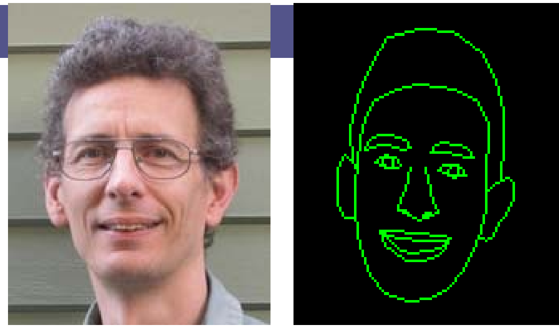
Shape Modelling

Shape Modelling: overview

- Aim:
  - A popular approach in order to distinguish objects from background: **solve the segmentation problem.**
  - There are some other approaches:
    - **Threshold** techniques, which make decisions based on local pixel information.
    - **Edge-based** methods: focused around contour detection.
    - **Region-based** methods: the image is partitioned into connected regions by grouping neighboring pixels of similar intensity levels.
    - Connectivity-preserving relaxation-based segmentation method: The main idea is to start with some initial boundary shape represented in the form of spline curves, and iteratively modify it by applying various shrink/expansion operations according to some energy function: ***active contour model***.

Shape Modelling: approaches



- Tim Cootes
  - Professor of Computer Vision
  - Imaging Sciences and the Biomedical Imaging Institute
  - The University of Manchester
  - Research Interests:
    - Statistical models of shape and appearance
    - Model matching algorithms
    - Methods of computing correspondence across sets of shapes and images
    - Applications in medical image analysis and face image interpretation

Shape Modelling: approaches

- Statistical Models.
- Active Shape Models (ASMs).
- Active Appearance Models (AAMs).
- Active Contour Models, also called snakes.

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Shape Modelling: approaches

- Statistical Models.

Initial Image Processing

↓

Training Data Set

↓

Variation Model

↓

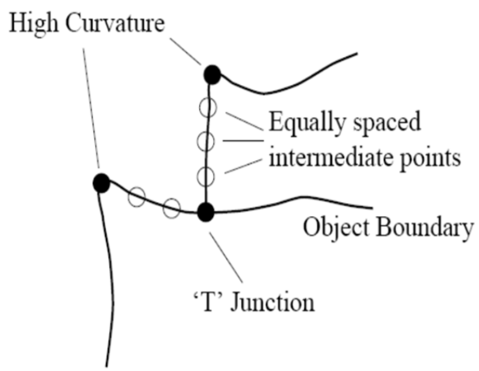
Experimental Data Set

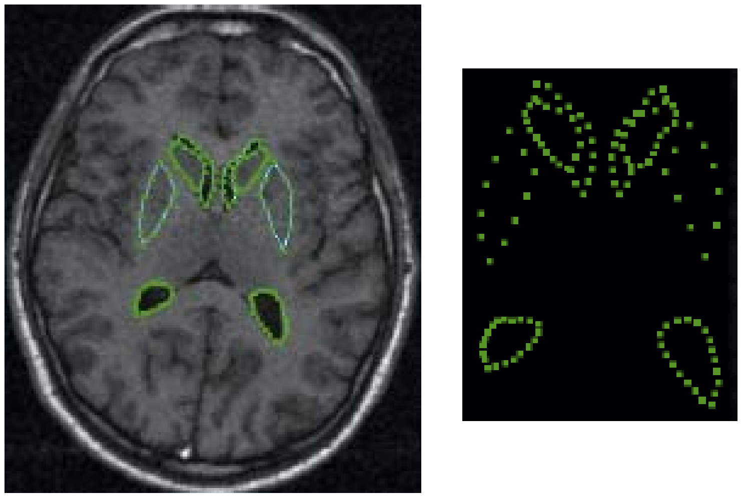
↓

Evaluation

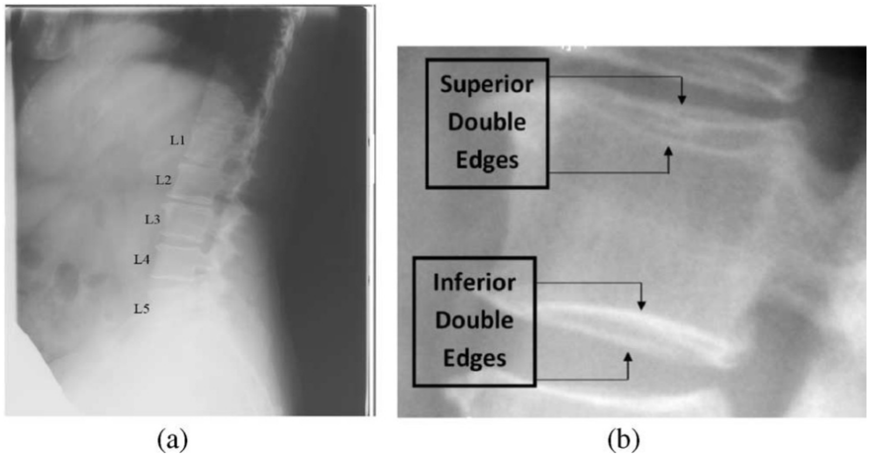
Shape Modelling: approaches

- Statistical Models: initial image processing
  - Selection of landmarks:
    - Well defined corners
    - 'T' junctions
    - Easily located biologic landmarks
  - Creation of a set of vectors for each image (data set)





Cootes (Active Shape tutorial)

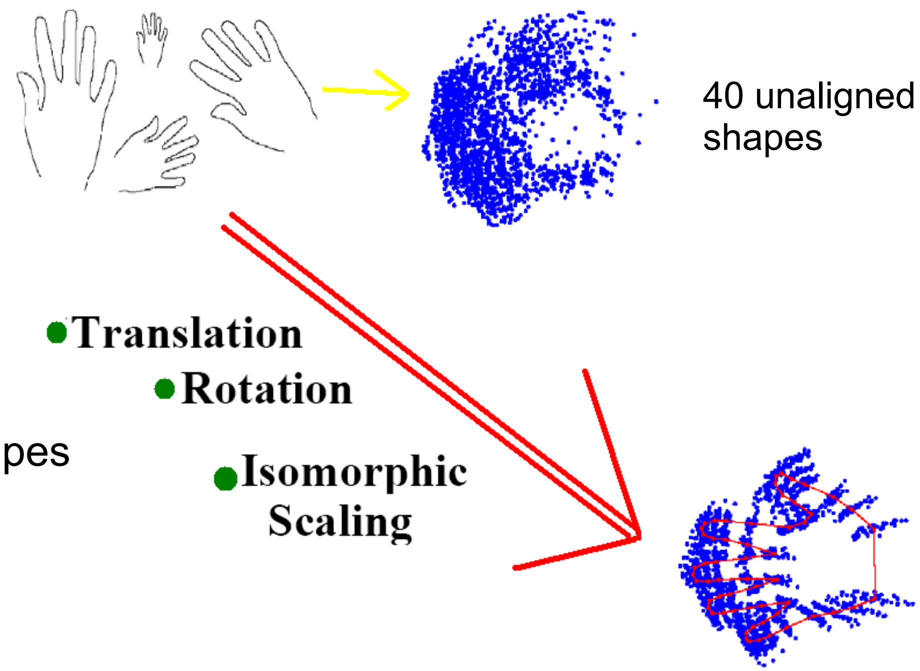


Double-Edge Detection of Radiographic Lumbar Vertebrae Images Using Pressurized Open DGVF Snakes. Texas Tech University, Lubbock, Texas, USA.

Shape Modelling: approaches

- Statistical Models: initial image processing
  - Alignment. Basic coregistration of the shape in the same coordinate system.
  - Procrustes analysis.
  - Creation of parameterised model.

Aligning shapes



Shape Modelling: approaches

- Statistical Models: initial image processing
  - Procrustes analysis:
    - The Procrustes distance is a least-squares type shape metric that requires two aligned shapes with one-to-one point correspondence.
    - The alignment part involves four steps:
      - Compute the centroid of each shape.
      - Re-scale each shape to have equal size.
      - Align w.r.t. position the two shapes at their centroids.
      - Align w.r.t. orientation by rotation.

Shape Modelling: approaches

- Statistical Models: initial image processing
  - Procrustes analysis: for 2 vectors, find transformation which minimizes

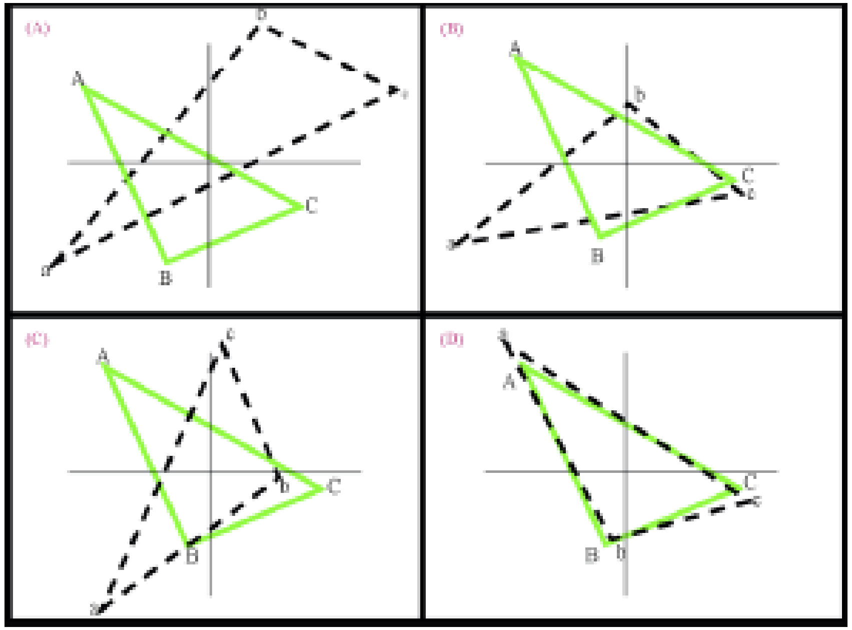
$$|x_1 - T(x_2)|^2$$

  - for n vectors, minimize:

$$\sum |m - Ti(x_i)|^2$$

$$\mathbf{m} = \frac{1}{n}\sum T_i(\mathbf{x}_i) \quad |\mathbf{m}| = 1$$

  - Resulting shapes have
    - Identical centre of gravity
    - Approximately the same scale and orientation



Shape Modelling: approaches

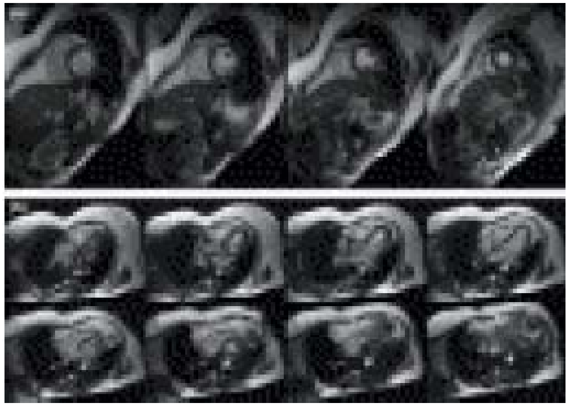
- Statistical Models: initial image processing
  - Parameterised model

$$\mathbf{x} = f_{shape}(\mathbf{b}) \longrightarrow \mathbf{x} = \overline{\mathbf{x}} + \mathbf{Pb}$$

- $\overline{\mathbf{x}}$ is calculated in procrustes analysis
- b is the model parameters
- We have only to find **P**

Shape Modelling: approaches

- Statistical Models: training data set



MR of heart, training set

Surfaces of data set images

"…a training set consists of an **input vector** and an answer vector, and is used together with a **supervised learning** method to train a knowledge database…"

- Wikipedia

Shape Modelling: approaches

- Statistical Models: variation model
  - Given: S – number of aligned vectors (shapes in the training set)
    N – number of landmarks
    D – dimensions of the problem (1D, 2D, or 3D)
  - Reduce dimension (PCA) form ND

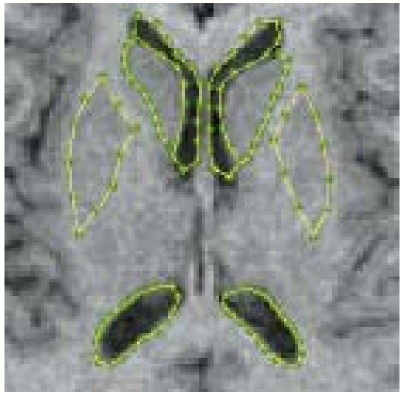
$$\mathbf{x} \approx \bar{\mathbf{x}} + \mathbf{Pb}$$

Where $\mathbf{P}$ contains the $\mathbf{t}$ eigenvectors, corresponding to $\mathbf{t}$ largest eigenvalues $\mathbf{b}$-vectors of parameters.

Shape Modelling: approaches

- Statistical Models: variation model
  - Example:

$$\mathbf{x} \approx \bar{\mathbf{x}} + \mathbf{Pb}$$



Labeled brain MR image

Varying the most significant parameter b1

Varying the most significant parameter b2

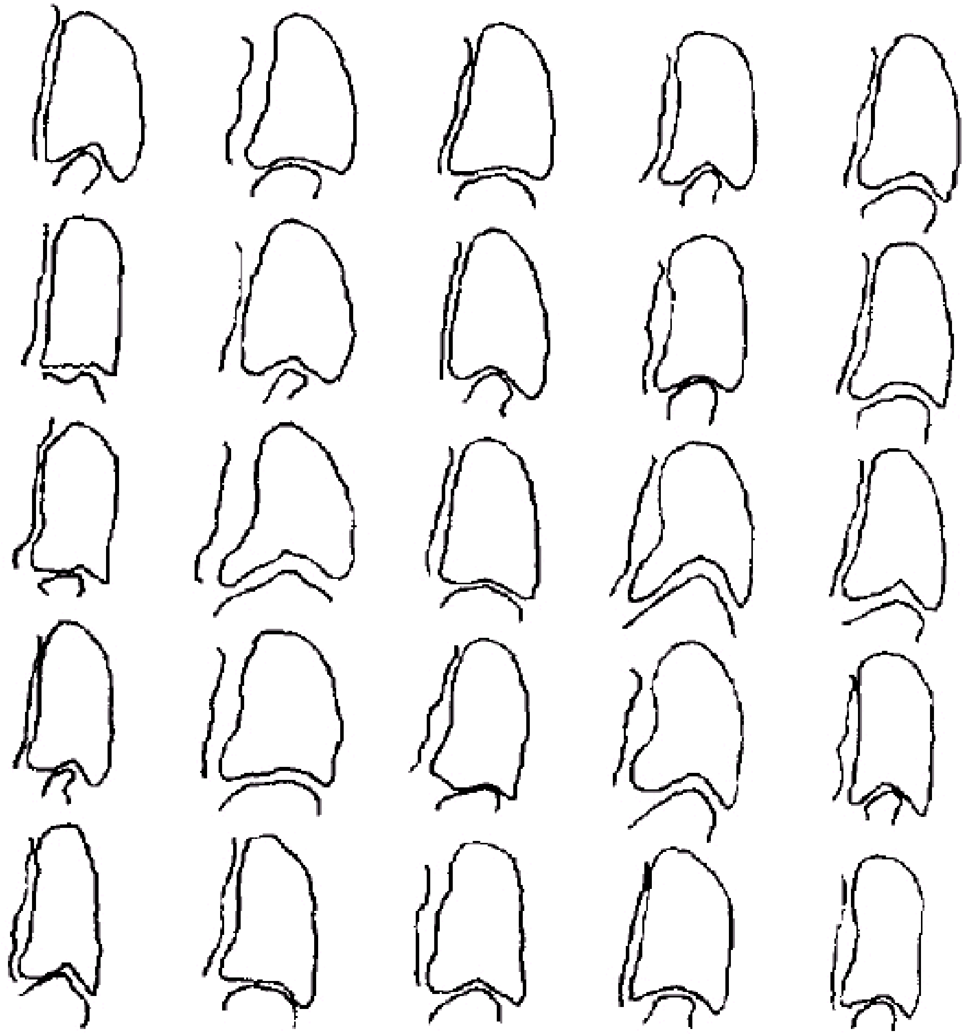
Shape Modelling: approaches

- Statistical Models: variation model
  - Generating new example shapes:
    - Shapes of training set approximated by $\mathbf{x} = \bar{\mathbf{x}} + \mathbf{Pb}$, where $\mathbf{P} = (\mathbf{p}_1 \mathbf{p}_2 \ldots \mathbf{p}_t)$ is the matrix of the first $\mathbf{t}$ eigenvectors and $\mathbf{b} = (b_1 b_2 \ldots b_t)^T$ is a vector of weights.
    - Vary $\boldsymbol{b_k}$ within suitable limits for similar shapes.

$$-3\sqrt{\lambda_k} \le b_k \le 3\sqrt{\lambda_k}$$

Shape Modelling: approaches

- Statistical Models: variation model
  - Heart Example
    - 66 examples
    - 96 points
      - Left ventricle
      - Right ventricle
      - Left atrium
    - Traced by cardiologists



Shape Modelling: approaches

- Statistical Models: variation model
  - Heart Example
    - 66 examples
    - 96 points
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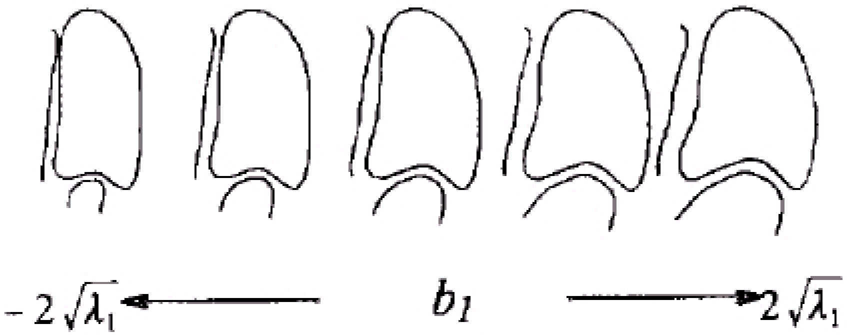
Eigenvalues of the Covariance Matrix Derived from a Set of Heart Ventricle Shapes

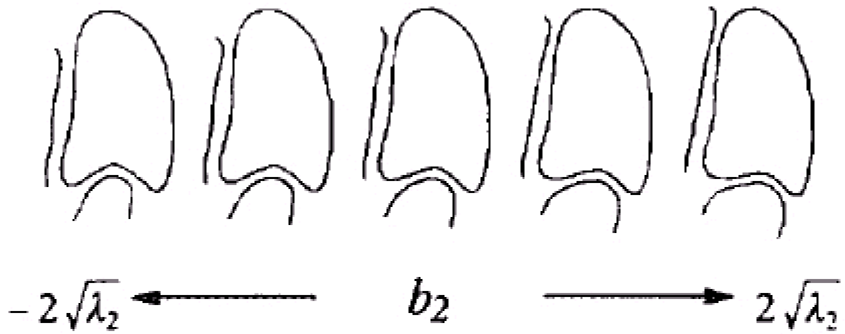
Eigenvalue	$\frac{\lambda_i}{\lambda_T} \times 100\%$
λ_1	37%
λ_2	17%
λ_3	13%
λ_4	7%
λ_5	6%
λ_6	4%

Shape Modelling: approaches

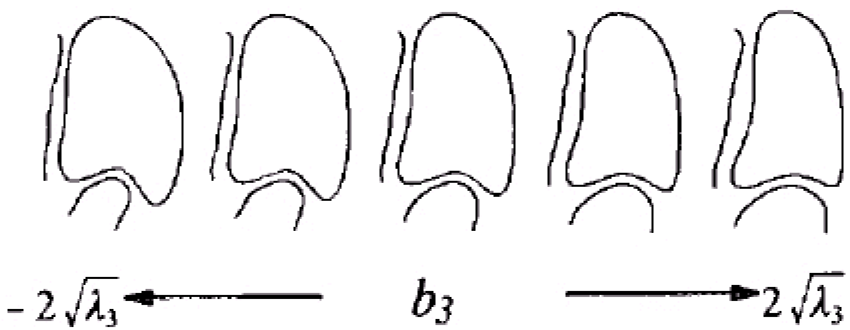
- Statistical Models: variation model
  - Heart Example



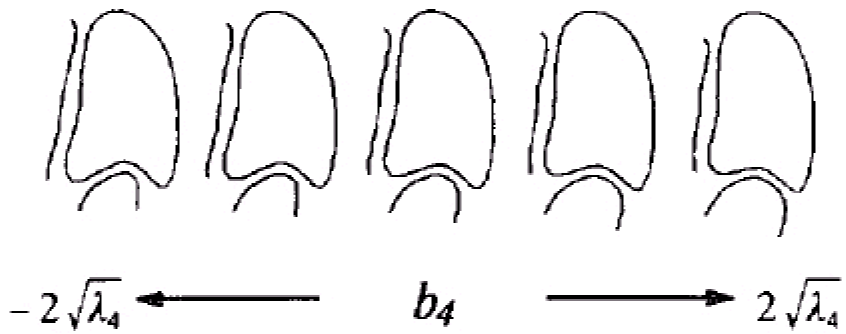
Varies Width



Varies Septum



Varies LV



Varies Atrium

Shape Modelling: approaches

- Statistical Models: experimental data set
  - Fitting a model to new points:
    - By minimizing the sum of square distances between corresponding model and image points in the iterative approach.
    - Iteratively generate plausible models and apply to find a match to the image.

Shape Modelling: approaches

- Statistical Models: evaluating
  - Testing how well the model generalizes:
    - Leave-one-out experiment
    - S training shapes: S-1 are used to generate the model.

      1 as an example to fit the model to and record the error.

      Repeat for all S shapes.

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Shape Modelling: approaches

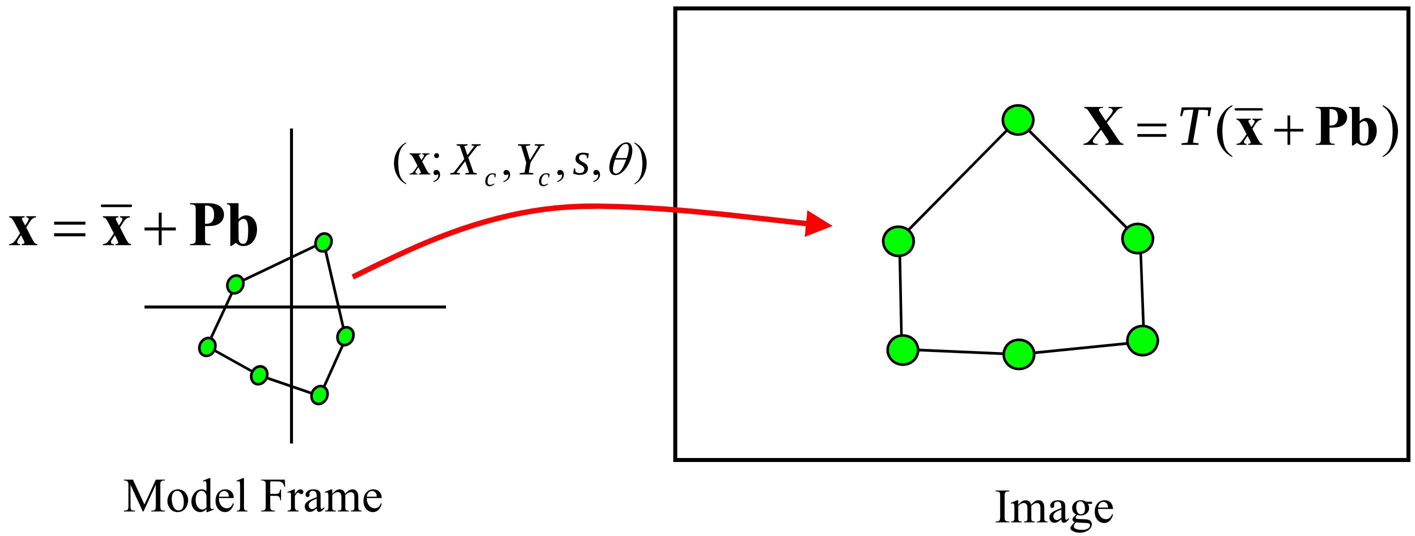
- Active Shape Models (ASMs)
  - Suppose we have a statistical shape model
    - Trained from sets of examples
  - How do we use it to interpret new images?
  - Use an “Active Shape Model”
  - Iterative method of matching model to image.

Shape Modelling: approaches

- Active Shape Models (ASMs)
  - Assume we have an initial estimate for the pose and shape parameters (e.g. the mean shape): $\mathbf{x} = \bar{\mathbf{x}} + \mathbf{P}\mathbf{b}$
  - The model points are defined in a model coordinate frame.
  - Must apply global transformation $\mathbf{T}$ to place in image.

Shape Modelling: approaches

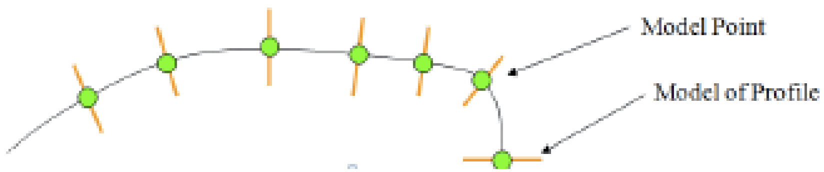
- Active Shape Models (ASMs)
  - Must apply global transformation $\mathbf{T}$ to place in image.

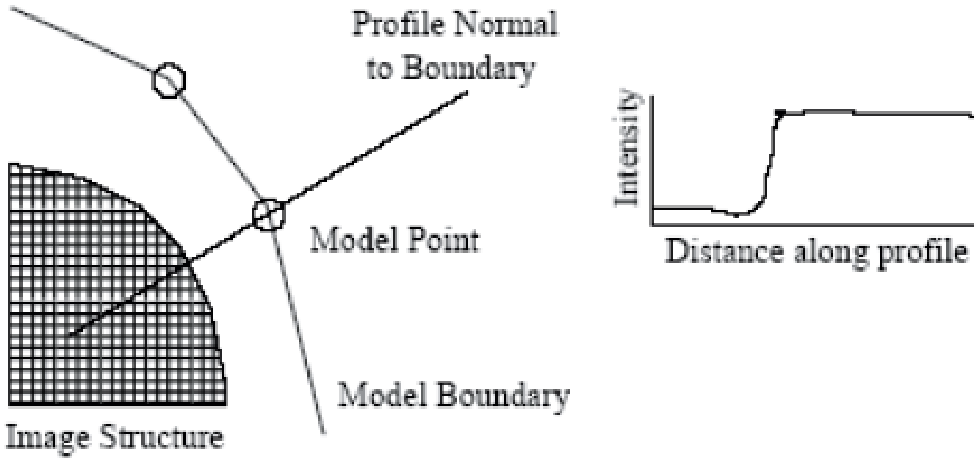


where $(\mathbf{X_c}, \mathbf{Y_c})$ is the position of the centre of the model in the image frame, rotated by $\boldsymbol{\theta}$ and scaled by $\mathbf{s}$.

Shape Modelling: approaches

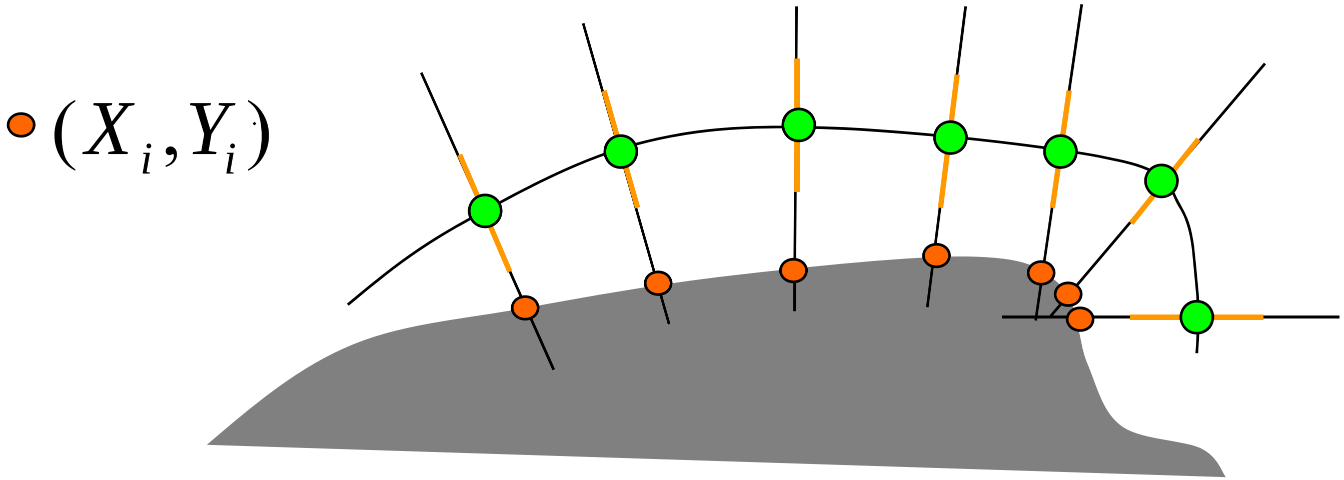
- Active Shape Models (ASMs): how it works
  - Match shape model to new image
  - Require:
    - Statistical shape model
    - Model of image structure at each point/training set





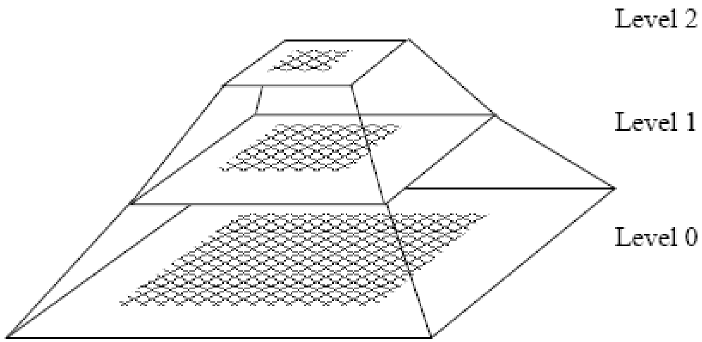
Shape Modelling: approaches

- Active Shape Models (ASMs): how it works
  - Algorithm
    - Examine a region of the image around each point (X_i, Y_i) to find the best nearby match.
    - Update parameters (X_t, Y_t, s, θ, b) to best fit the new found points.
    - Repeat until convergence.



Shape Modelling: approaches

- Active Shape Models (ASMs): how it works
  - Improved efficiency
    - Multi-resolution framework: gaussian image pyramid formed by repeated smoothing and sub-sampling.

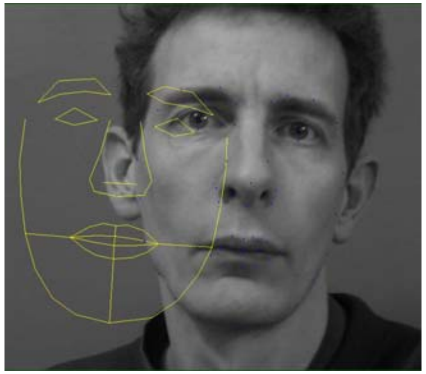


Each level half the size of the one below

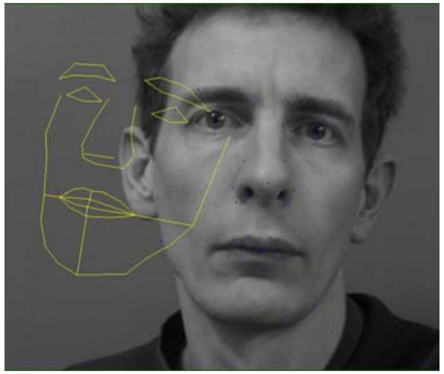
    - Allow rapid location of the boundary of objects with similar shapes.
    - Useful to classify objects based on shape or appearance, which have well defined shape.
    - Useful when approximate location of target object is known.

Shape Modelling: approaches

- Active Shape Models (ASMs): how it works
  - Limitations
    - Can fail if the initial guess is too far from the target.
    - Problems when position/size/orientation of targets is not known approximately.
    - Doesn't work with widely varying shapes.
    - The model can only deform in ways observed in the training set. If is not there, the model will not fit to it.



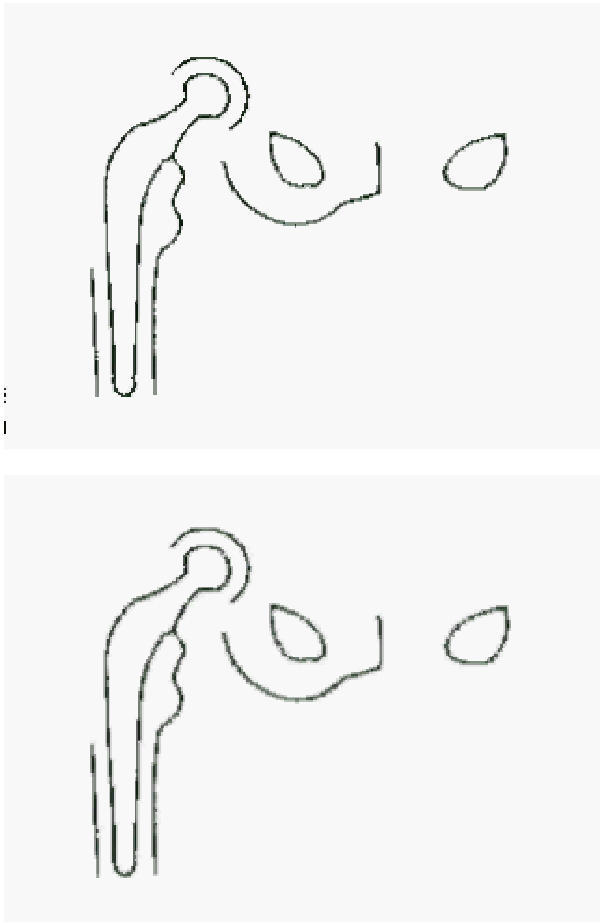
Initial guess



Search result

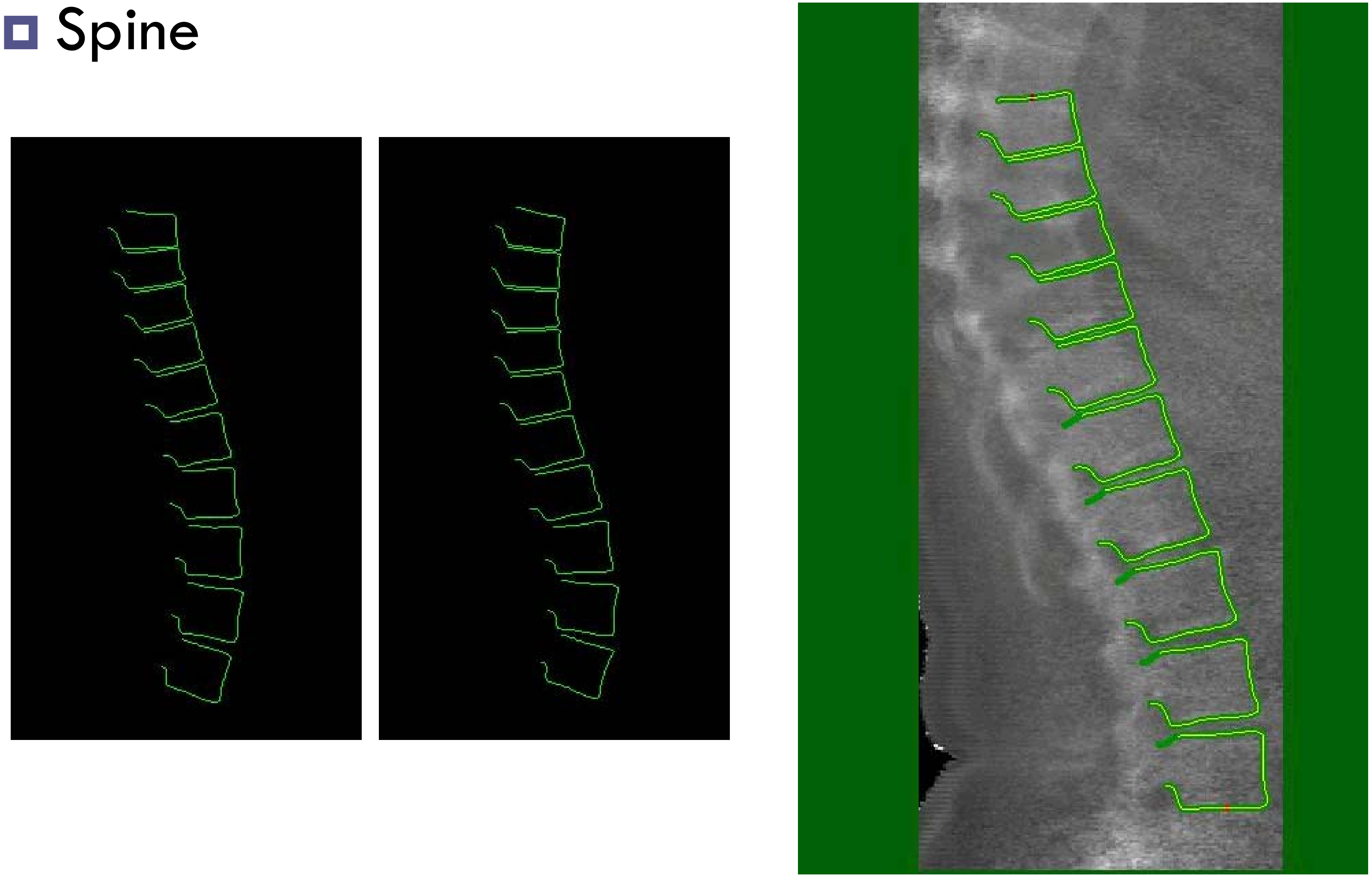
Shape Modelling: approaches

- Active Shape Models (ASMs): example
  - Hip radiograph



Shape Modelling: approaches

- Active Shape Models (ASMs): example
  - Spine



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Shape Modelling: approaches

- Active Appearance Models (AAMs)
  - Suppose we have a statistical shape model
    - Trained from sets of examples
  - How do we use it to interpret new images?
  - Use an "Active Appearance Model"
  - Iterative method of matching model to image.
  - The Active Appearance Model (AAM) is a **generalization** of the Active Shape Model approach, but uses **all the information in the image** region covered by the target object, rather than just near modeled edges.

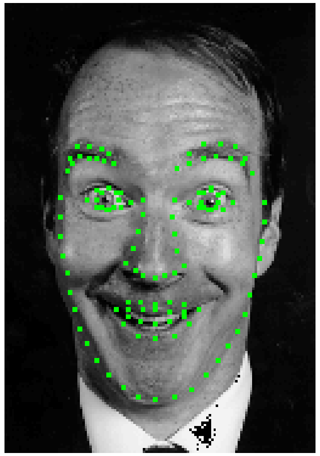
Shape Modelling: approaches

- Active Appearance Models (AAMs): Method
  - Since statistical shapes model the shape change of an object, why not construct a similar statistical model to represent the intensity variation across a region?
  - Method: given a set of training images, labeled with landmark points, we can use image warping to deform each image so that the object has the mean shape, then build a statistical model of the grey-levels across the object.

Ex: the central image is the mean

Shape Modelling: approaches

- Active Appearance Models (AAMs): Method
  - For each example, extract a shape vector

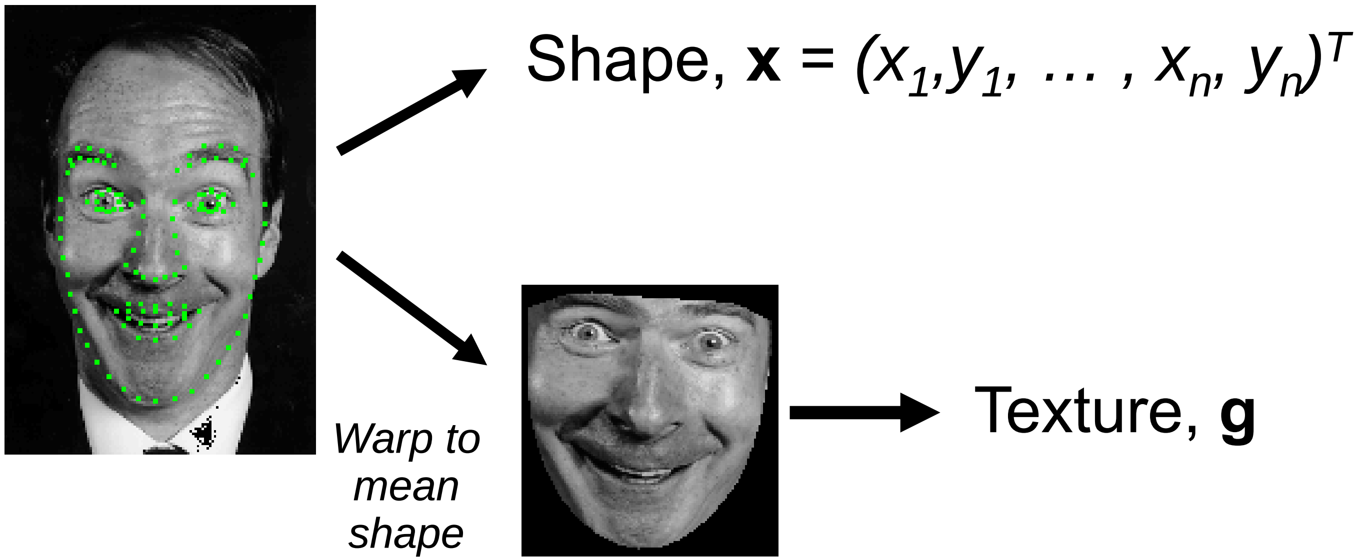


Shape, $\mathbf{x} = (x_1, y_1, \ldots, x_n, y_n)^T$

  - Built a statistical shape model: $\mathbf{x} = \bar{\mathbf{x}} + \mathbf{P}_s \mathbf{b}_s$

Shape Modelling: approaches

- Active Appearance Models (AAMs): Method
  - For each example, extract a texture vector

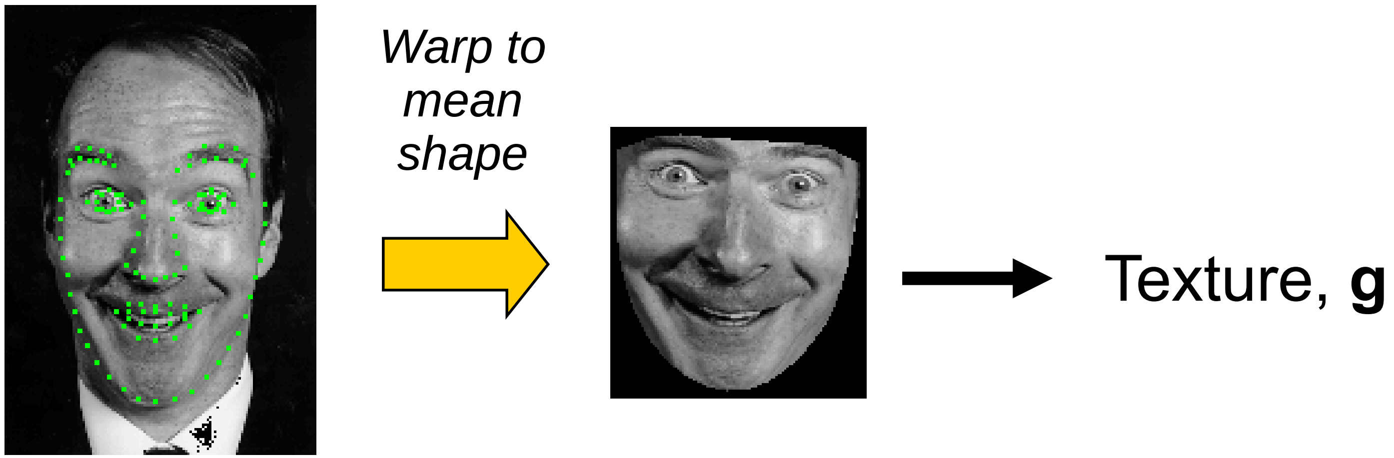


Shape Modelling: approaches

- Active Appearance Models (AAMs): Method
  - Warping texture
    - Problem:
      - Given corresponding points in two images, how do we warp one into the other?
    - Two common solutions:
      - Piece-wise linear using triangle mesh
      - Thin-plate spline interpolation

Shape Modelling: approaches

- Active Appearance Models (AAMs): Method
  - Warping texture



    - Normalise vectors (as for eigenfaces)
    - Build eigen-model $\mathbf{g} = \overline{\mathbf{g}} + \mathbf{P}_g \mathbf{b}_g$

Shape Modelling: approaches

- Active Appearance Models (AAMs): Method
  - Shape and texture often correlated
    - When smile, shadows change (texture) and shape changes
  - Learning this correlation leads to more compact (and specific) model.

$$\mathbf{x} = \overline{\mathbf{x}} + \mathbf{P}_s \mathbf{b}_s \qquad \mathbf{g} = \overline{\mathbf{g}} + \mathbf{P}_g \mathbf{b}_g$$

  - Varying $\mathbf{c}$ changes both shape and texture:

$$\boxed{\begin{aligned} \mathbf{x} &= \overline{\mathbf{x}} + \mathbf{Q}_s \mathbf{c} \\ \mathbf{g} &= \overline{\mathbf{g}} + \mathbf{Q}_g \mathbf{c} \end{aligned}}$$

Shape Modelling: approaches

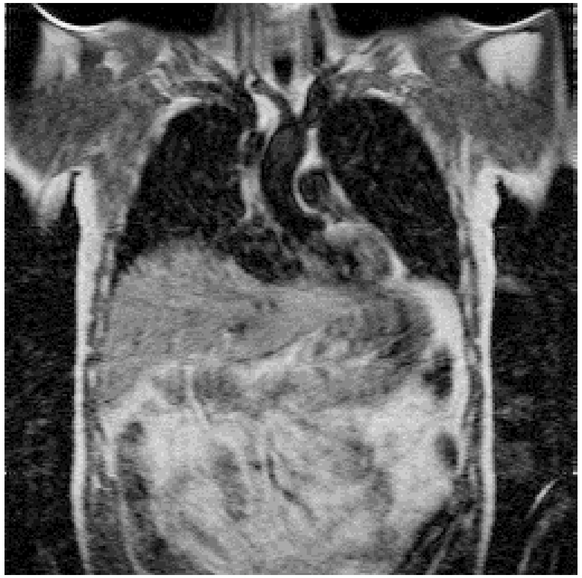
- Active Appearance Models (AAMs): Example

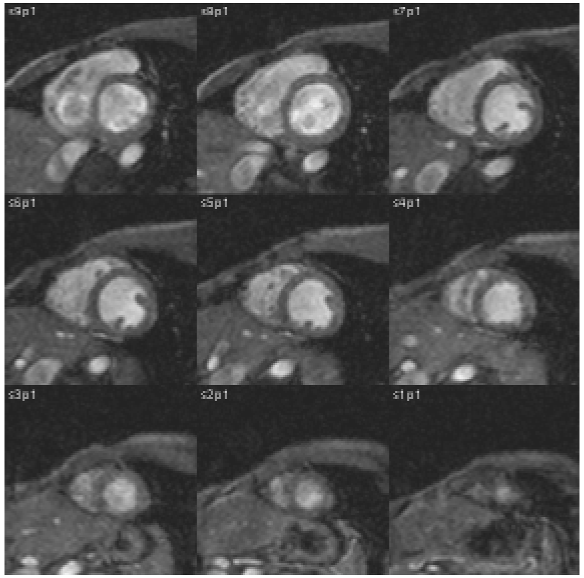
Initial | 2 its | 8 its | 14 its | 20 its | converged

A face model built from 400 images. The figures shows frames from an AAM search for a new face, each starting with the mean model displaced from the true face centre.

Shape Modelling: approaches

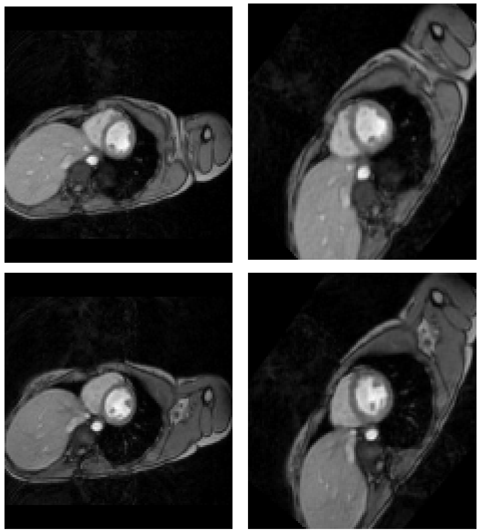
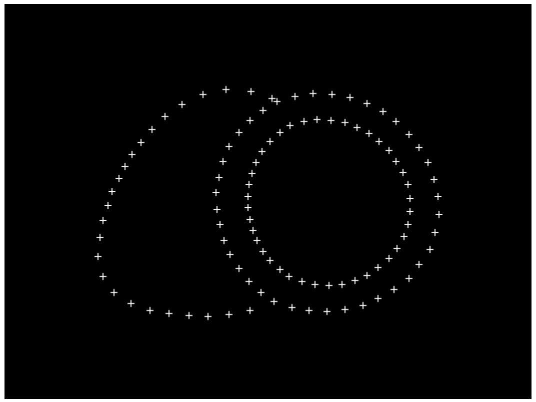
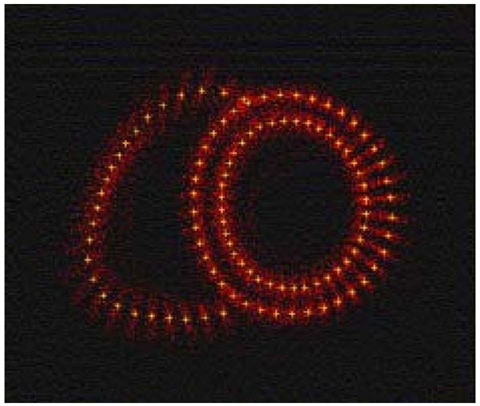
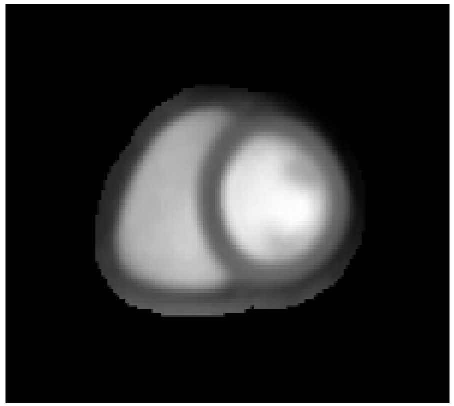
- Active Appearance Models (AAMs): Example
  - Cardiac MRI (courtesy of Milan Sonka, The University of Iowa).





Shape Modelling: approaches

- Active Appearance Models (AAMs): Example
  - Cardiac MRI: mean shape and mean grey level
    - Modelling of shape: represented by landmark points
    - Represent the shape borders of objects as a collection of corresponding points
    - Modelling of texture: represented by pixel intensities

Shape Modelling: approaches

- Active Appearance Models (AAMs): Example
  - Cardiac MRI: mean shape and mean grey level
    - Each example can be expressed by two vectors

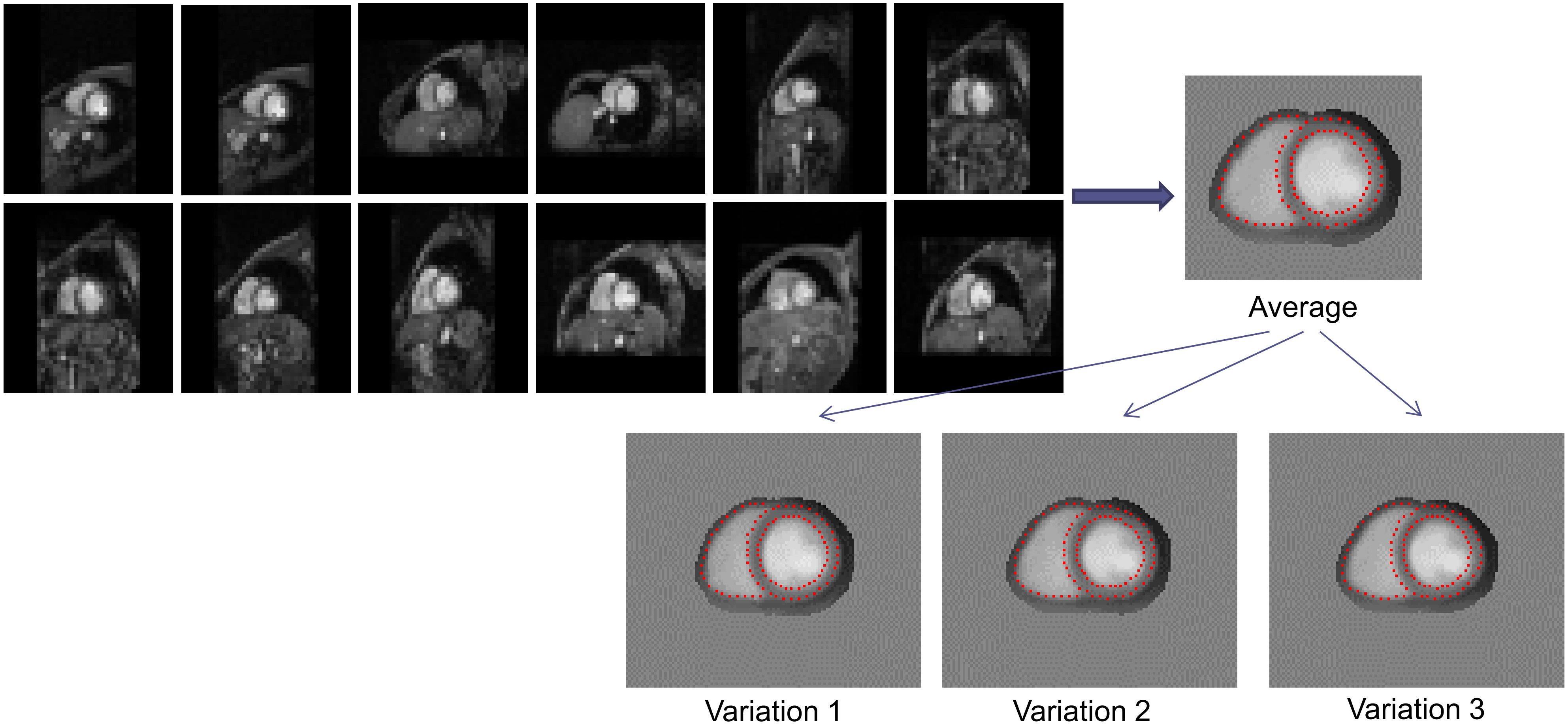
$$b_s \approx P_s^{\,T} (x - \bar{x})$$

$$b_g \approx P_g^{\,T} (g - \bar{g})$$

- b_s and b_g can be concatenated and PCA applied

Shape Modelling: approaches

- Active Appearance Models (AAMs): Example
  - Cardiac MRI: mean shape and mean grey level

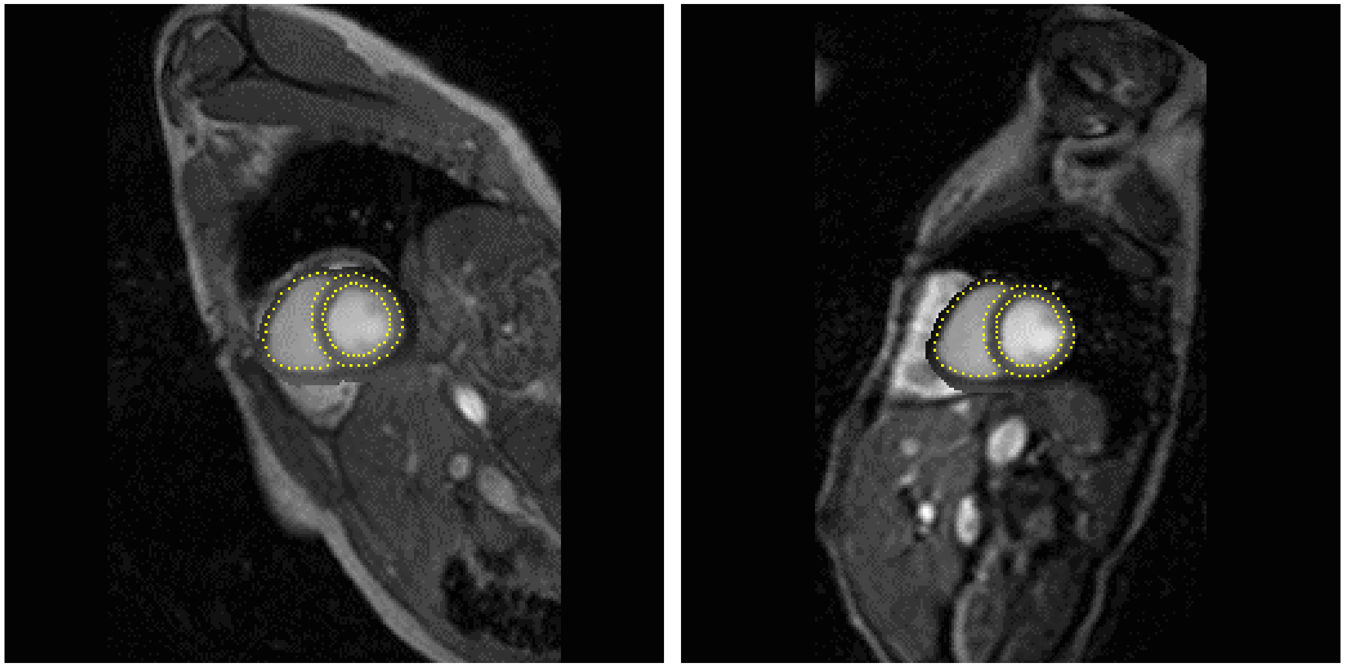


Shape Modelling: approaches

- Active Appearance Models (AAMs): Example
  - Cardiac MRI: matching
  - Matching the AAM to the image requires
    - A criterion function
      - The RMS error of the 'difference image' between the model and the underlying image patch
    - A minimization procedure
    - Derivatives of the criterion function with respect to all 'optimizable' parameters
      - Can be estimated using multiple linear regression
      - Using examples of derivative images

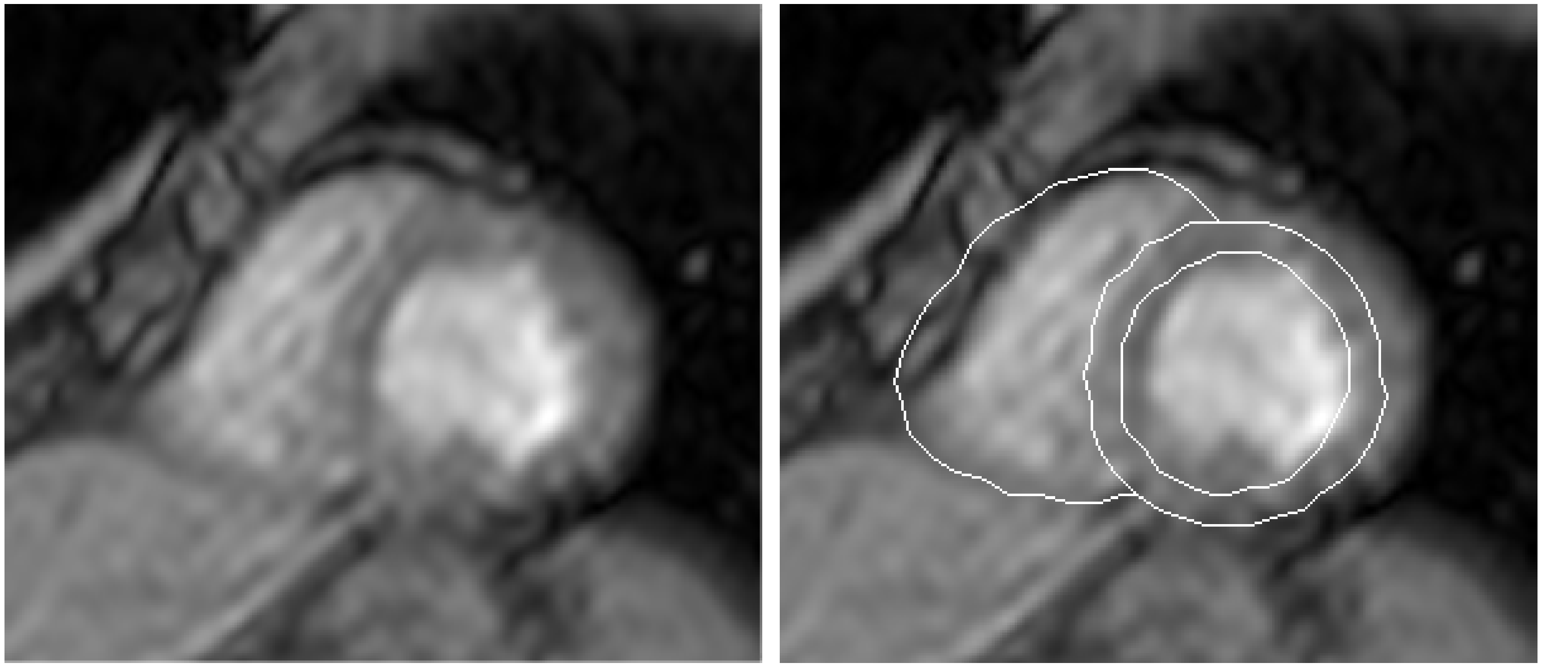
Shape Modelling: approaches

- Active Appearance Models (AAMs): Example
  - Cardiac MRI: results



Shape Modelling: approaches

- Active Appearance Models (AAMs): Example
  - Cardiac MRI: results
    - However, appearance matching may lock on incorrect features and get stuck in a local minimum.



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Shape Modelling: approaches

- Active Contour Models.
  - Connectivity-preserving relaxation-based segmentation method.
  - The main idea is to start with some initial boundary shape represented in the form of spline curves, and iteratively modify it by applying various shrink/expansion operations according to some energy function: ***active contour model***.

Shape Modelling: approaches

- Active Contour Models: energy minimization
  - Objective: find the minimum value for a given energy function.
  - Energy function contains two terms:
    - One term penalizes the solutions inconsistent with the observed data.
    - The other term enforces the spatial coherence.
  - Several energy minimization techniques:
    - Iterated Conditional Modes
    - Graph Cuts
    - Snakes
    - Gradient Vector Flow

Shape Modelling: approaches

- Active Contour Models: algorithm overview
  - Snakes (active contours) are curves defined within an image domain which can move under the influence of **internal forces coming from** within **the curve** itself, and **external forces computed from the image data**.
  - Algorithm overview:
    - First, a spline is initialized on the image.
    - Then, the energy function is minimized to make it cover only the desired object.
    - A snake falls into the closest local energy minima.
    - The process iterates.

Shape Modelling: approaches

- Active Contour Models: algorithm overview
  - Snakes are active models and exhibits dynamic behaviour.
  - Suitable for objects of changing shapes (e.g. lips, eyes,…)

Shape Modelling: approaches

- Active Contour Models: algorithm
  - The active contour model algorithm deforms a contour to lock onto features of interest within in an image.
  - Usually, the features are lines, edges, and/or object boundaries.
  - The authors (Kass et al.) named their algorithm "snakes" because the deformable contours resemble snakes as they move.
  - Given an approximation of the boundary of an object in an image, an active contour model can be used to find the "actual" boundary.

Shape Modelling: approaches

- Active Contour Models: algorithm
  - An active contour is an ordered collection of $\mathbf{n}$ points in the image plane:

$$V = \{v_1, \dots, v_n\}$$
$$v_i = (x_i\,, y_i) \quad i = \{1, \dots, n\}$$

  - The points in the contour iteratively approach the boundary of an object through the solution of an energy minimization problem. For each point in the neighborhood of v_i, an energy term is computed:

$$\boxed{E_i = \alpha E_{int}(v_i) + \beta E_{ext}(v_i)}$$

Shape Modelling: approaches

- Active Contour Models: algorithm
  - For each point in the neighborhood of v_i, an energy term is computed:

$$E_i = \alpha E_{int}(v_i) + \beta E_{ext}(v_i)$$

where $E_{int}(v_i)$ is an energy function **dependent on the shape of the contour**, and $E_{ext}(v_i)$ is an energy function **dependent on the image properties**, such as the gradient, near point v_i.

α and β are constants providing the relative weighting of the energy terms.

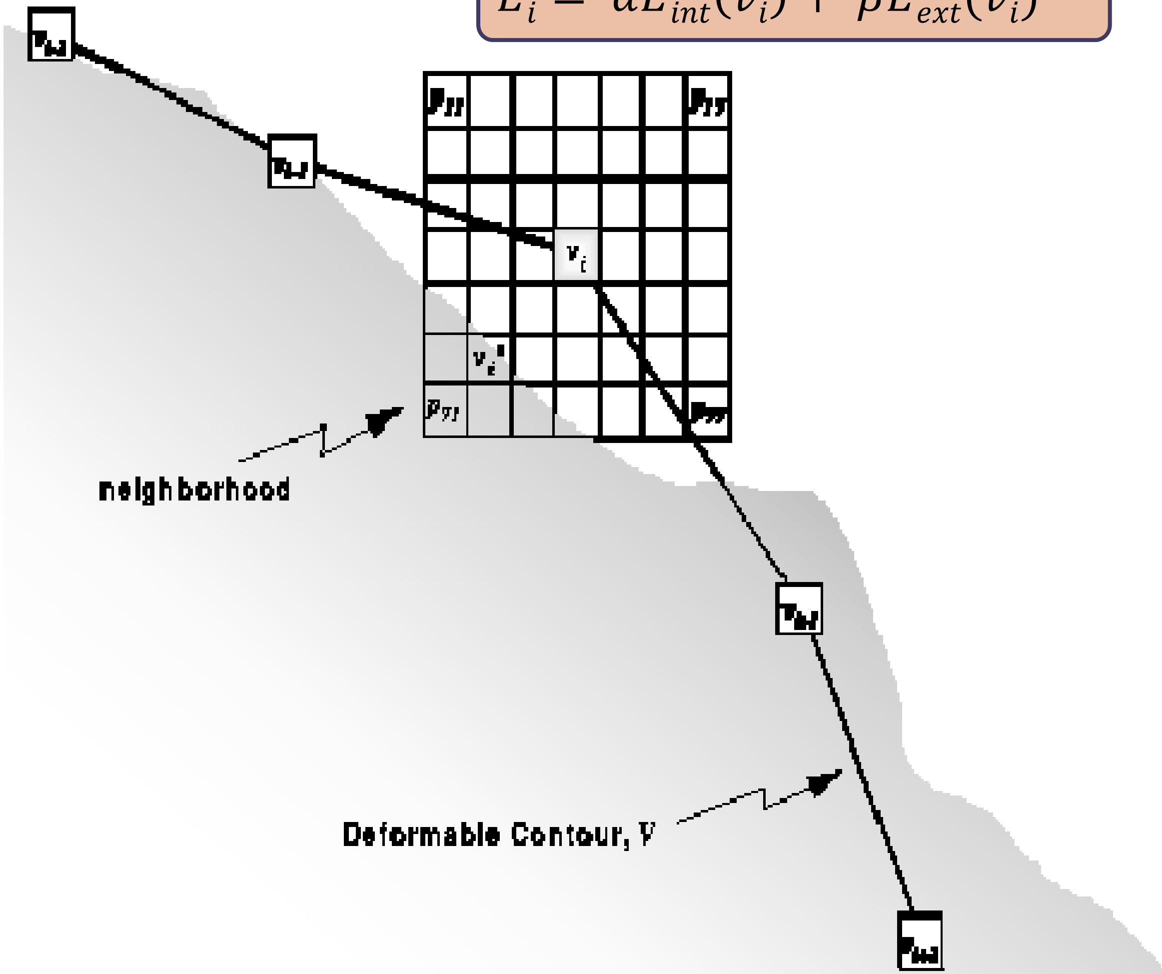
E_i, E_{int}, and E_{ext} are matrices. The value at the center of each matrix corresponds to the contour energy at point v_i.

Shape Modelling: approaches

Active Contour Models: algorithm

$$E_i = \alpha E_{int}(v_i) + \beta E_{ext}(v_i)$$

- Each point v_i is moved to the point $v_i{}'$ corresponding to the location of the minimum value in E_i.
- If the energy functions are chosen correctly, the contour V should approach and stop at the object boundary.



Shape Modelling: approaches

- Active Contour Models: internal energy
  - The **internal energy function** is intended to **enforce a shape on the deformable contour** and to maintain a constant distance between the points in the contour. Additional terms can be added to influence the motion of the contour.
  - An example of the internal energy function could be

$$\alpha E_{int}(v_i) = cEc_{ont}(v_i) + bE_{bal}(v_i)$$

where $E_{cont}(v_i)$ is the **continuity energy** that enforces the shape of the contour, and $E_{bal}(vi)$ is a **balloon force** that causes the contour to grow (balloon) of shrink. c and b provide the relative weighting of the energy terms.

Shape Modelling: approaches

- Active Contour Models: internal energy
  - Continuity energy: in the absence of other influences, the continuity energy term coerces an open deformable contour into a straight line and a closed deformable contour into a circle.

    The energy term $c_{jk}(v_i)$ for each element is defined as:

$$c_{jk}(v_i) = \frac{1}{l(V)} \|p_{jk}(v_i) - \gamma\,(v_{i-1} + v_{i+1}\|^2$$

where $p_{jk}(v_i)$ is the point in the image that corresponds to energy matrix $c_{jk}(v_i)$.

Shape Modelling: approaches

- Active Contour Models: internal energy
  - Continuity energy: $c_{jk}(v_i) = \frac{1}{l(V)} \|p_{jk}(v_i) - \gamma\,(v_{i-1} + v_{i+1}\|^2$

    where $l(V)$ is the normalization factor (the average distance between points in V) and γ a parameter:

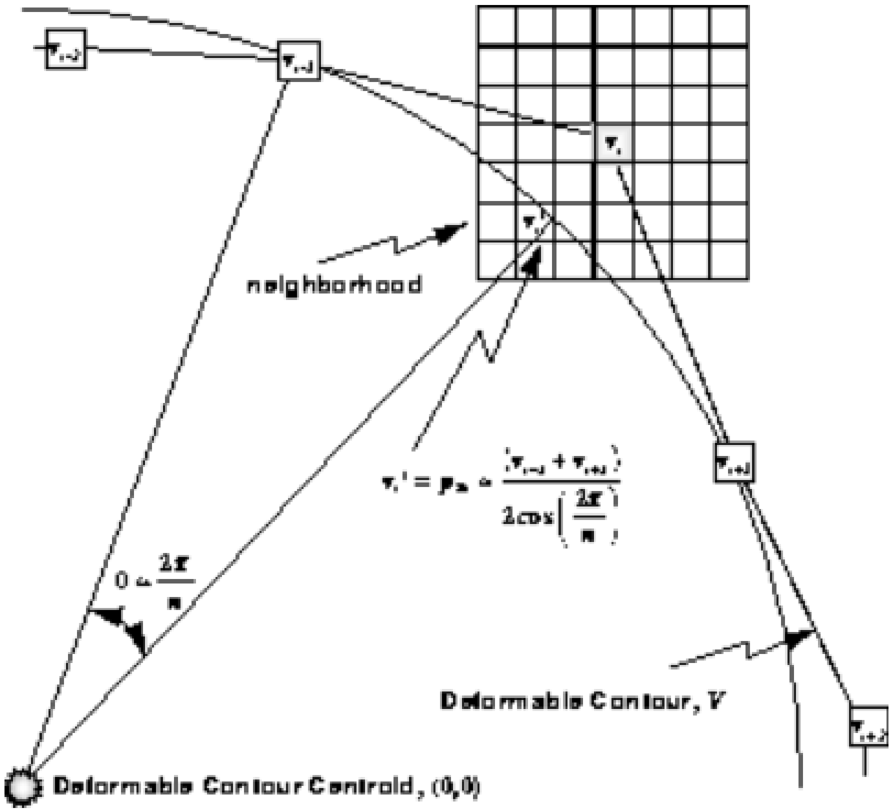
    - $\gamma = 0.5$ for an open contour. In this case, the minimum energy point is the point exactly half the way between v_{i-1} and v_{i+1}.
    - For the case of a closed contour, V is given a modulus of n. Therefore, $v_{n+i} = v_i$. γ is then defined as $\gamma = \frac{1}{2\cos(\frac{2\pi}{n})}$

Shape Modelling: approaches

- Active Contour Models: internal energy
  - Continuity energy: $c_{jk}(v_i) = \frac{1}{l(V)} \|p_{jk}(v_i) - \gamma\,(v_{i-1} + v_{i+1}\|^2$

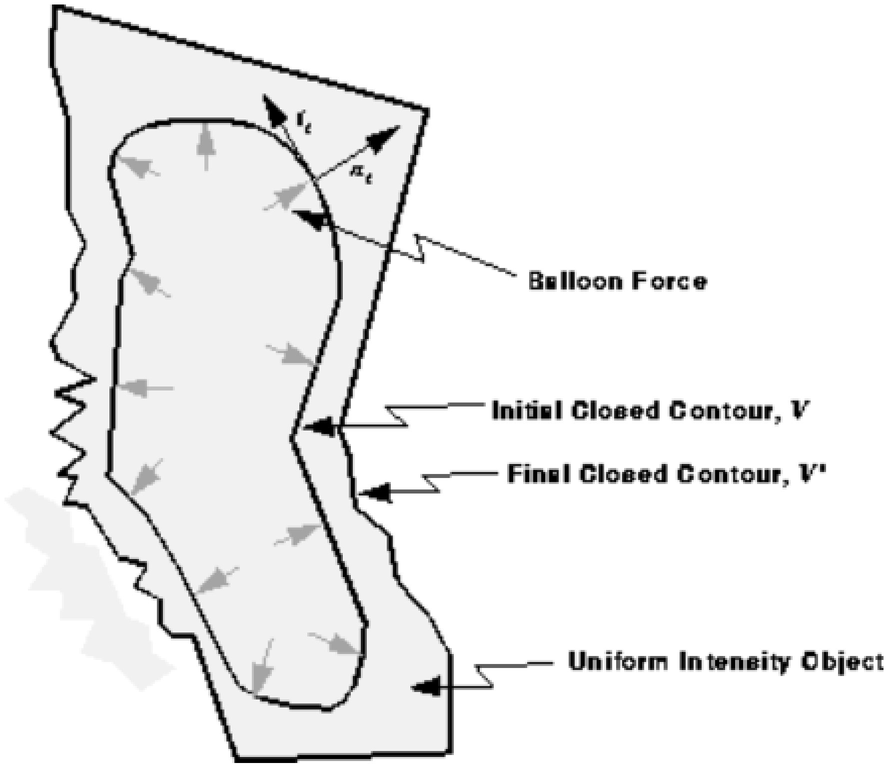
Here, the point of minimum energy of $E_{cont}(v_i)$ is pushed outwards so that V becomes a circle.

The point v_i' is the location of minimum energy because it lies on the circle connecting v_{i-1} and v_{i+1}.



Shape Modelling: approaches

- Active Contour Models: internal energy
  - Balloon force: a balloon force can be used on a closed deformable contour to force the contour to expand (or shrink) in the absence of external influences.
  - A contour initialized within a uniform image object will expand under the influence of a balloon force until it nears the object boundary (at which point the external energy function affects its motion).



Shape Modelling: approaches

- Active Contour Models: internal energy
  - Balloon force: usually an adaptive balloon force that varies inversely proportionally to the image gradient magnitude is used.
  - The adaptive balloon force is strong in homogeneous regions and weak near object boundaries, edges, and lines.
  - The energy term $c_{jk}(v_i)$ for each element in the matrix $E_{ball}(v_i)$ could be expressed as a dot product:

$$c_{jk}(v_i) = n_i \cdot (v_i - p_{jk}(v_i))$$

Shape Modelling: approaches

- Active Contour Models: internal energy
  - Balloon force:

$$c_{jk}(v_i) = n_i \cdot (v_i - p_{jk}(v_i))$$

where n_i is the outward unit normal of V at point v_i, and $p_{jk}(v_i)$ is the point in the neighborhood of v_i corresponding to entry $c_{jk}(v_i)$ in the energy matrix. Therefore, the balloon energy is smallest at points farthest from v_i in the direction of n_i .

n_i can be found by rotating the tangent vector t_i by 90^o

Adaptive balloon forces are scaled by the image gradient magnitude at point v_i

Shape Modelling: approaches

- Active Contour Models: external energy
  - The **external energy** function **attracts the deformable contour to interesting features**, such as object boundaries, in an image.
  - Any energy expression that accomplishes this attraction can be considered for use.
  - Image gradient and intensity are obvious and easy characteristics to look at.
  - Another characteristics could be object size or shape.

$$\beta E_{ext}(v_i) = mE_{mag}(v_i) + gE_{grad}(v_i)$$

Shape Modelling: approaches

- Active Contour Models: external energy

$$\beta E_{ext}(v_i) = mE_{mag}(v_i) + gE_{grad}(v_i)$$

where $E_{mag}(v_i)$ is an expression that attracts the contour to high or low intensity regions, and $E_{grad}(v_i)$ is an energy term that moves the contour towards edges.

m and g are constants provided to adjust the relative weights of the terms.

Shape Modelling: approaches

- Active Contour Models: external energy

$$\beta E_{ext}(v_i) = mE_{mag}(v_i) + gE_{grad}(v_i)$$

  - Image intensity energy: each element in the intensity energy matrix $E_{mag}(v_i)$ is assigned the intensity value of the corresponding image point in the neighborhood of v_i.

$$c_{jk}(v_i) = I\left(p_{jk}(v_i)\right)$$

Then, if m is positive, the contour is attracted to regions of low intensity, and viceversa.

Shape Modelling: approaches

- Active Contour Models: external energy

$$\beta E_{ext}(v_i) = mE_{mag}(v_i) + gE_{grad}(v_i)$$

  - Image gradient energy: the image gradient energy function attracts the deformable contour to edges in the image. An energy expression proportional to the gradient magnitude will attract the contour to any edge:

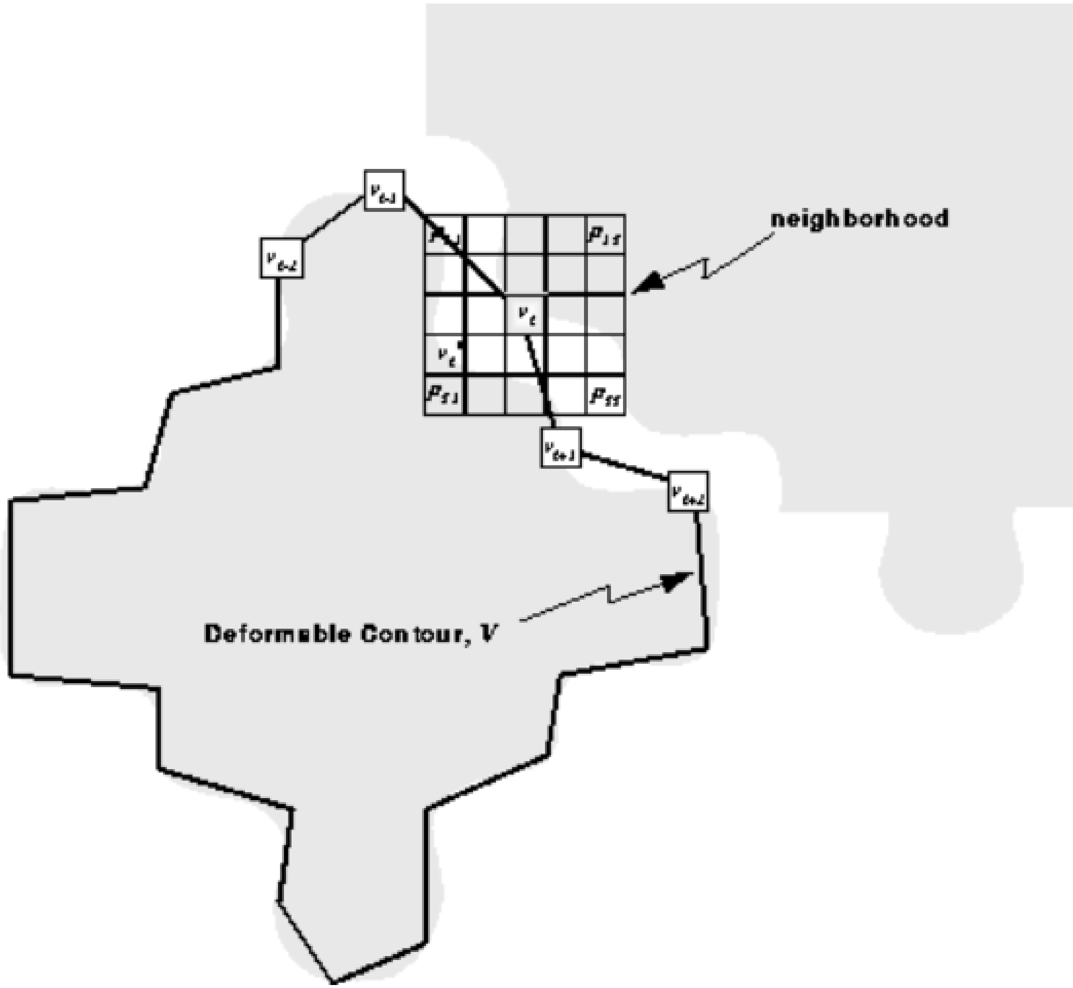
$$c_{jk}(v_i) = -|\nabla I(p_{jk}(v_i))|$$

When active contours are used to find object boundaries, an energy expression that discriminates between edges of adjacent objects is desirable. The key to such an expression is that the gradients at the edges of the objects have different directions.

Shape Modelling: approaches

- Active Contour Models: external energy

Because the gradient direction at the edge of the object of interest is similar to the outward unit normal direction of the contour, the active contour algorithm moves the snake point from v_i to v_i', even though the gradient magnitudes at both points are similar.



Shape Modelling: approaches

- Active Contour Models: external energy

  - Image gradient energy: the value for each element in the directional gradient energy matrix $E_{grad}(v_i)$ can therefore be defined by a dot product between the unit normal of the deformable contour and the image gradient:

$$c_{jk}(v_i) =- -n_i \cdot |\nabla I(p_{jk}(v_i))|$$

where n_i is the unit normal of the contour at point v_i, as defined.

Shape Modelling: approaches

- Active Contour Models: regularization

$$E_i = \alpha E_{int}(v_i) + \beta E_{ext}(v_i)$$

  - The energy functions should be scaled so that the neighborhood matrices contain comparable values. This process is referred to as regularization.

$$\alpha E_{int}(v_i) = cE_{cont}(v_i) + bE_{bal}(v_i)$$
$$\beta E_{ext}(v_i) = mE_{mag}(v_i) + gE_{grad}(v_i)$$

  - Each of the energy functions is adjusted to the range [0,1].

Shape Modelling: approaches

- Active Contour Models: regularization
  - Continuity energy: at each point in the deformable contour, the elements in neighborhood matrix for the continuity energy are simply scaled to the range [0,1].

$$c'_{jk}(v_i) = \frac{c_{jk}(v_i) - c_{min}(v_i)}{c_{max}(v_i) - c_{min}(v_i)}$$

where $c_{min}(v_i)$ and $c_{max}(v_i)$ are the minimum and maximum valued elements, respectively, in $E_{cont}(v_i)$.

Shape Modelling: approaches

- Active Contour Models: regularization
  - Balloon energy: the balloon energy is scaled to the range [0,1], then adapted to the image gradient intensity:

$$c'_{jk}(v_i) = \frac{c_{jk}(v_i) - c_{min}(v_i)}{c_{max}(v_i) - c_{min}(v_i)} \cdot \left(1 - \frac{|\nabla I(v_i)|}{|\nabla I|_{max}}\right)$$

where $|\nabla I|_{max}$ is the maximum gradient magnitude in the entire image.

Shape Modelling: approaches

- Active Contour Models: regularization

  - Intensity energy: a parameter δI is added to the intensity energy term for regularization:

$$c'_{jk}(v_i) = \frac{c_{jk}(v_i) - c_{min}(v_i)}{\max(c_{max}(v_i) - c_{min}(v_i), \delta I \cdot I_{max})}$$

where I_{max} is the maximum intensity in the entire image and δI has a range of $[0, \infty]$. Therefore, δI determines the sensitivity of the active contour to local variations in image intensity.

Shape Modelling: approaches

Active Contour Models: regularization

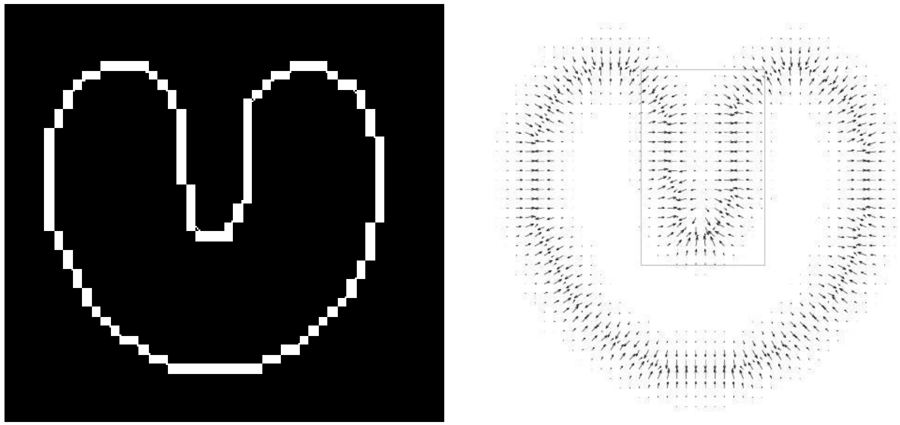
- Gradient energy: this term is regularized in the same manner as the intensity energy term:

$$c'_{jk}(v_i) = \frac{c_{jk}(v_i) - c_{min}(v_i)}{\max(c_{max}(v_i) - c_{min}(v_i), \delta G \cdot |\nabla I|_{max})}$$

where δG has a range of $[0, \infty]$. A large δG results in an active contour that is insensitive to weak edges.

Shape Modelling: approaches

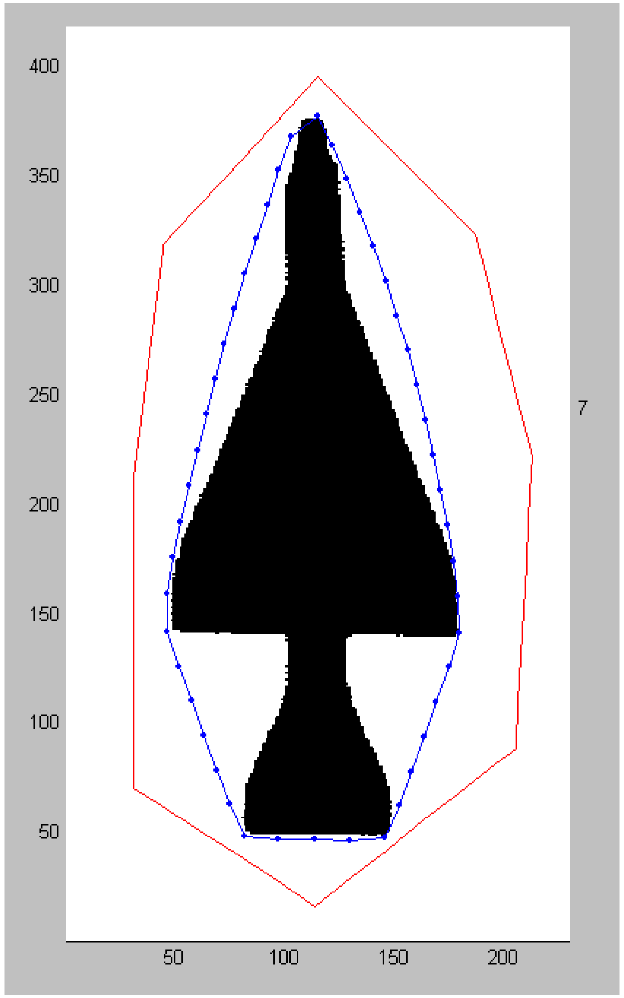
- Active Contour Models: drawbacks
  - Extremely sensitive to parameters
  - Small capture range



  - No external force acts on points which are far away from the boundary
  - Convergence is dependent on initial position

Shape Modelling: approaches

- Active Contour Models: drawbacks
  - Fails to detect concave boundaries. External energy can't pull control points into boundary concavity.



Shape Modelling: approaches

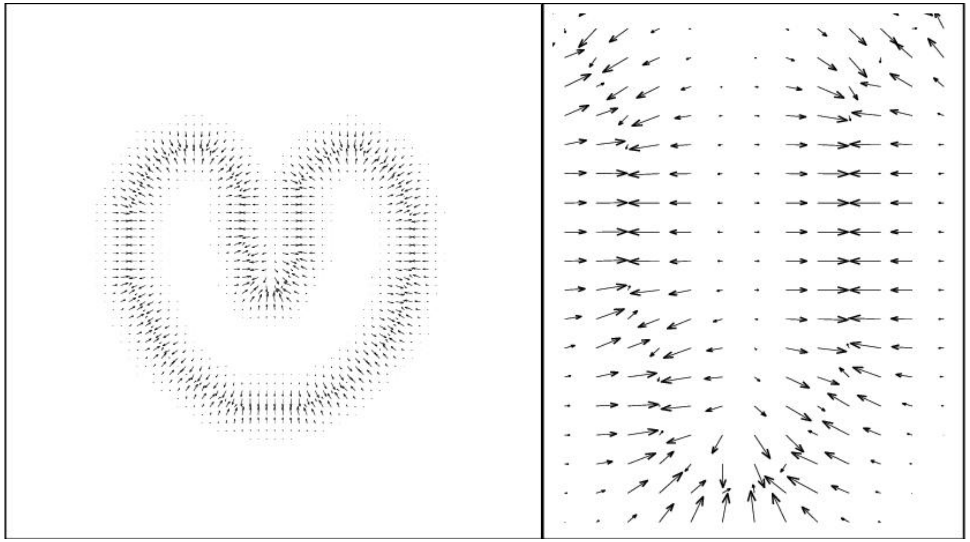
- Active Contour Models: modifications

  - Gradient Vector Flow (GVF): a new external force for snakes.
  - It detects shapes with boundary concavities.
  - Large capture range.

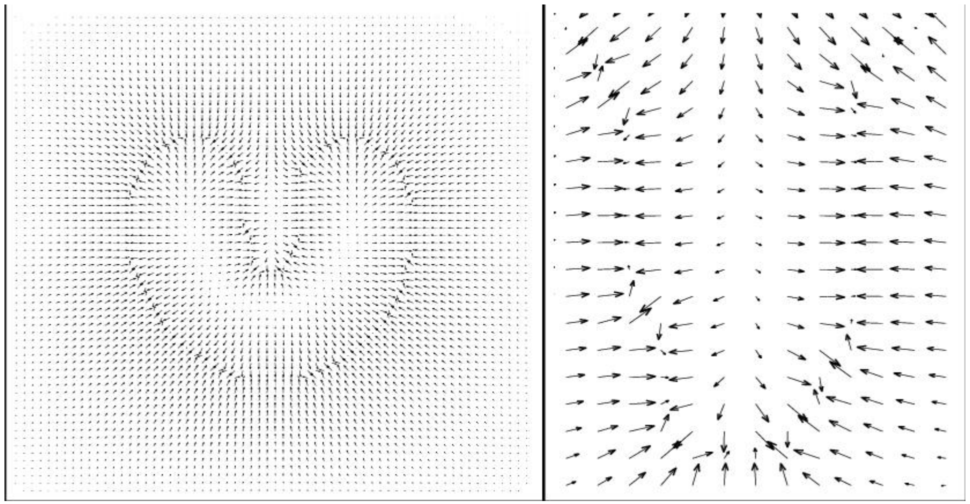
    - Chenyang Xu and Jerry L. Prince , "Snakes, Shape, and Gradient Vector Flow“, IEEE Transactions on Image Processing, 1998.
    - C. Xu and J.L. Prince, “Gradient Vector Flow: A New External Force for Snakes”, Proc. IEEE Conf. on Comp. Vis. Patt. Recog. (CVPR), Los Alamitos: Comp. Soc. Press, pp. 66-71, June 1997.

Shape Modelling: approaches

- Active Contour Models: modifications
  - Traditional force



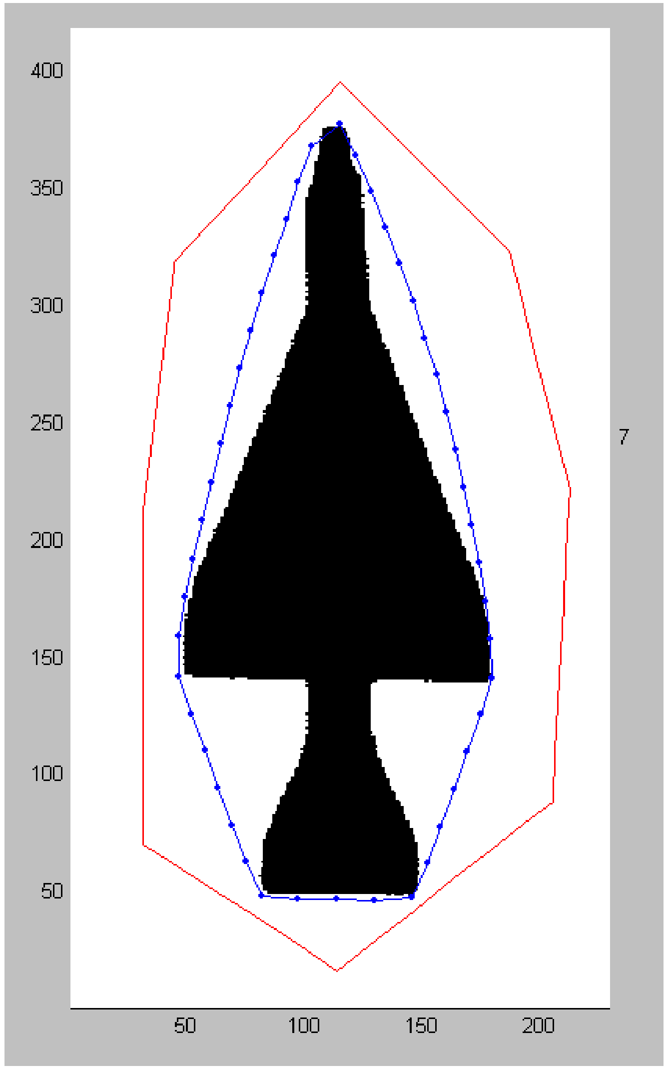
  - GVF force

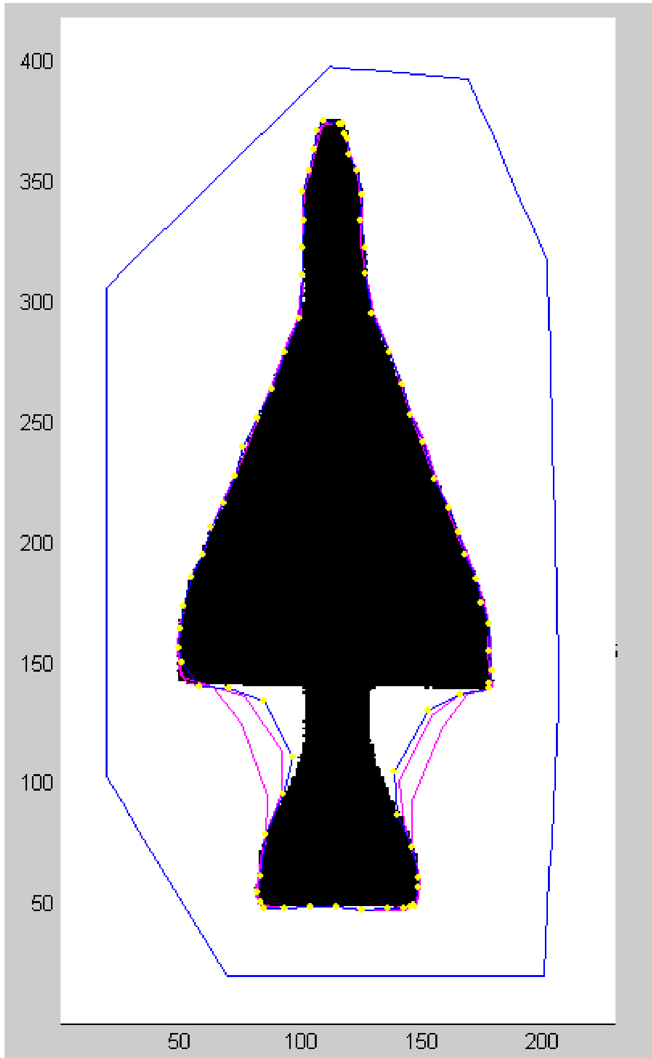


Shape Modelling: approaches

- Active Contour Models: modifications

Traditional snake

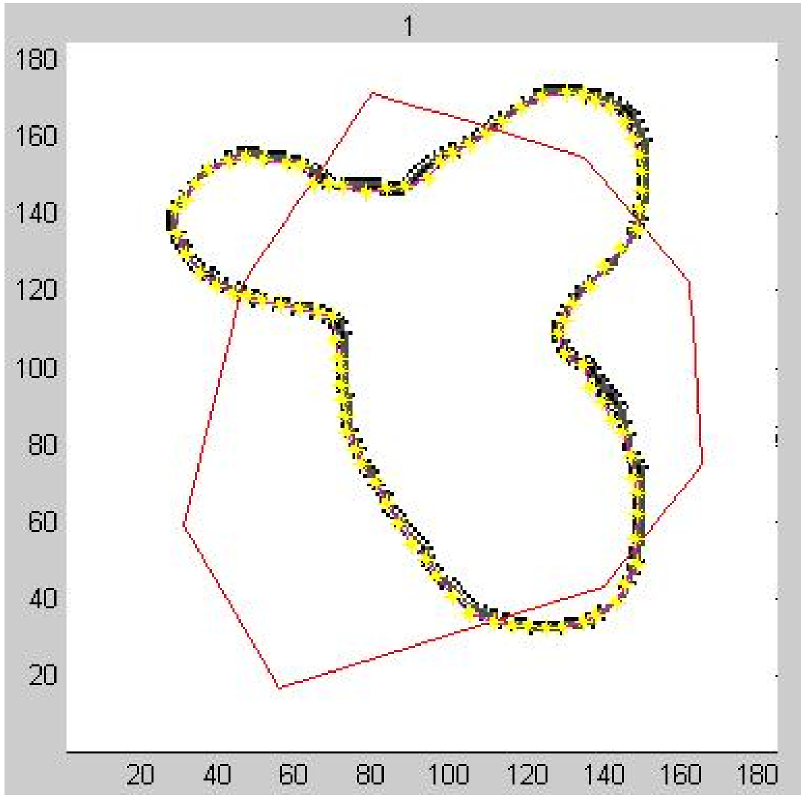


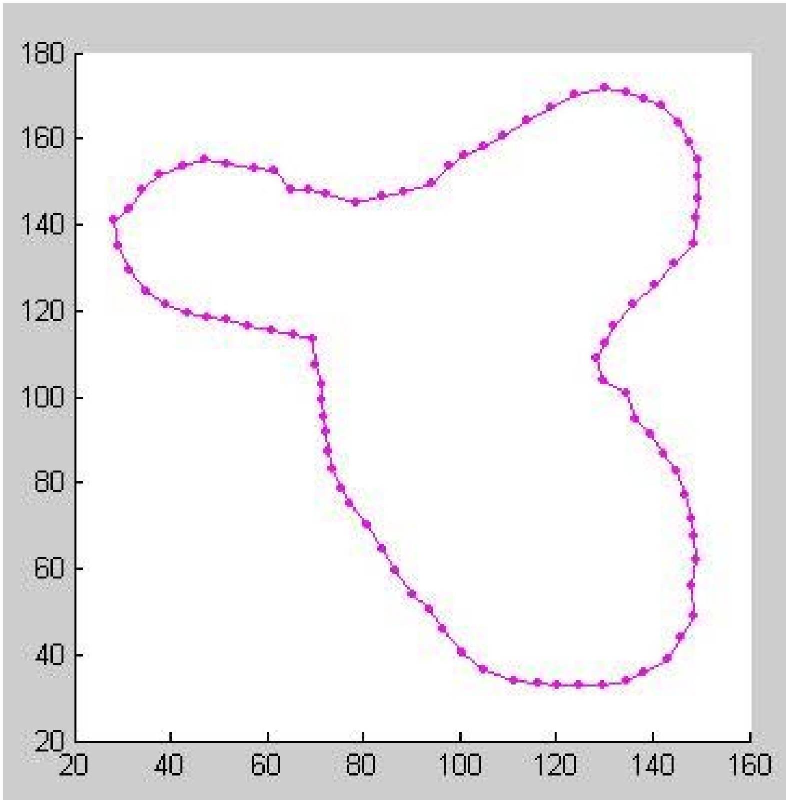


GVF snake

Shape Modelling: approaches

- Active Contour Models: modifications
  - The contour can also be initialized across the boundary of object (something not possible with traditional snakes).





Shape Modelling: approaches

- Active Contour Models: modifications
  - However, using GVF makes the final result even more sensitive to the choice of parameters.
  - It is extremely dependent on the initial location of the contour.
  - It is very slow. Finding GVF field is computationally expensive.

Shape Modelling: approaches

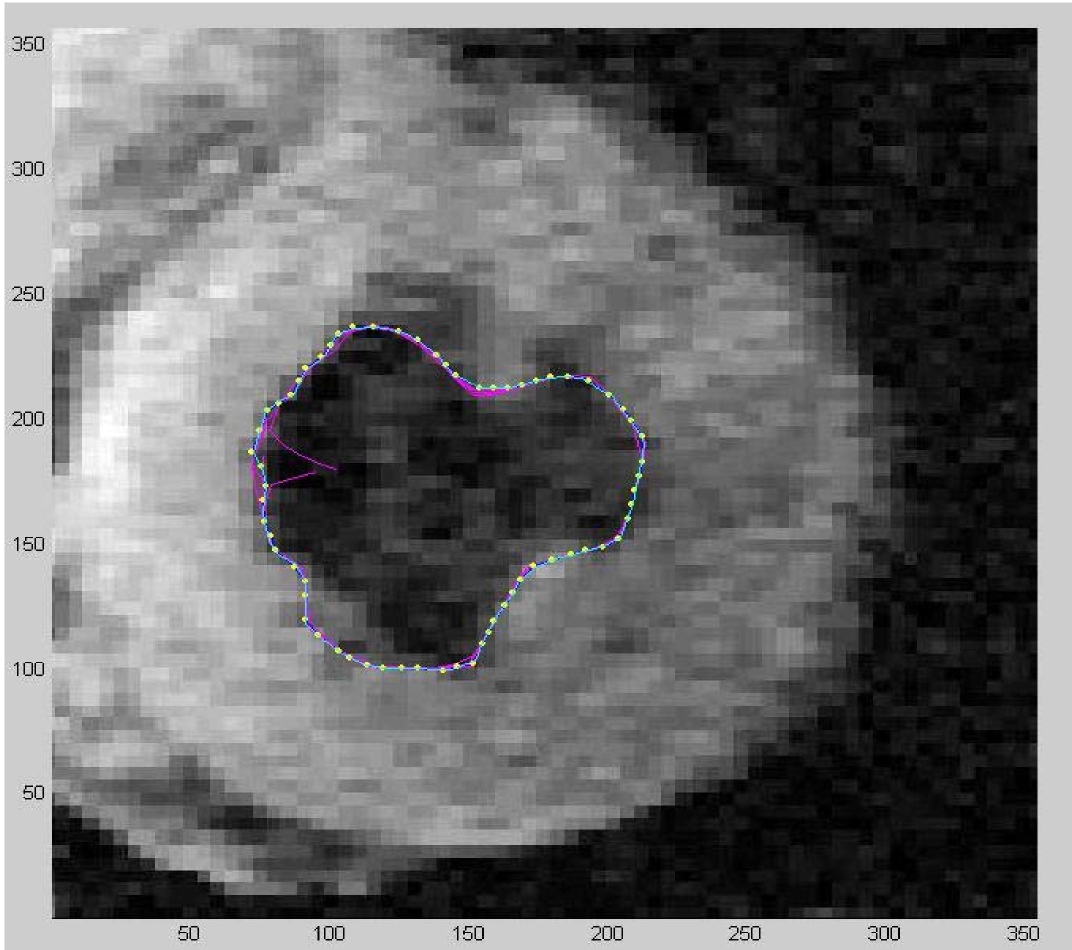
- Active Contour Models: applications
  - Segmentation of contours
  - Medical image segmentation from MRI, X-Ray or CT images
  - Lip tracking for lip synchronization and speech recognition
  - Real-time object tracking
  - Face recognition and expression generation

Shape Modelling: approaches

- Active Contour Models: examples

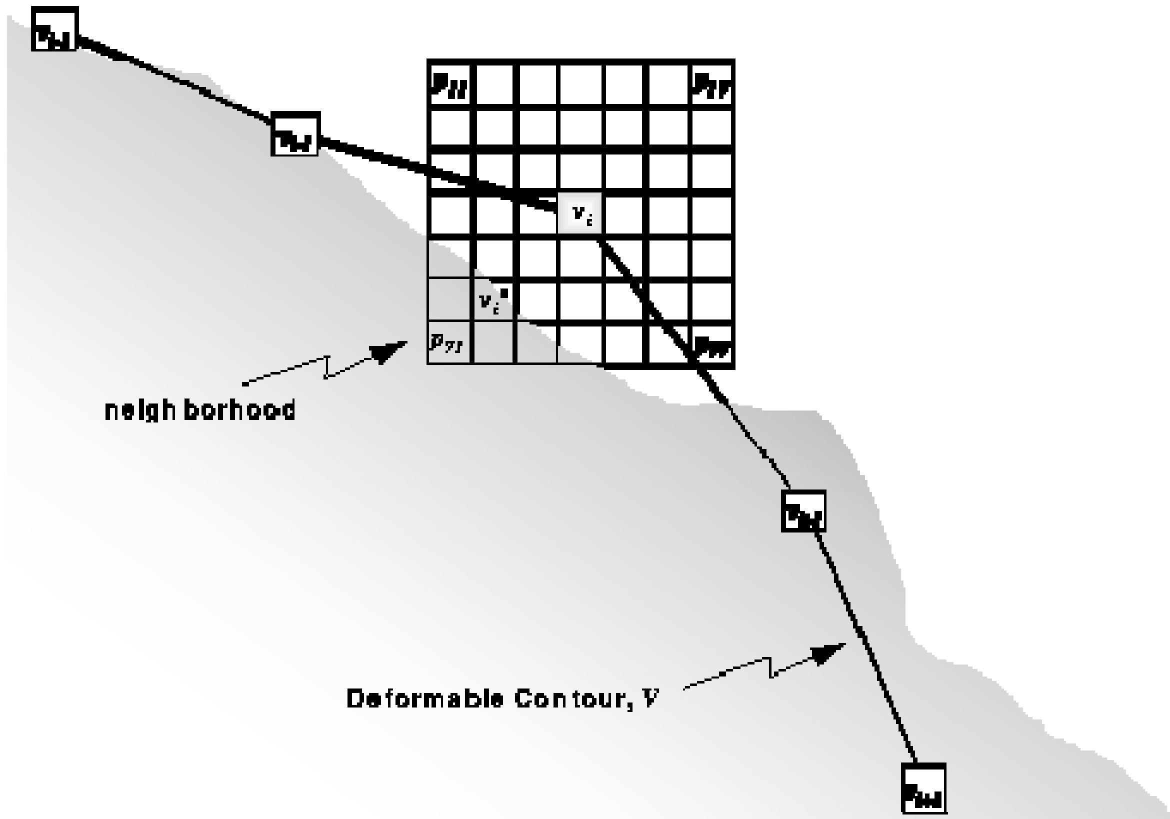
Shape Modelling: approaches

- Active Contour Models: examples
  - Magnetic resonance image of the left ventricle of human heart.
  - Notice the poor quality of the image, even with sampling artifacts.



Shape Modelling: approaches

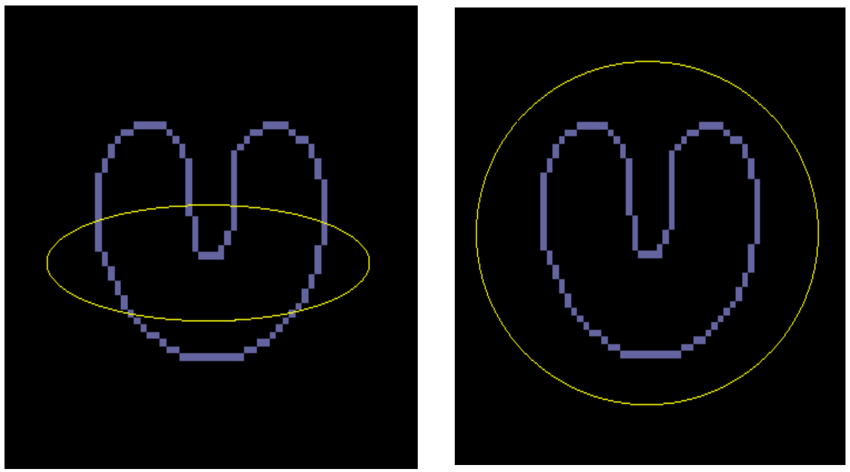
- Active Contour Models: summary
  - An energy minimizing spline guided by external constraint forces and pulled by image forces towards features:
    - Edge detection
    - Subjective contours
    - Motion tracking
    - Stereo matching



Shape Modelling: approaches

- Active Contour Models: summary
  - A snake falls into the closest local energy minimum.
  - Rely on some mechanism to place them near the desired contour.
  - Snakes try to match a deformable model to an image by means of energy minimization.
  - In the end, it completely "shrink-wraps" around the object.

Images taken from the GVF website: http://iacl.ece.jhu.edu/projects/gvf/



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Shape Modelling: assignment

- Material and Methods:
  - Implementation details:
    - The GUI implementation is an adaptation of Witkin and Terzopolous incorporating E_{line}, E_{edge} en E_{term} energy factors.
    - The user can select initialization and snake parameters:
      - Values for the Gaussian smoothing
      - Control parameters for the snake: for the Gaussian smoothing (**σ**), the initial position, the elasticity (**α**), the rigidity (β), the step size (γ), the scaling factor (**κ**), and the weighting factors W for E_{line}, E_{edge} and E_{term} and the number of iterations.
  - Use the provided images (medical and uniform images) to test and understand the algorithm