I. for Bern.

- Binom: Conj prior $\rightarrow Beta(\alpha, \beta)$
  $\alpha' = \alpha + \sum x_i$, $\beta' = \beta + n - \sum x_i$
  $mean = \frac{\alpha+r}{\alpha+\beta+n}$, $var = \frac{\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}$ (need to substitute α' and β')

  If m and s of the Beta are given
  $\alpha = \left(\frac{1-m}{s^2} - \frac{1}{m}\right)m^2$, $\beta = \alpha\left(\frac{1}{m} - 1\right)$
- Geom: Beta prior $\Rightarrow$ Beta post w/ $\alpha' = \alpha + n \quad , \quad \beta' = \beta + \sum(k_i - 1)$

I. for Pois.

The likelihood is $\propto$ to a $Gamma(y, \alpha, \lambda)$ with $\alpha = \sum y_i + 1$, $\lambda = n$

Priors

- Uniform $\Rightarrow$ post is Gamma w/ $\alpha = \sum y_i + 1 \quad , \quad \lambda = n$
- Jeffrey $\Rightarrow$ post is Gamma w/ $\alpha = \sum y_i + \frac{1}{2} \quad , \quad \lambda = n$
- **Gamma** $\rightarrow$ conj prior $\rightarrow \alpha = r + 1 \quad , \quad \lambda = n$
  - single obs: $\Rightarrow$ post is Gamma w/ $\alpha' = \alpha + y \quad , \quad \lambda' = \lambda + 1$
  - n obs $\{y_i\}$: $\Rightarrow$ post is Gamma w/ $\alpha' = \alpha + \sum y_i \quad , \quad \lambda' = \lambda + n$

$$E[n|y] = \frac{\alpha'}{\lambda'} \quad , \quad var = \frac{\alpha'}{\lambda'^2}$$

  If m and s of the Gamma are given
  $\alpha = \left(\frac{m}{s}\right)^2 \quad , \quad \lambda = \frac{m}{s^2}$
  If likelihood is Exp and we use $Gamma$ prior $\Rightarrow$ post is Gamma w/ $\alpha' = \alpha + n \quad , \quad \beta' = \beta + n\bar{x}$

I. for Norm.

σ^2 is known, while μ is a parameter. Likelihood is $Norm(\mu, \sigma^2)$

Priors

- Uniform $\Rightarrow$ post is Norm w/ $\mu_0 = \frac{1}{N}\sum y_i$ (true empirical mean) , $s^2 = \frac{\sigma^2}{N}$
  The inference will be $\mu = \mu_0 \pm \frac{\sigma}{\sqrt{N}}$
  - If the data has errors $\rightarrow \mu_0 = \frac{\sum y_i/\sigma_i^2}{\sum 1/\sigma_i^2} \quad , \quad s^2 = \left(\sum 1/\sigma_i^2\right)^{-1}$
- $Norm(m, s^2) \Rightarrow$ post is Norm w/
  - single obs: $m' = \frac{\sigma^2 m + s^2 y}{\sigma^2 + s^2} \quad , \quad s'^2 = \frac{\sigma^2 s^2}{\sigma^2 + s^2}$
  - n obs $\{y_i\}$: $m' = \frac{1/s^2}{n/\sigma^2 + 1/s^2}m + \frac{n/\sigma^2}{n/\sigma^2 + 1/s^2}\bar{y} \quad , \quad s'^2 = \frac{\sigma^2 s^2}{\sigma^2 + ns^2}$

μ is known, while σ^2 is a parameter.
$\rightarrow$ prior is the Inverse Gamma $\rightarrow E[x] = \frac{\beta}{\alpha-1} \quad , \quad var(x) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$
$\Rightarrow$ post is inverse Gamma w/ $\alpha' = \alpha + \frac{n}{2} \quad , \quad \beta' = \beta + \frac{\sum(x_i - \mu)^2}{2}$

Distributions

- Binom: x successes in n trials. $E[x] = np \quad , \quad var = np(1-p)$
- Geom: x failures to get the 1st success. $E[x] = \frac{1}{p} \quad , \quad var = \frac{1-p}{p^2}$
- Multinomial: generalize Binom $\rightarrow$ outcome A_i with prob p_i. $E[x_i] = np_i \quad , \quad var(x) = np_i(1-p_i)$
- Pois: $E[x] = \lambda \quad , \quad var = \lambda$
- NegBinom: prob of obtaining r-th success in n trials. $E[x] = \frac{r}{p} \quad , \quad var(x) = \frac{r(1-p)}{p^2}$

- Exp: prob of the distance between events in a Pois process. $E[x] = \frac{1}{\lambda} \quad , \quad var = \frac{1}{\lambda^2}$. $Exp(\lambda) \sim Gamma(1, \lambda)$
- Erlang: prob of waiting time x for the n-th event to occur. $E[x] = \frac{n}{\lambda} \quad , \quad var(x) = \frac{n}{\lambda^2}$. $Eral(n, \alpha) \sim Gamma(n, \lambda)$
- Gamma: $E[x] = \frac{\alpha}{\beta} \quad , \quad var(x) = \frac{\alpha}{\beta^2}$
- Beta: $E[x] = \frac{\alpha}{\alpha+\beta} \quad , \quad var(x) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

Combinatory

- Unique pairs (no order): $\frac{n(n-1)}{2}$
- Unique ordering: $n!$
- Permutations (order <u>matter</u>):
  - yes rep: n^r (seq of r obj from n)
  - no rep: $\frac{n!}{(n-r)!}$ (unique selection of r obj among n)
- Combinations (order doesn't matter):
  - yes rep: $\frac{(n+r+1)!}{r!(n-1)!}$ (n° of ways of choosing r obj from n, w/ replacement)
  - no rep: $\frac{n!}{r!(n-r)!}$ (n° of ways of choosing r obj from n, w/o regard of order)