University of Illinois **Spring 2021**
ECE 310 **Profs. Moon, Katselis, and Shomorony**

Midterm Exam 1

7:00-8:50 PM, Thursday, March 11, 2021

Please do not start the exam until the starting time.

- You may not use any books, electronic devices, or notes other than **one** handwritten two-sided sheet of 8.5" x 11" paper.
- You should solve the problems on blank sheets of paper, take pictures of your solutions, and upload them to Gradescope before the end of the exam time.

GOOD LUCK!

1. **(15 Pts.)** Answer **True** or **False** to each of the following statements:

   (a) An LTI system with transfer function $H(z) = \dfrac{1 - z^{-1}}{1 - 2z^{-1}}$ cannot be stable. **True/False**

   (b) Let $X_1(z), X_2(z)$ be the rational z-transforms of $x_1[n], x_2[n]$. Then the poles of $X_1(z)$ and $X_2(z)$ must be poles of the z-transform of $x[n] = x_1[n] + x_2[n]$. **True/False**

   (c) Cascade of two BIBO *unstable* LTI systems cannot be stable. **True/False**

   (d) A causal LTI system with transfer function $H(z) = \dfrac{z^{-1}}{1 - z^{-1}}$ produces an unbounded output for input $x[n] = u[n]$. **True/False**

   (e) Suppose $\sum_{n=-\infty}^{\infty} x[n]\delta[4\cos(2n\pi + \frac{\pi}{2}) - 6\sin(n\pi)] = 4$. Then, $\sum_{n=-\infty}^{\infty} x[n] = 3$. **True/False**

2. **(10 Pts.)** Calculate the result of the following convolution: $\{-\underset{\uparrow}{1}, -2, 3, -3, 2, 1\} * \{-1, \underset{\uparrow}{0}, 1\}$.

3. **(24 Pts.)** For each of the systems with input $x[n]$ and output $y[n]$ in the table, indicate with YES or NO whether the properties indicated apply to the system.

	Linear	Time-Invariant	Causal	Stable
$y[n] = x[n]\cos(\frac{\pi}{3}(n-2))$				
$y[n] = x[3]x[n]$				
$y[n] = (0.8 + 0.8j)^n x[n]$				

4. **(10 Pts.)** Given the z-transform pair $x[n] \leftrightarrow X(z) = 1/(1 - 0.5z^{-1})$ with ROC: $|z| > 0.5$, determine the z-transform of $(n+1)x[n]$ and its ROC.

5. **(12 Pts.)** Suppose an LTI system has transfer function $H(z) = \frac{3z^{-1}}{1+z^{-2}}$ with ROC: $|z| > 1$.

   (a) **(8 Pts.)** The impulse response $h[n]$ of this system can be expressed as

   $$h[n] = A\sin(\omega_0 n + \theta)u[n].$$

   Find the constants A, ω_0 and θ. **Hint:** One approach is to use the z-transform pair

   $$\sin(\omega_0 n)u[n] \longleftrightarrow \frac{(\sin\omega_0)z^{-1}}{1 - 2(\cos\omega_0)z^{-1} + z^{-2}}, \quad |z| > 1.$$

   (b) **(4 Pts.)** Find a bounded real or complex-valued input signal $x[n]$ for which the corresponding output $y[n]$ is unbounded.

6. **(8 Pts.)** The output $y[n]$ and input $x[n]$ of a causal LTI system are related by the equation below. The system is initially at rest. Find the values of the impulse response $h[n]$ for the indicated values of n.

$$y[n] = 2y[n-3] - x[n] + x[n-3]$$

(a) $h[0] =$

(b) $h[1] =$

(c) $h[3] =$

(d) $h[4] =$

7. **(21 Pts.)** Suppose that the input to a causal LTI system is

$$x[n] = \frac{1}{4}\left(-\frac{1}{3}\right)^n u[n] - 3 \cdot 4^n u[-n-1]$$

and the z-transform of the output $y[n]$ is

$$Y(z) = \frac{13/4}{(1-\frac{1}{2}z^{-1})(1-z^{-1})(1+\frac{1}{3}z^{-1})}.$$

(a) Find the z-transform of $x[n]$ and its ROC.

(b) Find the transfer function $H(z)$ and its ROC.

(c) Determine the ROC of $Y(z)$.

(d) Find the impulse response of the system.

(e) Is the system stable?