Question 1:

Determine the power of each current sources and voltage sources in the following circuit in Figure 1.

assume $V_a = 0$

use KCL at d, c, a, b

$$i_1 + 5 + 1 = 0$$
$$3 + i_3 + 2 = 1$$
$$1 + i_2 + 3 + i_1 + 5 = 0$$
$$1 + i_2 = i_3 + 2$$

$$\Rightarrow \begin{cases} i_1 = -6A \\ i_3 = -4A \\ i_2 = -3A \end{cases}$$

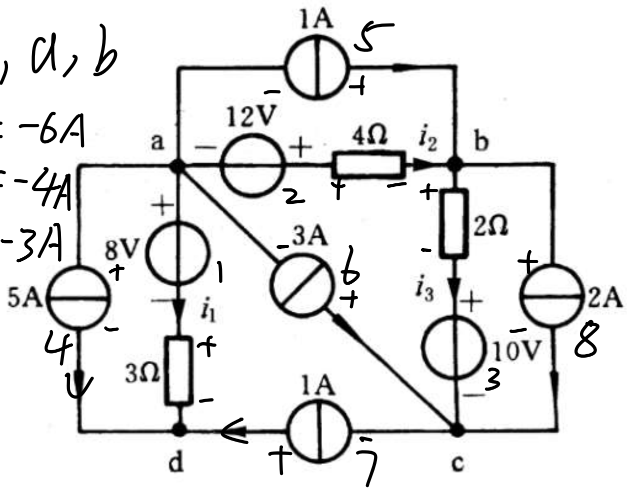


Figure 1

$V_7 = -22 - 8 + 18$

Use KVL

$$V_4 = 8 + 3 \cdot i_1$$
$$V_5 + i_2 \cdot 4 = 12$$
$$V_6 + 10 + 2 \cdot i_3 + i_2 \cdot 4 = 12$$
$$V_6 + V_7 + 3 i_1 + 8 = 0$$
$$V_8 = 2 i_3 + 10$$

$$\Rightarrow \begin{cases} V_4 = -10V \\ V_5 = 24V \\ V_6 = 22V \\ V_7 = -12V \\ V_8 = 2V \end{cases}$$

∴ for voltage source:

$P_1 = -8 \cdot 6 \text{ W} = -48\text{W}$

$P_2 = 12 \cdot 3 \text{ W} = 36\text{W}$

$P_3 = -4 \cdot 10 \text{ W} = -40\text{W}$

$$\Rightarrow \begin{cases} P_4 = -10 \cdot 5 \text{ W} = -50\text{W} \\ P_5 = -24 \cdot 1 \text{ W} = -24\text{W} \\ P_6 = -22 \cdot 3 \text{ W} = -66\text{W} \\ P_7 = 12 \cdot 1 \text{ W} = 12\text{W} \end{cases}$$

$P_8 = 2 \cdot 2 \text{ W} = 4\text{W}$

Question 2:

$\frac{V_{oc}^2}{\cancel{2}\cdot 4} = 4.5$ $\quad 9 \quad$ $U_{oc} = 6V$

In the following circuit in Figure 2, R_L is an adjustable resistor. The R_L can get maximum power of P_{max}= 4.5 W when R_L equals to 2 Ω. Determine R and U_S in the circuit.

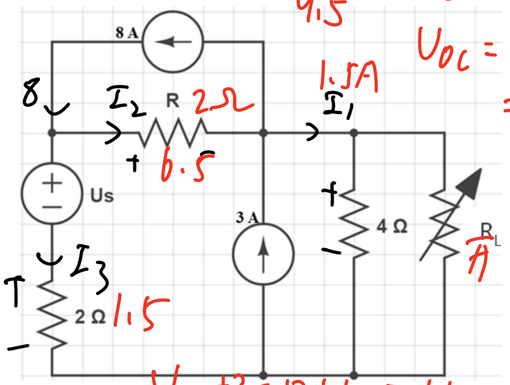
$V_{oc} + 2I_2 = U_s + 2I_3$

9.5

$U_{oc} = 4(I_2 - 5)$
$= 4I_2 - 20$

1.5A I_1

$8 = I_2 + I_3$
$3 + I_2 = 8 + I_1$
$I_2 + I_3 = 8$
$I_2 - I_1 = 5$



$\frac{(2+R)4}{2+R+4} = 2$
$8 + 4R = 12 + 2R$
$2R = 4$
$\boxed{R=2}$

$U_s + 3 = 13 + 6 \Rightarrow U_s = 16V$

Figure 2

when open : $\begin{cases} I_2 + I_3 = 8 \\ I_2 - I_1 = 5 \\ U_s + 2I_3 = RI_2 + 4I_1 \end{cases}$ $\quad 5 + I_1 + I_3 = 8 \quad I_3 = 3 - I_1$ $\quad I_2 = 5 + I_1$

$\Rightarrow U_s + 6 - 2I_1 = 5R + RI_1 + 4I_1$

$\Rightarrow U_s + 6 - 5R = (6+R)I_1 \Rightarrow I_1 = \frac{U_s+6-5R}{6+R}$

$\Rightarrow U_{out} = 4I_1 = \frac{4U_s + 24 - 20R}{6+R}$, $I_{out} = 0$

when short : $\begin{cases} I_2 + I_3 = 8 \\ I_2 - I_1 = 5 \\ U_s + 2I_3 = RI_2 \end{cases}$ $\quad I_3 = 3 - I_1$ $\quad I_2 = 5 + I_1$

$\Rightarrow U_s + 6 - 2I_1 = 5R + RI_1$

$\Rightarrow I_1 = \frac{U_s+6-5R}{2+R}$

$\Rightarrow V_{out} = 0$

$I_{out} = \frac{U_s+6-5R}{2+R}$

$\therefore R_{equal} = \frac{U_{out}}{I_{out}}$

$= \frac{4U_s+24-20R}{6+R} \cdot \frac{2+R}{U_s+6-5R} = 2$

$(U_s-4)^2 = 8\cdot 9$

$U_s - 4 = \pm 3\cdot 2\sqrt{2}$
$= \pm 6\sqrt{2}$

$\frac{\cancel{8}(U_s-4)}{(4U_s-16)(U_s-4)} \over \cancel{64}\ 16$ $= 4.5$ $\quad \boxed{U_s = 4 \pm 6\sqrt{2}}$

$P_{max} = \frac{1}{2}\cdot U_{out} I_{out} = \frac{1}{2} \cdot \frac{4U_s+24-20R}{6+R} \cdot \frac{U_s+6-5R}{2+R} = 4.5$

$8U_s + 48 - 40R + 4U_sR + 24R - 20R^2 = 12U_s + 72 - 60R + 2RU_s + 12R - 10R^2$

$4U_s + 24 - 20R - 2U_sR - 12R + 10R^2 = 0$

$4U_s^2 + 24U_s - 20RU_s + 24U_s + 144 - 120R - 20U_sR - 120R + 100R^2 = 108 + 72R + 9R^2$

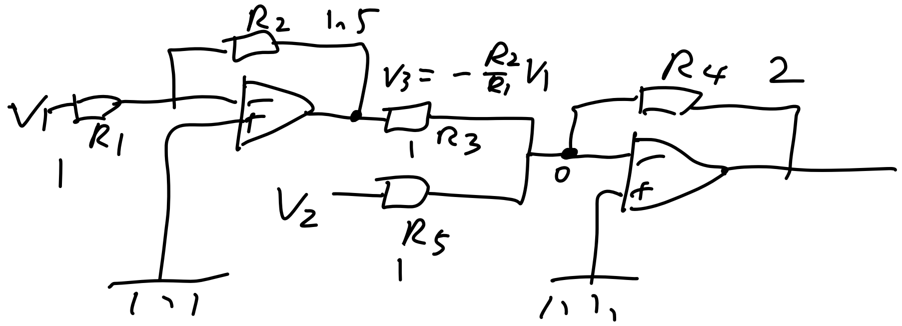
$4V_s^2 + 48U_s - 40RU_s + 36 - 312R + 91R^2 = 0$

Question 3:

Design an op-amp circuit that performs the following operation

$$\mathbf{v_0 = 3v_1 - 2v_2}$$

All resistances must $\leq$ 100kΩ?



$$-\frac{R_4}{R_3}\cdot\left(-\frac{R_2}{R_1}\right)V_1 = \frac{R_4 R_2}{R_3 R_1}V_1 = 3V_1$$

$$-\frac{R_4}{R_5}\cdot V_2 = -2V_2$$

$R_4 = 2k\Omega$

$R_5 = 1k\Omega$

$R_2 = 1.5k\Omega$

$R_1 = R_3 = 1k\Omega$

Question 4:

Determine $v_o(t)$ for $t > 0$ in Figure 4. Let $i_s = 10u(t)\ \mu A$, and assume that capacitor is initially uncharged?

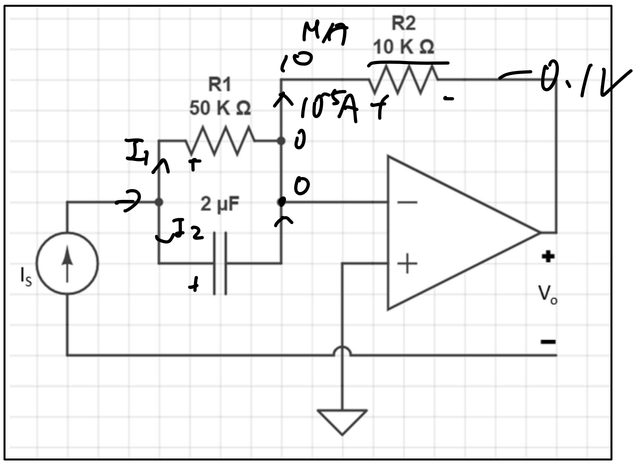


Figure 4

$V_o = -10^{-5} \cdot 10^4$
$= -0.1V$

$Q = CV$

$I = C\frac{dV}{dt}$ $\qquad V = 50k I_1$

$\begin{cases} I_2 = I_s - I_1 \\ I_2 = C\frac{50k\, dI_1}{dt} = \end{cases}$ $\qquad 2\times10^{-6} \cdot 5\times10^4$ $\qquad 5\times10^4$

$\therefore 0.1 I_1' = I_s - I_1 \Rightarrow I_1' = -10 I_1 + 10 I_s$ $\qquad 10^{-5}$

$= -10 I_1 + 10^{-4} u(t)$

$= -10 I_1 + 10^{-4}$

$\therefore I_1(t)_h = C e^{\int -10 dt} = C e^{-10t}$ $\qquad \therefore V_o = 0.5(1 - e^{-10t})V$

$I_1(t)_p = e^{\int -10 dt} \cdot \int 10^{-4} e^{10t} dt$

$= e^{-10t} \cdot \frac{10^{-4}}{10} e^{10t} = 10^{-5}$

$\therefore I_1(t) = C e^{-10t} + 10^{-5}$

$I_1(0) = 0 \Rightarrow C = -10^{-5}$

$\therefore I_1(t) = 10^{-5}(1 - e^{-10t})$

Question 5:

5.1 In Figure 5(a),

a) **Determine the equivalent impedance of the network if the frequency f=60 Hz.**

b) **Compute the current $i(t)$ if the voltage source is $v(t) = 50\cos(\omega t + 30°)$.**

c) **Calculate the equivalent impedance if the frequency f=400 Hz.**

$j \cdot 0.02 \cdot 400$

$50\angle 30°$

5.2 Sketch the phasor diagram for the network shown in Figure 5(b)

$2\pi \cdot 60$

(b): $50\cos(120\pi t + \frac{\pi}{6})$

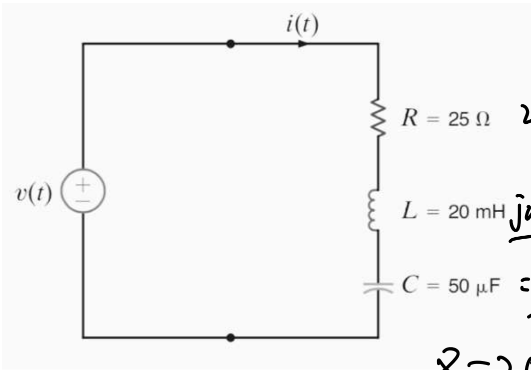
$$\frac{50\angle 30°}{25 + 2.4\pi j - \frac{500}{3\pi} j}$$

$$\approx \frac{50\angle 30°}{25 - 45.5118 j}$$

$$= \frac{50\angle 30°}{51.9261\angle -61.21965}$$

$$= 0.9624\angle 91.21965$$

$\therefore i(t) = 0.9624\cos(120\pi t + 91.21965°)$



(a) $f = 60Hz$ $\omega = 120\pi$

25Ω

$j\omega L = 2.4\pi j\,\Omega$

$\frac{1}{j\omega C} = \frac{500}{3\pi j}\,\Omega$

$Z = 25 + 2.4\pi j + \frac{500}{3\pi j}$

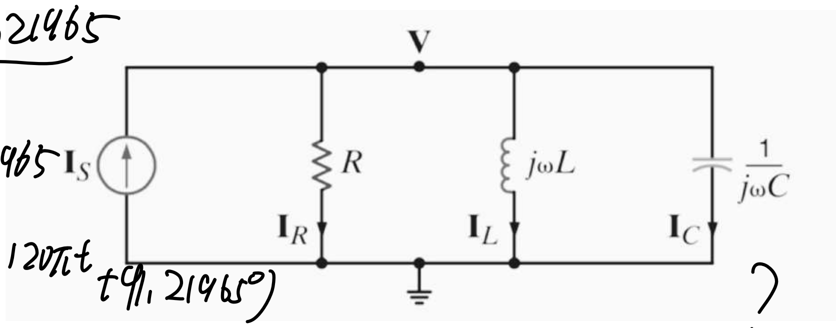
(c) $\omega = 800\pi$

25Ω

$16 j\pi$

$\frac{25}{\pi j}$

Figure 5(a)



?

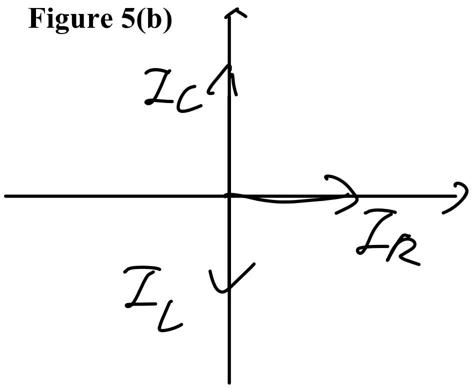
$j\omega L$

Figure 5(b)

I_C

I_C

for V



(b)

$(-4-28j)(6+8j)$ $\quad -8j-42j$

$\frac{(-1-7j)(6+8j)}{25}$ $\quad \frac{-6+56-50j}{25}$

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for left source

Question 6:

$\frac{(4-12j)(2+4j)}{6-8j} = \frac{(56-8j)(6+8j)}{100} = \frac{400+400j}{100} = 4+4j$

In Figure 6,

$2-2j$

$V_1 = -1$

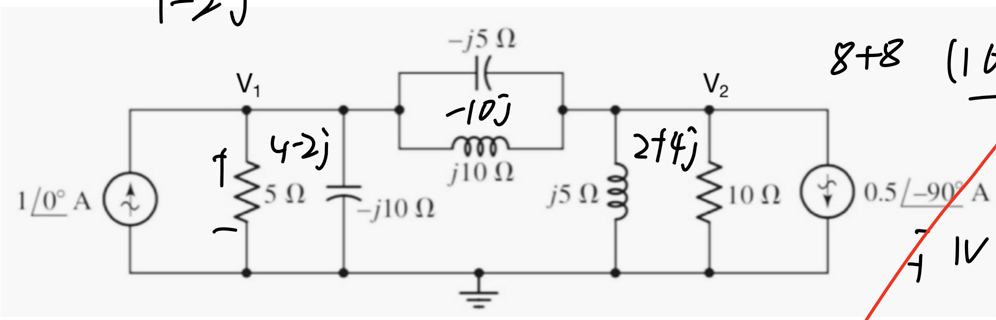
a) **Find the time-domain node voltages $v_1(t)$ and $v_2(t)$.**

b) **Use superposition to find V_1.**

$\frac{2+4j}{6-8j} \cdot 4-2j \cdot (-0.5j)$

$1-2j$

$8+8$ $\quad \frac{(16+12j)(6+8j)}{100} \cdot (-0.5j)$



$2j$ $\quad 1$

$+\ 1V$

(a) use KCL for node 1 **Figure 6**

$\frac{24j - \cancel{4} + 2V_2 - 2V_1 + V_1 - V_2}{10j}$

$$1\angle 0^\circ = \frac{V_1}{5} + \frac{V_1}{-10j} + \frac{V_1 - V_2}{-5j} + \frac{V_1 - V_2}{10j} = 1 = \frac{2V_1 j - 2V_1 + V_2}{10j}$$

$$-0.5\angle -90^\circ = \frac{V_2}{10} + \frac{V_2}{5j} + \frac{V_2 - V_1}{-5j} + \frac{V_2 - V_1}{10j} = 0.5j = \frac{V_2 j + \cancel{2V_2} + 2V_1 - \cancel{2V_2} + V_2 - V_1}{10j}$$

$$= \frac{V_2 j + V_1 + V_2}{10j}$$

$\therefore \ 10j = 2V_1 j - 2V_1 + V_2$

$5j = V_2 j + V_1 + V_2$

$V_1 = \frac{10j - V_2}{2j - 2}$

$-5 = V_2 j + \frac{10j - V_2}{2j-2} + V_2$

$10 - 10j = -2V_2 - \cancel{2V_2 j} + 10j - V_2 + \cancel{2V_2 j} - 2V_2$

$10 - 20j = -5V_2$

$V_2 = -2 + 4j$

$-12 + 4$

$\frac{(2+6j)(2j+2)}{-8} = \frac{-8+16j}{-8} = 1-2j$

Question 7:

(a) Apply source transformation to find v_x in the circuit of Figure 1(a).

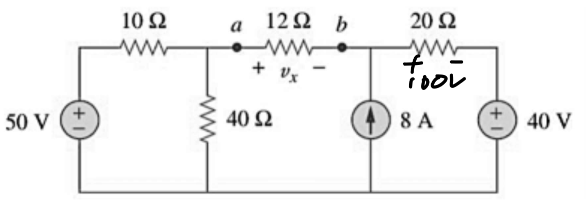
(b) In the circuit of Figure 7(b) find $i(t)$ for $t > 0$.

(a)

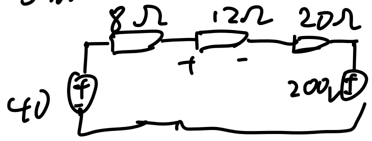
open 40V 0A

Short 0V 5A

8Ω



2A



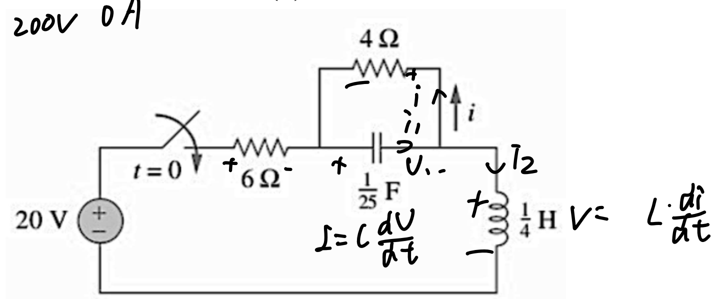
short 0V 10A

open 200V 0A

(a)

$-160V \cdot \frac{12}{40}$

$\simeq -48V$



(b)

Figure 7

(b) KVL: $20 = 6i_2 + V + L\cdot\frac{di_2}{dt}$

$V = -4i$

KCL: $i = C\frac{dV}{dt} - i_2$ $\quad i = -\frac{1}{4}V$

$i_2 = -i + C\frac{dV}{dt} = \frac{1}{4}V + C\frac{dV}{dt}$

$\frac{di_2}{dt} = \frac{1}{4}\frac{dV}{dt} + C\frac{d^2V}{dt^2}$

$20 = \frac{3}{2}V + 6C\frac{dV}{dt} + V + CL\frac{d^2V}{dt^2} + \frac{1}{16}\frac{dV}{dt}$

$\Rightarrow 0.01\frac{d^2V}{dt^2} + \frac{121}{400}\frac{dV}{dt} + \frac{5}{2}V = 20$

$\frac{6}{25} + \frac{1}{16} = \frac{96+25}{400} = \frac{121}{400}$

$0.01x^2 + \frac{121}{400}x + \frac{5}{2} = 0$

$x^2 + \frac{121}{4}x + 250 = 0$

$(\frac{121}{4})^2 - 1000$

$x = \frac{-\frac{121}{4} \pm 9.2161543j}{2}$

$V_s = 0$

$x_1 = -\frac{121}{8} - 4.608j$

$\alpha < \omega$

$x_2 = -\frac{121}{8} + 4.608j$

$\alpha = \frac{121}{8}$

$\omega_d = 4.608$

$\alpha = -\frac{121}{8}$

$V_1(t)_{-8}\cdot e^{-\frac{121}{8}t}(B_1\cos 4.608t + B_2\sin 4.608t)$

$V_1(0) = 0 \Rightarrow B_1 = 8$

$\frac{d^2V}{dt^2} + \frac{121}{4}\frac{dV}{dt} + 250V = 2000$

$x^2 + \frac{121}{4}x + 2500 = 0$

$x = \frac{-\frac{121}{4} \pm \sqrt{(\frac{121}{4})^2 - 10000}}{2}$

$x_1 = -\frac{121}{8} - 4.608j$

$x_2 = -\frac{121}{8} + 4.608j$

$e^{(-\frac{121}{8} - 4.608j)t}, e^{(-\frac{121}{8} + 4.608j)t}$

$Y(t) = C_1 e^{-\frac{121}{8} - 4.608jt} + C_2 e^{(-\frac{121}{8} + 4.608j)t}$

$2000,\ \mu = 0,\ n = 0 \Rightarrow m = 0$

$Y_p(t) = C_0$

$Y(t) = C_0 + C_1 e^{(-\frac{121}{8} - 4.608j)t} + C_2 e^{(-\frac{121}{8} + 4.608j)t}$

$= C_0 + e^{-\frac{121}{8}t}(C_1 e^{-4.608jt} + C_2 e^{4.608jt})$

$t \to \infty \quad Y(t) = 8 \Rightarrow C_0 = 8V$

$Y(t) = 8 + e^{-\frac{121}{8}t}(C_1\cos 4.608t + C_2\sin 4.608t)$

Question 8:

(a) Find the transfer function for the active filters in Figure 8.

(b) The filter in Figure 8 has a 3-dB cutoff frequency at 1 kHz. If its input is connected to a 120-mV variable frequency signal, find the output voltage at: 200 Hz, 2 kHz and 10 kHz.

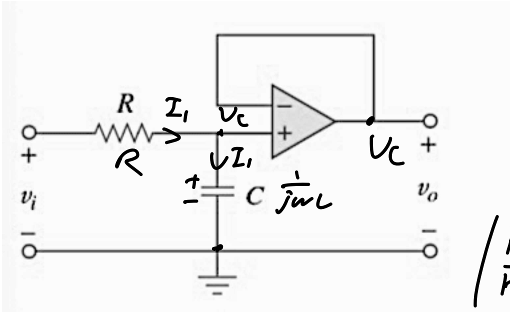


Figure 8

$I_1 = C\frac{dV_c}{dt}$

$\frac{V_i - V_c}{R} = C\frac{dV_c}{dt}$

$C\frac{dV_o}{dt} + \frac{1}{R}V_o - \frac{V_i}{R} = 0$

$\frac{dV_o}{dt} = -\frac{1}{RC}V_o + \frac{V_i}{RC}$

$H(\omega)\ \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \cdot V_i = \frac{1}{1+Rj\omega C}$

$\left|\frac{H(1k)}{H(0)}\right|^2 = \frac{1}{2} = \frac{1}{1+}$

$1+1000RCj = \cancel{2}\ 1$

$RC = \frac{1}{1000j}$

$V_o = 0.12 \cdot \frac{1}{1+\frac{\omega}{1000}}$

$j\omega \cdot \frac{1}{1000j}$

$= 0.12 \cdot \frac{1000}{1000+\omega}$

$= \frac{120}{1000+\omega}$