**University of Illinois** Fall 2022

# ECE 210/211: Exam 3

Wednesday, November 16, 2022

**Name: (in BLOCK CAPITALS)** ______________________________

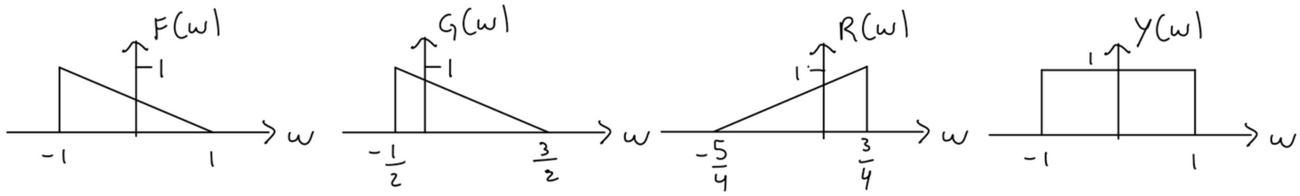
**UIN:** ____________________ **NetID:** ____________________

## Instructions

- Clearly PRINT your name in CAPITAL LETTERS.
- Cleary write your UIN and netID.
- This is a closed book and closed notes exam.
- Calculators and other electronic devices are not allowed.
- To get credit, you must SHOW ALL your work and provide explanations.
- To get full credit, simplify your answers.
- All answers must INCLUDE UNITS whenever appropriate.
- Angles must be expressed in the range $(-\pi, \pi]$ rad.
- Write your final answers in the spaces provided or points may be deducted.
- The exam is printed double-sided.
- There is an empty page at the end of the exam in case you need additional space.

1. **[30 points]** The four parts of this problem are unrelated

(a) [15 pts] This question uses Fourier transform properties. The signal $f(t)$ has Fourier transform $F(\omega)$, plotted below. Determine the inverse Fourier transforms of $G(\omega)$, $R(\omega)$ and $Y(\omega)$, in terms of $f(t)$.




$f(-t) \Leftrightarrow F(w)$

$f(t)e^{\frac{1}{2}jt} \Leftrightarrow G(w) = F(w - \frac{1}{2})$

$f(-t)e^{-\frac{jt}{4}} \Leftrightarrow R(w) = F(-(w + \frac{1}{4}))$

$f(t) + f(-t)$

$y(t) = \frac{1}{2\pi} \int_{-1}^{1} e^{jnwt} \, dw$

$= \frac{1}{2\pi} \frac{e^{jnt} - e^{-jnt}}{jnt}$

$= \frac{2j\sin(t)}{2\pi j \, t} - \frac{\sin(t)}{\pi \, t}$

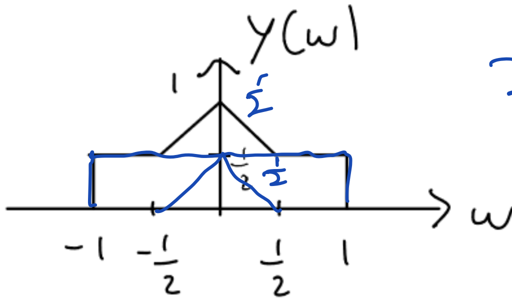
?

$g(t) =$ ______________________

$r(t) =$ ______________________

$y(t) =$ ______________________

(b) [5 pts] Determine the inverse Fourier transform of $Y(\omega)$, plotted below.




$\frac{1}{2}\text{rect}\left(\frac{w}{2}\right) + \frac{1}{2}\Delta(w)$

$\Downarrow$

$\frac{1}{2\pi}\text{sinc}\, t + \frac{1}{8\pi}\text{sinc}^2\left(\frac{t}{4}\right)$

$w+1$

$-w+1$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(w)\, e^{jwt}\, dw$$

$$= \frac{1}{2\pi}\left(\int_{-1}^{-\frac{1}{2}} e^{jwt}dw + \int_{-\frac{1}{2}}^{0} we^{jwt}dw + \int_{-\frac{1}{2}}^{0} e^{jwt}dw + \int_{0}^{\frac{1}{2}} -we^{jwt}dw + \int_{0}^{\frac{1}{2}} e^{jwt}dw + \int_{\frac{1}{2}}^{1} e^{jwt}dw\right)$$

$e^{jwt}$

$$= \frac{1}{2\pi}\left(\frac{e^{jt}-e^{-jt}}{jt} + \frac{1}{jt}\left[(jwt-1)e^{jwt}\Big|_{-\frac{1}{2}}^{0}\right] - \frac{1}{jt}\left[(jwt-1)e^{jwt}\Big|_{0}^{\frac{1}{2}}\right]\right)$$

$$y(t) = \frac{1}{\pi}\text{sinc}\, t - \frac{j}{2\pi}\sin\left(\frac{t}{2}\right) + \frac{1}{\pi jt}\cos\left(\frac{t}{2}\right)$$

$xe^{t}$

$(t-1)e^{t}$

$e^{t} + (t-1)e^{t}$

$\frac{1}{jt}\left[\cancel{1} - \left(-\frac{jt}{2}-1\right)e^{-\frac{1}{2}jt} - \left(\frac{jt}{2}-1\right)e^{\frac{jt}{2}} \cancel{+1}\right]$

$\frac{1}{2\pi}\cdot\frac{1}{jt}\left[\frac{jt}{2}\left(-2j\sin\left(\frac{t}{2}\right) + 2\cos\left(\frac{t}{2}\right)\right)\right]$

$\frac{-2j}{4\pi}$

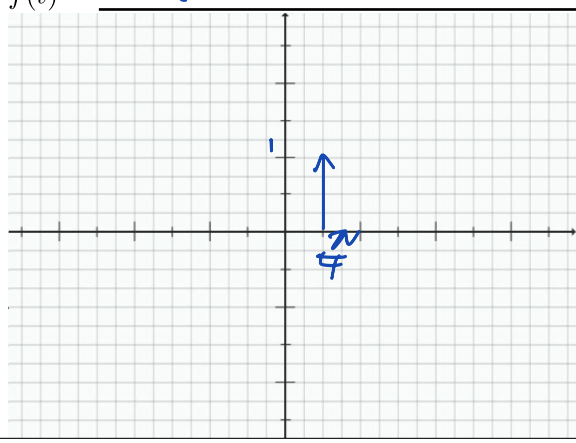
$y(t) =$ ______________________

$\frac{\pi}{4}$ rect $(\frac{t}{4})$

(c) [5 pts] Let $f(t) = rect(\frac{t}{\pi})[\delta(t-\frac{\pi}{4})-\delta(t+2\pi)]$. Obtain a simplified time-domain expression and plot the function on the axes provided. Be sure to clearly label the figure axes.

$f(t) =$

$f(t) =$ $\delta(t-\frac{\pi}{4})$




(d) [5 pts] Let $g(t) = rect(\frac{t}{\pi}) * [\delta(t-\frac{\pi}{4}) - \delta(t+2\pi)]$. Obtain a simplified time-domain expression and plot the function on the axes provided. Be sure to clearly label the figure axes.

$k = \frac{t}{\pi}$ $\quad$ $rect(\frac{t}{\pi}) * [\delta(t-\frac{\pi}{4})$ $\quad$ $\Omega = 0$ $\quad$ $w = \frac{\pi}{4}$

$t = k\pi$

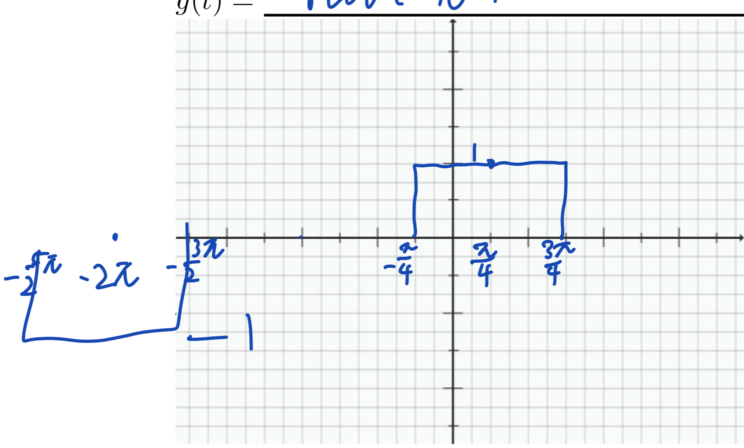
$\int_{-\infty}^{\infty} rect(\frac{\Omega}{\pi})\delta(w-\Omega-\frac{\pi}{4})\,d\Omega$

$w = \Omega + \frac{\pi}{4}$

$\frac{\pi}{2}$

$g(t) =$ $rect(\frac{t}{\pi}-\frac{1}{4}) - rect(\frac{t}{\pi}+2)$

$t - 2\pi$



2. **[20 points]** The two parts of this problem are not directly dependent upon each other.

(a) [12 pts] Let $f_1(t) = \text{rect}(\frac{t-1}{2})$ and $h_1(t) = e^{-2t}\text{rect}(\frac{t-1}{2})$, and let $y_1(t) = f_1(t) * h_1(t)$. Determine $y_1(t)$ for all $-\infty < t < \infty$.

$Y_1(t) = f_1(t) * h_1(t)$

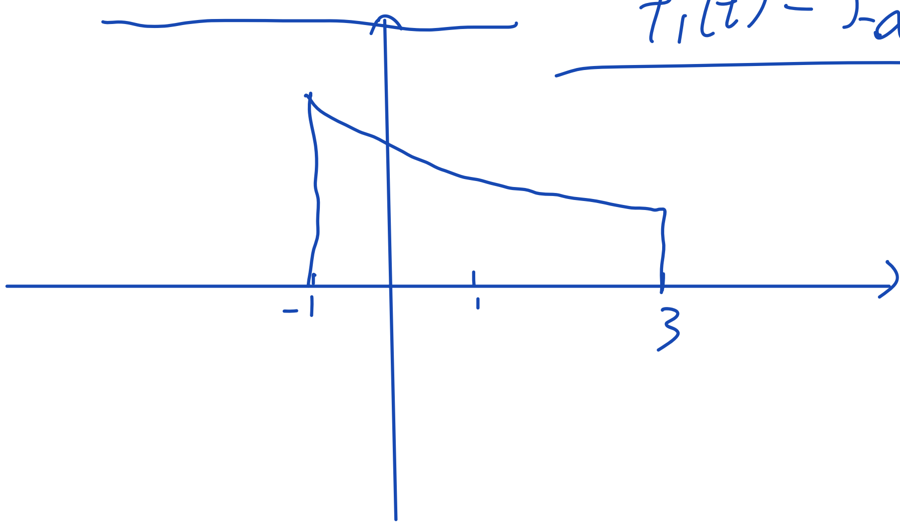
$\updownarrow$

$Y(w) = F(w)H(w)$

$F(w) = 2\,\text{Sinc}(w) \cdot e^{-jw}$

$h_1(t) = e^{-2t} f_1(t)$ ?

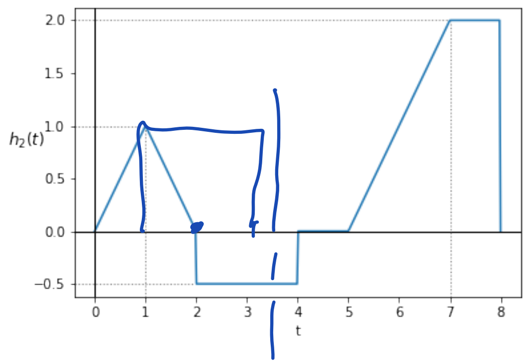
$f_1(t) = \int_{-\infty}^{\infty} h_1(t-\Omega) \cdot f_1(\Omega)\, d\Omega$




$y_1(t) =$ ____________________

$f(t)$

(b) [8 pts] Let $f_2(t) = rect(\frac{t-2}{2})$ (please note the delay) be the input to an LTI system. Let $h_2(t)$ be the impulse response of the LTI system as sketched below. The output of the system is given by $y_2(t) = f_2(t) * h_2(t)$. Determine $y_2(7)$, that is, $y_2(t)$ at $t = 7$ s.




$Y(w) = F(w) \cdot H(w)$

$f(7-\Omega) \quad h(\Omega)$

$H(w): 2rect(t-7.5) \Longleftrightarrow 2sinc(\frac{w}{2}) e^{-7.5jw}$

$F(w): rect(\frac{t-2}{2}) \Longleftrightarrow 2sinc(w)\, e^{-2jw}$

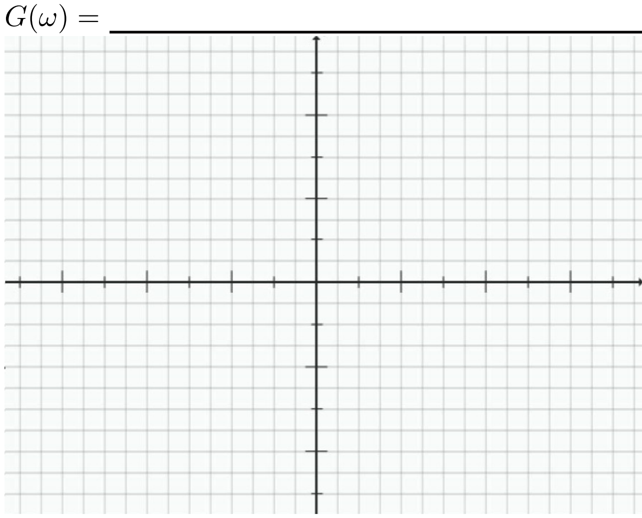
$\therefore Y(w) =$

$y_2(7) =$ ________________

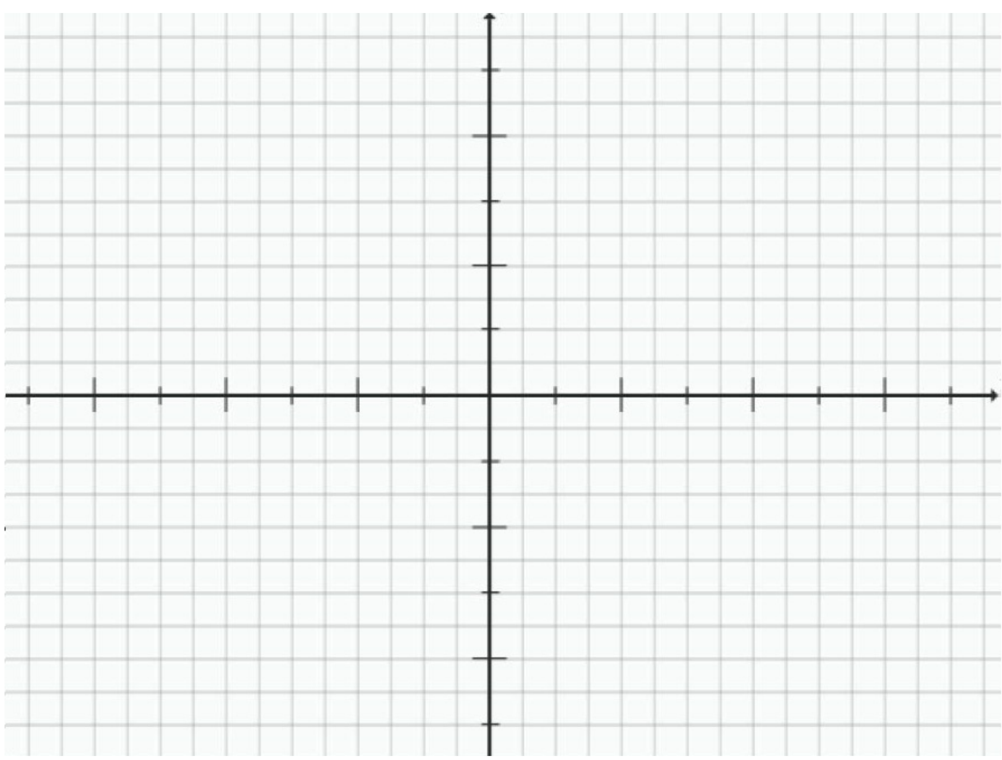
3. **[25 points]** An analog signal $g(t) = 4\cos(2\pi t) + 2\cos(5\pi t)$ is sampled every $T = 0.25$ seconds.

(a) [6 pts] Write an expression for $G(\omega)$, the Fourier transform of $g(t)$ and plot $G(\omega)$ over the range $-16\pi \leq \omega \leq 16\pi$. Clearly label the figure axes.

$G(\omega) =$ ________________________________________



(b) [14 pts] Plot $G_T(\omega)$, the Fourier transform of the sampled signal over the range $-16\pi \leq \omega \leq 16\pi$.

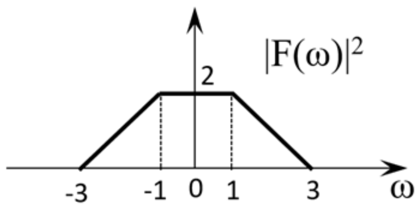


(c) [5 pts] If the sampled signal passed through an ideal (rect) low pass filter with bandwidth of $5.5\pi \frac{\text{rad}}{\text{s}}$ and amplitude of $\frac{1}{4}$, what is the output signal $y(t)$?

$y(t) =$ ______________________________

4. **[25 points]** The three parts of this problem are unrelated.

(a) [3 pts] Determine the 3-dB bandwidth of the signal $f(t)$ having the energy spectrum shown below:

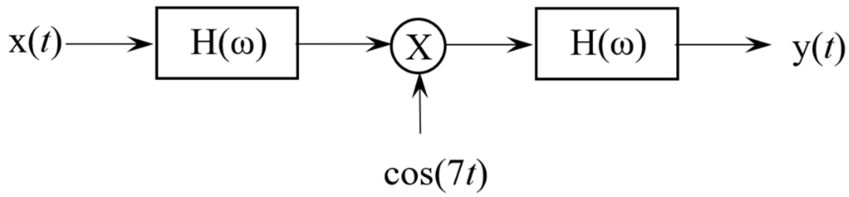


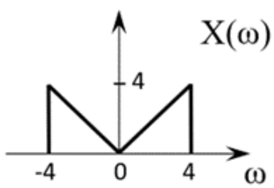

$\Omega_{3dB}$ = ______________________

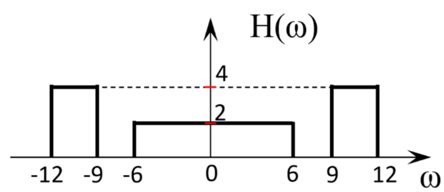
(b) [10 pts] Calculate the total energy of the signal $z(t) = \text{sinc}(2\pi t)$.

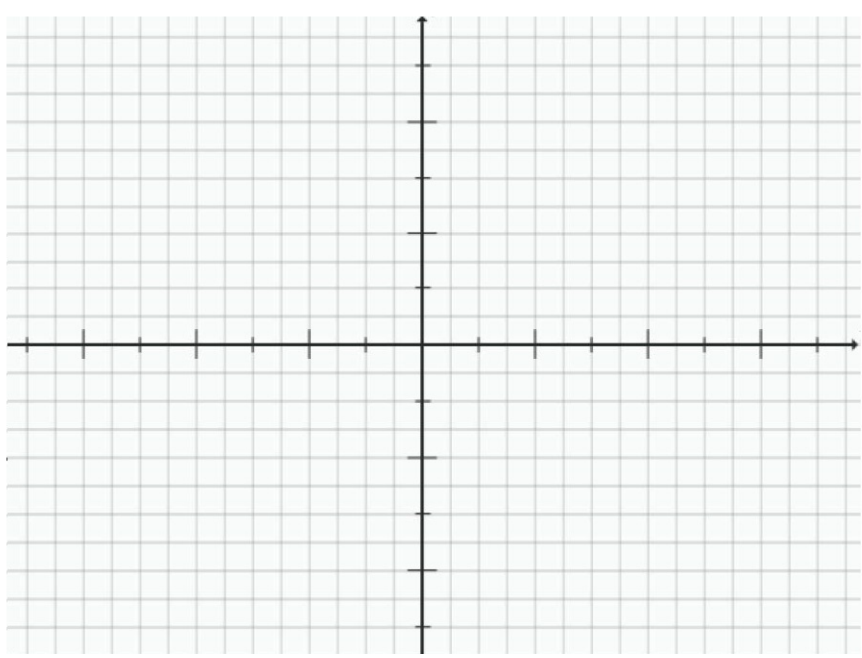
$W$ = ______________________

(c) [12 pts] Consider the system below. The Fourier transform of the input function and the frequency response, $H(\omega)$, are given. Plot the Fourier transform of the output function. Clearly label the critical points on both axes.



You may use this sheet for additional calculations but do not separate this sheet from the rest of the exam.

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