University of Illinois
ECE 310

Spring 2018
Profs. Do and Liang

Final Exam

8:00-11:00AM, Wednesday, May 9, 2018

Name: ____________________________________

Section: 9:00 AM 12:00 PM 3:00 PM

NetID: _____________

Score: _____________

Problem	Pts.	Score
1	6	
2	4	
3	8	
4	4	
5	4	
6	6	
7	4	
8	6	
9	6	
10	6	
11	4	
12	6	
13	8	
14	6	
15	6	
16	6	
Total	**90**	

Instructions

- You may not use any books, calculators, or notes other than four handwritten two-sided sheets of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

(6 Pts.)

1. Answer "True" or "False" for the following statements.

   (a) If the unit pulse response, $h[n]$, of an arbitrary LSI system is zero for $n < 0$, the system must be causal. **T/F**

   (b) A longer FIR filter can be designed to achieve the transition band width at least as narrow as another shorter FIR filter. **T/F**

   (c) If a system is BIBO stable then it must be causal. **T/F**

   (d) The response of a BIBO *unstable* LSI system to any non-zero input is always unbounded. **T/F**

   (e) A serial connection of two BIBO stable systems is necessarily stable. **T/F**

   (f) A parallel connection of two BIBO stable systems is necessarily stable. **T/F**

(4 Pts.)

2. A sequence $x[n]$ has the z-transform

$$X(z) = \frac{z}{(z-\frac{1}{2})(z-2)}, \quad \text{ROC} : |z| > 2.$$

   Circle all correct $x[n]$ in the following list of answers.

   (a) $x[n] = -\frac{1}{3}(\frac{1}{2})^n u[n] + \frac{4}{3}2^n u[n]$

   (b) $x[n] = -\frac{1}{3}(\frac{1}{2})^{n-1} u[n-1] + \frac{4}{3}2^{n-1} u[n-1]$

   (c) $x[n] = \frac{1}{3}(\frac{1}{2})^{n-1} u[-n] - \frac{4}{3}2^{n-1} u[-n]$

   (d) $x[n] = \frac{1}{3}(\frac{1}{2})^{n-1} u[n-1] + \frac{4}{3}2^{n-1} u[n-1]$

(8 Pts.)

3. Given a **causal** LTI system with the transfer function:

$$H(z) = \frac{4}{1 - \frac{1}{2}z^{-1}} + \frac{2}{1 - \frac{1}{4}z^{-1}},$$

   (a) Determine each of the following:

      i. Locations of the all poles and zeros of the system

      ii. Region of convergence of $H(z)$

      iii. Is the system BIBO stable?

   (b) What is the impulse response $\{h[n]\}$ of the given system?

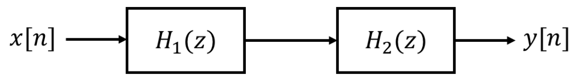
   (c) Determine a *causal* linear constant coefficient difference equation (LCCDE) whose transfer function is as above.

(4 Pts.)

4. Consider the following cascaded system with two causal subsystems:

$$H_1(z) = \frac{z - 0.25}{z^2 - 1.5z + 0.5}, \quad \text{and}$$
$$H_2(z) = \frac{z + c}{(z - 0.25)^2}.$$

Determine c so that the following cascaded system is BIBO stable.



(4 Pts.)

5. Recall that the convolution of two discrete signals $\{x[n]\}$ and $\{h[n]\}$ is denoted as:

$$(x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$

Prove that $((x * h_1) * h_2)[n] = (x * (h_1 * h_2))[n]$ for all n. Draw equivalent system block diagrams for the left side and right side of the equation.

(6 Pts.)

6. An FIR filter is described by the difference equation

$$y[n] = x[n] - x[n-6]$$

(a) Determine the frequency response of the system, $H_d(\omega)$.

(b) The filter's response to the input

$$x[n] = \cos\left(\frac{\pi}{10}n\right) + 3\sin\left(\frac{\pi}{3}n + \frac{\pi}{10}\right).$$

is given by

$$y[n] = A_1 \cos\left(\frac{\pi}{10}n + \phi_1\right) + A_2 \sin\left(\frac{\pi}{3}n + \phi_2\right).$$

Determine A_1, A_2, ϕ_1, and ϕ_2.

$A_1 =$	$A_2 =$	$\phi_1 =$	$\phi_2 =$

(4 Pts.)

7. Let $(X[k])_{k=0}^{99}$ be the 100−point **DFT** of a **real-valued** sequence $(x[n])_{n=0}^{99}$ and $X_d(\omega)$ be the **DTFT** of $x[n]$ zero-padded to infinite length. Circle all correct equations in the following list.

   (a) $X[70] = X_d\left(-\frac{6\pi}{10}\right)$

   (b) $X[70] = X_d\left(\frac{70\pi}{50}\right)$

   (c) $|X[70]| = |X_d\left(\frac{70\pi}{100}\right)|$

   (d) $\angle X[70] = -\angle X_d\left(\frac{3\pi}{5}\right)$

(6 Pts.)

8. Let $\{x[n]\}_{n=0}^{N-1}$ be a **real-valued** N-point sequence with N-point DFT $\{X[k]\}_{n=0}^{N-1}$.

   (a) Show that $X[N/2]$ is real-valued if N is even.

   (b) Show that $X[\langle N-k\rangle_N] = X^*[k]$ where $\langle n\rangle_N$ denotes n modulo N.

(6 Pts.)

9. The z-transform of $x[n]$ is

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, \quad \text{ROC:}|z| > \frac{1}{2}.$$

Compute the DTFT of the following signals:

(a) $\{y[n]\}$ where $y[n] = x[n]\, e^{-j\pi n/4}$, for all n.

(b) $\{v[n]\}$ where $v[n] = x[2n+1]$, for all n.

(6 Pts.)

10. The signal $x_a(t) = 3\sin(40\pi t) + 2\cos(60\pi t)$ is sampled at a sampling period T to obtain the discrete-time signal $x[n] = x_a(nT)$.

    (a) Compute and sketch the magnitude of the continuous-time Fourier transform of $x_a(t)$.

    (b) Compute and sketch the magnitude of the discrete-time Fourier transform of $x[n]$ for: (1) $T = 10\,\text{ms}$; and (2) $T = 20\,\text{ms}$.

    (c) For $T = 10\,\text{ms}$ and $T = 20\,\text{ms}$, determinate whether the original continuous-time signal $x_a(t)$ can be recovered from $x[n]$.

(4 Pts.)

11. An analog signal $x_a(t) = \cos(200\pi t) + \sin(500\pi t)$ is to be processed by a digital signal processing (DSP) system with a digital filter sandwiched between an ideal A/D and an ideal D/A converters with the sampling frequency 1 kHz. Suppose that we want to pass the second component but stop the first component of $x_a(t)$. Sketch the specification of the digital filter in such a DSP system, and identify the transition band of that desired digital filter.

(6 Pts.)

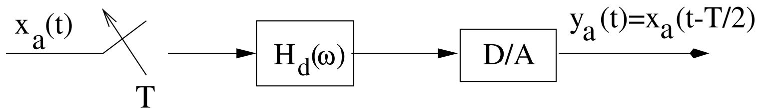
12. Let $x[n]$ be the input to a D/A convertor with $T = 5\text{ms}$. Sketch the output signal $x_a(t)$ for the following cases. **Label your axis tick marks and units clearly**.

    (a) The D/A convertor is a ZOH and $x[n] = 2\delta[n] + 3\delta[n-7]$.

    (b) The D/A convertor is an "ideal" D/A and $x[n] = 3\delta[n-7]$

(8 Pts.)

13. Consider the following system:



where the D/A convertor is an ideal D/A. Assume that $x_a(t)$ is bandlimited to $\Omega_{\max}$ (rad/sec), T is chosen to be $T < \frac{\pi}{\Omega_{\max}}$ and the impulse response of overall system is $h(t) = \delta(t - T/2)$ (or $H_a(\Omega) = e^{-j\Omega T/2}$).

(a) Determine the frequency response $H_d(\omega)$ of the **desired** digital filter.

(b) Determine the unit pulse response $h[n]$ of the **desired** digital filter.

(c) Determine a length-2 FIR filter $g[n]$ that approximates the above desired filter $h[n]$ using a rectangular window design. Is this **designed** FIR filter $g[n]$ LP or GLP?

(6 Pts.)

14. Consider the system in the figure below. For each of the following statements determine whether they are true or false. If false, give an example of $x[n]$ and the corresponding $y[n]$ that violate the property.

$$x[n] \longrightarrow \boxed{z^{-1}} \longrightarrow \boxed{\uparrow 2} \longrightarrow \boxed{\downarrow 3} \longrightarrow y[n]$$

(a) The system is linear

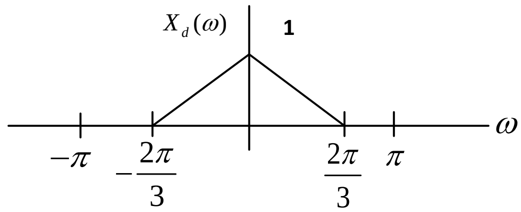
(b) The system is time-invariant

(c) The system is causal

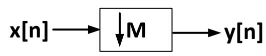
(d) The system is BIBO stable.

(6 Pts.)

15. Figure below shows the DTFT of a sequence $x[n]$.

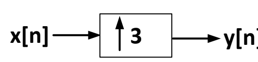


(a) Consider the downsampling operation shown below,



Compute the maximum integer value of M you can use so that no aliasing occurs.

(c) Consider the upsampling operation shown below. Sketch the DTFT of $y[n]$.



(6 Pts.)

16. Let $\mathcal{S}$ be a **stable LSI** system that maps an input signal $\{x[n]\}$ to a output signal $\{y[n]\}$ as:

$$\{x[n]\} \longrightarrow \boxed{\ \mathcal{S}\ } \longrightarrow \{y[n]\}$$

Suppose that instead of $\{x[n]\}$, we input $\{\widetilde{x}[n]\}$ into the system $\mathcal{S}$, where $\{\widetilde{x}[n]\}$ differs from $\{x[n]\}$ only at one sample; that is, for some finite constants n_0 and E,

$$\widetilde{x}[n] = \begin{cases} x[n] & \text{for all } n \neq n_0, \\ x[n] + E & \text{for } n = n_0. \end{cases}$$

This produces the output signal $\{\widetilde{y}[n]\}$. Show that for sufficiently large n, $\widetilde{y}[n]$ approach $y[n]$, which means

$$\lim_{n \to +\infty} |\widetilde{y}[n] - y[n]| = 0.$$

Hint: $\sum_{n \in \mathcal{Z}} |h[n]| < +\infty$ implies $\lim_{n \to \infty} |h[n]| = 0$.