ZJU-UIUC INSTITUTE

Online Final Examination

For Students, please read and sign the honor statement on a sheet of paper with your name, student ID number, date, and read the specific requirements and instructions below before starting your exam.

(For instructors, please complete the form below)

Course Code: **PHYS 213**	Semester: **Spring 2020**	Instructor: **O. Penkov**	
Exam Type: Closed-book √ Open-book ☐ Partly Open-book ☐ Take Home ☐			
Exam Date: 2020-05-28	Start Time: 14:00	End Time: 17:00	Duration: 3 hrs
Total pages: 12		Total questions: 30	

Specific requirements and instructions to students:

1) You can use a self-made formula sheet (1 page, A4)
2) You can use a calculator
3) You CAN NOT use any software, eg. MATLAB, Mathcad etc.
4) The answer could be submitted in any format. For your convenience, the question booklet is provided in both *pdf* and *docx* format.

Please clearly mark the right answer and provide a solution or brief explanation (where applicable). Each question has **only one** right answer.

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Honor Statement

I have read the academic integrity statement. I promise to abide by the exam rules and regulations and agree to comport myself during the remotely administered exam in the same manner as if I were in a proctored examination room.

Please write "**I have read and will follow the Honor Statement**" on a sheet of paper and include the following information, then submit along with your answer sheet in the end.

Name:

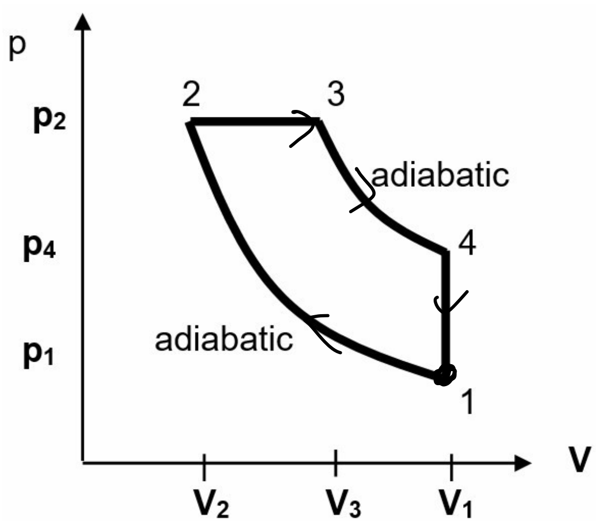
Student ID number:

Date:

(Please go on to the next page for questions)

The next three questions pertain to the situation described below.

Consider the thermodynamic cycle shown below.



a.

1) In order to use this cycle as a heat engine to do useful work, we should run the cycle in a

a. clockwise direction
b. counter-clockwise direction
c. this cycle cannot be used as a heat engine

a.

2) Assume that the working ideal gas is one mole of the molecule H_2, at a temperature such that the translational and rotational degrees of freedom are active. $V_1 = 1\ m^3$, $V_2 = 0.25\ m^3$, $V_3 = 0.5\ m^3$, and p_1=1 atm. What is the magnitude of the work done for the adiabatic process 1-2 (represented by the line connecting points 1 and 2)?

The equation of the line is P V^{γ}=constant, where $\gamma = \dfrac{N_{DOF}/2+1}{N_{DOF}/2}$

a. 1.9×10^5 J
b. 1.9 J
c. 7.5×10^4 J
d. 3.8×10^5 J
e. 3.8×10^4 J

$\gamma = \frac{7}{5}$ $\quad P_1 V_1^{\frac{7}{5}} = 101300 \cdot 1 = 1 \quad P = V_1^{-\frac{7}{5}} \cdot 101300$

$\int p\,dV = 101300 \int V_1^{-\frac{7}{5}}\,dV = 101300 \cdot \left(-\frac{5}{2}\right) V_1^{-\frac{2}{5}} \Big|_1^{0.25}$

1.9×10^5

a.

3) As the gas expands in step 3-4, from an initial volume V_3 to a final volume of V_1, by what factor does the average speed (rms velocity) of the gas molecules change?

$PV = nTR \quad n = 1$

a. $v_{rms}^{FINAL} = 2^{-1/5}\ v_{rms}^{INITIAL}$
b. $v_{rms}^{FINAL} = 2^{-1/2}\ v_{rms}^{INITIAL}$
c. there is no change to the average molecule speed

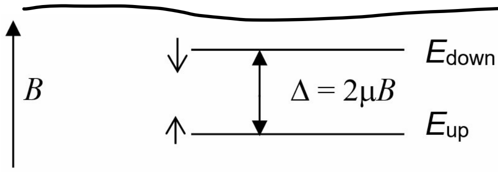
$V_1 T_1^{\frac{5}{2}} = V_2 T_2^{\frac{5}{2}}$

$\left(\frac{T_1}{T_2}\right) = \left(\frac{V_2}{V_1}\right)^{\frac{2}{5}}$

$\frac{v_1}{v_2} = \left(\frac{V_2}{V_1}\right)^{\frac{1}{5}} = \left(\frac{0.5}{1}\right)^{\frac{1}{5}} = \left(\frac{1}{2}\right)^{\frac{1}{5}} = 2^{-\frac{1}{5}}$

The next three questions pertain to the situation described below.

Electrons placed in a magnetic field of strength B have energy levels as shown below, relating to the two allowed directions of the electron's spin magnetic moment. N_{up} and N_{down} relate to the total number of electrons in each state. The total number of electrons is $N = N_{up} + N_{down}$. The electrons have a magnetic moment of magnitude $\mu = 9.27 \times 10^{-24}$ J/T (joules per tesla).



a. 4) What is the ratio N_{down}/N_{up} for a temperature of 5 K and a magnetic field of B = 2 T?

a. N_{down}/N_{up}=0.584
b. N_{down}/N_{up}=0.526
c. N_{down}/N_{up}=0.438
d. N_{down}/N_{up}=0.292
e. N_{down}/N_{up}=0.0584

$$\frac{n_1}{n_2} = e^{\frac{-\Delta}{kT}} = e^{-\frac{9.27\times10^{-24}\cdot 2\cdot 2}{1.381\times10^{-23}\cdot 5}}$$

$$\simeq 0.584$$

$\frac{n_{down}}{n_{up}} = 0 \quad n_{up}$

a. 5) If the temperature is decreased such that $kT \ll \mu B$, what value will the total magnetic moment $M = \mu(N_{up} - N_{down})$ approach?

a. $M = N\mu$
b. $M = -N\mu$
c. $M = 0$

$M =$

a. 6) If we instead increase the temperature such that $kT \gg \mu B$, what value will the heat capacity C_V of this system approach?

$n_1 = n_2$

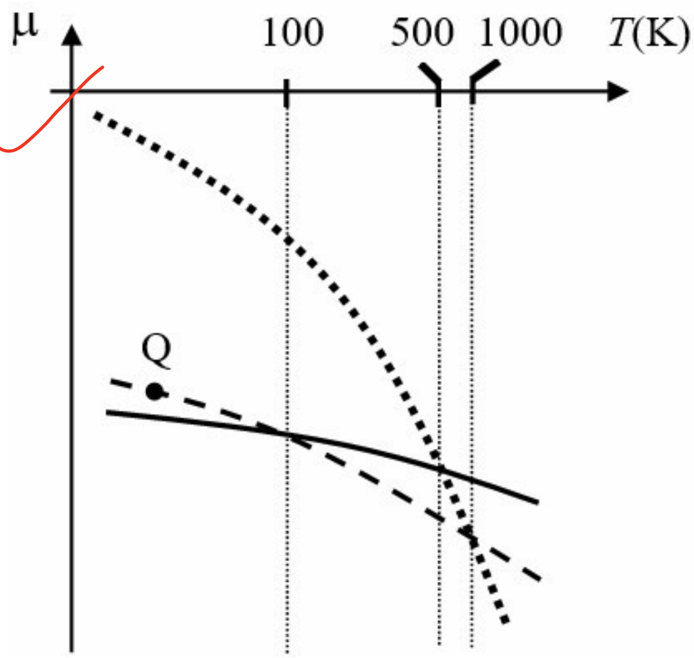
kT

a. $C_V = 0$
b. $C_V = Nk / 2$
c. $C_V = Nk \ln(2)$

$C_V = \frac{\Delta Q}{\Delta T}$

7) A substance is determined to have the following chemical potential (μ) versus temperature (T) diagram at a particular pressure. The three lines represent solid, liquid, and gas.

a.



What will happen to the substance after a long time if it starts out in the state labeled **Q** and the temperature is kept constant?

a. It will freeze. ✓
b. It will melt.
c. It will sublimate.
d. Nothing will happen.
e. It will boil.

a. 8) At 1 atm, nitrogen (N_2) molecules become a liquid below 77 K. Thus the vapor pressure of liquid nitrogen is 1 atm at 77 K.

Use this information to estimate the binding energy of a nitrogen molecule within the liquid.

Note: You may assume that the equilibrium condition for this system is $N_G/V = n_T e^{-\Delta/kT}$, with $n_T =$ 2 $\times 10^{31}\ m^{-3}$, and that the nitrogen gas is ideal.

✓a. 0.081 eV
b. 81 eV
c. 0.0081 eV
d. 0.81 eV
e. 8.1 eV

$PV = NkT$

$\frac{N}{V} = \frac{P}{kT}$

$\frac{P}{kTn_T} = e^{-\frac{\Delta}{kT}}$

$-\frac{\Delta}{kT} = \ln\frac{P}{kTn_T}$

$\Delta = -kT\ln\frac{P}{kTn_T}$

9) An analysis of Pluto's atmosphere has shown that the concentration of nitrogen molecules (N_2) at a height of 10 km is three-fourths (75%) that of the surface concentration.

a.

What is the local acceleration due to gravity g_{Pluto} near the surface of Pluto?

Note: Assume that the atmosphere is in thermal equilibrium, at a temperature of 50 K.

$e^{-\frac{E_H}{KT} - \frac{E_0 L}{KT}} = e^{-\frac{1\times10^4 \cdot mg}{KT}} = \frac{3}{4}$

$-\frac{1\times10^4 \cdot mg}{KT} = \ln\frac{3}{4}$

$\Rightarrow g = -\frac{KT\ln\frac{3}{4}}{10^4 \cdot m}$

≈ 0.427

a. 0.427 m/s^2
b. 0.854 m/s^2
c. 0.214 m/s^2
d. 1.71 m/s^2
e. 0.595 m/s^2

28 g/mol

$\frac{0.028 kg/mol}{6.02\times10^{23}}$

The next two questions pertain to the situation described below.

A block of aluminum has a heat capacity of 15 J/K at 300 K.

10) Approximately how many atoms are in this block of aluminum?

a.

a. 3.62×10^{23}
b. 1.09×10^{24}
c. 7.24×10^{23}

$3nR = 15$

$n = \frac{15}{3R} \approx 3.62\times10^{23}$

11) The aluminum block is initially at a temperature of 300 K. It is then brought into contact with 1 mole of gas particles at a temperature of 400 K and constant volume. After some time, the block and the gas thermalize to a common final temperature of 362 K. Of which of the following substances could the gas be composed?

A.

a. NH_3
b. H_2
c. Ar

15 J/K

$\frac{5}{2} \cdot 1 \cdot 8.31$

$15 \cdot \Delta T_1 \quad 20.775(100 - \Delta T_1)$

$35.775 \cdot \Delta T_1 = 2077.5$

$\Delta T_1 = 58.07$

The next two questions pertain to the situation described below.

Consider a system consisting of 6 distinct simple harmonic oscillators (SHOs) that are coupled together and can exchange energy. This system has a total energy of U=4ε stored amongst all of the oscillators, relating to four energy quanta (where ε is the quantized energy spacing between levels of the individual harmonic oscillators).

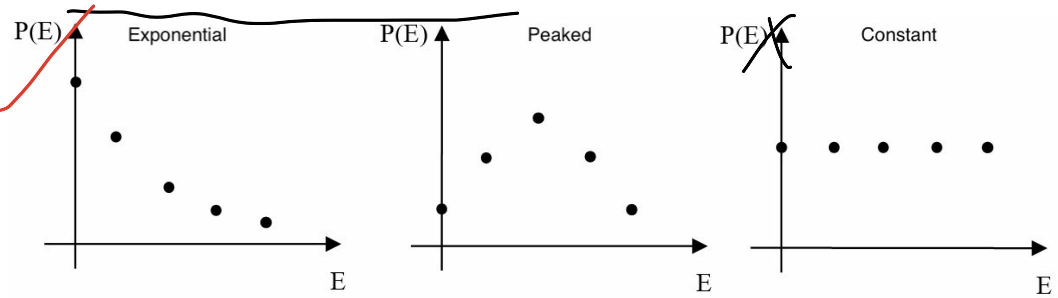
a. 12) How many unique ways can the four quanta of energy (4ε) be stored in this system?

a. 126
b. 210
c. 84
d. 1296
e. 54

$$\frac{9!}{4!\cdot 5!} = \frac{9\cdot \cancel{8}^{2}\cdot 7\cdot \cancel{6}}{\cancel{4}\cdot 3\cdot \cancel{2}\cdot 1} = 126$$

$\frac{18}{7}$

a. 13) Which of the following sketches most closely depicts the probability distribution P(E) of finding the first oscillator with energy E in equilibrium?



$$\frac{e^{-\frac{E}{kT}}}{e^{-\frac{E}{kT}}+e^{-\frac{U-E}{kT}}}$$

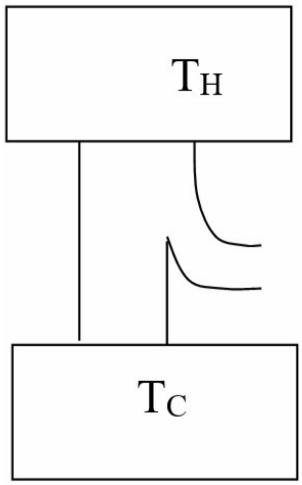
$$\frac{1}{1+e^{\frac{2E-U}{kT}}}$$

a. Exponential
b. Peaked
c. Constant

$\sim e^{-\frac{2E+U}{kT}}$

The next two questions pertain to the situation described below.

Consider the heat engine shown below, operated in contact with a cold thermal reservoir at temperature T_C = 200 K and a hot thermal reservoir at temperature T_H = 300 K.



1kJ

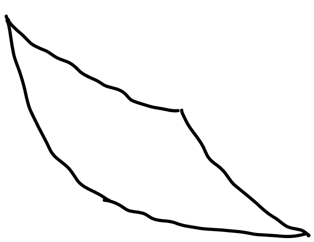
$W_{by} = 1000 \cdot \left(1 - \frac{200}{300}\right)$

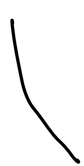
a. 14) If heat is extracted from the hot reservoir at a rate of 1 kJ every second, what is the maximum useable power that could be output by the heat engine?

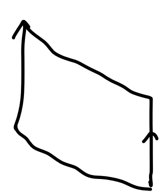
a. 333 W
b. 10^3 W
c. 167 W
d. 6.67 W
e. 33.3 W

a. 15) Ignoring any heat leaks or other sources of inefficiency, which of the following combinations of thermodynamic processes could be used cyclically by the heat engine to achieve the maximum useable power output?

a. adiabatic compression--> isothermal expansion --> adiabatic expansion --> isothermal compression
b. adiabatic compression--> adiabatic expansion--> adiabatic compression --> adiabatic expansion
c. isochoric heating--> isothermal expansion--> isochoric cooling--> isothermal compression

a. 

b. 

c. 

A 1-dimensional harmonic oscillator with frequency f = 7 x 10^{11} Hz is in equilibrium with a thermal reservoir of temperature T. The spacing between levels of the harmonic oscillator is given by ε = hf, and the ground state is defined to have energy $E_0 = 0$.

16) At what temperature does $P_1/P_0 = 1/3$, where P_1 is the probability that the oscillator has an energy E = ε (the first excited state) and P_0 is the probability to be in the ground state with an energy E_0?

a.

a. 30.6 K
b. 61.1 K
c. 153 K
d. 48.5 K
e. 7.71 K

$$\frac{P_1}{P_0} = e^{-\frac{E_1}{kT} + \frac{E_0}{kT}} = e^{\frac{-hf}{kT}} = \frac{1}{3}$$

$$\frac{-hf}{kT} = \ln\frac{1}{3} \quad T = \frac{-hf}{k\ln\frac{1}{3}} \approx 30.6K$$

a.

17) What happens to the ratio P_1/P_0 as the temperature is increased towards infinity?

a. It tends towards a value of 1
b. It tends towards a value of 1/2
c. It tends toward infinity, i.e., all the atoms in the excited state

$T \to \infty \quad 1$

The next two questions pertain to the situation described below.

About 4.3×10^{13} Watts of heat flow from the Earth's center to its surface, about half from radioactive decay and half left over from Earth's formation. The Earth is well-approximated as a sphere with radius 6370 km.

18) Suppose that this were Earth's *only* source of heat (i.e., neglect the Sun), and treat it as an ideal blackbody emitter. What temperature would Earth's surface equilibrate at?

a、

a. 34.9 K
b. 3300 K
c. 0.539 K
d. 1220 K
e. 49.4 K

$$4\pi r^2 \cdot \sigma_B T^4 = 4.3\times10^{13}$$

$$\Rightarrow T = 34.9K$$

19) Suppose that I somehow doubled the power flow to Earth's surface. How would this change the *wavelength* at which Earth's emission peaked? As before, assume Earth is an ideal blackbody emitter.

b.

a. It would increase (longer wavelength)
b. It would decrease (shorter wavelength)
c. It would stay the same

$P \nearrow 2 \quad T \nearrow 2^{\frac{1}{4}}$

λ_{max}

Suppose that we use a heater to boil liquid nitrogen (N_2 molecules). 4480 J of heat turns 20 g of liquid nitrogen into gas. Note that the latent heat is equal to the change in enthalpy, and that liquid nitrogen boils at 77 K. The system is kept at a constant pressure of 1 atm.

d. 20) Assuming that you can treat the gas as ideal gas and that the volume of the liquid is approximately zero, compute the binding energy of a nitrogen molecule in the liquid. (the binding energy is the difference in internal energy per molecule between the liquid and gas)

a.

$PV=nRT$

a. 9.4×10^{-21} J
b. 3.8×10^{27} J
c. 4.2×10^{-18} J
d. 10^{-20} J
e. 2.1×10^{-19} J

$4480 = \frac{20}{28} \cdot 6.02 \times 10^{23} \cdot V_1 + \frac{20}{28} \cdot 8.31 \cdot 77$

$H = U + PV = N\Delta + NkT = \Delta Q$

A hot copper sphere (mass = 5 kg, thermal conductivity κ_{Cu} = 385 W/m-K, specific heat c_{Cu} = 385 J/kg-K) at 200 °C is cooled by immersion in a cold container of water (mass = 50 kg, κ_{water} = 0.6 W/m-K, c_{water} = 4182 J/kg-K) at 5 °C. There are no heat leaks out of the system of copper sphere and water.

c. 21) What is the entropy change of the copper sphere during this process?

a. 0
b. 1010.4 J/K
c. -1010.4 J/K

$\Delta S = C_v \ln\left(\frac{T_f}{T_i}\right) = -1010.4$

$5 \cdot 385 \cdot \Delta T_1 = 50 \cdot 4182 \cdot (195 - \Delta T_1)$ $\quad T_f = 5 + (195 - \Delta T_1$

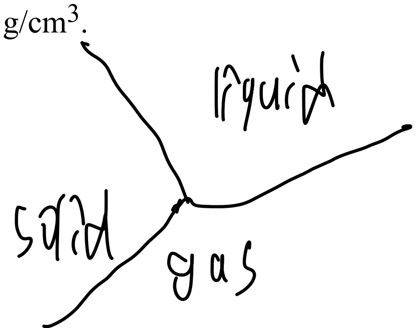
$\Rightarrow T_f = 6.77881\ °C$

d. 22) The latent heat (the energy it takes to turn from solid to liquid) of melting ice is 333 kJ/kg. Calculate the entropy difference of 5 kg of water and ice $\Delta S = S_{water} - S_{ice}$ at T = 0 °C.

a. 6.1 kJ/K
b. 9.14 kJ/K
c. 24.4 kJ/K
d. 0.203 kJ/K
e. 76.2 kJ/K

$\Delta S = \frac{\Delta Q}{T} = \frac{333 \cdot 5}{273.15}$

Suppose we tune the temperature and pressure of a container of gallium to its triple point at a temperature T=302 K, and pressure p=101 kPa. The densities of the phases of gallium are (i) solid: 5.91 g/cm^3 (ii) liquid: 6.05 g/cm^3 (iii) gas: 0.116 g/cm^3.



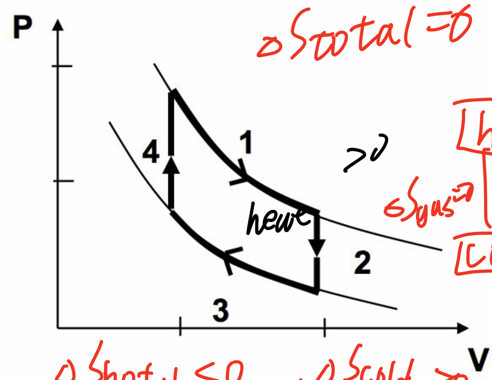
nTR

$\frac{5}{2} nR + nR$

23) If we slightly increase the pressure, which phase is stabilized in equilibrium?

b.

a. Solid
b. Liquid
c. Gas



$\Delta S_{total} = 0$

>0

hot

$\Delta S_{gas} =$ gas $\Rightarrow$ work

cold

Q_s

$\Delta S_{hot,1} < 0$, $\Delta S_{cold,2} > 0$

$\Delta S_{cold,3} > 0$, $\Delta S_{hot,4} < 0$

S_{hot} $\quad S_{hot} < 0$ $\quad T_{cold} > 0$

$\Delta S_{cold} > 0$

Consider a heat engine being driven in a Stirling cycle, as shown. Specifically, processes 1 and 3 are isothermal expansion and compression, respectively, and processes 2 and 4 are isochoric cooling and heating, respectively.

24) Which of the following statements is true about changes in entropy of the hot and cold reservoirs (ΔS_{hot}, ΔS_{cold}) during the different labelled processes?

b.

ΔS_{cold}

a. $\Delta S_{hot,1} = \Delta S_{cold,3} = 0$ ✗
b. $\Delta S_{hot,1} < 0$, $\Delta S_{cold,3} > 0$ ✓
c. $\Delta S_{hot,1} > 0$, $\Delta S_{cold,3} < 0$ ✗
d. $\Delta S_{hot,4} = \Delta S_{cold,2} = 0$ ✗
e. $\Delta S_{hot,total} = \Delta S_{hot,1} + \Delta S_{hot,2} + \Delta S_{hot,3} + \Delta S_{hot,4} > 0$ < 0

A stone with heat capacity C = 15 J/K is left outside on a warm day to reach a temperature of 312.15 K. The stone is then brought inside where the air temperature is 293.15 K. The stone is used to run a reversible engine.

Σ_{carnot}

25) What is the maximum work that can be accomplished?

b.

$\Delta F =$

a. 276 J
b. 8.86 J
c. 17.3 J
d. 285 J
e. -285 J

$\Delta U - T_e C_v \ln\left(\frac{T_f}{T_i}\right) = 8.86$

Consider a non-ideal gas, such that $U = \frac{3}{2}NkT - \frac{aN^2}{V} + const$ and S= N·k·ln(V/V$_0$), where a=10^{-46} J·m^3 and V_0=0.1 m^3. 3 moles of the gas are kept at constant temperature 343 K.

c.

26) What is the change in chemical potential if the gas expands from 0.01 to 0.02 m^3?

$= -\left(\frac{2aN}{V_2} - \frac{2aN}{V_1}\right) - Tk\ln\left(\frac{V_2}{V_1}\right)$

$= 1.48 \times 10^{-20}$

d.

a. 0 J
b. -2.13 × 10^{-20} J
c. -3.28 × 10^{-21} J
d. 1.48 × 10^{-20} J
e. 3.28 × 10^{-21} J

$\mu = \frac{dF}{dN} = \frac{3}{2}kT - \frac{2aN}{V} - Tk\ln\left(\frac{V}{V_0}\right)$

$= -3.283 \times 10^{-21}$

You find that by dissolving some amount of sugar into water, the melting point of the water is reduced by 3 degrees Celsius.

27) Which of the following best describes how the sugar lowered the melting point?

a.

a. The mixing of the sugar lowered the chemical potential of the liquid water.
b. The presence of the sugar keeps the system from reaching equilibrium. X
c. The mixing of the sugar raised the chemical potential of the solid ice.

Consider a non-ideal gas in a cylinder with a piston. The piston can move freely. There is a fixed external pressure p and temperature T.

28) Which statement is true about the **gas**?

a.

a. The Gibbs free energy is minimized. ✓
b. The Helmholtz free energy is minimized. x
c. The entropy is maximized. x

Suppose that a system has fixed volume and is in an environment with temperature T. Suppose also that you know the free energy F for that system.

b. 29) What is the equilibrium condition given by $\left(\frac{\partial F}{\partial V}\right)_{T,N}$? (That is, what does this derivative equal in equilibrium?)

a. $p - \frac{S}{T}$
b. p
c. $\frac{p}{T}$
d. $\frac{p}{T} + U$
e. $\frac{p}{T} - S$

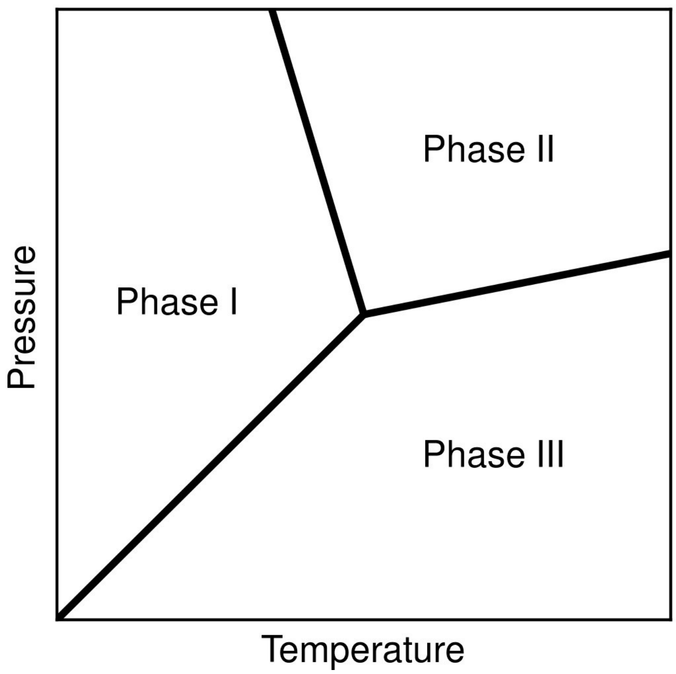
$-TdS$

$dF = dV - T_e\, dS$

$= -T_e \frac{dS}{dV}$

$= -T_e \cdot \frac{p}{T}$

$= -p_1 = -p_2$



Consider the phase diagram in the image.

b. 30) Which statement is true about the phases near the phase transition lines between the respective phases?

a. Phase I has more entropy per particle than Phase II
b. Phase III has more entropy per particle than Phase II
c. Phase III is denser than Phase II
d. Phase I is denser than Phase II
e. Phase I has more entropy per particle than Phase III