Sample Midterm 1 Questions

Note These 4 questions were never asked together in a written exam and don't constitute a full set of Math257 Midterm 1 questions.

Question 1 Depending on the parameter $t \in \mathbb{R}$, determine the solution of

$$\begin{array}{rcrcrcl} 2t \cdot x_1 & - & x_2 & - & x_3 & = & 2t-2 \\ 2x_1 & - & x_2 & - & t \cdot x_3 & = & 0 \\ -4x_1 & + & (3-t) \cdot x_2 & + & (1+t) \cdot x_3 & = & 1-t \end{array}$$

Question 2 a) Compute the determinant of the matrix

$$\begin{pmatrix} 1 & 6 & 0 & -4 \\ -1 & 0 & 1 & 3 \\ 3 & 1 & -2 & 0 \\ -6 & -1 & 1 & 1 \end{pmatrix}$$

.

$dx + e = 0$

$d = 0, e = 0$

b) True or false? $\det(\lambda \mathbf{A}) = \lambda \det(\mathbf{A})$ holds for all matrices $\mathbf{A} \in \mathbb{R}^{4\times 4}$ and $\lambda \in \mathbb{R}$.

Question 3 a) Find the inverse matrix of

$$\mathbf{A} = \begin{pmatrix} 2 & 2 & 1 \\ -3 & -4 & -1 \\ 4 & 1 & 2 \end{pmatrix}.$$

$B \cdot A^{-1}$

b) True or false? Changing the middle entry $a_{22} = -4$ of $\mathbf{A}$ changes the top left entry b_{11} of the inverse $\mathbf{B} = \mathbf{A}^{-1}$.

$a = 0, e = 0$ x^4

Question 4 Consider the map $f\colon P_4(\mathbb{R}) \to P_4(\mathbb{R})$ defined by

$(ax + e) = 0$

$$f(p) = p(1) + p'(1)(x-1).$$

a) Show that f is linear.

b) Determine a basis of the kernel of f.

$ax + b = 0$

c) Verify the rank-nullity formula for f and show that $P_4(\mathbb{R}) = \ker(f) \oplus \operatorname{range}(f)$.

d) True or false? For every linear map $g\colon P_4(\mathbb{R}) \to P_4(\mathbb{R})$ we have $P_4(\mathbb{R}) = \ker(g) \oplus \operatorname{range}(g)$.

assume $P_1(R) = a_1x^4 + b_1x^3 + c_1x^2 + d_1x + e_1$

$P_2(R) = a_2x^4 + b_2x^3 + c_2x^2 + d_2x + e_2$

(2) $f(P) = 0 \Rightarrow a_1 + b_1 + c_1 + d_1 + e_1 + (4a_1 + 3b_1 + 2c_1 + d_1)(x-1)$

$= 4a_1x + 3b_1x + 2c_1x + d_1x - 3a_1 - 2b_1 - c_1 + e_1 = 0$

$\therefore\ d_1 = -4a_1 - 3b_1 - 2c_1$

$e_1 = 3a_1 + 2b_1 + c_1$

$\therefore$ basis: $x^4 - 4x + 3,\ x^3 - 3x + 2,\ x^2 - 2x + 1$

base of range f: $x, 1$

(c) $\dim(P_4(R)) = 5$, $\dim(\ker f) = 3$, $\dim(\operatorname{range} f) = 2$

$\ker(f) \cap \operatorname{range}(f) = 0$

(d) ~~True~~ False

Q1: extended matrix

$$\begin{pmatrix} 2t & -1 & -1 & 2t-2 \\ 2 & -1 & -t & 0 \\ -4 & 3-t & 1+t & 1-t \end{pmatrix}$$

$$\begin{array}{l} r_2 \leftrightarrow r_1 \\ \sim \\ r_2 \times \frac{1}{2} \end{array} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{t}{2} & 0 \\ 2t & -1 & -1 & 2t-2 \\ -4 & 3-t & 1+t & 1-t \end{pmatrix}$$

$$\begin{array}{l} r_2 - 2tr_1 \\ \sim \\ r_4 + 4r_1 \end{array} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{t}{2} & 0 \\ 0 & t-1 & t^2-1 & 2t-2 \\ 0 & 1-t & 1-t & 1-t \end{pmatrix}$$

$$\begin{array}{l} r_3 + r_2 \\ \sim \end{array} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{t}{2} & 0 \\ 0 & t-1 & t^2-1 & 2t-2 \\ 0 & 0 & t(t-1) & t-1 \end{pmatrix}$$

① when $t-1=0 \Rightarrow t=1$ solution $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = c_1 \begin{pmatrix} \frac{1}{2} \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} \frac{1}{2} \\ 0 \\ 1 \end{pmatrix}$

when $t-1 \neq 0$

$$\begin{array}{l} r_1 + \frac{\frac{1}{2}}{t-1} r_2 \\ \sim \\ r_2 \cdot \frac{1}{t-1} \\ r_3 \cdot \frac{1}{t-1} \end{array} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & t+1 & 2 \\ 0 & 0 & t & 1 \end{pmatrix}$$

② when $t=0$ no solution

when $t \neq 0$

$$\begin{array}{l} r_1 - \frac{\frac{1}{2}}{t} r_3 \\ \sim \end{array} \begin{pmatrix} 1 & 0 & 0 & 1-\frac{1}{2t} \\ 0 & 1 & 0 & 1-\frac{1}{t} \end{pmatrix}$$

③ when $t \neq 0$ and $t \neq 1$
only one solution

$$r_2 - \frac{2t-1}{t} r_3 \quad \begin{pmatrix} 0 & 0 & 1 & \frac{1}{t} \end{pmatrix} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1-2t \\ 1-\frac{1}{t} \\ \frac{1}{t} \end{pmatrix}$$

$$r_3 \cdot \frac{1}{t}$$

Q2. (1) 18D

(2) False

Disprove assume $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\lambda = 2$

$$\therefore \det(\lambda A) = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$$\nparallel$$

$$\lambda \det(A) = 2$$

Q3: (a) $A = \begin{pmatrix} 2 & 2 & 1 \\ -3 & -4 & -1 \\ 4 & 1 & 2 \end{pmatrix}$, $A^* = \begin{pmatrix} -7 & -3 & 2 \\ 2 & 0 & -1 \\ 13 & 6 & -2 \end{pmatrix}$

$|A| = 3$ $\qquad \therefore A^{-1} = \dfrac{A^*}{|A|} = \begin{pmatrix} -\frac{7}{3} & -1 & \frac{2}{3} \\ \frac{2}{3} & 0 & -\frac{1}{3} \\ \frac{13}{3} & 2 & -\frac{2}{3} \end{pmatrix}$

(b) True