University of Illinois
ECE 310

Spring 2021
Profs. Moon, Katselis, and Shomorony

Final Exam

8:00-11:00 AM, Wednesday, May 12, 2021

Please do not start the exam until the starting time.

- You may not use any books, electronic devices, or notes other than **four** handwritten two-sided sheet of 8.5" x 11" paper.
- You should solve the problems on blank sheets of paper, take pictures of your solutions, and upload them to Gradescope before the end of the exam time.

GOOD LUCK!

1. **(10 Pts.)** Answer **True** or **False** to each of the following statements:

   (a) The phase response of a real-valued filter has odd symmetry. **T/F**

   (b) Strictly linear phase filters are always GLP filters. **T/F**

   (c) Applying transposition to a filter structure with transfer function $H(z)$ may change $H(z)$. **T/F**

   (d) If a lowpass filter is designed using the window FIR filter design method, same errors (ripples) will be achieved in both passband and stopband. **T/F**

   (e) In spectral analysis, to improve the ability to distinguish between two closely spaced frequencies, one should use more samples of the signal because a longer window has a narrower main lobe in the frequency domain. **T/F**

   (f) The upsampling operation

   $$y[n] = \begin{cases} x[n/3] & \text{if } n = 3k \text{ for some integer } k \\ 0 & \text{otherwise} \end{cases}$$

   for $-\infty < n < \infty$, is linear. **T/F**

   (g) Downsampling can cause aliasing. **T/F**

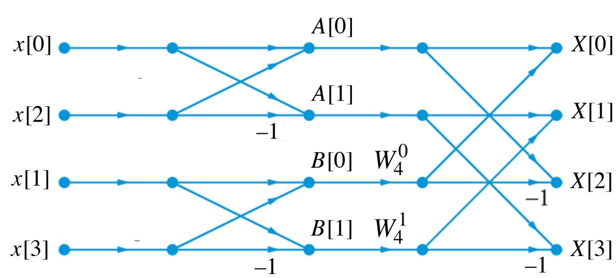
   (h) Type IV GLP FIR filters can be used to implement lowpass filters. **T/F**

   (i) Consider a radix-2 decimation in time FFT with size $N = 32$. Then the FFT will have five stages of computation. **T/F**

   (j) Direct Form II implementations require fewer multiplications per output sample than Direct Form I implementations. **T/F**

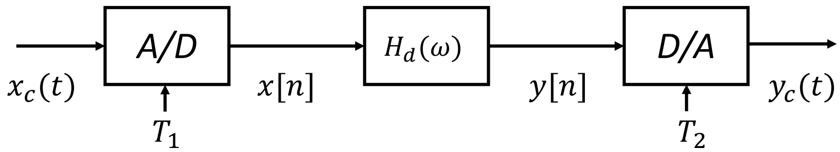
2. **(6 Pts.)** A real continuous-time signal $x_c(t)$ is bandlimited to frequencies below 8kHz, i.e., $X_c(\Omega) = 0$ for $|\Omega| \geq 2\pi(8000)$. Suppose that $x_c(t)$ is sampled with sampling period $T_s = 8 \cdot 10^{-5}$s to produce $x[n] = x_c(nT_s)$ and 4000 samples are extracted corresponding to $n = 0, 1, \ldots, 3999$. Let $\{X[k]\}_{k=0}^{3999}$ be the 4000-point DFT of $\{x[n]\}_{n=0}^{3999}$. To what continuous-time frequencies (in Hz) do the indices $k = 500$ and $k = 2800$ correspond?

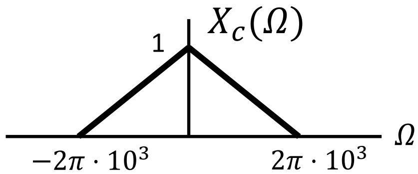
3. **(4 Pts.)** You would like to compute the DFT of $\{x[n]\}_{n=0}^{3} = \{3, 3, 1, 7\}$ using a radix-2 decimation-in-time FFT, with the following flow graph:

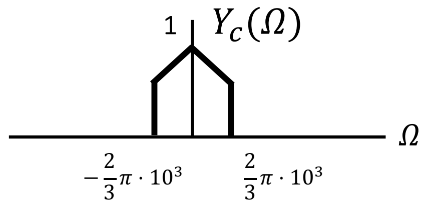


   Find the values of $A[0], A[1], B[0], B[1], X[0], X[1], X[2], X[3]$.

4. **(8 Pts.)** Consider the following system with an ideal A/D and an ideal D/A. The input signal $x_c(t)$ and output signal $y_c(t)$ have the continuous-time Fourier transforms shown in the figure below.





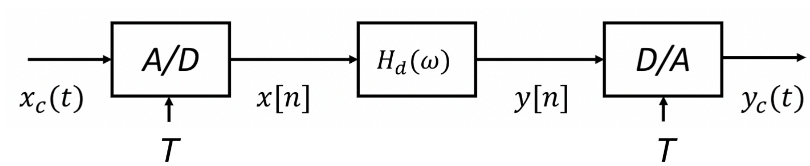


Sketch the desired frequency response of the digital filter $H_d(\omega)$ for this system for each of the following cases.

(a) $T_1 = T_2 = \frac{1}{2 \cdot 10^3}$.

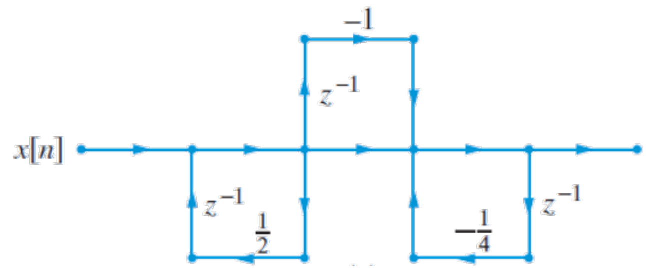
(b) $T_1 = \frac{1}{2 \cdot 10^3}$ and $T_2 = \frac{1}{10^3}$

5. **(6 Pts.)** A speech signal $x_c(t)$ is assumed to be bandlimited to 30 kHz. You would like to use the following setup in order filter this signal:



You would like to pass all frequencies between 7.5 kHz and 15 kHz, while suppressing all other frequencies. Determine the maximum sampling rate T that can be used to perform this task and sketch the frequency response $H_d(\omega)$ of the ideal filter needed.

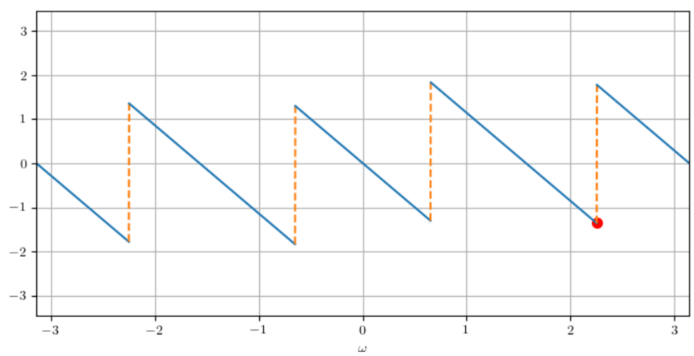
6. **(8 Pts.)** A discrete-time system is described by the following signal flow graph:



(a) Determine the transfer function of the filter.

(b) Apply transposition in order to obtain an equivalent filter structure.

7. **(6 Pts.)** Consider the following phase response $\angle H_d(\omega)$ of an FIR filter with $\omega \in [-\pi, \pi]$:



(a) Has this filter strict linear phase? Justify your answer.

(b) Has this filter (a) even symmetry, (b) odd symmetry, (c) neither even nor odd symmetry. Justify your answer.

(c) Determine $H_d(\omega_0)$ at the red point in the plot.

8. **(5 Pts.)** Let $x[n]$ be a sequence that is zero outside the interval $[0, N-1]$ with DTFT $X_d(\omega)$. Define the signals

$$y[n] = \begin{cases} x[n], & n = 0, 1, \ldots, N-1 \\ 0, & n = N, \ldots, 3N-1 \end{cases} \quad \text{and} \quad g[n] = \begin{cases} x\left[\frac{n}{3}\right], & n = 0, 3, 6, \ldots \\ 0, & \text{otherwise} \end{cases}, n = 0, 1, \ldots, 3N-1.$$

Let $Y[k]$ and $G[k]$ be the $3N$-point DFTs of $y[n]$ and $g[n]$. Select the correct expression for the DFT of each sequence in terms of samples of $X_d(\omega)$.

(a) $Y[k] = X_d\left(\frac{2\pi}{N}k\right)$ and $G[k] = X_d\left(\frac{2\pi}{3N}k\right)$ for $k = 0, 1, \ldots, 3N-1$

(b) $Y[k] = X_d\left(\frac{2\pi}{3N}k\right)$ and $G[k] = X_d\left(\frac{6\pi}{N}k\right)$ for $k = 0, 1, \ldots, 3N-1$

(c) $Y[k] = X_d\left(\frac{2\pi}{3N}k\right)$ and $G[k] = X_d\left(\frac{2\pi}{N}k\right)$ for $k = 0, 1, \ldots, 3N-1$

(d) $Y[k] = X_d\left(\frac{2\pi}{3N}k\right)$ and $G[k] = \begin{cases} X_d\left(\frac{2\pi}{N}k\right), & k = 0, 3, 6, \ldots \\ 0, & \text{otherwise} \end{cases}, k = 0, 1, \ldots, 3N-1$

(e) None of the above

9. **(8 Pts.)** Let $\{x[n]\}_{n=0}^{3} = \{1, 3, -2, 4\}$ and $\{h[n]\}_{n=0}^{2} = \{-1, 1, 3\}$.

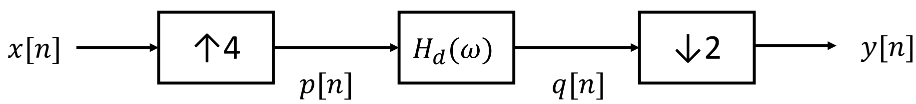
   (a) Compute the circular convolution $x[n] \circledast_4 h_{zp}[n]$ where $\{h_{zp}[n]\}_{n=0}^{3} = \{h[0], h[1], h[2], 0\}$.

   (b) Compute the linear convolution $x[n] * h[n]$.

   (c) What is the smallest value of N so that N-point circular convolution is equal to the linear convolution?

   (d) If you would like to compute the linear convolution $x[n] * h[n]$ using the DFT method employing the radix-2 decimation-in-time FFT algorithm, how many zeros should be padded to $x[n]$ and $h[n]$, respectively?

10. **(9 Pts.)** Consider the system in the figure below.



where the frequency response of $H_d(\omega)$ is

$$H_d(\omega) = \begin{cases} 1, & \frac{1}{4}\pi \le |\omega| \le \frac{1}{2}\pi \\ 0, & \text{otherwise.} \end{cases}$$

Suppose that an input $x[n]$ has the following DTFT

$$X_d(\omega) = \begin{cases} 1, & 0 < \omega \le \pi \\ \frac{\omega}{\pi} + 1, & -\pi < \omega \le 0. \end{cases}$$

   (a) Sketch $P_d(\omega)$.

   (b) Sketch $Q_d(\omega)$.

   (c) Sketch $Y_d(\omega)$.

11. **(8 Pts.)**

   (a) Determine the unit impulse response $h_1[n]$ of an ideal discrete-time lowpass filter with cutoff frequency $\omega_c = \pi/5$.

   (b) Determine the unit impulse response $h_2[n]$ of an ideal discrete-time bandpass filter satisfying the following specification:

$$H_d(\omega) = \begin{cases} 1, & \frac{3}{10}\pi \le |\omega| \le \frac{7}{10}\pi \\ 0, & \text{otherwise.} \end{cases}$$

   Hint: Use the result from (a) and the DTFT pair,

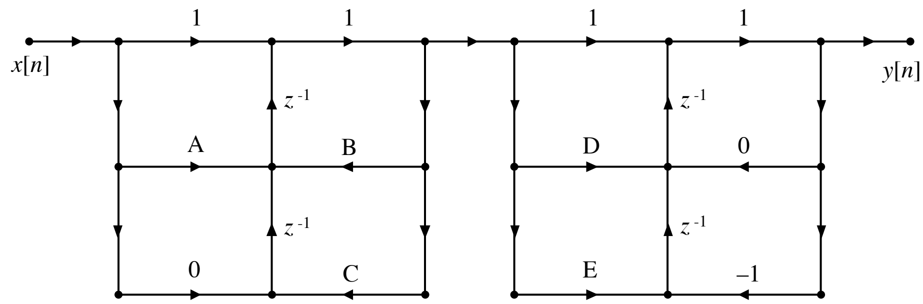
$$\cos(\omega_0 n) \cdot x[n] \longleftrightarrow \frac{1}{2}X_d(\omega - \omega_0) + \frac{1}{2}X_d(\omega + \omega_0).$$

   (c) Determine a length-7 causal GLP FIR filter $g[n]$ that approximates the above desired bandpass filter $h_2[n]$ using a rectangular window design.

12. **(10 Pts.)** Consider a causal LTI system with the following transfer function:

$$H(z) = \frac{(1 - z^{-3})}{(1 - jz^{-1})(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z^{-1})(1 + jz^{-1})}$$

You would like to implement it in cascade form using second-order transpose form II sections, according to the flow graph shown below. Determine the value of A, B, C, D and E.



13. **(6 Pts.)** The impulse response of a filter is given by

$$h[n] = \delta[n] + 3\delta[n-1] + \delta[n-2].$$

(a) Sketch the magnitude and phase of the filter's frequency response.

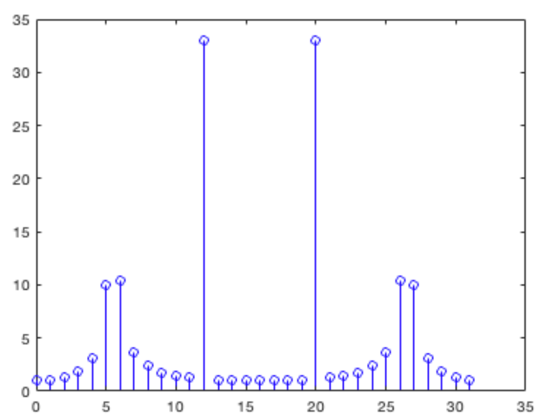
(b) Does the filter have strictly linear phase? Justify your answer.

(c) What is the output $y[n]$ when the input is $x[n] = 5 + e^{j\pi n/4}$ for all n?

14. **(6 Pts.)** Suppose we have a discrete-time signal $x[n]$ defined as

$$x[n] = A_1 \cos\left(\frac{k_1\pi}{32}n\right) + A_2 \cos\left(\frac{3\pi}{4}n\right),$$

where A_1, A_2 and k_1 are known to be positive integers. You compute the 32-point DFT of $\{x[n]\}_{n=0}^{31}$ and observe that the magnitude of $\{X[k]\}_{k=0}^{31}$ is as shown below:



Find the values of A_1, A_2 and k_1.