I. for Bern.

- Binom: Conj prior $\rightarrow Beta(\alpha, \beta)$

$\alpha' = \alpha + \sum x_i \quad , \quad \beta' = \beta + n - \sum x_i$

$mean = \frac{\alpha+r}{\alpha+\beta+n} \quad , var = \frac{\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}$ (need to substitute α' and β')

If m and s of the Beta are given

$\alpha = \left(\frac{1-m}{s^2} - \frac{1}{m}\right)m^2 \quad , \quad \beta = \alpha\left(\frac{1}{m} - 1\right)$

- Geom: Beta prior $\Rightarrow$ Beta post w/ $\quad \alpha' = \alpha + n \quad , \quad \beta' = \beta + \sum(k_i - 1)$

I. for Pois.

The likelihood is $\propto$ to a $Gamma(y, \alpha, \lambda)$ with $\alpha = \sum y_i + 1 \quad , \quad \lambda = n$

Priors

- Uniform $\Rightarrow$ post is Gamma w/ $\quad \alpha = \sum y_i + 1 \quad , \quad \lambda = n$
- Jeffrey $\Rightarrow$ post is Gamma w/ $\quad \alpha = \sum y_i + \frac{1}{2} \quad , \quad \lambda = n$
- **Gamma** $\rightarrow$ conj prior $\rightarrow \alpha = r + 1 \quad , \quad \lambda = n$

- single obs: $\Rightarrow$ post is Gamma w/ $\quad \alpha' = \alpha + y \quad , \quad \lambda' = \lambda + 1$

- n obs $\{y_i\}$: $\Rightarrow$ post is Gamma w/ $\quad \alpha' = \alpha + \sum y_i \quad , \quad \lambda' = \lambda + n$

$$\begin{equation}

E[n|y] = \frac{\alpha'}{\lambda'} \quad,\quad var=\frac{\alpha'}{\lambda'^2}

\end{equation}$$

If m and s of the Gamma are given

$\alpha = \left(\frac{m}{s}\right)^2 \quad , \quad \lambda = \frac{m}{s^2}$

If likelihood is Exp and we use $Gamma$ prior $\Rightarrow$ post is Gamma w/ $\quad \alpha' = \alpha + n \quad , \quad \beta' = \beta + n\bar{x}$

I. for Norm.

σ^2 is known, while μ is a parameter. Likelihood is $Norm(\mu, \sigma^2)$

Priors

- Uniform $\Rightarrow$ post is Norm w/ $\quad \mu_0 = \frac{1}{N}\sum y_i$ (true empirical mean) $\quad , \quad s^2 = \frac{\sigma^2}{N}$

The inference will be $\mu = \mu_0 \pm \frac{\sigma}{\sqrt{N}}$

- If the data has errors $\rightarrow \mu_0 = \frac{\sum y_i/\sigma_i^2}{\sum 1/\sigma_i^2} \quad , \quad s^2 = \left(\sum 1/\sigma_i^2\right)^{-1}$

- $Norm(m, s^2) \Rightarrow$ post is Norm w/

- single obs: $m' = \frac{\sigma^2 m + s^2 y}{\sigma^2 + s^2} \quad , \quad s'^2 = \frac{\sigma^2 s^2}{\sigma^2 + s^2}$

- n obs $\{y_i\}$: $m' = \frac{1/s^2}{n/\sigma^2 + 1/s^2} m + \frac{n/\sigma^2}{n/\sigma^2 + 1/s^2}\bar{y} \quad , \quad s'^2 = \frac{\sigma^2 s^2}{\sigma^2 + ns^2}$

μ is known, while σ^2 is a parameter.

$\rightarrow$ prior is the Inverse Gamma $\rightarrow E[x] = \frac{\beta}{\alpha-1} \quad , \quad var(x) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$

$\Rightarrow$ post is inverse Gamma w/ $\alpha' = \alpha + \frac{n}{2} \quad , \quad \beta' = \beta + \frac{\sum(x_i-\mu)^2}{2}$

Distributions

- Binom: x successes in n trials. $E[x] = np \quad , \quad var = np(1-p)$
- Geom: x failures to get the 1st success. $E[x] = \frac{1}{p} \quad , \quad var = \frac{1-p}{p^2}$
- Multinomial: generalize Binom $\rightarrow$ outcome A_i with prob p_i. $E[x_i] = np_i \quad , \quad var(x) = np_i(1-p_i)$
- Pois: $E[x] = \lambda \quad , \quad var = \lambda$
- NegBinom: prob of obtaining r-th success in n trials. $E[x] = \frac{r}{p} \quad , \quad var(x) = \frac{r(1-p)}{p^2}$
- Exp: prob of the distance between events in a Pois process. $E[x] = \frac{1}{\lambda} \quad , \quad var = \frac{1}{\lambda^2}. Exp(\lambda) \sim Gamma(1, \lambda)$
- Erlang: prob of waiting time x for the n-th event to occur. $E[x] = \frac{n}{\lambda} \quad , \quad var(x) = \frac{n}{\lambda^2}. Eral(n, \alpha) \sim Gamma(n, \lambda)$
- Gamma: $E[x] = \frac{\alpha}{\beta} \quad , \quad var(x) = \frac{\alpha}{\beta^2}$
- Beta: $E[x] = \frac{\alpha}{\alpha+\beta} \quad , \quad var(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

Combinatory

- Unique pairs (no order): $\frac{n(n-1)}{2}$
- Unique ordering: $n!$
- Permutations (order <u>matter</u>):

- yes rep: n^r (seq of r obj from n)

- no rep: $\frac{n!}{(n-r)!}$ (unique selection of r obj among n)

- Combinations (order doesn't matter):

- yes rep: $\frac{(n+r+1)!}{r!(n-1)!}$ (n° of ways of choosing r obj from n, w/ replacement)

- no rep: $\frac{n!}{r!(n-r)!}$ (n° of ways of choosing r obj from n, w/o regard of order)

Ineq.

Very useful when we don't have enough infos about the distr of random variables, but for which we can calculate $E[x]$ and/or $var(x)$

- Markov: $X \geq 0$ w/ $E[x] = \mu \Rightarrow P(X \geq k) \leq \frac{\mu}{k}$
- Jensen: $E[x^2] \geq (E[x])^2$ since $var(c) \geq 0 \rightarrow X$ w/ $E[x] = \mu$ and $g(x)$ is a convex func $\Rightarrow g(E[x]) \leq E[g(x)]$
- Cheby: $X \geq 0$ w/ $E[x] = \mu$ and $var(x) = \sigma^2 \Rightarrow P(|X-\mu| \geq k) \leq \frac{\sigma^2}{k^2}$
  if $k = r\sigma \quad \rightarrow \quad P(|X-\mu| \geq r\sigma) \leq \frac{1}{r^2}$