ECE 210/211 **Spring 2022**

University of Illinois **Alvarez, Mironenko, Mironov, Schmitz**

Analog Signal Processing

Thursday, April 21, 8:45-10pm

Exam III

Last Name:	

First Name:	

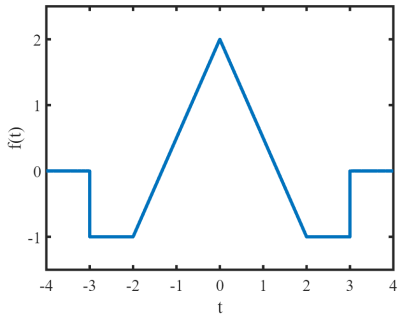
UIN:		**netID:**	

instructions :

- Clearly PRINT your name in CAPITAL LETTERS.
- Clearly write your UIN and netID.
- This is a closed book and closed notes exam.
- Calculators are not allowed.
- To get credit, you must SHOW ALL your work and/or reasoning. Answers without any work or reasoning may receive no credit.
- To get full credit, simplify your answers.
- Write your final answers in the spaces provided or points may be deducted.
- All answers should INCLUDE UNITS whenever appropriate.
- The exam is printed **double-sided.**

1. (25 pts) The three parts of this problem can be solved independently of each other.

   (a) [15 pts] Find the Fourier Transform of the signal $f(t)$ plotted below. Full credit is awarded only if your solution is expressed as the sum of two terms.



$F(\omega) =$ _____

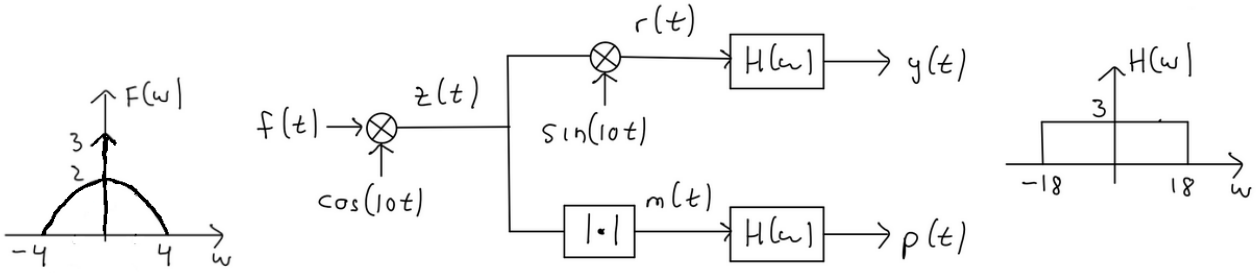
(b) [5 pts] A filter has an impulse response $h(t)$ with Fourier transform $H(\omega)$, whose amplitude is given by $|H(\omega)| = \sqrt{3}e^{-\frac{|\omega|}{2}}$. Find its 3-dB energy bandwidth, $\Omega_{3_{dB}}$. If convenient, you may leave your answer as a function of the natural logarithm (base-e), $\ln(x)$.

Ω_{3dB} = _____

(c) [5 pts] A filter has an impulse response $h(t)$ with Fourier transform $H(\omega)$, whose amplitude is given by $|H(\omega)| = \sqrt{3}e^{-\frac{|\omega|}{2}}$. Find its 95% energy bandwidth, $\Omega_{95\%}$. If convenient, you may leave your answer as a function of the natural logarithm (base-e), $\ln(x)$.

$\Omega_{95\%}$ = _____

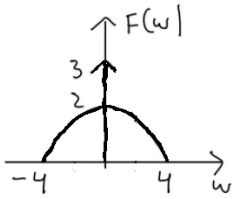
2. (25 pts) The input $f(t)$ with the frequency spectrum $F(\omega)$ goes through the system below.

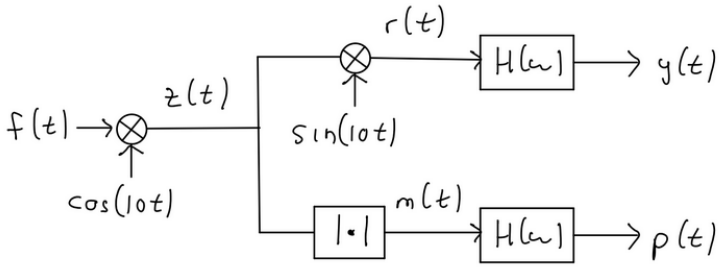


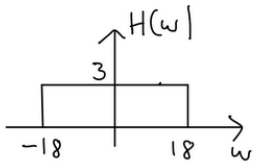
Plot $Z(\omega)$, $R(\omega)$, $Y(\omega)$, $M(\omega)$, and $P(\omega)$ between $-50 < \omega < 50$ rad/s, labeling all important points. You may plot with complex-valued amplitudes, or plot real and imaginary parts separately, or plot magnitude and phase separately. If needed,

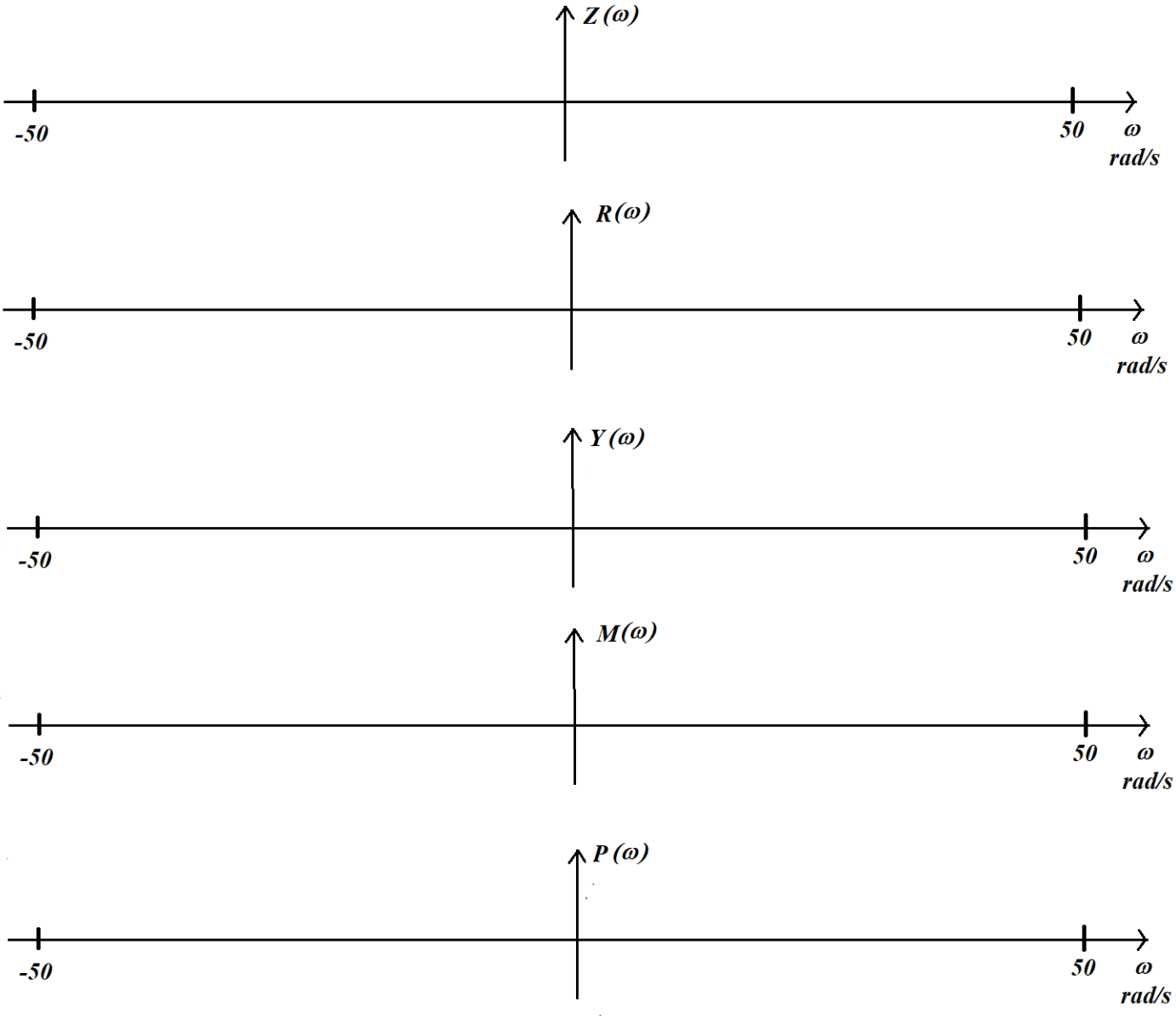
$$|\cos(\omega_0 t)| = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{1}{\pi\left(n^2 - \frac{1}{4}\right)} \cos\left(2n\omega_0 t + (1-n)\pi\right)$$

There is space for the plots on the next page.


F(ω)
3
2
-4
4
ω


f(t)
z(t)
cos(10t)
r(t)
sin(10t)
H(ω)
y(t)
|·|
m(t)
H(ω)
p(t)


H(ω)
3
-18
18
ω


Z(ω)
-50
50
ω
rad/s
R(ω)
-50
50
ω
rad/s
Y(ω)
-50
50
ω
rad/s
M(ω)
-50
50
ω
rad/s
P(ω)
-50
50
ω
rad/s

3. (25 pts) The four parts of this problem are unrelated.

(a) [5 pts] Simplify the following expression: $f(t) = (1+t)(\delta(t) - 2\delta(4-t))$

$f(t) =$ _____

(b) [10 pts] Find $g(t) = e^{-|t|} * cos(2t)$

$g(t) =$ _____

(c) [5 pts] Consider a superheterodyne AM receiver. If we want to listen to the radio station with carrier frequency $f_c = 600$kHz, what is the required local oscillator frequency and the resulting image station frequency, assuming that the intermediate frequency is 455kHz?

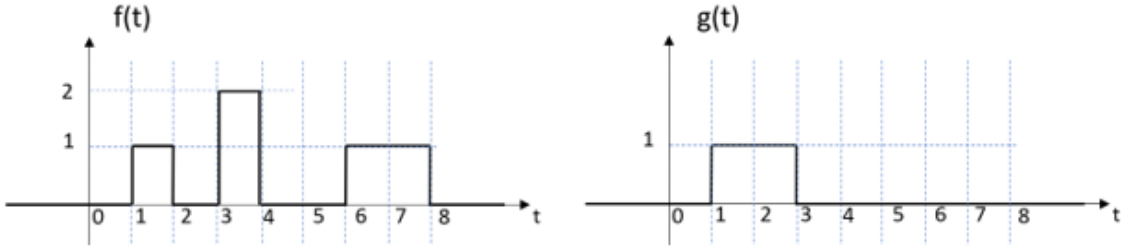
f_{LO} = _____ f_{IM} = _____

(d) [5 pts] Let signal $x(t)$ be a low-pass signal with a bandwidth of 500 rad/s. Determine the minimum sampling frequency in Hz needed to sample signal $z(t) = x(t)cos(500t)$ without causing aliasing error.

$f_s >$ _____

4. (25 pts) The two parts of this problem are not related. Although you are welcome to use a Fourier-based solution to check your answers, all graded calculations and plotting (if needed) must be clearly performed in the time domain.

(a) Functions $f(t)$ and $g(t)$ are given below and $y(t) = f(t) * g(t)$.



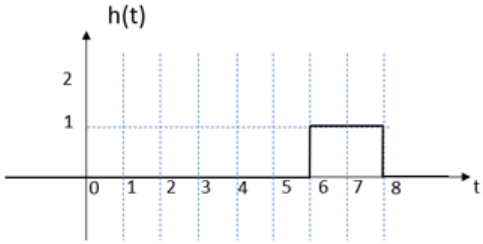
i. [6 pts] Determine $y(4.5)$

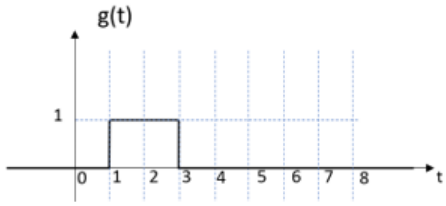
$y(4.5) =$ _____

ii. [10 pts] Find a mathematical expression for $y(t)$ for $0 < t < 7$. You may leave your answer as a piecewise function.

$y(t) =$ _____

(b) [9 pts] Functions $h(t)$ and $g(t)$ are given below and $z(t) = h(t) * g(t)$. Plot $z(t)$ for $7 < t < 12$ and indicate all critical points of the graph. It is not necessary to write a mathematical expression for $y(t)$.





You may use this sheet for additional calculations but **do not** separate this sheet from the rest of the exam.

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