Network structure and systemic risk in banking systems

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Importance

- Aim as a security to measure risk involved when/if certain financial institutions default.
- Rely on counterparty exposures and compounded effects of correlated market shocks, and not only balance sheet size or the number of financial institutions one is connected to
- Emphasize efficient risk mitigation by enforcing capital requirements which depend on exposures, and thus localize regulations to systemically important institutions.
- Examine individual impact on select institutions, and not just take the average outcome of a macroeconomic shock.

Network structure of banking systems

Definition

Represent a financial system with counterparty relations as a network $I = (V, E, c)$, consisting of

- a set V of *financial institutions*, whose number we denote by n,
- a matrix E of *bilateral exposures*: E_{ij} represents the exposure of node i to node j defined as the market value of all liabilities of institution j to institution i at the date of computation.
- $c = (c(i), i \in V)$ where $c(i)$ is the *capital* of the institution i, representing its capacity for absorbing losses.

Network structure of banking systems

Definition

The in-degree $k_{in}(i)$ of a node $i \in V$ is the number of its *debtors*, and the out-degree $k_{out}(i)$ is the number of its *creditors*:

$$k_{in}(i) = \sum_{j \in V} \mathbf{1}_{\{E_{ij} > 0\}}, \qquad k_{out}(i) = \sum_{j \in V} \mathbf{1}_{\{E_{ji} > 0\}}.$$

Let $k(i) = k_{in}(i) + k_{out}(i)$ denote the degree of node i.

Definition

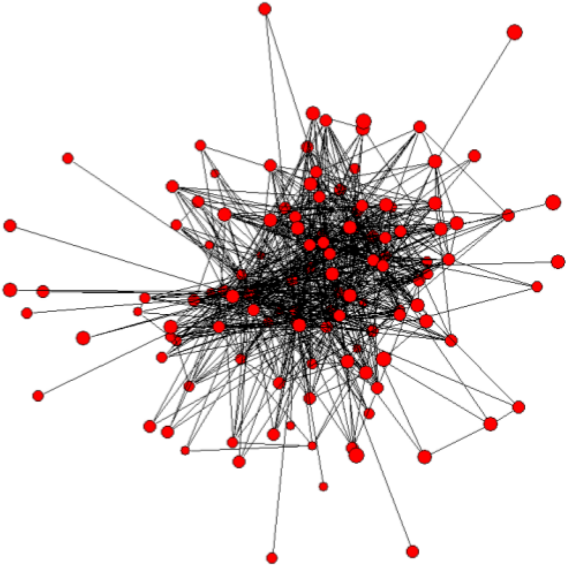
The *interbank assets* $A(i)$ and *interbank liabilities* $L(i)$ of financial instution i are

$$A(i) = \sum_{j \in V} E_{ij}, \qquad L(i) = \sum_{j \in V} E_{ji}.$$

Network structure of banking systems

In-Degree	Jun-07	Dec-07	Mar-08	Jun-08	Sep-08	Nov-08
Mean	8.56	8.58	8.75	8.98	8.99	7.88
Standard Deviation	10.84	10.86	10.61	11.15	11.32	11.02
5% quantile	0	0	0	0	0	0
95% quantile	30.50	29.30	30.45	31	32	30.60
Maximum	54	54	51	57	60	62
Out-Degree	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
Mean	8.56	8.58	8.75	8.98	8.99	7.88
Standard Deviation	8.71	8.82	9.02	9.43	9.36	8.76
5% quantile	0	0	0	0	0	0
95% quantile	26	26	27.90	29.25	30.20	27.40
Maximum	36	37	39	41	39	44
Exposures (in billions of BRL)	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
Mean	0.07	0.05	0.05	0.05	0.05	0.08
Standard Deviation	0.77	0.32	0.32	0.30	0.38	0.54
5% quantile	0.00	0.00	0.00	0.00	0.00	0.00
95% quantile	0.20	0.17	0.17	0.18	0.19	0.35
Maximum	23.22	9.89	9.90	9.36	12.50	15.90
Relative Exposures ($E_{ij}/c(i)$)	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
Mean	0.23	0.20	0.04	0.04	0.03	0.05
Standard Deviation	1.81	1.62	0.16	0.17	0.06	0.21
5% quantile	0.00	0.00	0.00	0.00	0.00	0.00
95% quantile	0.70	0.59	0.20	0.21	0.16	0.18
Maximum	49.16	46.25	4.57	5.17	0.69	6.02
Distance	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**
Mean	2.42	2.42	2.38	2.38	2.33	2.35
Standard Deviation	0.84	0.85	0.84	0.82	0.77	0.78
5% quantile	1	1	1	1	1	1
95% quantile	4	4	4	4	3	4
Maximum (Diameter)	5	6	6	6	5	6

Network structure of banking systems



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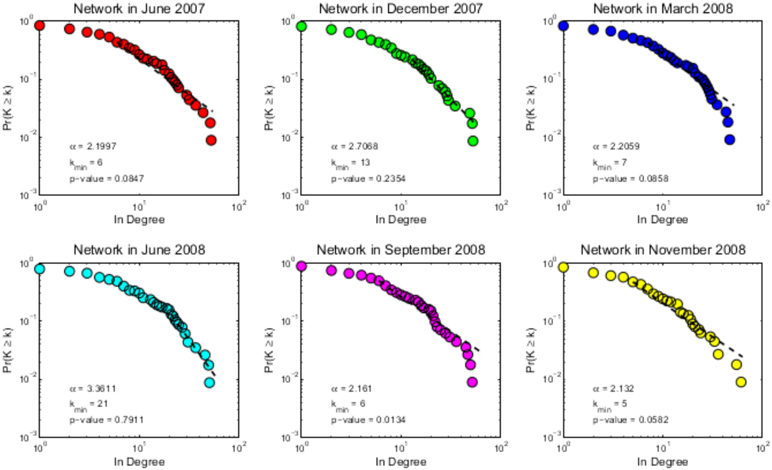

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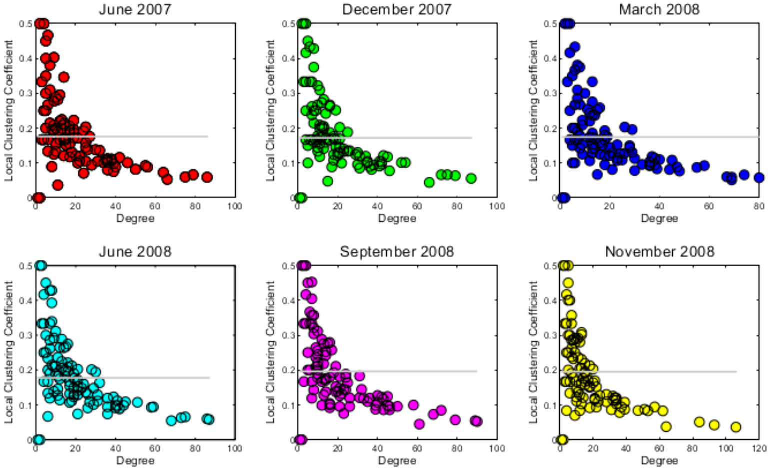


Systemic risk and default contagion

Definition

Consider an initial configuration of capital reserves $(c(j), j \in V)$. Define *loss cascade* as the sequence $(c_k(j), j \in V)_{k \geq 0}$ as

$$c_0(j) = c(j), \qquad c_{k+1}(j) = \max\{c_0(j) - \sum_{\{i, c_k(i)=0\}} (1 - R_i) E_{ji}, 0\},$$

where R_i is the recovery rate at the default of institution i.
$(c_{n-1}(j), j \in V)$, where $n = |V|$ is the number of nodes in the network, then represents the remaining capital once all counterparty losses have been accounted for. The *set of insolvent institutions* is given by

$$\mathbb{D}(c, E) = \{j \in V : c_{n-1}(j) = 0\}.$$

Systemic risk and default contagion

The set $\mathbb{D}(c, E)$ of insolvent (defaulted) institutions may be partitioned into subsets

$$\mathbb{D}(c, E) = \underbrace{\{j \in V : c_0(j) = 0\}}_{Fundamental\ \ defaults} \bigcup \underbrace{\{j \in V : c_0(j) > 0, c_{n-1}(j) = 0\}}_{Defaults\ \ by\ \ contagion}.$$

Definition

The *Default Impact* $DI(i, c, E)$ of a financial institution $i \in V$ is defined as the total loss in capital in the cascade triggered by the default of i:

$$DI(i, c, E) = \sum_{j \in V} c_0(j) - c_{n-1}(j),$$

where $(c_k(j), j \in V)_{k \geq 0}$ is is the loss cascade.

Systemic risk and default contagion

Consider a co-monotonic model for macroeconomic shocks

$$\epsilon(i, Z) = c(i) f_i(Z),$$

where Z is a (negative) random variable which represents the magnitude of such a shock, and f_i are strictly increasing functions taking values in $(-1, 0]$.

A *macroeconomic stress scenario* corresponds to a scenario where Z takes very large negative values. A plausible set of stress for scenarios may be using a low quantile α of Z:

$$\mathbb{P}(Z < \alpha) = q, \qquad q = 5\% \text{ or } 1\%.$$

Systemic risk and default contagion

Definition

The *Contagion Index* $CI(i, c, E)$ (at confidence level q) of institution $i \in V$ is defined as its expected Default Impact in a market stress scenario:

$$CI(i, c, E) = \mathbb{E}[DI(i, c + \epsilon(Z), E) \mid Z < \alpha].$$

- The Contagion Index measures the systemic impact of the failure of an institution; computation is by the expected loss–measured in terms of capital–inflicted to the network in the default cascade triggered by the initial default of i.
- The definition involves *conditioning* on stress scenarios.
- Computation of this index depends on the joint distribution of shocks which affect the balance sheets.
- Since $f_i(Z) > -1$, $c(i) + \epsilon(i, Z) > 0$. Hence defaults will never be caused by market shocks alone.

Systemic risk and default contagion

Model Z as a negative random variable with a heavy-tailed distribution F and an exponential function for f_i:

$$\epsilon(i, Z) = c(i)(\exp(\sigma_i Z) - 1),$$

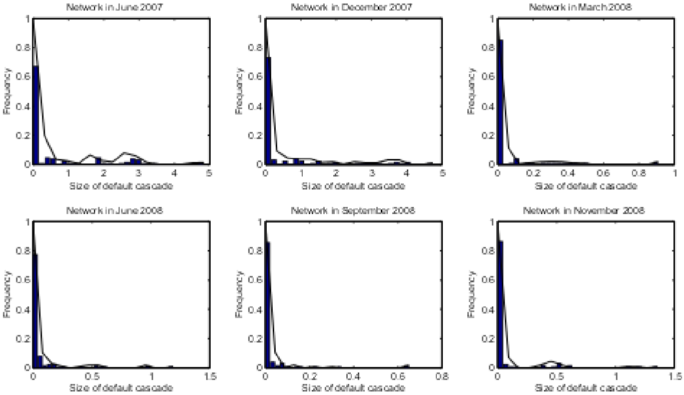
where σ_i is a scale factor depending on the creditworthiness, or probability of default p_i, of institution i.

One possible specification is to choose σ_i such that p_i corresponds to the probability of losing 90% of the capital in a market stress scenario:

$$\sigma_i = -\frac{\log(10)}{F^{-1}(p_i)}.$$

The default probabilities p_i are obtained from historical default rates given by credit ratings for the firms at the date corresponding to the simulations.

Is default contagion a significant source of systemic risk?



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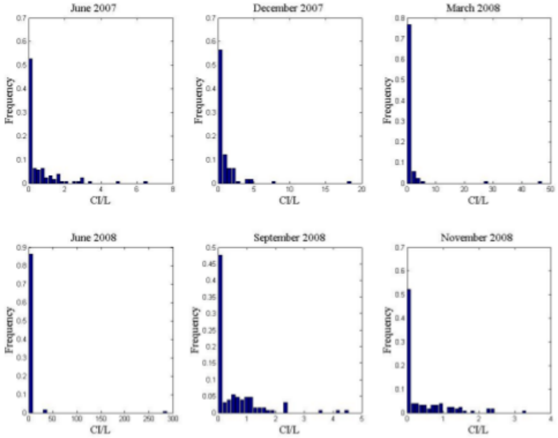

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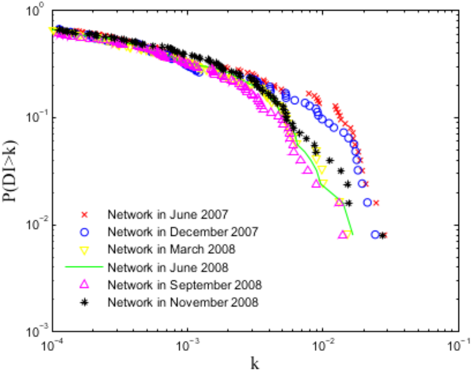

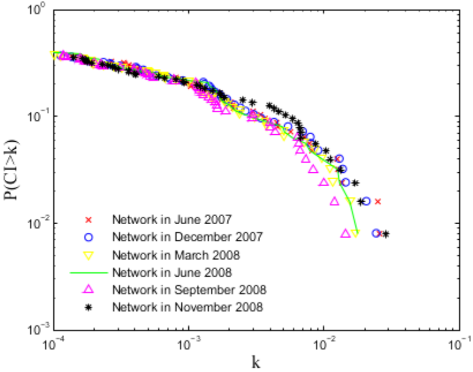

What makes an institution systemically important?

- Size of interbank liabilities. For June 2007, a linear regression of the logarithm of the Contagion Index and the logarithm of the interbank liabilities: interbank liabilities explains 27% of the cross-sectional variability of the Contagion Index.
- Number of creditors.
- Centrality and counterparty susceptibility.

Rank	Contagion Index (Billions BRL)	Number of creditors	Interbank liability (Billions BRL)
1	3.48	25	1.64
2	3.40	21	0.97
3	2.09	20	1.10
4	1.78	20	0.60
5	1.45	34	1.59
Network median	0.0007	20	0.52
90%-quantile	0.53	28	2.07

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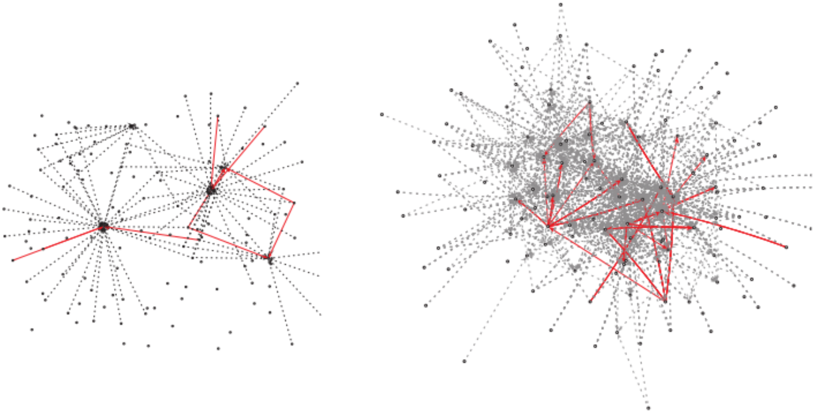

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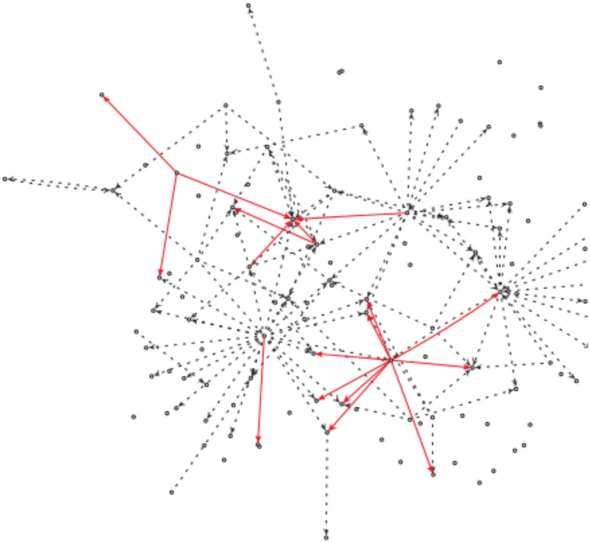

The case for targeted capital requirements

Implement capital regulations which either (1) impose more stringent requirements on the most systemic institutions in the network, or (2) strengthen "weak links" in the network which correspond to exposures which constitute a high fraction of capital.

- Minimum capital ratio: require institutions to hold a capital equal to or higher than a portion θ of their aggregate interbank exposure:

$$\overline{c}(i) = \max\{c(i), \theta A(i)\}.$$

- Minimum capital-to-exposure ratio: require institutions to hold a level of capital which covers a portion γ of their *largest* interbank exposure:

$$\overline{c}(i) = \max\left\{c(i), \frac{\max_{j \neq i}(E_{ij})}{\gamma}\right\}.$$