ECE 310 Exam 1

Boyao Wang

TOTAL POINTS

97 / 100

QUESTION 1

1 Question 1 12 / 12

✓ **- 0 pts** *All items correct*

- 2 pts Part (a) incorrect

- 2 pts Part (b) incorrect

- 2 pts Part (c) incorrect

- 2 pts Part (d) incorrect

- 2 pts Part (e) incorrect

- 2 pts Part (f) incorrect

QUESTION 2

2 Question 2 12 / 12

✓ **- 0 pts** *First system correct*

✓ **- 0 pts** *Second system correct*

✓ **- 0 pts** *Third system correct*

- 1 pts First system, linearity

- 1 pts 1st system, time-invariant

- 1 pts 1st system, causal

- 1 pts First system, stability

- 1 pts 2nd system, linear

- 1 pts 2nd system, time-invariant

- 1 pts 2nd system, causal

- 1 pts 2nd system, stable

- 1 pts 3rd system, linear

- 1 pts 3rd system, time-invariant

- 1 pts 3rd system, causal

- 1 pts 3rd system, stable

QUESTION 3

3 Question 3 12 / 12

✓ **- 0 pts** *Part (a): Correct result*

- 2 pts Part (a): Correct procedure, minor math errors

- 1 pts Part (a): Missing start point (where n=0)

- 6 pts Part (a): Incorrect result

✓ **- 0 pts** *Part (b): Correct result*

- 2 pts Part (b): Correct procedure, minor math errors

- 6 pts Part (b): Incorrect result

- 4 pts Part (a): Some step in the answer is correct, but wrong answer

- 4 pts Part b) Some step correct, but incorrect answer

- 4 pts Part b): Correct impulse signal and formula, not get to final equation

- 1 pts Part b): Correct answer but wrong simplification

QUESTION 4

4 Question 4 5 / 5

✓ **- 0 pts** *Correct*

- 3 pts Some correct work shown, incorrect result

- 5 pts Incorrect

QUESTION 5

5 Question 5 9 / 9

✓ **+ 9 pts** *Correct result*

+ 3 pts Correct use of derivative property

+ 3 pts Correct use of time shifting property

+ 2 pts Correct use of z-transform pair for $$u[n]$$

+ 1 pts Correct algebraic simplification

QUESTION 6

6 Question 6 20 / 20

✓ **+ 4 pts** *Part (a) Correct PFD coefficients*

✓ **+ 4 pts** *Part (a): Correct ROC and $$h[n]$$ for both right-sided*

✓ **+ 4 pts** *Part (a): Correct ROC and $$h[n]$$ for both left-sided*

✓ **+ 4 pts** *Part (a): Correct ROC and $$h[n]$$ for one right-sided, one left-sided*

+ 2 pts Part (a): ROC or $$h[n]$$ correct for both right-sided

+ 2 pts Part (a): ROC or $$h[n]$$ correct for both left-sided

+ 2 pts Part (a): ROC or $$h[n]$$ correct for one right-sided, one left-sided

✓ **+ 4 pts** *Part (b): Correct LCCDE for system*

+ 2 pts Part (b): Incorrect LCCDE for system up to some minor math or sign errors.

QUESTION 7

7 Question 7 18 / 20

✓ **- 0 pts** *Part (a): Correct*

- 2 pts Part (a): Incorrect poles

- 1 pts Part (a): Partially incorrect poles

- 2 pts Part (a): Incorrect zeros

- 1 pts Part (a): Partially incorrect zeros

- 3 pts Part (a): Incorrect ROC

- 0 pts Part (a): Incorrect ROC, but correct with respect to given poles

✓ **- 0 pts** *Part (b): Correct result*

- 3 pts Part (b): Incorrect PFD coefficients

- 3 pts Part (b): Incorrect inverse z-transform by inspection

- 2 pts Part (b): Incorrect expression for Y(z)

- 9 pts Part (b): Incorrect result

- 0 pts Part (c): Correct

- 0 pts Part (c): Correct with respect to answers in part (a)

✓ **- 2 pts** *Part (c): Identified system as unstable, but invalid bounded input to yield unbounded output*

- 4 pts Part (c): Identified system as stable

QUESTION 8

8 Question 8 9 / 10

- 0 pts Part (a): Correct $$\alpha=-4$$

- 3 pts Part (a): Incorrect, but some correct work

- 5 pts Part (a): Incorrect

- 0 pts Part (b): Correct all $$\alpha\in\mathbb{R}$$ are valid

✓ **- 1 pts** *Slightly wrong*

- 3 pts Part (b): Incorrect, but some correct work

- 5 pts Part (b): Incorrect

All alpha in (b) since unit circle in ROC.

University of Illinois
ECE 310

Profs. Do, Snyder, Moustakides

Midterm Exam 1

7:00-9:00pm, Wednesday, September 27, 2023

Name: Wang Boyao

Section: 9:00 AM   12:00 PM   3:00 PM

NetID: boyaow2

Score: ____________

Problem	Pts.	Score
1	12	
2	12	
3	12	
4	5	
5	9	
6	20	
7	20	
8	10	
Total	100	

Instructions

- You may not use any books, calculators, or notes other than one handwritten two-sided sheet of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

$\frac{5z^{-1}}{z^{-1}}$ $= u[n] - 2u[n-1]$

ROC $|z|>1$

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(12 Pts.)

1. Select **True** or **False** to each of the following statements:

   (a) Any causal discrete-time system must also be time-invariant. **True/False**

   (b) The ROC of a given z-transform cannot contain any poles or zeros. **True/False**

   (c) Two BIBO stable systems connected in parallel always form a BIBO stable system. **True/False**

   $H(z) = H_1(z) + H_2(z)$

   $h[n] = h_1[n] + h_2[n]$

   (d) For a discrete-time system with impulse response $h[n]$, the system output $y[n]$ to any input signal $x[n]$ is always given by $y[n] = x[n] * h[n]$. **True/False**

   $h[n] = n$ $z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4}$

   (e) Two LTI systems given by impulse responses $h_1[n]$ and $h_2[n]$ are connected in series in some order, i.e. $h_1[n]$ or $h_2[n]$ may come first. If we pass an input signal $x[n]$ to this system, we will receive the same output signal $y[n]$ for either ordering of $h_1[n]$ and $h_2[n]$. **True/False**

   $X(z)$ erics $H_1(z)H_2(z)$

   (f) A BIBO stable LTI system with transfer function $H(z) = \frac{1}{1-3z^{-1}}$ must be non-causal. **True/False**

   $=3$ pole

   $|z|<3$

(12 Pts.)

2. For each of the systems with input $x[n]$ and output $y[n]$ shown in the table, indicate by "**yes**" or "**no**" whether the properties indicated apply to the system. You will only be graded on your answers in the boxes and not on any work you show.

	Linear	Time-invariant	Causal	Stable
$y[n] = \log(\lvert n \rvert + 1)x[n]$ $ax_1[n]$ $bx_2[n]$	yes	no	yes	no
$y[n] = x[n] * u[n+1]$ $= \sum_{k=-\infty}^{\infty} x[k]u[n-k+1]$ $= \sum_{k=-\infty}^{\infty} x[n-k]u[k+1]$ $k \geq -1$	yes	yes	no	no
$y[n] = x[n] + 3$ $ax_1[n] + bx_2[n] + 3$	no	yes	yes	yes

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$\frac{2-z^{-1}-z^{-2}}{(1-z^{-1})}-1$ $\quad \begin{matrix}2 & 1\\ 1 & -1\end{matrix} \quad$ $\frac{(2+z^{-1})(1-z^{-1})}{(1-z^{-1})}$

(12 Pts.)

3. For each of the following parts, calculate the convolution $y[n] = x[n] * h[n]$.

(a) $\{x[n]\}_{n=0}^{4} = \{\underset{\uparrow}{1}, 2, 3, 2, 1\}$, $\{h[n]\}_{n=0}^{1} = \{\underset{\uparrow}{-1}, 1\}$.

$\because$ $x[n]$ starts at $n_s = 0$, ends at $n_e = 4$

$h[n]$ starts at $m_s = 0$, ends at ~~ne~~ $m_e = 1$

$\therefore$ $y[n] = x[n] * h[n]$ starts at $n_s + m_s = 0$, ends at $n_e + m_e = 5$

$\because y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$\therefore y[0] = \sum_{k=-\infty}^{\infty} x[k]\, h[-k] = x[0]h[0] = -1$

$y[1] = \sum_{k=-\infty}^{\infty} x[k]\, h[1-k] = x[0]h[1] + x[1]h[0] = 1 + 2\cdot(-1) = -1$

$y[2] = \sum_{k=-\infty}^{\infty} x[k]\, h[2-k] = \cancel{x[0]h[2] + x[1]h[1]}\ x[1]h[1] + x[2]h[0] = 2\cdot 1 + 3\cdot(-1) = -1$

$y[3] = \sum_{k=-\infty}^{\infty} x[k]\, h[3-k] = x[2]h[1] + x[3]h[0] = 3\cdot 1 + 2\cdot(-1) = 1$

$y[4] = \sum_{k=-\infty}^{\infty} x[k]\, h[4-k] = x[3]h[1] + x[4]h[0] = 2\cdot 1 + 1\cdot(-1) = 1$

$y[5] = \sum_{k=-\infty}^{\infty} x[k]\, h[5-k] = x[4]h[1]\ \cancel{+x[5]h[0]} = 1\cdot 1 = 1$

$\therefore y[n] = \{\underset{\uparrow}{-1}, -1, -1, 1, 1, 1\}$

(b) $x[n] = \left(\frac{3}{4}\right)^n u[n]$, $\quad h[n] = 2u[n] - u[n-1] - u[n-2]$

$\because x[n] = \left(\frac{3}{4}\right)^n u[n]$

$a^n u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-az^{-1}}, \ |z| > a$

$\therefore X(z) = \frac{1}{1-\frac{3}{4}z^{-1}}, \ |z| > \frac{3}{4}$

$\because h[n] = 2u[n] - u[n-1] - u[n-2]$

$u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-z^{-1}}, \ |z| > 1$

$x[n-k] \overset{z}{\longleftrightarrow} z^{-k} X(z)$, ~~ROC: Rx except~~

$\therefore H(z) = \frac{2}{1-z^{-1}} - \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-2}}{1-z^{-1}}, \ |z| > 1$

$= \frac{(2+z^{-1})(1-z^{-1})}{1-z^{-1}} = 2 + z^{-1}, \ |z| > 0$

$\because y[n] = x[n] * h[n]$

$x_1[n] * x_2[n] \overset{z}{\longleftrightarrow} X_1(z) X_2(z)$

$\therefore Y(z) = X(z)H(z) = \frac{2+z^{-1}}{1-\frac{3}{4}z^{-1}} = \frac{2}{1-\frac{3}{4}z^{-1}} + \frac{z^{-1}}{1-\frac{3}{4}z^{-1}}, \ |z| > \frac{3}{4}$

$\because a^n u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-az^{-1}}, \ |z| > a$

~~$u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-z^{-1}}, |z| > 1$~~

$x[n-k] \overset{z}{\longleftrightarrow} z^{-k} X(z)$

$\therefore y[n] = 2\left(\frac{3}{4}\right)^n u[n] + \left(\frac{3}{4}\right)^{n-1} u[n-1]$

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$$-z \cdot (-1) \frac{z^{-2}}{(1-z^{-1})^{2}}$$

$$z \cdot \frac{z^{-1}}{(1-z^{-1})^2}$$

(5 Pts.)

4. Suppose we have an LTI system described by impulse response $h[n]$. We know that the system response to input $x[n] = u[n]$ is given by $y[n] = \delta[n] + \delta[n-1]$. Which of the following sequences correctly expresses $h[n]$? **(Circle one)**

   (a) $h[n] = \{\underset{\uparrow}{1}, 0, 1\}$

   (b) $h[n] = \{\underset{\uparrow}{1}, 0, -1\}$

   (c) $h[n] = \{\underset{\uparrow}{1}, 1\}$

   (d) $h[n] = \{\underset{\uparrow}{1}, -1\}$

$Y(z) = 1 + z^{-1}$

$X(z) = \frac{1}{1-z^{-1}} \quad |z|>1$

$\therefore H(z) = \cancel{\frac{1+z^{-1}}{1-z^{-1}}} (1+z^{-1})(1-z^{-1}) = 1 - z^{-2}$

$h(n) = \cancel{u[n]+u[n-1]}$

$\delta[n] - \delta[n-2]$

$= \{\underset{\uparrow}{1}, 0, -1\}$

(9 Pts.)

5. Calculate the z-transform $X(z)$ of $x[n] = nu[n+1]$.

$x[n] = nu[n+1] = (n+1)u[n+1] - u[n+1] = x_1[n] - x_2[n]$

$\because x_1[n] = (n+1)u[n+1]$

$u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-z^{-1}}, |z|>1$

$nx[n] \overset{z}{\longleftrightarrow} -z\frac{dX(z)}{dz}$

$x[n-k] \overset{z}{\longleftrightarrow} z^{-k}X(z)$

$\therefore nu[n] \overset{z}{\longleftrightarrow} (-z)(-1)\cdot\frac{z^{-2}}{(1-z^{-1})^2} = \frac{z^{-1}}{(1-z^{-1})^2}$

$X_1(z) = z^{-(-1)} \cdot \frac{z^{-1}}{(1-z^{-1})^2} = \frac{1}{(1-z^{-1})^2}, |z|>1$

$\because x_2[n] = u[n+1]$

$u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-z^{-1}}, |z|>1$

$x[n-k] \overset{z}{\longleftrightarrow} z^{-k}X(z)$

$\therefore X_2(z) = z \cdot \frac{1}{1-z^{-1}} = \frac{z}{1-z^{-1}}, |z|>1$

$\because x[n] = x_1[n] - x_2[n]$

$ax_1[n] + bx_2[n] \overset{z}{\longleftrightarrow} aX_1(z) + bX_2(z)$

$\therefore X(z) = X_1(z) - X_2(z) = \frac{1}{(1-z^{-1})^2} - \frac{z}{1-z^{-1}}$

$$\frac{\overset{\frac{1}{3}}{A}}{1-\frac{1}{2}z^{-1}} + \frac{\overset{\frac{2}{3}}{B}}{1-2z^{-1}}$$

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$A+B=1 \quad 2A+2B=2$

$-2A-\frac{1}{2}B=-1 \quad B=\frac{2}{3}$

$\frac{3}{2}B=1 \quad A=\frac{1}{3}$

(20 Pts.)

6. We are given the following transfer function $H(z)$ of an LTI system

$$H(z) = \frac{1-z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}.$$

(a) Determine all possible ROCs and the corresponding impulse response $h[n]$ for each ROC.

$\because H(z) = \frac{1-z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})} = \frac{\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{2}{3}}{1-2z^{-1}} = \frac{1}{3}\cdot\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{2}{3}\cdot\frac{1}{1-2z^{-1}}$

$\therefore H(z)$ has poles at $z_1=\frac{1}{2}$, $z_2=2$

$\therefore$ all possible ROCs: ① $|z|<\frac{1}{2}$, ② $\frac{1}{2}<|z|<2$, ③ $2<|z|$

① when $|z|<\frac{1}{2}$

$\because -a^n u[-(n+1)] \xleftrightarrow{z} \frac{1}{1-az^{-1}}$, $|z|<|a|$

$\therefore h[n] = \frac{1}{3}\cdot(-1)(\frac{1}{2})^n u[-(n+1)] + \frac{2}{3}\cdot(-1)\cdot 2^n u[-(n+1)]$

$= [(-\frac{1}{3})(\frac{1}{2})^n + (-\frac{2}{3})\cdot 2^n]\, u[-(n+1)]$

② when $\frac{1}{2}<|z|<2$

$\because -a^n u[-(n+1)] \xleftrightarrow{z} \frac{1}{1-az^{-1}}$, $|z|<|a|$

$a^n u[n] \xleftrightarrow{z} \frac{1}{1-az^{-1}}$, $|z|>|a|$

$\therefore h[n] = \frac{1}{3}\cdot(\frac{1}{2})^n u[n] + \frac{2}{3}\cdot(-1)\cdot 2^n u[-(n+1)] = \frac{1}{3}\cdot(\frac{1}{2})^n u[n] - \frac{2}{3}\cdot 2^n u[-(n+1)]$

③ when $2<|z|$

$\because a^n u[n] \xleftrightarrow{z} \frac{1}{1-az^{-1}}$, $|z|>|a|$

$\therefore h[n] = \frac{1}{3}\cdot(\frac{1}{2})^n u[n] + \frac{2}{3}\cdot 2^n u[n] = [\frac{1}{3}\cdot(\frac{1}{2})^n + \frac{2}{3}\cdot 2^n]\, u[n]$

(b) Determine the corresponding difference equation with transfer function $H(z)$.

$\because H(z) = \frac{1-z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})} = \frac{1-z^{-1}}{1-\frac{5}{2}z^{-1}+z^{-2}} = \frac{Y(z)}{X(z)}$

$\therefore (1-z^{-1})X(z) = (1-\frac{5}{2}z^{-1}+z^{-2})Y(z)$

$\therefore X(z) - z^{-1}X(z) = Y(z) - \frac{5}{2}z^{-1}Y(z) + z^{-2}Y(z)$

$\because x[n-k] \xleftrightarrow{z} z^{-k}X(z)$

$\therefore x[n] - x[n-1] = y[n] - \frac{5}{2}y[n-1] + y[n-2]$

$\therefore y[n] = \frac{5}{2}y[n-1] - y[n-2] + x[n] - x[n-1]$

$\frac{1+z^{-1}}{1-z^{-1}} \cdot \frac{1}{1-\frac{9}{4}z^{-1}}$

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$\frac{2z^{-2}-6z^{-1}-8+3z^{-1}+9}{-\frac{1}{4}z^{-2}+\frac{3}{4}z^{-1}+1}$ -8 $-6\cdot\frac{4}{3}$ $\frac{(1+\frac{1}{2}z^{-1})(\quad\frac{1}{2}z^{-1})}{(1-\frac{1}{4}z^{-1})(1+z^{-1})}$

$A+B=1$ $\quad \frac{5}{4}B=3$

$A-\frac{1}{4}B=-2$ $\quad B=\frac{12}{5}$

$-\frac{7}{5}$ $-\frac{3}{5}$

(20 Pts.)

7. Consider a **causal** LTI system with the following transfer function:

$\frac{1-\cos(w_0)z^{-1}}{1-2\cos(w_0)z^{-1}+z^{-2}}$ $\cos(w_0)=1$ $\frac{1-z^{-1}}{(1-z^{-1})^2}$

$$H(z) = \frac{1-3z^{-1}+2z^{-2}}{1+\frac{3}{4}z^{-1}-\frac{1}{4}z^{-2}}. = \frac{(1-z^{-1})(1-2z^{-1})}{(1-\frac{1}{4}z^{-1})(1+z^{-1})}$$

(a) Determine the poles, zeros, and ROC of the system.

$\frac{1-\frac{1}{4}z^{-1}}{1-\frac{1}{2}z^{-1}+z^{-2}}$

$H(z) = \frac{1-3z^{-1}+2z^{-2}}{1+\frac{3}{4}z^{-1}-\frac{1}{4}z^{-2}} = -8+\frac{9+3z^{-1}}{1+\frac{3}{4}z^{-1}-\frac{1}{4}z^{-2}} = -8+\frac{9+3z^{-1}}{(1-\frac{1}{4}z^{-1})(1+z^{-1})}$

$\because H(z) = \frac{(1-z^{-1})(1-2z^{-1})}{(1-\frac{1}{4}z^{-1})(1+z^{-1})}$

$\therefore$ poles at $z_1=\frac{1}{4}$, $z_2=-1$

zeros at $z_3=1$, $z_4=2$

$\because$ system is causal $\therefore$ ROC: $|z|>|P_{max}| \Rightarrow |z|>1$

Poles: $\frac{1}{4}$, -1

Zeros: 1, 2

ROC: $\cancel{|z|>\frac{1}{4}}$ $|z|>1$

(b) Calculate the system response $y[n]$ to input signal $x[n] = u[n]$.

$\because x[n]=u[n]$

$u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-z^{-1}}$, $|z|>1$

$\therefore X(z)=\frac{1}{1-z^{-1}}$

$\therefore Y(z)=X(z)H(z)=\frac{1}{1-z^{-1}}\cdot\frac{(1-z^{-1})(1-2z^{-1})}{(1-\frac{1}{4}z^{-1})(1+z^{-1})} = \frac{1-2z^{-1}}{(1-\frac{1}{4}z^{-1})(1+z^{-1})} = \frac{-\frac{7}{5}}{1-\frac{1}{4}z^{-1}}+\frac{\frac{12}{5}}{1+z^{-1}}$, $|z|>1$

$\because a^n u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-az^{-1}}$, $|z|>|a|$

$\therefore y[n] = -\frac{7}{5}\cdot(\frac{1}{4})^n u[n]+\frac{12}{5}(-1)^n u[n]$

$4A+4B=4$

$A+B=1$

$-4A-B=1$

$\boxed{x[n]=\cos n\, u[n]}$

$3B=6$

$B=\frac{5}{3}$ $\frac{7}{2}$

(c) Is the system described by $H(z)$ BIBO stable? If the system is not BIBO stable, give an example of a bounded input that will produce an unbounded output.

No. It's not BIBO stable as its ROC does not contain unit circle

~~let $X(z)=\frac{1+z^{-1}}{(1-z^{-1})(1+4z^{-1})}=\frac{-\frac{2}{3}}{1-z^{-1}}+\frac{\frac{5}{3}}{1+4z^{-1}} \Rightarrow x[n]$~~

let $x[n]=\cos(n)\,u[n]$

from part (a) $H(z)=-8+\frac{\frac{21}{5}}{1-\frac{1}{4}z^{-1}}+\frac{\frac{24}{5}}{1+z^{-1}} \Rightarrow h[n]=-8\delta[n]+\frac{21}{5}(\frac{1}{4})^n u[n]+\frac{24}{5}(-1)^n u[n]$

$\therefore y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]=\sum_{k=0}^{\infty}(\cos k)[-8\delta[n-k]+\frac{21}{5}(\frac{1}{4})^{n-k}u[n-k]+\frac{24}{5}(-1)^{n-k}u[n-k])$

the term $\cos k\cdot\frac{24}{5}(-1)^{n-k}u[n-k]$ is unbounded $\Rightarrow$ output unbounded

$(1-\frac{3}{2}z^{-1})(1-2z^{-1})$ $1-\frac{7}{2}z^{-1}+3z^{-2}$ $a=2$ $w_0=0$ $a=\sqrt{3}$ $2\sqrt{3}\cos w=\frac{7}{2}$ $\cos w=\frac{7}{4\sqrt{3}}$

$B = 3 - A$

$A + B = 3$

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$1 \quad + \frac{2}{3}$ $\quad -\frac{4}{3}A + \frac{2}{3}B = \alpha$

(10 Pts.)

$1 \quad -\frac{4}{3}$ $\quad -\frac{4}{3}A + 2 - \frac{2}{3}A = \alpha$ $\quad B = 3 - 1 + \frac{1}{2}\alpha$

8. Suppose we have an LTI system described by $2 - 2A = \alpha$ $\quad 2A = 2 - \alpha \quad A = 1 - \frac{1}{2}\alpha$

$$y[n] - \frac{2}{3}y[n-1] - \frac{8}{9}y[n-2] = 3x[n] + \alpha x[n-1],$$

where α is a finite, real-valued constant and there exists a transfer function $H(z)$ for the system.

(a) Assuming the system is **causal**, for what value(s) of α is this system BIBO stable?

$\because x[n-k] \overset{z}{\longleftrightarrow} z^{-k}X(z)$

$\therefore Y(z) - \frac{2}{3}z^{-1}Y(z) - \frac{8}{9}z^{-2}Y(z) = 3X(z) + \alpha z^{-1}X(z)$

$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{3 + \alpha z^{-1}}{1 - \frac{2}{3}z^{-1} - \frac{8}{9}z^{-2}} = \frac{3 + \alpha z^{-1}}{(1 + \frac{2}{3}z^{-1})(1 - \frac{4}{3}z^{-1})} = \frac{1 - \frac{1}{2}\alpha}{1 + \frac{2}{3}z^{-1}} + \frac{2 + \frac{1}{2}\alpha}{1 - \frac{4}{3}z^{-1}}$

if $1 - \frac{1}{2}\alpha$ and $2 + \frac{1}{2}\alpha$ both not zero

$H(z)$ has poles at $z_1 = -\frac{2}{3}$, $z_2 = \frac{4}{3}$

and ROC: $|z| > |P_{max}|$ as system is causal

to let system BIBO stable, ROC ~~should~~ should contain unit circle

$\therefore z_2$ pole should not exit

$\therefore 2 + \frac{1}{2}\alpha = 0 \Rightarrow \alpha = -4$ $\qquad \boxed{\alpha = -4}$

$\therefore$ ROC: $|z| > \frac{2}{3}$ $\Rightarrow$ ROC contains unit circle $\Rightarrow$ system BIBO stable

(b) Assuming the system is **non-causal** and **two-sided**, for what value(s) of α is this **non-causal** system BIBO stable?

same to (a) $H(z) = \frac{1 - \frac{1}{2}\alpha}{1 + \frac{2}{3}z^{-1}} + \frac{2 + \frac{1}{2}\alpha}{1 - \frac{4}{3}z^{-1}}$

$\because$ system is non-causal and two-sided

$\therefore$ ROC has form: $a < |z| < b$

according to (a) to has this form

$1 - \frac{1}{2}\alpha$ and $2 + \frac{1}{2}\alpha$ should both not zero

$\therefore$ $\alpha \neq 2$ and $\alpha \neq -4$ so that ROC: $\frac{2}{3} < |z| < \frac{4}{3}$

thus ROC contains unit circle $\Rightarrow$ system BIBO stable

$\therefore \alpha = \{\alpha \in R \mid \alpha \neq 2 \text{ and } \alpha \neq -4\}$