ANALOG SIGNAL PROCESSING

ECE 210

Exercise 5

18th May, 2023

Prof. Yang Xu （徐杨）

yangxu-isee@zju.edu.cn

Yue Dai, yuedai@zju.edu.cn
Jinfeng Huang, huangjinfeng@zju.edu.cn
Yance Chen, 12031045@zju.edu.cn
Yuan Ma, 22241064@zju.edu.cn
Youshui He, 22241003@zju.edu.cn
Zongwen Li, zongwen_li@zju.edu.cn
Muhammad Malik, mmalik@zju.edu.cn
Munir Ali, Munir_li@zju.edu.cn
Muhammad Abid, abid_anwar@zju.edu.cn

ZJU-UIUC Institute
Zhejiang University / University of Illinois at Urbana-Champaign Institute

Question 1: Basic System Properties

(1) Time invariant
(2) Linear
(3) Causal
(4) Stable

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

(a) $y(t) = x(t-2) + x(2-t)$ linear stable

(b) $y(t) = [\cos(3t)]x(t)$ linear causal stable

(c) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$ linear ~~stable~~

(d) $y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$ linear causal stable ~~time invariant~~

(e) $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$ linear causal stable time invarent

(f) $y(t) = x(t/3)$ linear stable

(g) $y(t) = \frac{dx(t)}{dt}$ linear causal time invarent

Question 2: LTI Systems

(a) Consider an LTI system with input and output related through the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau.$$

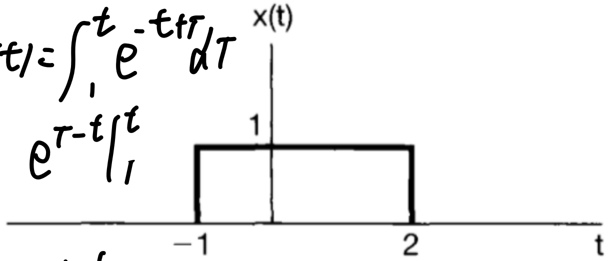
$$\int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2)\, d\tau = \begin{cases} 0, & t<2 \\ e^{-(t-2)}, & t \geq 2 \end{cases} = u(t-2)e^{-(t-2)}$$

What is the impulse response $h(t)$ for this system?

(b) Determine the response of the system when the input $x(t)$ is as shown in Figure

when $t<1$, $y(t)=0$

when $1 \leq t \leq 4$, $y(t) = \int_{1}^{t} e^{-t+\tau}\, d\tau$



$= e^{\tau-t}\Big|_{1}^{t}$

$= 1 - e^{1-t}$

when $t>4$, $y(t) = \int_{1}^{4} e^{-t+\tau}\, d\tau$

$= e^{\tau-t}\Big|_{1}^{4}$

$= e^{4-t} - e^{1-t}$

Question 3: The Properties of FS

Suppose we are given the following information about a signal $x(t)$:

1. $x(t)$ is real and odd.
2. $x(t)$ is periodic with period $T = 2$ and has Fourier coefficients a_k.
3. $a_k = 0$ for $|k| > 1$. $w = \frac{2\pi}{T} = \pi$
4. $\frac{1}{2}\int_0^2 |x(t)|^2\, dt = 1$.

Specify two different signals that satisfy these conditions.

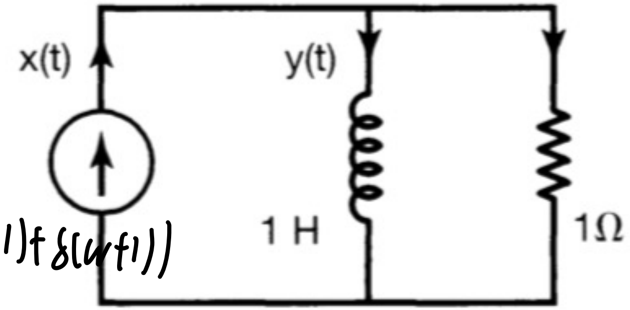
$x(t) = a_{-1} e^{-j\pi t} + 0 + a_1 e^{j\pi t}$

$\frac{1}{T}\int_0^T |x(t)|^2 dt = |a_1|^2 + |a_{-1}|^2 = 1 \Rightarrow |a_1|^2 = \frac{1}{2}$ $a_1 = \pm \frac{\sqrt{2}}{2}$

Question 4: LTI Systems with FS

Consider a causal LTI system implemented as the *RL* circuit .
A current source produces an input current $x(t)$, and the system output is considered to be the current $y(t)$ flowing through the inductor.

(a) Find the differential equation relating $x(t)$ and $y(t)$.
(b) Determine the frequency response of this system by considering the output of the system to inputs of the form $x(t) = e^{j\omega t}$.
(c) Determine the output $y(t)$ if $x(t) = \cos(t)$.



(c) $U_1 = L \cdot \frac{dy(t)}{dt}$, $I_2 = \frac{U_1}{R} = \frac{V_1}{1}$

$\therefore x(t) = V_1 + y(t) = L \cdot \frac{dy(t)}{dt} + y(t)$

$\therefore$ equation: $\frac{dy(t)}{dt} = -\frac{1}{L} y(t) + \frac{1}{L} x(t)$

(b) $x(t) = e^{jwt}$

$\cos(t) \leftrightarrow \pi\{\delta(w-1) + \delta(w+1)\}$

$H(jw) = \frac{1}{1+jw}$

$Y(w) = X(w) \cdot H(w)$

$\Updownarrow$

$y(t) = x(t) * h(t)$

$Y(w) = \frac{\pi}{1+jw}(\delta(w-1) + \delta(w+1))$

Question 5: LTI systems with FT

ans 错误

$\frac{1}{2\pi}\int_{-\infty}^{\infty} Y(w) e^{jwt} dt$

$= \frac{1}{2\pi}\left(\frac{\pi}{1+j}e^{jt} + \frac{\pi}{1-j}e^{-jt}\right)$

$= \frac{1}{2}\left(\frac{1-j}{2}e^{jt} + \frac{1+j}{2}e^{-jt}\right)$

The input and the output of a stable and causal LTI system are related by the differential equation

(a) $-w^2Y(w) + 6jwY(w) + 8Y(w) = 2X(w)$

$= \frac{1}{2}\left(\frac{1}{2} 2\cos t + \frac{1}{2}[2\sin t]\right)$

$\therefore H(w) = \frac{Y(w)}{X(w)} = \frac{2}{-w^2+6jw+8}$

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

$\frac{1}{2}(\cos t + \sin t)$

$= \frac{2}{(jw+2)(jw+4)} = \frac{1}{jw+2} - \frac{1}{jw+4}$

$\frac{\sqrt{2}}{2}\cos(t - \frac{\pi}{4})$

(a) Find the impulse response of this system.

(b) What is the response of this system if $x(t) = te^{-2t}u(t)$?

$\therefore h(t) = e^{-2t}u(t) - e^{-4t}u(t)$

(c) Repeat part (a) for the stable and causal LTI system described by the equation

$$\frac{d^2y(t)}{dt^2} + \sqrt{2}\frac{dy(t)}{dt} + y(t) = 2\frac{d^2x(t)}{dt^2} - 2x(t)$$

(b) $x(t) = te^{-2t}u(t) \leftrightarrow \frac{1}{(2+jw)^2}$

$\therefore Y(w) = \frac{2}{(2+jw)^3(jw+4)}$

$= \frac{-\frac{1}{4}}{jw+4} + \frac{\frac{1}{4}}{jw+2} + \frac{-\frac{1}{2}}{(jw+2)^2} + \frac{1}{(jw+2)^3}$

(c) $-w^2Y(w) + \sqrt{2}jwY(w) + Y(w) = -2w^2X(w) - 2X(w)$

$\therefore H(w) = \frac{Y(w)}{X(w)} = \frac{-2w^2-2}{-w^2+\sqrt{2}jw+1}$

$= \frac{-2w^2+2\sqrt{2}jw+2-2\sqrt{2}jw-4}{-w^2+\sqrt{2}jw+1}$

$= 2 - \frac{2\sqrt{2}jw+4}{\ldots}$

$= -\frac{1}{4}e^{-4t}u(t) + \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + \frac{1}{2}t^2e^{-2t}u(t)$

$= 2 - \frac{2\sqrt{2}j\omega + 2}{(j\omega + \frac{\sqrt{2}}{2})^2 + \frac{1}{2}} + 2\sqrt{2}\,\frac{\frac{\sqrt{2}}{2}}{(j\omega + \frac{\sqrt{2}}{2})^2 + \frac{1}{2}}$

$= 2\delta(t) - 2\sqrt{2}e^{-\frac{\sqrt{2}}{2}t}\cos(\frac{\sqrt{2}}{2}t)u(t) + 2\sqrt{2}e^{-\frac{\sqrt{2}}{2}t}\sin(\frac{\sqrt{2}}{2}t)u(t)$

Question 6: Laplace Transform & Its Properties

$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(s)e^{st}\,ds$

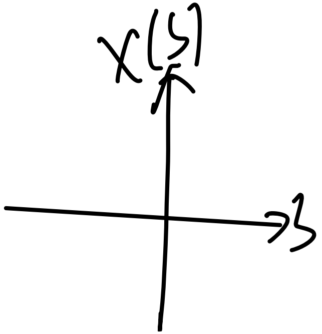
Suppose we are given the following information about a causal and stable LTI system S with impulse response $h(t)$ and a rational system function $H(s)$:

1. $H(1) = 0.2$.
2. When the input is $u(t)$, the output is absolutely integrable. $\Rightarrow H(0) = 0$
3. When the input is $tu(t)$, the output is not absolutely integrable.
4. The signal $d^2h(t)/dt^2 + 2\,dh(t)/dt + 2h(t)$ is of finite duration. $= p(t)$
5. $H(s)$ has exactly one zero at infinity. ✓

Determine $H(s)$ and its region of convergence.

$\longrightarrow u(t) \leftrightarrow \frac{1}{s}$

$tu(t) \leftrightarrow \frac{1}{s^2}$



$s^2H(s) + 2sH(s) + 2H(s)$

$H(s)(s^2 + 2s + 2)$

$H(s) = \frac{A\prod_{i=1}^{N}(s - z_i)}{s^2 + 2s + 2} = \frac{s}{s^2 + 2s + 2}$

$(s+1)^2 + 1 = 0$

$s + 1 = \pm j$

Re(s) > -1 $s = -1 \pm j$

Question 7: Sampling

Figure 2(a) shows the overall system for filtering a continuous-time signal using a discrete-time filter. If $X_c(j\omega)$ and $H(e^{j\omega})$ are as shown in Figure 2(b), with $1/T = 20$ kHz, sketch $X_p(j\omega)$, $X(e^{j\omega})$, $Y(e^{j\omega})$, $Y_p(j\omega)$, and $Y_c(j\omega)$.

$T = \frac{1}{20000}$

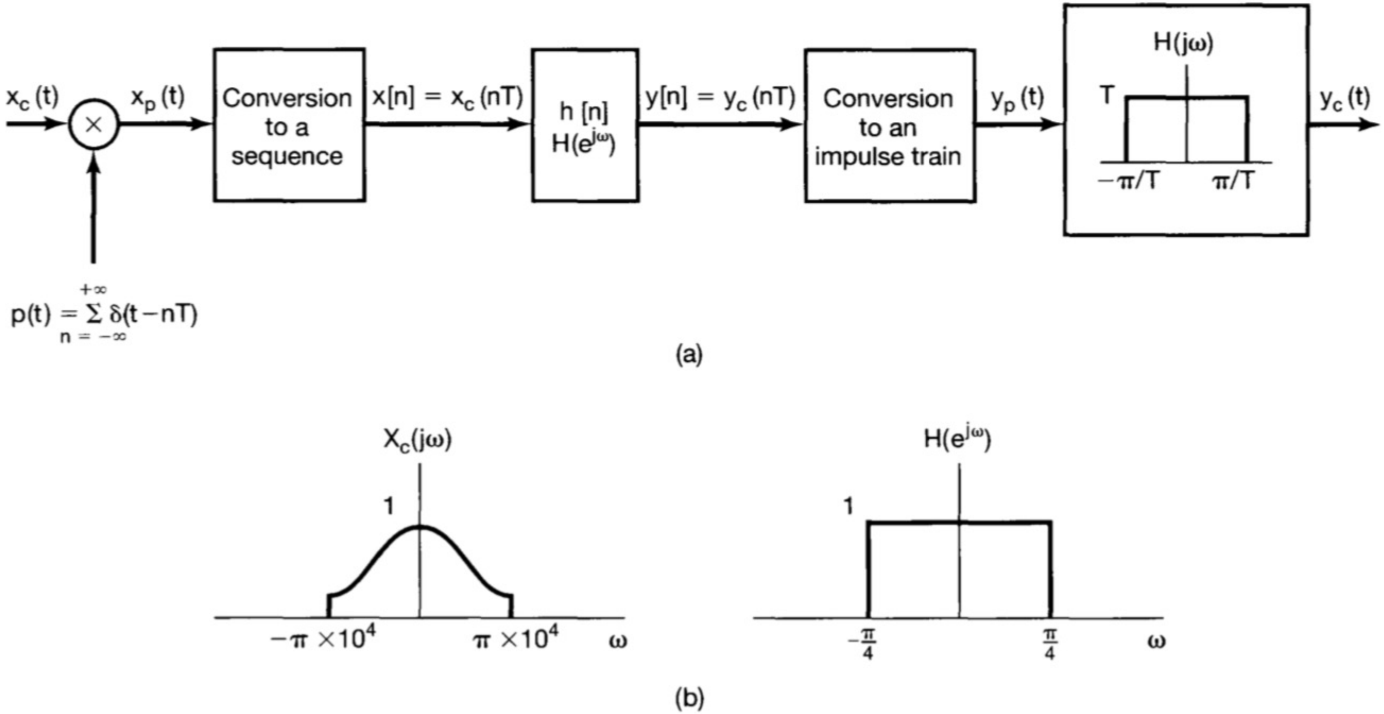
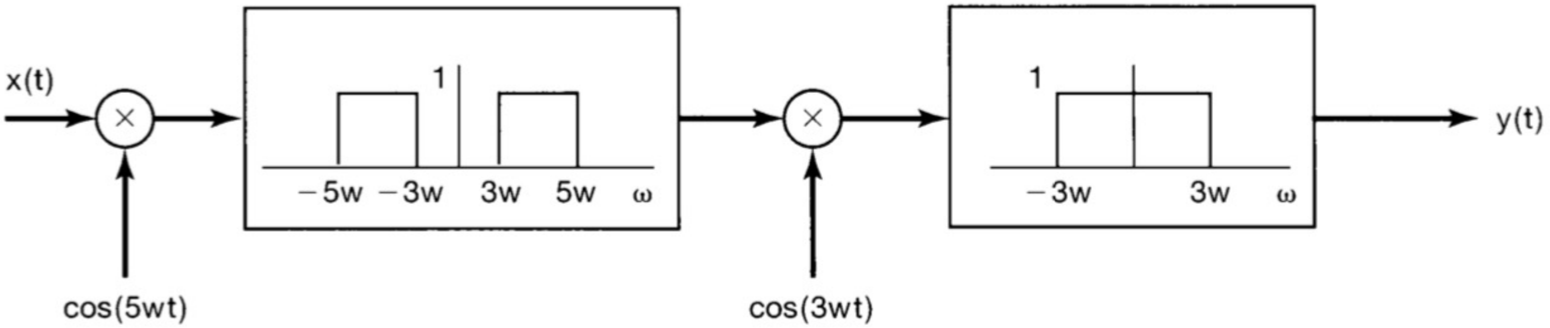


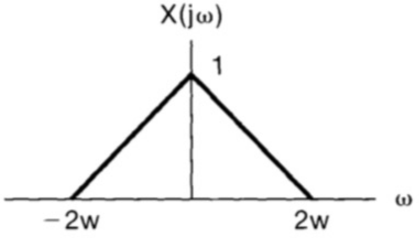
Figure 2

Question 8: Modulation

In Figure 3(a), a system is shown with input signal $x(t)$ and output signal $y(t)$. The input signal has the Fourier transform $X(j\omega)$ shown in **Figure 3(b)**. Determine and sketch $Y(j\omega)$, the spectrum of $y(t)$.



(a)



(b)

Figure 3