University of Illinois
ECE 310

Profs. Liang, Moon, Snyder

Midterm Exam

7:00-9:00pm, Wednesday, March 1, 2023

Name: ______________________________

Section: 9:00 AM 12:00 PM 3:00 PM

NetID: __________

Score: __________

Problem	Pts.	Score
1	10	
2	12	
3	15	
4	6	
5	20	
6	20	
7	9	
8	4	
9	4	
Total	100	

Instructions

- You may not use any books, calculators, or notes other than one <u>handwritten</u> two-sided sheet of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

(10 Pts.)

1. Answer **True** or **False** to each of the following statements: *Grading:* Correct answer = 2 pt.; Incorrect answer = -1 pt. No answer = 0 pts.

   (a) If the system response $y[n]$ of a discrete-time system to any possible input signal $x[n]$ is fully described by its unit pulse response, then the system must be LTI. **True/False**

   (b) If a system is BIBO stable, any unbounded input will produce an unbounded output. **True/False**

   (c) If a system has a right-sided unit pulse response, then it must be causal. **True/False**

   (d) The DTFT, $X_d(\omega)$, of a sequence $x[n]$ is always related to its z-transform $X(z)$ by $X_d(\omega) = X(z)|_{z=e^{j\omega}}$. **True/False**

   (e) If $x[n]$ is a real-valued sequence, then $|X_d(\omega)|$ is an even function. **True/False**

(12 Pts.)

2. For each of the systems with input $x[n]$ and output $y[n]$ shown in the table, indicate by "**yes**" or "**no**" whether the properties indicated apply to the system. Note: you do not need to provide proofs/justification.

	Linear	Shift-Invariant	Causal	Stable
$y[n] = x[n] * (-1)^n u[n]$				
$y[n] = \frac{x[n]}{x[2]}$				
$y[n] = \cos^2\left(\frac{\pi}{2}n\right) x[n]$				

(15 Pts.)

3. For each of the following parts, compute the convolution $x[n] * h[n]$ between the given sequences.

   (a) $x[n] = \{\underset{\uparrow}{1}, -2, 2\}$, $h[n] = \{3, 1, \underset{\uparrow}{0}, 3, 1, 1\}$

   (b) $x[n] = \left(-\frac{1}{2}\right)^n u[n]$, $h[n] = \left(\frac{2}{3}\right)^n u[n-1]$

   (c) $x[n] = \log\left(|n| + 1\right)$, $h[n] = u[n+1] - u[n-2]$

(6 Pts.)

4. An LTI system with unit pulse response $h[n]$ has the following input-output relationships:

$$x_1[n] * h[n] = y_1[n]$$
$$x_2[n] * h[n] = y_2[n],$$

where

$$x_1[n] = \{\underset{\uparrow}{1}, 3, 6, -3\}$$
$$x_2[n] = \{\underset{\uparrow}{1}, 2, -1, 0\}.$$

Determine $h[n]$ in terms of $y_1[n]$ and $y_2[n]$.

(20 Pts.)

5. Consider a causal LTI system described by the following LCCDE:

$$y[n] = y[n-1] + \frac{3}{4}y[n-2] + x[n] - 4x[n-2].$$

(a) Determine the transfer function $H(z)$ and state the poles, zeros, and the ROC of this system.

(b) Calculate the system's response $y[n]$ to input $x[n] = 2\delta[n] - 3\delta[n-1]$.

(c) Is the system given by $H(z)$ BIBO stable? Justify your reasoning.

(20 Pts.)

6. Suppose that the input $x[n]$ to **a causal and stable LTI system** produces the output $y[n]$. The z-transform of $x[n]$ and $y[n]$ is given below:

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})}$$

$$Y(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})(1-\frac{1}{4}z^{-1})}$$

   (a) Find the transfer function $H(z)$ and its ROC.

   (b) Find the unit pulse response $h[n]$.

   (c) Determine the difference equation of the system.

(9 Pts.)

7. The transfer function of a causal LTI system is given below:

$$H(z) = \frac{z-3}{z-4}, \quad \text{ROC: } |z| > 4$$

   (a) Find a **bounded** input $x[n]$ that will produce an **unbounded** output $y[n]$.

   (b) Find an **unbounded** input $x[n]$ that will produce a **bounded** output $y[n]$.

   (c) Find a **bounded** input $x[n]$ that will produce a **bounded** output $y[n]$.

(4 Pts.)

8. Determine the signal $x[n]$ whose DTFT is $X_d(\omega) = 1 + 2\cos(2\omega) - 2j\sin(4\omega)$. (Circle one of the following) **Note**: The arrow indicates $n = 0$.

   (a) $x[n] = \{0.5\pi, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, -0.5\pi\}$

   (b) $x[n] = \{0.5, 0, 0, 0, \underset{\uparrow}{1}, -3j, 0, 0, -0.5\}$

   (c) $x[n] = \{-j, 0, 1, 0, \underset{\uparrow}{j}, 0, 1, 0, j\}$

   (d) $x[n] = \{-1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

   (e) $x[n] = \{-1, 0, j, 0, \underset{\uparrow}{1}, 0, j, 0, 1\}$

   (f) None of the above

(4 Pts.)

9. Consider the sequence $\{x[n]\}_{n=-1}^{2} = \{1-j,\ 1,\ -1-j,\ 2j\}$. Determine the values of A, B, C, and D of the following calculations without explicitly evaluating $X_d(\omega)$ for every ω.

   (a) $X_d(0) = A + jB$.

   (b) $X_d(\frac{\pi}{2}) = C + jD$.