310

1. basic

discrete signal 的n只能取整数，非整数的n对应样本舍弃.

仅知H(z)无法知h[n]，只知H(z) ROC几种可能情况，定ROC后才能确定h[n]

2. 复数

Polar form / rectangular form

$x = Re^{j\theta} = a + bj$, $\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$

$a = R\cos\theta$, $b = R\sin\theta$, $\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$

$\text{Magnitude}(x) = |x| = \sqrt{a^2+b^2} = R$

$\text{phase}(x) = \angle x = \theta = \begin{cases} \tan^{-1}(\frac{b}{a}), & a \geq 0 \\ \tan^{-1}(\frac{b}{a}) + \pi, & a < 0 \end{cases}$

3. discrete-time signal

$\{x[n]\}_{n=-1}^{5} \Leftrightarrow \begin{cases} x[n] = \{0, \overset{\uparrow}{4}, -2, 0, 3, 1, 0\} \\ x[n] = \{0, \underline{4}, -2, 0, 3, 1, 0\} \end{cases}$ $x[-1]=0$, $x[0]=4$, $x[1]=-2$

unit impulse: $\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$

unit step function: $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

4. discrete-time system

$x[n] \xrightarrow{T} y[n] \Leftrightarrow y[n] = T(x[n])$

① linear if $T(ax_1[n] + bx_2[n]) = aT(x_1[n]) + bT(x_2[n])$

② Time-invariant if $y[n-n_0] = T(x[n-n_0])$

③ Causal if output $n \geq$ system term $n \Leftrightarrow h[n] = 0$ for $n<0$

④ BIBO stable: for any $x[n]$ that $|x[n]| < \beta$ $\forall n$,

$|T(x[n])| < \alpha$, $\forall n$, $0 < \alpha < \infty$, $0 < \beta < \infty$

or one of below is true

4.1 $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$ iff

4.2 ROC of H(z) contains the unit circle, i.e. $|z|=1$ iff

ROC shape of H(z)	Causal	Right-Sided	Left-Sided	Condition for stability
$\lvert z\rvert > p_{max}$	✓	✓	X	$p_{max} < 1$
$\lvert z\rvert < p_{min}$	X	X	✓	$p_{min} > 1$
$p_{max} < \lvert z\rvert < \infty$	X	✓	X	$p_{max} < 1$
$a < \lvert z\rvert < b$	X	✓	✓	$p_{max} = a < 1$ for $h_r[n]$ $p_{min} = b > 1$ for $h_l[n]$

	impulse response h[n]	Transfer function H(z)
Parallel	$h_1[n] + h_2[n]$	$H_1(z) + H_2(z)$
series	$h_1[n] * h_2[n]$	$H_1(z) H_2(z)$

5. convolution

For any discrete-time signal

$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

For LTI system T

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$= h[n] * x[n]$

if $x[n]$ starts at n_s, end at n_e

$h[n]$ starts at m_s, end at m_e

then $y[n]$ starts at $m_s + n_s$

ends at $n_e + m_e$

$\begin{cases} h_1[n] * h_2[n] = h_2[n] * h_1[n] \\ (x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n]) \\ x[n] * \delta[n] = x[n] \\ x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n] \end{cases}$

6. z transform

$x[n] \xleftrightarrow{z} X(z)$

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$

ROC: values of z where z-transform sum converges.

(a) right sided: $x[n] = 0$ for $n < n_0$

ROC: $\begin{cases} a < |z|, & n_0 \geq 0 \\ a < |z| < \infty, & n_0 < 0 \end{cases}$

(b) left sided: $x[n] = 0$ for $n > n_0$

ROC: $\begin{cases} |z| < a, & n_0 \leq 0 \\ 0 < |z| < a, & 0 < n_0 \end{cases}$

(c) two sided

ROC: $a < |z| < b$

z-transform table

x[n]	X(z)	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-(n+1)]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-(n+1)]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\cos(w_0 n)u[n]$	$\frac{1-\cos(w_0)z^{-1}}{1-2\cos(w_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$\sin(w_0 n)u[n]$	$\frac{\sin(w_0)z^{-1}}{1-2\cos(w_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$a^n\cos(w_0 n)u[n]$	$\frac{1-a\cos(w_0)z^{-1}}{1-2a\cos(w_0)z^{-1}+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$a^n\sin(w_0 n)u[n]$	$\frac{a\sin(w_0)z^{-1}}{1-2a\cos(w_0)z^{-1}+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

Z transform property

Signal	Z transform	ROC
$x[n-k]$	$z^{-k}X(z)$	R_x except $z=0$ or ∞
$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$	at least $R_{x_1} \cap R_{x_2}$
$x_1[n]*x_2[n]$	$X_1(z)X_2(z)$	at least $R_{x_1} \cap R_{x_2}$
$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x
$x^*[n]$	$X^*(z^*)$	R_x
$x[-n]$	$X(z^{-1})$	$\frac{1}{R_x}$
$a^n x[n]$	$X(\frac{z}{a})$	$\lvert a \rvert R_x$
$Re\{x[n]\}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	at least R_x
$Im\{x[n]\}$	$\frac{1}{2}[X(z)-X^*(z^*)]$	at least R_x