ZJU-UIUC INSTITUTE

Online Final Examination

For Students, please read and sign the honor statement on a sheet of paper with your name, student ID number, date, and read any instructions below before starting your exam.

Course Code: ECE210	Semester: Spring 2020		Instructor: Prof. Yang Xu
Exam Type: **Closed-book** ☐ Open-book ☐ Partly Open-book ☐ Take Home ☐			
Exam Date: May 24th 2020	Start Time: 9:00 am	End Time: 12:00 am	Duration: 3 hour
Total pages: 9		Total questions: 7	

Specific requirements and instructions to students:

1. Sign the honor statement before starting your exam on time and attach it with your solution sheets.
2. Write your solutions on A4 size blank sheets of paper and submit the clear scan version of them in the Blackboard before the end of the exam.
3. Approved calculators are permitted.
4. Access to blackboard/internet/lecture materials is not permitted except for downloading and uploading the exam paper and solutions.
5. The formula sheet is attached on the last page of the exam paper.
6. If you have any questions, please ask or type them via WebEx.

You have only one chance to submit your solutions!

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Honor Statement

I have read the announcement concerning exam administration, the exam instructions, the academic integrity statement repeated from the course syllabus, and provided my reflection statement on why integrity and honesty are important. I promise to abide by the exam rules and regulations and agree to comport myself during the remotely administered exam in the same manner as if I were in a proctored examination room.

Please write "**I have read and will follow the Honor Statement**" on a sheet of paper and include the following information, then submit along with your answer sheet.

Signature:

Student ID number:

Date:

(Please go on to the next page for questions)

Question 1: **(15')**

(a) Apply source transformation to find v_x in the circuit of Figure 1(a).

(b) In the circuit of Figure 1(b) find $i(t)$ for $t > 0$.

(a) for left source

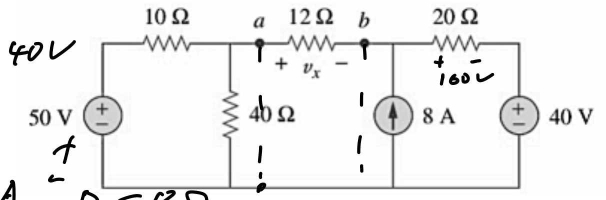
open circuit: $V = \frac{40}{10+40} \cdot 50 = 40V$

$I = 0$

short circuit: $V = 0$

$I = \frac{50}{10} A = 5A$

$R_1 = 8\Omega$



(a)

for right source

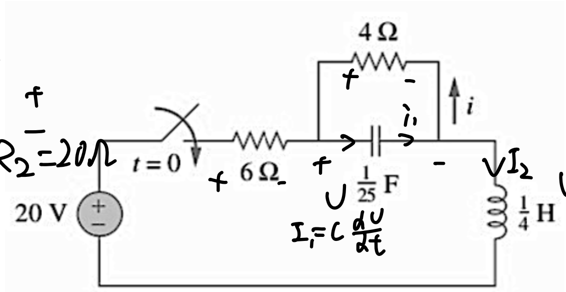
open circuit: $V = 200V$

$I = 0$

short circuit: $V = 0$

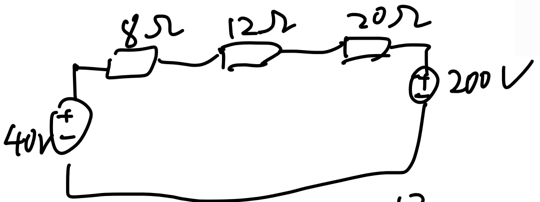
$I = 10A$

$R_2 = 20\Omega$



(b)

$V_3 = L \cdot \frac{dI_2}{dt}$



$20 \cdot \frac{4}{10}$

Figure 1

$V_x = (40-200) \cdot \frac{12}{40} = -48V$

(b) KVL: $20 = 6i_2 + U + \frac{1}{4} \cdot \frac{di_2}{dt}$

$i_2 = i_1 - i = \frac{1}{25}\frac{dU}{dt} + \frac{U}{4}$

$\frac{di_2}{dt} = \frac{1}{25}\frac{d^2U}{dt^2} + \frac{dU}{4dt}$

$20 = \frac{6}{25}\frac{dU}{dt} + \frac{3}{2}U + U + \frac{1}{100}\frac{d^2U}{dt^2} + \frac{1}{16}\frac{dU}{dt}$

$20 = \frac{1}{100}\frac{d^2U}{dt^2} + \frac{96+25}{400} \cdot \frac{dU}{dt} + \frac{5}{2}U$

$\frac{d^2U}{dt^2} + \frac{121}{4}\frac{dU}{dt} + 250U = 2000$

$x^2 + \frac{121}{4}x + 250 = 0$

$x = \frac{-\frac{121}{4} \pm \sqrt{-84.9375}}{2}$

$x_1 = -\frac{121}{8} - 4.608i$

$x_2 = -\frac{121}{8} + 4.608i$

$m_1 = m_2 = 1$

$e^{x_1 t}, \quad e^{x_2 t}$

$Y_1(t) = C_1 e^{(-\frac{121}{8} - 4.608i)t} + C_2 e^{(-\frac{121}{8} + 4.608i)t}$

$\mu = 0 \quad m = 0$

$t = 0$

$= e^{-\frac{121}{8}t}(C_1 e^{-4.608it} + C_2 e^{4.608it})$

$= e^{-\frac{121}{8}t}(C_3 \cos 4.608t + C_4 \sin 4.608t)$

$Y_p(t) = C_0$

$Y(t) = 8 + e^{-\frac{121}{8}t}(C_3 \cos 4.608t + C_4 \sin 4.608t)$

$Y(0) = 8 + C_3 = 0 \qquad C_3 = -8$

$Y'(t) = 121 e^{-\frac{121}{8}t} \cos 4.608t$

$+ e^{-\frac{121}{8}t} \cdot 4.608 \, C_4 \cos(4.608t$

$= 121 + 4.608 C_4 = 0 \qquad C_4 = -26.25868$

$Y(t) = 8 + e^{-\frac{121}{8}t}(-8\cos 4.608t - 26.25868 \sin 4.608t)$

$i(t) = \frac{-Y(t)}{4} = -2 + e^{-\frac{121}{8}t}(2\cos 4.608t + 6.564673 \sin 4.608t)$

Question 2: **(10')**

(a) Find the transfer function for the active filters in Figure 2.

(b) The filter in Figure 2 has a 3-dB cutoff frequency at 1 kHz. If its input is connected to a 120-mV variable frequency signal, find the output voltage at: 200 Hz, 2 kHz and 10 kHz.

(a) $H(w) = \frac{V_o}{V_i} = \frac{\frac{1}{jwC}}{R+\frac{1}{jwC}} = \frac{1}{1+jwCR}$

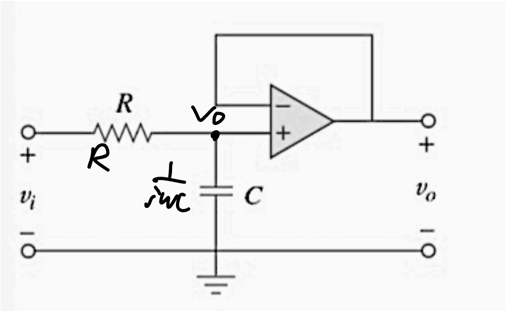


Figure 2

(b) $w = 2\pi B = 2000\pi$

$\therefore \frac{|H(1kHz)|^2}{|H(0)|^2} = \frac{1}{1+C^2R^2w^2} = \frac{1}{2}$

$\Rightarrow 2 = 1+C^2R^2w^2$

$\Rightarrow 1 = C^2R^2w^2$

$CR = \frac{1}{w} = \frac{1}{2000\pi}$

$H(w) = \frac{1}{1+\frac{jw}{2000\pi}} = \frac{2000\pi}{2000\pi + jw}$

$w_1 = 200\cdot 2\pi = 400\pi$

$H(400\pi) = \frac{2000\pi}{2000\pi + 400\pi j} = \frac{5}{5+j} = \frac{5(5-j)}{26} = \frac{25-5j}{26}$

$V_{out} = 0.12\cdot|H(400\pi j)| = 0.11767V$

$w_2 = 2000\cdot 2\pi = 4000\pi$

$H(4000\pi) = \frac{2000\pi}{2000\pi+4000\pi j} = \frac{1}{1+2j}$

$V_{out} = 0.12\cdot|H(4000\pi j)| = 0.053665V$

$w_3 = 10^4\cdot 2\pi = 2\times10^4\pi$

$H(2\times10^4\pi) = \frac{2000\pi}{2000\pi+20000\pi j} = \frac{1}{1+10j}$

$V_{out} = 0.12\cdot|H(20000\pi j)| = 0.01194V$

$F(w) = \frac{4-2\cos w}{5+jw} \leftrightarrow f(t)$

$\Updownarrow$

$F_2(w) = \frac{2-\cos\frac{w}{2}}{5+j\frac{w}{2}} e^{-\frac{jw}{2}} \leftrightarrow f(2t-1)$

Question 3: **(15')**

Consider a causal LTI system implemented as the RLC circuit shown in Figure 3. In this circuit, $x(t)$ is the input voltage. The voltage $y(t)$ across the capacitor is considered the system output.

(a) Find the differential equation relating $x(t)$ and $y(t)$

(b) Determine the frequency response of this system by considering the output of the system to inputs of the form $x(t) = e^{j\omega t}$.

(c) Determine the output $y(t)$ if $x(t) = sin\,(t)$.

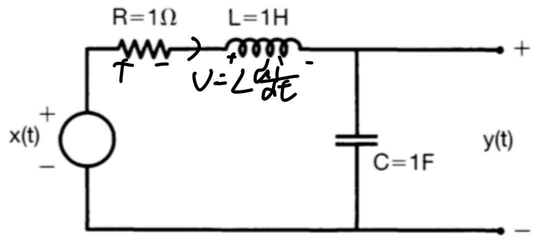


Figure 3

(a) $i = C\frac{dy(t)}{dt}$

KVL: $x(t) = C\frac{d(y(t))}{dt}\cdot R + L\cdot C\frac{d^2y(t)}{dt^2} + y(t)$

$\Rightarrow \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = x(t)$

(b) $y(t) = H(jw)e^{jwt}$

$\Rightarrow -w^2H(jw)e^{jwt} + jwH(jw)e^{jwt} + H(jw)e^{jwt} = e^{jwt}$

$\Rightarrow H(jw) = \frac{1}{-w^2+jw+1}$

(c) $x(t) = \sin t \leftrightarrow j\pi[\delta(w+1) - \delta(w-1)] = X(w)$

$Y(w) = X(w)\cdot H(jw) = \frac{j\pi\,\delta(w+1)}{-w^2+jw+1} + \frac{j\pi}{w^2-jw-1}\delta(w-1)$

$\therefore y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(w)e^{jwt}dt$

$= \frac{1}{2\pi}\left(\frac{j\pi}{-j}e^{-jt} + \frac{j\pi}{-j}e^{jt}\right)$

$= -\frac{1}{2}(2\cos t) = -\cos t$

Question 4: **(10')**

Let

$$g(t) = x(t)cos^2 t * \frac{sin\, t}{\pi t}.$$

($*$ is a convolution operation)

Assuming that $x(t)$ is real and $X(j\omega) = 0$ for $|\omega| \geq 1$, show that there exists an LTI system S such that

$$x(t) \xrightarrow{S} g(t).$$

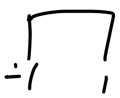
$$x(t)\cos t \longleftrightarrow \frac{1}{2}X(w-1) + \frac{1}{2}X(w+1)$$

$$x(t)\cos^2 t \longleftrightarrow \frac{1}{4}X(w-2) + \frac{1}{4}X(w) + \frac{1}{4}X(w) + \frac{1}{4}X(w+2)$$

$$= \frac{1}{4}X(w-2) + \frac{1}{2}X(w) + \frac{1}{4}X(w+2)$$

$$G(w) = \left[\frac{1}{4}X(w-2) + \frac{1}{2}X(w) + \frac{1}{4}X(w+2)\right] \cdot \frac{1}{\pi} \cdot \pi\, rect\left(\frac{w}{2}\right)$$

$$= \left[\frac{1}{4}X(w-2) + \frac{1}{2}X(w) + \frac{1}{4}(w+2)\right] \cdot rect\left(\frac{w}{2}\right)$$



$$= \frac{1}{2}X(w) \cdot rect\left(\frac{w}{2}\right)$$

$$\therefore H(s) = \frac{G(w)}{X(w)} = \frac{1}{2} rect\left(\frac{w}{2}\right) \longleftrightarrow \frac{1}{2\pi} sinc(t) = h(t)$$

Question 5: **(15')**

Consider the signal $x(t)$ in Figure 5.

(a) Find the Fourier transform $X(j\omega)$ of $x(t)$.

(b) Sketch the signal

$$\tilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k).$$

(c) Find another signal $g(t)$ such that $g(t)$ is not the same as $x(t)$ and

$$\tilde{x}(t) = g(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k).$$

$(b) = \sum_{k=-\infty}^{\infty} x(t-4k)$

$\tilde{x}(t)$

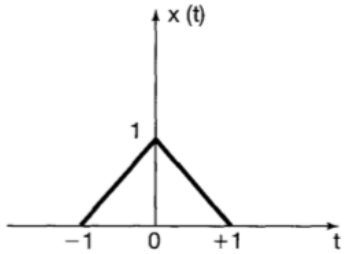


Figure 5

$(a)\ x(t) = \Delta\left(\frac{t}{2}\right) \leftrightarrow X(j\omega) = \mathrm{sinc}^2\left(\frac{\omega}{2}\right)$

$(b)\ \because x(t) \leftrightarrow \mathrm{sinc}^2\left(\frac{\omega}{2}\right)$

$\sum_{k=-\infty}^{\infty} \delta(t-4k) \leftrightarrow \frac{\pi}{2} \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{\pi}{2}\right)$

$\frac{\pi}{2}$ $\qquad \mathrm{sinc}^2\left(\frac{n\pi}{4}\right)$

$\therefore \tilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k) \leftrightarrow \mathrm{sinc}^2\left(\frac{\omega}{2}\right) \cdot \frac{\pi}{2} \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{\pi}{2}\right)$

$= \frac{\pi}{2} \sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{\omega}{2}\right) \delta\left(\omega - n\frac{\pi}{2}\right)$

$(c) \therefore \tilde{x}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{X}(\omega) e^{j\omega t} d\omega$

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\pi}{2} \sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{\omega}{2}\right) \delta\left(\omega - n\frac{\pi}{2}\right) e^{j\omega t} d\omega$

$= \frac{1}{4} \sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{n\pi}{4}\right) e^{\frac{n\pi}{2} j t}$

Question 6: **(15')**

Consider the system S characterized by the differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t). \qquad t>0$$

(a) Determine the zero-state response of this system for the input $x(t) = e^{-4t}u(t)$.

(b) Determine the zero-input response of this system for $t > 0^-$, given that

$$y(0^-) = 1, \qquad \left.\frac{dy(t)}{dt}\right|_{t=0^-} = -1$$

(c) Determine the output of S when the input is $x(t) = e^{-4t}u(t)$ and the initial conditions are the same as those specified in part (b).

$t>0$

(a) $\lambda^2 - \lambda - 2 = 0 \Rightarrow (\lambda-2)(\lambda+1) = 0 \Rightarrow \lambda_1 = 2,\ \lambda_2 = -1,\ m_1 = m_2 = 1$

fundamental: $e^{2t},\ e^{-t}$

$\therefore y_0(t) = c_1 e^{2t} + c_2 e^{-t}$

$e^{-4t},\ \mu = -4,\ k = 0 \Rightarrow m = 0$

$\therefore y_p(t) = c_0 e^{-4t}$

$16c_0 \cancel{e^{-4t}} + 4c_0 \cancel{e^{-4t}} - 2c_0 \cancel{e^{-4t}} = \cancel{e^{-4t}}\ 1$

$18c_0 = 1 \Rightarrow c_0 = \frac{1}{18}$

$\therefore y(t) = c_0 e^{-4t} + c_1 e^{2t} + c_2 e^{-t} = \frac{1}{18} e^{-4t} + \overset{\frac{1}{18}}{c_1} e^{2t} + \overset{-\frac{1}{9}}{c_2} e^{-t}$

$\cancel{y(0) = \frac{1}{18} + c_1 + c_2 = 0}$

$\cancel{y'(0) = -\frac{2}{9} e^{-4t} + 2c_1 e^{2t} - c_2 e^{-t} = 0}$

$\Rightarrow y(0) = \frac{1}{18} + c_1 + c_2 = 0$

$y'(0) = -\frac{2}{9} + 2c_1 - c_2 = 0$

$-\frac{1}{6} + 3c_1 = 0$

$c_1 = \frac{1}{18}$

$c_2 = -\frac{1}{9}$

(b) $y_0(t) = c_1 e^{2t} + c_2 e^{-t}$

$y(0) = c_1 + c_2 = 1$

$y'(t) = 2c_1 e^{2t} - c_2 e^{-t}$

$y'(0) = 2c_1 - c_2 = -1$

$c_1 = 0,\ c_2 = 1$

$y_0(t) = e^{-t}$

(c) $y(t) = \frac{1}{18} e^{-4t} + \frac{1}{18} e^{2t} + \frac{8}{9} e^{-t}$

Question 7: **(20')**

Figure 7 shows a system to be used for sinusoidal amplitude modulation. $x(t)$ is band-limited with a maximum frequency ω_M so that $X(j\omega) = 0, |\omega| > \omega_M$. As indicated, the signal $s(t)$ is a periodic impulse train with period T and with an offset from $t = 0$ of Δ. The system $H(j\omega)$ is a band-pass filter.

(a) With $\Delta = 0, \omega_M = \frac{\pi}{2T}, \omega_l = \frac{\pi}{T}$ *and* $\omega_h = \frac{3\pi}{T}$, determine $y(t)$ and sketch $Y(j\omega)$.

(b) If ω_M, ω_l and ω_h are the same as given in part (a), but Δ is non-zero, determine $y(t)$.

($y(t)$ is represented by Δ and T)

$w_M = \frac{\pi}{2T}$

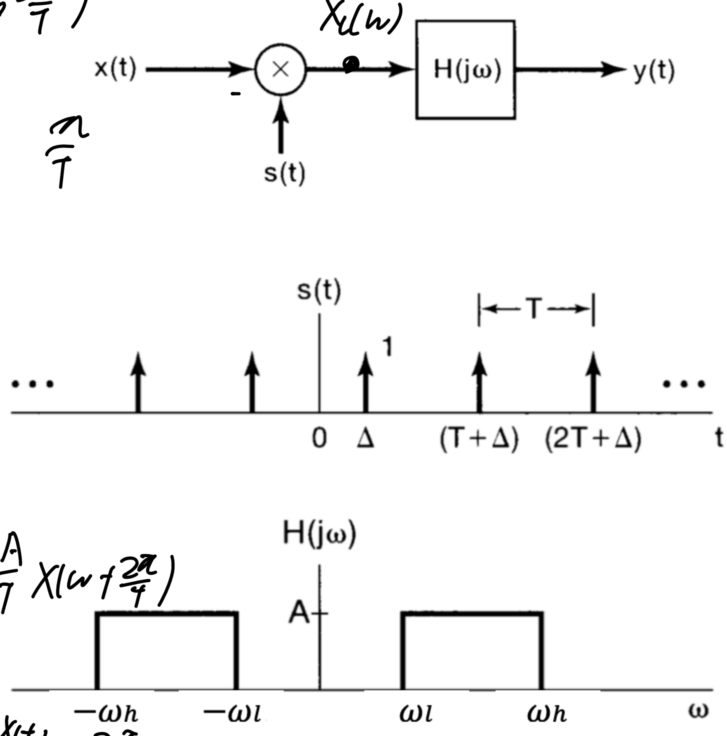
(a) $X_c(w) = \sum_{n=-\infty}^{\infty} \frac{1}{T} X(w - n\frac{2\pi}{T})$

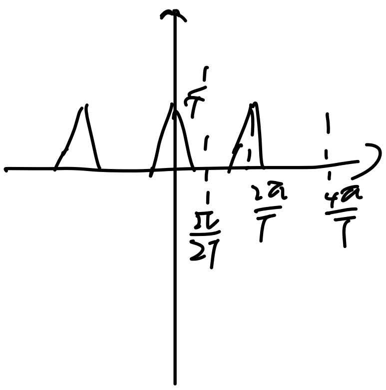
(b) $X_c(w) = \sum_{n=-\infty}^{\infty} \frac{1}{T} X(w - n\frac{2\pi}{T}) \cdot e^{-j\Delta w}$

$= \sum_{n=-\infty}^{\infty} \frac{e^{-j\Delta w}}{T} X(w - n\frac{2\pi}{T})$

$\therefore Y(jw) = \frac{Ae^{-j\Delta w}}{T} X(w - \frac{2\pi}{T}) + \frac{Ae^{-j\Delta w}}{T} X(w + \frac{2\pi}{T})$

$\frac{\pi}{T}$





$Y(jw) = \frac{A}{T} X(w - \frac{2\pi}{T}) + \frac{A}{T} X(w + \frac{2\pi}{4})$

$Y(t) = \frac{A}{2\pi T} x(t) e^{j\frac{2\pi}{T}t} + \frac{A}{2\pi T} x(t) e^{-\frac{2\pi}{T}t}$

Figure 7

M

$\therefore y(t) = \frac{A}{T} e^{j\frac{2\pi}{T}(t-\Delta)} x(t-\Delta) + \frac{A}{T} e^{-j\frac{2\pi}{T}(t-\Delta)} x(t-\Delta)$

$= \frac{A}{T}$

Formula Sheet

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t) e^{-jn\omega_o t} dt$
$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\lvert F_n \rvert$ $\theta_n = \angle F_n$

Table 1: Fourier series forms.

	Name:	Condition:	Property:
1	Scaling	Constant K	$K\, f(t) \leftrightarrow K\, F_n$
2	Addition	$f(t) \leftrightarrow F_n,\ g(t) \leftrightarrow G_n,\ \ldots$	$f(t) + g(t) + \ldots \leftrightarrow F_n + G_n + \ldots$
3	Time shift	Delay t_o	$f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
8	Average power		$P \equiv \frac{1}{T}\int_T \lvert f(t)\rvert^2 dt = \sum_{n=-\infty}^{\infty} \lvert F_n \rvert^2$

Table 2: Fourier series properties

	Name:	Condition:	Property:
1	Amplitude scaling	$f(t) \leftrightarrow F(\omega)$, constant K	$Kf(t) \leftrightarrow KF(\omega)$
2	Addition	$f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$, $\cdots$	$f(t)+g(t)+\cdots \leftrightarrow F(\omega)+G(\omega)+\cdots$
3	Hermitian	Real $f(t) \leftrightarrow F(\omega)$	$F(-\omega)=F^*(\omega)$
4	Even	Real and even $f(t)$	Real and even $F(\omega)$
5	Odd	Real and odd $f(t)$	Imaginary and odd $F(\omega)$
6	Symmetry	$f(t) \leftrightarrow F(\omega)$	$F(t) \leftrightarrow 2\pi f(-\omega)$
7	Time scaling	$f(t) \leftrightarrow F(\omega)$, real s	$f(st) \leftrightarrow \frac{1}{\lvert s\rvert}F(\frac{\omega}{s})$
8	Time shift	$f(t) \leftrightarrow F(\omega)$	$f(t-t_o) \leftrightarrow F(\omega)e^{-j\omega t_o}$
9	Frequency shift	$f(t) \leftrightarrow F(\omega)$	$f(t)e^{j\omega_o t} \leftrightarrow F(\omega-\omega_o)$
10	Modulation	$f(t) \leftrightarrow F(\omega)$	$f(t)\cos(\omega_o t) \leftrightarrow \frac{1}{2}F(\omega-\omega_o)+\frac{1}{2}F(\omega+\omega_o)$
11	Time derivative	Differentiable $f(t) \leftrightarrow F(\omega)$	$\frac{df}{dt} \leftrightarrow j\omega F(\omega)$
12	Freq derivative	$f(t) \leftrightarrow F(\omega)$	$-jtf(t) \leftrightarrow \frac{d}{d\omega}F(\omega)$
13	Time convolution	$f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$	$f(t)*g(t) \leftrightarrow F(\omega)G(\omega)$
14	Freq convolution	$f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$	$f(t)g(t) \leftrightarrow \frac{1}{2\pi}F(\omega)*G(\omega)$
15	Compact form	Real $f(t)$	$f(t)=\frac{1}{2\pi}\int_0^\infty 2\lvert F(\omega)\rvert\cos(\omega t+\angle F(\omega))d\omega$
16	Parseval, Energy W	$f(t) \leftrightarrow F(\omega)$	$W \equiv \int_{-\infty}^{\infty}\lvert f(t)\rvert^2 dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert F(\omega)\rvert^2 d\omega$

Table 3: Important properties of the Fourier transform.

	$f(t) \leftrightarrow F(\omega)$		
1	$e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega}$, $a>0$	14	$\delta(t) \leftrightarrow 1$
2	$e^{at}u(-t) \leftrightarrow \frac{1}{a-j\omega}$, $a>0$	15	$1 \leftrightarrow 2\pi\delta(\omega)$
3	$e^{-a\lvert t\rvert} \leftrightarrow \frac{2a}{a^2+\omega^2}$, $a>0$	16	$\delta(t-t_o) \leftrightarrow e^{-j\omega t_o}$
4	$\frac{a^2}{a^2+t^2} \leftrightarrow \pi a e^{-a\lvert\omega\rvert}$, $a>0$	17	$e^{j\omega_o t} \leftrightarrow 2\pi\delta(\omega-\omega_o)$
5	$te^{-at}u(t) \leftrightarrow \frac{1}{(a+j\omega)^2}$, $a>0$	18	$\cos(\omega_o t) \leftrightarrow \pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$
6	$t^n e^{-at}u(t) \leftrightarrow \frac{n!}{(a+j\omega)^{n+1}}$, $a>0$	19	$\sin(\omega_o t) \leftrightarrow j\pi[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]$
7	$\mathrm{rect}(\frac{t}{\tau}) \leftrightarrow \tau\mathrm{sinc}(\frac{\omega\tau}{2})$	20	$\cos(\omega_o t)u(t) \leftrightarrow \frac{\pi}{2}[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]+\frac{j\omega}{\omega_o^2-\omega^2}$
8	$\mathrm{sinc}(Wt) \leftrightarrow \frac{\pi}{W}\mathrm{rect}(\frac{\omega}{2W})$	21	$\sin(\omega_o t)u(t) \leftrightarrow j\frac{\pi}{2}[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]+\frac{\omega_o}{\omega_o^2-\omega^2}$
9	$\triangle(\frac{t}{\tau}) \leftrightarrow \frac{\tau}{2}\mathrm{sinc}^2(\frac{\omega\tau}{4})$	22	$\mathrm{sgn}(t) \leftrightarrow \frac{2}{j\omega}$
10	$\mathrm{sinc}^2(\frac{Wt}{2}) \leftrightarrow \frac{2\pi}{W}\triangle(\frac{\omega}{2W})$	23	$u(t) \leftrightarrow \pi\delta(\omega)+\frac{1}{j\omega}$
11	$e^{-at}\sin(\omega_o t)u(t) \leftrightarrow \frac{\omega_o}{(a+j\omega)^2+\omega_o^2}$, $a>0$	24	$\sum_{n=-\infty}^{\infty}\delta(t-nT) \leftrightarrow \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\frac{2\pi}{T})$
12	$e^{-at}\cos(\omega_o t)u(t) \leftrightarrow \frac{a+j\omega}{(a+j\omega)^2+\omega_o^2}$, $a>0$	25	$\sum_{n=-\infty}^{\infty}f(t)\delta(t-nT) \leftrightarrow \sum_{n=-\infty}^{\infty}\frac{1}{T}F(\omega-n\frac{2\pi}{T})$
13	$e^{-\frac{t^2}{2\sigma^2}} \leftrightarrow \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$		

Table 4: Important Fourier transform pairs. The left-hand column includes only "energy signals" $f(t)$, while the right-hand column includes "power signals" and distributions.

Name	Property
Commutative	$h(t) * f(t) = f(t) * h(t)$
Distributive	$f(t) * (g(t) + h(t)) = f(t) * g(t) + f(t) * h(t)$
Associative	$f(t) * (g(t) * h(t)) = (f(t) * g(t)) * h(t)$
Shift	$h(t) * f(t) = y(t) \Rightarrow h(t - t_0) * f(t) = h(t) * f(t - t_0) = y(t - t_0)$
Derivative	$h(t) * f(t) = y(t) \Rightarrow (\frac{d}{dt}h(t)) * f(t) = h(t) * (\frac{d}{dt}f(t)) = \frac{d}{dt}y(t)$
Reversal	$f(t) * h(t) = y(t) \Rightarrow h(-t) * f(-t) = y(-t)$
Start-point	If $h(t) = 0$ for $t < t_{sh}$ and $f(t) = 0$ for $t < t_{sf}$ then $y(t) = h(t) * f(t) = 0$ for $t < t_{sy} = t_{sh} + t_{sf}$.
End-point	If $h(t) = 0$ for $t > t_{eh}$ and $f(t) = 0$ for $t > t_{ef}$ then $y(t) = h(t) * f(t) = 0$ for $t > t_{ey} = t_{eh} + t_{ef}$.
Width	$h(t) * f(t) = y(t) \Rightarrow T_y = T_h + T_f$ where T_h, T_f, and T_y denote the widths of $h(t)$, $f(t)$, and $y(t)$.

Table 5: Convolution properties.

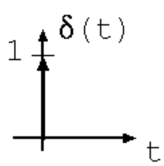
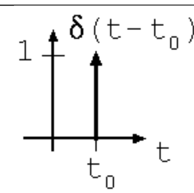
Name	Impulse properties	Shifted-impulse properties
Convolution	$\delta(t) * f(t) = f(t)$	$\delta(t - t_0) * f(t) = f(t - t_0)$
Sifting	$\int_{-\infty}^{\infty} \delta(t) f(t) dt = f(0)$ and $\int_a^b \delta(t) f(t) dt = \begin{cases} f(0) & \text{if } a < 0 < b \\ 0, & otherwise \end{cases}$	$\int_{-\infty}^{\infty} \delta(t - t_0) f(t) dt = f(t_0)$ and $\int_a^b \delta(t - t_0) f(t) dt = \begin{cases} f(t_0) & \text{if } a < t_0 < b \\ 0, & otherwise \end{cases}$
Sampling	$f(t)\delta(t) = f(0)\delta(t)$	$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$
Symmetry	$\delta(-t) = \delta(t)$	$\delta(t_0 - t) = \delta(t - t_0)$
Scaling	$\delta(at) = \frac{1}{\lvert a \rvert}\delta(t),\ a \neq 0$	$\delta(a(t - t_0)) = \frac{1}{\lvert a \rvert}\delta(t - t_0),\ a \neq 0$
Area	$\int_{-\infty}^{\infty} \delta(t) dt = 1$ and $\int_a^b \delta(t) dt = \begin{cases} 1 & \text{if } a < 0 < b \\ 0, & otherwise \end{cases}$	$\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1$ and $\int_a^b \delta(t - t_0) dt = \begin{cases} 1 & \text{if } a < t_0 < b \\ 0, & otherwise \end{cases}$
Definite integral	$\int_{-\infty}^{t} \delta(\tau) d\tau = u(t)$	$\int_{-\infty}^{t} \delta(\tau - t_0) d\tau = u(t - t_0)$
Unit-step derivative	$\frac{d}{dt}u(t) = \delta(t)$	$\frac{d}{dt}u(t - t_0) = \delta(t - t_0)$
Derivative	$(\frac{d}{dt}\delta(t)) * f(t) = \frac{d}{dt}f(t)$	$(\frac{d}{dt}\delta(t - t_0)) * f(t) = \frac{d}{dt}f(t - t_0)$
Fourier transform	$\delta(t) \leftrightarrow 1$	$\delta(t - t_0) \leftrightarrow e^{-j\omega t_0}$
Graphical symbol		

Table 6: Properties of the impulse and shifted impulse.

1	$\delta(t) \leftrightarrow 1$	7	$\delta'(t) \leftrightarrow s$
2	$e^{pt}u(t) \leftrightarrow \frac{1}{s-p}$	8	$u(t) \leftrightarrow \frac{1}{s}$
3	$te^{pt}u(t) \leftrightarrow \frac{1}{(s-p)^2}$	9	$tu(t) \leftrightarrow \frac{1}{s^2}$
4	$t^n e^{pt}u(t) \leftrightarrow \frac{n!}{(s-p)^{n+1}}$	10	$t^n u(t) \leftrightarrow \frac{n!}{s^{n+1}}$
5	$\cos(\omega_o t)u(t) \leftrightarrow \frac{s}{s^2+\omega_o^2}$	11	$\sin(\omega_o t)u(t) \leftrightarrow \frac{\omega_o}{s^2+\omega_o^2}$
6	$e^{-\alpha t}\cos(\omega_d t)u(t) \leftrightarrow \frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$	12	$e^{-\alpha t}\sin(\omega_d t)u(t) \leftrightarrow \frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Table 7: Laplace transforms pairs $h(t) \leftrightarrow \hat{H}(s)$ involving frequently encountered causal signals — α, ω_o, and ω_d stand for arbitrary real constants, n for non-negative integers, and p denotes for an arbitrary complex constant.

	Name:	Condition:	Property:
1	Multiplication	$f(t) \to \hat{F}(s)$, constant K	$Kf(t) \to K\hat{F}(s)$
2	Addition	$f(t) \to \hat{F}(s)$, $g(t) \to \hat{G}(s) \cdots$	$f(t) + g(t) + \cdots \to \hat{F}(s) + \hat{G}(s) + \cdots$
3	Time scaling	$f(t) \to \hat{F}(s)$, real $a > 0$	$f(at) \to \frac{1}{a}\hat{F}(\frac{s}{a})$
4*	Time delay	$f(t) \leftrightarrow \hat{F}(s)$, $t_o \geq 0$	$f(t - t_o) \leftrightarrow \hat{F}(s)e^{-st_o}$
5	Frequency shift	$f(t) \to \hat{F}(s)$	$f(t)e^{s_o t} \to \hat{F}(s - s_o)$
6	Time derivative	Differentiable $f(t) \to \hat{F}(s)$	$f'(t) \to s\hat{F}(s) - f(0^-)$ $f''(t) \to s^2\hat{F}(s) - sf(0^-) - f'(0^-)$ $\cdots$ $f^{(n)}(t) \to s^n\hat{F}(s) - \cdots - f^{(n-1)}(0^-)$
7	Time integration	$f(t) \to \hat{F}(s)$	$\int_{0^-}^{t} f(\tau)d\tau \to \frac{1}{s}\hat{F}(s)$
8	Freq. derivative	$f(t) \to \hat{F}(s)$	$-tf(t) \to \frac{d}{ds}\hat{F}(s)$
9*	Time convolution	$h(t) \leftrightarrow \hat{H}(s)$, $f(t) \leftrightarrow \hat{F}(s)$	$h(t) * f(t) \leftrightarrow \hat{H}(s)\hat{F}(s)$
10	Freq. convolution	$f(t) \to \hat{F}(s)$, $g(t) \to \hat{G}(s)$	$f(t)g(t) \to \frac{1}{2\pi j}\hat{F}(s) * \hat{G}(s)$
11	Poles	$f(t) \to \hat{F}(s)$	Values of s such that $\lvert\hat{F}(s)\rvert = \infty$
12	ROC	$f(t) \to \hat{F}(s)$	Portion of $s-$ plane to the right of rightmost pole $\neq \infty$
13*	Fourier transform	$f(t) \leftrightarrow \hat{F}(s)$	$F(\omega) = \hat{F}(j\omega)$ if and only if ROC includes $s = j\omega$
14	Final value	Poles of $s\hat{F}(s)$ in LHP	$f(\infty) = \lim_{s\to 0} s\hat{F}(s)$
15	Initial value	Existence of the limit	$f(0^+) = \lim_{s\to\infty} s\hat{F}(s)$

Table 8: Important properties of the *one-sided* Laplace transform. Properties marked by * in the first column hold only for causal signals.