Generalised Linear (Mixed) Models for data of multiple species

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Species correlation

If we fit a GLM to data of multiple species, we assume **independence**

But, observations of the same species form groups. Co-occurring species have more similar observations than for other species

In GLMM language: observations of species exhibit correlation

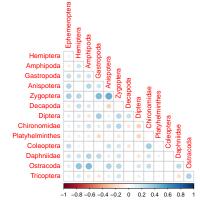
- Part of this can be explained by shared environmental responses
- 2) The other part remains

The previous model

If we look at the correlations in the residuals of this model, we can see that.

Correlated residuals

```
corrplot::corrplot(cor(residuals(model4)$resi), type = "lower", diag = FAL
```



If you have presence-absence data of a species, you fit a Species

$$y_{ij} = \alpha + \mathbf{x}_i^{\top} \boldsymbol{\beta} \tag{1}$$

 $ightharpoonup \mathbf{x}_i$ is usually a bioclimatic variable

Distribution Model of the form:

- Then you want to predict where a species may occur
- Potentially based on future climate scenarios
- But what if you have data on multiple species?
 - Fitting multiple SDMs requires independence

A decade ago, Joint Species Distribution Models (JSDM) were introduced to model binary data of multiple species

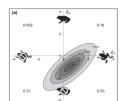
- Pollock et al. (2015): co-occurrence of frogs and trees
- ► Clark et al. (2015): co-occurrence of trees

The goal: to incorporate covariation of species

Species associations

What induces covariation between species?

- Shared environmental responses (abiotic conditions)
- Biotic interactions



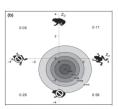
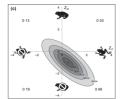


Figure 1: Pollock et al. (2015, fig. 1)



Interactions and co-occurrence

Ecology Letters

Ecology Letters, (2020) 23: 1050-1063

doi: 10.1111/ele.13525

IDEAS AND PERSPECTIVES

Co-occurrence is not evidence of ecological interactions

Interactions induce correlation, but we cannot infer interactions from (non-temporal) co-occurrence data.

 \mathbf{Y} community

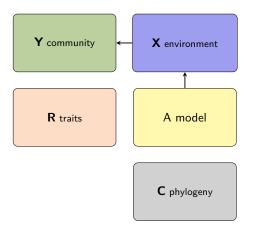
X environment

R traits

C phylogeny

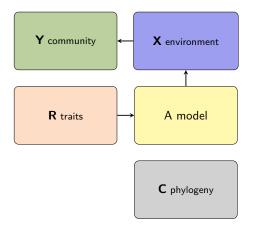
Summary

Typical questions in the framework

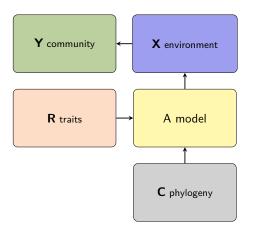


Q: How does the environment structure the community? environmental filtering

Typical questions in the framework



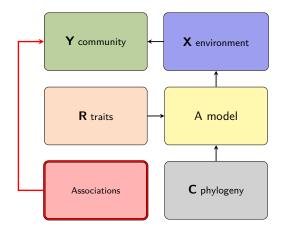
Q: How do traits affect species' responses to the environment? environmental filtering



Q: Do species with shared evolutionary history co-occur? (phylogenetic structuring) environmental filtering

Summary

Typical questions in the framework



Q: Do species co-occur **after** the environment has been considered? **biotic filtering**

Summary

Joint Species Distribution Modeling

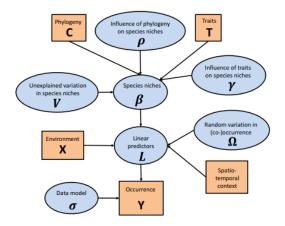


Figure 2: Figure from Ovaskainen et al. (2017)

Joint Species Distribution Modeling

The idea of JSDMs is to incorporate species associations

- Species may co-occur due to biotic interactions
- Due to similar environmental preferences
- Or because they have a similar history

Either how, it results in correlations between responses

Summary

Joint Species Distribution Model (JSDM)

- For community data, we want to incorporate correlation of species
- We have Multivariate data (in contrast to multivariable)

$$g\{\mathbb{E}(\mathbf{y}_i|\boldsymbol{\epsilon}_i)\} = \boldsymbol{\beta}_0 + \mathbf{x}_i^{\top} \boldsymbol{\beta} + \boldsymbol{\epsilon}_i$$
 (2)

- \triangleright we add ϵ_i relative to the VGLM(M)
- This random effect takes care of the left-over (co)variation of species
- ightharpoonup so we assume $\epsilon_i \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$
- \triangleright Σ is the matrix of species associations

JSDM: the model

$$\frac{\eta_{ij}}{\eta_{ij}} = \frac{\beta_{0j}}{\eta_{0j}} + \dots + \frac{\epsilon_{ij}}{\eta_{0j}}$$

$$\uparrow$$
(3)

- $\epsilon_i \sim \mathcal{N}(0, \Sigma)$
- $\triangleright \Sigma$ is the matrix of species associations
- So we expect a positive values of species co-occur, and negative if they do not

JSDM: it is a mixed-effects model

The JSDM is "just" a mixed-effects model. So we can fit it with available software:

In lme4:

```
glmer(abundance ~ species + x:species + (0+species sites), data = data)`
```

- ▶ There are p(p+1)/2 correlations between species
- This model becomes (very) large very quickly
- ▶ Will usually not fit
- So we need to do something smart!

Species associations

- Difficult to estimate: there are usually too many parameters
- Can only fit this way when there are (much) more sites than species
- The number of pairwise associations grows quadratically
 - 2 with 2 species, 6 for 4 species, 45 for 10 species, 4950 for 100

$$\Sigma = \begin{bmatrix} 1 & sp_{12} & \cdots & sp_{1j} \\ sp_{21} & 1 & \cdots & sp_{2j} \\ \vdots & & \ddots & \vdots \\ sp_{j1} & sp_{j2} & \cdots & 1 \end{bmatrix}$$
(4)

This very quickly becomes an issue for fitting models

Summary

Ordination to the rescue

- GLIVMs were introduced as a technical solution to this problem
- We represent the covariance matrix with fewer **dimensions**: $\Sigma \approx \Gamma \Gamma^{\top}$

"The factor analytic solution" because factor analysis (Spearman, 1904) is the precursor of GLLVMs

	Classic	JSDM		
Focus	Low-dimensional	Distributions		
	space			
Goal	Inference	Prediction Binary		
Data type	Usually quantitative			
Scale	Local	Regional		
Covariates	Environmental	Bioclimatic		
Presentation	Ordination diagram	Correlation plot/map		
Audience	Community ecologists	Macro ecologists		

JSDM vs classical multivariate analysis

	Classic	JSDM		
Focus	Low-dimensional	Distributions		
	space			
Goal	Inference	Prediction Binary		
Data type	Usually quantitative			
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Covariates	Environmental	Bioclimatic		
Presentation	Ordination diagram	Correlation plot/map		
Audience	Community ecologists	Macro ecologists		

That is not to say JSDMs cannot be used for non-binary data, for inference, or for local scales

JSDM software implementations

There are many!

JSDM software implementations

There are many!

- Boral (Bayesian, slow and somewhat outdated)
- sJSDM (Bayesian, relatively slow, but faster than Boral)
- ► Hmsc (Bayesian, generally slow, loads of functionality)
- ecoCopla (Frequentist, very fast but limited functionality)
- ► CBFM (Frequentist, geared towards spatio-temporal analysis)
- sjSDM (Frequentist, very fast but limited functionality, requires python)
- glmmTMB (Frequentist, fast and very versatile, not purpose-coded)
- allym (Frequentist fast and very versatile nurpose coded)

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Something practical

Presence-absence models in gllvm tend to be a bit finicky:

- The data has low information content: hard to find good starting values
- Try different starting values
- Try different approximation
- I.e., play around with the settings a little

Example with Eucalyptus data (Pollock et al. 2015)

```
Y <- read.csv("../data/eucalyptY.csv")[,-1]
X <- read.csv("../data/eucalyptX.csv")[,-1]
X <- data.frame(lapply(X, function(x))f(is.numeric(x)){scale(x)}else{as.factor(x)}))
knitr:kable(head(X), format="latex", booktabs = T)</pre>
```

0.7306972

MGS1 0.1972540 1.0236658 -1.1686010 0.3254470 -1.551741 1.0315338 -0.5939667 -0.407 MGS5 0.1972540 0.7778142 -1.1686010 0.3254470 -1.551741 1.4558834 -0.5939667 -0.407 MGS4 0.1757452 1.0236658 -0.8419304 -0.5684498 -1.551741 1.3144335 -0.5939667 -0.163 MGS3 0.1757452 0.4090369 -0.8419304 -0.5684498 -1.426354 0.4657344 -0.5939667 -0.476									
MGS5 0.1972540 0.7778142 -1.1686010 0.3254470 -1.551741 1.4558834 -0.5939667 -0.407 MGS4 0.1757452 1.0236658 -0.8419304 -0.5684498 -1.551741 1.3144335 -0.5939667 -0.163 MGS3 0.1757452 0.4090369 -0.8419304 -0.5684498 -1.426354 0.4657344 -0.5939667 -0.476	IDENT	elev	PerSlope	aspect	Northness	Sdclass	Rockiness	VallyBotFlat	topowe
	MGS5 MGS4	0.1972540 0.1757452	0.7778142 1.0236658	-1.1686010 -0.8419304	0.3254470 -0.5684498	-1.551741 -1.551741	1.4558834 1.3144335	-0.5939667 -0.5939667	-0.407019 -0.407019 -0.163936 -0.476998 -1.003677

0.0144213

-1.426354

1.5973333

-0.5939667

-0.889502

▶ 20 species

0.6381843

▶ 458 sites

MGN1

Soil covariates and a few bioclimatic

1.3924432

Eucalyptus: fit a model

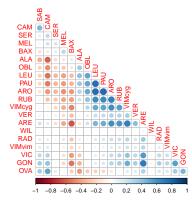
```
jsdm1 <- gllvm::gllvm(Y, X = X, formula = ~ Sandiness + cvTemp,
  family = "binomial", num.lv = 2, method = "EVA", starting.val = "zero")</pre>
```

- method: controls the approximation (LA, VA (default), EVA)
- starting.val: how to generate initial values (multimodality)
- n.init: rerun multiple times

The gllvm R-package is fast, but might can take a minute. Models should be refitted with 'n.init'.

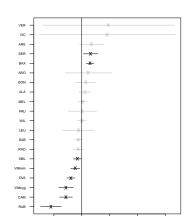
Eucalyptus: residual assocations

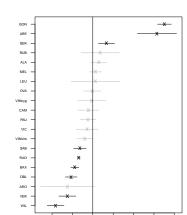
```
corrplot::corrplot(gllvm::getResidualCor(jsdm1), order = "AOE", type = "lower", diag = FALSE)
```



Eucalyptus: environment effects

gllvm::coefplot(jsdm1)





Eucalyptus: explained variation

[1] 0.6172353

Eucalyptus: with random effects

Many random effects can take long. Speed up with:

- sd.errors = FALSE (turn off standard error calculation)
- ▶ Ab.struct = "diagonal" (simplify the approximation)
- TMB::openmp(parallel::detectCores()-1, autopar = TRUE, DLL =
 "gllvm") (parallel computation)
- optim.method = "L-BFGS-B"

Eucalyptus: with random effects

Formula: ~(0 + Sandiness + cvTemp | 1)

```
summary(jsdm3)
```

Informed LVs: 0
Constrained LVs: 0
Unconstrained LVs: 2

Call:

##

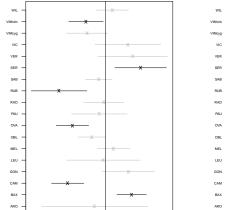
##

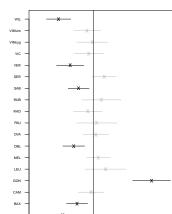
```
## gllvm::gllvm(y = Y, X = X, formula = ~(0 + Sandiness + cvTemp |
## 1), family = "binomial", num.lv = 2, method = "EVA", starting.val =
##
## Family: binomial
##
```

AIC: 4486.426 AICc: 4487.341 BIC: 4942.273 LL: -2179.2 df: 64

Eucalyptus: with random effects

gllvm::randomCoefplot(jsdm3)





Fourth corner analysis

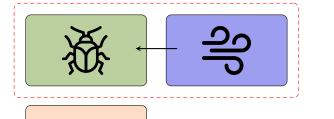


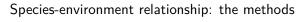




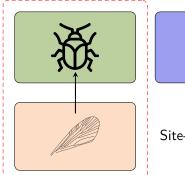
- Y: community data
- X: environmental variables
- TR: species traits

Fourth corner analysis





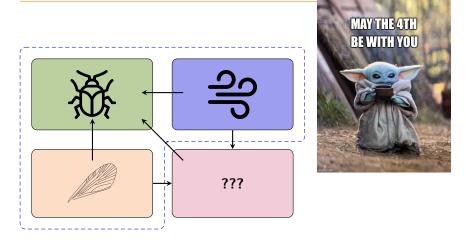
Fourth corner analysis





Site-trait relationship: VGLM(M)

Fourth corner analysis



Trait-based analysis

- 1. CWM + RDA Doledec et al. (1996)
- Double constrained ordination. Lebreton et al. (1988), ter Braak et al. (2018)
- 3. Fourth corner (LV) Models Brown et al. (2014), Ovaskainen at al. (2017), Niku et al. (2021)

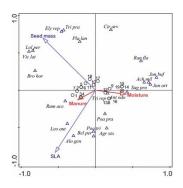


Figure 3: Quadriplot ter Braak et al. (2018)

Fourth-corner LVMs

A kind of LVM (JSDM) that also includes traits

Received: 19 February 2020 Revised: 6 April 2021 Accepted: 9 April 2021

DOI: 10.1002/env.2683

SPECIAL ISSUE PAPER

WILEY

Summary

Analyzing environmental-trait interactions in ecological communities with fourth-corner latent variable models

Jenni Niku¹ | Francis K. C. Hui² | Sara Taskinen¹ | David I. Warton³

Fourth-corner LVMs

The model is very similar to before:

$$\eta_{ij} = \beta_{0j} + \mathbf{x}_i^{\top} \boldsymbol{\beta}_j + \epsilon_{ij}$$
 (5)

Fourth-corner LVMs

The model is very similar to before:

$$\eta_{ij} = \beta_{0j} + \mathbf{x}_i^{\top} \boldsymbol{\beta}_j + \epsilon_{ij}$$
 (5)

but now, we are hierarchically modelling species' effects β_i

Traits (6) $\Rightarrow \beta_x$ species-common effects $lackbrack \mathbf{b}_i$ species-specific effects ϵ B_{rtr} 4th-corner coefficients←

Fourth-corner IVMs

$$\boldsymbol{\beta}_j = \boldsymbol{\beta}_x + \mathbf{B}_{xtr} \mathbf{tr}_j + \mathbf{b}_j \tag{7}$$

We are modeling our species coefficients of covariates

- \triangleright With an species-common effect: β_r
- \triangleright Slopes for traits \mathbf{B}_{rtr} : these describe how our species' environment responses depend on traits
 - Can be used nicely for hypothesis testing
- \triangleright An error term \mathbf{b}_i to capture everything left

Fourth-corner IVMs

- With a 4th corner LVM we can examine trait-environment relationships
- Figure out why species prefer particular conditions In contrast to "just" which conditions they prefer
- While still incorporating other reasons for co-occurrence

TR <- read.csv("../data/eucalyptTR.csv");row.names(TR) <- TR\$TAXON

The formula formulation follows the equation: the first two terms are β_x , the next terms represent \mathbf{B}_{xtr} , and b_{ki} last

Estimate Std. Error z value Pr(>|z|)

0.02195 0.06392 0.343 0.7313

0.02825 0.13767 0.205 0.8374

0.08374 -1.054

0.08903 -2.464

1.126 0.2604

0.17758 -0.452 0.6516

0.14613 -1.723 0.0848 .

0.2919

0.0137 *

-0.08825

-0.08018

-0.25183

0.07646 0.06793

Variance Std.Dev Corr

0.3502 0.5918 0.0000

Sandiness: MedianSeedMass.mg. -0.21936

Sandiness 0.0575 0.2398 cvTemp

Coefficients predictors:

Sandiness: MedianSLA

cvTemp:MaxHeight.m.

cvTemp:MedianSLA

Sandiness:MaxHeight.m.

Sandiness

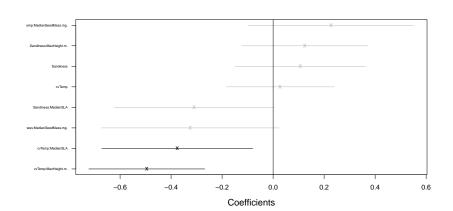
cvTemp

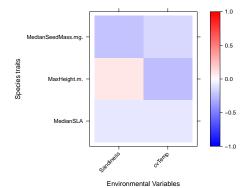
summary(jsdm4)

Call:

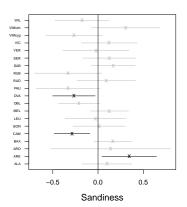
```
## gllvm::gllvm(y = Y, X = X, TR = TR, formula = ~Sandiness + cvTemp +
       (Sandiness + cvTemp): (MedianSLA + MaxHeight.m. + MedianSeedMass.mg.),
       family = "binomial", method = "EVA", randomX = ~Sandiness +
          cvTemp. starting.val = "zero")
## Family: binomial
## AIC: 4401.794 AICc: 4402.888 BIC: 4900.377 LL: -2130.9 df: 70
## Informed LVs: 0
## Constrained LVs: 0
## Unconstrained LVs: 2
## Formula: -Sandiness+cvTemp+Sandiness:MedianSLA+Sandiness:MaxHeight.m.+Sandiness:MedianSeedMass.mg.+cvTemp:MedianSLA+cvTemp:MaxHeight.m.+cvTemp:MedianSeedMass.mg
## LV formula: ~ 0
## Row effect: ~ 1
## Random effects:
```

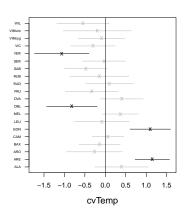
gllvm::coefplot(jsdm4)

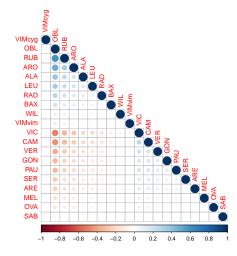




gllvm::randomCoefplot(jsdm4)







Example with Eucalyptus data: hypothesis testing

```
## Model 1 : v ~ NULL
```

```
## Model 2: ~ Sandiness + cvTemp + (Sandiness + cvTemp):(MedianSLA + Ma

## Resid.Df D Df.diff P.value
```

```
## 1 9096 0.00000 0
## 2 9090 96.63163 6 0
```

anova(jsdm4, jsdm3)

We accept the alternative hypothesis: species responses to the environment are structured by traits

So far, we have been discussing **unstructured** species associations But what if we have information to provide?

- \blacktriangleright We can structure species random effects \mathbf{b}_i by Phylogeny
- We assume that more closely related species have similar responses to the environment

The Phylogeny provides more information and makes for more accurate estimation

(and we can predict for species without data)

Here I will omit traits and LVs for brevity. So our model is:

$$\boldsymbol{\eta} = \mathbf{1}\beta_{0j}^{\top} + \mathbf{X}\mathbf{B} \tag{8}$$

- **B** are the random effects for covariates
- $lackbox{ We assume } \mathbf{B} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_m \otimes \mathbf{\Sigma}_r)$
- \sum_{r} covariance matrix of random effects
- $\triangleright \; \Sigma_m$ correlation matrix due to phylogeny

We assume that all our random effects are structured by the Phylogeny

$$\Sigma_m = \mathbf{C}\rho + (1 - \rho)\mathbf{I} \tag{9}$$

- 2) $0 \le \rho \le 1$ is Pagel's λ : the Phylogenetic signal parameter

$$\Sigma_m = \mathbf{C}\rho + (1 - \rho)\mathbf{I} \tag{9}$$

- 1) **C** is a correlation matrix due to the Phylogeny (ape::vcv(., corr = TRUE))
- 2) $0 \le \rho \le 1$ is Pagel's λ : the Phylogenetic signal parameter

This model only generates positive species associations.

Phylogenetic signal

- ▶ 1: Fully phylogenetically structured responses
- 0: Normal ("iid") random effects

When it is 0, it does not mean there is nothing going on.

Absence of phylogenetic signal:

- Scale mismatch
- Evolution moves very fast
- Too little information
- Traits are phylogenetically structured
- There are other (flexible) terms in the model
- Model misspecification

Presence of phylogenetic signal:

- ▶ Related species have similar "traits" (environmental response)
- Occupy similar environments

Model limitation

This phylogenetic model assumes traits evolve following the Brownian motion model of evolution. This can only generate positive associations.

But, competitive exclusion tells us that species evolve to differentiate resource.

- Similar species can (stably) co-occur if they utilize a different resource
- Similar species that utilize the same resource should not (stably) co-occur

The latter results in negative correlations, but no corresponding model for trait evolution has been developed

unless species do not stably co-occur and/or evolution is still ongoing

Example with fungi data (Abrego 2021)

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DOI: 10.1111/1365-2745.13839

RESEARCH ARTICLE

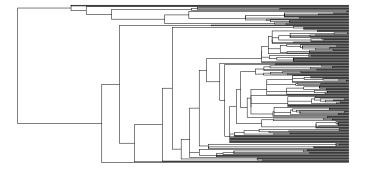


Traits and phylogenies modulate the environmental responses of wood-inhabiting fungal communities across spatial scales

Nerea Abrego^{1,2} | Claus Bässler^{3,4} | Morten Christensen⁵ | Jacob Heilmann-Clausen⁶ o

```
Y = read.csv("../data/fungiY.csv",)[,-1]
X = read.csv("../data/fungiX.csv")[,-1]
tree = ape::read.tree("../data/fungiTree.txt")
```

- 215 species (after cleaning)
- ▶ 1666 sites
- ▶ 19 covariates of various kinds



Phylogenetic models in ${\tt gllvm}$ use a **nearest neighbour** approximation

- We need to set the number of tips to consider on the tree
- ► The ordering of species matters!

```
covMat <- ape::vcv(tree)
e <- eigen(covMat)
distMat <- ape::cophenetic.phylo(tree)
ord <- gllvm:::findOrder(covMat = covMat, distMat = distMat,
species <- colnames(covMat)[ord]
Y <- Y[, species]
covMat <- covMat[species, species]
distMat <- distMat[species, species]</pre>
```

Summary

Ordering species

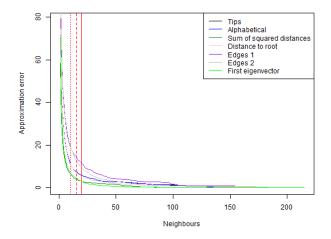


Figure 4: See vignette 7

Summary

Example with fungi data

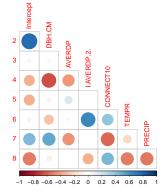
```
TMB::openmp(parallel::detectCores()-1, autopar = TRUE, DLL = "gllvm")
jsdm5 <- gllvm::gllvm(y = Y, X=X, family = "binomial", num.lv = 0, betaOcom = TRUE,
row.eff = -(1 | REGION/RESERVE), studyDesign = X[,c("REGION", "RESERVE")],
formula = -(DBH.CM+AVERDP+I(AVERDP-2)+CONNECT10+TEMPR+PRECIP+log.AREA|1),
colMat = list(covMat, dist = distMat), nn.colMat = 15, max.iter = 10e3, optim.method = "L-BFGS-B")</pre>
```

This takes a while to fit, but is really incredibly fast (it is just a complex model)

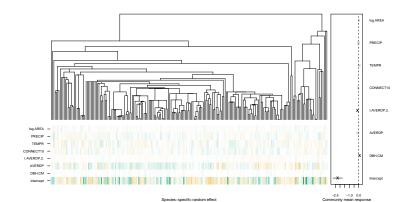
```
summary(jsdm5)
```

```
## Call:
## gllvm::gllvm(y = Y, X = X, formula = ~(DBH.CM + AVERDP + I(AVERDP^2) +
      CONNECTIO + TEMPR + PRECIP + log.AREA | 1), family = "binomial",
      num.lv = 0, studyDesign = X[, c("REGION", "RESERVE")], colMat = list(covMat,
          dist = distMat), row.eff = -(1 | REGION/RESERVE), beta0com = TRUE,
      nn.colMat = 15, max.iter = 10000, optim.method = "L-BFGS-B")
## Family: binomial
## AIC: 103171.8 AICc: 103171.8 BIC: 103678.9 LL: -51539 df: 47
## Informed LVs: 0
## Constrained LVs: 0
## Unconstrained LVs: 0
## Formula: -(DBH.CM + AVERDP + I(AVERDP^2) + CONNECTIO + TEMPR + PRECIP + log.AREA | 1)
## I.V formula: ~ 0
## Row effect: ~(1 | REGION/RESERVE)
## Random effects:
              Signal Variance Std.Dev Corr
   Intercept 0.6037 1.0495 1.0244
   DBH.CM
              0.6037 0.0051 0.0715 0.7642
  AVERDP
              0.6037 0.1796 0.4238 0.0529 0.0458
   I.AVERDP.2. 0.6037 0.0066 0.0815 -0.3550 -0.6454 -0.4397
   CONNECT10 0.6037 0.0401 0.2003 -0.3544 -0.0711 0.1790 -0.0091
  TEMPR
              PRECIP
              0.6037 0.0440 0.2098 0.4461 0.5139 -0.4465 0.0148 -0.5701
   log.AREA 0.6037 0.0140 0.1184 -0.5196 -0.0889 0.0173 -0.3518 0.4538
```

corrplot::corrplot(cov2cor(jsdm5\$params\$sigmaB), type = "lower", diag = FA



gllvm::phyloplot(jsdm5, tree)



There is phylogenetic structuring; species environmental responses are more similar if they have a shared evolutionary history.

Of course, this might be covariate dependent (colMat.rho.struct = "term")

- ▶ JSDMs is a framework for analysing species co-occurrence data
- Focussed on prediction, but also suitable for inference
- We can also fit models with non-binary data (e.g., counts or biomass)
- The GLLVM framework is used here to implement JSDM efficiently
- We can incorporate random effects
- Phylogenetically structure species' effects
- Above all: we incorporate correlation of species