I. for Bern.

• Binom: Conj prior o Beta(lpha,eta)

$$lpha'=lpha+\sum x_i$$
 , $eta'=eta+n-\sum x_i$ $mean=rac{lpha+r}{lpha+eta+n}$, $var=rac{lphaeta(lpha+eta+n)}{(lpha+eta)^2(lpha+eta+1)}$ (need to substitute $lpha'$ and eta')

If m and s of the Beta are given

$$lpha = ig(rac{1-m}{s^2} - rac{1}{m}ig)m^2$$
 , $eta = lphaig(rac{1}{m} - 1ig)$

• Geom: Beta prior \Rightarrow Beta post w/ lpha' = lpha + n , $eta' = eta + \sum (k_i - 1)$

I. for Pois.

The likelihood is \propto to a $Gamma(y, lpha, \lambda)$ with $lpha = \sum y_i + 1$, $\lambda = n$

Priors

- Uniform \Rightarrow post is Gamma w/ $lpha = \sum y_i + 1 \quad , \quad \lambda = n$
- Jeffrey \Rightarrow post is Gamma w/ $lpha = \sum y_i + rac{1}{2} \quad , \quad \lambda = n$
- Gamma o conj prior o lpha=r+1 , $\lambda=n$
 - single obs: \Rightarrow post is Gamma w/ $lpha' = lpha + y \quad , \quad \lambda' = \lambda + 1$
 - n obs $\{y_i\}$: \Rightarrow post is Gamma w/ $lpha'=lpha+\sum y_i \quad , \quad \lambda'=\lambda+n$

$$E[n|y] = rac{lpha'}{\lambda'} \quad , \quad var = rac{lpha'}{\lambda'^2}$$

If m and s of the Gamma are given

$$lpha = \left(rac{m}{s}
ight)^2 \quad , \quad \lambda = rac{m}{s^2}$$

If likelihood is Exp and we use Gamma prior \Rightarrow post is Gamma w/ $\alpha' = \alpha + n$, $\beta' = \beta + n\bar{x}$

I. for Norm.

 σ^2 is known, while μ is a parameter. Likelihood is $Norm(\mu, \sigma^2)$

Priors

- Uniform \Rightarrow post is Norm w/ $\mu_0=rac{1}{N}\sum y_i$ (true empirical mean) , $s^2=rac{\sigma^2}{N}$ The inference will be $\mu=\mu_0\pmrac{\sigma}{\sqrt{N}}$
 - If the data has errors $o \mu_0=rac{\sum y_i/\sigma_i^2}{\sum 1/\sigma_i^2}$, $s^2=\left(\sum 1/\sigma_i^2
 ight)^{-1}$
- $Norm(m, s^2) \Rightarrow \mathsf{post}$ is Norm w/

 - single obs: $m'=rac{\sigma^2 m + s^2 y}{\sigma^2 + s^2}$, $s'^2=rac{\sigma^2 s^2}{\sigma^2 + s^2}$ • n obs $\{y_i\}$: $m'=rac{1/s^2}{n/\sigma^2 + 1/s^2}m + rac{n/\sigma^2}{n/\sigma^2 + 1/s^2}ar{y}$, $s'^2=rac{\sigma^2 s^2}{\sigma^2 + ns^2}$

 μ is known, while σ^2 is a parameter.

- o prior is the Inverse Gamma o $E[x]=rac{eta}{lpha-1} \quad , \quad var(x)=rac{eta^2}{(lpha-1)^2(lpha-2)}$
- \Rightarrow post is inverse Gamma w/ $lpha'=lpha+rac{n}{2}$, $eta'=eta+rac{\sum(x_i-\mu)^2}{2}$

Distributions

- Binom: x successes in n trials. E[x] = np , var = np(1-p)
- Geom: x failures to get the 1st success. $E[x] = rac{1}{p}$, $var = rac{1-p}{p^2}$
- Multinomial: generalize Binom o outcome A_i with prob $p_i.$ $E[x_i] = np_i$, $var(x) = np_i(1-p_i)$
- Pois: $E[x] = \lambda$, $var = \lambda$
- NegBinom: prob of obtaining r-th success in n trials. $E[x]=rac{r}{p}$, $var(x)=rac{r(1-p)}{p^2}$

- Exp: prob of the distance between events in a Pois process. $E[x]=rac{1}{\lambda}$, $var=rac{1}{\lambda^2}.$ $Exp(\lambda)\sim Gamma(1,\lambda)$
- Erlang: prob of waiting time x for the n-th event to occur. $E[x]=rac{n}{\lambda}$, $var(x)=rac{n}{\lambda^2}$. $Eral(n,lpha)\sim Gamma(n,\lambda)$
- ullet Gamma: $E[x]=rac{lpha}{eta}$, $var(x)=rac{lpha}{eta^2}$
- Beta: $E[x]=rac{lpha}{lpha+eta}$, $var(x)=rac{lphaeta}{(lpha+eta)^2(lpha+eta+1)}.$

Combinatory

- Unique pairs (no order): $\frac{n(n-1)}{2}$
- Unique ordering: n!
- Permutations (order <u>matter</u>):
 - yes rep: n^r (seq of r obj from n)
 - no rep: $\frac{n!}{(n-r)!}$ (unique selection of r obj among n)
- Combinations (order doesn't matter):
 - yes rep: $\frac{(n+r+1)!}{r!(n-1)!}$ (n° of ways of choosing r obj from n, w/ replacement)
 - no rep: $\frac{n!}{r!(n-r)!}$ (n° of ways of choosing r obj from n, w/o regard of order)