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given: $y = mx + b$, Initial values: $m = -1, b = 1$

learning rate $\alpha = 0.1$

data points: $(x_1, y_1) = (1, 3), (x_2, y_2) = (3, 6)$

gradient formulas: $\frac{\partial J}{\partial m} = -\frac{2}{n}\sum (y_i - \hat{y}_i) x_i$

$$\frac{\partial J}{\partial b} = -\frac{2}{n}\sum (y_i - \hat{y}_i)$$

$m_{new} = m_{old} - \alpha \cdot \frac{\partial J}{\partial m}$, $b_{new} = b_{old} - \alpha \frac{\partial J}{\partial b}$.

as we are 3 group members, we will do 3 iterations.

1. Compute predictions.
2. Compute errors.
3. Compute gradients.
4. Update m and b.

## 1st Iteration

$m = -1, b = 1$: $\hat{y}_1 = -1(1) + 1 = 0$

$\hat{y}_2 = -1(3) + 1 = -2$

Step 2: errors $e_1 = y_1 - \hat{y}_1 = 3 - 0 = 3$

$e_2 = y_2 - \hat{y}_2 = 6 - (-2) = 8$

Step 3: gradients (n=2)

$$\frac{\partial J}{\partial m} = \frac{-2}{2}\left[(3)(1)+(8)(3)\right] = -(3+24) = -27$$

$$\frac{\partial J}{\partial b} = \frac{-2}{2}(3+8) = -11.$$

Step 4: updates: $m = -1 - 0.1(-27) = -1+2.7 = 1.7$

$b = 1 - 0.1(-11) = 1+1.1 = 2.1$

2nd iteration

Step 1: Predictions: $m = 1.7, b = 2.1$

$\hat{y}_1 = 1.7(1)+2.1 = 3.8$

$\hat{y}_2 = 1.7(3)+2.1 = 7.2$

Step 2: errors:

$e_1 = 3 - 3.8 = -0.8$

$e_2 = 6 - 7.2 = -1.2$

Step 3: gradients.

$$\frac{\partial J}{\partial m} = -1.\left[(-0.8)(1)+(-1.2)(3)\right] = -(-0.8-3.6) = 4.4$$

$$\frac{\partial J}{\partial b} = -1.(-0.8-1.2) = 2.0$$

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Step 4: updates; $m = 1.7 - 0.1(4.4) = 1.7 - 0.44 = 1.26$

$b = 2.1 - 0.1(2.0) = 2.1 - 0.2 = 1.9$

## 3rd iteration

Step 1: predictions, $m = 1.26, b = 1.9$

$\hat{y}_1 = 1.26(1) + 1.9 = 3.16$

$\hat{y}_2 = 1.26(3) + 1.9 = 5.58$

Step 2: errors; $e_1 = 3 - 3.16 = -0.16$

$e_2 = 6 - 5.58 = 0.42$

Step 3: gradients

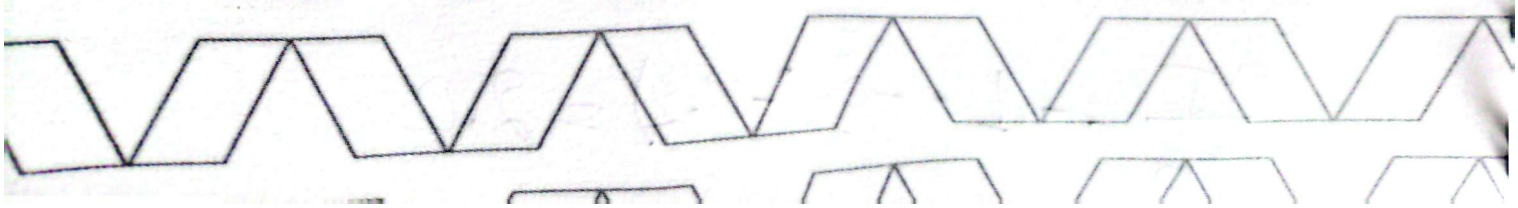
$\frac{\partial J}{\partial m} = -1.[(-0.16)(1) + (0.42)(3)] = -(-0.16 + 1.26) = -1.10$

$\frac{\partial J}{\partial b} = -1.(-0.16 + 0.42) = 0.26$

Step 4: updates

$m = 1.26 - 0.1(-1.10) = 1.26 + 0.11 = 1.37$

$b = 1.9 - 0.1(0.26) = 1.9 + 0.026 = 1.926$



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## Final Summary

| Iterations | m | b |
|---|---|---|
| 0 (initial) | -1.00 | 1.00 |
| 1 | 1.70 | 2.10 |
| 2 | 1.26 | 1.90 |
| 3 | 1.37 | 1.926 |

**Trend observation:**

After three updates, the values of m and b are stabilizing and moving toward values that reduce the prediction error. The gradients get smaller over time, indicating the model is converging toward an optimal fit.