Math 241 Calculus III (Prof. Honold) **Midterm Exam 3** 2020/12/16

Name: _____ Student No.: _____ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. The value of $\int_{[0,1]^2} (x+y)^2 \,\mathrm{d}^2(x,y)$ is

☐ $\frac{1}{6}$ ☐ $\frac{1}{2}$ ☐ $\frac{5}{6}$ ☑ $\frac{7}{6}$ ☐ $\frac{3}{2}$

2. With $D = \{(x,y) \in \mathbb{R}^2; x^2+y^2 \le 9,\ 0 \le x \le y\}$ the integral $\int_D y - x \,\mathrm{d}^2(x,y)$ is equal to

☐ $9+9\sqrt{2}$ ☐ $9-9\sqrt{2}$ ☑ $-9+9\sqrt{2}$ ☐ $-9-9\sqrt{2}$ ☐ 0

3. The volume of $\{(x,y,z) \in \mathbb{R}^3; y^2+z^2 \le x(1-x)\}$ is

☐ 0 ☐ $\pi/4$ ☑ $\pi/6$ ☐ $\pi/8$ ☐ $+\infty$

4. $\lim_{k\to\infty} \int_0^1 \frac{t(1+k^2t^2)}{(1+kt)^2}\,\mathrm{d}t$ is equal to

☐ 0 ☐ $\frac{1}{3}$ ☑ $\frac{1}{2}$ ☐ 1 ☐ $+\infty$

5. For $F(x) = \int_0^\infty \frac{\sin(xt)}{1+t^3}\,\mathrm{d}t$ the derivative $F'(0)$ is

☐ 0 ☐ $\int_0^\infty \frac{\mathrm{d}t}{1+t^3}$ ☐ $\int_0^\infty \frac{\mathrm{d}t}{(1+t^3)^2}$ ☐ undefined ☑ $\int_0^\infty \frac{t}{1+t^3}\,\mathrm{d}t$

6. The function $f(x,y) = x^2+3xy+y^2$ has at $(x,y)=(0,0)$

☐ a non-critical point ☐ a local maximum ☐ a local minimum
☑ a saddle point ☐ none of the foregoing

7. The graph of $f(x,y) = \mathrm{e}^{2xy}$ has at the point $(1,2)$ in north-western direction the slope

☐ $-\mathrm{e}^4/\sqrt{2}$ ☐ $\mathrm{e}^4\sqrt{2}$ ☑ $-\mathrm{e}^4\sqrt{2}$ ☐ $-2\mathrm{e}^4$ ☐ $\mathrm{e}^4/\sqrt{2}$

8. The surface in $\mathbb{R}^3$ with equation $xy+yz+z=1$ is a

☐ ellipsoid ☐ hyperboloid of 1 sheet ☐ hyperboloid of 2 sheets
☐ elliptic paraboloid ☑ hyperbolic paraboloid

9. The tangent plane to the surface $z = x^2-y^2$ in $(1,2,-3)$ contains the point

☑ $(1,1,1)$ ☐ $(-1,1,1)$ ☐ $(1,-1,1)$ ☐ $(1,1,-1)$ ☒ $(0,0,0)$

10. The function $z = g(x,y)$ implicitly defined by the equation $x^3y+y^3z+z^3x = -1$ and $g(-1,1)=1$ has $\nabla g(-1,1)$ equal to

☐ $(1,-1)$ ☐ $(1,-2)$ ☑ $(2,1)$ ☐ $(1,2)$ ☐ $(2,-1)$

Time allowed: 40 min CLOSED BOOK ***Good luck!***

$$\nabla g(-1,1) = \begin{pmatrix} \frac{\partial x}{\partial z} \\ \frac{\partial z}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{F_x}{F_z} \\ \frac{F_y}{F_z} \end{pmatrix} = \begin{pmatrix} \frac{3x^2y+z^3}{y^3+3z^2x} \\ \frac{x^3+3y^2z}{y^3+3z^2x} \end{pmatrix}$$

x^3+3y^2z

$$-1,1,1 \quad = \begin{pmatrix} \frac{4}{-2} \\ \frac{-1+3}{-2} \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$\nabla = \begin{pmatrix} 2x \\ -2y \\ -1 \end{pmatrix} \quad \begin{pmatrix} 2 \\ -4 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \quad \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} \frac{F_x}{F_z} \\ \frac{F_y}{F} \end{pmatrix} \quad \begin{matrix} y^3+3z^2x \\ 1-3 \\ -1+3 \\ =2 \end{matrix} = -1$$