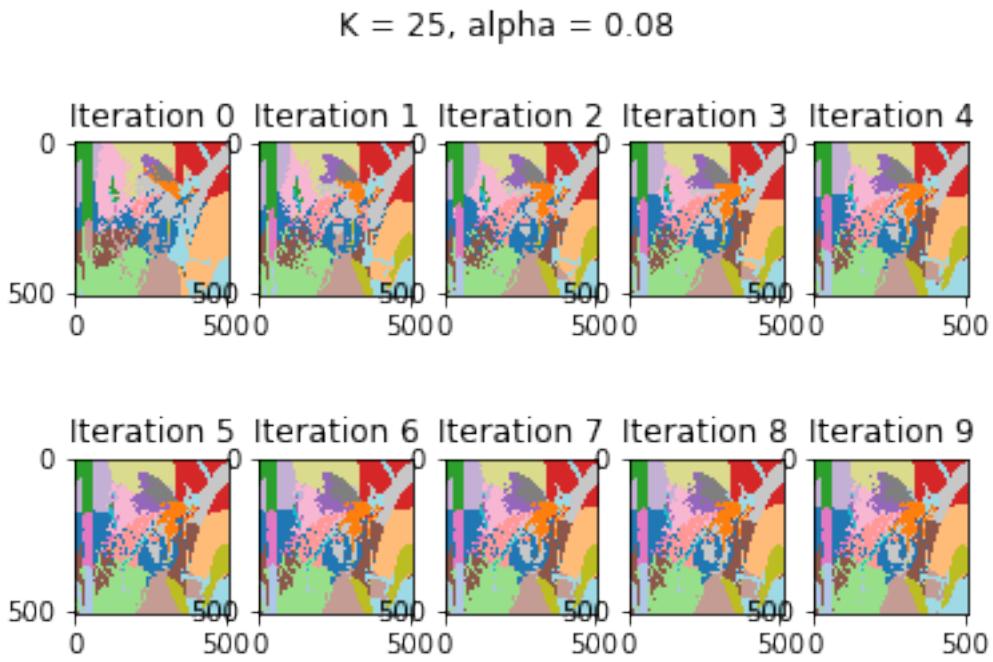


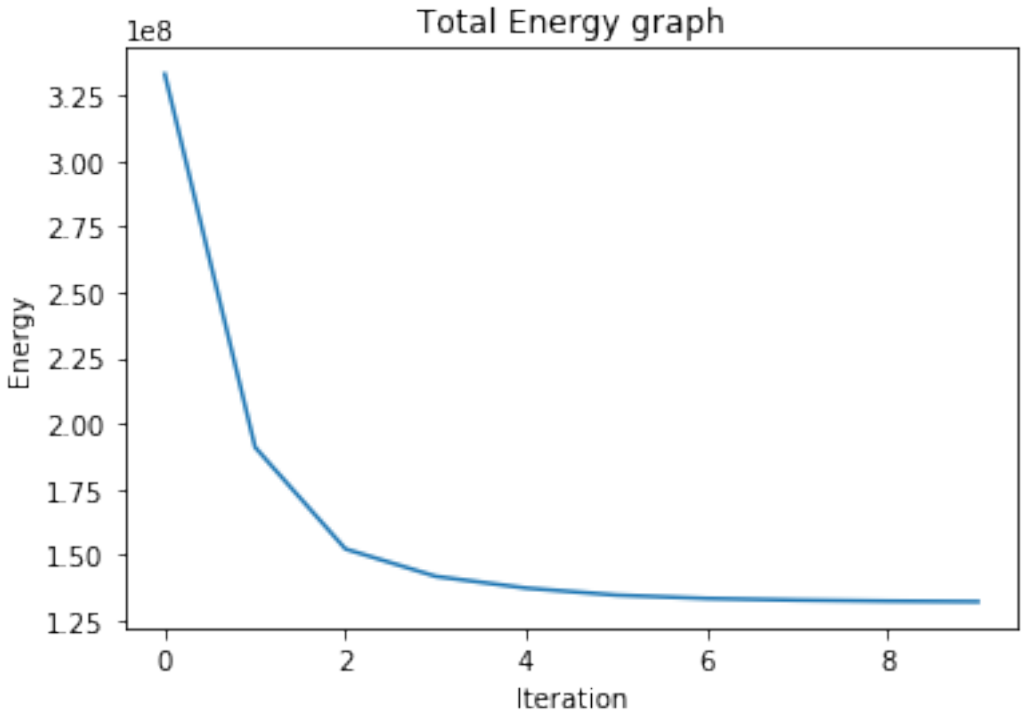

Total Energy graph
1e8
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

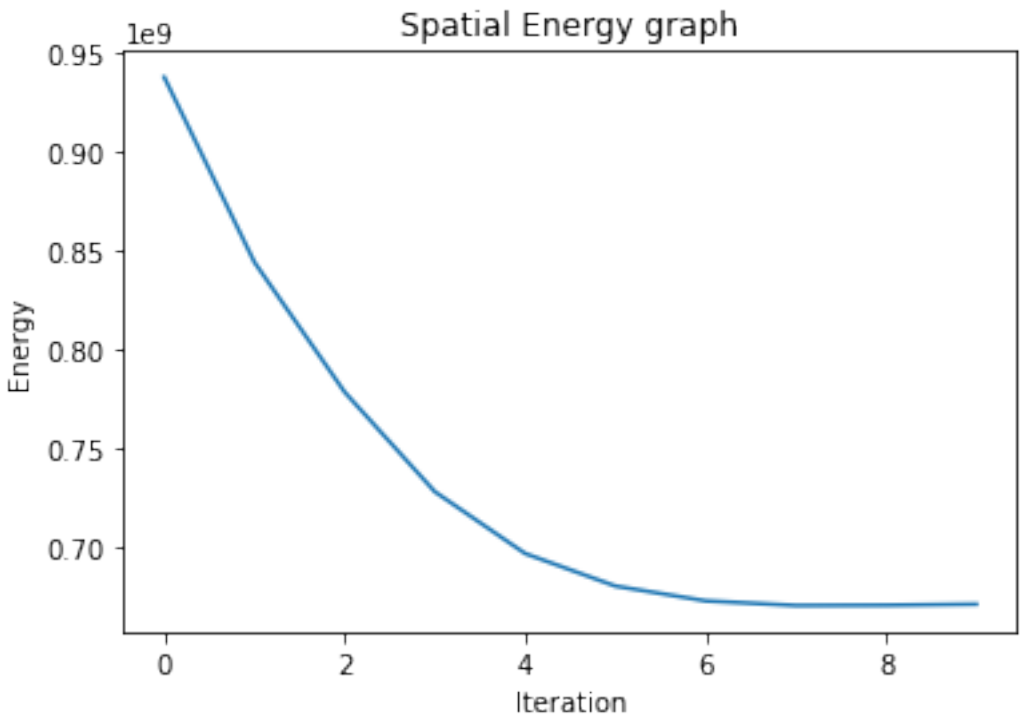

Spatial Energy graph
1e8
9.0
8.5
8.0
7.5
7.0
6.5
Energy
0
2
4
6
8
Iteration

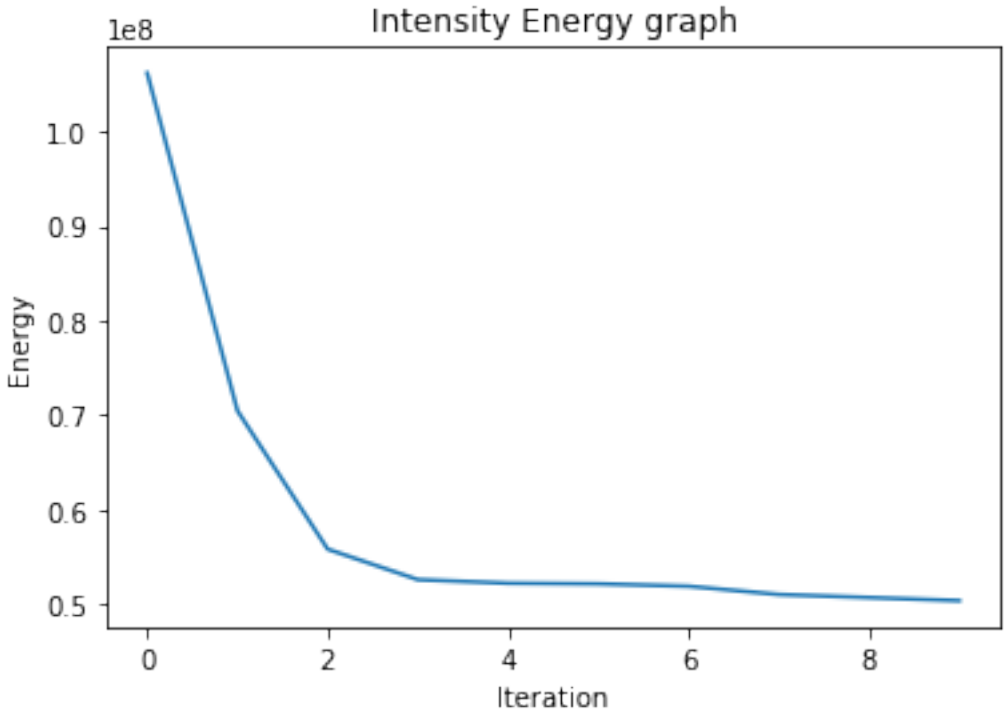



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

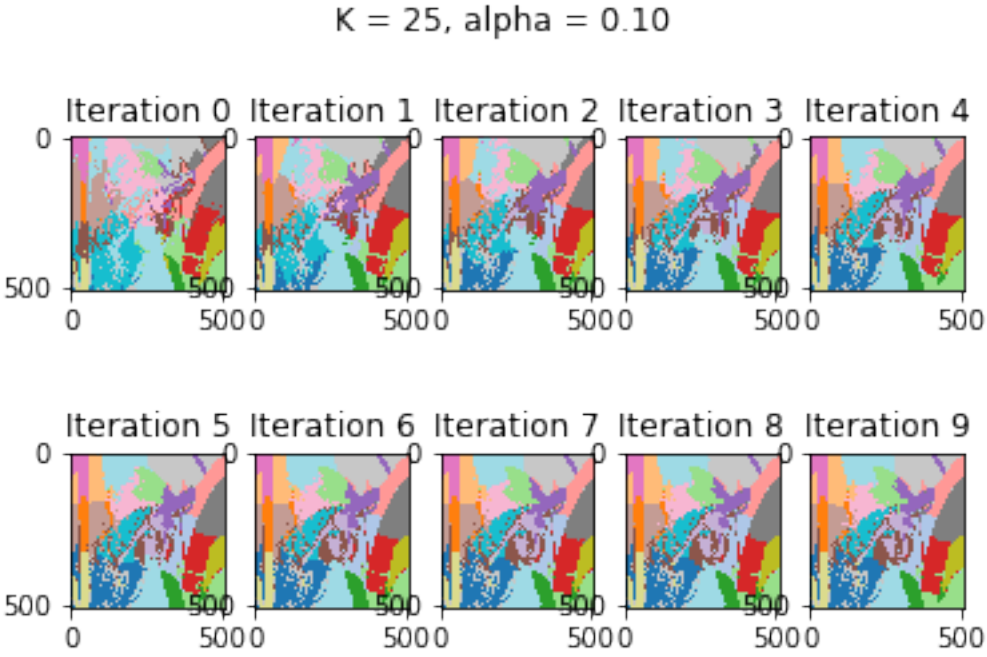

K = 25, alpha = 0.08
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

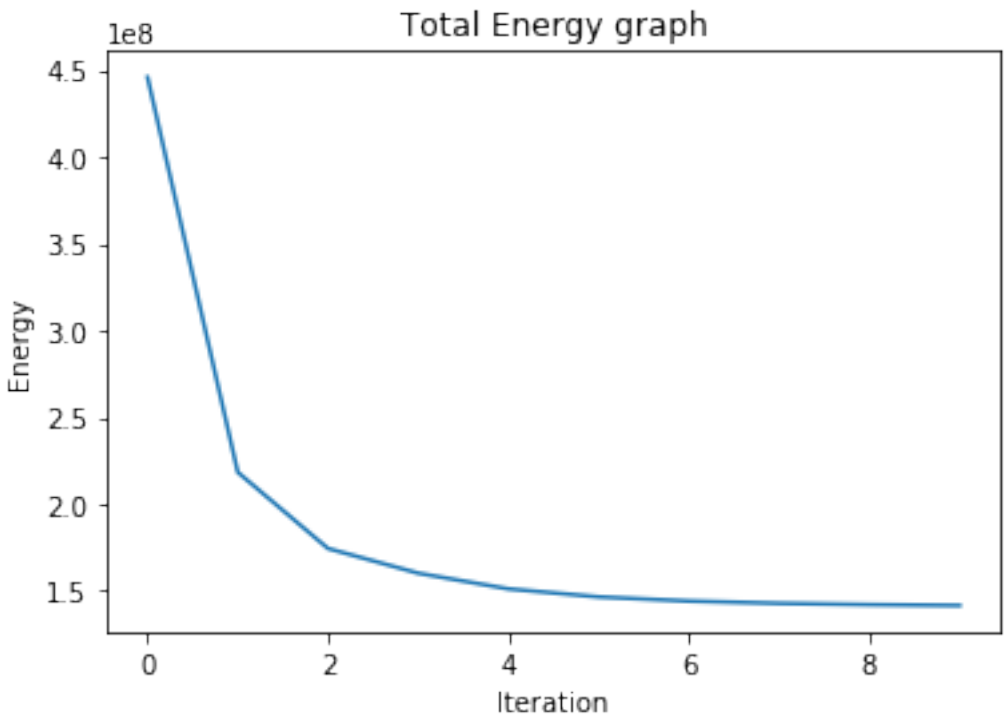

Total Energy graph
1e8
3.25
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
Energy
0
2
4
6
8
Iteration

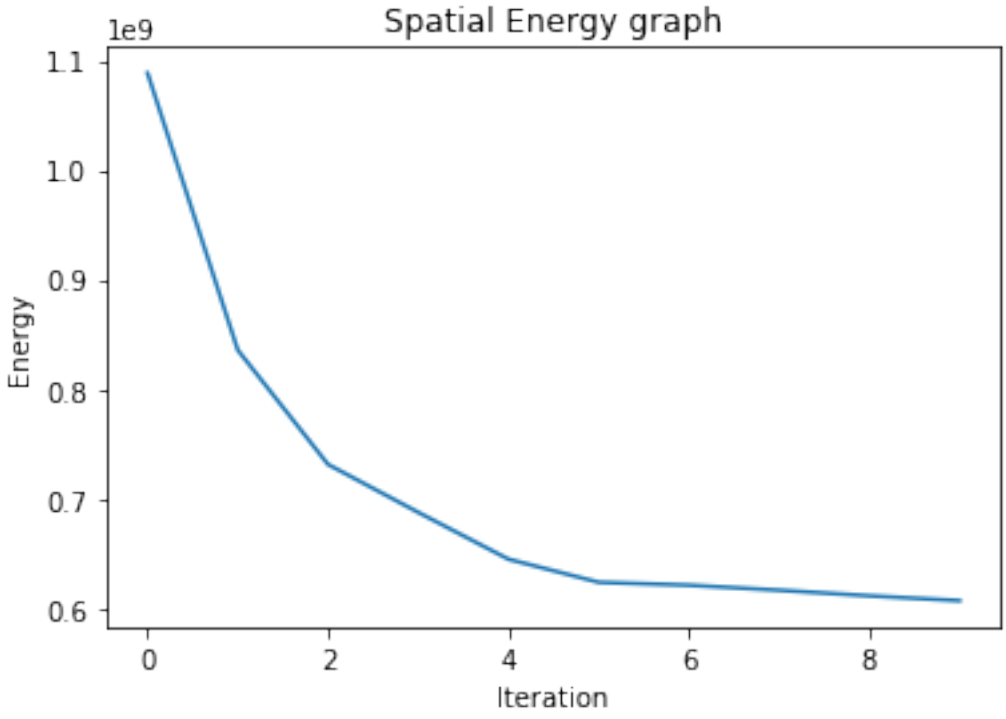

Spatial Energy graph
1e9
0.95
0.90
0.85
0.80
0.75
0.70
Energy
0
2
4
6
8
Iteration

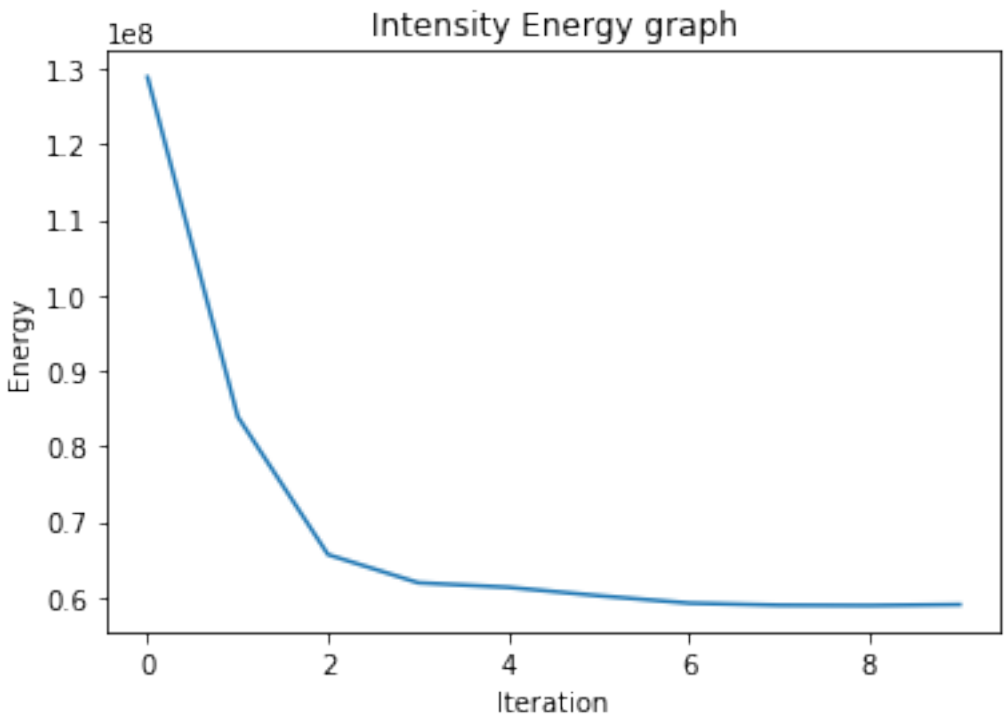

Intensity Energy graph
1e8
1.0
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

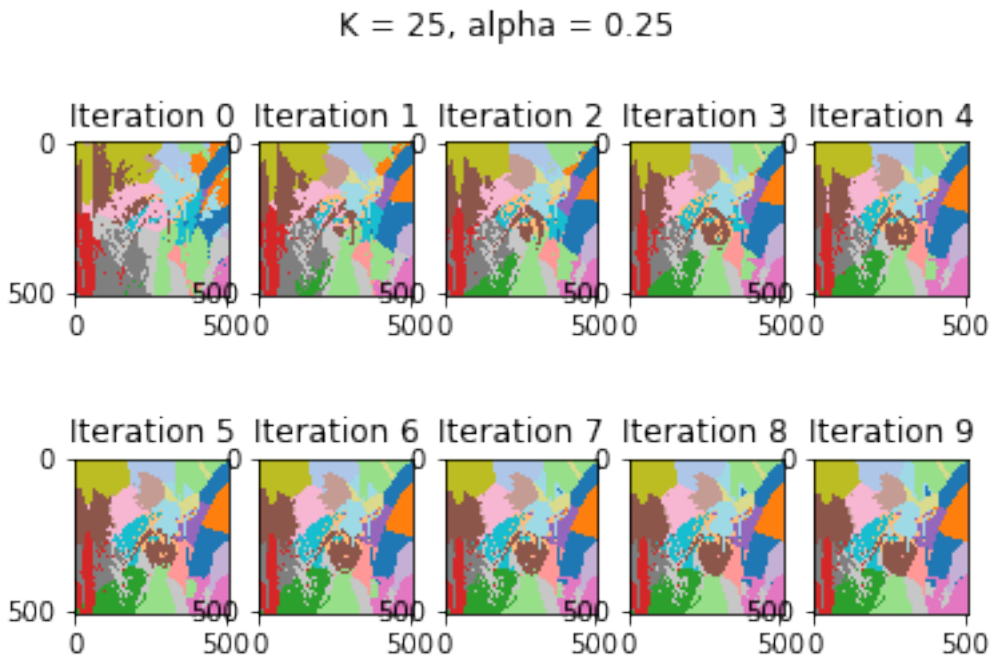


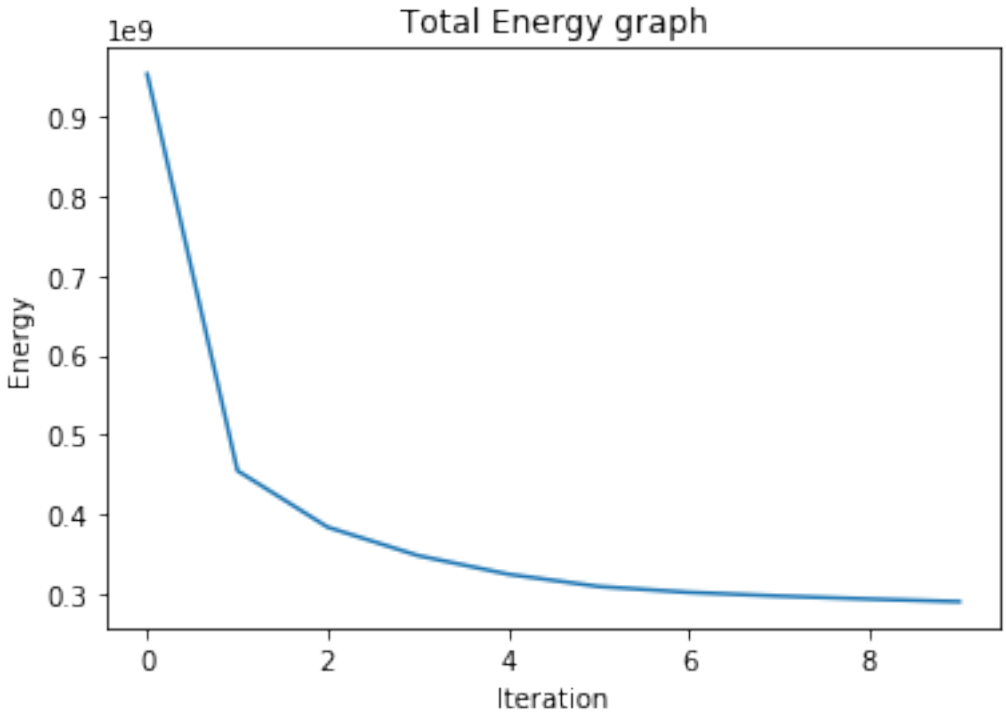

Total Energy graph
1e8
4.5
4.0
3.5
3.0
2.5
2.0
1.5
Energy
0
2
4
6
8
Iteration

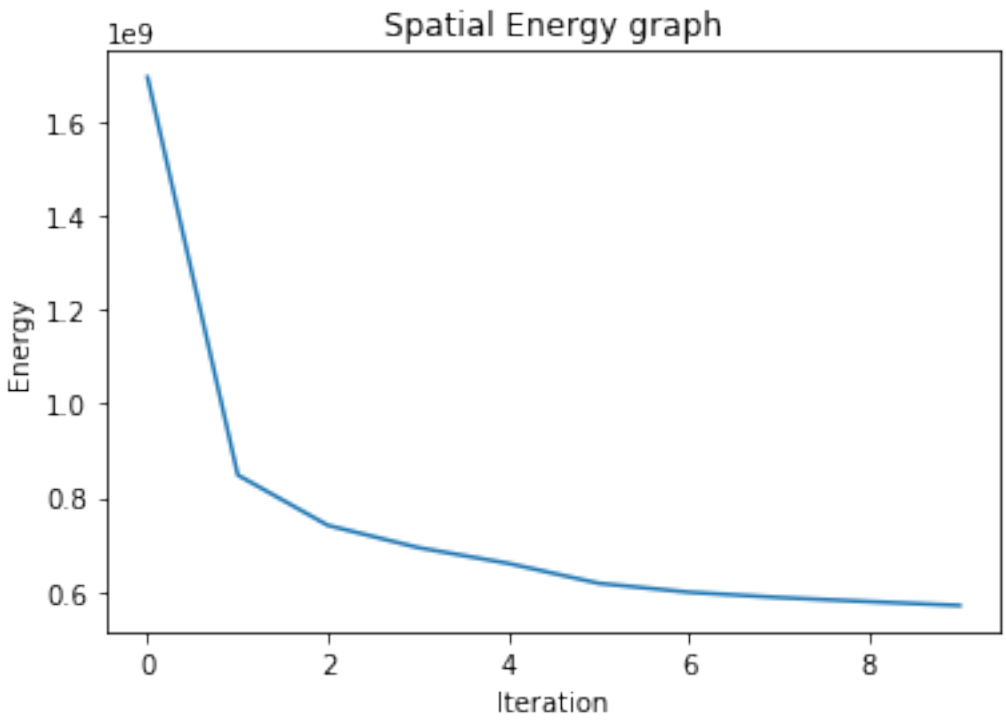

Spatial Energy graph
1e9
1.1
1.0
0.9
0.8
0.7
0.6
Energy
0
2
4
6
8
Iteration

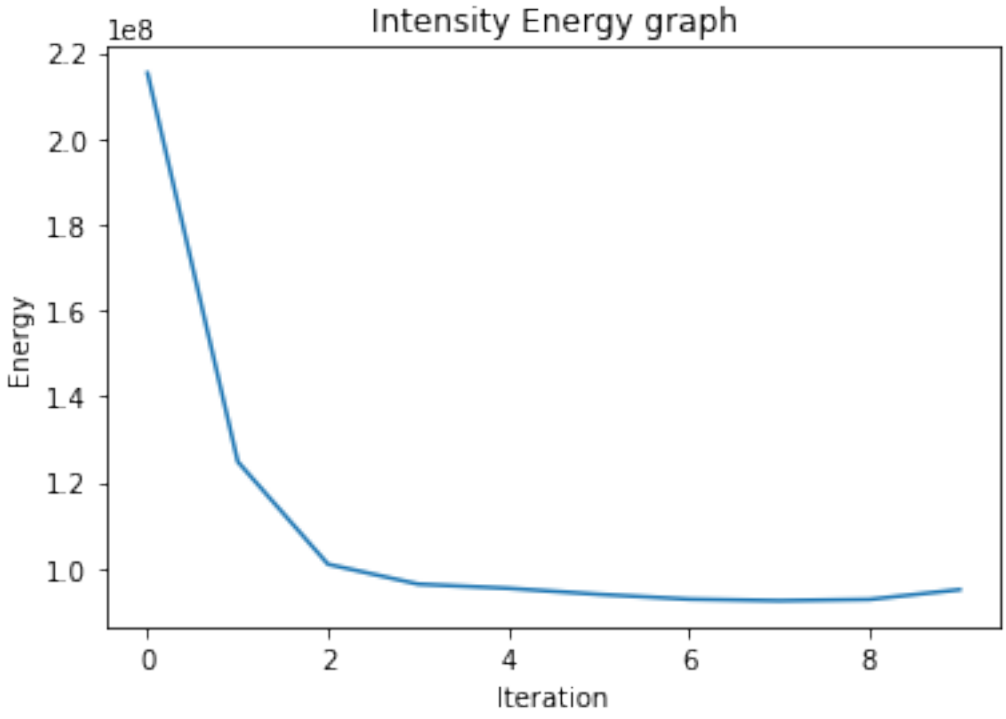



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

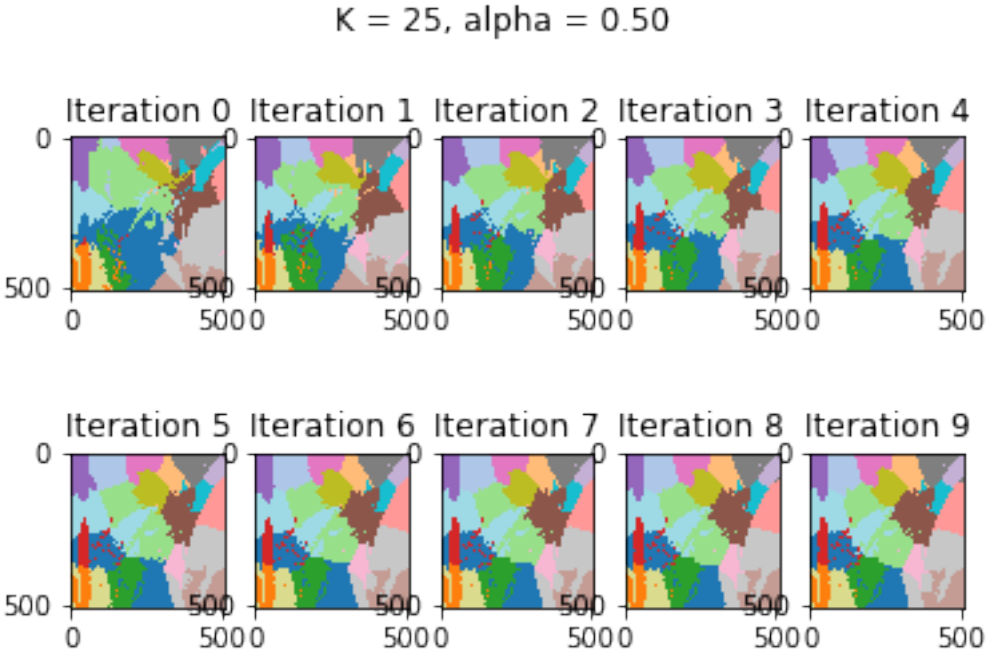

K = 25, alpha = 0.25
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

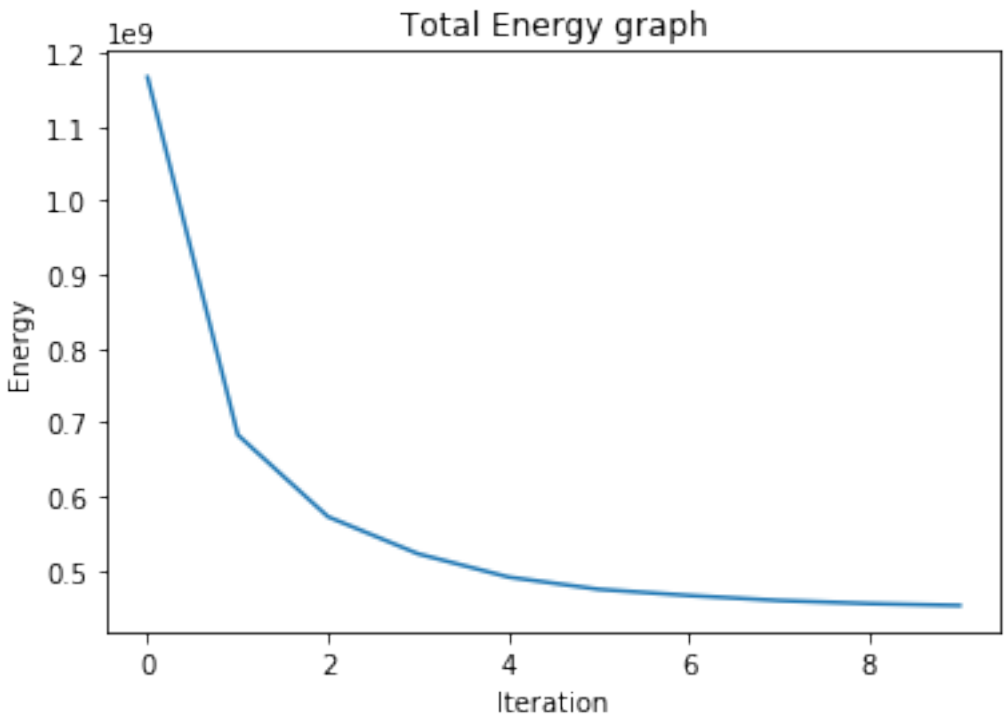

Total Energy graph
1e9
0.9
0.8
0.7
0.6
0.5
0.4
0.3
Energy
0
2
4
6
8
Iteration

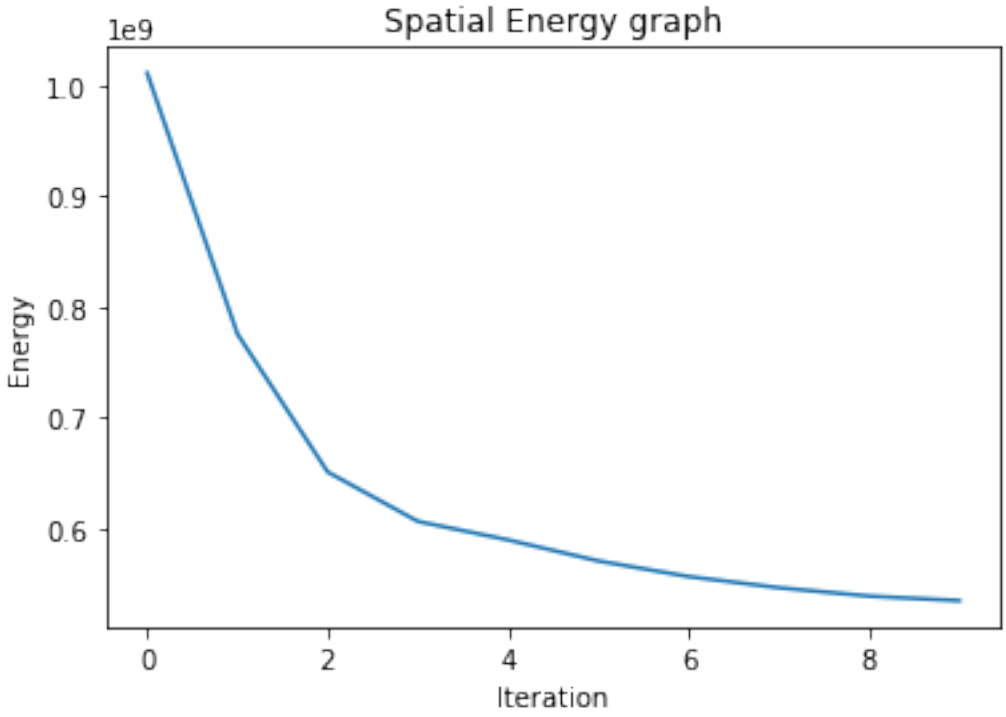

Spatial Energy graph
1e9
1.6
1.4
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

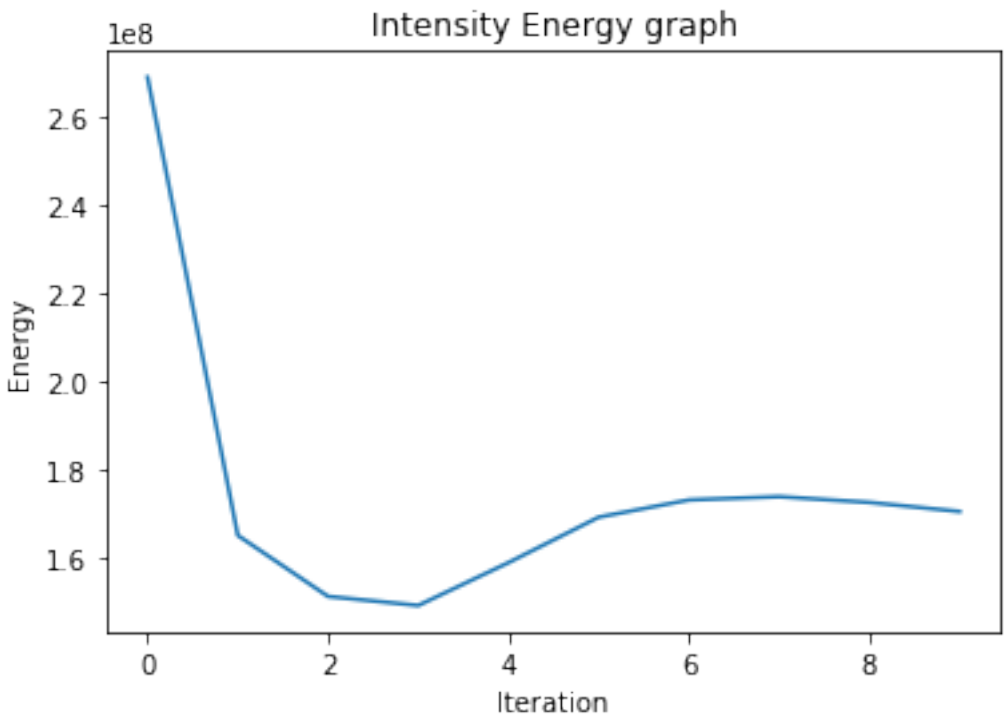

Intensity Energy graph
1e8
2.2
2.0
1.8
1.6
1.4
1.2
1.0
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

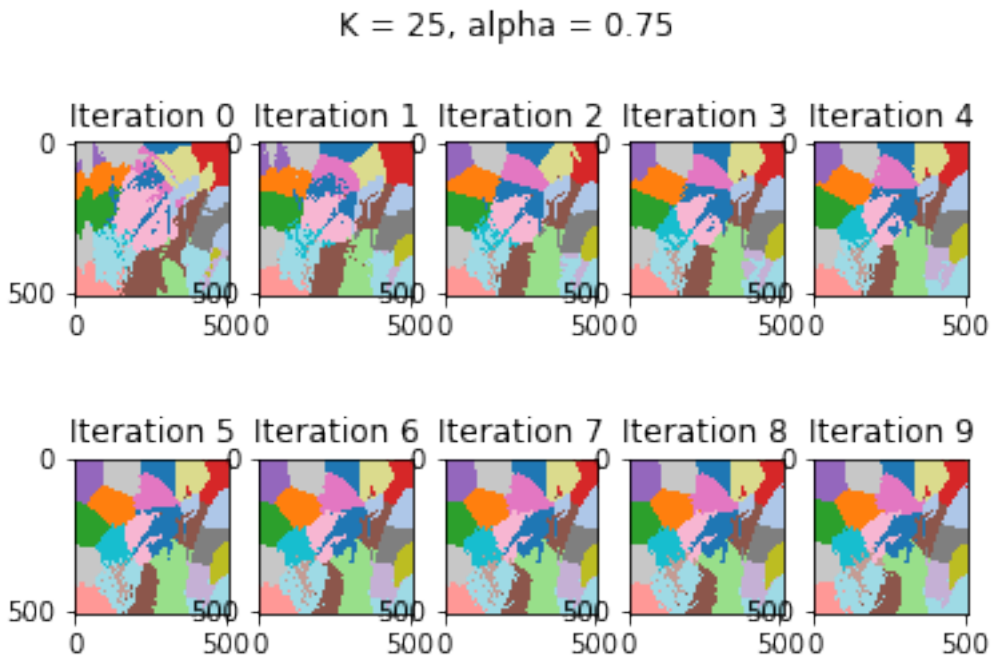


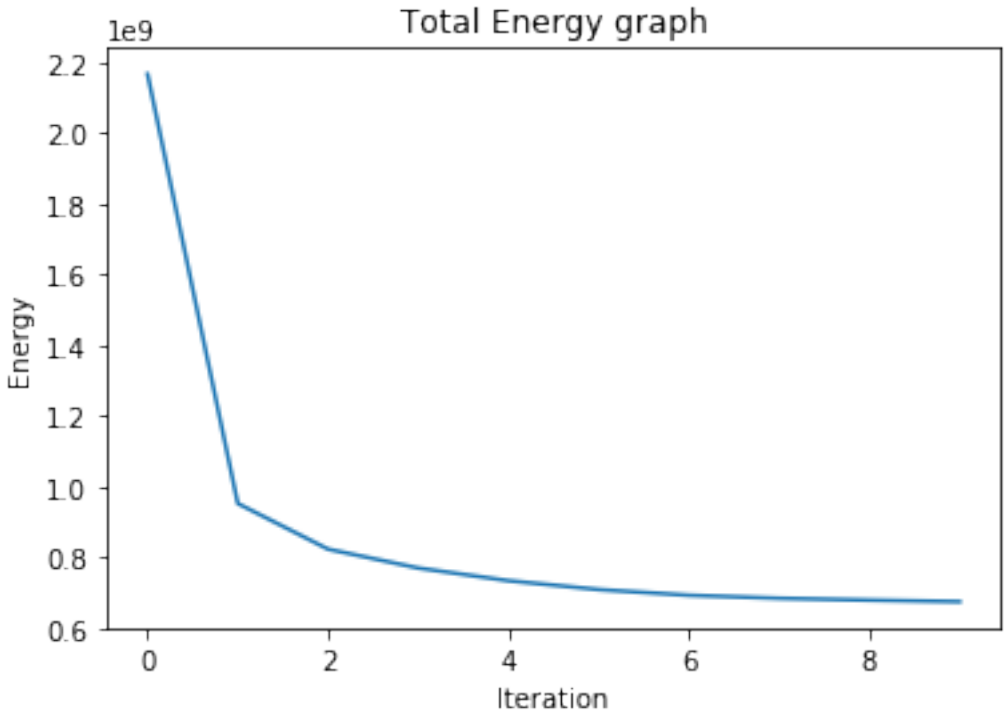

Total Energy graph
1e9
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

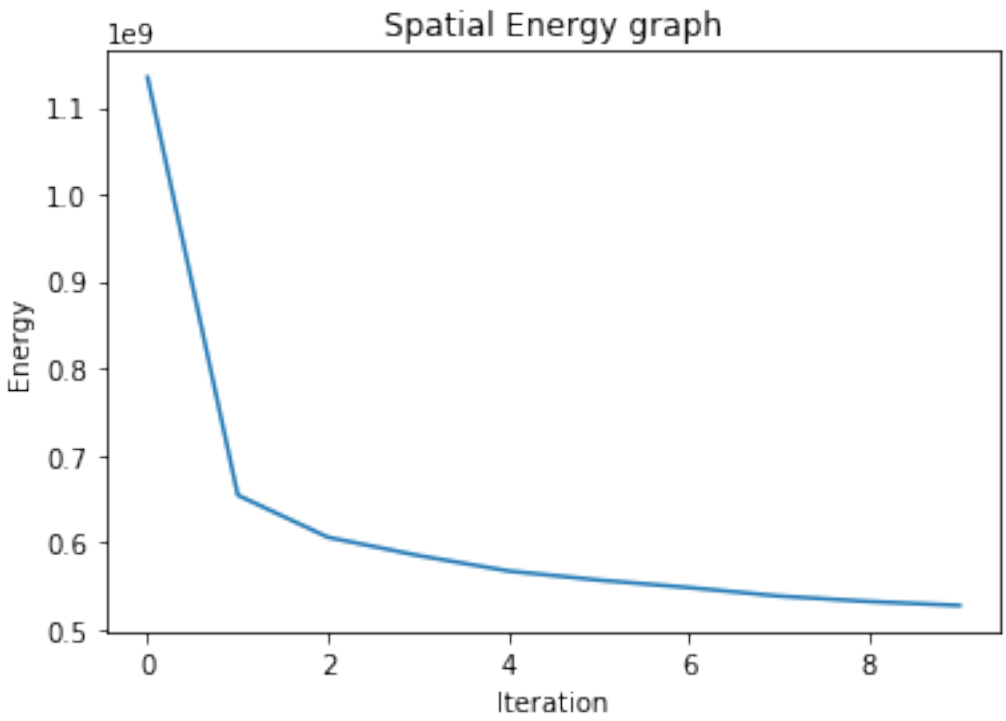

Spatial Energy graph
1e9
1.0
0.9
0.8
0.7
0.6
Energy
0
2
4
6
8
Iteration

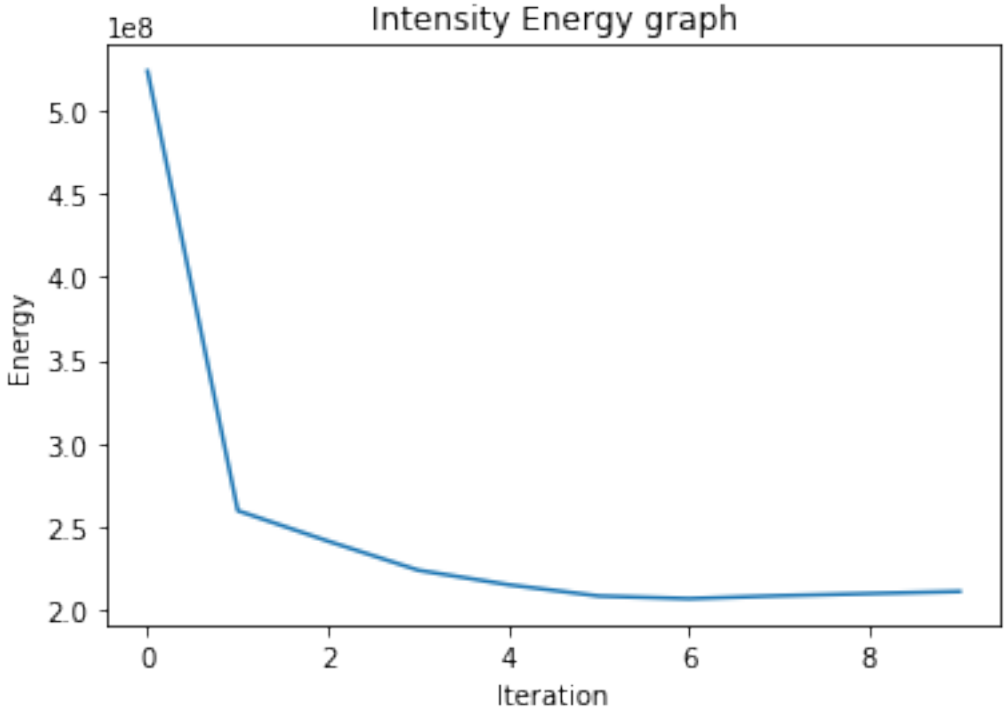



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

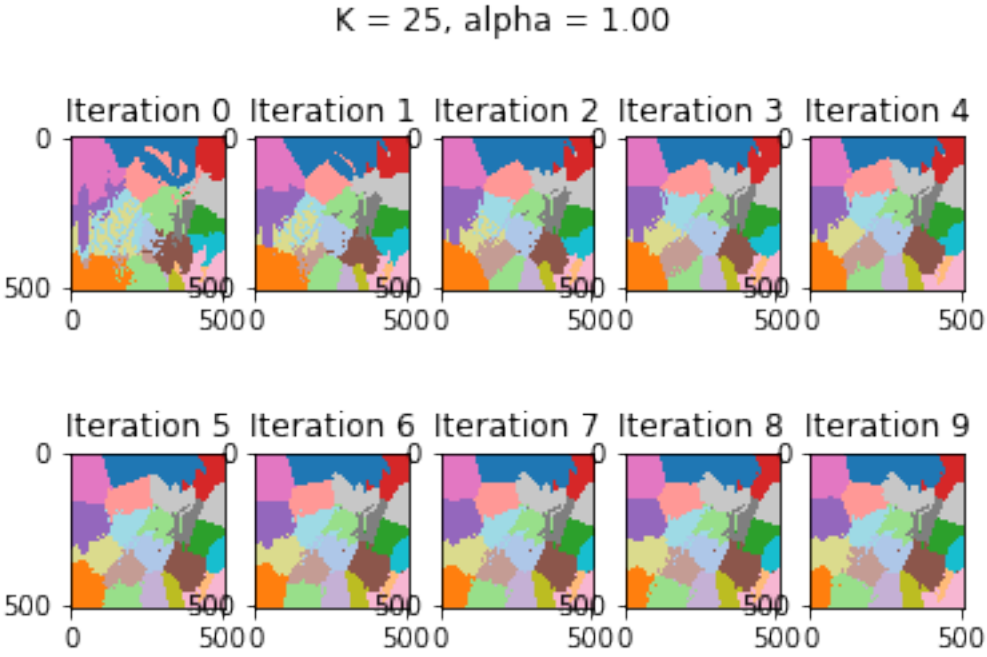

K = 25, alpha = 0.75
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

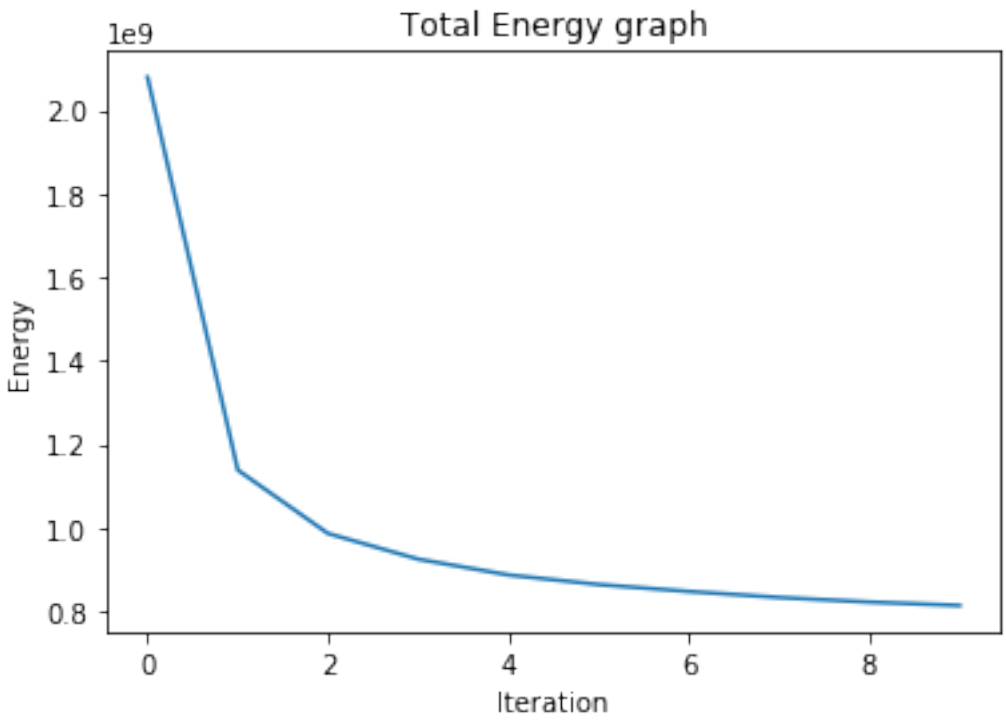

Total Energy graph
1e9
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

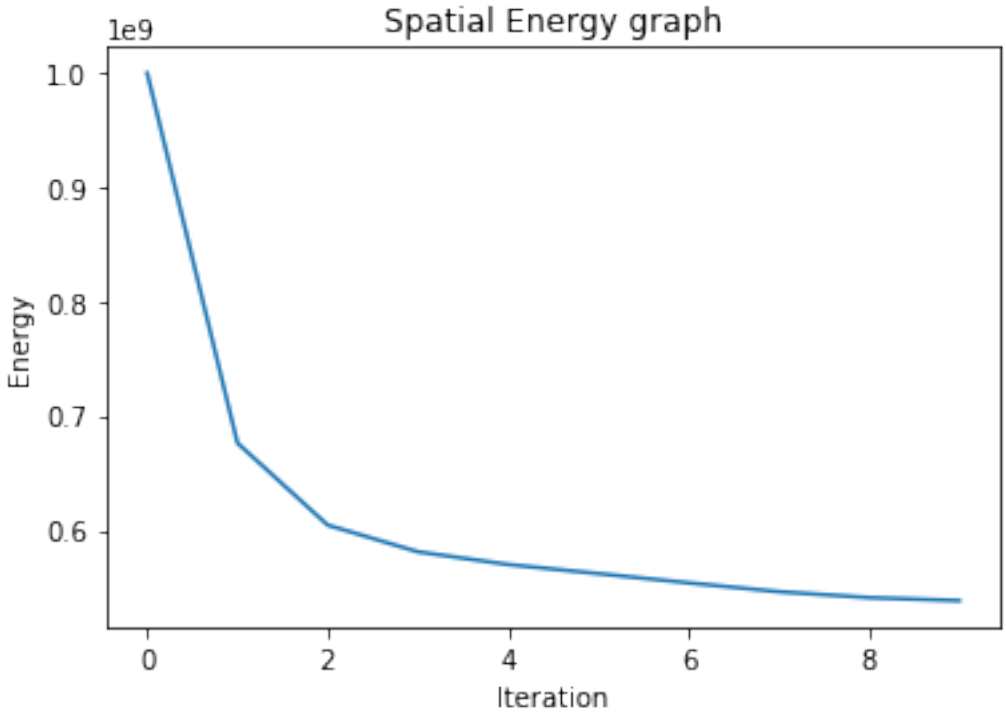

Spatial Energy graph
1e9
1.1
1.0
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

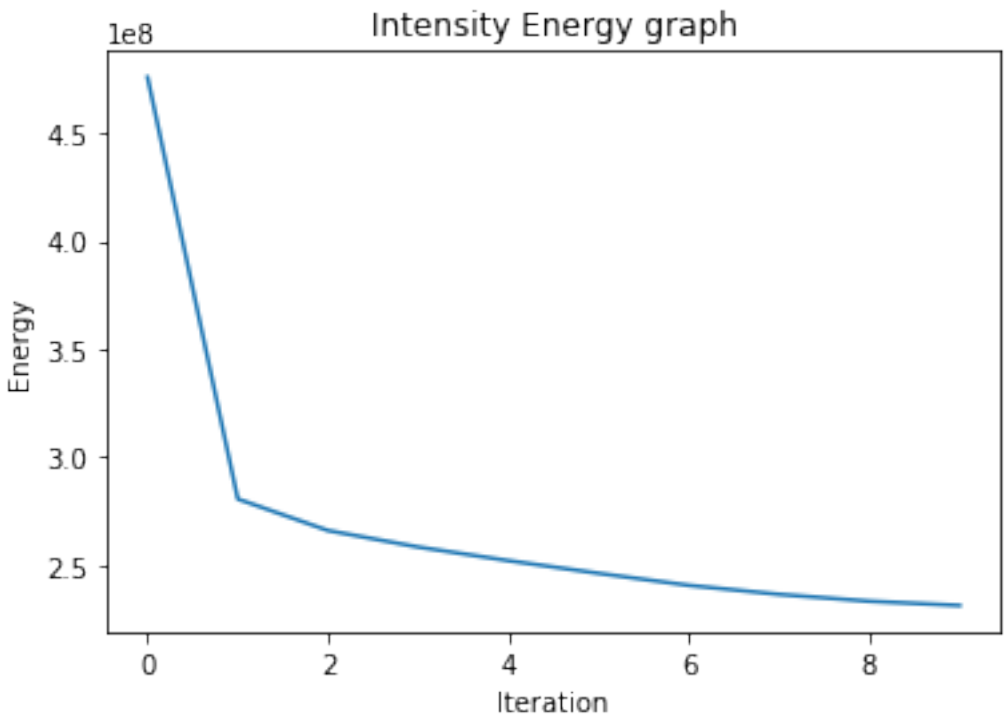

Intensity Energy graph
1e8
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```




Total Energy graph
1e9
2.0
1.8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration


Spatial Energy graph
1e9
1.0
0.9
0.8
0.7
0.6
Energy
0
2
4
6
8
Iteration



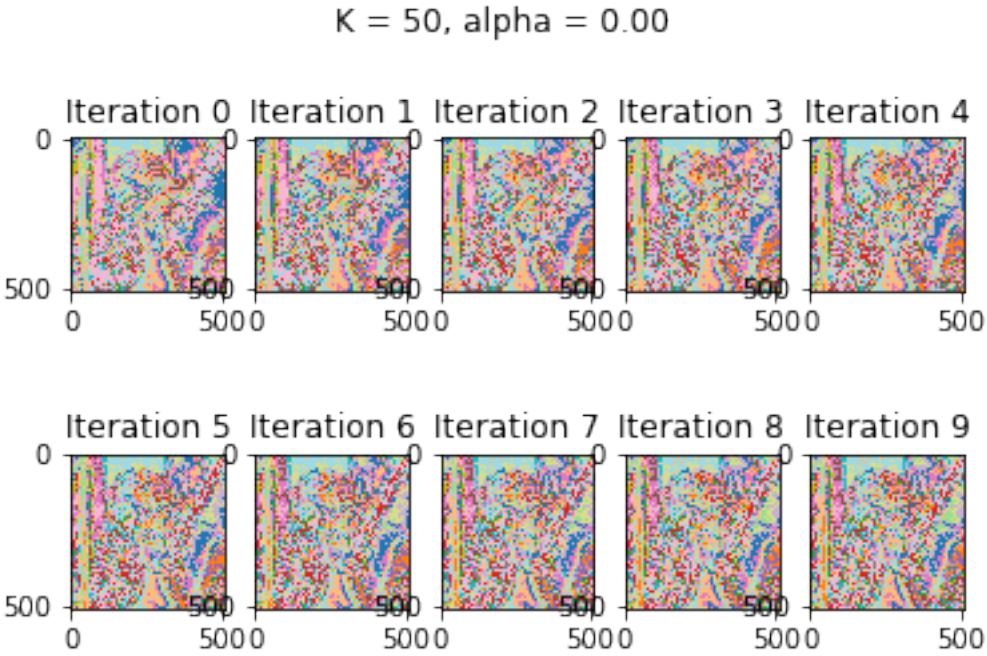
8 K = 50

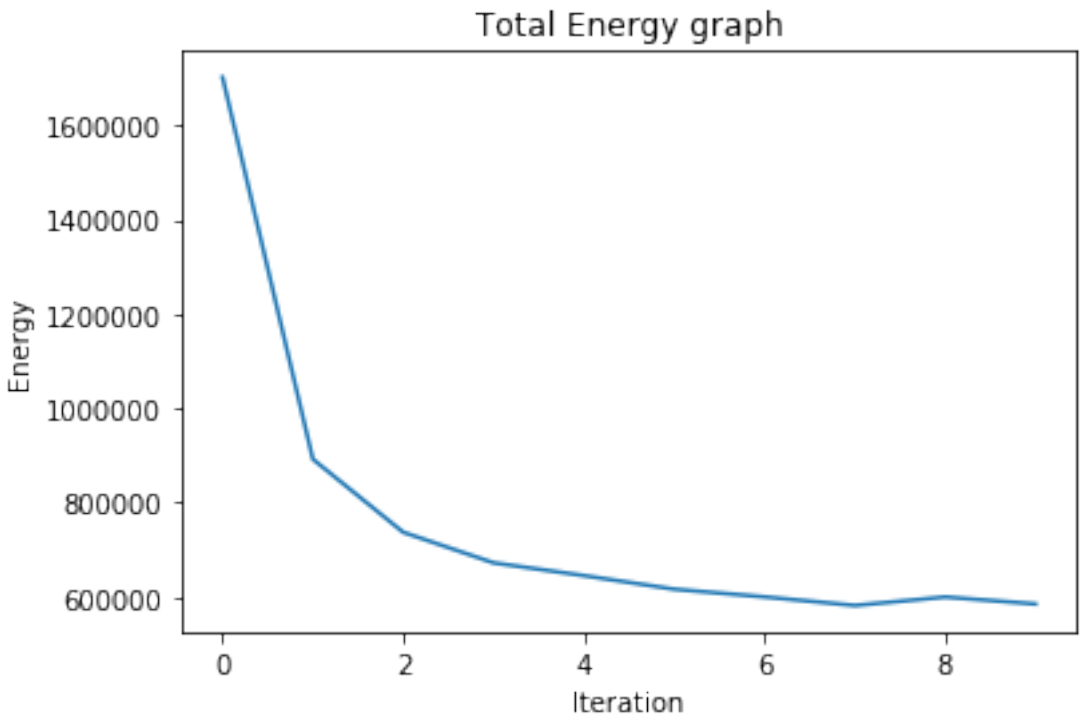
```
In [249]: K = 50
          KMeans(img_matrix, img_rows, img_cols, K, 0)
          KMeans(img_matrix, img_rows, img_cols, K, 0.01)
          KMeans(img_matrix, img_rows, img_cols, K, 0.03)
          KMeans(img_matrix, img_rows, img_cols, K, 0.05)
          KMeans(img_matrix, img_rows, img_cols, K, 0.07)
          KMeans(img_matrix, img_rows, img_cols, K, 0.08)
          KMeans(img_matrix, img_rows, img_cols, K, 0.10)
          KMeans(img_matrix, img_rows, img_cols, K, 0.25)
          KMeans(img_matrix, img_rows, img_cols, K, 0.50)
          KMeans(img_matrix, img_rows, img_cols, K, 0.75)
          KMeans(img_matrix, img_rows, img_cols, K, 1)
```

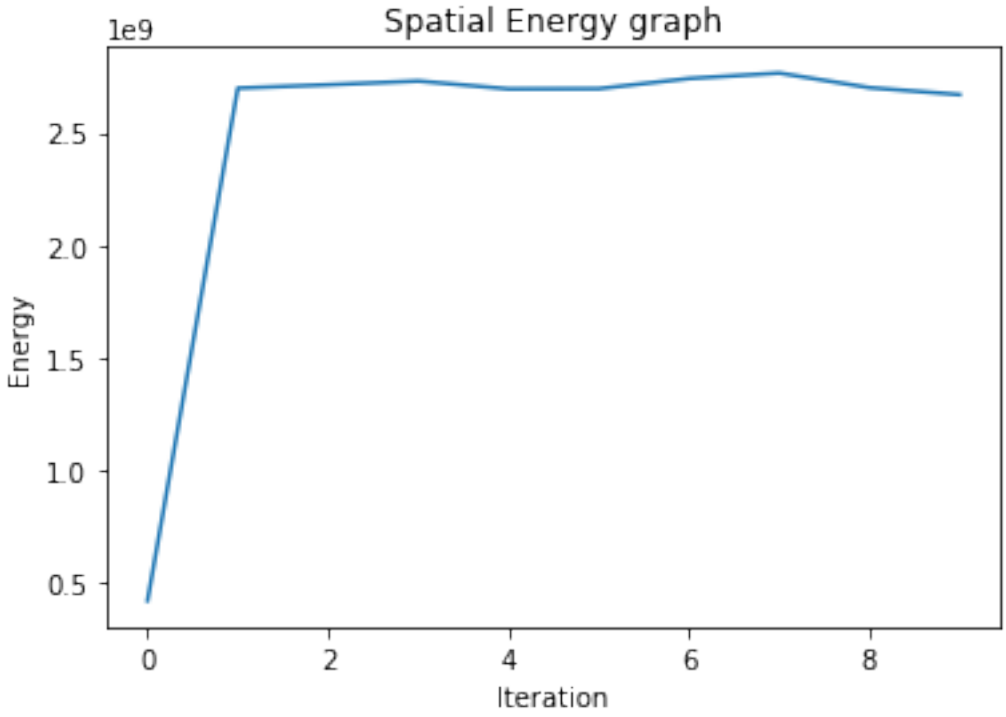
```
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:66: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:67: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:68: RuntimeWarning: inval
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:32: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:33: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:35: RuntimeWarning: overi
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:36: RuntimeWarning: overi
```

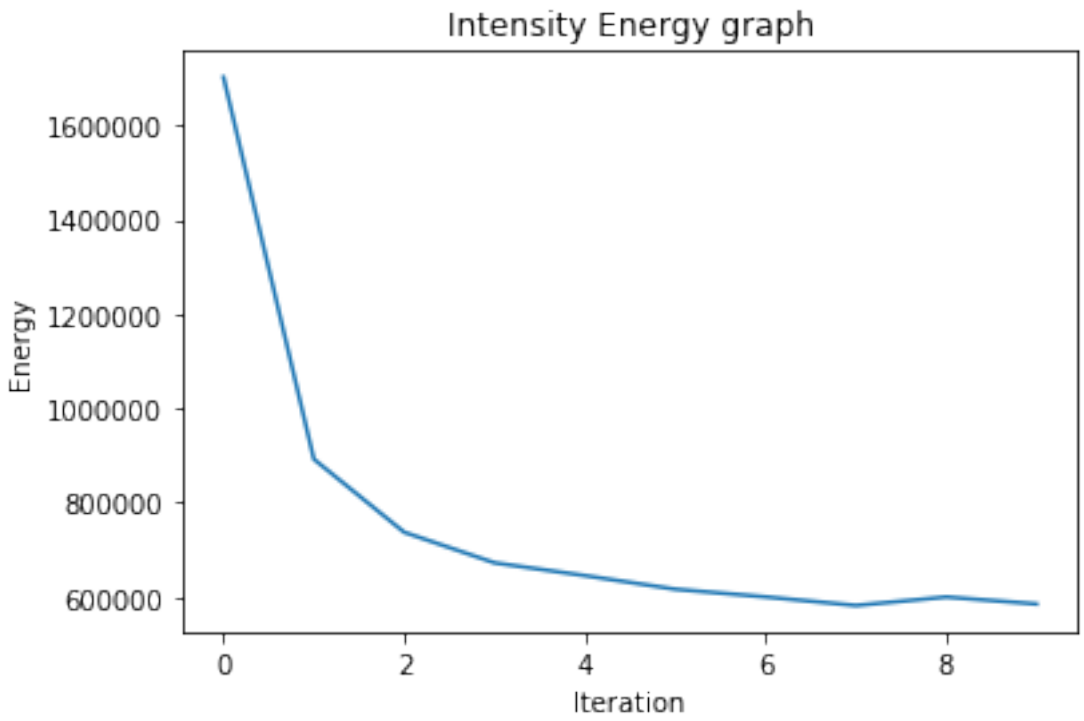
```
/home/nam/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:39: RuntimeWarning: over
```

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

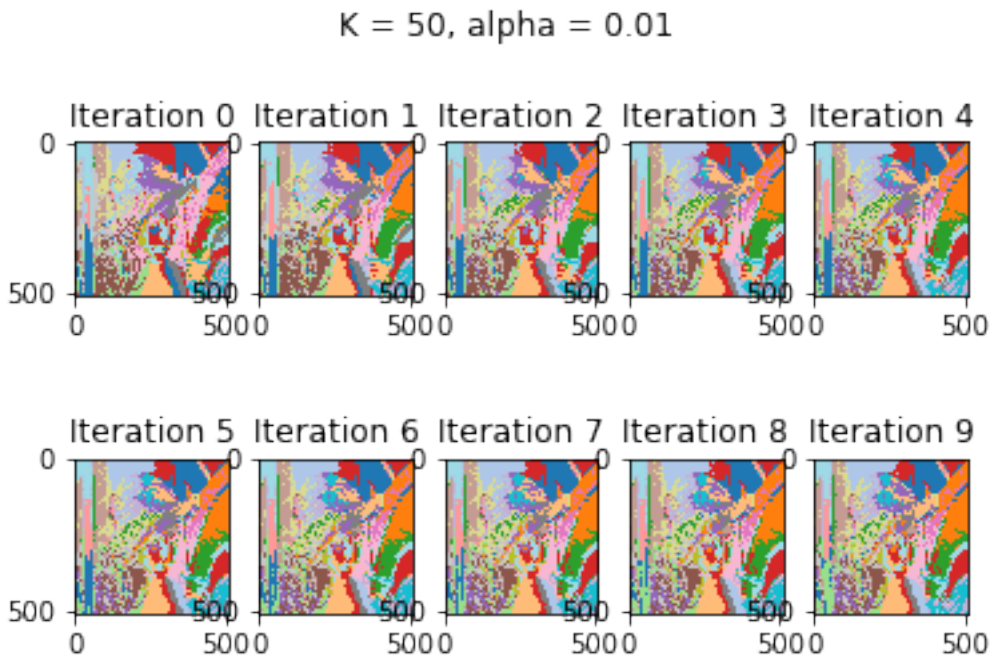


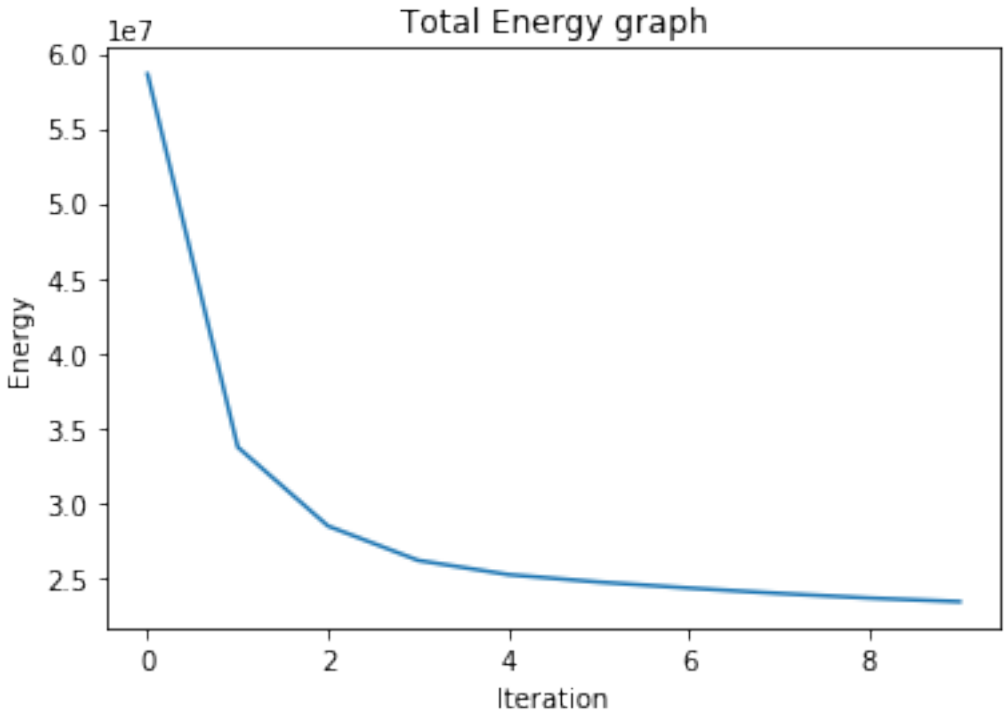

Total Energy graph
1600000
1400000
1200000
1000000
800000
600000
Energy
0
2
4
6
8
Iteration

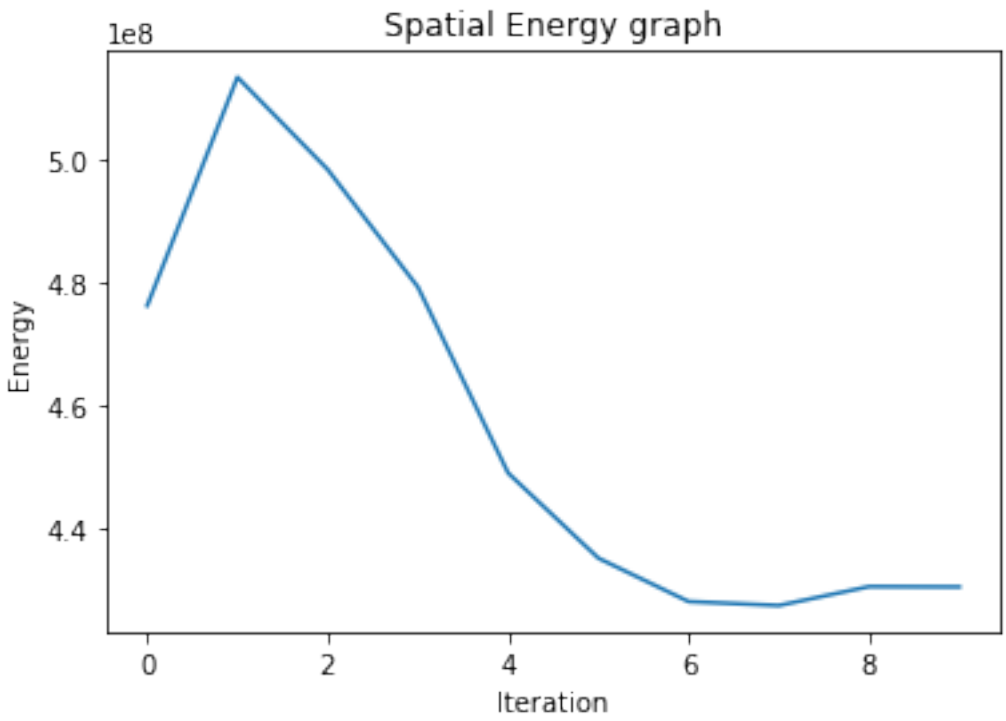

Spatial Energy graph
1e9
2.5
2.0
1.5
1.0
0.5
Energy
0
2
4
6
8
Iteration

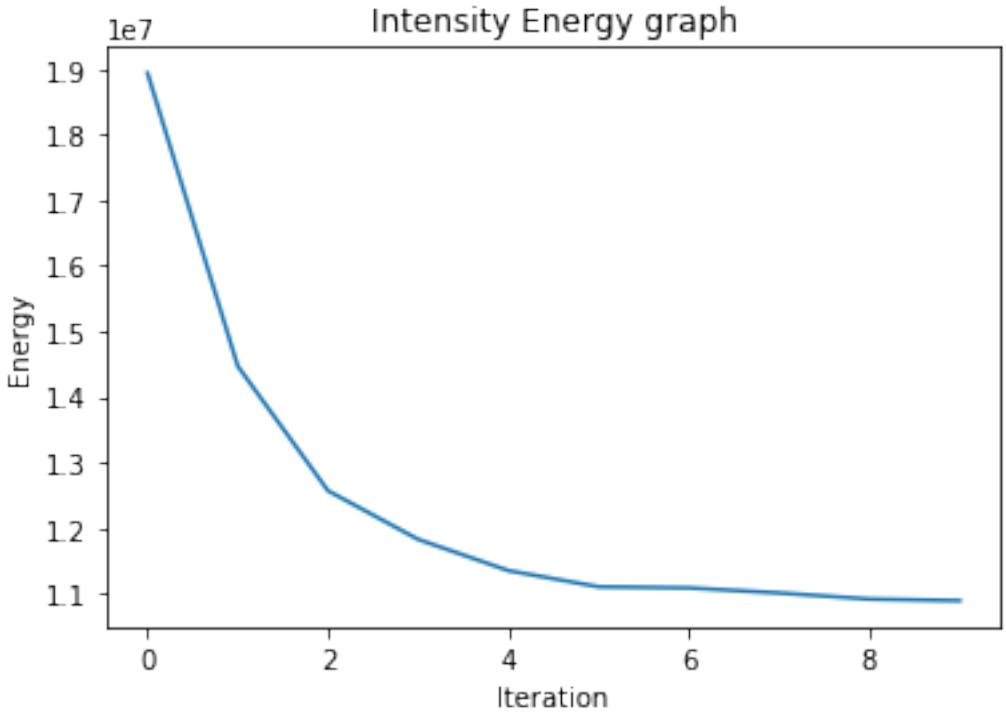



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

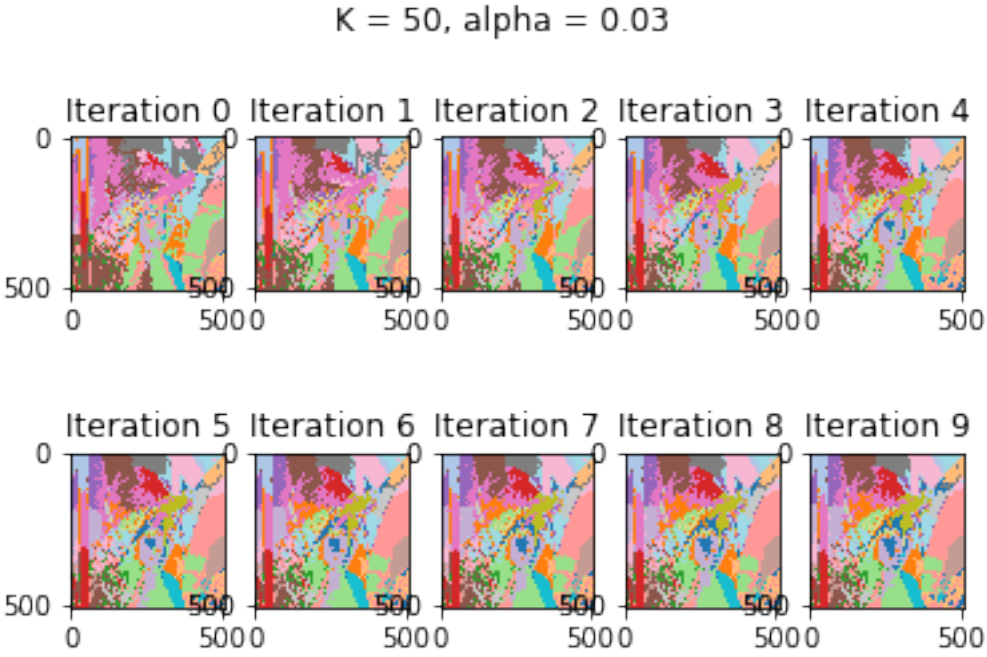

K = 50, alpha = 0.01
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

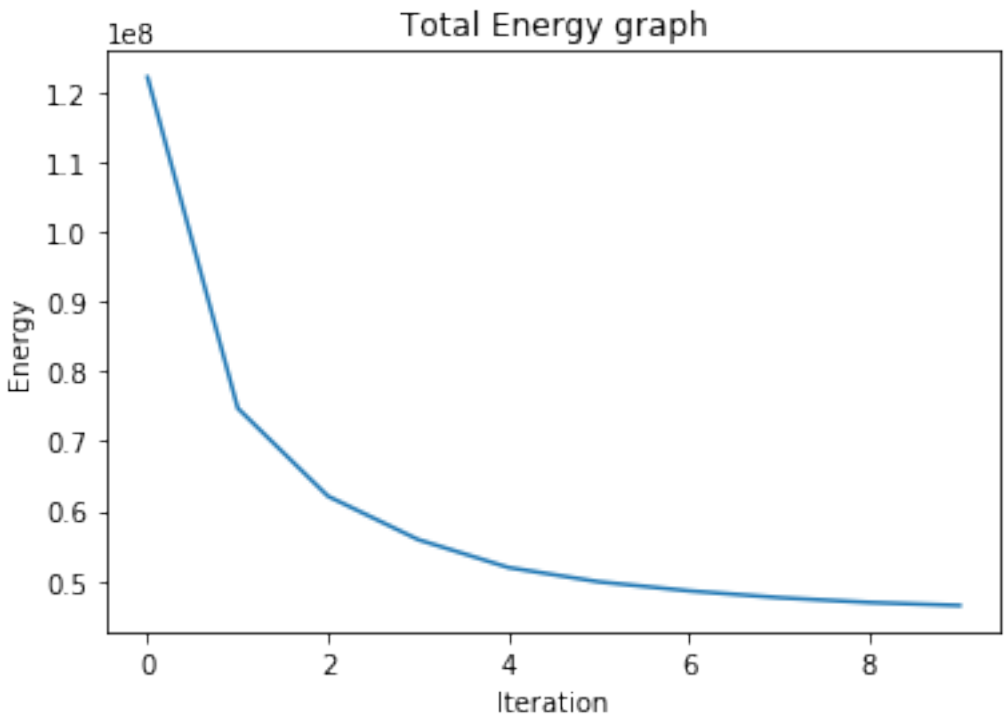

Total Energy graph
1e7
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

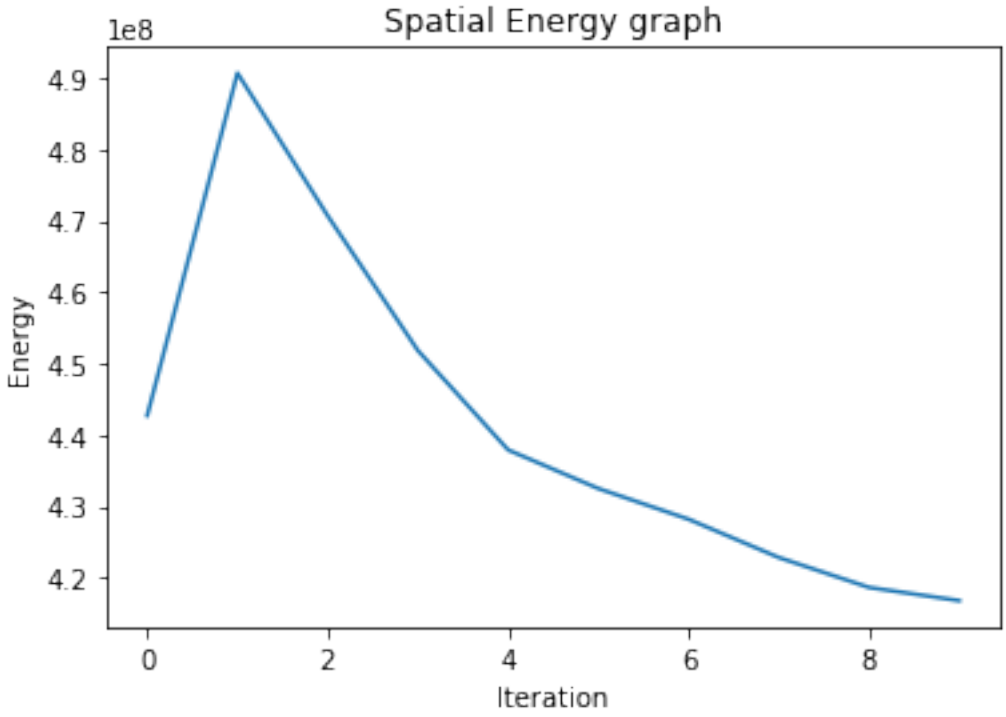

Spatial Energy graph
1e8
5.0
4.8
4.6
4.4
Energy
0
2
4
6
8
Iteration

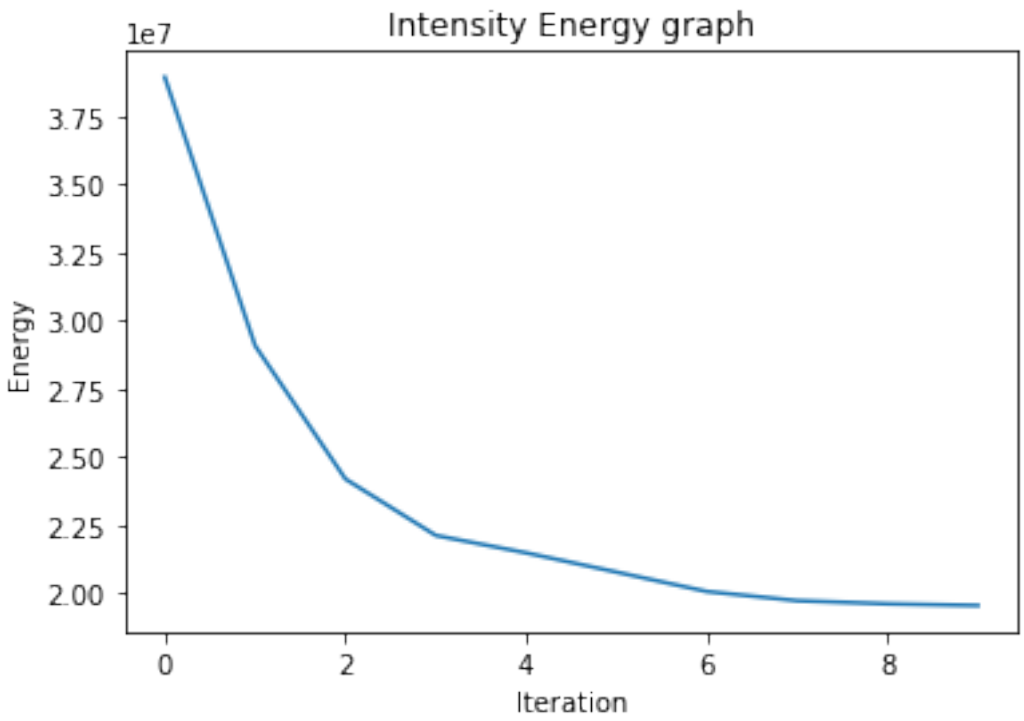

Intensity Energy graph
1e7
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

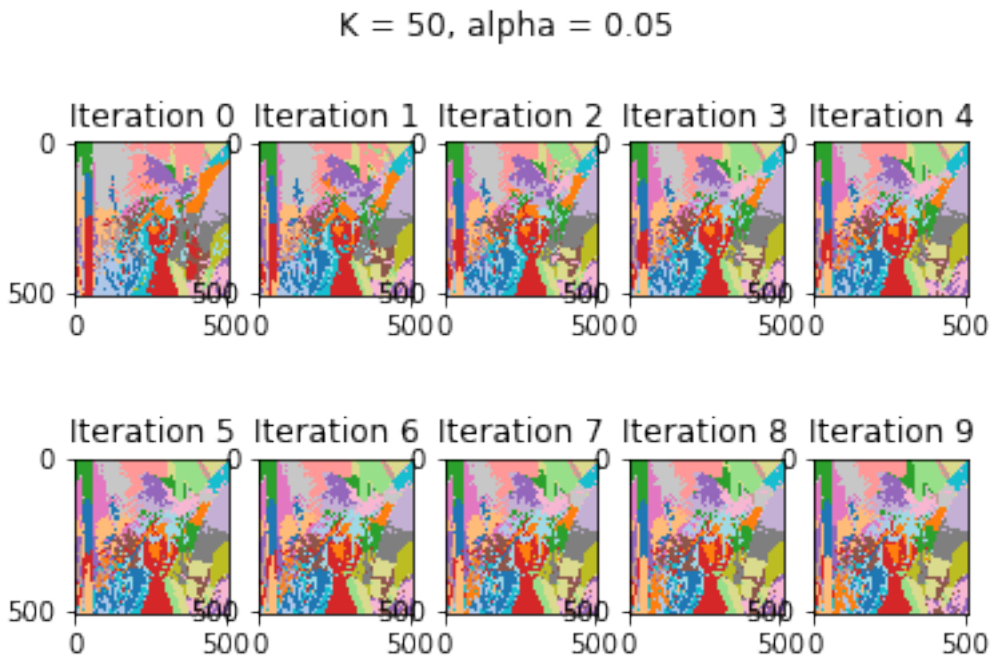


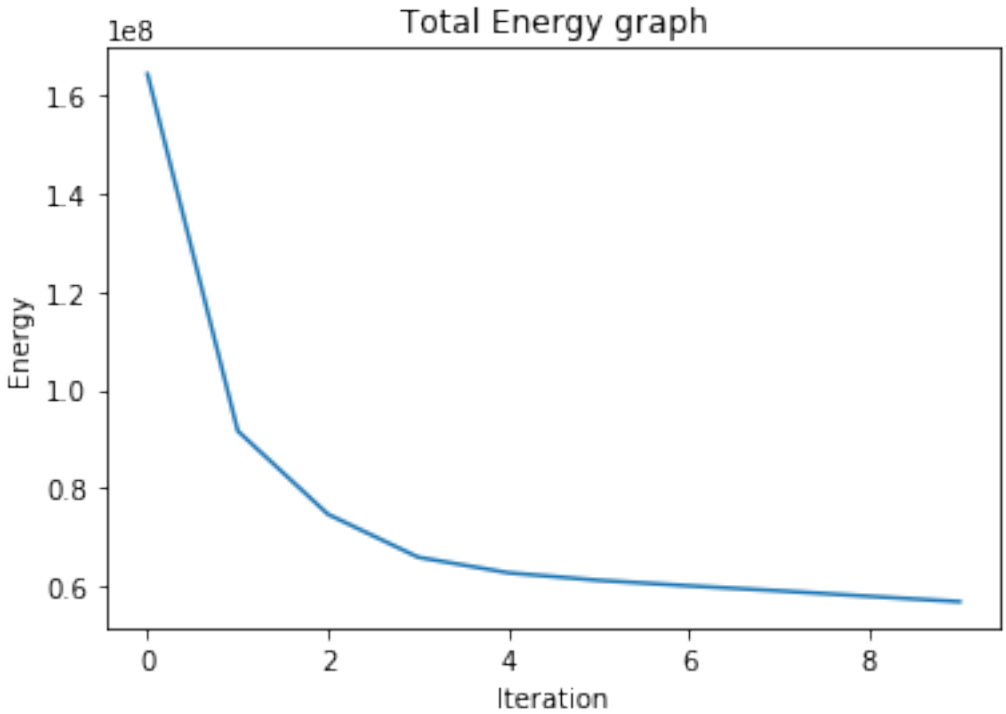

Total Energy graph
1e8
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
Energy
0
2
4
6
8
Iteration

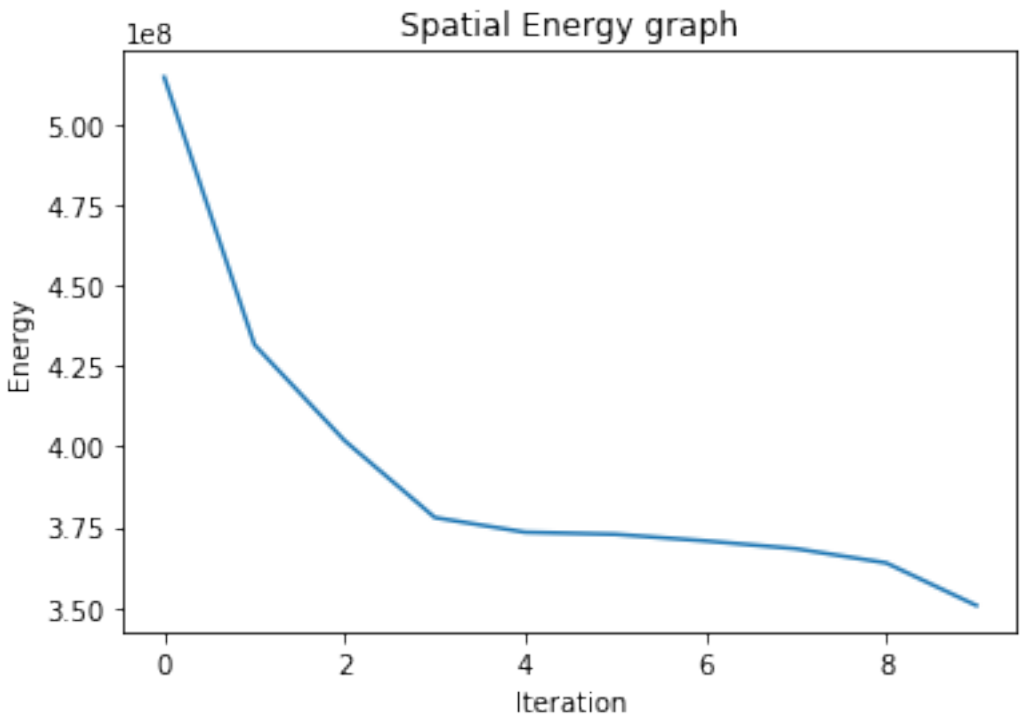

Spatial Energy graph
1e8
4.9
4.8
4.7
4.6
4.5
4.4
4.3
4.2
Energy
0
2
4
6
8
Iteration

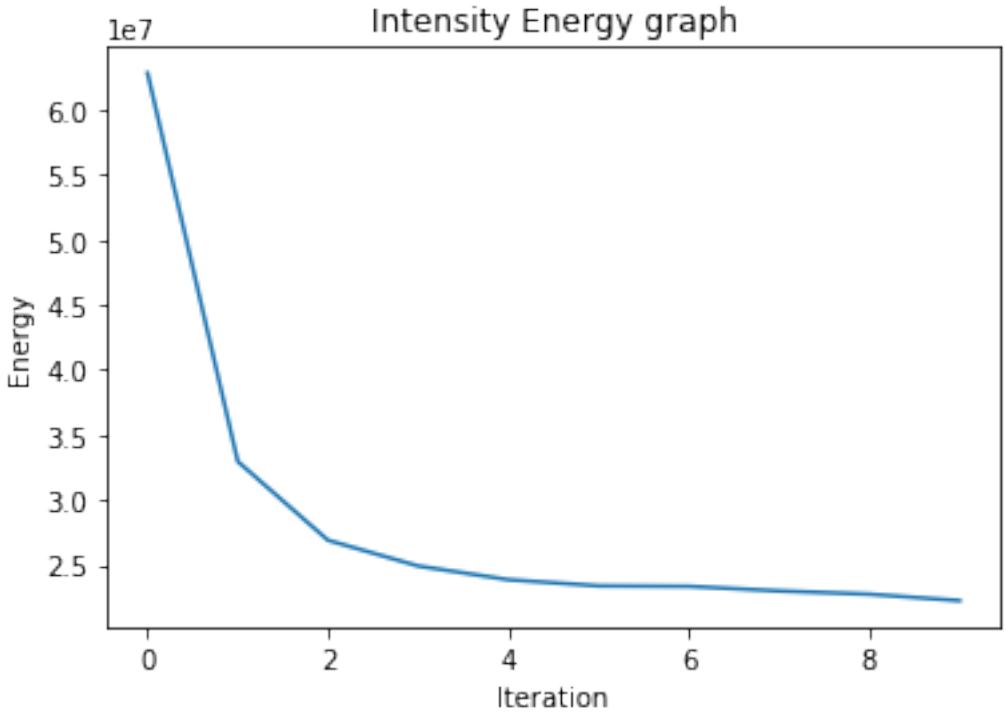



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

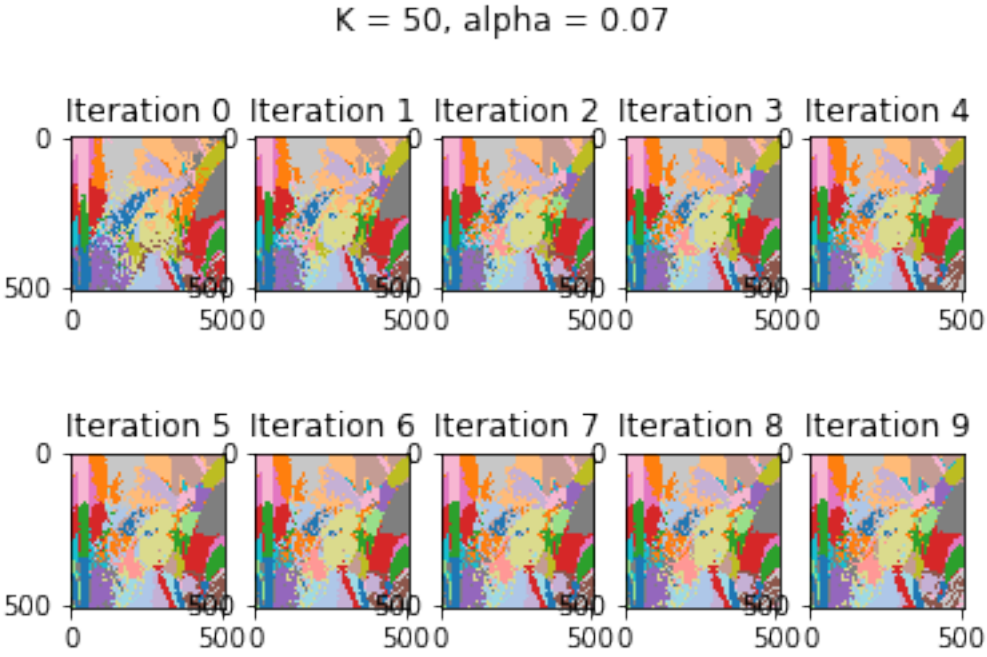

K = 50, alpha = 0.05
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

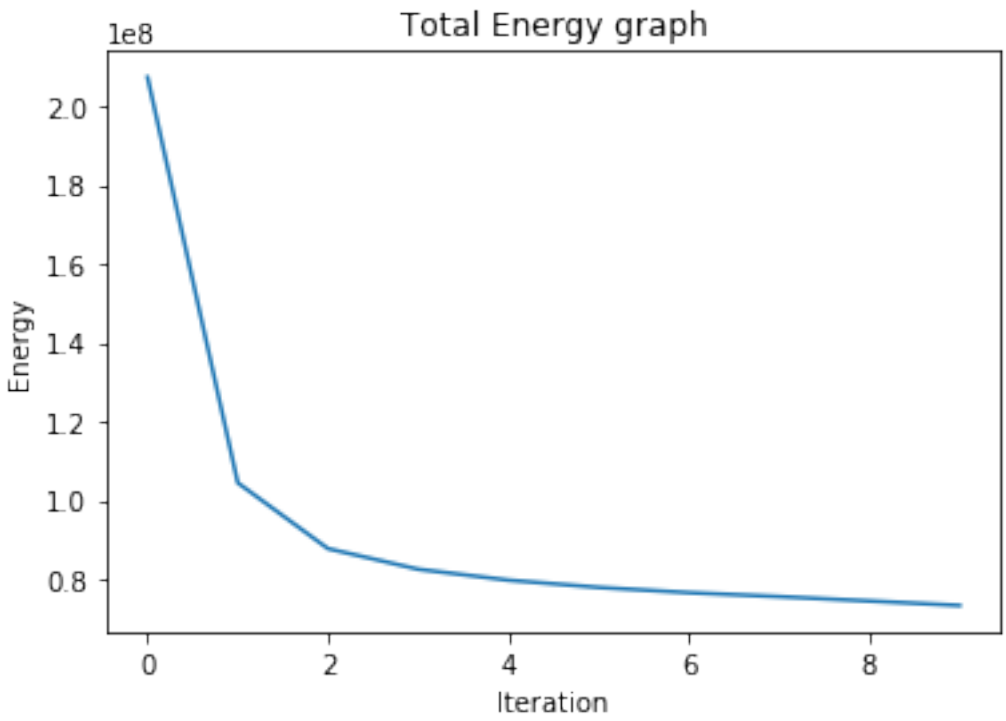

Total Energy graph
1e8
1.6
1.4
1.2
1.0
0.8
0.6
Energy
0
2
4
6
8
Iteration

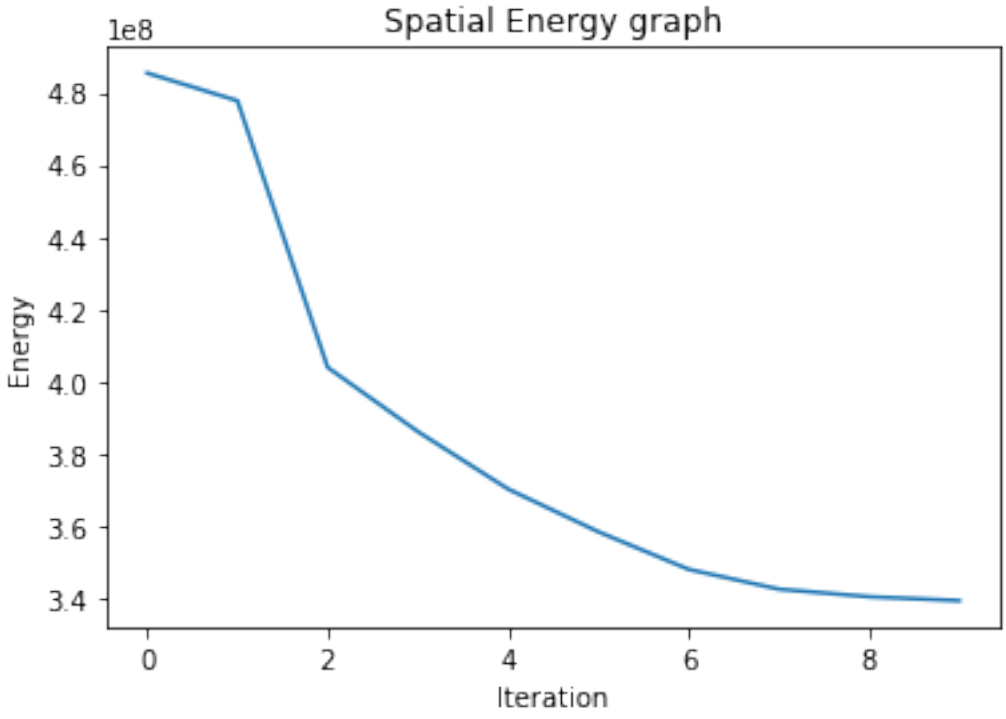

Spatial Energy graph
1e8
5.00
4.75
4.50
4.25
4.00
3.75
3.50
Energy
0
2
4
6
8
Iteration

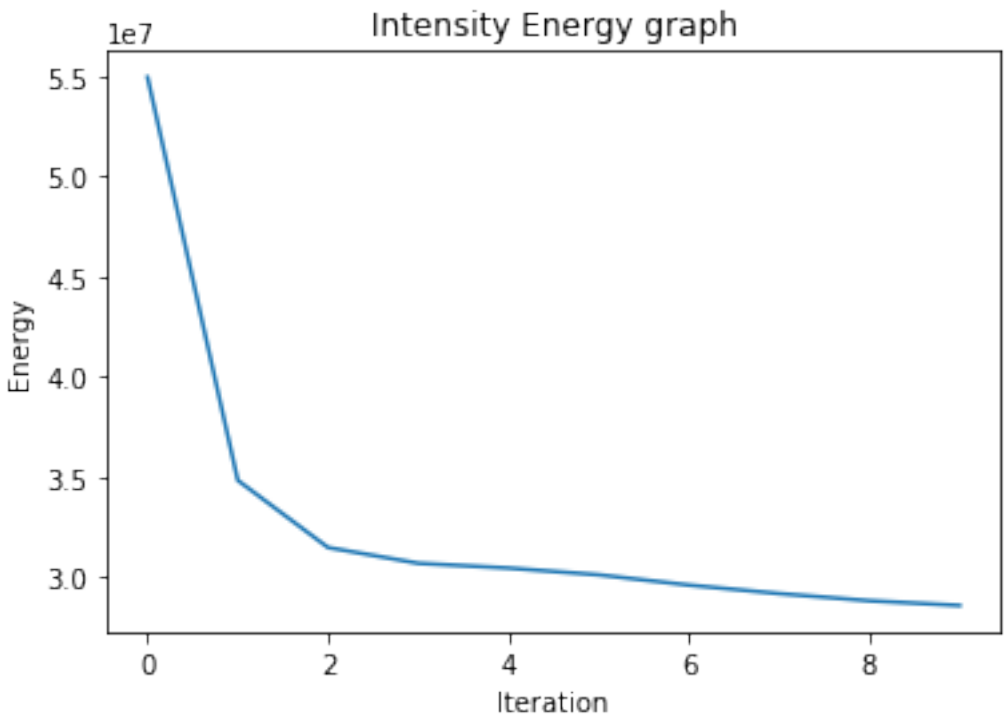

Intensity Energy graph
1e7
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

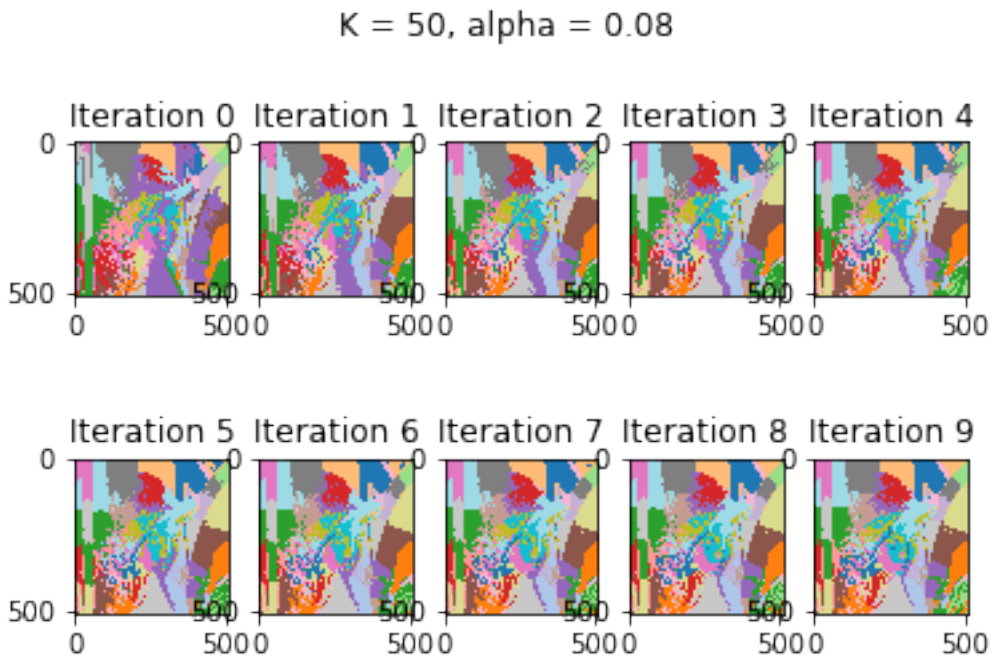


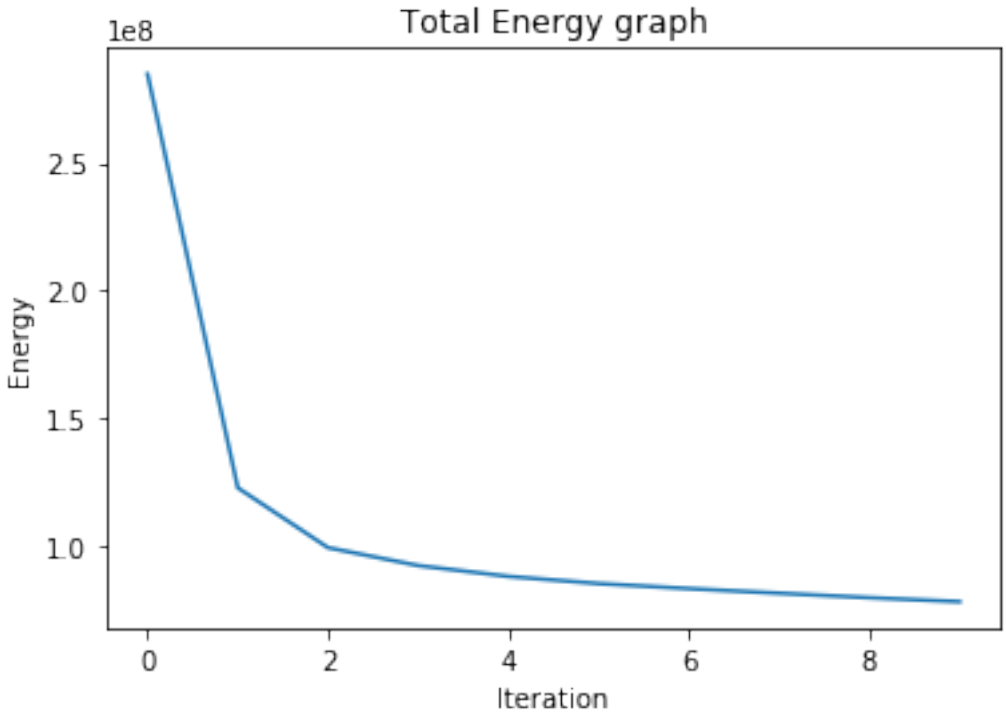

Total Energy graph
1e8
2.0
1.8
1.6
1.4
1.2
1.0
0.8
Energy
0
2
4
6
8
Iteration

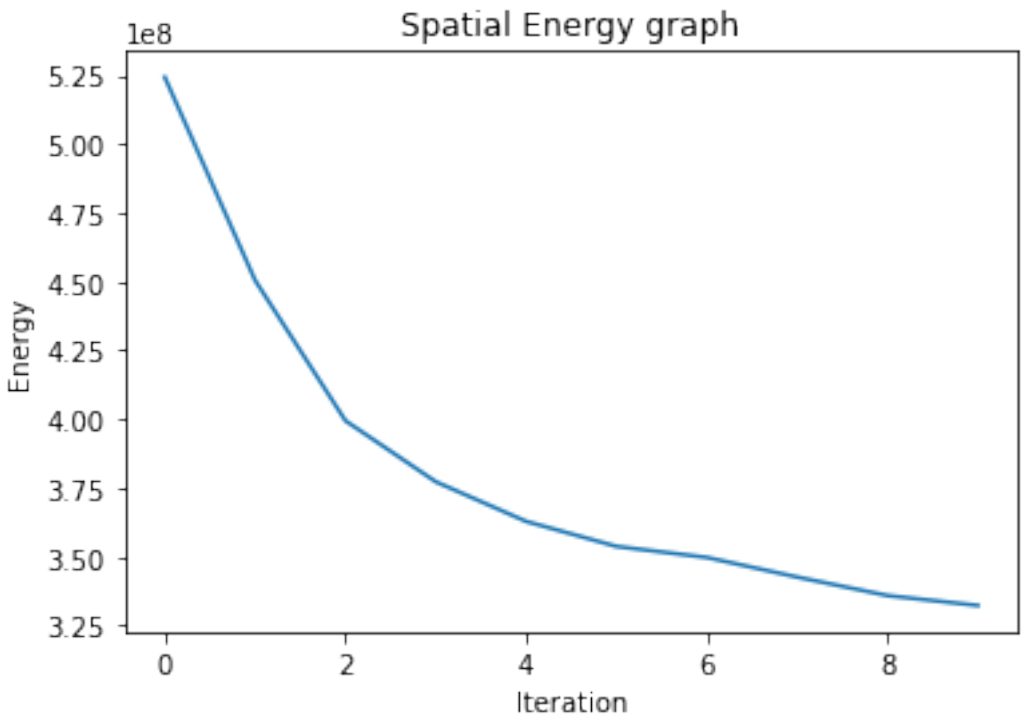

Spatial Energy graph
1e8
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
Energy
0
2
4
6
8
Iteration

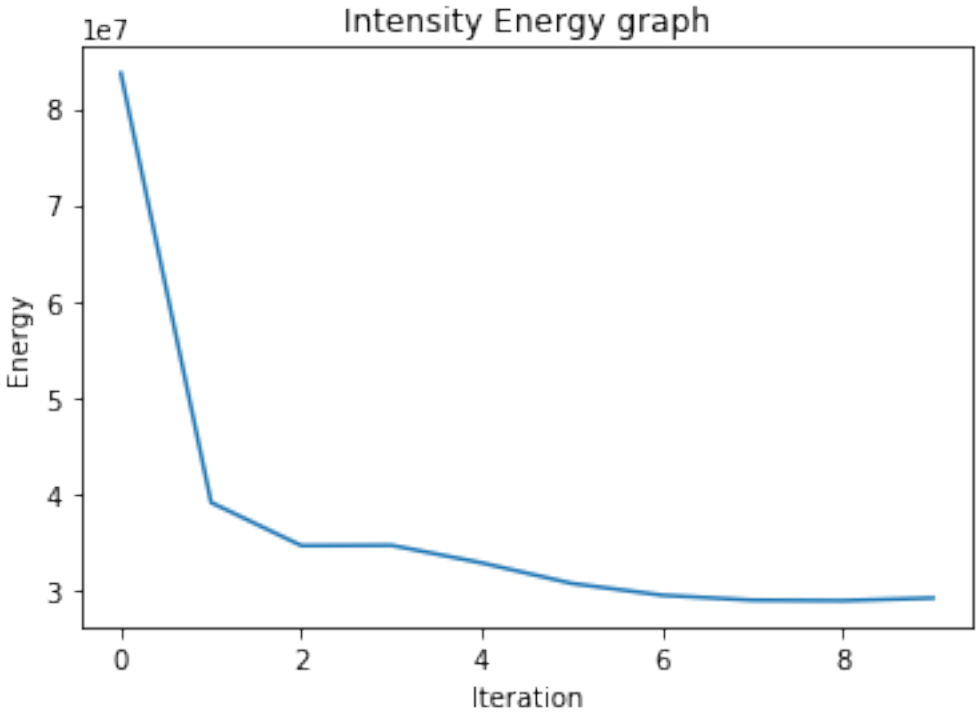



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

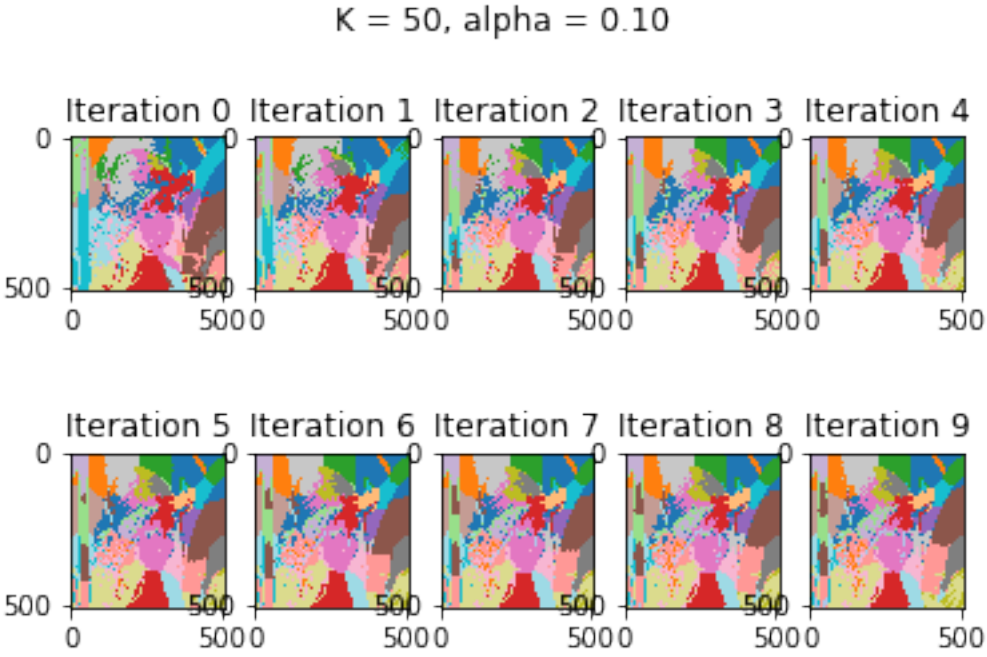

K = 50, alpha = 0.08
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500
0
500

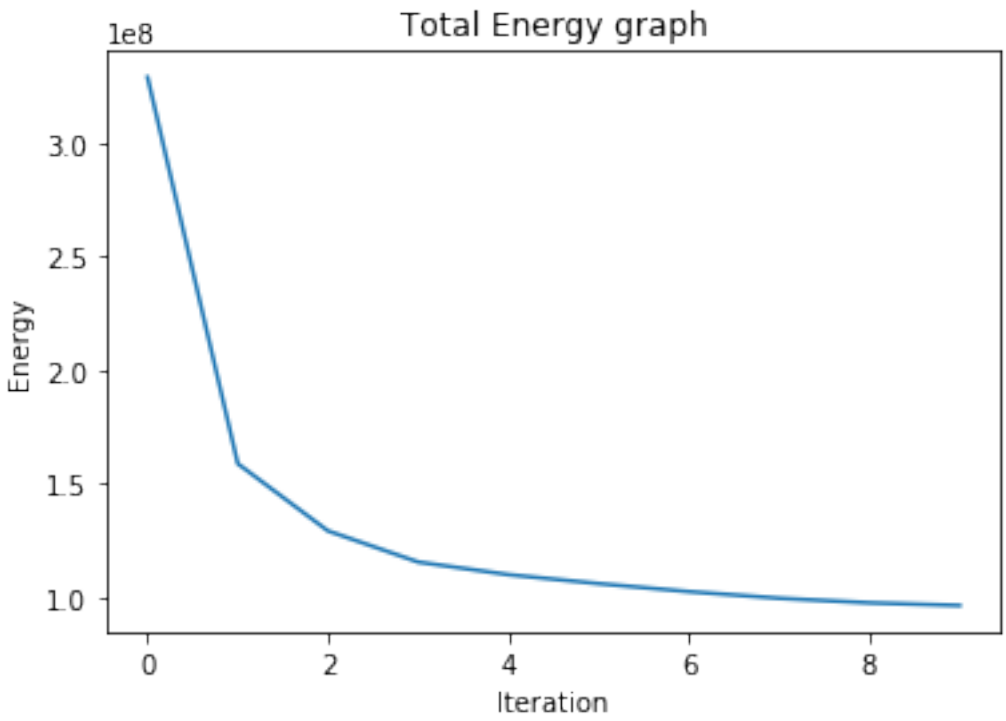

Total Energy graph
1e8
2.5
2.0
1.5
1.0
Energy
0
2
4
6
8
Iteration

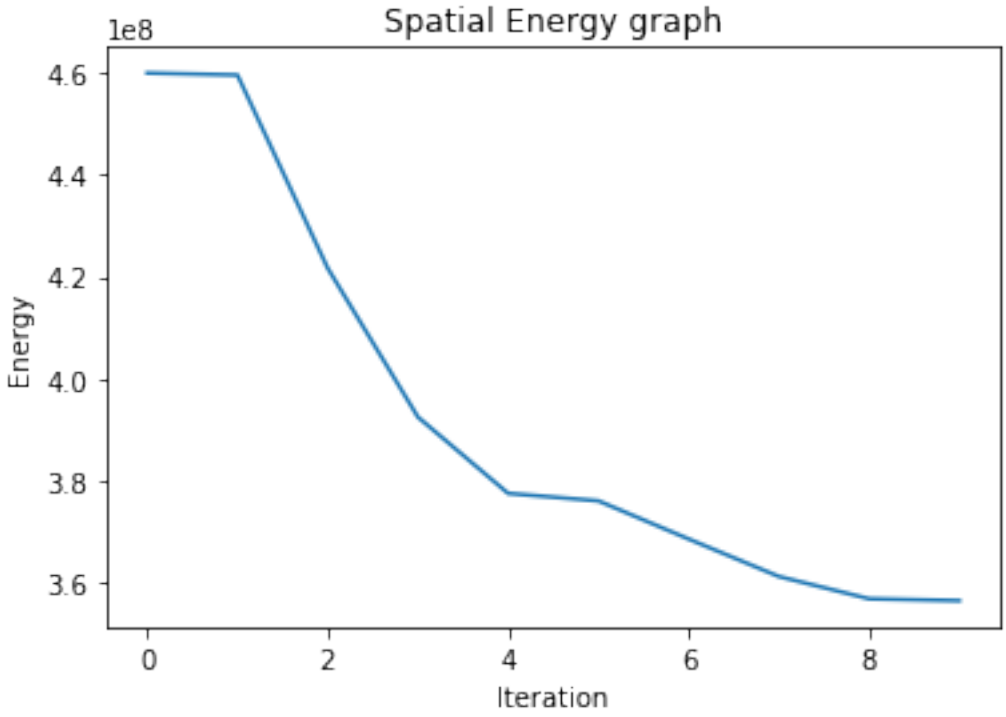

Spatial Energy graph
1e8
5.25
5.00
4.75
4.50
4.25
4.00
3.75
3.50
3.25
Energy
0
2
4
6
8
Iteration

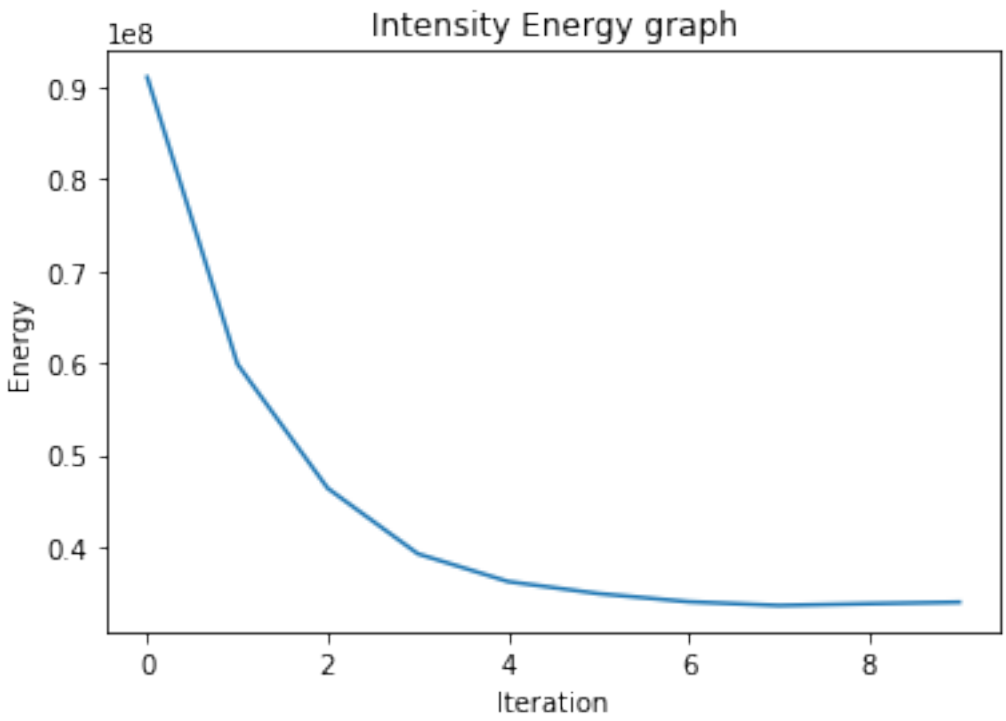

Intensity Energy graph
1e7
8
7
6
5
4
3
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

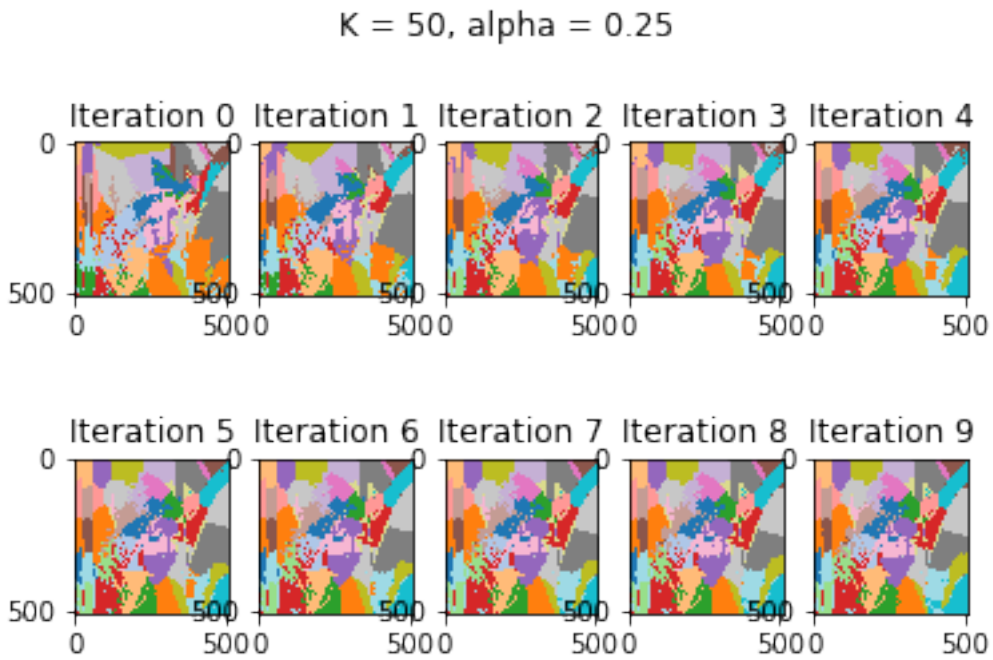


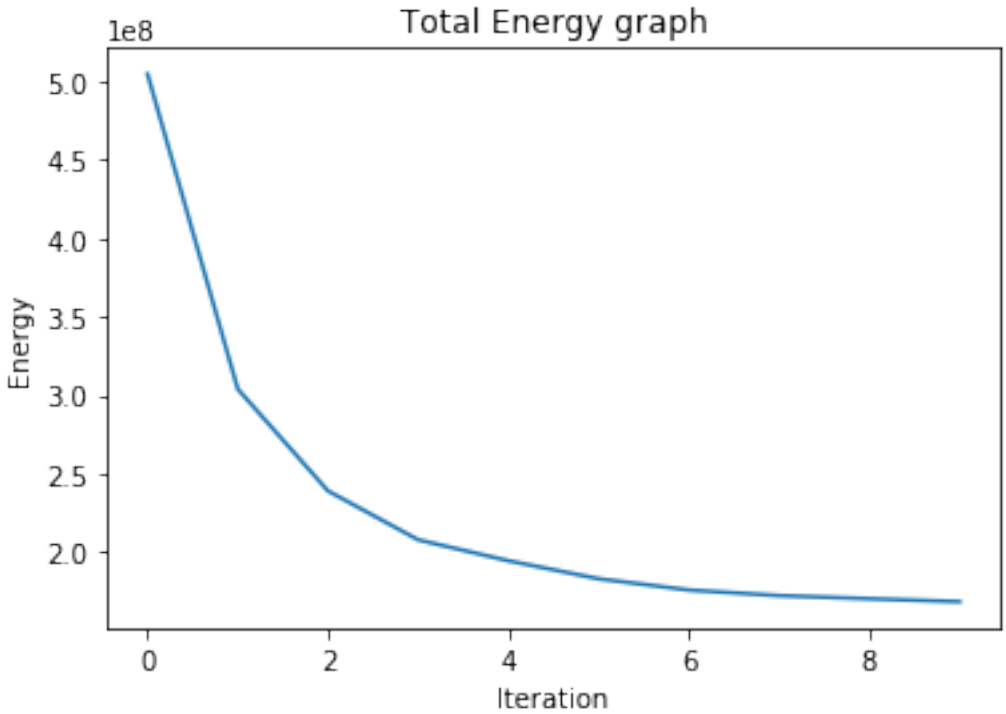

Total Energy graph
1e8
3.0
2.5
2.0
1.5
1.0
Energy
0
2
4
6
8
Iteration

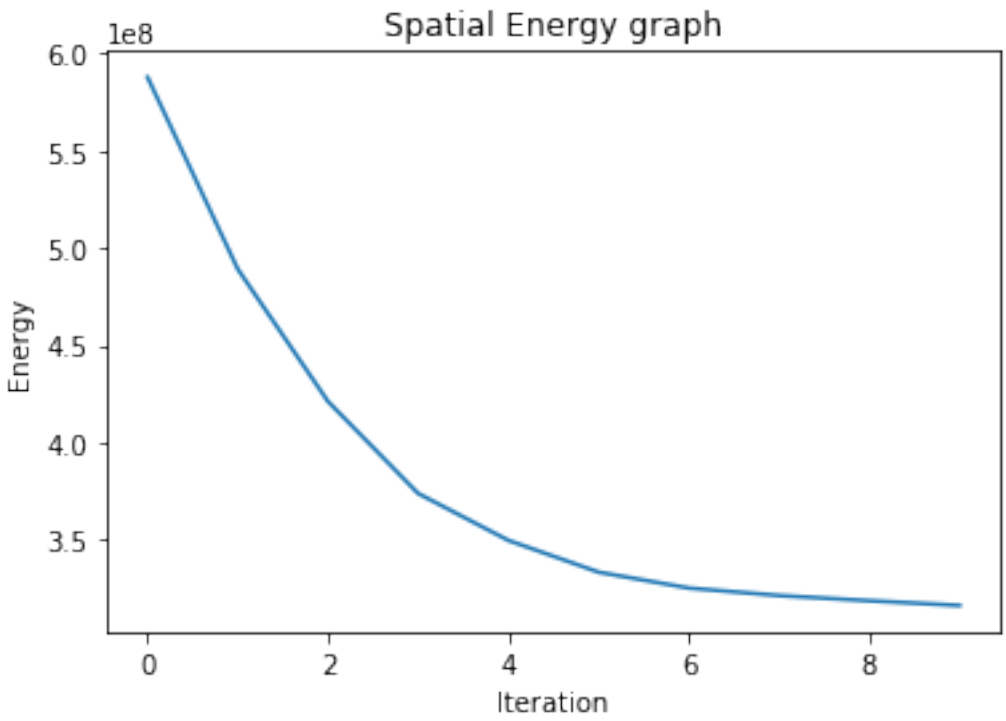

Spatial Energy graph
1e8
4.6
4.4
4.2
4.0
3.8
3.6
Energy
0
2
4
6
8
Iteration

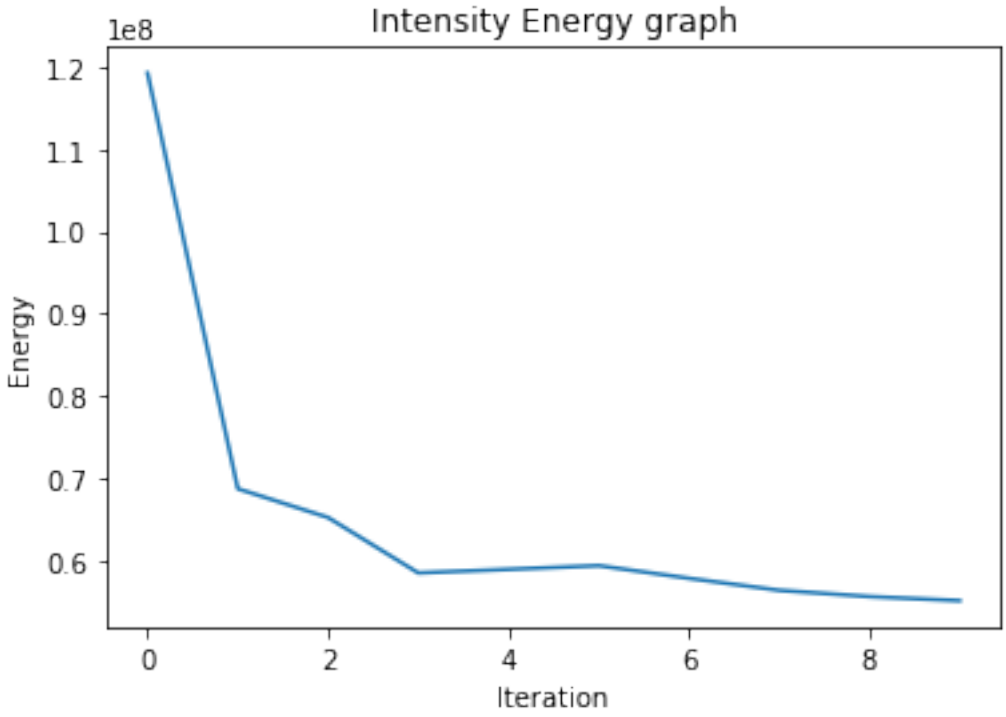



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

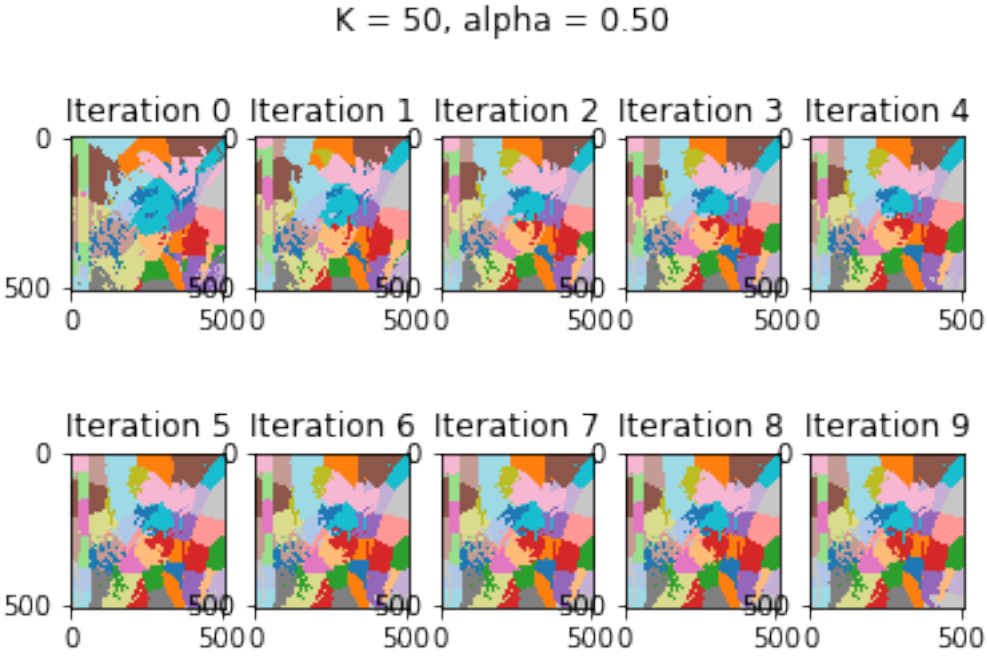

K = 50, alpha = 0.25
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

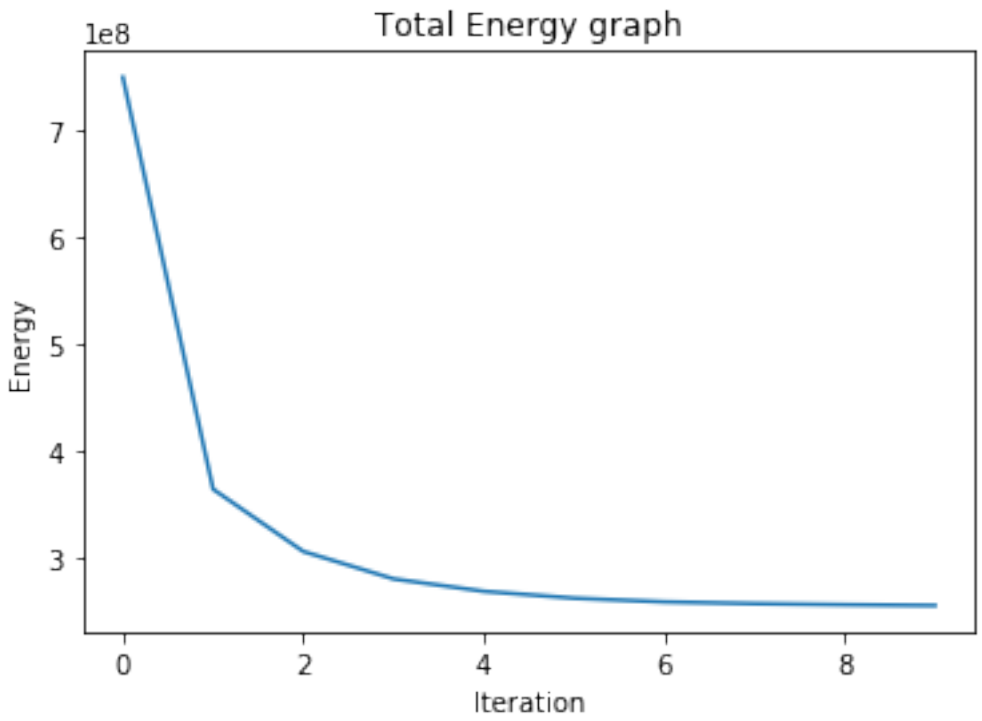

Total Energy graph
1e8
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Energy
0
2
4
6
8
Iteration

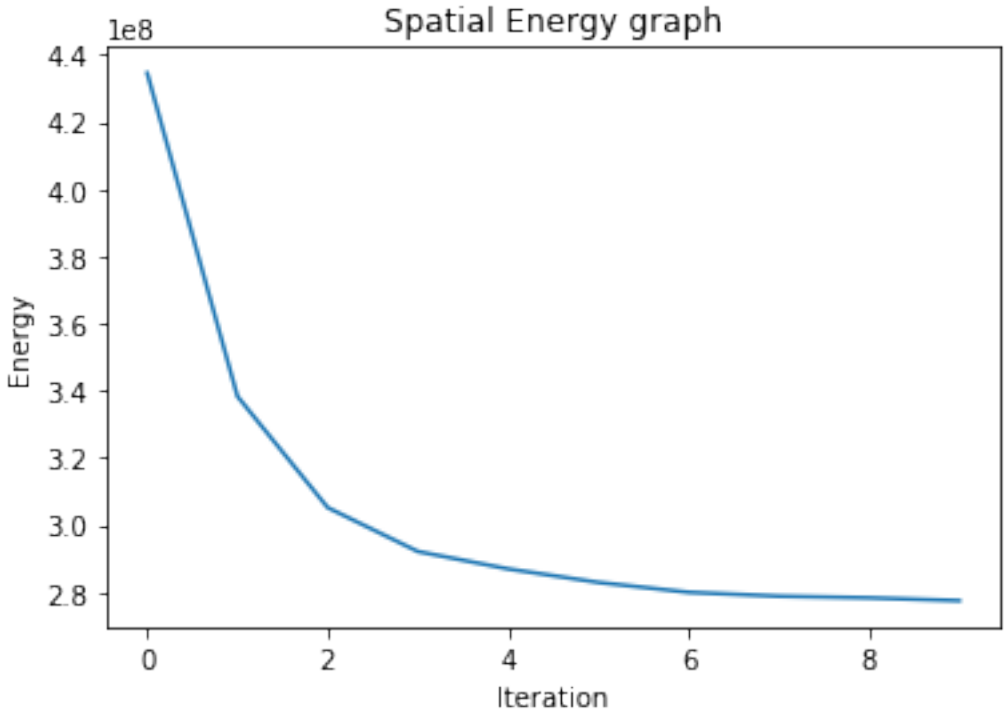

Spatial Energy graph
1e8
6.0
5.5
5.0
4.5
4.0
3.5
Energy
0
2
4
6
8
Iteration

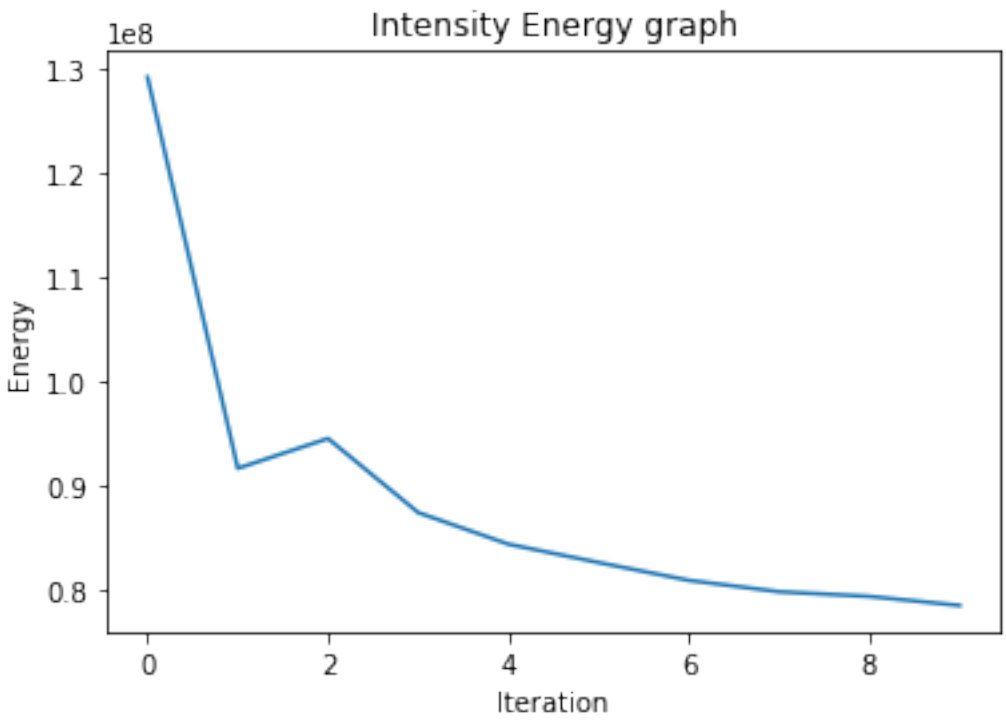

Intensity Energy graph
1e8
1.2
1.1
1.0
0.9
0.8
0.7
0.6
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

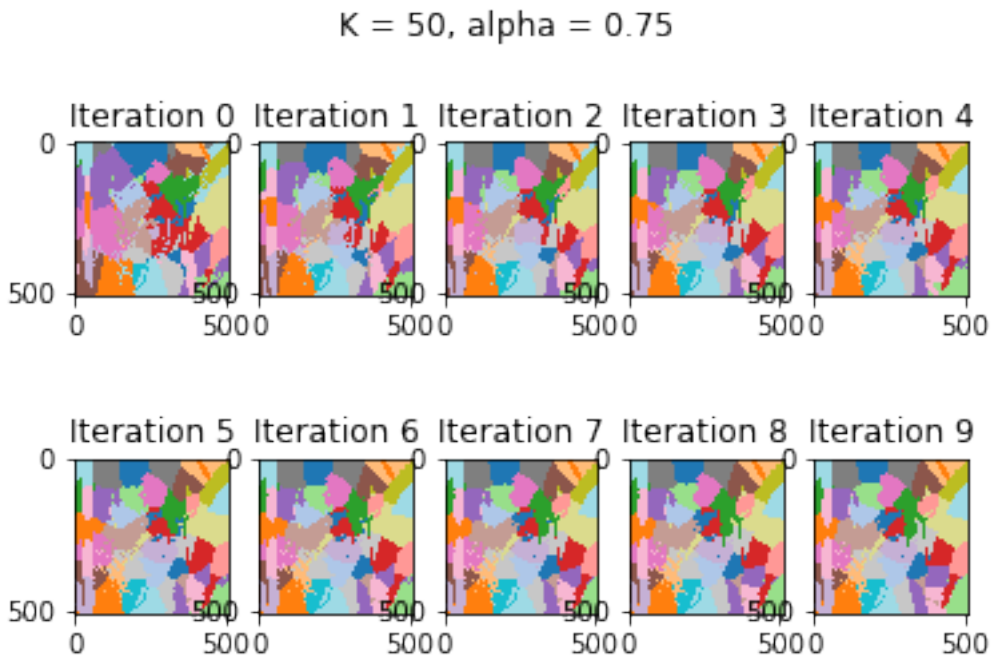


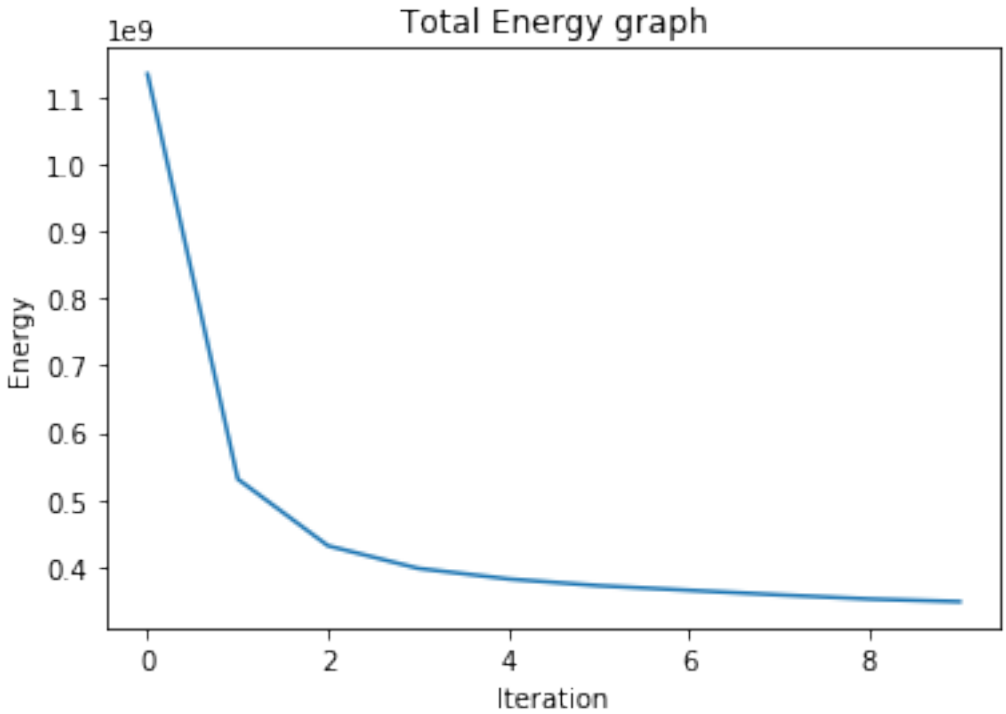

Total Energy graph
1e8
7
6
5
4
3
Energy
0
2
4
6
8
Iteration

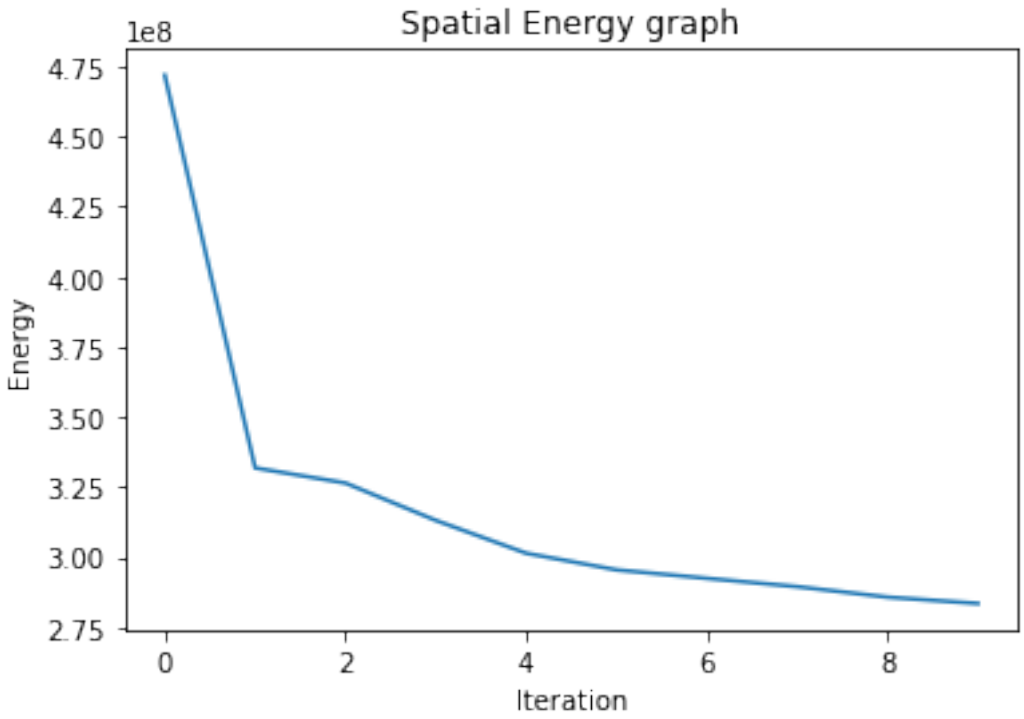

Spatial Energy graph
1e8
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
Energy
0
2
4
6
8
Iteration

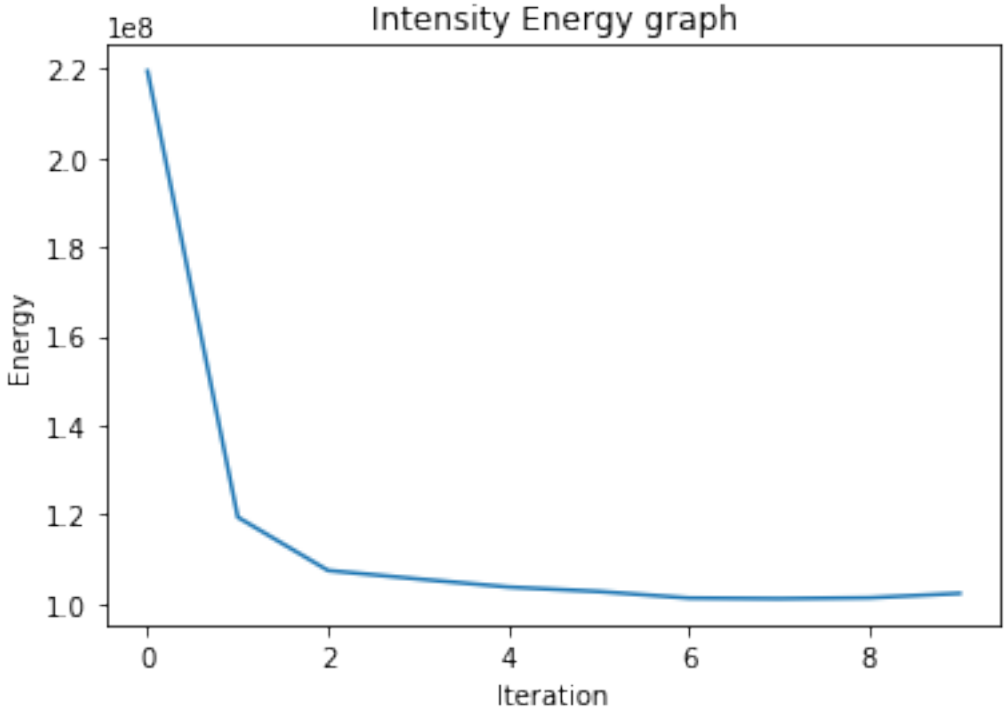



```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```

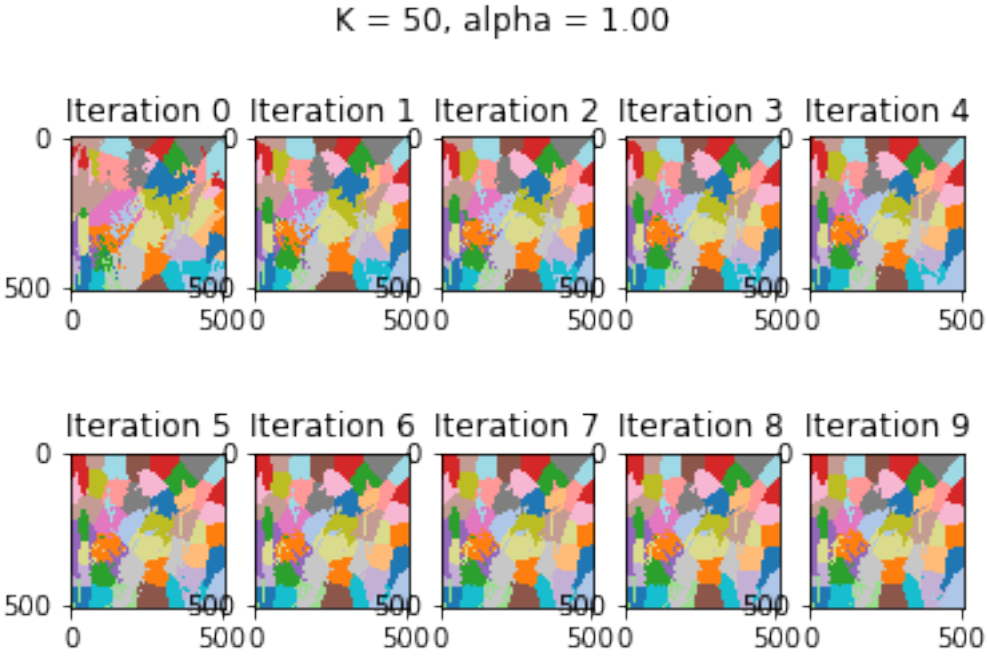

K = 50, alpha = 0.75
Iteration 0
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
0
500

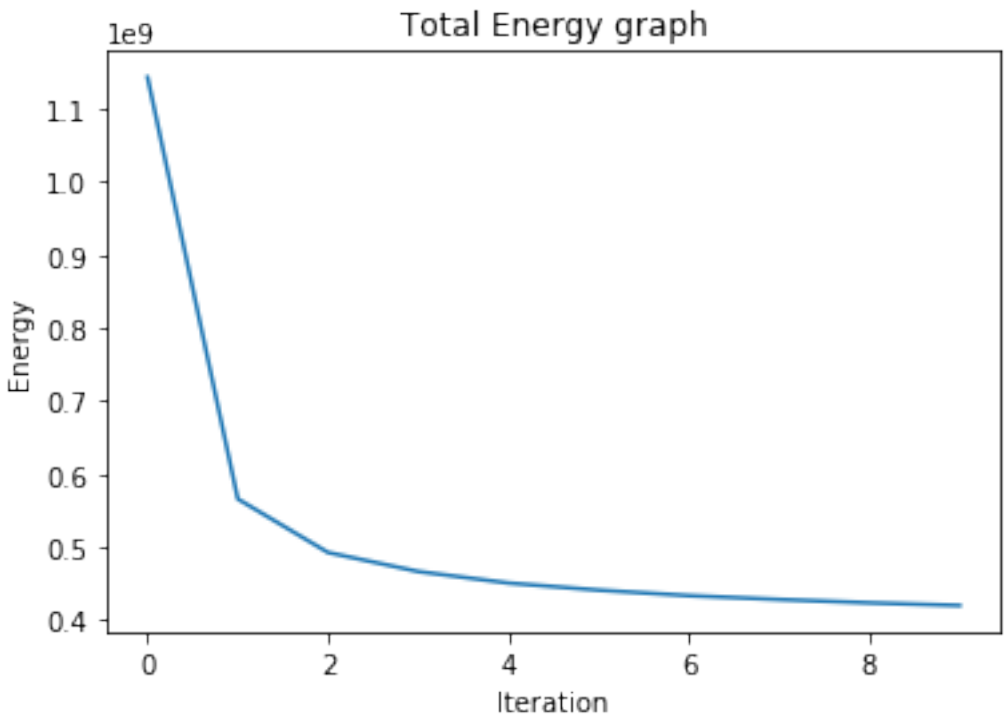

Total Energy graph
1e9
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration

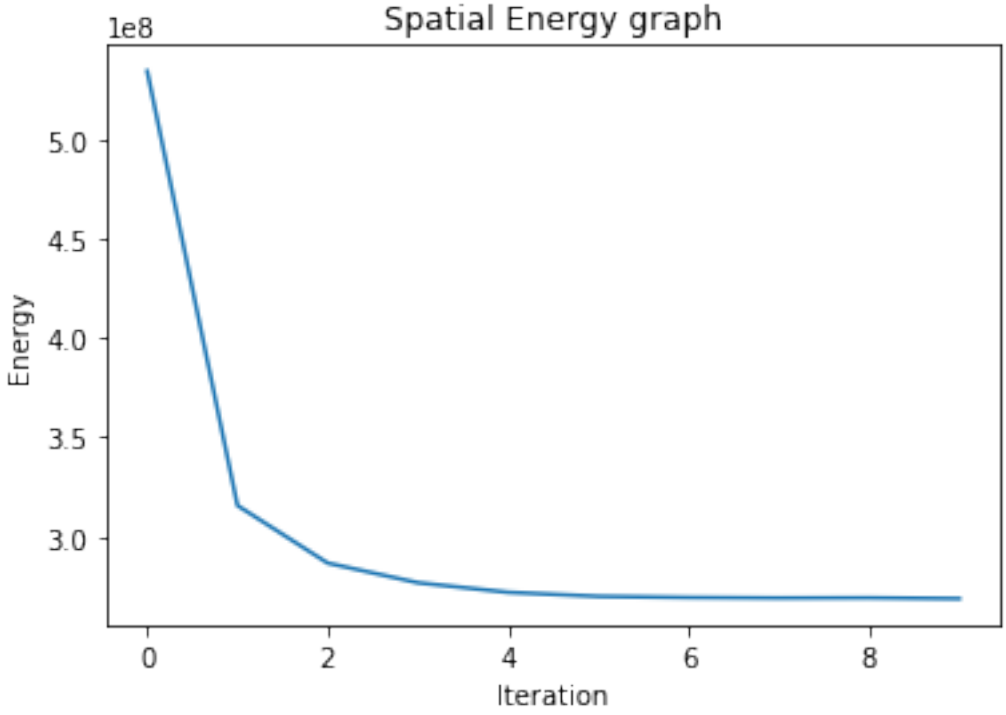

Spatial Energy graph
1e8
4.75
4.50
4.25
4.00
3.75
3.50
3.25
3.00
2.75
Energy
0
2
4
6
8
Iteration

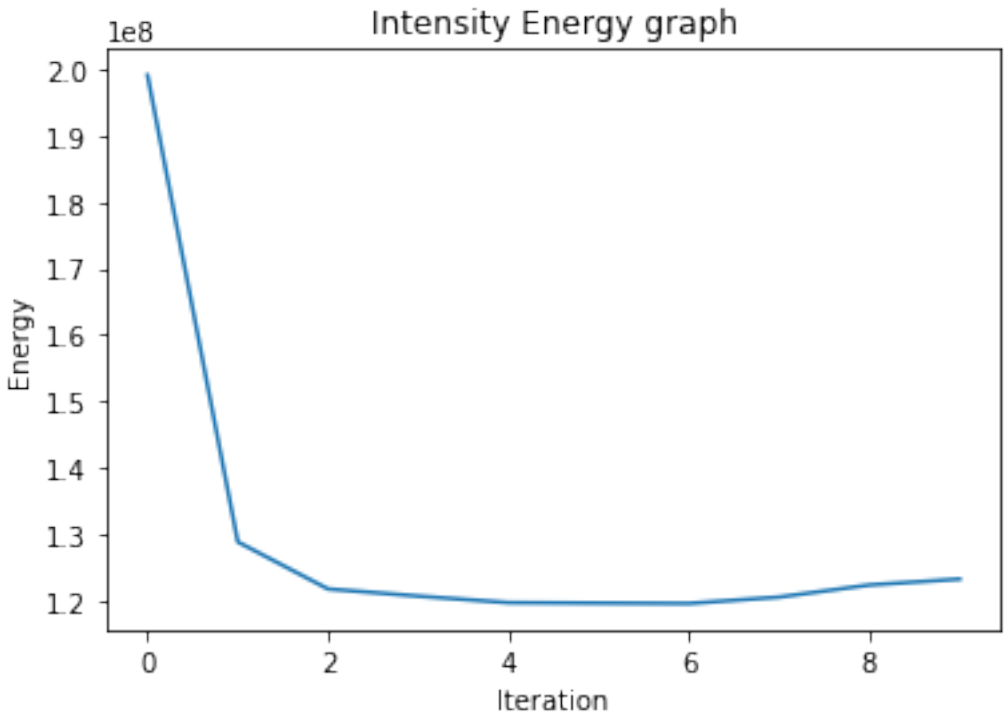

Intensity Energy graph
1e8
2.2
2.0
1.8
1.6
1.4
1.2
1.0
Energy
0
2
4
6
8
Iteration

```
0 th iteration
1 th iteration
2 th iteration
3 th iteration
4 th iteration
5 th iteration
6 th iteration
7 th iteration
8 th iteration
9 th iteration
```




Total Energy graph
1e9
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Energy
0
2
4
6
8
Iteration


Spatial Energy graph
1e8
5.0
4.5
4.0
3.5
3.0
Energy
0
2
4
6
8
Iteration


Intensity Energy graph
1e8
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
Energy
0
2
4
6
8
Iteration