University of Illinois | **Spring 2016**
ECE 310 | **Profs. Do & Liang**

Final Exam

Problem	Pts.	Score
1	12	
2	6	
3	6	
4	4	
5	6	
6	6	
7	4	
8	6	
9	6	
10	6	
11	10	
12	5	
13	5	
14	6	
15	6	
16	6	
Total	100	

Name ______________________________

Section: 9:00 AM 3:00 PM

Score ____________

Please do not turn this page over until told to do so.

You may not use any books, calculators, or notes other than three handwritten two-sided sheets of 8.5" x 11" paper.

GOOD LUCK!

(12 Pts.)

1. Answer "True" or "False" for the following statements. **Justify your answers**.

   (a) FIR filters are always BIBO stable. T/F

   (b) The DTFT of a sequence always exists as long as its z-transform exists. T/F

   (c) The bilinear transformation of an analog pole-zero filter with transfer function $H_L(s)$ results in a digital filter with transfer function $H(z)$. The number of poles and zeros in $H_L(s)$ is the same as the number of poles and zeros in $H(z)$. T/F

   (d) $\frac{\omega_c}{\pi}\text{sinc}(\omega_c n) * \frac{\omega_c}{\pi}\text{sinc}(\omega_c n) = \frac{\omega_c}{\pi}\text{sinc}(\omega_c n)$ T/F

   (e) The maximum overshoot/undershoot in the resulting frequency response of a low-pass FIR filter designed using the windowing design method with a rectangular window will be reduced with an increase in the filter length. T/F

   (f) In comparison with the rectangular window, a lowpass FIR filter design the Hamming window would have a **wider/narrower** (circle one) transition band and **larger/smaller** (circle one) passband ripples.

(6 Pts.)

2. For each of the systems shown in the table, indicate by "**yes**" or "**no**" whether the properties indicated apply to the system.

	Linear	SI	Causal	BIBO stable
$y[n] = 10x[n^2]$				
$y[n] - \frac{1}{2}y[n-1] = x[n+1]$				
$y[n] = x[n]\sin^2(e^{x[n]})$				

(6 Pts.)

3. Let $x[n] = \{\overset{\downarrow}{2}, -1, 3, 1\}$, $y[n] = \{\overset{\downarrow}{2}, 0, -1, 0, 3, 0, 1, 0\}$, $z[n] = \{\overset{\downarrow}{2}, -1, 3, 1, 0, 0, 0, 0\}$,
$X_d(\omega) = \text{DTFT}\{x[n]\}$, $Y_d(\omega) = \text{DTFT}\{y[n]\}$, $Z_d(\omega) = \text{DTFT}\{z[n]\}$,
$X[k] = \text{DFT}\{x[n]\}$, $Y[k] = \text{DFT}\{y[n]\}$, $Z[k] = \text{DFT}\{z[n]\}$.
Mark all the valid statements below (-1pt for each incorrect answer).

(a) $Y_d(\omega) = X_d(2\omega)$
(b) $Z_d(\omega) = X_d(2\omega)$
(c) $Y_d(\omega) = X_d(\omega)$
(d) $Z_d(\omega) = X_d(\omega)$
(e) $Y_d(\omega) = X_d(\omega/2)$
(f) $Z_d(\omega) = X_d(\omega/2)$
(g) $Y[2k] = X[k]$, for $k = 0, 1, 2, 3$
(h) $Z[2k] = X[k]$, for $k = 0, 1, 2, 3$
(i) $Y[k] = X[k]$, for $k = 0, 1, 2, 3$
(j) $Z[k] = X[k]$, for $k = 0, 1, 2, 3$

(4 Pts.)

4. Assume $x[n]$ is a finite-duration sequence of length 20, and $y[n]$ is obtained by zero-padding $x[n]$ to length 32. Let $X[k]$ and $Y[k]$ be the DFT of $x[n]$ and $y[n]$, respectively. Mark all the correct statements below (-1pt for each incorrect answer).

(a) $X[0] = Y[0]$
(b) $X[1] = Y[1]$
(c) $X[10] = Y[16]$
(d) $X[5] = Y[6]$
(e) None of the above

(6 Pts.)

5. Consider a causal linear shift-invariant system that is specified by the following difference equation:

$$y[n] = \frac{5}{2}y[n-1] - y[n-2] + x[n] + 2x[n-1].$$

   (a) Find the transfer function $H(z)$ of the system.

   (b) Find the impulse response $h[n]$ of the system.

   (c) Sketch the zero-pole plot of the system.

   (d) Draw the Direct Form I implementation of the system.

   (e) Is the system BIBO stable?

(6 Pts.)

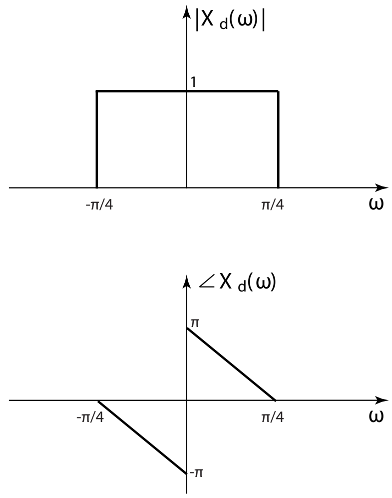
6. Let $\{x_n\}_{n=0}^{3} = \{1, 2, 3, 0\}$ and $\{h_n\}_{n=0}^{3} = \{1, -1, 0, 0\}$.

   (a) Evaluate circular convolution: $\{\tilde{y}_n\}_{n=0}^{3} = \{x_n\}_{n=0}^{3} \circledast \{h_n\}_{n=0}^{3}$ as a matrix-vector multiplication $\widetilde{\mathbf{y}} = \widetilde{\mathbf{C}}_h\, \mathbf{x}$. Show $\widetilde{\mathbf{C}}_h$ and $\{\tilde{y}_n\}_{n=0}^{3}$.

   (b) Evaluate linear convolution: $\{y_n\}_{n=0}^{6} = \{x_n\}_{n=0}^{3} * \{h_n\}_{n=0}^{3}$ as a matrix-vector multiplication $\mathbf{y} = \mathbf{C}_h\, \mathbf{x}$. Show $\mathbf{C}_h$ and $\{y_n\}_{n=0}^{6}$.

(4 Pts.)

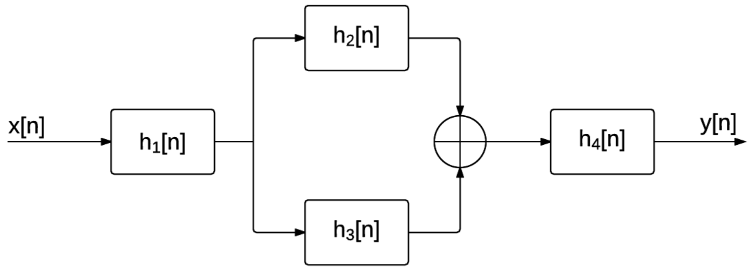
7. Let $X_d(\omega)$ be defined as in the figure below.



Determine $\text{DTFT}^{-1}\{X_d(\omega)\}$. Your answer should not contain any complex numbers.

(6 Pts.)

8. Consider the following system consisting of four subsystems in both serial and parallel connections, in which $h_1[n] = 2^{-n}u[n]$, $h_2[n] = 4^{-n}u[n]$, $h_3[n] = -\frac{1}{2}4^{-(n-1)}u[n-1]$, and $h_4[n] = (-4)^{-n}u[n]$



(a) Determine the transfer function of the system.

(b) Determine the frequency response of the system.

(c) Determine the system's difference equation in the form:

$$y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2],$$

for $n = 0, 1, 2, \ldots$

(6 Pts.)

9. Consider the following filter

$$H(z) = 1 + z^{-1} + z^{-2}.$$

   (a) Derive and sketch the magnitude response and phase response of the filter.

   (b) Is this a linear phase (LP) filter? How about GLP? Explain your answers.

   (c) Derive and plot the output of the filter given the following input signal

$$x[n] = \cos\left(\frac{\pi}{3}n\right) + 2\cos\left(\frac{2\pi}{3}n\right).$$

(6 Pts.)

10. Marie Threeten just got an "A" from ECE 310 and she wanted to use her filter design skills in her senior project. She successfully designed a length-36 filter using the windowing method. The unit pulse response of her filter is given by

$$\{h[n]\}_{n=0}^{35} = (-1)^n \frac{1}{6} \text{sinc}\left[\frac{\pi}{6}\left(n - \frac{35}{2}\right)\right]$$

(a) Is this a low-pass or high-pass filter? | Low-pass/High-pass |

(b) What is the value of $h[36]$ for the filter? | $h[36] =$ |

(c) What kind of window function $w[n]$ did Marie use in her design? | Rectangular/Hamming |

(d) Let

$$g[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} G_d(\omega) e^{j\omega n} d\omega, \qquad n = 0, 1, \cdots, 35.$$

such that $h[n] = g[n]w[n]$. It can be shown that

$$|G_d(\omega)| = \begin{cases} 1, & \frac{5\pi}{6} < \omega < \pi, \\ 0, & 0 \leq \omega \leq \frac{5\pi}{6}. \end{cases}$$

Determine $G_d(\omega)$ for $-\pi \leq \omega < \pi$.

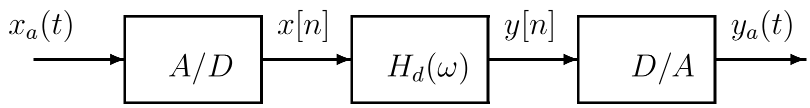
(e) How is $H_d(\omega)$ related to $G_d(\omega)$ and $W_d(\omega)$?

| $H_d(\omega) =$ |

(10 Pts.)

11. After completing ECE 310, Jimmy and Johns decided to start up a new company specialized in DSP. Their first contract was to develop a lowpass system that would filter out all frequencies above 3 kHz in speech signals, which are assumed to be bandlimited to 5 kHz.

    (a) The first started with an ideal design using the following DSP system:

    where $x_a(t)$ is the input analog speech signal, $y_a(t)$ is the output analog speech signal, A/D is an analog-to-digital converter with sampling interval T, D/A is an ideal digital-to-analog converter with the same interpolating interval T, and $H_d(\omega)$ is a digital filter.

    Determine T for the Nyquist sampling frequency and sketch the desired frequency response of the digital filter $H_d(\omega)$ for this system.

    (b) Suppose that an input speech signal has the following Fourier transform

    $$X_a(\Omega) = \begin{cases} 1 - \frac{|\Omega|}{10^4\pi}, & \text{if } |\Omega| \leq 10^4\pi \\ 0 & \text{else.} \end{cases}$$

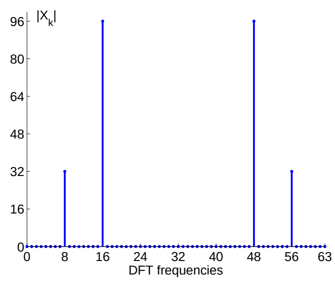
    Sketch $X_d(\omega)$, $Y_d(\omega)$, and $Y_a(\Omega)$ with the T and $H_d(\omega)$ found in part (a).

(c) Jimmy and Johns then realized that instead of an ideal D/A, they have only a zero-order-hold (ZOH) D/A. Sketch the new output $Y_a(\Omega)$ of part (b) when the ideal D/A is replaced by a ZOH D/A with the same T.

(d) To obtain the desired output, Jimmy and Johns add a compensated analog filter $F_a(\Omega)$ after the ZOH D/A. Sketch the magnitude response of this filter $F_a(\Omega)$ and specify its transition bandwith.

(5 Pts.)

12. Asssume that $x_a(t) = \sum_{\ell=1}^{L} A_\ell \cos(\Omega_\ell t)$, where the A_ℓ have positive values. We further assume that $x_a(t)$ is measured at $t = nT$ for $T = 1/8$ second and $n = 0, 1, \ldots, 63$ to obtain $\{x_n\}_{n=0}^{63} = \{x_a(nT)\}_{n=0}^{63}$. The 64-point DFT of $\{x_n\}_{n=0}^{63}$ is represented by $\{X_k\}_{k=0}^{63}$, whose magnitude is shown in the figure below.

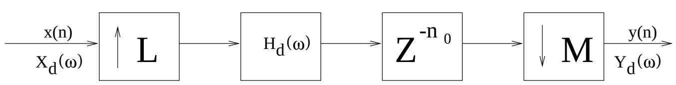


Determine L, and A_ℓ and Ω_ℓ for $\ell = 1, 2, \ldots, L$.

$L =$	$\{A_\ell\}_{\ell=1}^{L} =$	$\{\Omega_\ell\}_{\ell=1}^{L} =$

(5 Pts.)

13. For the following system, determine the smallest integer values for L, M, and n_0, respectively and the corresponding $H_d(\omega)$ such that $Y_d(\omega) = X_d(\omega)e^{-j4.5\omega}$.



$L =$	$M =$	$n_0 =$

(6 Pts.)

14. The linear convolution $\{x_n\}_{n=0}^{8} * \{h_n\}_{n=0}^{10}$ is to be evaluated using the DFT method. Namely, $\mathrm{DFT}^{-1}\{\mathrm{DFT}\{x_n\} \cdot \mathrm{DFT}\{h_n\}\}$.

    (a) Determine the minimum number of zeros that should be padded to $\{x_n\}$ and $\{h_n\}$, respectively, before the DFTs are applied.

Number of zeros padded to $\{x_n\}$ =	Number of zeros padded to $\{h_n\}$ =

    (b) If the DFTs are to be calculated with a radix-2 FFT algorithm, how many zeros should now be padded to $\{x_n\}$ and $\{h_n\}$, respectively.

Number of zeros padded to $\{x_n\}$ =	Number of zeros padded to $\{h_n\}$ =

    (c) In (a), can the zeros be padded at the beginning (instead of the end) of the sequences? If so, how do you obtain $\{x_n\}_{n=0}^{8} * \{h_n\}_{n=0}^{10}$ from $\mathrm{DFT}^{-1}\{\mathrm{DFT}\{x_n\} \cdot \mathrm{DFT}\{h_n\}\}$?

(6 Pts.)

15. The FFT algorithm successively divides-and-conquers a length N-DFT into two length $(N/2)$-DFTs.

    (a) Use the radix-2 decimation-in-time FFT algorithm to write the length-4 DFT as a combination of two length-2 DFTs.

    (b) Write the length-4 DFT as a multiplication with a 4×4 matrix and use the above FFT algorithm to express that 4×4 length-4 DFT matrix as a product of several matrices that correspond to reordering input signal samples, applying two length-2 DFTs, and combing the results into the desired length-4 DFT.

(6 Pts.)

16. Following is a MATLAB implementation of the LMS algorithm.

```
w  = zeros(K+1, 1);          % Initial filter
for n = 1:N-K
    xn = x(n+K:-1:n);
    en = d(n+K) - xn * w;    % Error
    w = w + mu * en * xn';   % Update filter
end
```

(a) Write out the update equation of the above LMS implementation and specify each variable.

(b) Identify the true and approximated gradient expressions that the above LMS implementation uses for gradient descent optimization.