#1 Considere (

$$f(x) = \frac{1 - 2^{0,5x+1}}{5}$$

a) Calcule el polinomio de interpolación $p_4(x)$.
intervalo $[0,8]$ $S = \{0, 2, 4, 6, 8\}$

$x_0 = 0 \Rightarrow \frac{-1}{5} \Rightarrow (0, \frac{-1}{5})$

$x_1 = 2 \Rightarrow \frac{-3}{5} \Rightarrow (2, -\frac{3}{5})$

$x_2 = 4 \Rightarrow \frac{-7}{5} \Rightarrow (4, -\frac{7}{5})$

$x_3 = 6 \Rightarrow -3 \Rightarrow (6, -3)$

$x_4 = 8 \Rightarrow \frac{-31}{5} \Rightarrow (8, \frac{-31}{5})$

Lagrange

$$P_4(x) = \sum_{n=0}^{4} y_n \cdot L_n(x)$$

$$= y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x) + y_4 L_4(x)$$

$$L_0(x) = \prod_{j=0,\, j\neq 0}^{4} \frac{x - x_j}{x_n - x_j} = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} \cdot \frac{x - x_3}{x_0 - x_3} \cdot \frac{x - x_4}{x_0 - x_4}$$

$$\left(\frac{x-2}{-2}\right) \cdot \left(\frac{x-4}{-4}\right) \cdot \left(\frac{x-6}{0-6}\right) \left(\frac{x-8}{-8}\right) = \frac{A}{384}$$

$$L_1(x) = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} \cdot \frac{x - x_3}{x_1 - x_3} \cdot \frac{x - x_4}{x_1 - x_4}$$

$$\left(\frac{x}{2}\right) \cdot \left(\frac{x-4}{-2}\right) \cdot \left(\frac{x-6}{-4}\right) \cdot \left(\frac{x-8}{-6}\right) = \frac{B}{-96}$$

$$L_2(x) = \frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1} \cdot \frac{x - x_3}{x_2 - x_3} \cdot \frac{x - x_4}{x_2 - x_4} = \left(\frac{x}{4}\right) \cdot \left(\frac{x-2}{2}\right) \left(\frac{x-6}{-2}\right) \left(\frac{x-8}{-4}\right) = \frac{C}{64}$$

$$L_3(x) = \frac{x - x_0}{x_3 - x_0} \cdot \frac{x - x_1}{x_3 - x_1} \cdot \frac{x - x_2}{x_3 - x_2} \cdot \frac{x - x_4}{x_3 - x_4} = \left(\frac{x}{6}\right) \cdot \left(\frac{x-2}{4}\right) \left(\frac{x-4}{2}\right) \left(\frac{x-8}{-2}\right) = \frac{D}{-96}$$

$$L_4(x) = \frac{x - x_0}{x_4 - x_0} \cdot \frac{x - x_1}{x_4 - x_1} \cdot \frac{x - x_2}{x_4 - x_2} \cdot \frac{x - x_3}{x_4 - x_3} = \left(\frac{x}{8}\right)\left(\frac{x-2}{6}\right)\left(\frac{x-4}{4}\right)\left(\frac{x-6}{2}\right)$$

$$L_4(x) = \frac{D}{384}.$$

a) $$P_4(x) = \frac{-1}{5} \cdot \frac{A}{384} + \frac{-3}{5} \cdot \frac{B}{-96} + \frac{-7}{5} \cdot \frac{C}{64} + -3 \cdot \frac{D}{96} + \frac{-31}{5} \cdot \frac{D}{384}$$

b) Calcule la cota de error de polinomio $p_2(x)$ intervalo $[0, 1]$, $S_2 = \{0, 0.5, 1\}$

$n = 2$

$$|f(x) - P_2(x)| \leq \left| \frac{f^{(3)}(\varepsilon_x)}{3!} (x-0)(x-0{,}5)(x-1) \right|.$$

$$= \frac{1}{3!} \left| f^{(3)}(\varepsilon) \right| \cdot \left| (x-0)(x-0{,}5)(x-1) \right|$$

$$f^{3}(\varepsilon) = \max_{x \in [0,1]} \left| f^{3}(x) \right|$$

$$f^{3}(x) = \left| -0{,}0166 \cdot e^{0{,}3465x} \right|$$

- Puntos críticos

$$f^{4}(x) = 0$$

$$0{,}00577\, e^{0{,}3465x} = 0. \quad \text{nunca se hace cero.}$$

- Evaluo en los extremos

$$\underbrace{f^{3}(1) = -0{,}0234}_{\max} \qquad f(0) = -0{,}0166.$$

$$|f(x) - P_2(x)| \leq \frac{1}{6} \cdot 0{,}0234 \left| x(x-0{,}5)(x-1) \right|$$

No lo evaluo porque eso daría 0.

$f(x) = e^x$

#2

(a) Aproxime $\int_0^1 f(x)\,dx$ regla compuesto de Simpson

7 puntos

$a = 0$ $\quad h = \frac{b-a}{7-1} = \frac{1}{6} = 0{,}1666$

$b = 1$

$x_0 = 0$ $\quad x_3 = \frac{1}{2}$ $\quad x_6 = 1$

$x_1 = \frac{1}{6}$ $\quad x_4 \;\frac{2}{3}$

$x_2 = \frac{1}{3}$ $\quad x_5 = \frac{5}{6}$

$$= \frac{h}{3}\left[f(x_0) + 2\sum_{i=1}^{\frac{n}{2}-1} f(x_{2i}) + 4\sum_{i=1}^{\frac{n}{2}} f(x_{2i-1}) + f(x_n) \right]$$

Indices pares

$\Sigma_p = e^{x_2} + e^{x_4}$

$= 6{,}0616$

Indices impares

$\Sigma_{im} = e^{x_1} + e^{x_3} + e^{x_5}$

$= 5{,}1310$

T

$$= \frac{\frac{1}{6}}{3}\left[1 + 2 \cdot 3{,}3433 + 4 \cdot 5{,}1310 + 2{,}7182 \right]$$

$\int_0^1 e^x dx = 1{,}7182$

(b) Cota de error

$|I - A| = \frac{h^2}{12} \cdot \alpha_{max}$

$\alpha_{max} = \max_{x \in [0,1]} |f''(x)|$

$g(x) = e^x$

$g(x) = 0$ $\quad$ = nunca se hace cero

Evalio

$g(0) = e^0 = 1$

$g(1) = e^1 = 2,7182 \rightarrow Max$

$$h = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$$

$$n = \frac{1}{h}$$

$$\frac{h^2}{12} \cdot \alpha max$$

$$\frac{\left(\frac{1}{n}\right)^2}{12} \cdot e^1 < 10^{-8}$$

$$\left(\frac{1}{n}\right)^2 < \frac{10^{-8} \cdot 12}{e^1}$$

$$\frac{1}{n} < \sqrt{\frac{10^{-8} \cdot 12}{e^1}}$$

$$\frac{1}{2,1010 \times 10^{-4}} < n$$

$$\boxed{n > 4759,44}$$