UNIVERSITY OF PADOVA
Department of Physics and Astronomy ‘Galileo Galilei’
MSc in Physics of Data

Binary Stellar Evolution - Dormant BH in binary system

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Introduction

- Study binaries with a compact object → **GAIA DR3** → large amount of such candidates
- In our project we are interested in **BH-MS binaries**

Introduction
SEVN

Difficult to observe these systems → deal with Binary System Evolution (BSE) simulations ➔ **SEVN** (Stellar EVolution for N-body)

- Rapid binary population synthesis code
- Initial conditions → evolution → interpolating PARSEC stellar tracks
- Analytic and semi-analytic prescriptions
- Flexible, low computation time → easy to change or update it

Goal of the project

- Understand what kind of processes these systems are likely to experience during their lifetime → DNN and XGBoost
  - understand the **importance** that each feature has had in the evolution of systems
  - **classify** the GAIA candidates
  - retrieve the guessed full **evolution history** of candidates

Simulated data
Selection

- Systems with **metallicities** of 0.02 (solar-like systems), 0.001 (metal-poor), and 0.0001 (very metal-poor)
- Three different **α values**: 0.1 (indicating low efficiency), 1 (intermediate efficiency), and 5 (high efficiency).

- We utilized `Dask` to deal with the large amount of data

Simulated data
Selection

- Select **BH+MS** systems

```
mask = (
          ((outputs['RemnantType_0'] == 6) & (outputs['RemnantType_1'] == 0) &
          (outputs['Phase_1'] == 1)) | ((outputs['RemnantType_1'] == 6) &
          (outputs['RemnantType_0'] == 0) & (outputs['Phase_0'] == 1))
        )
bh_star_df = outputs[mask]
```

Simulated data
Selection

- Dividing between interacting and non-interacting systems → **Roche Lobe** radius

$$\frac{r1}{a} = \frac{0.49\,q^{2/3}}{0.6\,q^{2/3} + \ln\left(1+q^{1/3}\right)}$$

- If the radius of the star or the BH in the system is greater than the Roche Lobe radius r1, then the binary is classified as interacting. Otherwise, for $r \leq r1$ it is non-interacting.

Simulated data
Selection

- Add **period** and converted in log

```
period = (2*m.pi*np.sqrt( semimaj**3 / (const.G * mass_tot) )) / s_to_day

int_binsys['logP'] = np.log10(period)
```

Simulated data
Elapsed BWorld time

- For **visualization purposes** only

- Restricting to non-interacting → retain only the first row → initial time of the BH+MS phase

- From initial dataset → systems (bound and non-interacting) composed of a BH and a star no more in MS phase (`Phase_* > 1`) → retain only the first row → end time of the BH+MS phase

- Calculate the difference

Simulated data
Evolution channel selection

- SEVN records each phase the two objects underwent

- Restricting to the selection to:
  - Non-Interacting (**NI**)
  - (stable) Mass Transfer (**MT**)
  - (at least one) Common Envelope (**CE**)

- Create series of masks → apply them in succession

Simulated data
Evolution channel selection

- Selection based on `BEvent` column

- ```
  # Selecting systems that are in the evolutionary phase of two stars
  mask_3 = (((outputs['RemnantType_0'] == 0)) & (outputs['RemnantType_1'] == 0))
  ```

- ```
  # Creating a mask to select systems that went through at least 1 Common Envelope
  mask_CE = msms_df['BEvent'].isin([7, 9, 11, 12, 14, 15])
  ```

- ```
  # Creating a mask to select systems that went through a stable Mass Transfer AND
  NOT a CE
  mask_MT1 = (out_masked['BEvent'].isin([4, 5, 6, 8, 10, 13, 16, 17, 18, 19, 20]))
  ```

- ```
  # Filling the remaining rows (NaN) with the Non Interacting flag
  out_masked_1['type'] = out_masked_1['type'].fillna('NI')
  ```

Simulated data
Evolution channel selection

- **Merge** the new dataframes

  - non-interacting BH+MS systems (`non_int_binsys`)

  - evolution types (`type_df`)

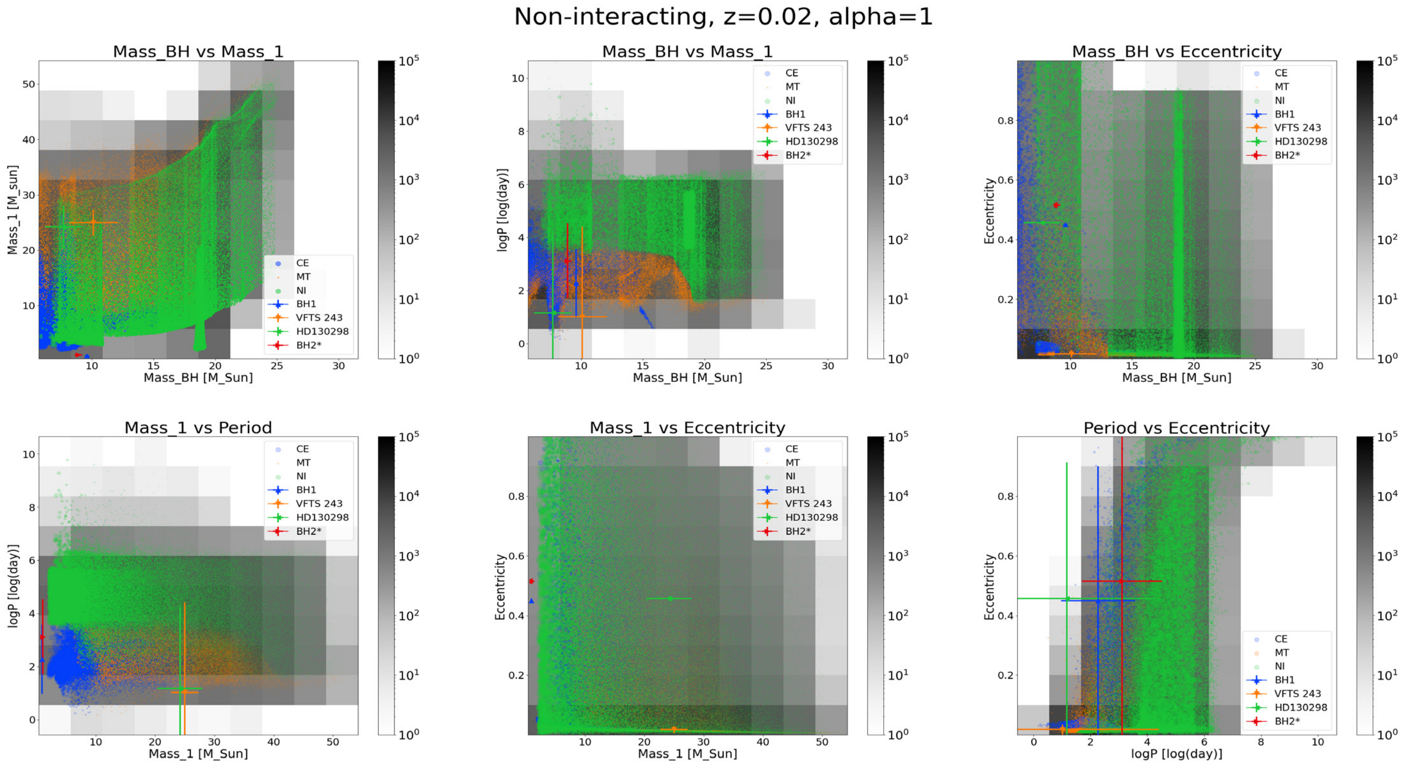
  - elapsed time (`df_bwt`)

	ID	name	Mass_BH	Mass_1	Eccentricity	logP	z	alpha	BEvent	type	elapsed_bwt	tot_mass
0	80	0_201673565337120	17.66923	28.74325	0.212435	4.221513	0.0200	0.5	-1	NI	1.621285	46.41248
1	87	0_929528790266714	23.50375	44.82448	0.752931	5.343398	0.0200	0.5	-1	NI	0.180697	68.32823
2	109	0_889165204871227	21.41187	34.20442	0.227290	4.468927	0.0200	0.5	-1	NI	0.026888	55.61629
3	122	0_111281974618981	17.45101	19.13807	0.326498	5.457667	0.0200	0.5	-1	NI	4.488835	36.58908
4	226	0_363554648261798	16.08790	28.28335	0.112235	3.496335	0.0200	0.5	-1	NI	1.450930	44.37125

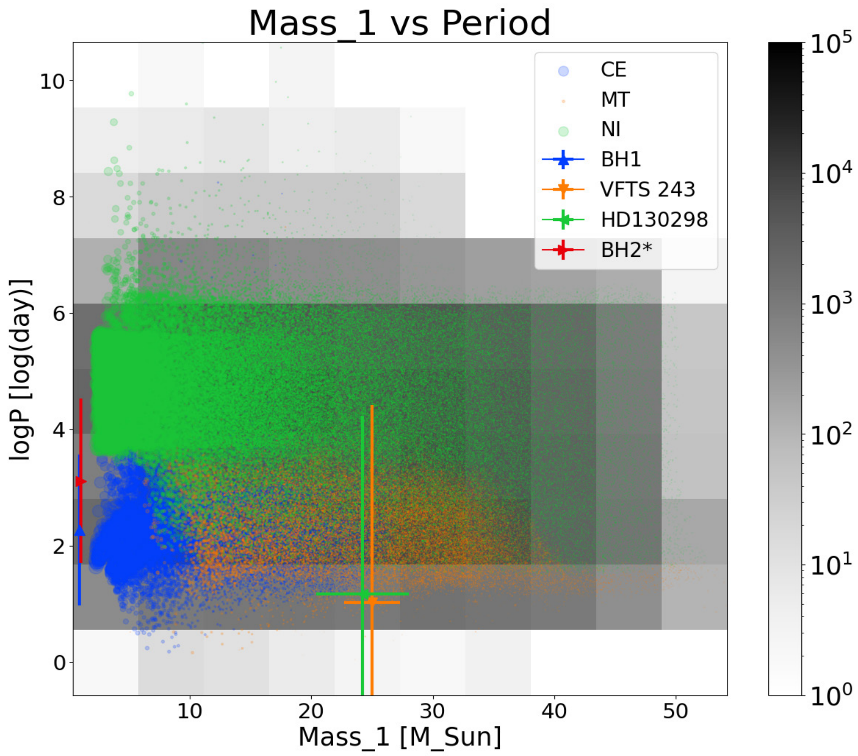
Confirmed BH-MS systems

	ID	name	Mass_BH	dMass_BH	Mass_1	dMass_1	Period	dPeriod	logP	dlogP	Eccentricity	dEccentricity	loc	system	z
0	R00	BH1	9.6	0.18	0.93	0.05	185.59000	0.05000	2.268555	-1.301030	0.450	0.005	MW	disc-field	solar
1	R01	VFTS 243	10.1	2.00	25.00	2.30	10.40310	0.00040	1.017163	-3.397940	0.017	0.012	LMC	OC	sub-solar
2	R02	HD130298	7.7	1.50	24.20	3.80	14.62959	0.00085	1.165232	-3.070581	0.457	0.007	MW	runway-field	solar
3	R03	BH2*	8.9	0.30	1.07	0.19	1300.00000	26.00000	3.113943	1.414973	0.515	0.010	MW	field	solar

Data distribution



Data distribution



Classification models

- **Combine** dataset created before
- **Shuffle** to add more randomness → removes patterns or biases
- **Balance** the dataframe → select equal number of systems for each label

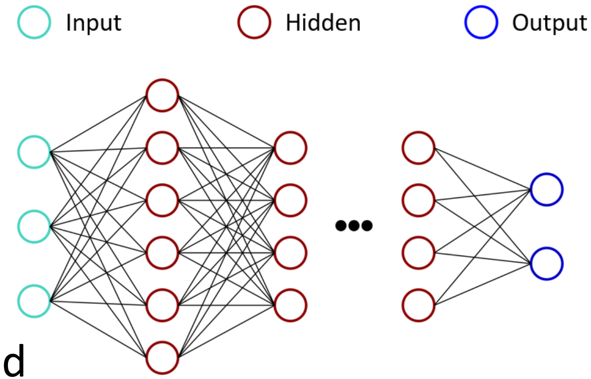
```
data_tot = pd.concat([data_a05, data_a1, data_a5])

NI = shuffle(data_tot[data_tot['type']=='NI']).head(430242)
MT = shuffle(data_tot[data_tot['type']=='MT']).head(430242)
CE = shuffle(data_tot[data_tot['type']=='CE'])

balanced_data = pd.concat([NI, MT,CE])

shuffle_data = shuffle(balanced_data)
```

Deep Neural Network (DNN)



- Type of **artificial neural network**
- Particularly effective for supervised learning tasks → classification
- Composed of:
  - Multiple **layers** of interconnected nodes called **neurons**
  - Neurons receive input from the previous layer and produce an output
  - Connections → assigned **weights** → strength/importance of the connection
- DNN learn by adjusting weights during training → optimization algorithms → e.g., gradient descent → ADAM

Deep Neural Network (DNN)

- “**Deep**” → multiple hidden layers → capture intricate patterns and relationships

- For classification tasks → output layer consists same number of neurons of the classes you want to classify

- DNN assign probability to each class based on the learned weights and biases

- The class with the highest probability is chosen as the predicted class

Deep Neural Network (DNN)
Our model

- Several tests and “gridsearches” → optimal structure
- **4** hidden layers, **20** neurons each
- Input layer → n° neurons = n° of input features
- Output layer → n° neurons = 3 (n° of classes)

- Activation function → **ReLU**
- Optimizer → **Adam**
- Loss function → **categorical cross-entropy**

Deep Neural Network (DNN)
Our model

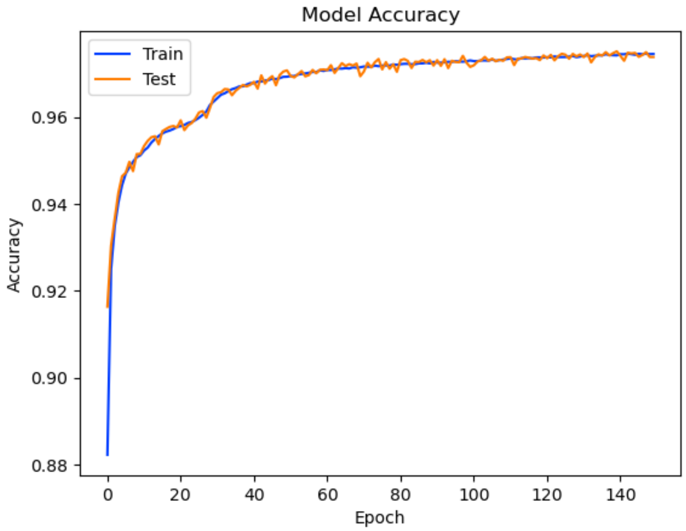
- **Epochs** → 100-200 → 150
- **Batch size** → 450-750 → 750

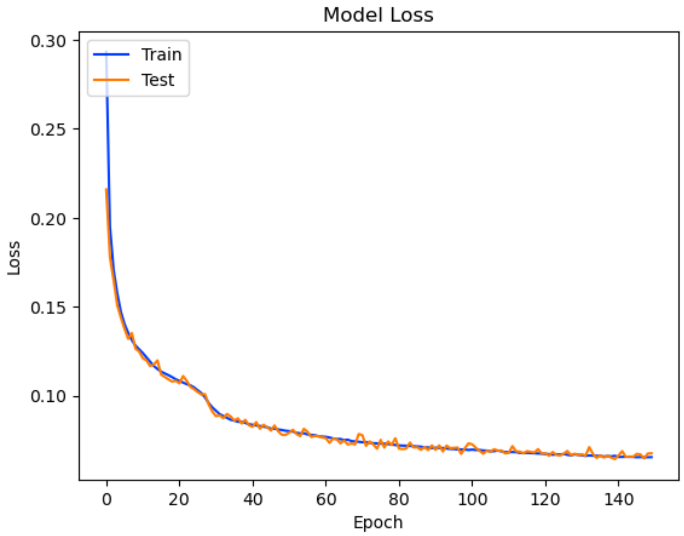
➔ balance between model performance and computational efficiency

- Training → all the **features**
  → `Mass_BH`, `Mass_1`, `Eccentricity`, `logP`, `z`, `alpha`
  → scaling of the first four features

Deep Neural Network (DNN)
Results

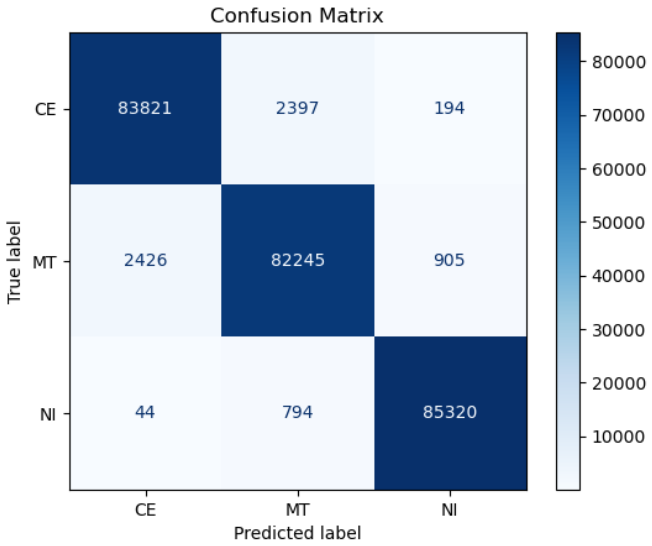
- Performance metrics → **accuracy** and **loss**





Deep Neural Network (DNN)
Results

- Confusion matrix



Deep Neural Network (DNN)

Results

- Classification report
  - **Precision** → TP/(TP+FP)
  - **Recall** → TP/(TP+FN)
  - **F1 score** → harmonic mean of precision and recall
  - **Accuracy** → (TP+TN)/(TP+FP+TN+FN)

	precision	recall	f1-score	support
CE	0.97	0.97	0.97	86412
MT	0.96	0.96	0.96	85576
NI	0.99	0.99	0.99	86158
accuracy			0.97	258146

Deep Neural Network (DNN)
Classification

- Same structure as before
- Added a **dropout layer** after each hidden layer → prevent overfitting
- **Epochs** → 100-200 → 150
- **Batch size** → 1000

- Training → **features** present in the GAIA dataset
  → `Mass_BH, Mass_1, logP`
  → scaling of the first four features

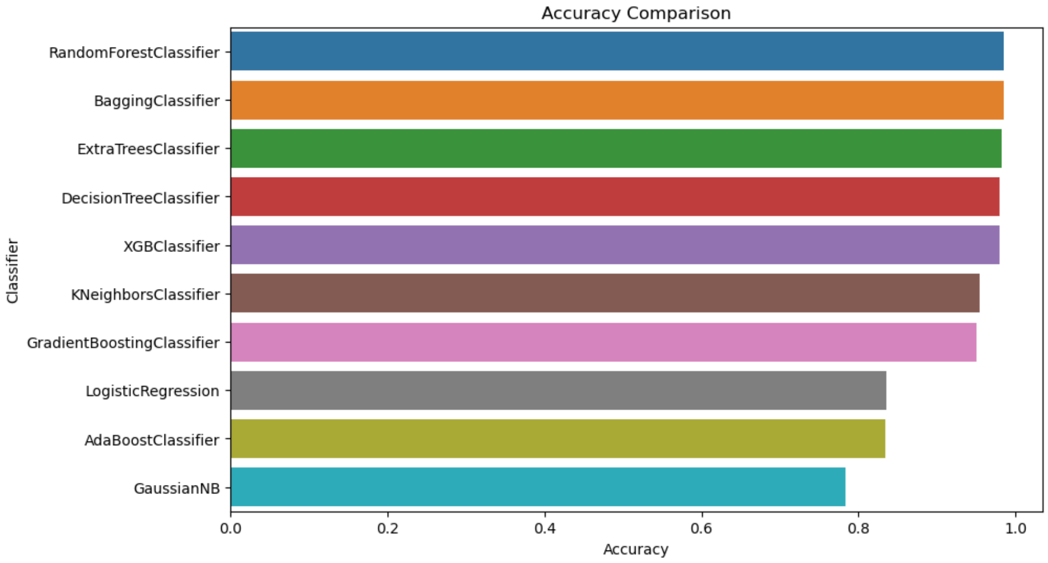
Deep Neural Network (DNN)
Classification

- Classification report

	precision	recall	f1-score	support
CE	0.89	0.88	0.88	86051
MT	0.87	0.88	0.87	86181
NI	0.97	0.98	0.98	85914
accuracy			0.91	258146

Different classifier

- Conducted experiments using various classification algorithms
- Evaluated and ranked based on their performance metrics



Different classifier

- `RandomForestClassifier`, `BaggingClassifier`, `ExtraTreesClassifier`, `DecisionTreeClassifier`, and `XGBClassifier` demonstrated similar performances
- We’ve chosen to utilize **`XGBoost`**
  - **Sequential Ensemble Building** → each subsequent model rectifies the errors made by the preceding models
  - **Gradient Boosting Framework** → iteratively optimizing a objective function to enhance performances
  - **High Predictive Power and Efficiency** → optimize computational efficiency → handle large datasets and complex problems

XGboost

- Algorithm used for **supervised** learning tasks
- Ensemble learning method → combine weak decision trees → create strong model

- Works by iteratively training and adding decision trees to the ensemble
- Identifies areas where the previous trees have made errors → correct them
- Minimize loss function by learning from gradients errors
- Final prediction → aggregating the predictions from all the trees

XGBoost
Our model

- Gridsearch analysis → default model is the best (when considering all features)

Results

```
Accuracy: 0.979038993437822
Precision: 0.9790641302747958
Recall: 0.979038993437822
F1-Score: 0.9790338071672348
```

Classification Report:

	precision	recall	f1-score	support
CE	0.98	0.97	0.97	85954
MT	0.97	0.97	0.97	86443
NI	0.99	0.99	0.99	85749
accuracy			0.98	258146

XGBoost
Classification

- Gridsearch analysis:
  - **Learning rate** → 1
  - **n° estimators** → 300
  - **Max depth** → 7

Results

```
Accuracy: 0.9226019384379383
Precision: 0.9225351950821429
Recall: 0.9226019384379383
F1-Score: 0.922555799194637
```

Classification Report:

	precision	recall	f1-score	support
CE	0.90	0.89	0.89	86117
MT	0.89	0.89	0.89	86190
NI	0.98	0.99	0.98	85839
accuracy			0.92	258146

Naïve Bayesian Classifier

- Simple **probabilistic** ML algorithm → Bayes' theorem
- Strong assumption on **independence** among features
  → presence or absence of one feature does not affect the model
- Calculates the probability of each class → selects the class with the highest probability
- ➔ **unsuitable** ➔ because of features independence
- Suboptimal results → accuracy < 80%

SHAP

- SHapley Additive exPlanations → explain the predictions of ML models

- Core idea → SHAP values → distributing the payout of a cooperative game among the players

- Based on cooperative game theory
→ allocate the contribution of each feature in a prediction model to the final prediction
→ quantify the impact of each feature on the prediction → considering all combinations

SHAP

Several advantages:

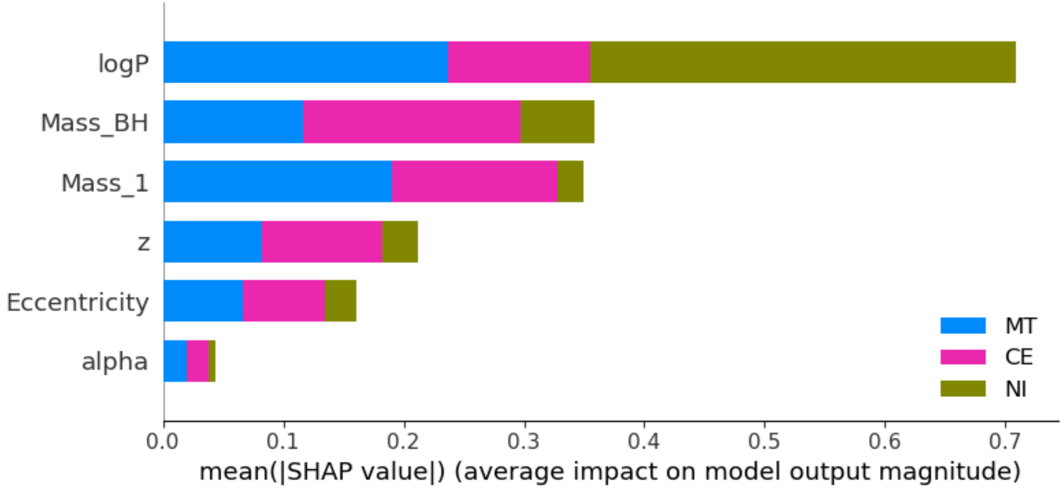
1. Unified framework for feature importance measurement

2. Consistent → if a feature is removed or added, the SHAP values change accordingly

3. Locally/Globally accurate → explain individual predictions/overview of features importance

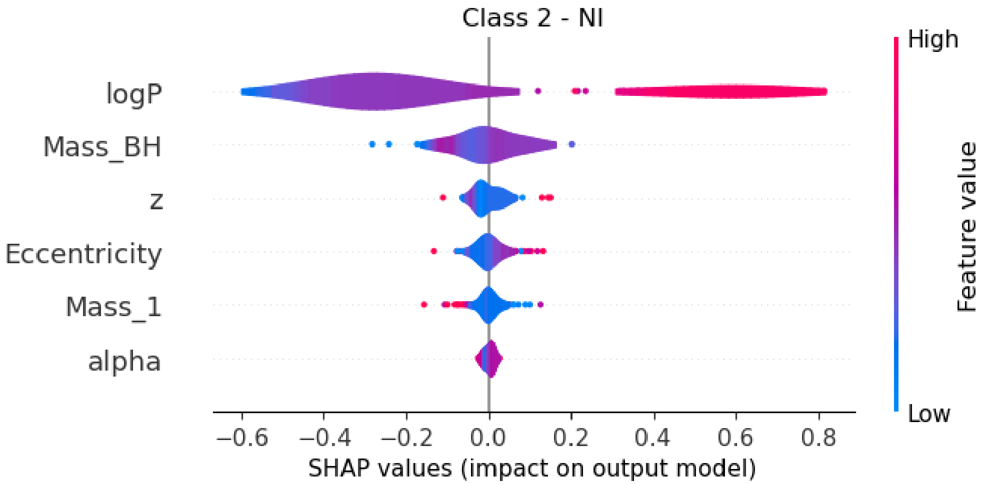
SHAP
DNN

- Mean absolute SHAP value → magnitude of each feature's contribution towards the predicted label

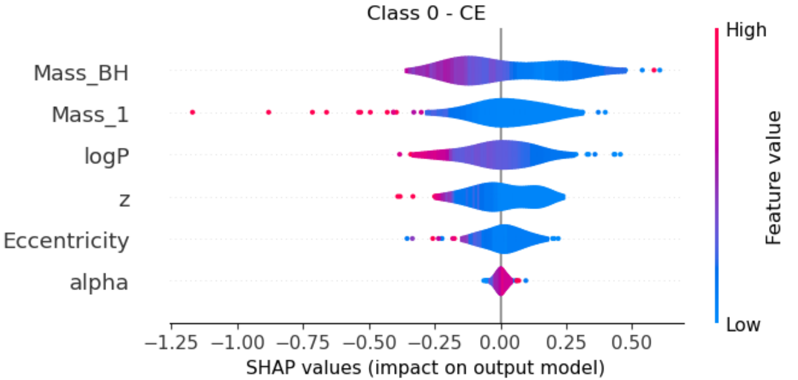


SHAP
DNN

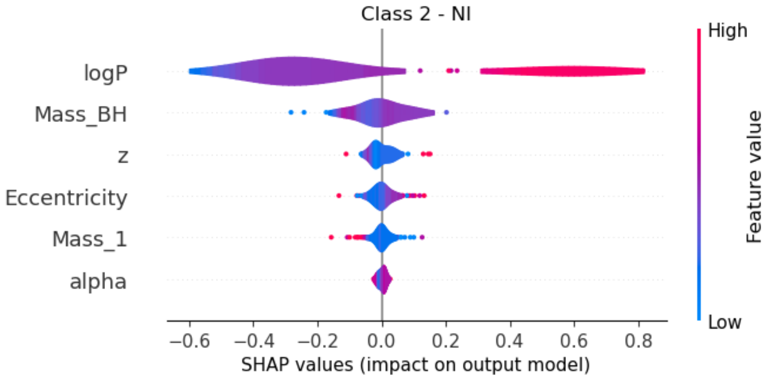
- Violin plot → most useful type
- Representation of the distribution of SHAP values for different features

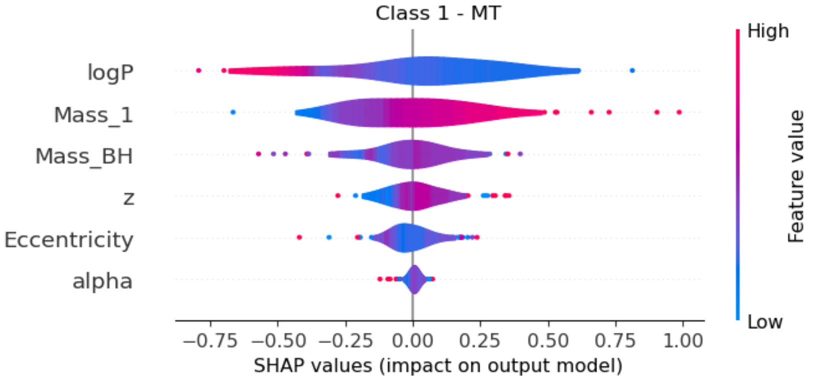


SHAP DNN

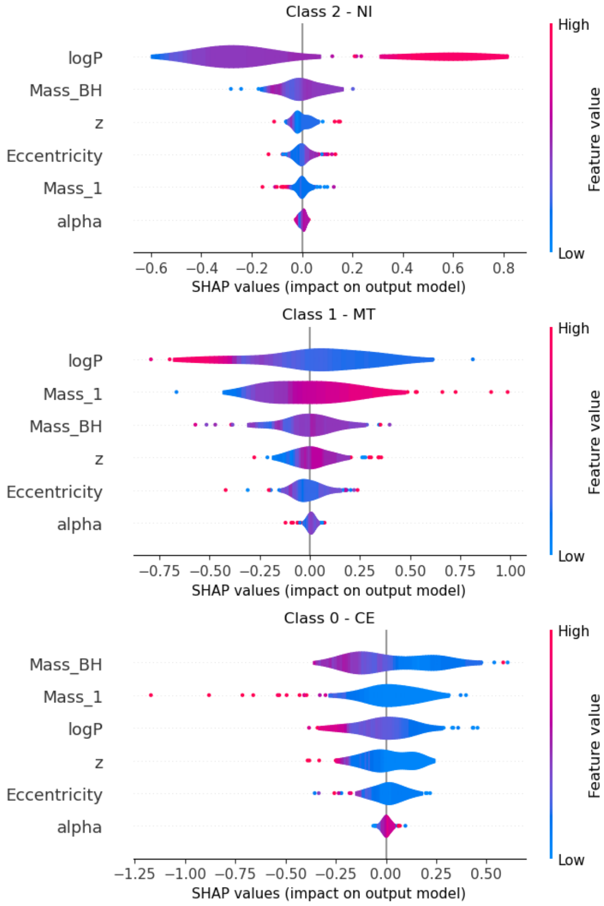


- **Vertical axis**: features in descending order of importance
- **Horizontal axis**: range of SHAP values
- **Violin shape**: density of the SHAP values for a specific feature
- **Color**: original value

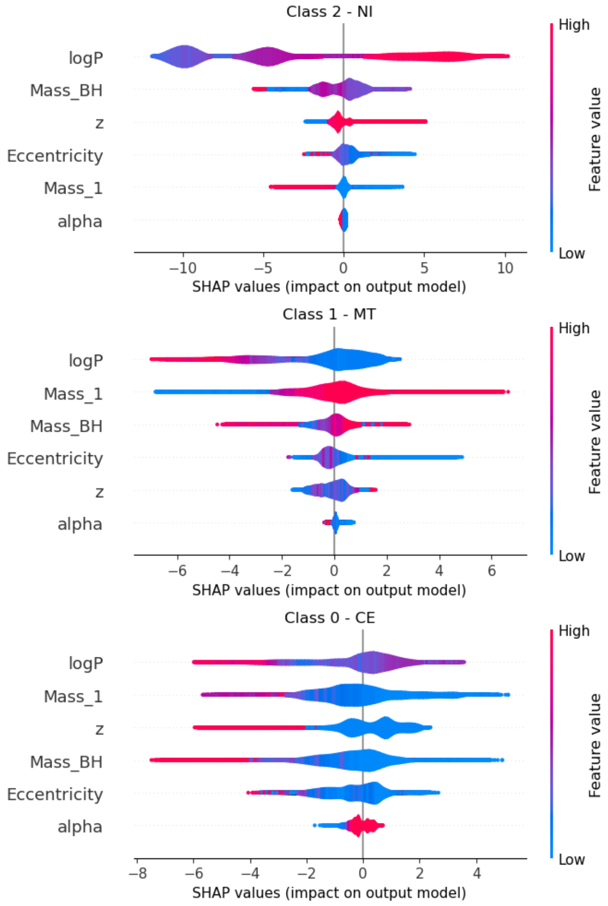




DNN


Class 2 - NI
logP
Mass_BH
z
Eccentricity
Mass_1
alpha
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6 0.8
SHAP values (impact on output model)
High
Feature value
Low
Class 1 - MT
logP
Mass_1
Mass_BH
z
Eccentricity
alpha
−0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
SHAP values (impact on output model)
High
Feature value
Low
Class 0 - CE
Mass_BH
Mass_1
logP
z
Eccentricity
alpha
−1.25 −1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50
SHAP values (impact on output model)
High
Feature value
Low

XGB


Class 2 - NI
logP
Mass_BH
z
Eccentricity
Mass_1
alpha
−10 −5 0 5 10
SHAP values (impact on output model)
High
Feature value
Low
Class 1 - MT
logP
Mass_1
Mass_BH
Eccentricity
z
alpha
−6 −4 −2 0 2 4 6
SHAP values (impact on output model)
High
Feature value
Low
Class 0 - CE
logP
Mass_1
z
Mass_BH
Eccentricity
alpha
−8 −6 −4 −2 0 2 4
SHAP values (impact on output model)
High
Feature value
Low

Results interpretation (DNN)

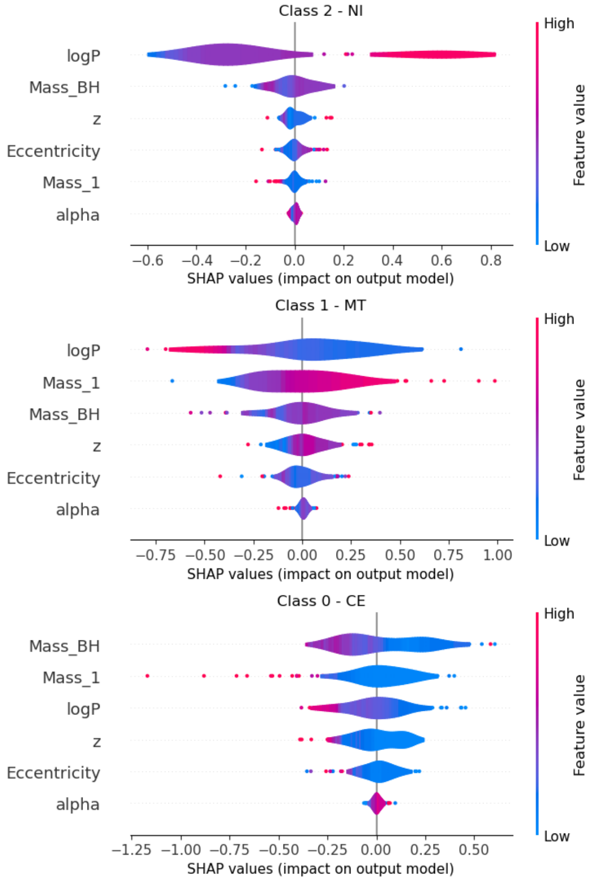
NI

- **logP**: High values of the period contribute greatly

- **Mass_BH**: intermediate values → no interaction → higher stellar mass → higher mass of the BH

- **Z**: based on stellar tracks → higher z values → higher radius during He-b → exceeding RL
  ➔ preference for low-medium values for NI
  → lacks definitive evidence

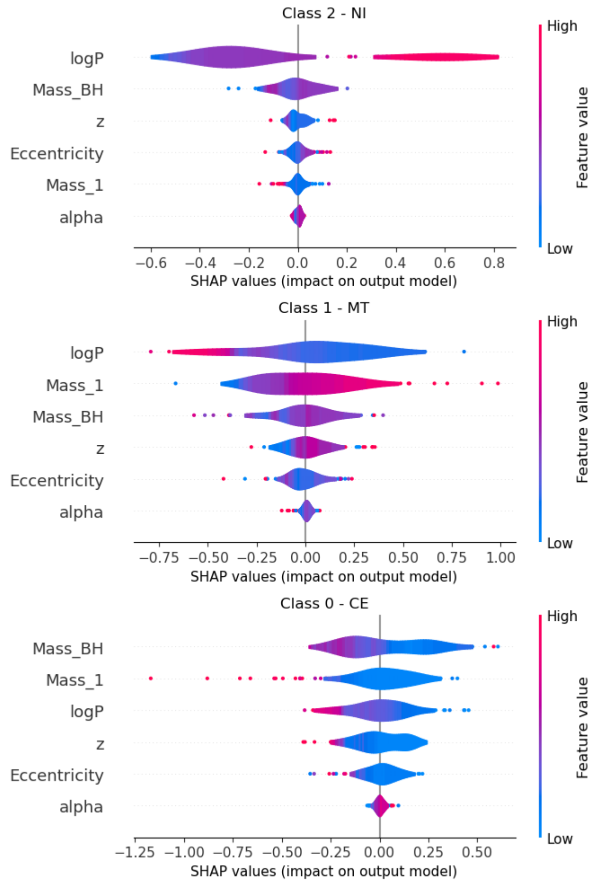
- **Eccentricity**: slight contribution
- **Mass_1**: seems not contribute
- **Alpha**: seems not contribute



Results interpretation (DNN)

MT

- **logP**: Low period values contribute significantly
- **Mass_1 & Mass_BH**: during stars evolution, mass ratio < QC → this info have influenced the mass values in the BH-MS phase
  → it's not crystal clear if the model had interpreted correctly these features
- **Z**: high metallicity values contribute
- **Eccentricity**: Slight contribution
- **Alpha**: seems not contribute



Results interpretation (DNN)

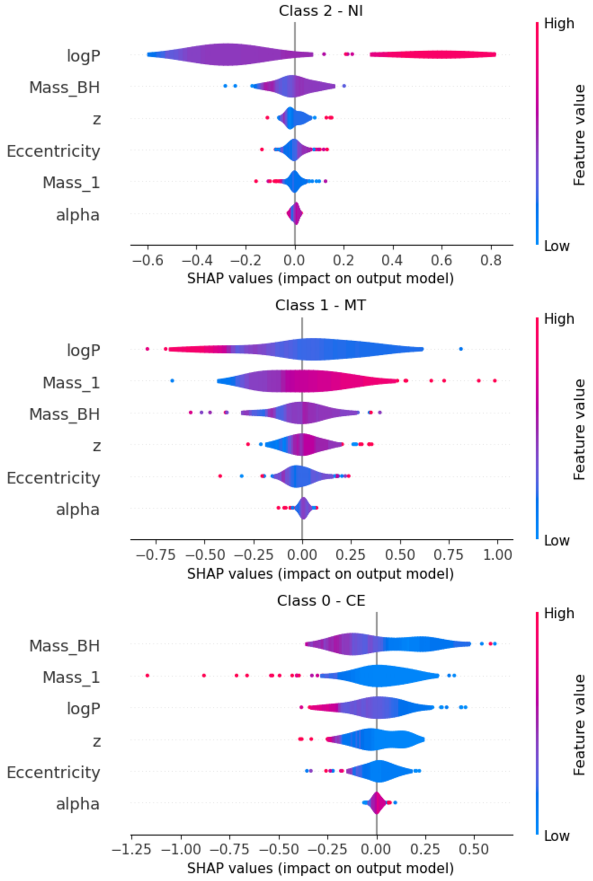
CE

- **Mass_1 & Mass_BH**: during stars evolution, mass ratio > QC → this info have influenced the mass values in the BH-MS phase
  → it's not crystal clear if the model had interpreted correctly these features

- **LogP & Eccentricity**: low values contribute greatly

- **Z**: inconsistent from an astrophysical point of view

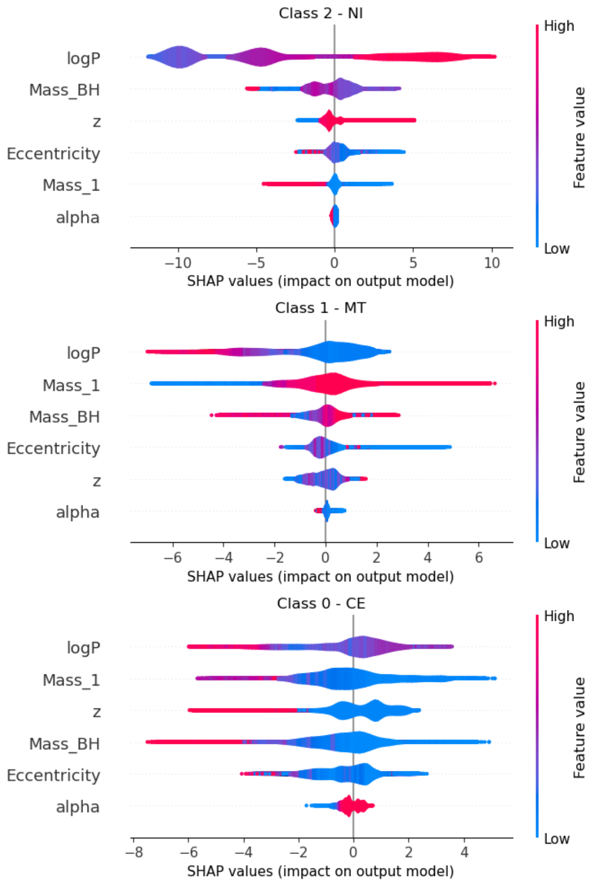
- **Alpha**: seems not contribute



Results interpretation (XGB)

NI

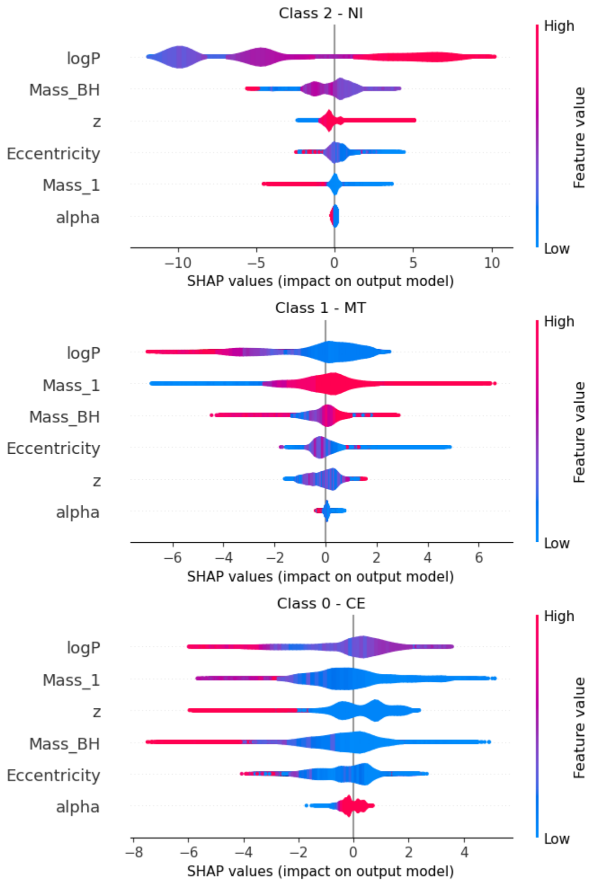
- **Z**: inconsistent from an astrophysical point of view
- **Eccentricity**: inconsistent from an astrophysical point of view
- **Mass_1**: not clear what is the contribution



Results interpretation (XGB)

MT

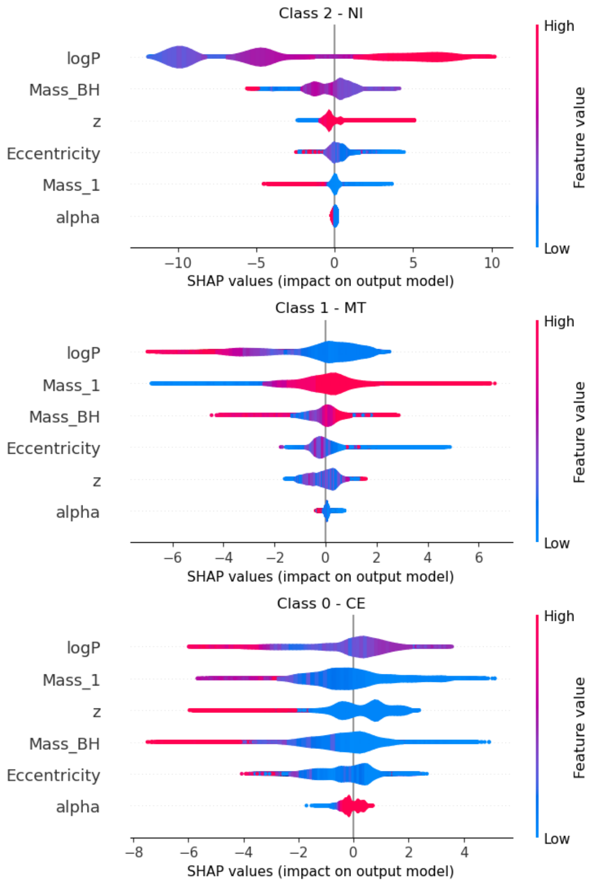
- **Eccentricity**: slight contribution given by low to medium values of eccentricity
- **Z**: no great contribution. Inconsistent from an astrophysical point of view



Results interpretation (XGB)

CE

Same as for the DNN



Results interpretation
Conclusions

- Enhance the model's understanding and recognition of the threshold (QC)
- Inclusion of additional training data → information regarding the initial mass ratio or the mass ratio in each phase experienced by the donor star.
- In our training experiments →including the mass ratio of the BH-MS → notable decrease in the model's accuracy.
  ➔ we excluded this feature

Candidates Classification

- Classify GAIA DR3 candidates → trained models
- For statistical analysis → **sampled** 10.000 gaussian samples for each system
- The classification generates a dataset → percentage for each label

pred_type	CE	MT	NI	total_count	CE_percentage	MT_percentage	NI_percentage
ID							
9720614798216320	7255.0	2561.0	184.0	10000.0	72.55	25.61	1.84
9881104840594176	1152.0	3607.0	5241.0	10000.0	11.52	36.07	52.41
14686108092825984	695.0	262.0	9043.0	10000.0	6.95	2.62	90.43
30994481168584576	0.0	109.0	9891.0	10000.0	0.00	1.09	98.91

Candidates Classification

- Setting a threshold based on statistical significance, such as 1, 2, or 3 standard deviations

DNN

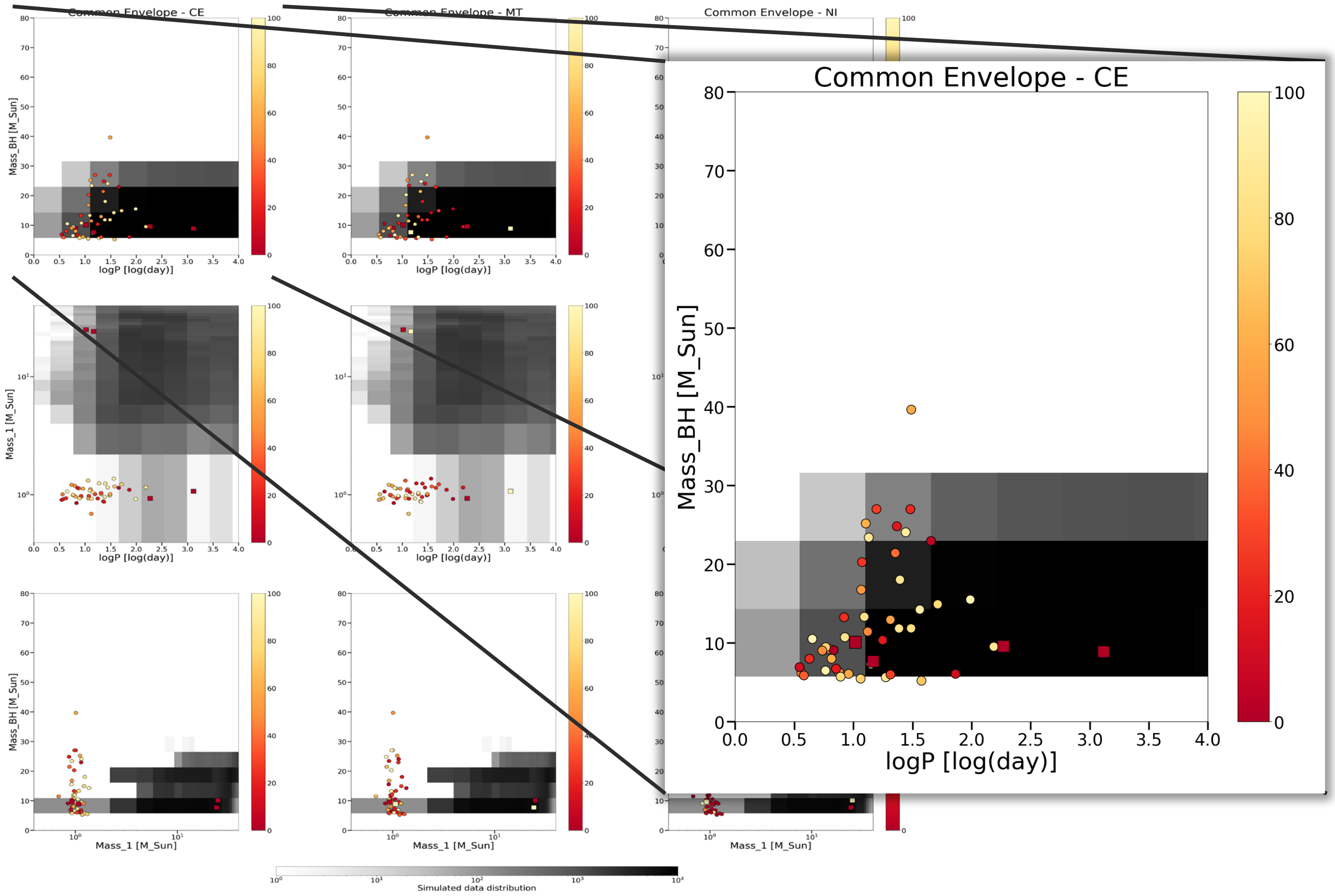
Label	$\sigma \geq 1$	$\sigma \geq 2$	$\sigma \geq 3$
NI	1301 (28.0%)	762 (16.4%)	418 (9.0%)
CE	527 (11.34%)	61 (1.31%)	17 (0.37%)
MT	679 (14.61%)	2 (0.04%)	2 (0.04%)

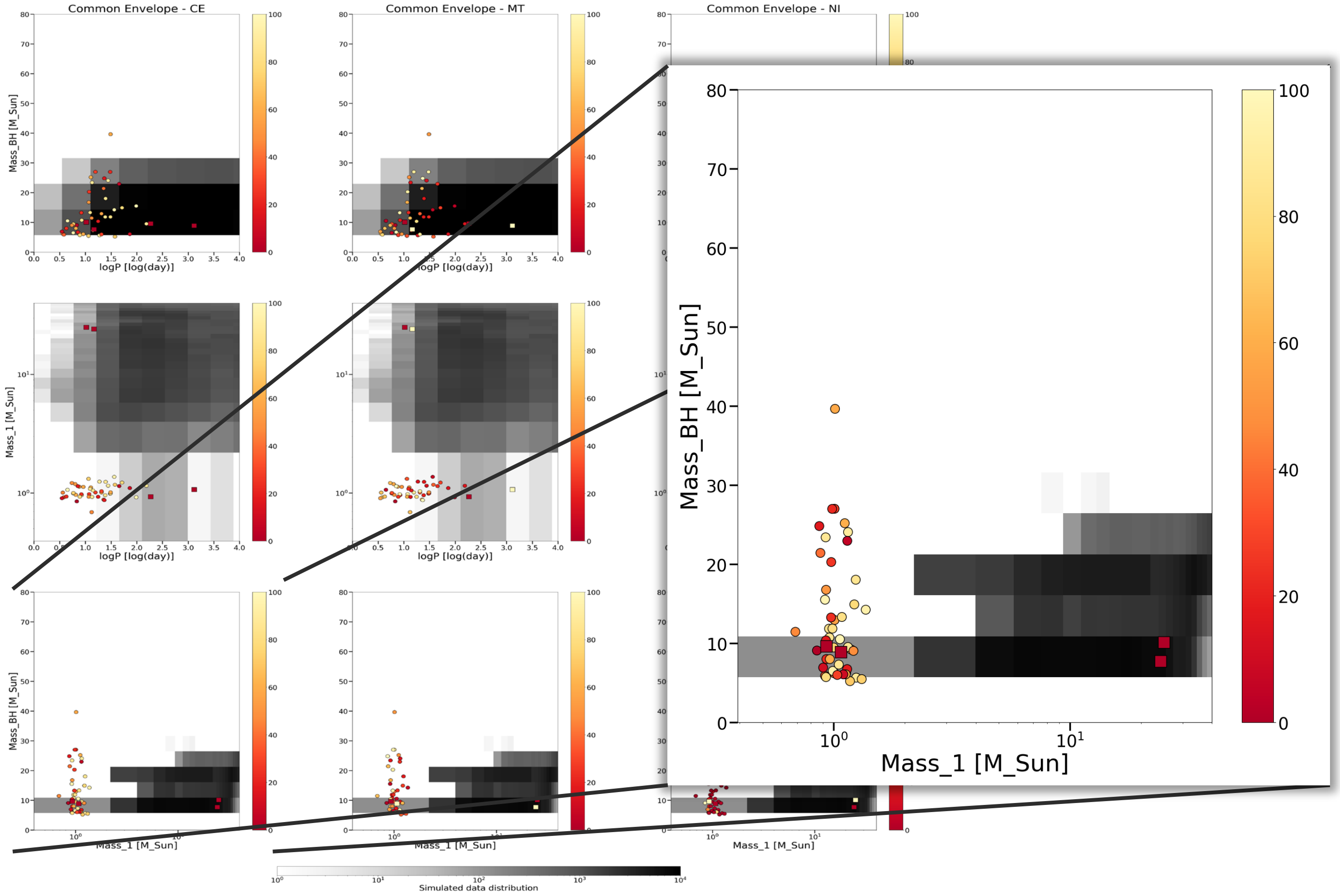
XGB

Label	$\sigma \geq 1$	$\sigma \geq 2$	$\sigma \geq 3$
NI	1271 (27.36%)	648 (13.95%)	231 (4.97%)
CE	393 (8.46%)	45 (0.97%)	6 (0.13%)
MT	543 (11.69%)	2 (0.04%)	2 (0.04%)

Candidates Classification

- Comparing simulated data with candidate systems
- Narrow down to “**gold**” plating → more reliable
- From simulated data → **z=0.02** → systems observed by GAIA exhibit solar-like Z
- Consistent agreement between the two models
- **Goal** → assess how well SEVN model capture the characteristics of real binary systems


Common Envelope - CE
Common Envelope - MT
Common Envelope - NI
Mass_BH [M_Sun]
logP [log(day)]
Mass_1 [M_Sun]
Simulated data distribution


Common Envelope - CE
Common Envelope - MT
Common Envelope - NI
Mass_BH [M_Sun]
Mass_1 [M_Sun]
logP [log(day)]
Simulated data distribution

Candidates Classification

- Close correspondence in the logP-Mass_BH phase space
- When considering Mass_1 (mass of the MSs) → significant discrepancy for values below 10 Msun
- Challenge in providing a definitive explanation → current models do not anticipate so lightweight MSs

Candidates Classification

Possible explanations:

- Reevaluating **mass transfer mechanisms**: mass transfer is more significant

- Limitations of current **stellar tracks**: min MZAMS = 2.2 Msun → absence of tracks for the objects observed by GAIA

Similar systems

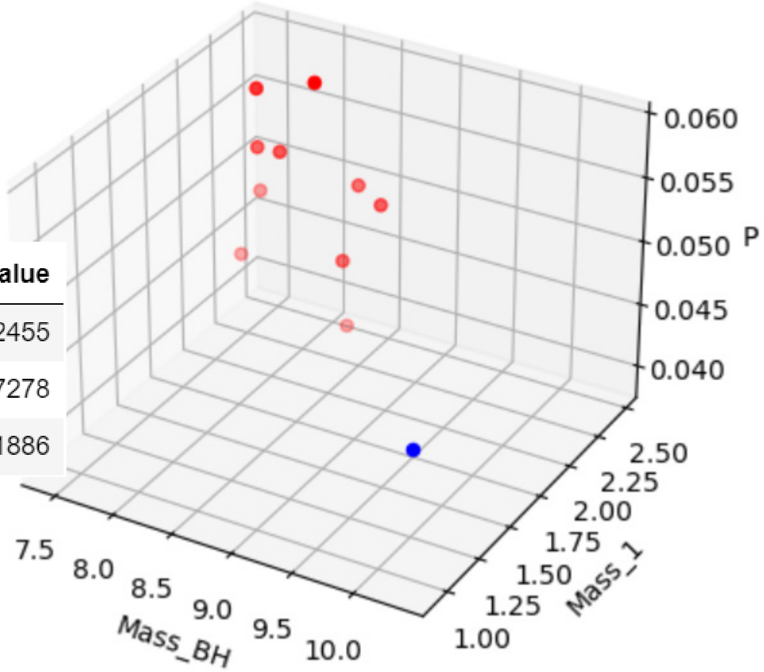
- Find similar systems to the real candidates of BH-MS binary systems and predicting the history of the evolution

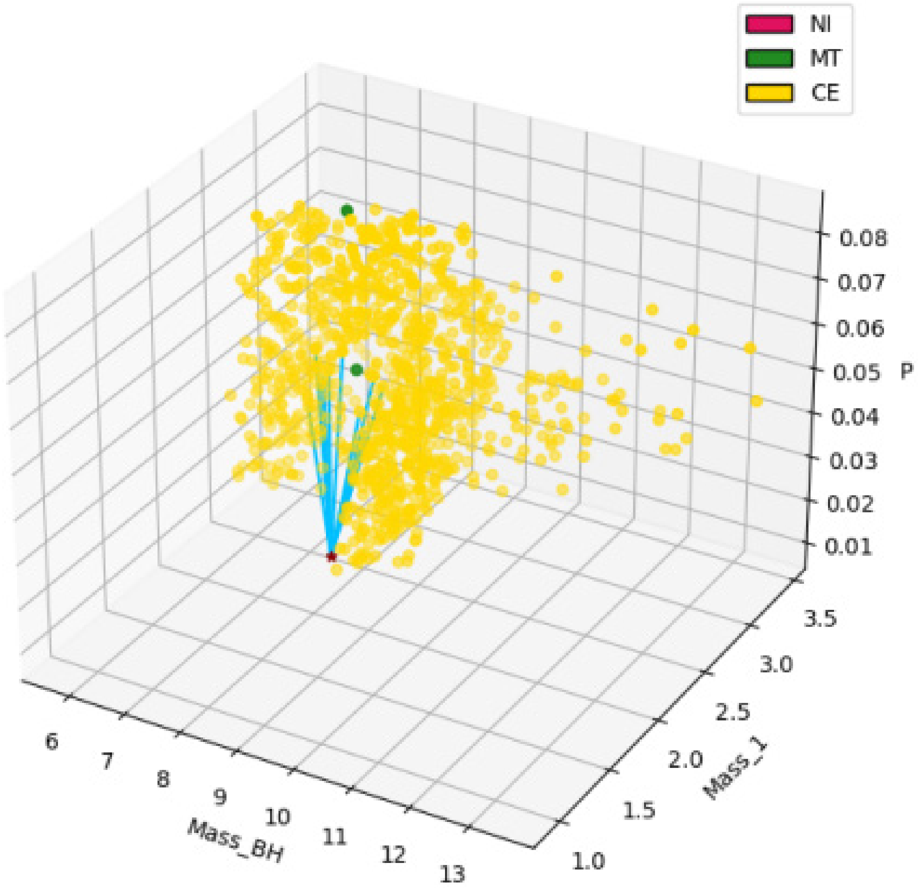
$$d_{\mathrm{SEVN},Real} = \sqrt{\left(\frac{Sec\_M_{SEVN} - Sec\_M_R}{S\_M_{upper} - S\_M_{lower}}\right)^2 + \left(\frac{MS\_M_{SEVN} - MS\_M_R}{MS\_M_{upper} - MS\_M_{lower}}\right)^2 + \left(\frac{P_{SEVN} - P_R}{P_{upper} - P_{lower}}\right)^2}$$

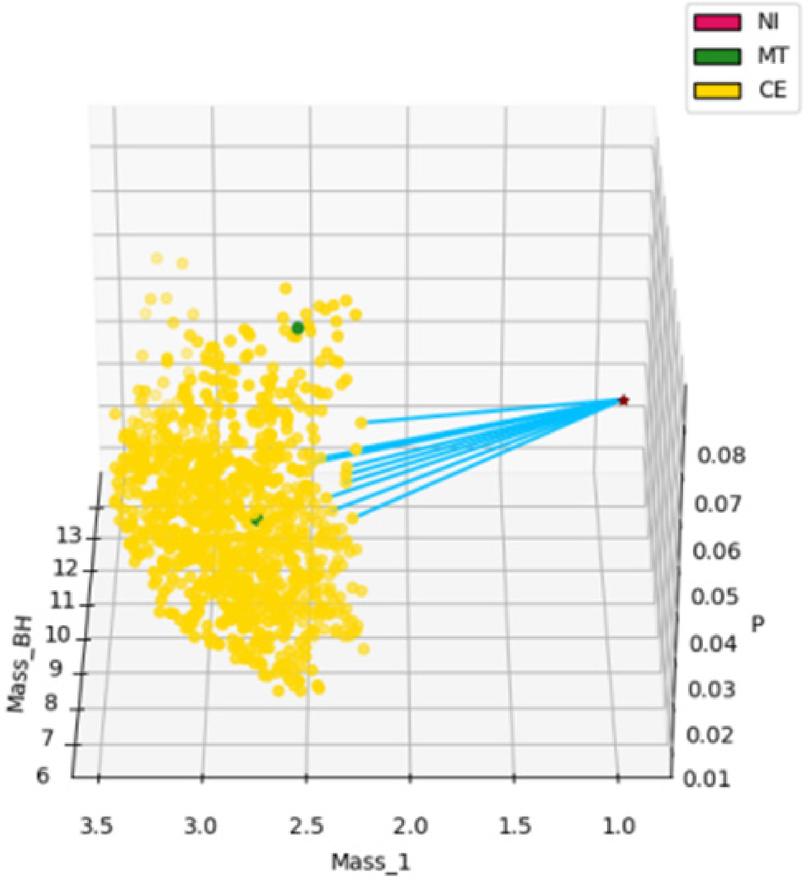
- Selected most similar 10 systems and 1000 systems for the analysis

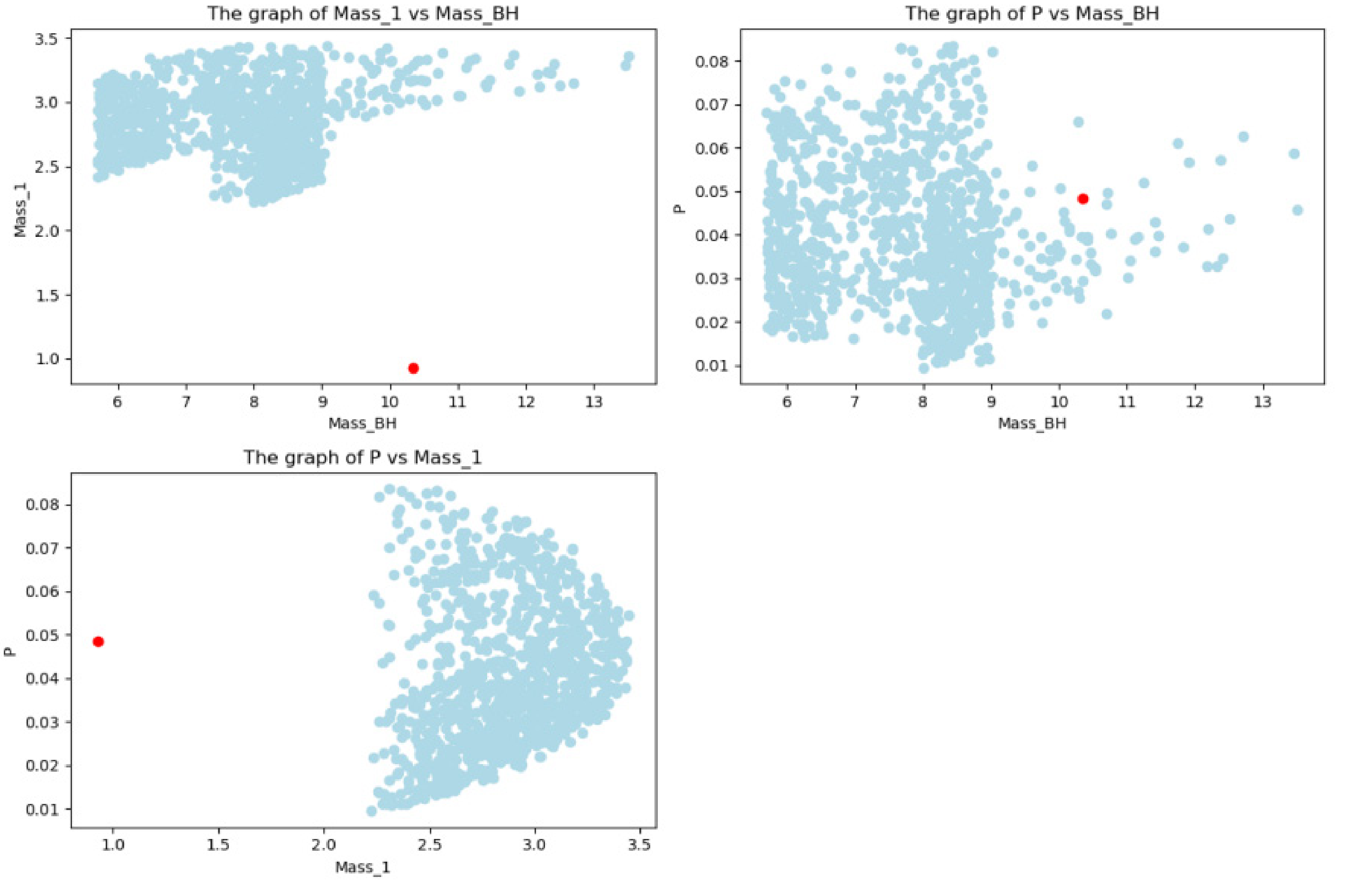
Index	Source Id	BH Mass	MS Mass	Period
44	6.70435e+18	10.34443	0.9273589	0.04849124

	Name	Mass_BH	Mass_1	P	z	alpha	type	value
0	0_420895437325839	8.291450	2.310986	0.044914	0.001	1.0	CE	0.622455
1	0_617684874163902	7.746563	2.305923	0.052374	0.020	0.5	CE	0.637278
2	0_478118807791845	7.422293	2.278359	0.043486	0.020	0.5	CE	0.641886

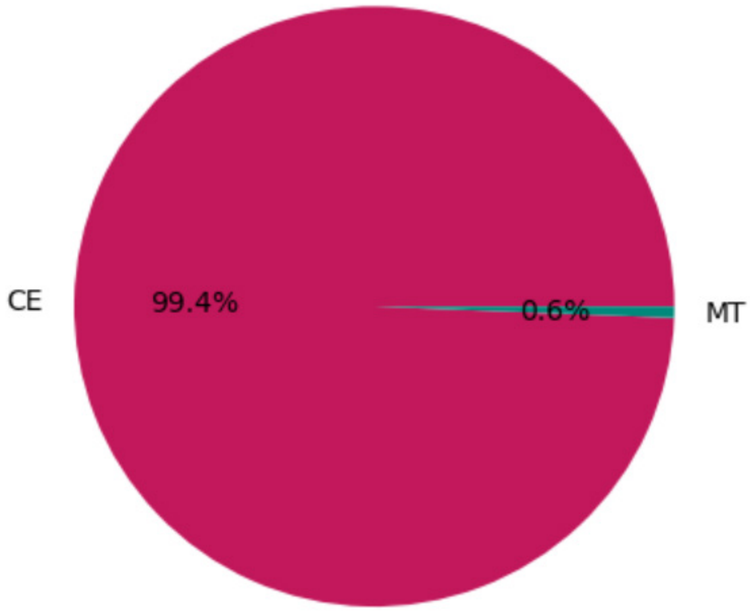



NI
MT
CE
P
Mass_BH
Mass_1
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
6
7
8
9
10
11
12
13
1.0
1.5
2.0
2.5
3.0
3.5


NI
MT
CE
P
Mass_BH
Mass_1
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
13
12
11
10
9
8
7
6
3.5
3.0
2.5
2.0
1.5
1.0


The graph of Mass_1 vs Mass_BH
Mass_1
Mass_BH
The graph of P vs Mass_BH
P
Mass_BH
The graph of P vs Mass_1
P
Mass_1

Percentage possibility of history type

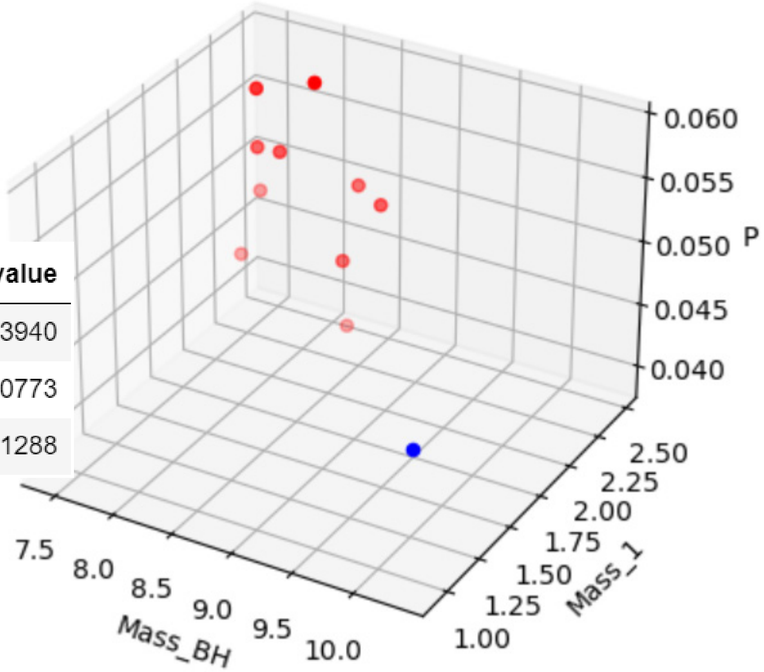


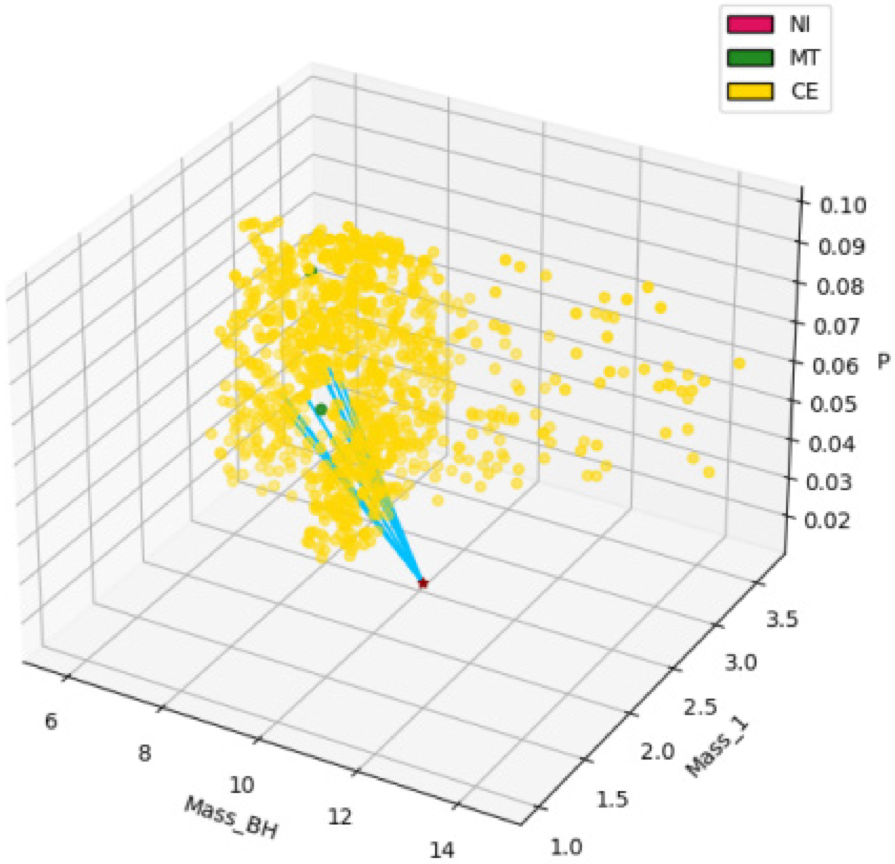
Predicted Evolution History

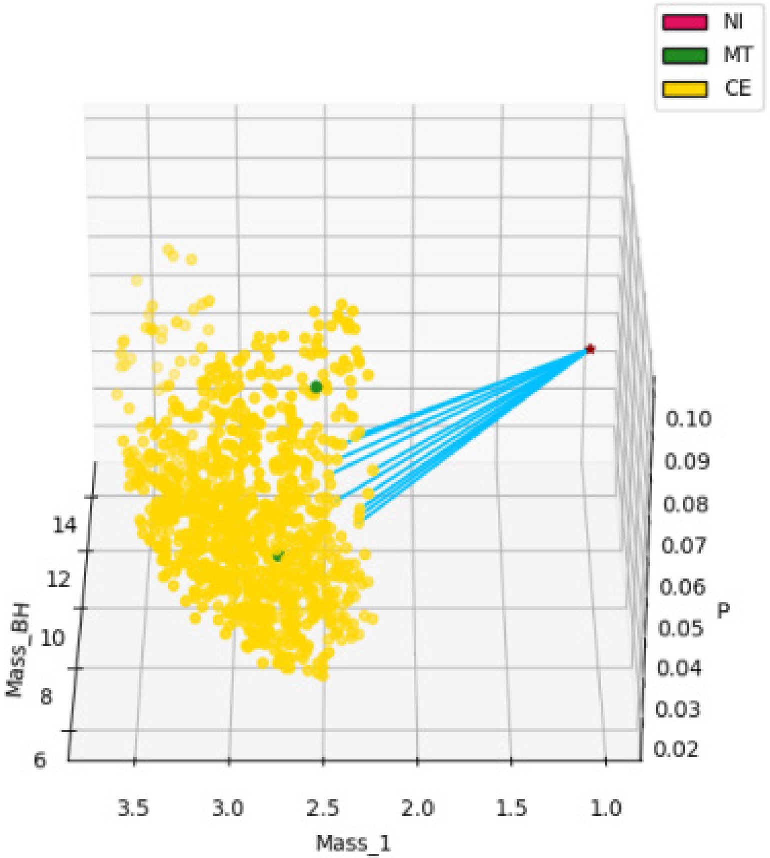
	Mass_0	Mass_1	Semimajor	Eccentricity	logP
0	22.343182	2.333571	2538.700000	2.439900e-01	5.224156
1	21.729472	2.333557	2633.953640	2.439899e-01	5.246389
2	21.725657	2.333557	2634.618130	2.439899e-01	5.246531
3	21.686325	2.333608	2345.928790	2.237674e-01	5.211639
4	17.379476	2.339919	1089.713271	8.820789e-02	4.422595
5	16.291458	2.339936	877.833962	8.819750e-02	4.206295
6	8.447135	2.339950	59.604493	0.000000e+00	3.094087
7	7.918371	2.339953	62.834400	4.882281e-02	3.139410
8	7.918571	2.337774	62.843623	4.882171e-02	3.139548
9	7.918572	2.337769	62.843637	4.882171e-02	3.139548
10	7.918663	2.337587	59.773876	0.000000e+00	3.106891
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

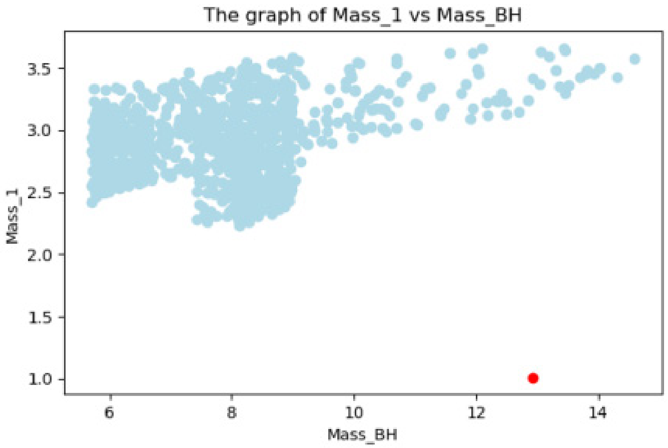
Index	Source Id	BH Mass	MS Mass	Period
15	1.84975e+18	12.92211	1.008930	0.05640179

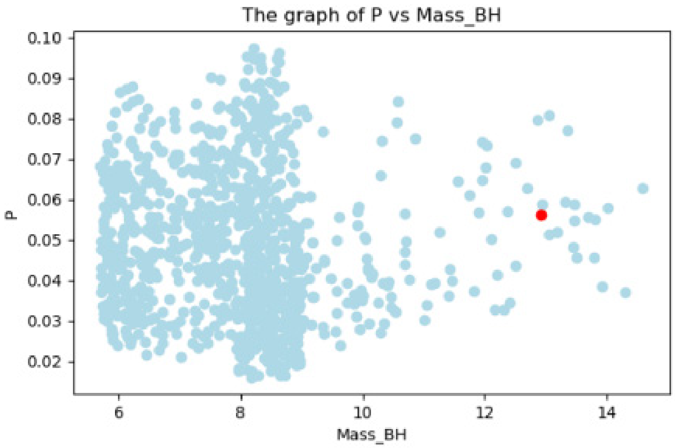
	Name	Mass_BH	Mass_1	P	z	alpha	type	value
0	0_941788181138103	8.131221	2.235984	0.059276	0.001	1.0	CE	0.573940
1	0_308481399850096	8.652071	2.331987	0.063940	0.001	1.0	CE	0.600773
2	0_251362050400014	7.595382	2.261225	0.057378	0.020	0.5	CE	0.601288

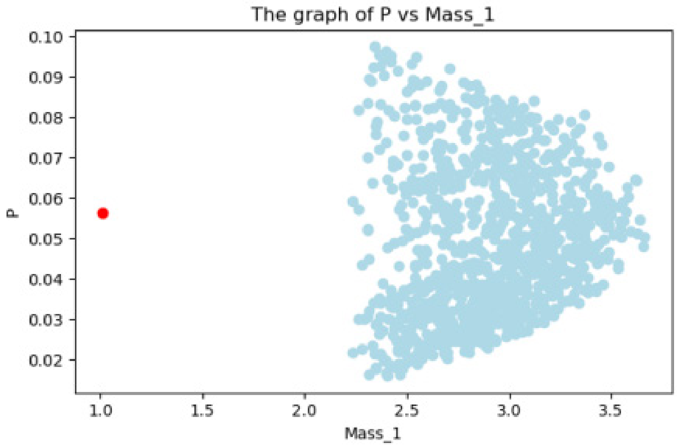



NI
MT
CE
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
P
6
8
10
12
14
Mass_BH
1.0
1.5
2.0
2.5
3.0
3.5
Mass_1

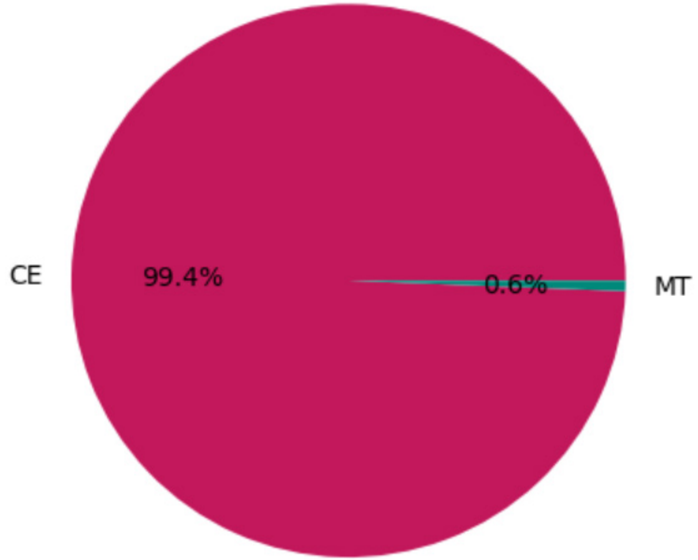

NI
MT
CE
14
12
10
8
6
Mass_BH
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
P
3.5
3.0
2.5
2.0
1.5
1.0
Mass_1


The graph of Mass_1 vs Mass_BH
Mass_1
Mass_BH
3.5
3.0
2.5
2.0
1.5
1.0
6
8
10
12
14


The graph of P vs Mass_BH
P
Mass_BH
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
6
8
10
12
14


The graph of P vs Mass_1
P
Mass_1
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
1.0
1.5
2.0
2.5
3.0
3.5

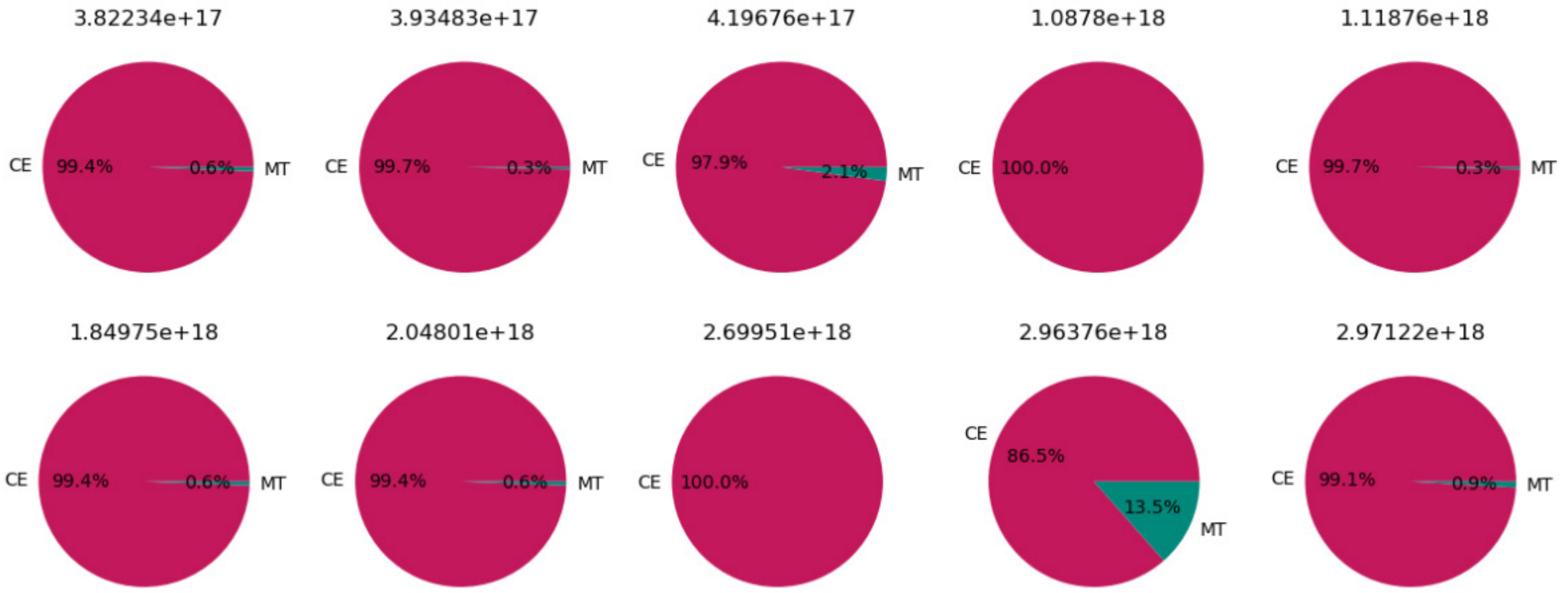
Percentage possibility of history type



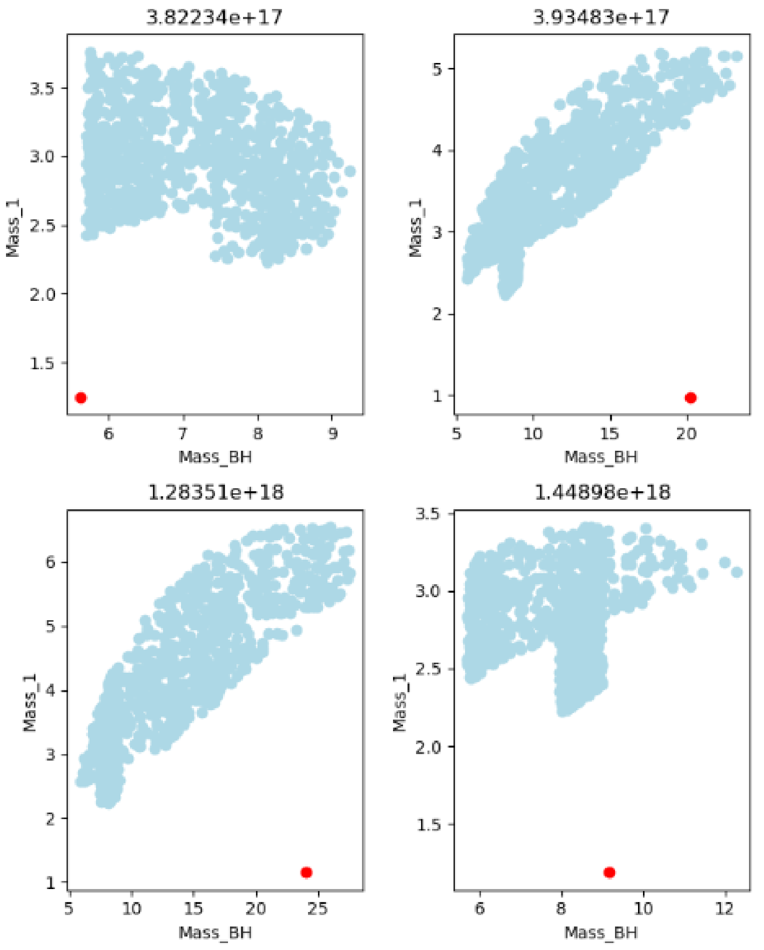
Predicted Evolution History

	Mass_0	Mass_1	Semimajor	Eccentricity	logP
0	22.777832	2.340671	2843.500000	2.328800e-01	5.338222
1	21.979220	2.340658	2962.345380	2.328799e-01	5.366404
2	21.973112	2.340658	2963.307780	2.328799e-01	5.366625
3	21.920654	2.340729	2677.593580	2.151050e-01	5.333032
4	16.240294	2.349304	1075.302656	8.352793e-02	4.331777
5	15.142083	2.349322	863.492979	8.351754e-02	4.116152
6	8.614823	2.349335	65.565025	0.000000e+00	3.152544
7	8.085015	2.349338	69.056809	4.798366e-02	3.197147
8	8.085213	2.347084	69.067184	4.798240e-02	3.197287
9	8.085214	2.347079	69.067200	4.798239e-02	3.197287
10	8.085324	2.346866	65.837928	0.000000e+00	3.165996
11	8.166855	1.900922	94.922475	0.000000e+00	3.406405

The percentage possibility of type for Real candidates : Gold



Why in almost all cases, the Mass_1 of the real candidate is significantly lower than the masses of similar systems in the SEVN simulation ?



Similar systems
Limitations

- **Initial Stellar Mass:** The model imposes a lower limit of **2.2 solar masses** for the initial stellar mass in binary systems

- **Individual System Variability**: Each binary system has its own unique characteristics and evolutionary path
- **External and Missing Factors**

- **Model Assumptions**: The methodology relies on the assumption that the selected similar systems from the SEVN simulation accurately represent the range of possible evolutionary paths for the real candidate system.
- **Unique Nature of Real Candidates**
- **Evolution History**: The lack of complete evolution history for real binary system

Similar systems
Future improvements

- **Enhanced Simulation Models** → better capture the complexities and variations of BSE

- **Use more parameters**