Analysing multivariate ecological data with Generalized Linear Latent Variable Models

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Welcome!

	Who	**Affiliation**	**Expertise**
	Bert van der Veen PhD candidate	Norwegian insitiute of Bioeconomy research & Norwegian univeristy of Science and Technology	- Statistical ecology - Ordination - Species distribution modeling
	Jenni Niku Postdoc	University of Jyväskylä	- Statistical ecology - Species distribution modeling
	Sam Perrin PhD candidate	Norwegian university of Science and Technology	- Fresh water ecology - Invasion ecology - Species distribution modeling
	Robert Brian O'Hara Professor	Norwegian University of Science and Technology	- Statistical ecology - Species distribution modeling - Data integration

Exercise material and package installation

https://github.com/BertvanderVeen/BES2020GLLVMworkshop

Questions

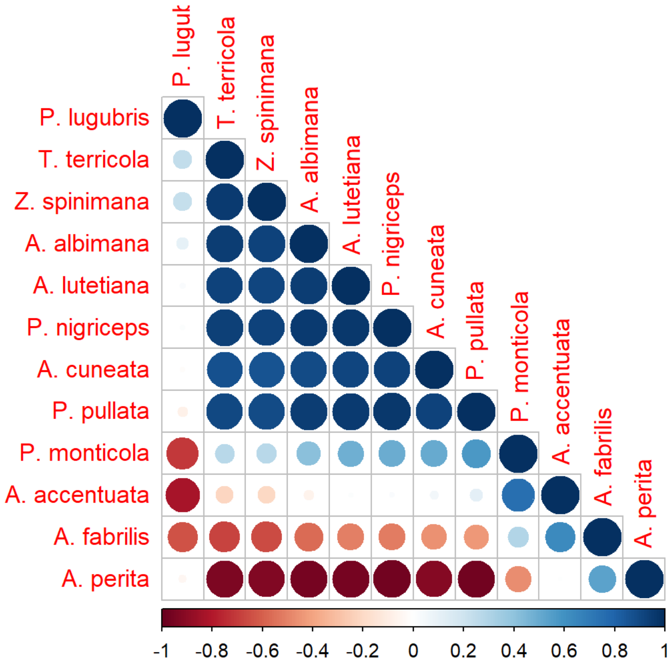
In zoom chat to **Bob** or

On twitter: #**GLLVMs**, @**vdVeenB** or @**J__Niku** or @**samperrinNTNU** or @**BobOHara**
On github: https://github.com/BertvanderVeen/BES2020GLLVMworkshop/discussions

Program day 1

Topic	Duration	Who
- Ecological gradients - Ordination - Generalized Linear Latent Variable models	20 minutes	
Break	5 minutes	
- `gllvm` R-package (Niku et al. 2019) - How to: model-based ordination with GLLVMs	20 minutes	
- Exercise: break out	10 minutes	
Break	5 minutes	
- Some ecology	20 minutes	
- Exercise: break out	15 minutes	
- Discuss results exercise		

Program day 2



Gathering data

We go out, register species at multiple sites

Gathering data

We go out, register species at multiple sites

"Multivariate"

- What does multivariate mean?
- Multivariate: multiple **responses**
- E.g. counts of species at sites

	Species 1	Species 2	Species 3	Species 4	Species 5
Site 1	25	10	0	0	0
Site 2	0	2	0	0	0
Site 3	15	20	2	2	0
Site 4	2	6	0	1	0
Site 5	1	20	0	2	0

"Multivariable"

- Multiple **predictors**
- E.g. measurements of the environment

	Predictor 1	Predictor 2	Predictor 3	Predictor 4	Predictor 5	Predictor 6
Site 1	2.3321	3.0445	0.0000	3.0445	4.4543	3.9120
Site 2	3.0493	3.2581	1.7918	1.0986	4.5643	1.6094
Site 3	2.5572	3.5835	0.0000	2.3979	4.6052	3.6889
Site 4	2.6741	4.5109	0.0000	2.3979	4.6151	2.9957
Site 5	3.0155	2.3979	0.0000	0.0000	4.6151	2.3026
Site 6	3.3810	3.4340	3.4340	2.3979	3.4340	0.6931

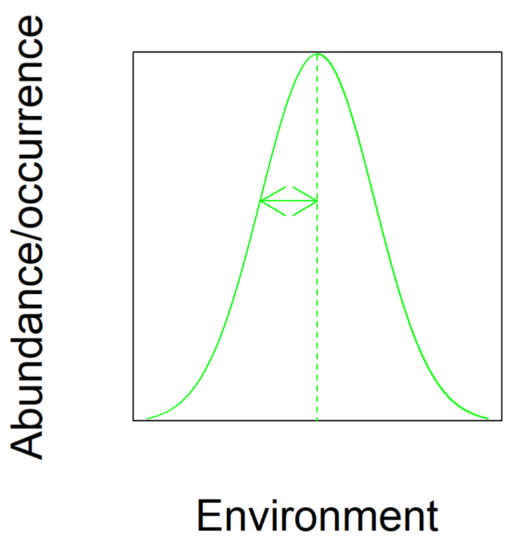
To clarify

- Both data and model can be univariate or multivariate
- Multivariate data can be analysed with both multivariate and univariate models (SDM, CA)
- Multivariable data can be used in multivariate or univariate analysis
  - Generally the same for all responses
  - (But, note that the model can of course set terms to zero)

Why analyse multivariate data?

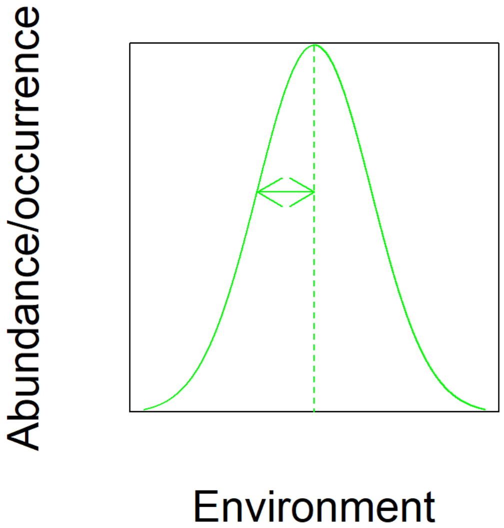
- Interest in **co**-occurrence patterns
  - In contrast to **occurrence** patterns (a species distribution)
- Why do species co-occur?
  - Similar environmental preferences
  - Similar history in the environment
  - Might results in **Interactions**
- Multiple species form a **community**

Occurrence pattern

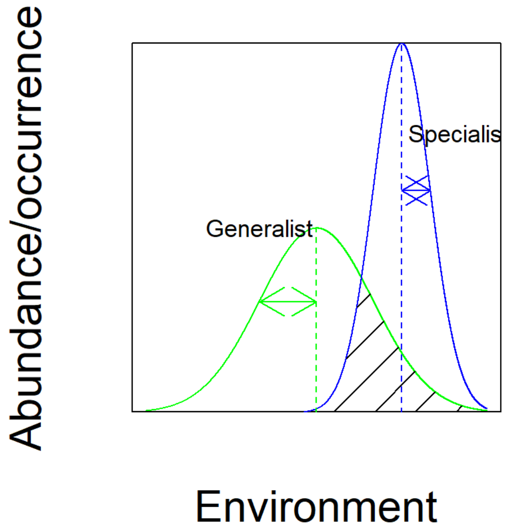


Why analyse multivariate data?

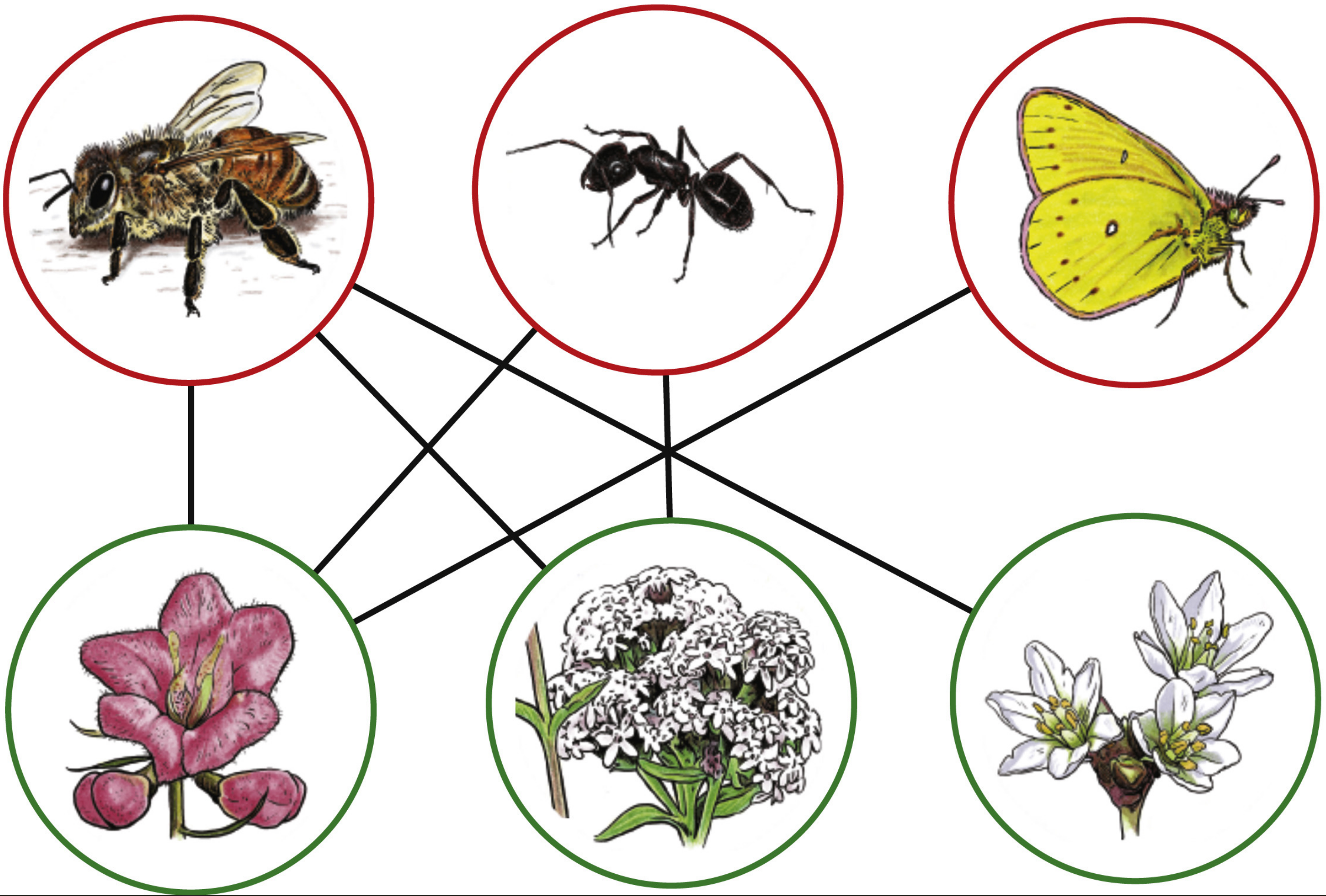
Occurrence pattern



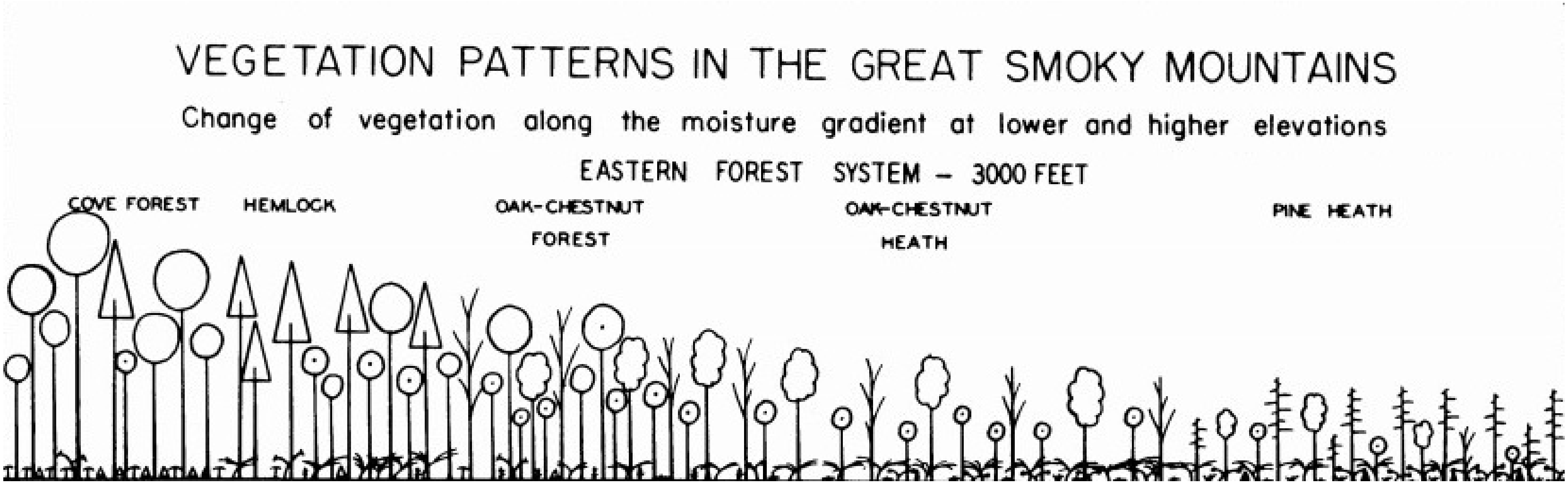
Co-occurrence pattern



But then for more species



Turnover along a gradient



(Whittaker, 1967)

Ecological gradient analysis

"Gradient analysis is a research approach for study of spatial patterns of species." **(Whittaker, 1967)**

Our sites describe the environment. Multiple gradients can form a **complex** gradient.

	Predictor 1	Predictor 2	Predictor 3	Predictor 4	Predictor 5
Site 1	**2.3321**	**3.0445**	0.0000	3.0445	4.4543
Site 2	**3.0493**	**3.2581**	1.7918	1.0986	4.5643
Site 3	**2.5572**	**3.5835**	0.0000	2.3979	4.6052
Site 4	**2.6741**	**4.5109**	0.0000	2.3979	4.6151
Site 5	**3.0155**	**2.3979**	0.0000	0.0000	4.6151

Ecological gradients

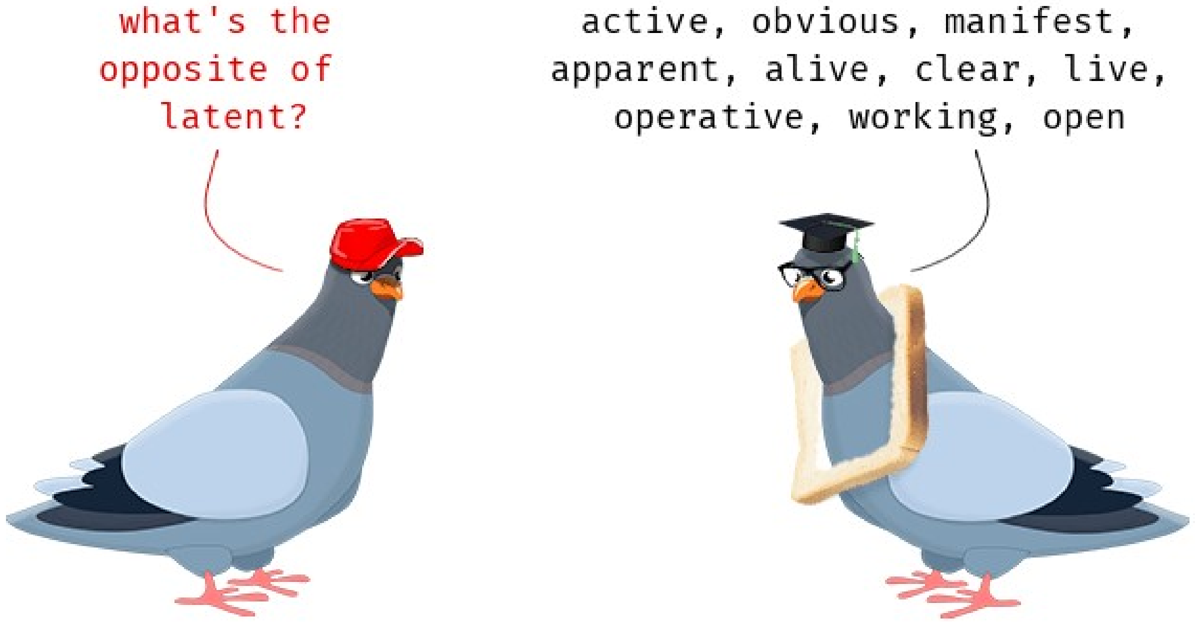
1) Ecological gradient: gradual change in the environment

- e.g. temperature

2) Complex gradient: change in several ecological gradients

- e.g. soil moisture and acidity on an elevation gradient
- Can be represented as a single factor, covariate, predictor, latent variable, ordination axis

Gradients can be **observed** or **latent**



Ecological gradients

"Few major complex ecological gradients normally account for most of the variation in species composition." **(Halvorsen, 2012)**

In essence:

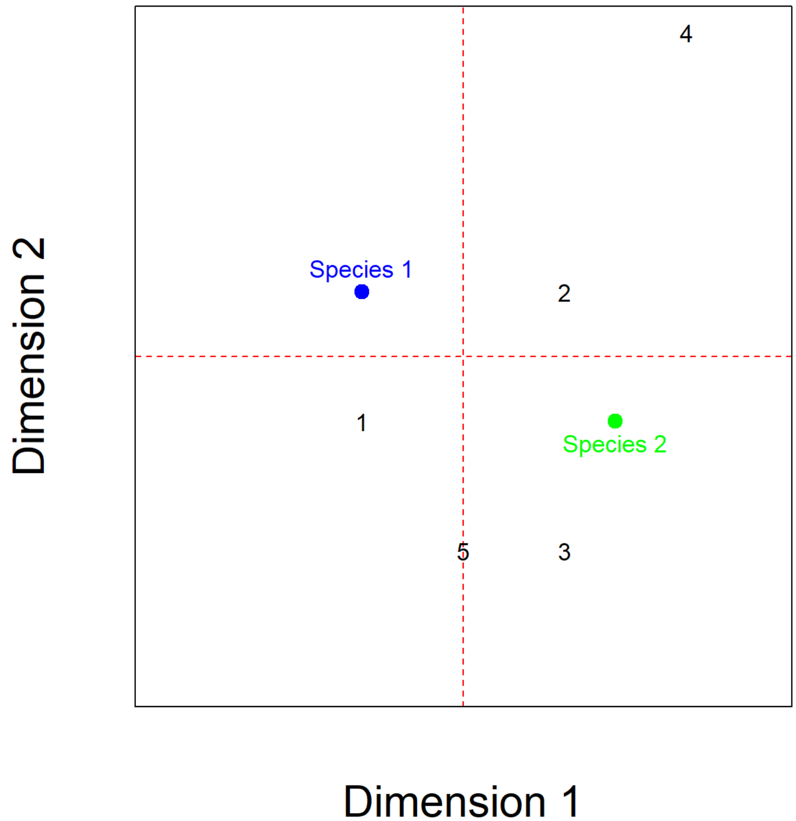
Community structure is generally low-dimensional.

Analysing multivariate data: ordination

- Termed by David Goodall (1954)
- Applied factor analysis to a community
- Reducing dimension of data
- ordering species or samples along an ecological gradient
- e.g.
    - Principal Component Analysis (PCA; `prcomp()`)
    - Correspondence Analysis (CA; `cca()` in **vegan**)
    - Multidimensional scaling (PCoA; `cmdscale()`, NMDS; `metaMDS()` in **vegan**)
    - **Factor analysis**: Precursor to GLLVMs (FA; `factanal()`)

Ordination: visual inspection

- Most common tool is the biplot Gabriel 1971
- Distance between species indicates dissimilarity
- Distance between sites indicates dissimilarity

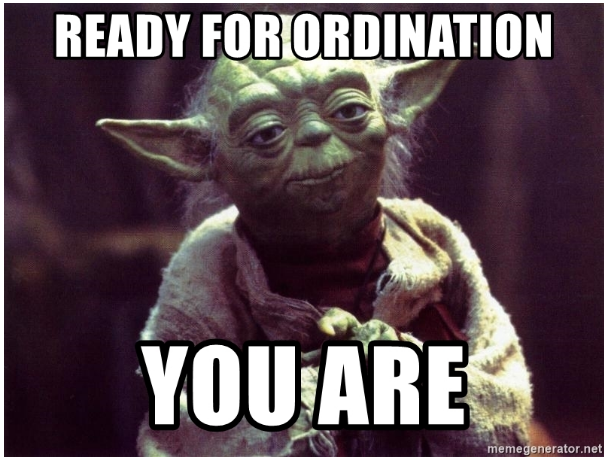


Classical methods have some issues..

- Ordination axis (ecological gradient) treated as fixed (parameter)
- Horshoe or arch effect (PCA, CA)
- Difficult (near to impossible) to check any assumptions
- Mean-variance relationships.. Warton and Hui 2017

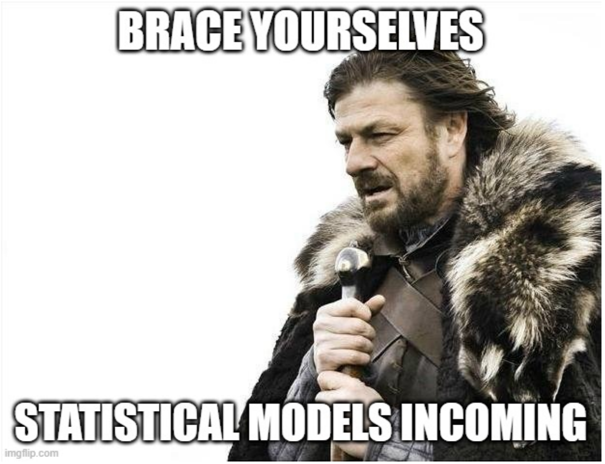
In general, not very flexible.

Questions so far?



Model-based thinking

- Concept: apply regression concepts to multivariate analysis Warton et al. 2015
    - Explicit statistical models
    - Residual diagnostics
    - Model selection
    - et cetera



Specifying a multivariate statistical model

- β_{0j} intercept per species
- X_{ik} site-specific covariates
- β_{0j} species-specific coefficients

$$g(\mathrm{E}(y_{ij})) = \beta_{0j} + \boldsymbol{X}_i^\top \boldsymbol{\beta}_j \tag{2}$$

- Stacked SDM or glm() function
- For example: `mvabund` Wang et al. 2012
- For observed gradients (predictors)
- Hypothesis testing for multivariate data

A Multivariate Mixed-effects model

- Add residual for $i = 1 \dots n$ sites and $j = 1 \dots p$ species
- Structure Σ by species
- A "joint species distribution model" Pollock et al. 2014
- Can fit using standard mixed-effects modeling software.

$$g(\mathrm{E}(y_{ij})) = \beta_{0j} + \boldsymbol{X}_i^\top \boldsymbol{\beta}_j + \epsilon_{ij}, \qquad \boldsymbol{\epsilon}_i \sim \mathcal{N}(0, \Sigma) \tag{3}$$

In `lme4`:

```
glmer(abundance~species+x:species+
(0+species|sites),family="poisson",data=data)
```

- Σ has $p(p+1)/2$ parameters (which increases quadratically with # species)

Model-based ordination to the rescue!

- Represent the latent complex ecological gradient
- A model like in regression

- "Model-based approaches to unconstrained ordination" Hui et al. 2015

All the benefits from regression and ordination!

e.g.

  - Procrustus analysis
  - Biplots
  - Model-selection
  - Residual diagnostics
  - Appropriate mean-variance relationships
  - No distance metrics
  - Hypothesis testing
  - etc.

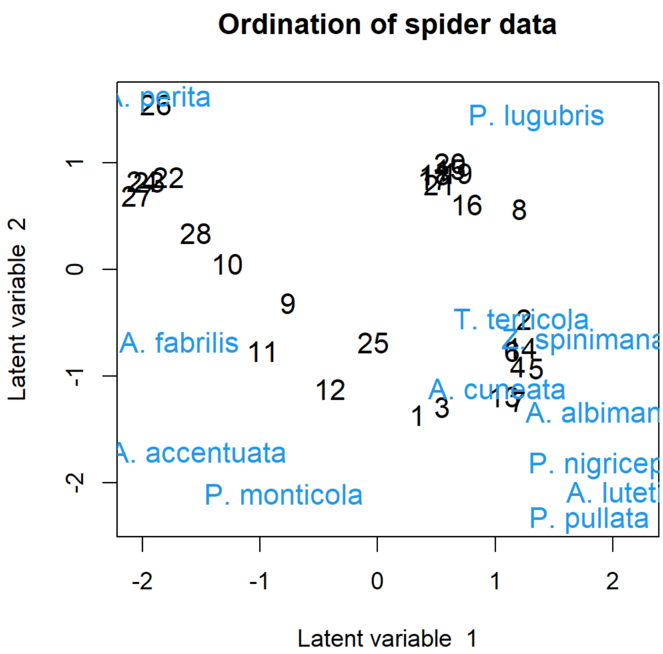
Generalized Linear Latent Variable Models

- GLLVM for short
- Add factor analytic structure to Σ
- Ordination = dimension reduction
- $\epsilon_{ij} = \boldsymbol{u}_i^\top \boldsymbol{\theta}_j$
  - i.e. $\boldsymbol{\epsilon}_i \sim \mathcal{N}(0, \boldsymbol{\theta}_j \boldsymbol{\theta}_j^\top)$
- Faster and fewer parameters:
  - Number of parameter doesn't grow so fast
  - More latent variables, better estimation of Σ

$$\Sigma = \begin{bmatrix} \theta_{11} & 0 & 0 \\ \theta_{12} & \theta_{22} & 0 \\ \vdots & \ddots & \vdots \\ \theta_{1j} & \cdots & \theta_{dj} \end{bmatrix} \begin{bmatrix} \theta_{11} & \theta_{12} & \cdots & \theta_{1j} \\ 0 & \theta_{22} & \ddots & \vdots \\ 0 & 0 & \cdots & \theta_{dj} \end{bmatrix} \tag{4}$$

Generalized Linear Latent Variable Models

- Still a mixed-effects model
- Low-rank approach to Σ
- d latent variables treated as random-effect
- Produces ordination
    - "site scores" : $\boldsymbol{u}_i$
    - "species scores" or "loadings": $\boldsymbol{\theta}_j$
    - No varimax



$$g(\mathrm{E}(y_{ij})) = \beta_{0j} + \boldsymbol{X}_i^\top \boldsymbol{\beta}_j + \boldsymbol{u}_i^\top \boldsymbol{\theta}_j, \qquad \boldsymbol{u}_i \sim \mathcal{N}(0, \boldsymbol{I}) \tag{5}$$

Compared to (unconstrained) classical ordination

- Principal Component Analysis is (essentially) a GLLVM with normal distribution
- Correspondence analysis
  - approximate GLLVM (with Poisson/binomial distribution) ter Braak 1985
  - With row-effect Hui et al. 2015
- Non-metric multidimensional scaling?
  - Relation is complicated (but produces similar results in practice)

- **GLLVMs are more flexible**
- **The statistical model can be extended**

More complex ordination?

e.g. spatially dependent sites

Methods in Ecology and Evolution

British Ecological Society

Methods in Ecology and Evolution 2016, **7**, 744–750 doi: 10.1111/2041-210X.12514

APPLICATION

BORAL – Bayesian Ordination and Regression Analysis of Multivariate Abundance Data in R

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