Math 241 Calculus III (Prof. Honold) **Midterm Exam 3** 2022/05/11

Name: _____ Student No.: _____ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. The value of $\int_{[0,1]^2} x(x-y)(x+y)\,\mathrm{d}^2(x,y)$ is

☐ $-\frac{1}{12}$ ☐ $-\frac{1}{6}$ ☐ $-\frac{1}{3}$ ☐ 0 ☑ $\frac{1}{12}$

2. Let Δ be the (solid) triangle in $\mathbb{R}^2$ with vertices $(0,1)$, $(1,0)$, $(2,2)$. The integral $\int_\Delta xy\,\mathrm{d}^2(x,y)$ equals

☐ $7/8$ ☑ $13/8$ ☐ $19/8$ ☐ $23/8$ ☐ $29/8$

3. The volume of $\{(x,y,z)\in\mathbb{R}^3; x^2+y^2+z^2\le 2x+3\}$ is contained in the interval

☐ $[0,20)$ ☑ $[20,40)$ ☐ $[40,60)$ ☐ $[60,80)$ ☐ $[80,100]$

4. For $F(x)=\int_0^1 \frac{\ln(t^2+2xt+1)}{t}\,\mathrm{d}t$ the derivative $F'(0)$ is equal to

☑ $\pi/2$ ☐ 0 ☐ π/e ☐ $2e$ ☐ $\ln(t^2+1)/t$

5. The value of the integral $\int_{[-1,1]^2} \frac{1}{1+xy}\,\mathrm{d}^2(x,y)$ is contained in the interval

☐ $[0,1)$ ☐ $[1,2)$ ☐ $[2,4)$ ☑ $[4,8)$ ☐ $[8,+\infty)$

6. The optimization problem "minimize $x^2+y^2+z^2$ subject to $x+2y+3z=1$" has the Lagrange multiplier

☑ $\lambda=\frac{1}{7}$ ☐ $\lambda=\frac{2}{7}$ ☐ $\lambda=\frac{4}{7}$ ☐ $\lambda=\frac{8}{7}$ ☐ $\lambda=\frac{16}{7}$

7. The function $f(x,y)=x^2-3xy+2y^2$ has at $(x,y)=(0,0)$

☐ a non-critical point ☐ a local maximum ☑ a local minimum
☐ a saddle point ☐ none of the foregoing

8. The function $g(x,y)=ax^2+bxy+cy^2$ satisfies Laplace's Equation $g_{xx}+g_{yy}=0$ if

☐ $c=0$ ☐ $a=-b$ ☐ $a=0$ ☐ $b=-c$ ☑ $c=-a$

9. The tangent plane to the surface $z=\sqrt{x^3+y^3}$ in $(2,2,4)$ contains the point

☑ $(0,0,-2)$ ☐ $(0,0,1)$ ☐ $(0,0,2)$ ☐ $(0,0,-1)$ ☐ $(0,0,0)$

10. The function $x=g(y,z)$ implicitly defined by the equation $x^3-xyz+1=0$ and $g(2,1)=1$ has $\nabla g(2,1)$ equal to

☐ $(-1,2)$ ☐ $(1,-2)$ ☑ $(-1,-2)$ ☐ $(-2,1)$ ☑ $(1,2)$

Time allowed: 45 min CLOSED BOOK ***Good luck!***

$$D = \begin{pmatrix} 3x^2 \\ 3y^2 \\ -2z \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \\ -8 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$$

$z^2 = x^3 + y^3$

$x^3 + y^3 - z^2 = 0$ 곡면

$$\begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} + t_1 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

$x = -1$

$x = 1$

$$D \begin{pmatrix} 3x^2 - yz \\ -xz \\ -xy \end{pmatrix} \qquad \begin{matrix} 3x^2 - z \\ -x \\ -2x \end{matrix} \qquad \begin{matrix} 1 \\ -1 \\ -2 \end{matrix}$$