Math 241 Calculus III (Prof. Honold) **Midterm Exam 1** 2021/10/20

Name: _____ Student ID : _____ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. The tangent to $f(t) = (t^5, t^3, t)$ in the point $(1,1,1)$ meets the plane $x - y + z = 0$ in
   ☐ $(0, \frac{2}{3}, \frac{2}{3})$ ☑ $(-\frac{2}{3}, 0, \frac{2}{3})$ ☐ $(\frac{2}{3}, \frac{2}{3}, 0)$ ☐ $(\frac{2}{3}, 0, -\frac{2}{3})$ ☐ no point.

2. The arc length of the curve $g(t) = \big(t\sin(18t), t\cos(18t), 4t^{3/2}\big)$, $t \in [0,3]$ is
   ☐ 10 ☐ 36 ☐ 38 ☑ 84 ☐ 148

3. For a C^1-curve $\mathbf{u}\colon I \to \mathbb{R}^3 \setminus \{\mathbf{0}\}$ and $t \in I$, the derivative $(\mathrm{d}/\mathrm{d}t)\,|\mathbf{u}(t)|^3$ is equal to
   ☐ $3\,|\mathbf{u}(t)|^2\,\mathbf{u}'(t)$ ☐ $3\,|\mathbf{u}(t)|^2$ ☒ $3\,|\mathbf{u}(t)|\,\mathbf{u}(t)\cdot\mathbf{u}'(t)$ ☐ $\frac{3}{2}\,|\mathbf{u}(t)|$ ☐ $3\,\mathbf{u}(t)\cdot\mathbf{u}'(t)$

4. The range of a parametric space curve $f\colon \mathbb{R} \to \mathbb{R}^3$ with nonzero curvature and $f''(t) = \mathbf{w} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$ (i.e., a nonzero constant vector) is
   ☒ a line ☐ an ellipse ☑ a parabola ☐ a hyperbola ☐ non-planar

5. The volume of the pyramid ("tetrahedron") with vertices $(2,0,-1), (3,-1,0), (0,a,-1), (0,3,1)$ is equal to 1 for
   ☐ $a = -3$ ☐ $a = -2$ ☐ $a = -1$ ☐ $a = 0$ ☑ $a = 1$

6. The inverse matrix of $\begin{pmatrix} 5 & 6 \\ 6 & 7 \end{pmatrix}$ is
   ☐ $\begin{pmatrix} -5 & 6 \\ 6 & -7 \end{pmatrix}$ ☐ $\begin{pmatrix} 5 & -6 \\ -6 & 7 \end{pmatrix}$ ☐ $\begin{pmatrix} -7 & -6 \\ -6 & -5 \end{pmatrix}$ ☑ $\begin{pmatrix} -7 & 6 \\ 6 & -5 \end{pmatrix}$ ☐ $\begin{pmatrix} 7 & -6 \\ -6 & 5 \end{pmatrix}$

7. The matrix $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ describes a reflection whose axis in polar coordinates is given by
   ☐ $\theta = 0°$ ☐ $\theta = 22.5°$ ☑ $\theta = 45°$ ☐ $\theta = 72°$ ☐ $\theta = 90°$

8. The maximum rank of $\mathbf{A} \in \mathbb{R}^{4\times 5}$ with entries ± 1 and exactly two entries equal to -1 is
   ☐ 1 ☐ 2 ☑ 3 ☐ 4 ☐ 5

9. The linear system
$$\begin{aligned} x_1 + x_2 + x_3 &= b \\ x_1 - x_3 &= -1 \\ 3x_1 - 2x_2 - 7x_3 &= 1 \end{aligned}$$
has a solution if
   ☐ $b = 0$ ☑ $b = -3$ ☐ $b = 3$ ☐ $b = -1$ ☐ $b = 1$

10. The distance from the point $(1,-1,1)$ to the line $x_1 - x_2 = 2x_2 - x_3 = 1$ is
    ☑ $\frac{1}{2}\sqrt{14}$ ☐ $\frac{11}{4}$ ☐ $\frac{1}{2}\sqrt{11}$ ☐ $\sqrt{5}$ ☐ $\frac{7}{2}$

Time allowed: 40 min CLOSED BOOK ***Good luck!***

$u(t) = (t^3, t^2, t)$

$(\sqrt{t^6 + t^4 + t^2})^3$

$3t^2 \quad 2t \quad 1$

$$(t^6+t^4+t^2)^{\frac{3}{2}}$$

$$\frac{3(6t^5+4t^3+2t)}{2}\cdot\sqrt{t^6+t^4+t^2}$$

$$(9t^5+6t^3+3t)\sqrt{t^6+t^4+t^2}$$

$$y=x^2$$

$$\begin{pmatrix} t \\ t^2 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2t \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$\frac{3}{2}\left(u_1^2(t)+u_2^2(t)+u_3^2(t)\right)^{\frac{1}{2}}\cdot\left(2u_1(t)u_1'(t)\right.$$