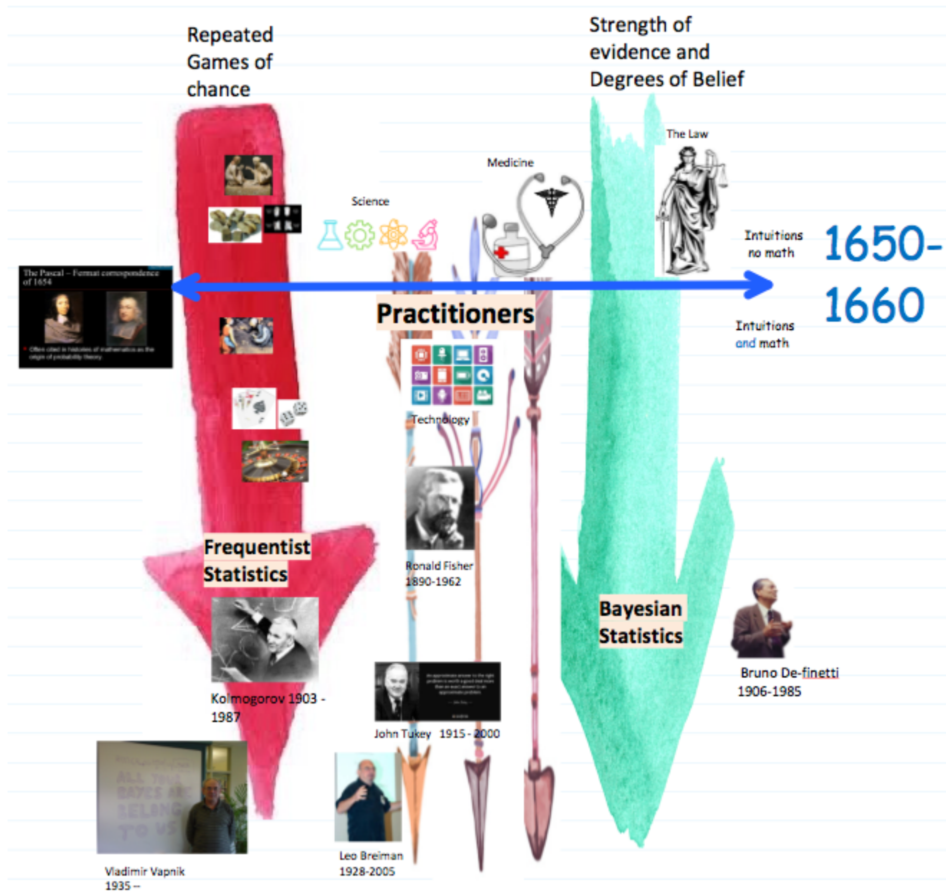
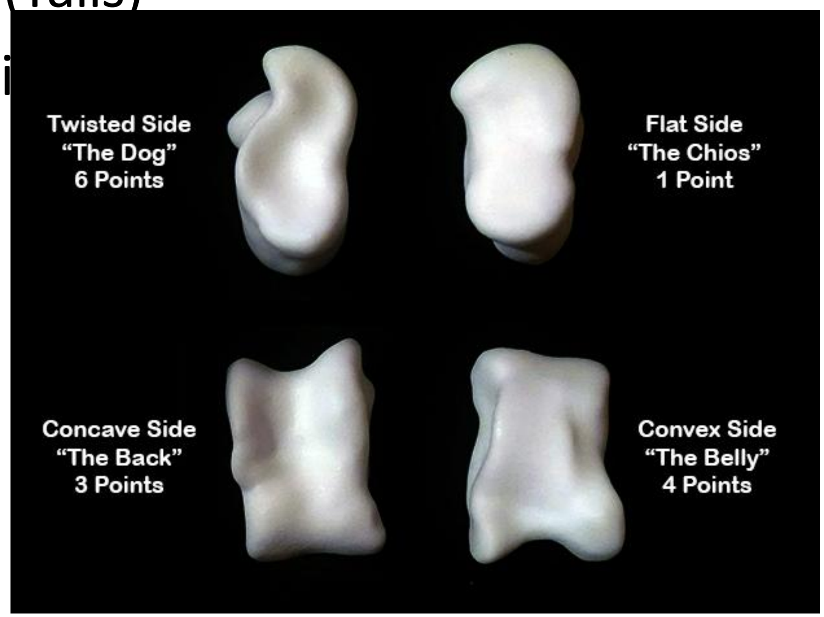
A short history of probability
And Statistics

Games of chance VS. Strength of evidence



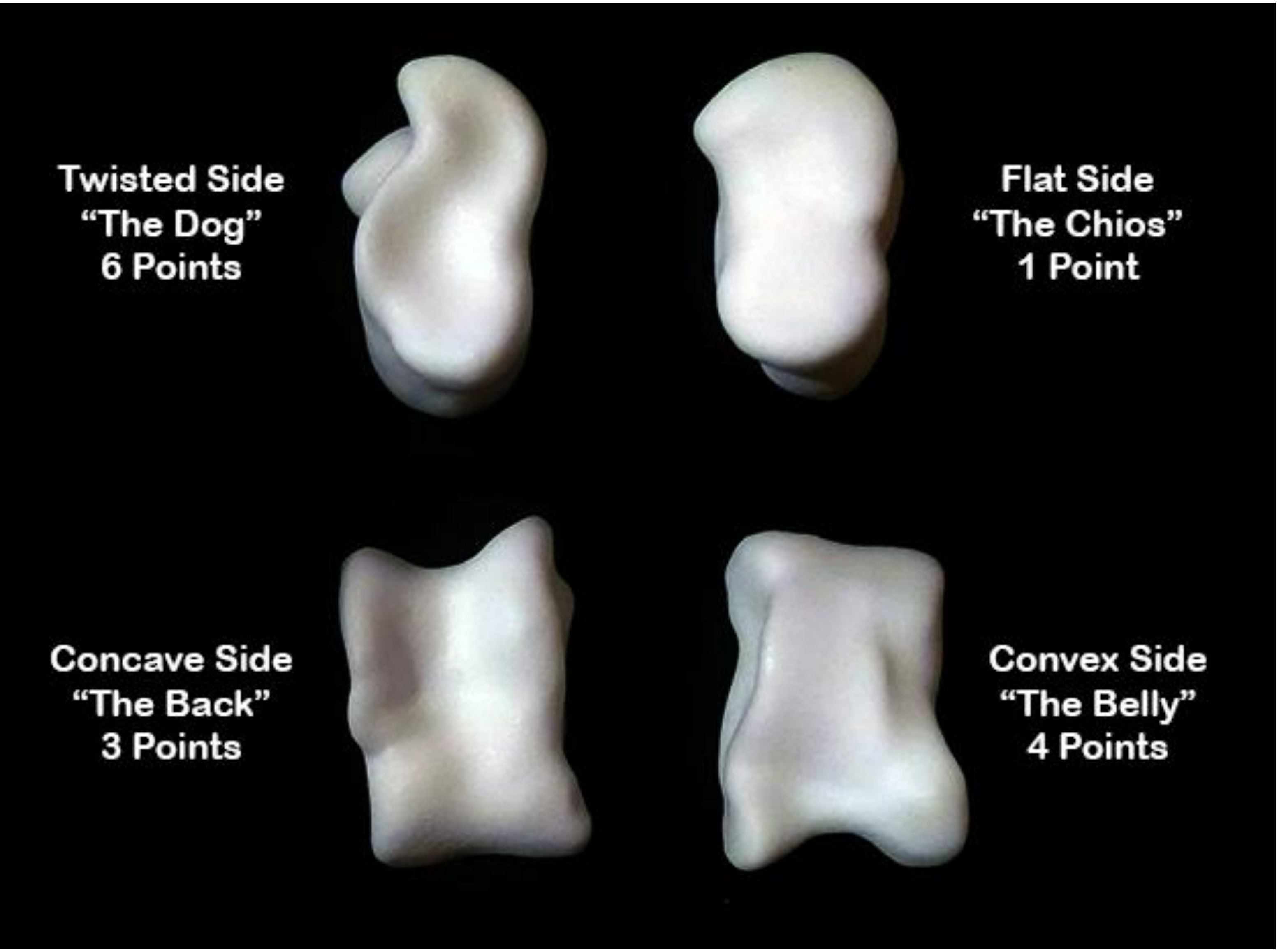
Games of chance

- Sumeria, Assyria, ancient Greece, ancient Rome
- Knuckle Bones (Talis)
- Repeat the basi



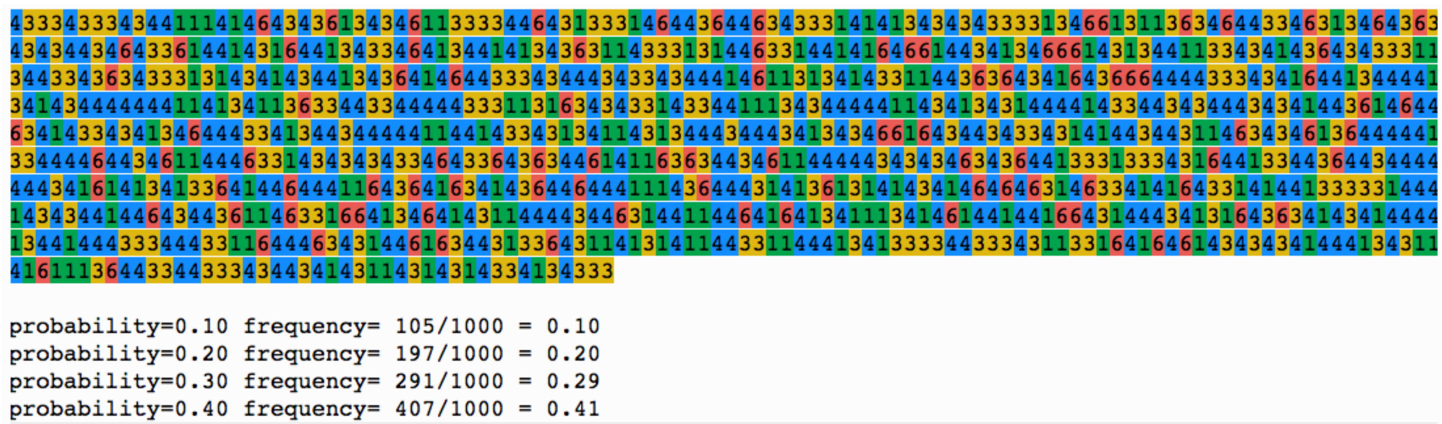
From knuckle bones to dice and cards

- Winning or losing is up to chance, luck, or god.
- **Equal probability Assumption**: all outcomes have the same probability.
- True for dice and roulette
- Not true for knuckle bones.


Twisted Side
“The Dog”
6 Points
Flat Side
“The Chios”
1 Point
Concave Side
“The Back”
3 Points
Convex Side
“The Belly”
4 Points

Long Term Frequencies

- The probability that a knucklebone lands on a narrow face is smaller than it lands on a wide face.
- Each knucklebone is different, the probabilities are different.
- Suppose we have P(**6**)=0.1, P(**1**)=0.2,p(**3**)=0.3, p(**4**)=0.4
- Flip 1000 times:



Long Term Frequencies

- The probability of landing on a narrow face is smaller than that of landing on a wide face.
- Each knucklebone is different, the probabilities are different.
- Suppose we have P(6)=0.1, P(1)=0.2,p(3)=0.3, p(4)=0.4
- Flip 100 times:

```
6311344631644443414341141436431364441134344344631343343334343114646346141433436434636311141413
```

```
6 probability=0.10 frequency= 12/100 = 0.12
1 probability=0.20 frequency= 21/100 = 0.21
3 probability=0.30 frequency= 29/100 = 0.29
4 probability=0.40 frequency= 38/100 = 0.38
```

Long Term Frequencies

- The probability of landing on a narrow face is smaller than that of landing on a wide face.
- Each knucklebone is different, the probabilities are different.
- Suppose we have P(6)=0.1, P(1)=0.2,p(3)=0.3, p(4)=0.4
- Flip 10 times:

```
6414114444

6 probability=0.10 frequency= 1/10 = 0.10
1 probability=0.20 frequency= 3/10 = 0.30
3 probability=0.30 frequency= 0/10 = 0.00
4 probability=0.40 frequency= 6/10 = 0.60
```

Stopping a game in the middle

- Simplified version of problem in famous letter from Pascal to Fermat in 1654
- Suppose a card game of pure chance is played until one side wins.
- Both players put in 1$.
- The winner takes the 2$
- Suppose the game is <u>**stopped**</u> before either side wins.
- How should the 2$ be split?
- What is the probability that player 1 will win given the cards currently held?

The frequentist point of view

- To assign a probabilities to the outcomes of a game/experiment is the same as saying that if we repeat the game many times, the long term frequencies of the outcomes converge to the probabilities.
- Provides a solid foundation on which probability theory is built.
- Makes sense in games and other situations where one can repeat the same random choice many times.
- Not always possible ….

Situations where repetition is hard

1. A meteorologist says that the probability of rain tomorrow is 10%.
   - What does that mean?
   - It will either rain or not rain.
   - Tomorrow happens only once.
2. Suppose a surgeon says that there is a 2% chance of complications with a particular surgery.
   - It might mean that 2% of the patients that underwent the surgery had complications.
   - What does it mean for <u>you</u> ?
   - Maybe most of the complications where with patients older than 90 (and you are 35) …

The colloquial meaning of probability

- The word “probable” was in use before 1650. But it’s meaning was not quantitative
- Even today the words “probable” and “probably” have common use meanings that is qualitative, not quantitative.

Definition of PROBABLY **Merriam Webster Dictionary**

: insofar as seems reasonably true, factual, or to be expected : without much doubt • is *probably* happy • it will *probably* rain

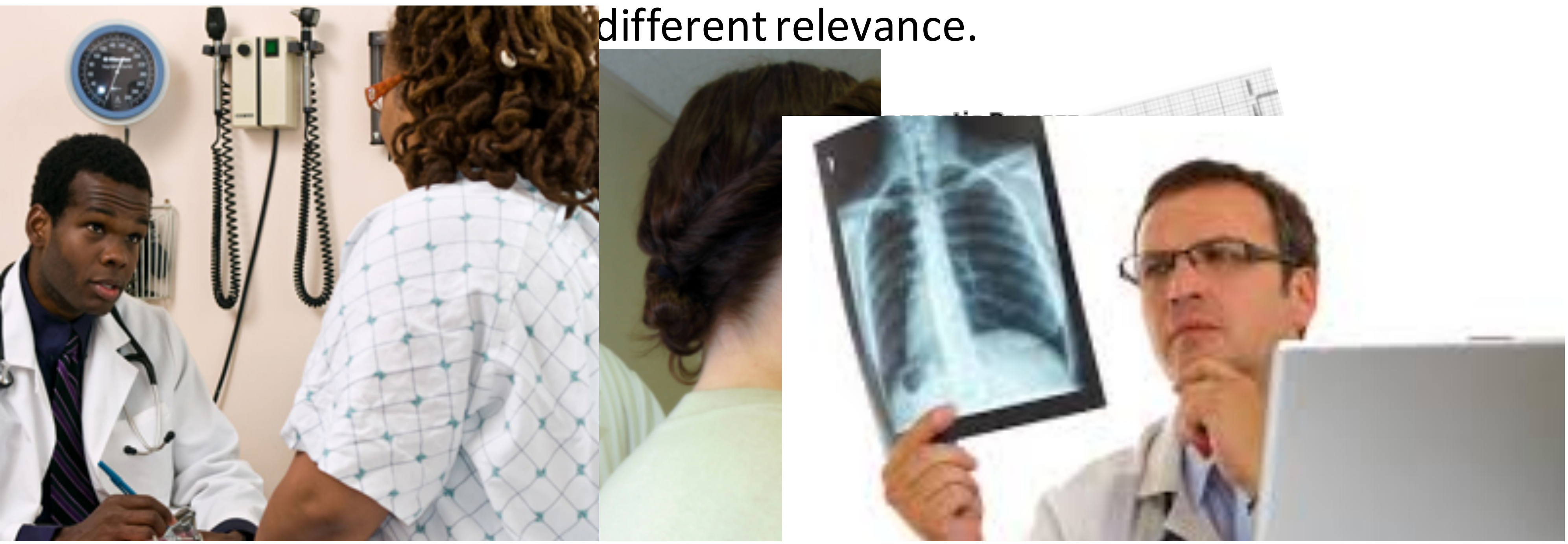
A probable doctor

- Before 1660 it was common to say that someone is a “probable doctor”.
- It meant that the doctor was <u>**approved**</u> by some authority.
- At the time, in Europe, the authority was usually the church.
- Today MDs are approved by a board, after passing the board exams.

Combining evidence for Diagnosis

- Diagnosing a patient requires combining pieces of information.
- Most information is uncertain (measurement error)

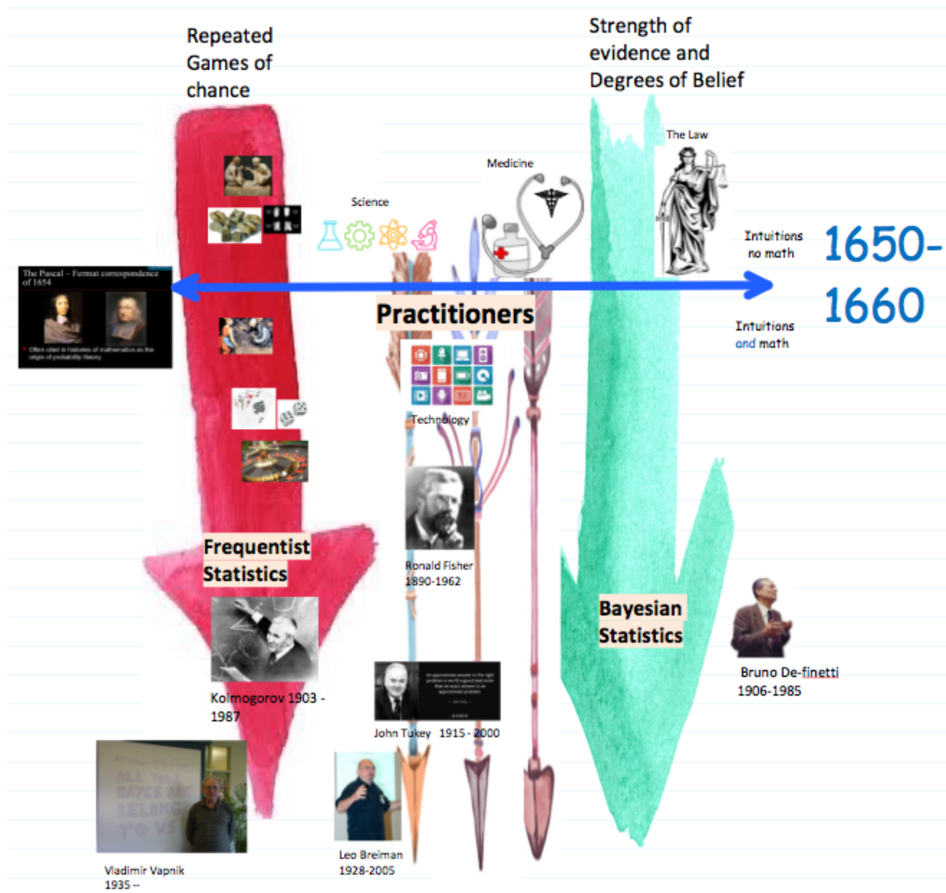
different relevance.



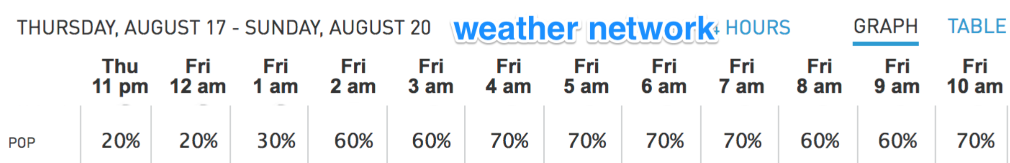
Combining evidence

- Central to many fields: Medicine, economics, investment, Law, Science, Technology …..
- Typically, you don’t repeat an experiment many times.
- The math used is probability theory, but much of the discussion is not mathematical.
- Closely related concepts: Fairness, pricing.
- A popular approach: Bayesian Statistics.

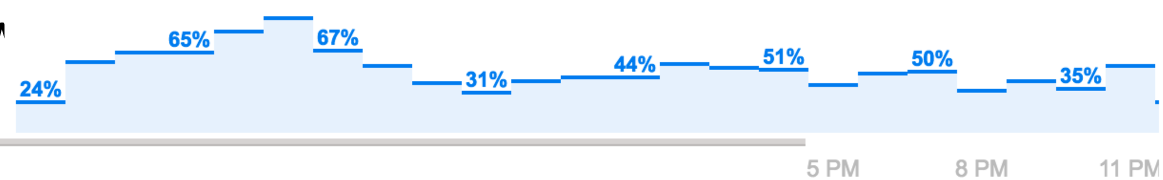
Next video: an exploration of duality

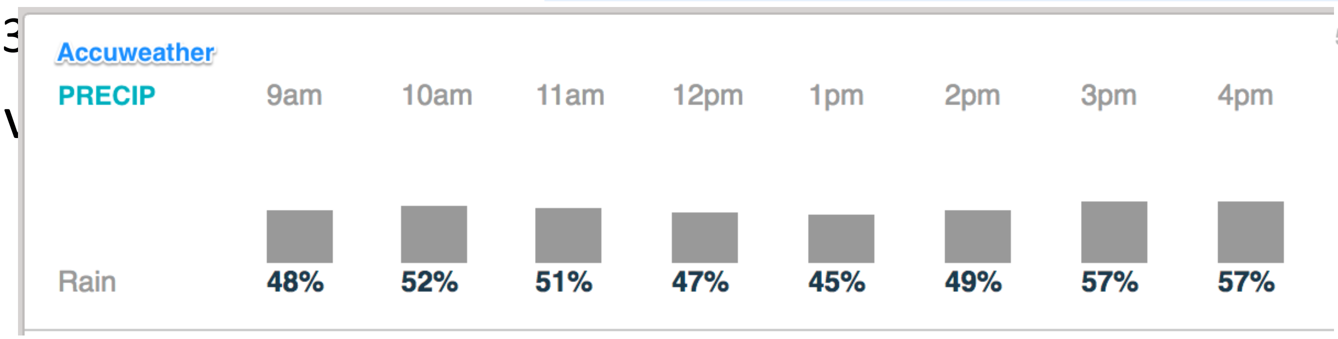


Combining e



- Each morning you need to choose one of several routes to your work.
  1. You consult several w
  2. You choose a route.
  3





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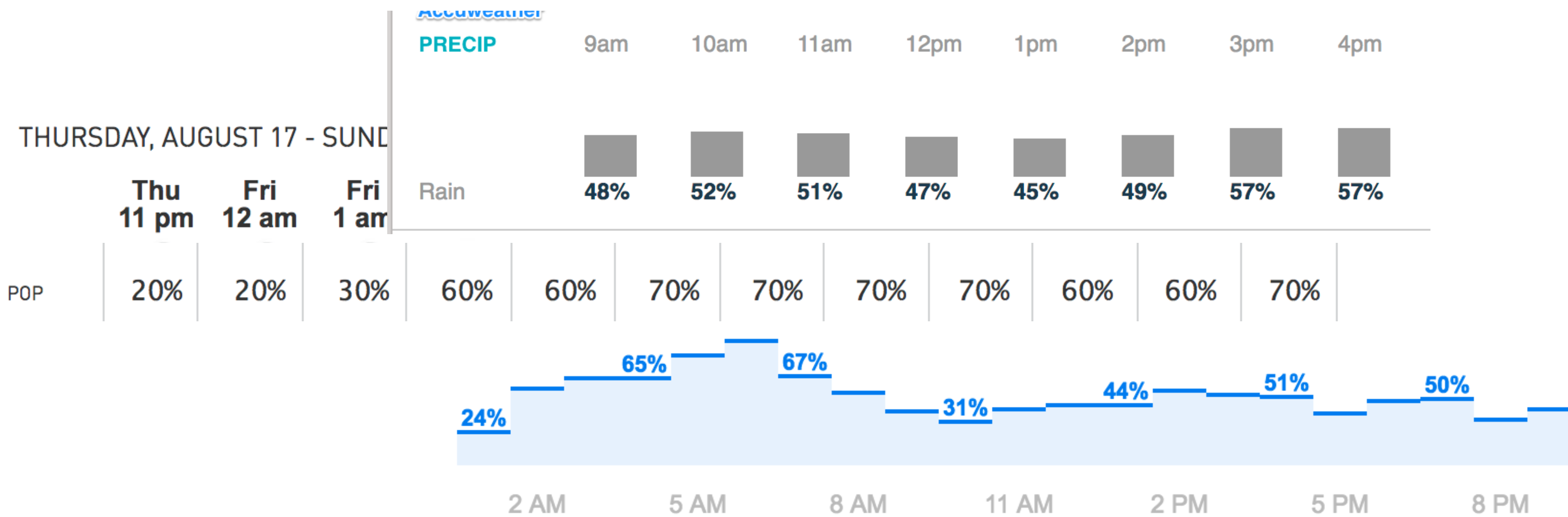


	Tue	Wed	Th u	Fri	Sat	Sun	Mon	Tue	Wed
Weather 1									
Weather 2									
Weather 3									
Rain?									

	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed
forecaster1									
forecaster2									
forecaster3									
Rain?									

Making rational decisions

- You want to know whether it will rain tomorrow.
- You consult different forecasters:
- Chance of rain 20%

Making probabilistic inferences

- You consult several weather prediction channels.
  - Each channel predict rain with a different probability.
  - What is your prediction?
- You consult several surgeons
  - You have access to their past performance
  - Who should you trust:
    - A young doctor who has done 10 surgeries and all were successful
    - Or
    - An oldr doctor who has done 100 surgeries and 95 were successful

The problem of stopping a game in the middle

Introduction to the

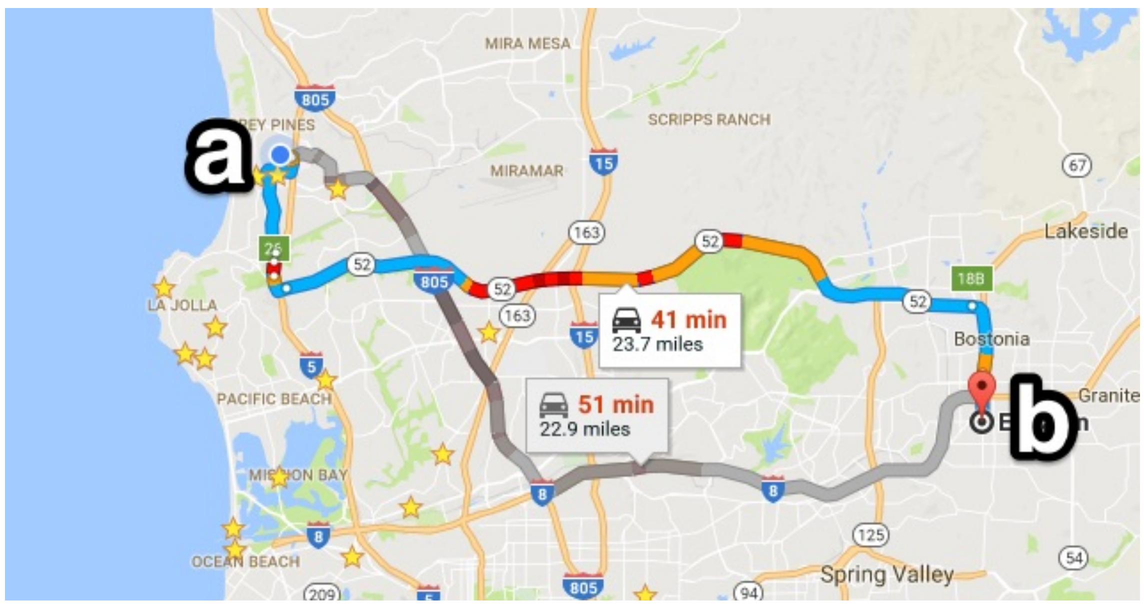
Introduction to Probability and Statistics

Probability

Statistics

Why should you care about prob&stat?

- Navigation software:
  - **Certainty**: Find the **shortest** route from a to b.
  - **Uncertainty**: Find the **fastest** route from from a to b.



Why should you care about prob&stat? II

- Search Engine:
  - **Certainty**: Find **all** web pages that contain the words “Trump”,”Hillary” and “debate”
  - **Uncertainty**: Find the 10 **most relevant** pages for the query “Trump, Hillary debate”

Why should you care about prob&stat? III

- Insurance Company:
  - **Certainty**: If a person with life insurance dies, the insurance company has to pay the family $X
  - **Uncertainty**: What is the minimal life insurance premium such that **the probability that** the life insurance company will be bankrupt in 10 years is smaller than 1% ?

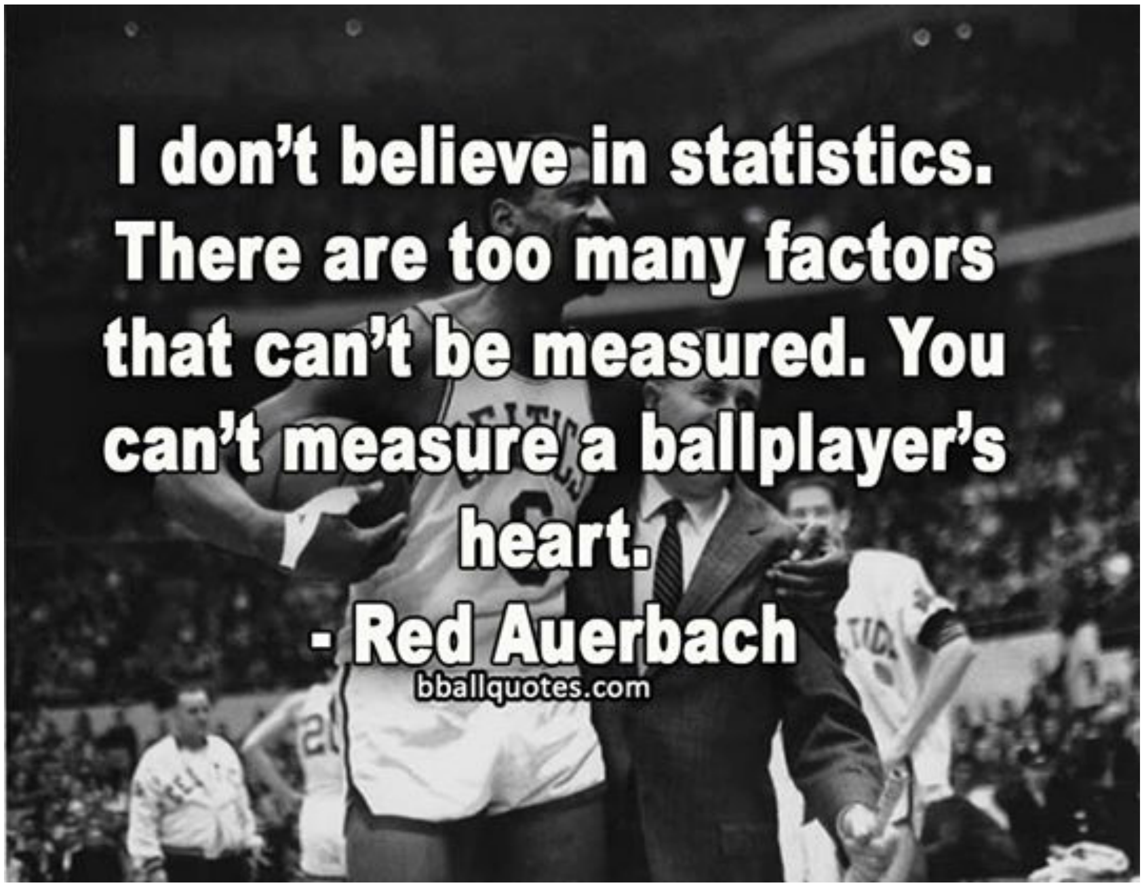
What will you learn in this course?

- Navigation and search engine problems are advanced, in this class you will learn the foundations.
- Solve basic problems of reasoning under uncertainty:
- Examples:
  - If you flip a coin 100 times, what is the probability of getting at most 10 "heads" ?
  - What is the probability of getting a "4 of a kind" hand in poker.

Computer science examples

- If you want to hash 1,000,000 elements and can allow more than 5 indirections for only 10 elements, how big does the table need to be?
- Suppose that the expected time between failures for a router is one year. What is the probability that the router will fail during the first month?

Some don't believe in statistics



Many do


TurboStats
Eastside High
76
1:02
Westside High
84
Large Chart
Mark Grant
15 Bobby B.
Game Statistics for Eastside High

Summary

- Uncertainty is all around us.
- Probability and Statistics provide a rational way to deal with uncertainty.

- Next:
  - What is probability?
  - What is statistics?