Implementation and comparrison exponential function algoritms

Andreas

1 Algoritms

The expotential function, defined by the sum. [1]

$$\exp(x) := \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots \ . \tag{1}$$

A numerical approximation of this function is found using the c built-in math.h related libary. Another numerical approximation of this function is presented in (2)

$$\exp(x) := 1 + x(1 + \frac{x}{2}(1 + \frac{x}{3}(1 + \frac{x}{4}(1 + \frac{x}{5} * (1 + \frac{x}{6}(1 + \frac{x}{7}(1 + \frac{x}{8}(1 + \frac{x}{9}(1 + \frac{x}{10}))))))))) \ . \tag{2}$$

By examination it is clear that this is the expanded formula of the function in (1):

$$\exp(x) := 1 + (x + \frac{x^2}{2}(1 + \frac{x}{3}(1 + \frac{x}{4}(1 + \frac{x}{5} * (1 + \frac{x}{6}(1 + \frac{x}{7}(1 + \frac{x}{8}(1 + \frac{x}{9}(1 + \frac{x}{10})))))))) \ . \tag{3}$$

$$\exp(x) := 1 + (x + \frac{x^2}{2} + \frac{x^3}{6}(1 + \frac{x}{4}(1 + \frac{x}{5} * (1 + \frac{x}{6}(1 + \frac{x}{7}(1 + \frac{x}{8}(1 + \frac{x}{9}(1 + \frac{x}{10}))))))) \ . \tag{4}$$

And so on, until the equation becomes

$$\exp(x) := 1 + (x + \frac{x^2}{2} + \frac{x^3}{6}(1 + ... \frac{x^{10}}{3628800})))))))))) \ . \tag{5}$$

For small x, it is reasonable that this function is a good approximation as x^{10} is much smaller than !10. In the algorithm this is explotied with the statement if(x¿1./8)return pow(exp(x/2),2). This ensures that the function is calculated precisely even as x becomes large.

The algorithm is plotted from x=0-5 in figure along with the algorithm from math.h.

2 Figures

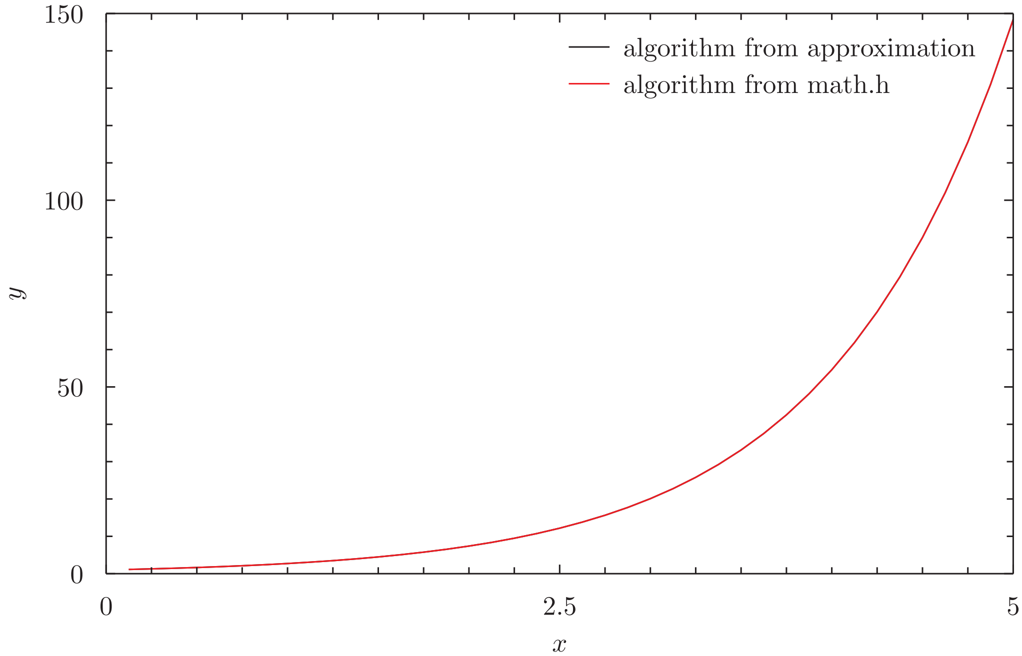

Figure 1: empty

There is clear overlap between the two algorithms, and they must be nealy identical.

References

[1] Eric W. "Exponential Function". mathworld.wolfram.com. Retrieved 2020-08-28