University of Illinois
ECE 310 **Profs. Liang, Moon, Snyder**

Final Exam

8:00-11:00am, Friday, May 5, 2023

Name: ________________________________

Section: 9:00 AM 12:00 PM 3:00 PM

NetID: ____________

Score: ____________

Problem	Pts.	Score
1	10	
2	6	
3	4	
4	4	
5	5	
6	4	
7	5	
8	6	
9	12	
10	8	
11	6	
12	4	
13	4	
14	6	
15	9	
16	7	
Total	100	

Instructions

- You may not use any books, calculators, or notes other than three handwritten two-sided sheets of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

(10 Pts.)

1. Answer **True** or **False** to each of the following statements: *Grading:* Each part is worth 1 pt.

   (a) The FFT is an efficient algorithm for computing the DFT of a length-N discrete-time signal in $\mathcal{O}(N \log N)$ time. **True/False**

   (b) All FIR filters have generalized linear phase. **True/False**

   (c) If the ADC sampling period T is 2, the amplitude of DTFT will always be half of its CTFT. **True/False**

   (d) If a bandlimited signal is sampled above the Nyquist rate, the discrete-time signal from the ADC is free of aliasing. **True/False**

   (e) Consider a radix-2 decimation in time FFT with size $N = 128$. Then the FFT will have seven stages of computation. **True/False**

   (f) Direct Form II implementations require fewer delay units per output sample than Direct Form I implementations. **True/False**

   (g) The response $y[n]$ of a BIBO *unstable* LTI system to any non-zero input $x[n]$ is always unbounded. **True/False**

   (h) Downsampling a digital signal may cause aliasing, thus we apply an anti-aliasing filter after downsampling to fix any aliasing when implementing a decimator system. **True/False**

   (i) $\frac{\omega_c}{\pi}\text{sinc}(\omega_c n) * \frac{\omega_c}{\pi}\text{sinc}(\omega_c n) = \frac{\omega_c}{\pi}\text{sinc}(\omega_c n)$ **True/False**

   (j) Circular convolution can be applied to two arbitrary periodic sequences. **True/False**

(6 Pts.)

2. For each of the systems shown in the table, indicate by "**yes**" or "**no**" whether the properties indicated apply to the system. (Each correct answer receives 0.5 pt; each wrong answer receives -0.5 pt. no negative points for the whole problem.)

	Linear	Time-invariant	Causal	BIBO stable
$y[n] = 5x[4n-3]$				
$y[n] = \frac{1}{4}y[n-1] + x[n+1]$				
$y[n] = x[n] * u[n-2]$				

(4 Pts.)

3. For each of the filters shown in the table, indicate by "**yes**" or "**no**" whether the properties indicated apply to the filter. (In each case, the remaining terms of the unit pulse response $h[n]$ of the filter are zero.) (Each correct answer receives 1 pt; each wrong answer receives -1 pt. no negative points for the whole problem.)

$h[n]$	GLP Type-1	GLP Type-2
$\{h_n\}_{n=0}^{4} = \{2,1,6,1,2\}$		
$\{h[n]\}_{n=0}^{21} = (-1)^n \frac{1}{6}\text{sinc}\frac{\pi}{6}\left(n - \frac{21}{2}\right)$		

(4 Pts.)

4. Consider the following discrete-time system where α is a real-valued constant:

$$x[n] \longrightarrow \boxed{\; e^{j\alpha\omega} \;} \longrightarrow y[n]$$

   (a) Specify the condition on α under which the system is LTI.

   (b) Specify the condition on α under which the system is causal.

(5 Pts.)

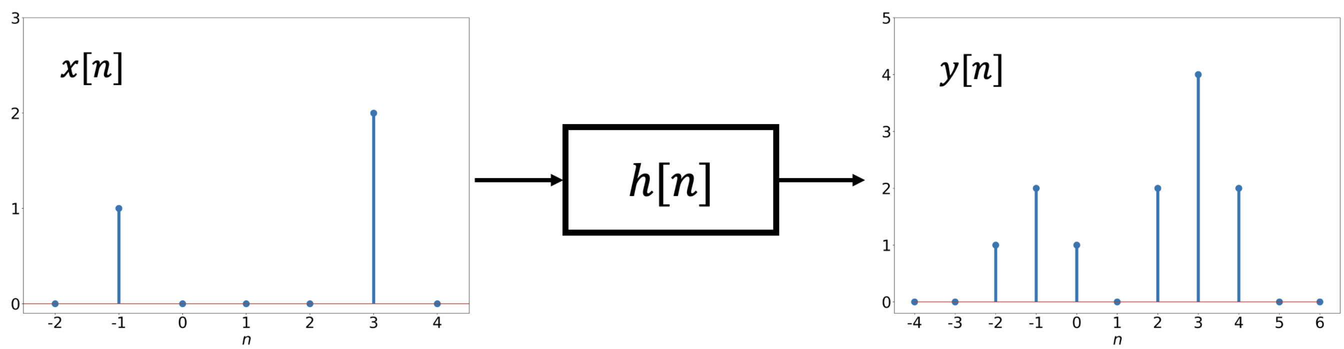
5. Let $y[n] = x[3n+2]$. Determine $Y_d(\omega)$ in terms of $X_d(\omega)$, where $X_d(\omega)$ and $Y_d(\omega)$ are the DTFT of $x[n]$ and $y[n]$, respectively.

(4 Pts.)

6. Assume that the unit pulse response, $h[n]$, of an LTI system is bounded. Is the system described by $h[n]$ guaranteed to be BIBO stable? Justify your answer.

(5 Pts.)

7. Suppose we have an LTI system described by unit pulse response $h[n]$. For an input signal $x[n]$ shown below we obtain the corresponding system response $y[n]$.



(a) Determine the unit pulse response $h[n]$ of the above system.

(b) Is this system causal?

(6 Pts.)

8. Consider a stable and causal LTI system. The transfer function of the system is

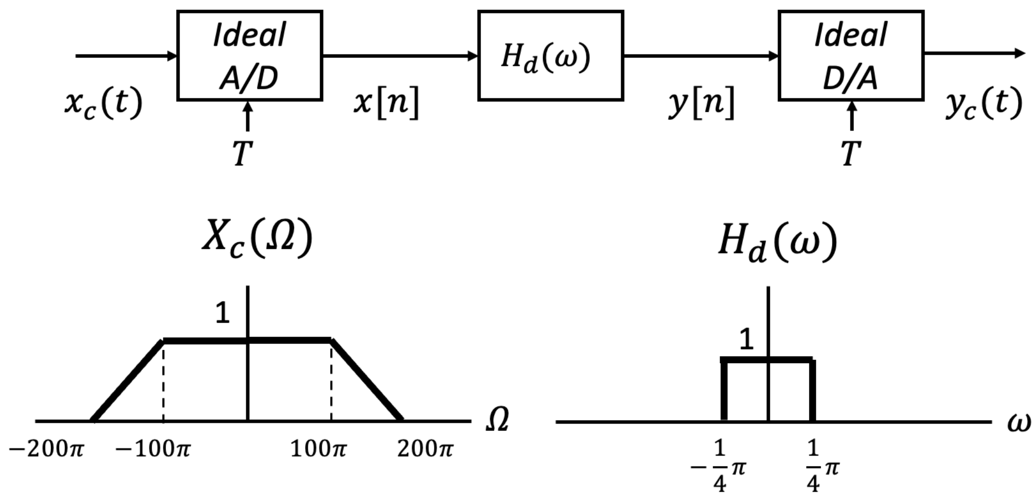
$$H(z) = \frac{z^{-1} - z^{-2}}{1 - \frac{1}{2}z^{-1}}$$

   (a) Suppose the input is a unit step sequence, $u[n]$. Find the z-transform of the output, $Y(z)$ and its ROC.

   (b) Find the unit pulse response of the system.

(12 Pts.)

9. Consider the following system with the given input and digital filter:



(a) Find the maximum value of T allowed without aliasing errors in $x[n]$.

(b) Find the maximum value of T allowed without aliasing errors in $y[n]$.

(c) Sketch $Y_d(\omega)$ and $Y_c(\Omega)$ for $T = \frac{5}{800}$. Please carefully label the frequencies and amplitudes.

(8 Pts.)

10. Let $x[n]$ and $h[n]$ be two length-5 sequences given below where A and B are unknown constants.

$$x[n] = \{\underset{\uparrow}{A}, -2, B, -2, 1\}, \quad h[n] = \{\underset{\uparrow}{2}, 3, 1, -1, -2\}.$$

Instead, we know that $y[n] = x[n] \circledast_5 h[n]$ is given by

$$y[n] = \{\underset{\uparrow}{-6}, -19, -1, 5, 9\}.$$

(a) Solve for A and B.

(b) Suppose we have another sequence $v[n]$:

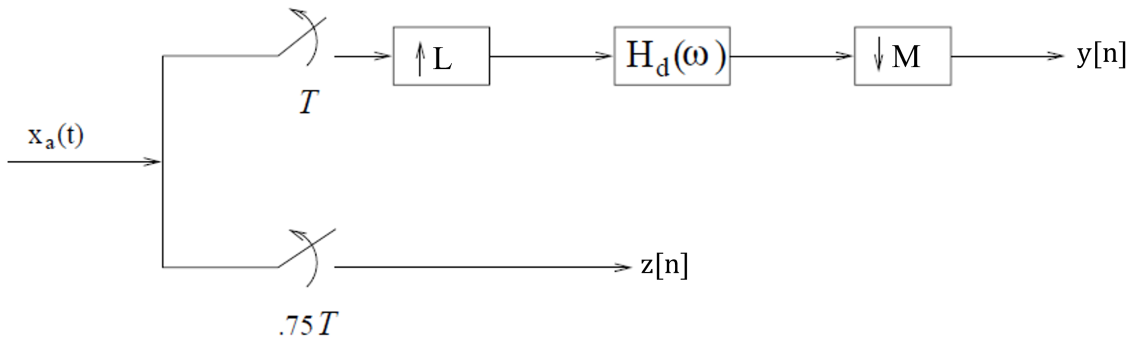
$$v[n] = \{\underset{\uparrow}{B}, -2, 1, A, -2\}.$$

Determine $z[n] = v[n] \circledast_5 h[n]$.

(c) What is the minimum N such that $x[n] \circledast_N h[n] = x[n] * h[n]$?

(6 Pts.)

11. Consider the following system consisting of two synchronized ideal A/D converters. Assume that the input analog signal $x_a(t)$ is bandlimited to π/T. Complete the digital rate conversion subsystem by determining M, L, and $H_d(\omega)$ such that $y[n] = z[n]$.



(4 Pts.)

12. Let $\{x[n]\}_{n=0}^{N-1} = \cos\left(\frac{\pi}{6}n\right) + \cos\left(\frac{\pi}{4}n\right), \ 0 \leq n \leq N-1$ be a length-N discrete-time signal. For which of the following values of N will the resulting DFT $X[k]$ have only four non-zero values? (Please circle one choice.)

    (a) $N = 8$

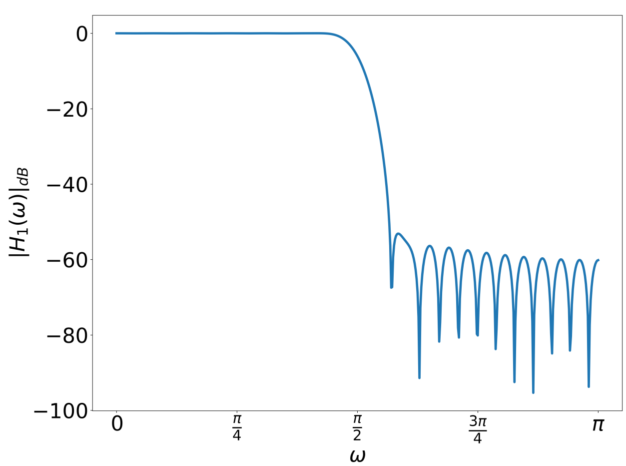
    (b) $N = 12$

    (c) $N = 24$

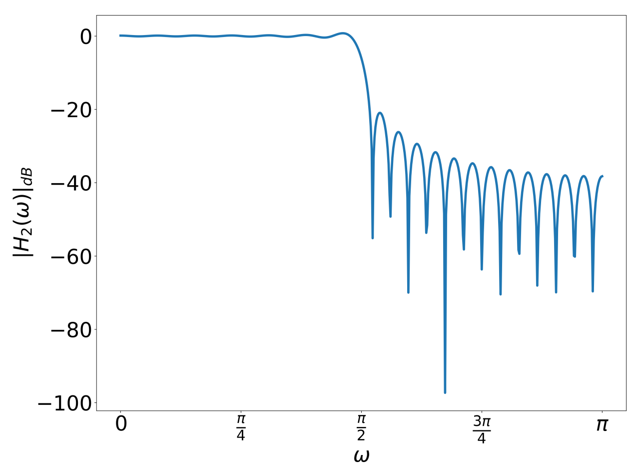
    (d) $N = 36$

(4 Pts.)

13. Suppose we use the window method for FIR filter design to create a length-51 low-pass filter with cutoff frequency $\omega_c = \frac{\pi}{2}$. We use a rectangular window and Hamming window to design two versions of this filter. The figure below depicts these two magnitude responses on a dB scale.



(a) $|H_1(\omega)|$

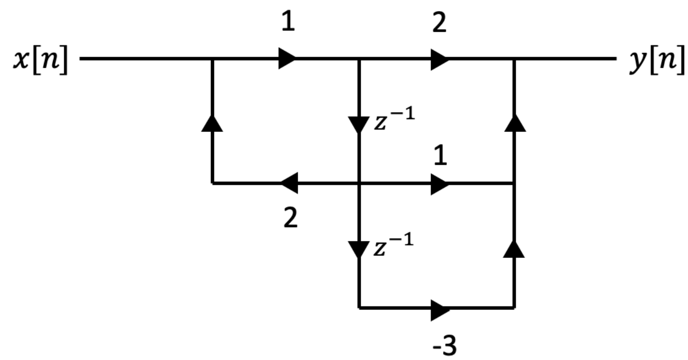


(b) $|H_2(\omega)|$

Circle the magnitude response corresponding to the filter designed with the **Hamming window** and **justify your reasoning**.

(6 Pts.)

14. Consider the causal LTI system shown below:



(a) Find the transfer function $H(z)$ for this system.

(b) Determine the LCCDE relating $x[n]$ and $y[n]$ for the flow graph.

(c) Is the system stable?

(9 Pts.)

15. An FIR filter is described by the below unit pulse response $h[n]$ where K is a real-valued constant.

$$h[n] = \{\underset{\uparrow}{-1}, K, 6, K, -1\}$$

(a) Determine $R(\omega)$ (in terms of K), α, and β such that the frequency response of this filter $H_d(\omega) = R(\omega)e^{j(-\alpha\omega+\beta)}$ of this filter where $R(\omega)$ is a real-valued function and α and β are real-valued constants.

$R(\omega) =$ ____________________________

$\alpha =$ ____________________________

$\beta =$ ____________________________

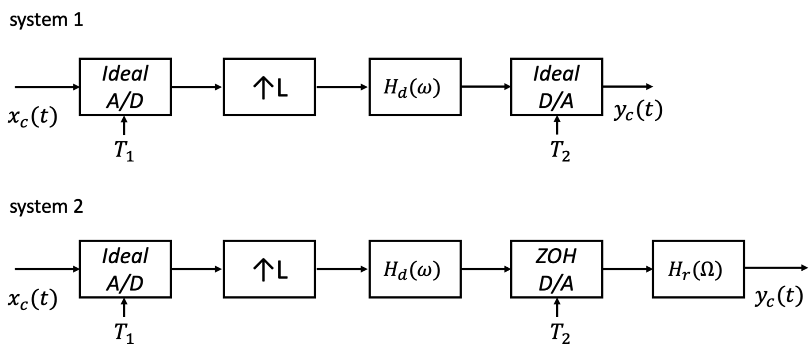
(b) For what value of K will $H_d(0) = 0$?

(c) For what value of K will $H_d(\pi) = 0$?

(d) For what values of K will $H_d(\omega)$ have strictly **linear phase**?

(7 Pts.)

16. Consider the two systems in the following figure, where $X_c(\Omega)$ is bandlimited to $4 \cdot 10^3\pi$, T_1 is $\frac{1}{4 \cdot 10^3}$ second, and $H_d(\omega)$ is an ideal LPF whose cut-off frequency is π/L.



(a) For System 1, express T_2 in terms of T_1 and L so that the signals $x_c(t)$ and $y_c(t)$ are identical.

(b) In System 2, suppose the analog compensation filter $H_r(\Omega)$ has a transition band starting at $\Omega_1 = 4 \cdot 10^3\pi$ radian/second and ending at $\Omega_2 = 76 \cdot 10^3\pi$ radian/second. Determine the minimum L so that System 2 functions as System 1.