University of Illinois Fall 2022

ECE 210/211: Exam 2

Wednesday, October 19, 2022

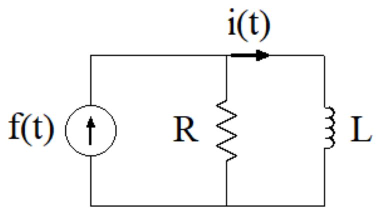
Name: (in BLOCK CAPITALS) ____

UIN: ______________________ **NetID:** ______________________

Instructions

- Clearly PRINT your name in CAPITAL LETTERS.
- Cleary write your UIN and netID.
- This is a closed book and closed notes exam.
- Calculators and other electronic devices are not allowed.
- To get credit, you must SHOW ALL your work and provide explanations when needed.
- To get full credit, simplify your answers.
- All answers must INCLUDE UNITS whenever appropriate.
- Angles must be expressed in the range $(-\pi, \pi]$ rad.
- Write your final answers in the spaces provided or points may be deducted.
- The exam is printed double-sided.
- There is an empty page at the end of the exam in case you need additional space.

1. **[25 points]** Consider the following circuit for $t > 0$.



(a) [5 pts] Obtain the ODE that governs this system for $t > 0$ in terms of R, L, $i(t)$ and $f(t)$ only (no other variables).

ODE = ____________________

For the rest of the problem, let $R = 8\ \Omega$, $f(t) = \frac{1}{2}\cos(\omega t) + 2\sin(\omega t)$ A and $i(0^-) = -1$ A. It is known that $i(t) = Be^{-4t} - \frac{2}{5}\cos(2t) + \frac{9}{5}\sin(2t)$ A for $t > 0$.

(b) [5 pts] Determine the values of L and ω.

L = ____________ ω = ____________

Recall that $R = 8\ \Omega$, $f(t) = \frac{1}{2}\cos(\omega t) + 2\sin(\omega t)$ A and $i\left(0^{-}\right) = -1$ A. It is known that for $t > 0$, $i(t) = Be^{-4t} - \frac{2}{5}\cos(2t) + \frac{9}{5}\sin(2t)$ A.

(c) [5 pts] If $i(t)$ is the zero-state response, what is the value of B?

B = ____________

(d) [5 pts] If $B = -\frac{3}{5}$ when $i(t)$ is the full solution of the ODE, determine $i_{tr}(t)$, the transient component of $i(t)$ and also determine $i_{ss}(t)$, the steady-state component of $i(t)$.

$i_{tr}(t)$ = ____________ $i_{ss}(t)$ = ____________

(e) [5 pts] Determine the steady-state phasor I.

I = ____________

2. **[12 points]** Let a dissipative LTI system have frequency response $H(\omega) = \frac{Y}{F} = \frac{j\omega}{1-\omega^2+j\omega}$ where F and Y are the phasors of the input $f(t)$ and output $y(t)$, respectively.

(a) [3 pts] Determine $|H(\omega)|$.

$$|H(w)| = \frac{|w|}{\sqrt{w^4 - w^2 + 1}}$$

$w^2 + 1 - 2w^2 + w^4$

$$\frac{2j}{-3+2j}$$

$|H(\omega)| =$ ____________________

(b) [9 pts] Let $f(t) = 2 + \cos(t + \frac{\pi}{4}) + 2\sin(2t)$. Determine the steady-state output, $y_{ss}(t)$.

$$\frac{j}{j}$$

for 2 $w=0$ $Y=0$

for $\cos(t+\frac{\pi}{4})$ $F(1) = \angle 45°$ $Y(1) = \angle 45°$

for $2\sin(2t)$ $F(2) = 2$ $Y(2) = \frac{4j}{-3+2j}$

=

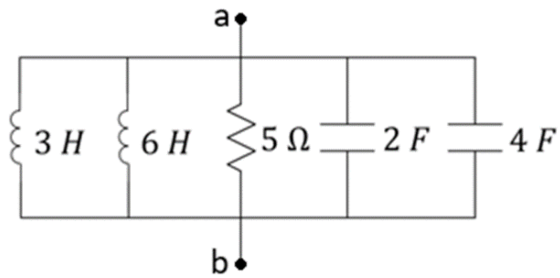
$Y(2)$ $\frac{4}{\sqrt{13}} \cdot \angle 90 + \arctan \frac{2}{3})$

$Y_{ss}(t) = \cos(t + 45°)$

$+ \frac{4}{\sqrt{13}} \cos(2t + 90 + \arctan \frac{2}{3})$

$y_{ss}(t) =$ ____________________

3. **[13 points]** Consider the circuit shown below.



(a) [3 pts] What is the impedance of just the two inductors (3 H and 6 H) in parallel as a function of ω?

$Z_{L,eq} =$ ____________________

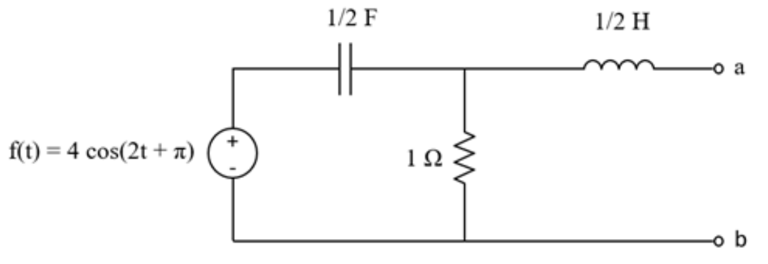
(b) [3 pts] What is the impedance of just the two capacitors (2 F and 4 F) in parallel as a function of ω?

$Z_{C,eq} =$ ____________________

(c) [7 pts] At what frequency $\omega > 0$ does the circuit shown above have a real-valued equivalent impedance between nodes **a** and **b**?

$\omega =$ __________

4. **[18 points]** Consider the circuit below.

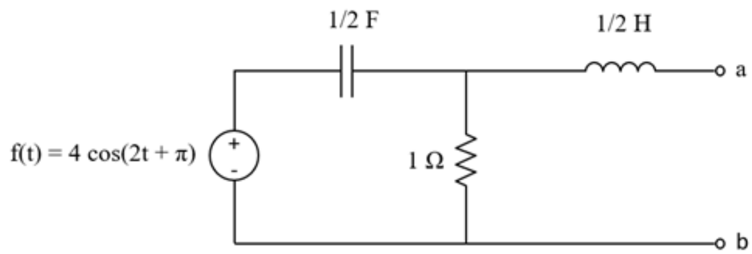


(a) [5 pts] Determine the voltage phasor across the capacitor, and identify its magnitude and phase.

$|V_C| =$ ______________________ $\angle V_C =$ ______________________

(b) [3 pts] Determine the average absorbed power in the capacitor, P_C.

$P_C =$ ______________________



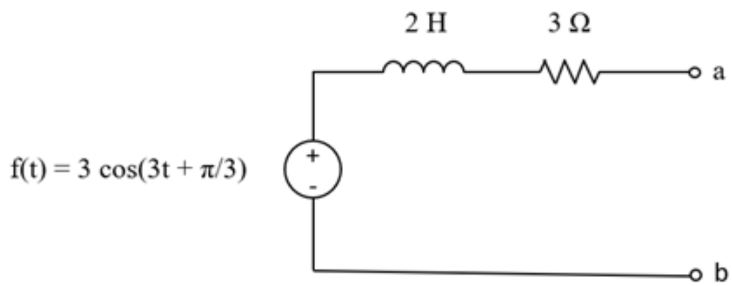
(c) [5 pts] Determine the Thevenin's equivalent voltage phasor of this circuit as seen from terminals a and b, and identify its magnitude and phase.

$|V_T| =$ ______________________ $\angle V_T =$ ______________________

(d) [5 pts] Determine the Thevenin's equivalent impedance, express it in rectangular form, and simplify it as much as you can.

$Z_T =$ ______________________

5. **[7 points]** Consider the circuit below.



(a) [5 pts] Determine the maximum average absorbed power that the circuit may deliver to a load.

$P_{max} =$ ______________________

(b) [2 pts] Determine the maximum average absorbed power that the circuit may deliver to a load if the inductor is replaced by a capacitor having capacitance $C = 5$ F.

$P_{max} =$ ______________________

6. **[6 points]** Given the periodic function $f(t) = 4\sin^2(3t - \frac{\pi}{3})$, write out all of the non-zero Fourier series coefficients F_n that describe its Fourier expansion in exponential form.
Hint: $\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$.

$\omega = 6 \quad T = \frac{2\pi}{6} = \frac{\pi}{3}$

$$F_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega t}\, dt$$

$$= \frac{3}{2\pi}\int_0^{\frac{\pi}{3}} 4\sin^2(3t - \tfrac{\pi}{3})\, e^{-3jnt}\, dt \qquad e^{6jt} e^{-\frac{\pi}{6}j}$$

$2 - 2\cos(6t - \frac{2\pi}{3})$

$$= \frac{3}{2\pi}\left(\int_0^{\frac{\pi}{3}} 2e^{-3jnt}\,dt + \int_0^{\frac{2\pi}{3}} -2\,\cancel{\cos(6t - \tfrac{2\pi}{3})}\, dt\right)$$

$\sin(6t - \frac{\pi}{6})\, e^{-3jnt}$

$\frac{1 - \cos(6t - \frac{2\pi}{3})}{2} \cdot 4$

$$= \frac{3}{2\pi}\left(-\frac{2}{3jn} e^{-3jnt}\Big|_0^{\frac{\pi}{3}} + \int_0^{\frac{2\pi}{3}} \cancel{4}j \cdot \frac{e^{j(6t - \frac{\pi}{6})} - e^{j(-6t + \frac{\pi}{6})}}{\cancel{2}j} e^{-3jnt}\, dt\right.$$

$\frac{2\pi}{3} - 6t$

$$= \frac{3}{2\pi}\left(-\frac{2}{3jn} e^{-\pi jn} + \frac{2}{3jn} + j\left(\frac{e^{-\frac{\pi}{6}j}}{6j}(e^{4\pi j} - 1) + \frac{e^{\frac{\pi}{6}j}}{6j}(e^{-4\pi j} - 1)\right)\right)$$

$\frac{\pi}{6} + \frac{\pi}{2} - 6t$

$\frac{\pi}{2} - (6t - \frac{\pi}{6})$

$$= \frac{3}{2\pi}\left(j\left(e^{-\frac{\pi}{6}j}\left(e^{(6j - 3jn)t}\right) - e^{\frac{\pi}{6}j} e^{-(6j + 3jn)t}\right) dt\right.$$

$$= \frac{3}{2\pi}\left(j\left(\frac{e^{-\frac{\pi}{6}j}}{6j - 3jn}\left(e^{2\pi j - 1\pi nj} - 1\right) + \frac{e^{\frac{\pi}{6}j}}{6j + 3jn}\left(e^{-2\pi j - 1\pi nj} - 1\right)\right)\right)$$

$\cos$

$n = 0$

$$= \frac{3}{2\pi}\left(\frac{e^{-\frac{\pi}{6}j}}{6 - 3n}\right. \qquad n = 1$$

$\cos(-n\pi)$

$$\frac{3}{2\pi}\left(\frac{-2e^{-\frac{\pi}{6}j}}{6 - 3n} + \frac{-2e^{\frac{\pi}{6}j}}{6 + 3n}\right)$$

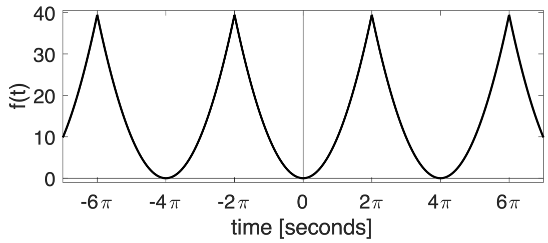
$2 - n \qquad 2 + n$

$$\begin{cases} \frac{\cancel{3}}{\cancel{2}\pi}\left(\frac{\cancel{4}2}{\cancel{3}jn}\right) & n \text{ is odd} \\ \frac{1}{\pi}\left(\frac{e^{-\frac{\pi}{6}j}}{n - 2} - \frac{e^{\frac{\pi}{6}j}}{2 + n}\right) & n \text{ is odd} \end{cases}$$

$F_n =$ ______________________________

$$\frac{2}{\pi jn} + \frac{1}{\pi}\left(\frac{e^{-\frac{\pi}{6}j}}{n - 2} - \frac{e^{\frac{\pi}{6}j}}{2 + n}\right) \quad n \text{ is odd}$$

7. **[19 points]** Consider the periodic signal $f(t)$ which equals t^2 over the range $-2\pi < t < 2\pi$ as shown in the figure below. This signal can be expressed as a Fourier series expansion having exponential coefficients F_0 and $F_{n\neq 0} = \frac{8}{n^2}(-1)^n$.



(a) [2 pts] What is the fundamental frequency, ω_0, of signal $f(t)$?

$T = 4\pi \qquad \omega = \frac{2\pi}{T} = \frac{1}{2}$

$\omega_0 =$ $\frac{1}{2}$

(b) [4 pts] What is F_0?

$$F_0 = \frac{1}{T}\int_{-2\pi}^{2\pi} t^2 \, dt$$

$$= \frac{1}{4\pi} \frac{8\pi^3}{3} \cdot 2 = \frac{4\pi^2}{3}$$

$F_0 =$ $\frac{4\pi^2}{3}$

(c) [5 pts] Write out the Fourier series expansion of $f(t)$ as an approximation comprised of the superposition of five terms corresponding to $n = -2, -1, 0, 1, 2$.

$$f(t) = \frac{4\pi^2}{3} - 8e^{\frac{1}{2}jt} - 8e^{-\frac{1}{2}jt} + 2e^{jt} + 2e^{-jt}$$

$f(t) \approx$ ____________

(d) [2 pts] Periodic signals can also be expressed in terms of trigonometric functions via the following general form:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

For the $f(t)$ shown above, which of the following statements is true? Briefly explain your answer.

i. $a_n = 0$ and $b_n \neq 0$ for all n

ii. $a_n \neq 0$ and $b_n = 0$ for all n ✓

iii. $a_n \neq 0$ and $b_n \neq 0$ for all n

Explain: ii even function

(e) [6 pts] Now consider this signal as input to an LTI system characterized by frequency response $H(\omega) = \frac{j\omega}{2+j\omega}$, such that the output $y(t)$ can also be expressed as a Fourier series having coefficients Y_n. What are the magnitude and phase of the Y_2 coefficient?

for $n=2$ $F_2 = 2$

$H(1) = \frac{j}{2+j}$

$Y_2 = \frac{4j}{2+2j}$

$\frac{4\cdot 2}{2\sqrt{2}}$

$|Y_2| = \sqrt{2}$

$90^\circ - 45^\circ$

$\phi = 45^\circ$

$\frac{2}{\sqrt{5}}$

$90^\circ - \arctan\frac{1}{2}$

$|Y_2| =$ ______ $\angle Y_2 =$ ______

You may use this sheet for additional calculations but do not separate this sheet from the rest of the exam.

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