Ex 1.

1. pseudo code

procedure EXACTBM1D (T, N)

$h = \frac{N}{h}$

$\hat{W}_0 \leftarrow 0$

For $i = 0, 1, \cdots, N-1$:

$Z \leftarrow N(0,1)$

$\hat{W}_{i+1} = \hat{W}_i + \sqrt{h} Z$

Return $(\hat{W}_0, \hat{W}_1, \cdots \hat{W}_N)$

2. Prove that $\hat{W}$ is an exact sampling.

Examine three conditions of BM.

① $\hat{W}_0 = 0$.

② $\hat{W}_{i+1} - \hat{W}_i = \sqrt{h} Z_i$, $\hat{W}_{i+2} - \hat{W}_{i+1} = \sqrt{h} Z_{i+1}$, $\cdots$

$\hat{W}_i$ is independently increment.

③ $\hat{W}_{t_{i+1}} - \hat{W}_{t_i} \sim N(0, t_{i+1} - t_i)$, for $i = 1, 2, \cdots$

$\hat{W}_{t_{i+1}} \sim N(0, t_{i+1})$

So $\hat{W}$ is a exact sampling of BM.

Ex3. 1. pseudocode

procedure ARASIAN (T, N)

$\hat{W}_0 = 0$, $h = \frac{T}{h}$, $Sum \leftarrow 0$

for $j = 0, 1, \cdots, n-1$

$Z \leftarrow N(0,1)$

$\hat{W}_{j+1} = \hat{W}_j + \sqrt{h} Z$

$S_j = S_0 \exp((r - \frac{1}{2}\sigma^2) j \frac{N}{T} + \sigma W(j))$

$$\check{S} = \frac{1}{n+1}(\check{S}_0 + \check{S}_1 + \check{S}_2 + \cdots + \check{S}_n)$$

if $(\check{S} - K)^+ > 0$

$\quad sum \Leftarrow sum + \check{S}$

Return $\frac{sum}{N}$.

3.2 Geometric BM

We denote by $GBM(s, \mu, \sigma^2)$ the dynamics

$$dS_t = \mu S_t dt + \sigma S_t dW_t, S_0 = s$$

Non-negativity of the GBM process is good for modeling stock price, namely BSM.

Example 1 *Find* $\log S_t$ *for* $S \sim GBM(s, \mu, \sigma^2)$.

By Ito's, $d\ln S_t = \frac{1}{S_t} dS_t + \frac{1}{2}\left(-\frac{1}{S_t^2}\right)(dS_t)^2$

$$= \frac{1}{S_t}(\mu S_t dt + \sigma S_t dW_t) - \frac{1}{2S_t^2}\sigma^2 S_t^2 dt$$

$$= \mu dt + \sigma dW_t - \frac{1}{2}\sigma^2 dt$$

$$= (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW_t$$

$$\ln S_t - \ln S_0 = \int_0^t (\mu - \frac{1}{2}\sigma^2) dp + \int_0^t \sigma dW_p$$

$$\ln S_t = \ln s + (\mu - \frac{1}{2}\sigma^2)t + \sigma W_t$$