University of Illinois
ECE 310

Profs. Liang, Moon, Snyder

Midterm Exam 2

7:00-9:00pm, Wednesday, April 5, 2023

Name: ________________________________

Section: 9:00 AM 12:00 PM 3:00 PM

NetID: ____________

Score: ____________

Problem	Pts.	Score
1	10	
2	3	
3	3	
4	5	
5	14	
6	8	
7	20	
8	10	
9	15	
10	12	
Total	100	

Instructions

- You may not use any books, calculators, or notes other than one <u>handwritten</u> two-sided sheet of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

(10 Pts.)

1. Answer **True** or **False** to each of the following statements: *Grading:* Correct answer = 2 pt.; Incorrect answer = -1 pt. No answer = 0 pts.

   (a) By sampling a continuous-time signal $x_c(t) = \cos(\pi^3 t)$ with some sampling period T, it is possible to obtain a discrete time signal $x[n] = \cos(3\pi n/4)$. **True/False**

   (b) If the Nyquist sampling rate for a continuous-time signal $x_c(t)$ is F_s, then the Nyquist sampling rate for $y_c(t) = x_c(3t)$ is $F_s/3$. **True/False**

   (c) Suppose $x[n]$ is a finite-length signal with DTFT $X_d(\omega)$. We zero-pad $x[n]$ with some number of zeros to obtain $y[n]$ with DTFT $Y_d(\omega)$. It follows then that $X_d(\omega) = Y_d(\omega)$. **True/False**

   (d) If $x[n]$ is the inverse DFT of $\{1, 2, 3, 4\}$, then $x[n]$ must be zero for $n < 0$ or $n > 3$. **True/False**

   (e) If $x_c(t)$ is a bandlimited continuous-time signal, then there must exist a finite $\Omega_{\max}$ such that $X_a(\Omega) = 0$ for $|\Omega| > \Omega_{\max}$, where $X_a(\Omega)$ is the CTFT of $x_c(t)$. **True/False**

(3 Pts.)

2. Let $X_d(\omega)$ be the DTFT of a *real-valued* sequence $x[n]$. We further assume that $X_d(\omega) = j\omega$ for $0 \leq \omega \leq \pi$. We then have (*select one answer only*):

   (a) $X_d(\omega) = \omega$ for $-\pi \leq \omega \leq 0$
   (b) $X_d(\omega) = -\omega$ for $-\pi \leq \omega \leq 0$
   (c) $X_d(\omega) = j\omega$ for $-\pi \leq \omega \leq 0$
   (d) $X_d(\omega) = -j\omega$ for $-\pi \leq \omega \leq 0$
   (e) None of the above

(3 Pts.)

3. Let $X_d(\omega)$ be the DTFT of an *arbitrary* sequence $x[n]$. We further assume that $X_d(\omega) = j\omega$ for $0 \leq \omega \leq \pi$. We then have (*select one answer only*):

   (a) $X_d(\omega) = \omega$ for $2\pi \leq \omega \leq 3\pi$
   (b) $X_d(\omega) = -\omega$ for $2\pi \leq \omega \leq 3\pi$
   (c) $X_d(\omega) = j\omega$ for $2\pi \leq \omega \leq 3\pi$
   (d) $X_d(\omega) = -j\omega$ for $2\pi \leq \omega \leq 3\pi$
   (e) None of the above

(5 Pts.)

4. Calculate the inverse DTFT, $x[n]$, of $X_d(\omega) = 5e^{j\pi\omega}\delta(\omega - \omega_o)$, where ω_0 is a constant.

(14 Pts.)

5. We have an LSI system with with the following unit pulse response given by $h[n]$:

$$h[n] = \delta[n] + \delta[n-6].$$

   (a) Determine the frequency response $H_d(\omega)$ of this LTI system.

   (b) Plot the magnitude response $|H_d(\omega)|$ and phase response $\angle H_d(\omega)$ for $-\pi \leq \omega \leq \pi$. Make sure to carefully label your axes.

(8 Pts.)

6. The frequency response of an LSI system is

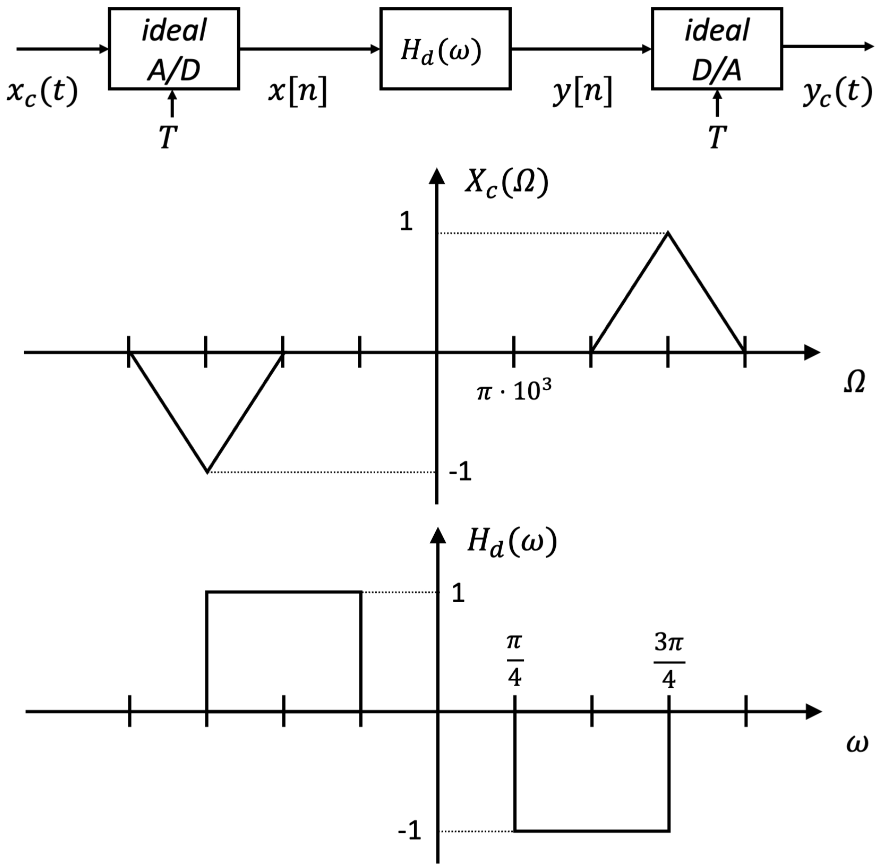
$$H_d(\omega) = \omega e^{j\pi \cos\omega}, \qquad |\omega| \le \pi \ .$$

Compute the response of this system $y[n]$ to the following input signal:

$$x[n] = 3 + e^{j\frac{\pi}{3}n} + \sin\left(\frac{\pi}{2}n + \frac{\pi}{4}\right).$$

(20 Pts.)

7. Consider the process of a bandlimited continuous-time signal $x_c(t)$ via the system shown below.



(a) What is the Nyquist rate for the signal $x_c(t)$ (i.e., no aliasing at the A/D converter)? Answer the rate in Hz.

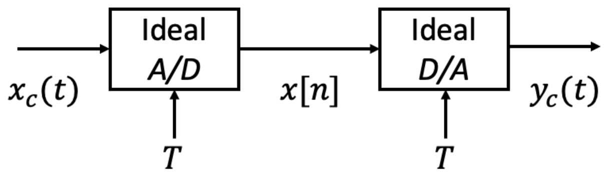
(b) Sketch the DTFT $X_d(\omega)$ and $Y_d(\omega)$ at $T = \frac{1}{4} \cdot 10^{-3}$ for $-\pi \leq \omega \leq \pi$. Please carefully label your axes.

(c) Sketch the DTFT $X_d(\omega)$ and $Y_d(\omega)$ at $T = \frac{1}{3} \cdot 10^{-3}$ for $-\pi \leq \omega \leq \pi$. Please carefully label your axes.

(10 Pts.)

8. Consider the sampling and reconstruction system shown below. The input to the ideal A/D is

$$x_c(t) = 2\cos\left(100\pi t + \frac{\pi}{4}\right) + \cos\left(300\pi t\right)$$



(a) If the output of A/D is

$$x[n] = 2\cos\left(\frac{1}{4}\pi n + \frac{\pi}{4}\right) + \cos\left(\frac{3}{4}\pi n\right)$$

determine two choices for T consistent with the information.

(b) If the output of the ideal D/A is

$$y_c(t) = 2\cos\left(100\pi t + \frac{\pi}{4}\right) + 1$$

determine the value of T.

(15 Pts.)

9. For all parts of this question, let $\{x[n]\}_{n=0}^{5} = \{1, 2, -3, -4, 5, 6\}$ be a length-6 signal with DFT $\{X[k]\}_{k=0}^{5}$ given below:

$$X[k] = \{X_0, X_1, X_2, X_3, X_4, X_5\}.$$

(a) Compute X_0 and X_3.

(b) The DFT of another length-6 signal $\{y[n]\}_{n=0}^{5}$ is given by $\{Y[k]\}_{k=0}^{5}$:

$$Y[k] = \{X_4, -X_5, X_0, -X_1, X_2, -X_3\}.$$

Determine the signal $y[n]$.

(c) Suppose we zero-pad $x[n]$ with 24 zeros to obtain $\{z[n]\}_{n=0}^{29}$ with corresponding DFT $\{Z[k]\}_{k=0}^{29}$. Which of the following relations is true? (Circle all that are true. More than one may be true.)

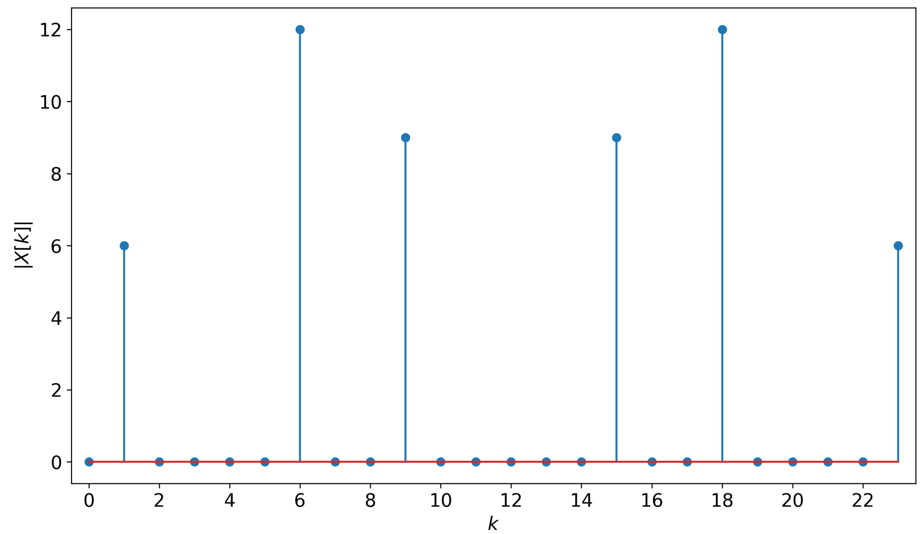
i. $X[0] = Z[0]$
ii. $X[1] = Z[4]$
iii. $X[2] = Z[10]$
iv. $X[1] = Z^*[25]$
v. $X[2] = Z^*[22]$

(12 Pts.)

10. An analog signal $x_a(t)$ is known to be composed of M spectral components such that

$$x_a(t) = \sum_{m=1}^{M} A_m \cos(\Omega_m t).$$

The analog signal is sampled with sampling period $T = \frac{1}{12{,}000}$ s to create digital signal $\{x[n]\}_{n=0}^{23} = \{x_a(nT)\}_{n=0}^{23}$ with DFT $X[k]$. The DFT magnitude spectrum $|X[k]|$ is given in the below image. You may assume that $x_a(t)$ is sampled above the Nyquist rate and thus no aliasing occurs.



(a) Determine the number of spectral components M.

(b) Determine the amplitudes of each spectral component $\{A_m\}_{m=1}^{M}$

(c) Determine the analog radial frequencies of each spectral component $\{\Omega_m\}_{m=1}^{M}$. (Please use consistent subscripts between parts (b) and (c))