1. pesudo.

Init $b = 0$, $\alpha = 0.01$

Procedure: SA(b, n)

for $i = 1, 2, \cdots n$ do

$x_i := N(b, \sigma^2)$

$b_{i+1} = b_i + \alpha(x_i - b_i)$

Return b_{n+1}

4. Prove $\lim_n E(b_n) = b$

$$b_{n+1} = (1-\alpha) b_n + \alpha x_n$$

$$E[b_{n+1}] = (1-\alpha) E[b_n] + \alpha E(x_n)$$

Since $x \sim N(b, \sigma^2)$, $E(x_n) = b$

$$E[b_{n+1}] = (1-\alpha) E(b_n) + \alpha b$$

Let $x = \lim_n E[b_n]$, then $x = (1-\alpha)x + \alpha b$

So $x = b = \lim_n E[b_n]$

5. Prove or disprove $b_n \to b$ in L^2.

$$\lim_n E[|b_n - b|^2] = \lim_n E[(b_n - E[b_n])^2] = \lim_n Var(b_n)$$

$b_n = \sum_{k=0}^{n-1} (1-\alpha)^{n-k-1} \alpha x_k$. $x \in N(b, \sigma^2)$.

$$Var(b_n) = \sigma^2 \alpha^2 \sum_{k=0}^{n-1} (1-\alpha)^{2(n-k-1)}$$

$$\lim_n Var(b_n) = \sigma^2 \alpha^2 \lim_{b\to\infty} \sum_{k=0}^{n-1} (1-\alpha)^b$$

$\because 1-\alpha < 1$ $\therefore \lim_{b\to\infty} \sum_{k=0}^{n-1} (1-\alpha)^b = 0 = \lim_n Var(b_n) = \lim_n E(|b_n - b|)^2$

Thus $b_n \to b$ in L^2.