

Structural Analysis Toolbox Report

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1 Overview

The investigated system has

- 15 constraints $\mathcal{C} = [c_1, c_2, c_3, c_4, c_5, c_6, m_{13}, m_{14}, m_{15}, d_7, d_8, d_9, d_{10}, d_{11}, d_{12}]$,
- 5 known variables $\mathcal{K} = [u_1, u_2, y_1, y_2, y_3]$ and
- 13 unknown variables $\mathcal{X} = [\theta_1, d\theta_1, \omega_1, \text{d}\omega_1, \theta_2, d\theta_2, \omega_2, \text{d}\omega_2, \theta_3, d\theta_3, \omega_3, \text{d}\omega_3, d]$.

The constraints of the system are as follows:

c_1	$0 = d\theta_1 - \omega_1$
c_2	$0 = d - u_1(t) + k_1 \cdot (\theta_1 - \theta_2) + J_1 \cdot \text{d}\omega_1 + b_1 \cdot \omega_1$
c_3	$0 = d\theta_2 - \omega_2$
c_4	$0 = k_2 \cdot (\theta_2 - \theta_3) - k_1 \cdot (\theta_1 - \theta_2) - u_2(t) + J_2 \cdot \text{d}\omega_2 + b_2 \cdot \omega_2$
c_5	$0 = d\theta_3 - \omega_3$
c_6	$0 = J_3 \cdot \text{d}\omega_3 - k_2 \cdot (\theta_2 - \theta_3) + b_3 \cdot \omega_3$
m_{13}	$0 = y_1(t) - \theta_1$
m_{14}	$0 = y_2(t) - \theta_2$
m_{15}	$0 = y_3(t) - \theta_3$
d_7	$0 = d\theta_1 - \frac{\partial}{\partial t} \theta_1$
d_8	$0 = \text{d}\omega_1 - \frac{\partial}{\partial t} \omega_1$
d_9	$0 = d\theta_2 - \frac{\partial}{\partial t} \theta_2$
d_{10}	$0 = \text{d}\omega_2 - \frac{\partial}{\partial t} \omega_2$
d_{11}	$0 = d\theta_3 - \frac{\partial}{\partial t} \theta_3$
d_{12}	$0 = \text{d}\omega_3 - \frac{\partial}{\partial t} \omega_3$

The analysis obtained 1 matchings that yield in total 2 parity equations.

2 Canonical Decomposition

The system consists of

- the over-determined subsystem \mathcal{S}^+ with $\mathcal{C}^+ = [c_3, c_4, c_5, c_6, m_{13}, m_{14}, m_{15}, d_9, d_{10}, d_{11}, d_{12}]$ and $\mathcal{X}^+ = [\theta_1, \theta_2, d\theta_2, \omega_2, \text{d}\omega_2, \theta_3, d\theta_3, \omega_3, \text{d}\omega_3]$,
- the just-determined subsystem \mathcal{S}^0 with $\mathcal{C}^0 = [c_1, c_2, d_7, d_8]$ and $\mathcal{X}^+ = [d\theta_1, \omega_1, \text{d}\omega_1, d]$ and
- the under-determined subsystem \mathcal{S}^- with $\mathcal{C}^- = []$ and $\mathcal{X}^+ = []$.

3 Incidence Matrix

Table 2 presents the incidence matrix of the investigated system.

#	\mathcal{K}					\mathcal{X}												
	u_1	u_2	y_1	y_2	y_3	θ_{a_1}	$d\theta_{a_1}$	ω_{a_1}	$d\omega_{a_1}$	θ_{a_2}	$d\theta_{a_2}$	ω_{a_2}	$d\omega_{a_2}$	θ_{a_3}	$d\theta_{a_3}$	ω_{a_3}	$d\omega_{a_3}$	d
c_1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
c_2	1	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	1
c_3	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
c_4	0	1	0	0	0	1	0	0	0	1	0	1	1	1	0	0	0	0
c_5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
c_6	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1	0
m_{13}	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
m_{14}	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
m_{15}	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
d_7	0	0	0	0	0	X	1	0	0	0	0	0	0	0	0	0	0	0
d_8	0	0	0	0	0	0	0	X	1	0	0	0	0	0	0	0	0	0
d_9	0	0	0	0	0	0	0	0	0	X	1	0	0	0	0	0	0	0
d_{10}	0	0	0	0	0	0	0	0	0	0	0	X	1	0	0	0	0	0
d_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	X	1	0	0	0
d_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	1	0

Table 2: Incidence matrix of the investigated system.

4 Matchings

Table 3 lists the obtained matchings. The fields either contain the matched unknown variables, zeros to indicate an unmatched constraints or nothing if constraints are not used in a matching.

	c_1	c_2	c_3	c_4	c_5	c_6	m_{13}	m_{14}	m_{15}	d_7	d_8	d_9	d_{10}
1	ω_1	d	ω_2	ω_2	ω_3	ω_3	θ_1	θ_2	θ_3	$d\theta_1$	ω_1	$d\theta_2$	0

Table 3: Matchings of the investigated system.

5 Parity Equations

$$0 = \frac{u_2(t) - b_2 \cdot \frac{\partial}{\partial t} y_2(t) + k_1 \cdot y_1(t) - k_1 \cdot y_2(t) - k_2 \cdot y_2(t) + k_2 \cdot y_3(t)}{J_2} - \frac{\partial^2}{\partial t^2} y_2(t)$$

$$0 = \frac{k_2 \cdot (y_2(t) - y_3(t)) - b_3 \cdot \frac{\partial}{\partial t} y_3(t)}{J_3} - \frac{\partial^2}{\partial t^2} y_3(t)$$

6 Detectability and isolability analysis

Table 4 lists the detectability and isolability properties of the parity equations separately and over all combined. Detectable (d), isolable (i) and non-failable constraints (n) are marked accordingly.

	c_1	c_2	c_3	c_4	c_5	c_6	m_{13}	m_{14}	m_{15}	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}
1			d	d	d	d	d	d	d	n	n	n	n	n	n
ALL			d	d	d	d	d	d	d	n	n	n	n	n	n

Table 4: Detectability and isolability of the investigated system.