ANALOG SIGNAL PROCESSING

ECE 210 & 211

Exercise 2

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$$\frac{dV}{dt} = p(t)\cdot V + q(t)$$

$$V(t)_0 = e^{\int -p(t)\,dt}$$

$$V(t)_p = e^{\int p(t)dt}\int q(t)\, e^{\int -p(s)ds}\, dt$$

Question 1:

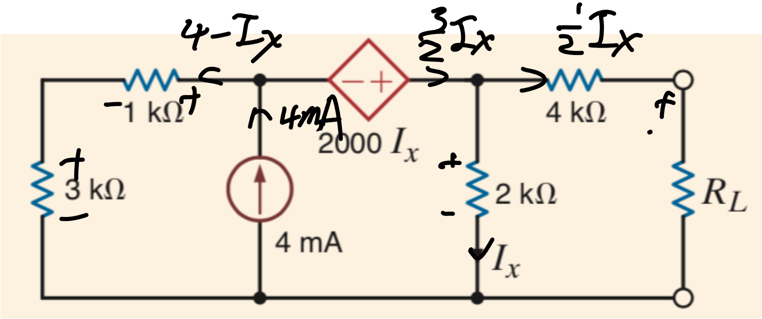
Let us find R_L for maximum power transfer and the maximum power transferred to this load in the circuit below.

when the load is open.

$4\times10^3\cdot(4-I_x)+2\times10^3 I_x = 2\times10^3 I_x$

$\Rightarrow I_x = 4\,mA$

$\Rightarrow \begin{cases} V_{out} = 8V \\ I = 0 \end{cases}$

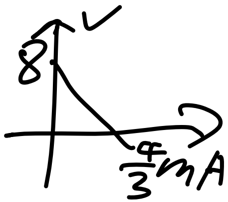


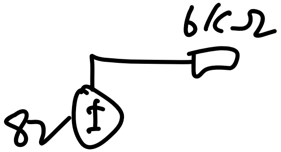
when the load is short

$(4-\frac{3}{2}I_x)\cdot 4+2I_x = 2I_x$

$4=\frac{3}{2}I_x \quad I_x = \frac{8}{3}\,mA$

$\Rightarrow \begin{cases} V_{out} = 0 \\ I_{out} = \frac{4}{3}\,mA \end{cases}$





$\frac{8}{R} = \frac{4}{3}$

$R = 6$

$P = \frac{16}{6} = \frac{8}{3}\,mW$

$R_L = 6k\Omega$

Question 2 : The circuit is in steady state prior to time t=0, when the switch is closed. Calculate the current i(t) for $t > 0$?

$V_0 + V_P$

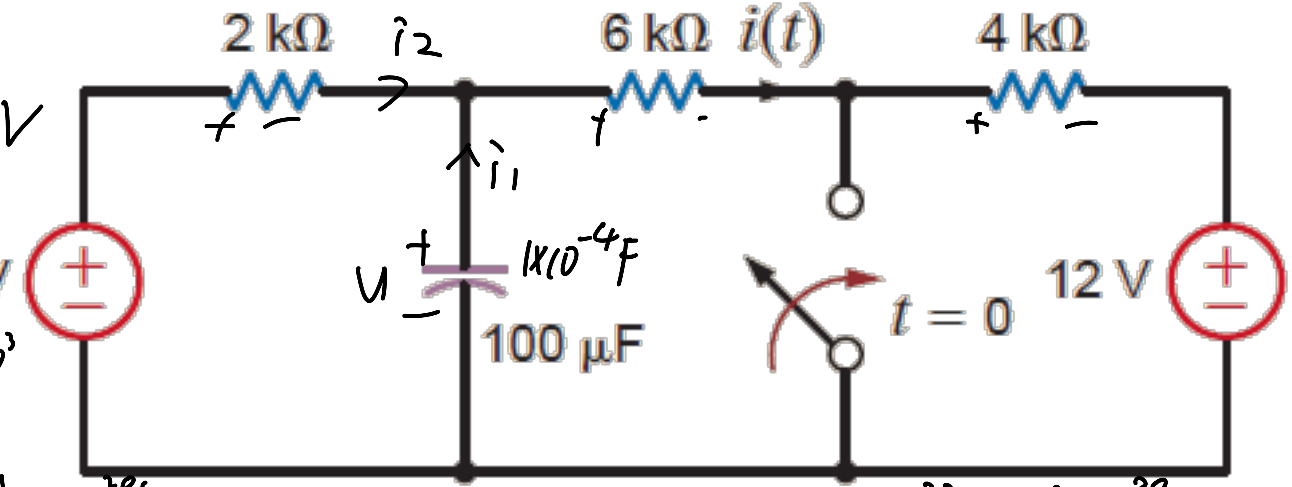
1.5

$\frac{\cancel{36}}{8}$

when $t<0$

$i(t) = \frac{36-12}{12} A = 2mA$, $u(t) = 36-2\times 2 = 32 V$

when $t \geq 0$



$\lim_{t\to\infty} u(t) = 36 \cdot \frac{6}{6+2} V$

$= 27V$

$\therefore \tau = R_{Th}C = \frac{12}{8} \cdot 1\times10^{-4} \times 10^3$

$= 0.15s$

$\therefore u(t) = 27 + (32-27) \cdot e^{-\frac{20}{3}t}$

$= 27 + 5e^{-\frac{20}{3}t}$

$I_2 = \frac{36-27-5e^{-\frac{20}{3}t}}{2}$

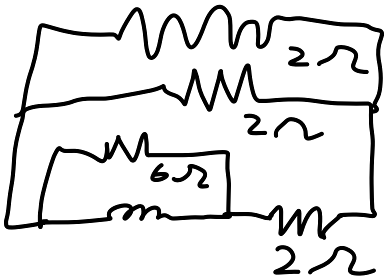
$= 4.5 - 2.5e^{-\frac{20}{3}t} mA$

$\therefore i_1 = -C\frac{du}{dt} = 1\times10^{-4} \cdot 5 \cdot (\frac{20}{3}) e^{-\frac{20}{3}t} = \frac{10}{3} e^{-\frac{20}{3}t}$

$\therefore i(t) = i_1 + i_2 = 4.5 - 2.5e^{-\frac{20}{3}t} + \frac{10}{3}e^{-\frac{20}{3}t}$

$= 4.5 + \frac{5}{6} e^{-\frac{20}{3}t}$

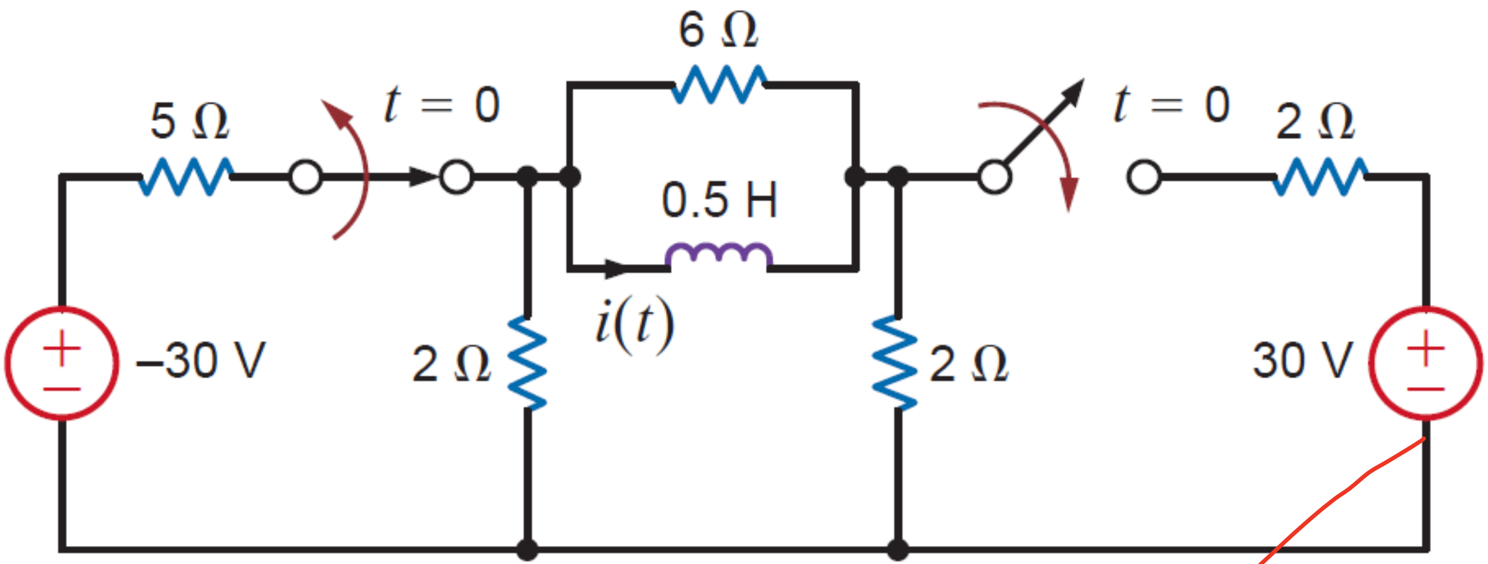
$\frac{5}{2}$ $\frac{15}{6}$ $\frac{20}{6}$



3Ω

Question 3: Find $i(t)$ for $t > 0$?

Note: The switch position is changed at $t = 0$



for $t < 0$

$i(t) = -\frac{30 \cdot \frac{1}{1+5}}{2} = -2.5A$

for $t \geq 0$ $R_{Th} = \frac{18}{9}\Omega = 2\Omega$, $\tau = \frac{L}{R} = \frac{0.5}{2} = 0.25s$

$i(\infty) = \frac{30V \cdot \frac{1}{1+2}}{2} = -5A$

$\therefore i(t) = -5 + (-2.5+5)e^{-4t} = -5 + 2.5e^{-4t} A$

Question 4:

Let $R = 1\ \Omega$, $C = 1\text{F}$, and

$\int u\,dv = uv - \int v\,du$

$$v_s(t) = \cos(t)$$ $\omega = 1$

$du = -\sin t\,dt$

$u = \cos s$, $dv = e^t dt$

$v = e^t$

in the RC circuit shown earlier in Figure . Then, for $t > 0$, the capacitor voltage $v(t)$ will be the solution to the ODE

$\int \cos s\, e^t dt = \cos s\, e^t + \int e^t \sin t\,dt$

$= \cos s\, e^t + e^t \sin t - \int e^t \cos t\,dt$

$$\frac{dv}{dt} + v(t) = \cos(t).$$

Determine $v(t)$ for $t > 0$, assuming zero initial state—that is, $v(0^-) = 0$.

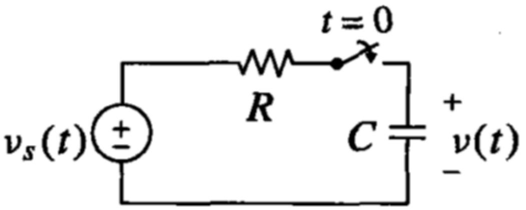
$\frac{dV}{dt} = -V(t) + \cos(t)$

$\therefore V(t)_o = ce^{\int -1\,dt} = ce^{-t}$

$V(t)_p = e^{\int -1\,dt} \int \cos s\, e^{\int 1\,ds} dt$

$= e^{-t} \int \cos s\, e^s dt$

$= e^{-t}\, \frac{1}{2}(\cos t\, e^t + \sin t\, e^t) = \frac{1}{2}\cos t + \frac{1}{2}\sin t$



$\therefore V(t) = ce^{-t} + \frac{1}{2}\cos t + \frac{1}{2}\sin t$

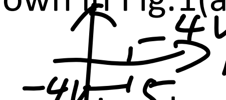
For c: $V(0^-) = V(0) = c + \frac{1}{2} = 0 \Rightarrow c = -\frac{1}{2}$

$\therefore v(t) = -\frac{1}{2}e^{-t} + \frac{1}{2}\cos t + \frac{1}{2}\sin t$

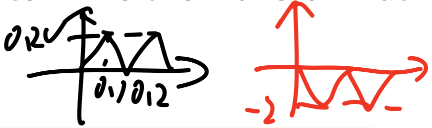
Question 5:

The capacitor is initially discharged.

(a) The waveform in Fig.1(b) is applied at the input of the differentiator circuit shown in Fig.1(a). If R =1K, and C=2uF, determine the waveform at the output of the op-amp. output



(b) If the integrator shown in Fig.2(b) has the parameters R1=5K and C2=0.2uF, determine the waveform at the op-amp output if the input waveform is given as in Fig.2(a).



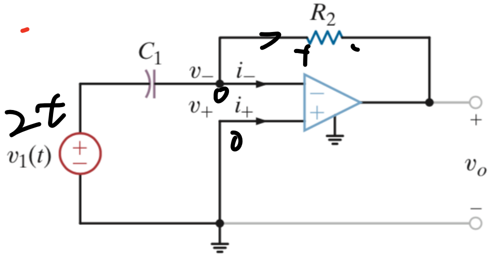
for C $Q = CV$

$I = C\frac{dV}{dt}$ 2×10^3

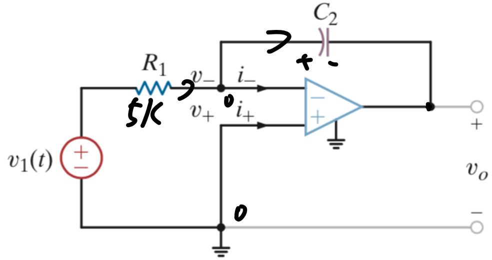
$I = C\frac{du}{dt}$ 2

$\frac{du}{dt} = +2$

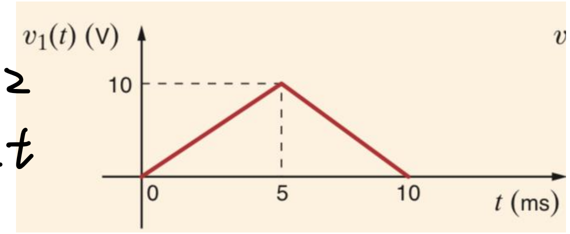
$u = 2t$



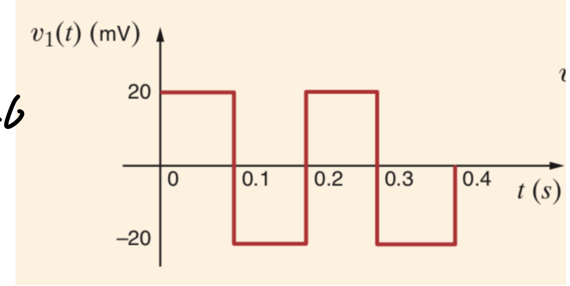
1(a)



2(a)



1(b)



2(b)

$\frac{4}{8\times10^{-3}}$ 2×10^3

$2\times10^3 \times 2\times10^{-6} = 4\times10^{-3}$

4 V

$\frac{20}{5}\times10^{-6} = 4\times10^{-6}$ 2×10^{-6}

$V_0 = 0 - V_2$

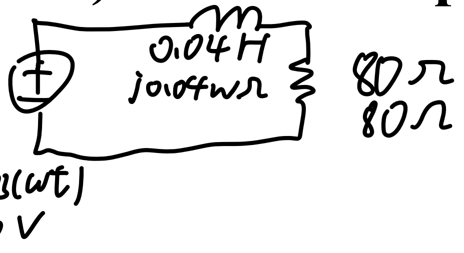
Question 6: A Variable frequency source having amplitude 6V connected to a series *RL* circuit of values *L*=40mH and *R*= 80 ohms.

a) At what frequency , does $V_R = V_L$?
b) What would be the phase angle ϕ at that frequency?
c) What will be the current amplitude and RMS value?
d) Write an expression for current at steady state in phasor form?

$I = \boxed{Ae^{-t/\tau}} + \boxed{B_1 \cos t}$

$B_1 \cos t + G \sin t$

(a)



$6 \cdot \frac{|j0.04w|}{|80+j0.04w|} = 6 \frac{|80|}{|80+j0.04w|}$

$\Rightarrow 0.04w = 80 \Rightarrow w = 2000$

$\Rightarrow f = \cancel{2\pi w = 4000\pi}$

(b) $\phi = \arctan\left(\frac{V_L}{V_R}\right) = 45°$ ✓

(c) $I = \frac{6}{80+80j}$ $\therefore |I| = \frac{\cancel{6}\,3}{_{40}\cancel{80}\sqrt{2}} = \frac{3\sqrt{2}}{80}$ A $= I_{max}$

RMS value $= |I| \cdot \frac{1}{\sqrt{2}} = \frac{3}{80}$ A

(d) $I = Re\{|I|e^{jwt}\} = \frac{3\sqrt{2}}{80}\cos w$

phasor form : $\frac{6}{80+80j}$

?

$= \frac{\omega}{2\pi} = 318Hz$

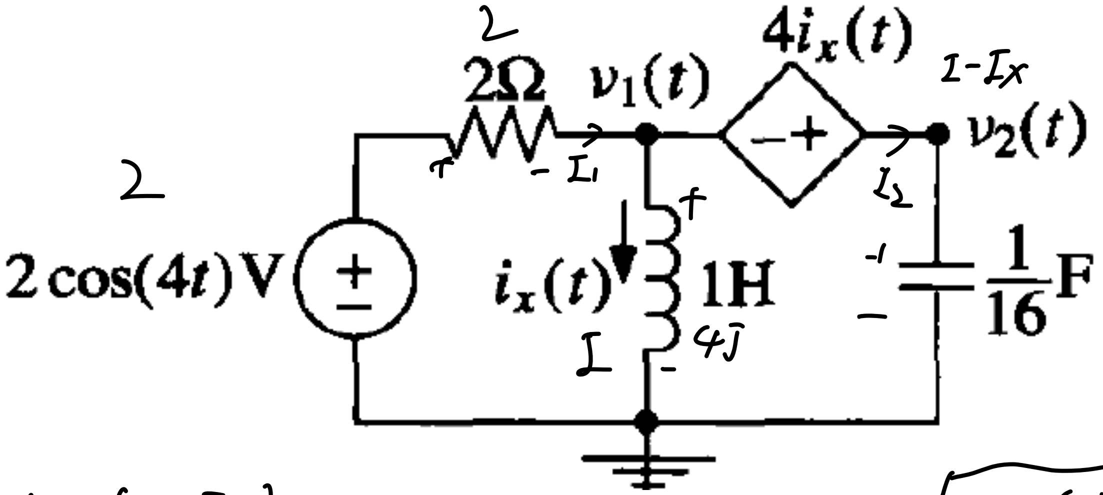
Question 7: Find $v_1(t)$ in the network using phasor method?

$\omega = 4$

$\frac{16}{j\omega}$

$4I$

$-\frac{16j}{\omega}$



2

$-\frac{16j}{\omega} = -4j$

$2 = 2I + I_x 4j$

$I_x \cdot 4j + 4I_x = -4j \cdot (I - I_x)$

$= -4jI + 4jI_x$

$1 = I + 2I_x j = jI_x + 2jI_x = 3jI_x \quad I_x = -\frac{1}{3}j$

$I_x = -jI \Rightarrow I = jI_x$

$\therefore V_1 = 0 + 4j\left(-\frac{1}{3}\right)j = \frac{4}{3}V$

$v_1(t) = \frac{4}{3}\cos(4t)$

Question 8:

Let us determine the phasor diagram for the series circuit shown below.

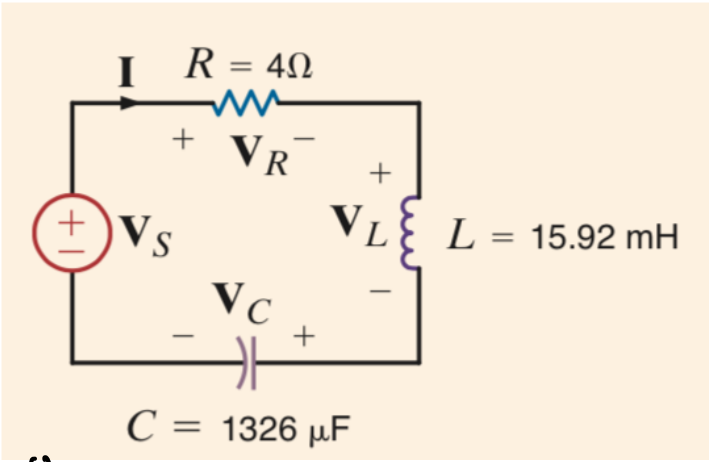
let $\omega = 377\ rad/s$

$$Z = 4 + j\omega L + \frac{1}{j\omega L}$$

$$= 4 + 6j + \frac{2}{j}$$

$$= 4 + 6j - 2j = 4 + 4j$$

$$\therefore I = \frac{V_s}{Z} =$$



assume use I as base line

$$\therefore V_R = 4I = 4I$$

$$V_L = 6jI = 6I\angle 90^\circ$$

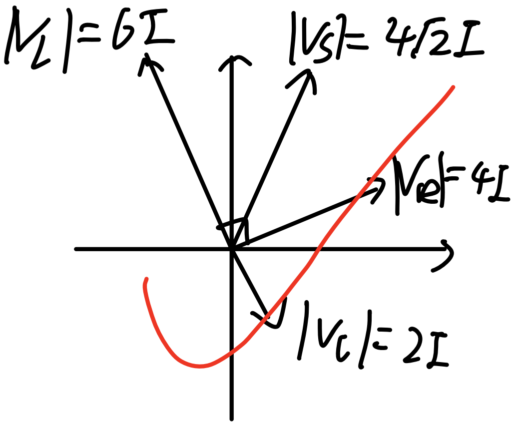
$$V_C = -2jI = 2I\angle -90^\circ$$

$$V_S = ZI = (4+4j)I = 4\sqrt{2}I\angle 45^\circ$$

Tips: assume $\omega = 377\ rad/s$, then $\omega L = 6$, $\frac{1}{\omega C} = 2$

$$\phi = \arctan\left(\frac{|V_L| - |V_C|}{|V_R|}\right) = \arctan 1 = 45^\circ$$

$\therefore$ phasor



Question 9:

Let us determine the current **Io** in the network below using nodal analysis, loop analysis, superposition, source exchange, Thévenin's theorem, and Norton's theorem.

$$I_1 = \frac{V_1}{1+j}$$

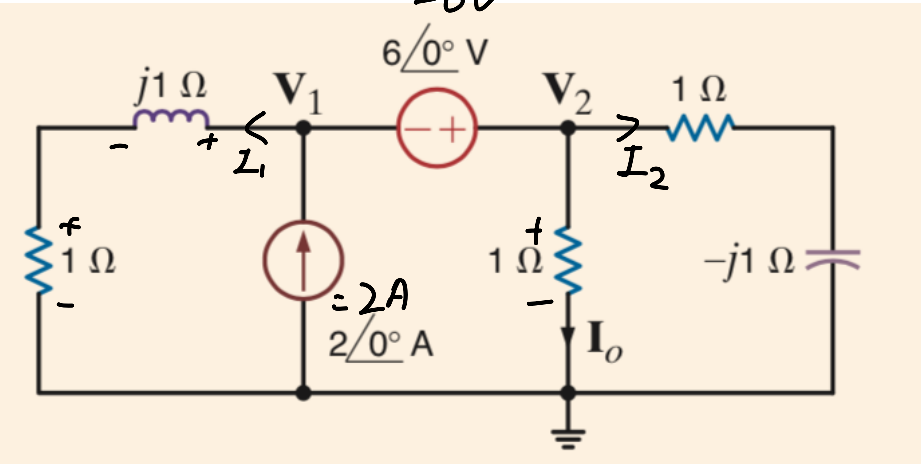
$$V_2 = V_1 + 6V \Rightarrow V_1 = V_2 - 6$$

$$= I_0 - 6$$

$$I_0 = \frac{V_2}{1\Omega}$$

$$I_2 = \frac{V_2}{1-j} A$$

$= 6V$



$$I_o = \frac{5}{2} - \frac{3}{2}j$$

$$2 = I_1 + I_2 + I_0 = \frac{I_0 - 6}{1+j} + \frac{I_0}{1-j} + I_0$$

$$2 = \frac{(I_0 - 6)(1-j)}{2} + \frac{I_0(1+j)}{2} + I_0 = 2I_0 + 3j - 3$$

$$I_o = \frac{5}{2} - \frac{3}{2}j$$

$$2I_o = 5 - 3j$$

$$\frac{I_0 - 6 \cancel{- I_0 j} + 6j + I_0 \cancel{+ I_0 j} + 2I_0}{2} = \frac{4I_0 + 6\hat{j} - 6}{2} = 2I_0 + 3\hat{j} - 3$$