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Attributes of GRB Pulses: An Improved Bayesian Blocks Algorithm for Binned Data

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Abstract. A procedure to estimate temporal locations, amplitudes, widths, rise and decay times of pulses occurring within a large sample of GRB light curves, using BATSE 64-ms concatenated data, is based on an improved version of Bayesian Blocks. It determines the maximum likelihood value of the number of blocks by marginalizing over all the other parameters (block locations and sizes). The blocks are then used to obtain objective, automatic estimates of pulse parameters, which can either be used to study pulse-attribute correlations or as the starting solution for iterative, nonlinear fits of parametric models.

THE PULSE PARADIGM

Our goal is to characterize burst light curves [6] in order to illuminate the underlying physical mechanisms. One common characteristic of GRB light curves that rises above their bewildering heterogeneity is the frequent presence of one or more sub-structures. Such *pulses* are distinct features, localized in time (possessing well-defined onset, a monotonic rising portion, peak, and monotonic declining portion), and with no apparent fine-time-scale structure. It seems natural that these structures signal the presence of underlying, physically independent events.

A previous Huntsville paper [10] dealt with the BATSE TTE data. Here we discuss four-channel concatenated data, consisting of photon counts in fixed-length time intervals — bins of length 1.024 s before trigger and 64 ms after. These data can be found at the Compton Gamma Ray Observatory Science Support Center (`http://cossc.gsfc.nasa.gov/cossc/batse/`). We ignore dead time and other possible departures from the ideal, essentially perfect Poisson counting process model.

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BAYESIAN BLOCKS

To address the fundamental difficulties of extracting overlapping pulses from noisy data, we have developed an analysis scheme called Bayesian Blocks [11]. The procedure determines the most likely partitioning of the data into segments, or *blocks*, in which the signal rate is sensibly constant[1]. The transition points between the pieces are conventionally called *changepoints* in the large literature on this topic [1,4,5,7–9,14–17].

The sizes and locations of the blocks are determined by the data in a rigorous statistical way that (1) imposes no limits on time resolution, (2) is semi-parametric (i.e., no parametric model of pulse shape), (3) effectively de-noises the data, and (4) yields immediately useful quantities, such as pulse locations, widths, and heights.

The fundamental step is determining whether a given interval of data should be subdivided into two subintervals or kept as one. The details are given in [11], where the problem of dealing with a whole data series was solved by applying this fundamental step first to the whole data interval, and then recursively to each subinterval until the Bayes criterion favors keeping all the subintervals whole. While this somewhat *ad hoc* procedure gives useful results, it is more correct to treat the multiple changepoint problem directly. Below is an outline of this approach; details will be described elsewhere [12].

THE ALGORITHM

Here or in the iterative "divide and conquer" method of [11], we use a Poisson Model with a constant photon rate for various intervals of time, or blocks. Denote such a model $M(\theta)$, with an array of parameters θ representing the Poisson rate parameters and the change point locations. The fundamental measure of the probability of a model, given the data is [11]

$$p(D|M) = \int p(D|\ \theta, M) p(\theta|M)\ \mathrm{d}\theta\ . \tag{1}$$

The integration in this formula implements the underlying idea of the straightforward Bayesian prescription for estimating the number of model parameters (e.g., [13], especially §4.2). This *marginalizing of nuisance parameters* provides an Occam razor that remarkably and automatically takes model complexity into account — allowing direct comparison of models of very different form, and with different numbers of parameters.

This *Bayes factor* is useful when comparing models of different form. That is, the relative probability of models M_1 and M_2, using Bayes theorem, is

$$\frac{p(M_1|D)}{p(M_2|D)} = \frac{p(D|M_1)}{p(D|M_2)}\ \frac{p(M_1)}{p(M_2)}\ , \tag{2}$$

[1] This means, not that the signal is actually believed constant over the block, but that the data at hand are statistically consistent with a signal rate that is constant.

where $p(M_1)$ and $p(M_2)$ are the prior probabilities of the two models. The idea is to marginalize all of the model parameters except N_p, the number of parameters, and then find the maximum likelihood as a function of N_p.

Two major simplifications result in a powerful, general purpose algorithm. First, the likelihood for a series of blocks is just the product of those for the blocks. Therefore *the only likelihood computation needed is that for a single block.* Second, the Poisson rate parameters can be marginalized exactly — not just for time intervals, but for an arbitrary volume B in a space of any dimension.

Assuming a constant-rate Poisson process throughout B (thus distributing points independently and uniformly), the counts in B — denoted X — follow the probability distribution

$$P(X) = \frac{(\lambda V)^X e^{-\lambda V}}{X!} \tag{3}$$

where λ is the event rate per unit volume, V is the volume of the set B. To specify any prior information about the rate parameter λ, we adopt the *conjugate prior* [2]

$$p(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} \tag{4}$$

The parameters α and β can be adjusted to represent the constraints placed on λ by the instrument. This prior yields the Bayes factor

$$p(D|M) = \frac{\beta^\alpha V^X}{\Gamma(\alpha) X!} \int_0^\infty \lambda^{X+\alpha-1} e^{-\lambda(V+\beta)} d\lambda \ , \tag{5}$$

or

$$p(D|M) = \frac{\beta^\alpha V^X}{\Gamma(\alpha)} \frac{\Gamma(X+\alpha)}{(V+\beta)^{X+\alpha}} \ , \tag{6}$$

where as usual the model-independent factor $X!$ is discarded.

The marginalization of the location parameters (i.e., the discrete bin indices) requires a sum over a high-dimensional space. A simple implementation of the standard tool in such cases, Markov Chain Monte Carlo [3], works surprisingly well — probably because these data, integer counts with low dynamic range and noise exactly describable by the Poisson distribution, are relatively well behaved.

The resulting posterior probability, as a function of N_p (here equal to the number of changepoints), peaks sharply at a good estimate of the number of changepoints — so sharply that any reasonable prior distribution for N_p suffices. Determining the values of the other parameters, namely block locations and heights, requires little more than keeping track of the maximum likelihood computed during the Monte Carlo exploration of parameter space. *Integration and optimization are the same problem, computationally.* More sophisticated procedures, such as averaging the model parameters with the likelihood acting as a weighting function, involve only more manipulations of the same computed quantities.

We will report elsewhere [12] on an extensive program of GRB modeling.

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