

**Instructor: Brian Harrington and Anna Bretscher****Last Name:** \_\_\_\_\_**First Name:** \_\_\_\_\_**UtorID (Markus Login):** \_\_\_\_\_

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***Do not turn this page until you have received the signal to start.***

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This exam consists of 6 questions on 20 pages (including this one). When you receive the signal to start, please make sure that your copy is complete. Proper documentation is required for all functions and code blocks. If you use any space for rough work, indicate clearly what you want marked. Please read all questions thoroughly before starting on any work.

We have provided you with grids for your answers, this is simply to help you show the indentation of your code and you are not required to adhere to the grids in any specific way. For your reference, we also give you an **extra copy of all code on the last two pages**. You may tear these two pages off the exam.

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# 1: \_\_\_\_\_ / 4

# 2: \_\_\_\_\_ / 6

# 3: \_\_\_\_\_ / 6

# 4: \_\_\_\_\_ / 5

# 5: \_\_\_\_\_ / 12

# 6: \_\_\_\_\_ / 7

**TOTAL:** \_\_\_\_\_ / 40

[Use the space below for rough work. This page will not be marked unless you clearly indicate the part of your work that you want us to mark.]

A large grid of squares, approximately 20 columns by 25 rows, intended for rough work. It is enclosed in a thin black border.

**Question 1.** [4 MARKS]

Sort the following functions (of positive integer  $n$ ) in big-Oh order. I.e., write them in an order so that each function is in big-Oh of the one immediately after it. If functions are big-Oh of each other, then group them together using “==”.

For example, given three functions  $f_1, f_2$  and  $f_3$  such that  $f_1 \in \mathcal{O}(f_2)$  and  $f_2 \in \mathcal{O}(f_1)$  and  $f_1, f_2 \in \mathcal{O}(f_3)$ , we would order them  $f_1 == f_2, f_3$ .

$$2n^3, 2^n + n^2 + \log n, 3^n, 4(\log n), 48, 7n^2, 92^n, \log(n^2)$$

**Question 2.** [6 MARKS]

Recall Assignment 2 on *regular expressions* and the rules we used to build regular expression trees.

**Part (a)** [3 MARKS]

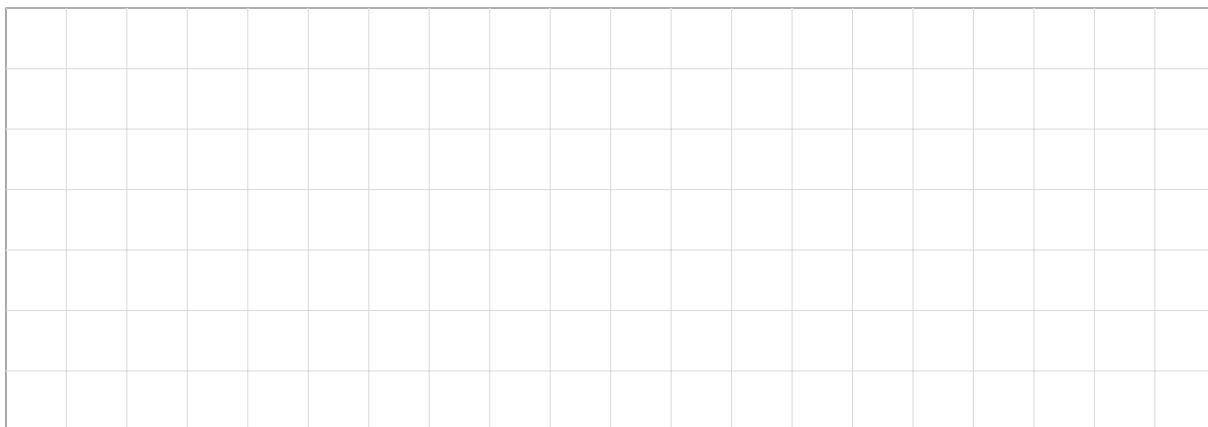
Draw the regex tree for the regex  $(0^*.(2^*|(0.1))^*)$ .

**Part (b)** [3 MARKS]

Complete the following function by adding **one** Python statement, plus a comment that explains that statement.

**Hint:** *No loops are needed. First work out how one can determine the number of nodes without forming a tree. Then describe this in a comment. Finally use str methods to write your Python statement.*

```
def num_nodes(r):
    ''' (str) -> int
        Return the number of nodes in the regex tree that represents the
        regex r.
        REQ: r is a valid regex.
    '''
```

A large rectangular grid consisting of 10 columns and 10 rows of small squares, intended for drawing the regular expression tree.

**Question 3.** [6 MARKS]

Consider inserting the integers 1 through 6 into an initially empty *max* heap using the algorithm described in class. Recall that during the bubble up process, a key within a node is sometimes swapped with the key in its parent node. We are interested in the ordering of the sequence of insertions, and the total number of key swaps.

**Part (a)** [2 MARKS]

What is the maximum number of key swaps that can be incurred by a sequence of operations that inserts the integers 1 through 6 into a max heap? Circle exactly one of the choices below.

5

6

7

8

9

**Part (b)** [2 MARKS]

Give a sequence of insertions that forces the maximum number of key swaps given in Part (a).

**Part (c)** [2 MARKS]

- c) Give two sequences of insertions of the integers 1 through 6 that incurs no key swaps. The first sequence must insert 4 as early as possible, and the second sequence must insert 4 as late as possible.

First sequence:

Second sequence:

```
from math import *

def f1(a, b, r):
    ''' (int, int, float) -> list of (int, int)
    REQ: 0 < r < 1.
    '''
    if a >= b:
        return [] # return empty list
    c = floor(a + (b-a)*r)
    return f1(a, c, r) + f1(c+1, b, r) + [(a, b)] # concatenate 3 lists
```

[Use the space below for rough work. This page will not be marked unless you clearly indicate the part of your work that you want us to mark.]



**Question 4.** [5 MARKS]

Consider the function  $f_1$  on the previous page<sup>1</sup>:

In the space below, write the output of the following code:

```
print("STEP 1:", f1(1, 4, 0.5))  
print("STEP 2:", f1(10, 12, 0.1))
```

---

<sup>1</sup>Boy, this sure looks familiar... good thing you had several weeks to review it, huh?

```
class CustomerNode:  
    def __init__(self, ticket_num, cust_name, server_name, next):  
        '''(IntNode, int, str, str, CustomerNode) -> NoneType  
        Create a new CustomerNode with ticket number ticket_num,  
        customer name cust_name, server name server_name and  
        pointing to next.  
        REQ: server_name must be either "Anna" or "Brian".  
        '''  
  
        self._ticket_num = ticket_num  
        self._cust_name = cust_name  
        self._server_name = server_name  
        self.next = next  
  
    def __repr__(self):  
        """(CustomerNode) -> str  
        Return a string representing self.  
        """  
  
        return ("CustomerNode(" + repr(self._ticket_num) + ", " +  
               repr(self._cust_name) + ", " + repr(self._server_name) +  
               ", " + repr(self.next) + ")")  
  
    def get_ticket_num(self):  
        return self._ticket_num  
  
    def get_cust_name(self):  
        return self._cust_name  
  
    def get_server_name(self):  
        return self._server_name
```

**Question 5.** [12 MARKS]

Anna's Deli has just two servers, Anna and Brian. It's a small operation. To control their long queues of customers, they use a roll of numbered tickets. Each successive ticket's number is one greater than its previous ticket's number. On entry every customer takes the next available ticket and lines up either in Anna's queue or in Brian's queue. Sometimes Anna or Brian must leave to deliver a lecture<sup>2</sup>. When this happens the two queues are merged into a single queue, and all customers are served in the order they arrived (i.e., ordered by their ticket number). When Anna or Brian is able to return, the queue splits again into Anna's queue and Brian's queue.

Nick decided to help Anna and Brian by writing some code to simulate the queues at Anna's Deli. He used linked lists to represent queues. First he created the class `CustomerNode` for the nodes in his linked list. Then he wrote a function called `reclistmerge()`, which takes two sorted (by ticket number) linked lists of customers, and returns a single sorted list of customers from the two input lists. His function is given here:

```
def reclistmerge(L1, L2):
    ''' (CustomerNode, CustomerNode) -> CustomerNode

    ...
    #
    #
    if L1 == None:
        return L2
    if L2 == None:
        return L1

    # debug printing
    print(L1.get_ticket_num(), L2.get_ticket_num())

    #
    #
    if L1.get_ticket_num() < L2.get_ticket_num():
        #
        #
        L1.next = reclistmerge(L1.next, L2)
        return L1
    else:
        #
        #
        L2.next = reclistmerge(L1, L2.next)
        return L2
```

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<sup>2</sup>obviously you all need to pay more tuition so your lecturers don't have to hold down 2 jobs



**Part (a) [3 MARKS]**

Nick is trying to convince himself that his code is working properly. Notice the debugging `print` line in `recmergelist()`. Give the output of the following lines of code:

```
L_A = CustomerNode(2, "Alice", "Anna",
                    CustomerNode(3, "Bob", "Anna",
                                CustomerNode(5, "Carol", "Anna", None)))
L_B = CustomerNode(1, "Ziggy", "Brian",
                    CustomerNode(4, "Yaz", "Brian",
                                CustomerNode(6, "Xavier", "Brian", None)))
L = reclistmerge(L_A, L_B)
```

**Part (b) [4 MARKS]**

Then disaster happened...the CODE DECOMMENTER struck!!! The `docstring` was erased and all internal comments were removed. Please help Nick to recomment his code in function `reclistmerge()`. You should complete the `docstring` and write an informative comment wherever there is a group of #s on the given code on the previous page.

```
def listsplit(L):
    ...
    (CustomerNode) -> (CustomerNode, CustomerNode)
    Split the linked list headed by L into 2 lists, first one with server
    "Anna" and second one with server "Brian".
    Within each list, the original order of nodes is preserved.
    Return the heads of the two lists.
    ...

# This version uses recursion (and no loops).

# if list is empty, then return empty lists
# initialize lists to return
# list is nonempty, so place first node in appropriate list, then
# return the split lists
# split the rest of the list, and make the first node be the head of
# the corresponding split list

(L_A.next, L_B) = listsplit(L.next)
(L_A, L_B.next) = listsplit(L.next)
else:
    if L == None:
        if L.get_server_name() == "Anna":
            L_A = L_B = None # A means Anna, B means Brian
            L_A = L
            L_B = L
    return (L_A, L_B)
return (L_A, L_B)
```

**Part (c) [5 MARKS]**

Whenever Anna or Brian return to serve customers, the single queue splits into two queues with customers returning to their preferred server line. Nick also wrote a function to split the merged queue back into two queues. The function `listsplit()` is the “*inverse*” of `listmerge()` in the sense that

- `listmerge(listsplit(L))` and `L` are equal,
- `listsplit(listmerge(L_A, L_B))` and `(L_A, L_B)` are equal.

Nick was having a really rotten day. Just as he finished re-commenting his `relistmerge()` code the CODE MANGER struck. His `listsplit()` method on the previous page had all lines scrambled. Help Nick correct `listsplit()`.

```
def listsplit(L):
    ''' (CustomerNode) -> (CustomerNode, CustomerNode)
    Split the linked list headed by L into 2 lists, first one with server
    "Anna" and second one with server "Brian".
    Within each list, the original order of nodes is preserved.
    Return the heads of the two lists.
    '''
```

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**Question 6.** [7 MARKS]

Consider the following `BTNode` class to hold integers in a binary tree.

```
class BTNode:  
    def __init__(self, i):  
        ''' (BTNode, int) -> None  
        Create a new BTNode with integer i.  
        '''  
        self.my_int = i  
        self.left = self.right = None # pointers to left and right subtrees
```

We also assume the availability of a standard queue module with methods to instantiate a new queue, enqueue an object, dequeue an object, and test if a queue is empty. Consider the following function.

```
def hmmmm(root):  
    ''' (BTNode) -> BTNode  
    '''  
    return_node = None  
    q = Queue()  
    if root != None:  
        q.enqueue(root)  
  
    while (not q.is_empty()):  
        node = q.dequeue()  
        print(node.my_int)  
        if node.left != None:  
            q.enqueue(node.left)  
        if node.right != None:  
            q.enqueue(node.right)  
        if node.my_int % 2 == 0:  
            return_node = node  
  
    return return_node
```

**Part (a) [2 MARKS]**

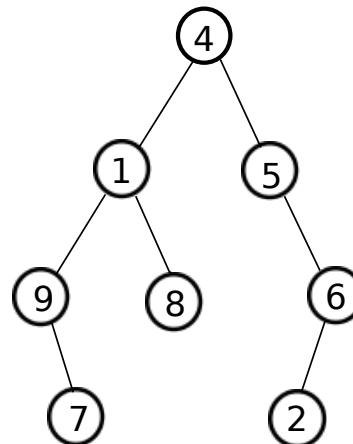
Suppose we run the line of code,

```
hmmm(mytree)
```

where mytree is the root of this tree.

Give the output that would be printed and circle the node on the tree that would be returned.

**Output:**

**Part (b) [3 MARKS]**

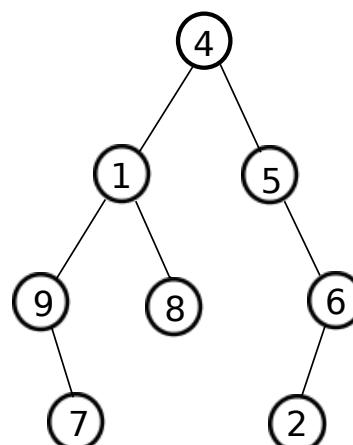
Describe what `hmmm(root)` returns.

**Part (c) [2 MARKS]**

Suppose that we now replace the `queue` ADT with a `stack` ADT.

Now each `enqueue` call is replaced by a `push` call and each `dequeue` call is replaced by a `pop` call. Repeat Part (a) using a `stack`. List your output below and circle the appropriate node on the tree that would be returned.

**Output:**



## Short Python function/method descriptions:

You may tear this page off, but if you do so, you must not include any work on it (front or back) that you wish to have marked.

```
__builtins__:  
abs(number) -> number  
    Return the absolute value of the given number.  
max(a, b, c, ...) -> value  
    With two or more arguments, return the largest argument.  
min(a, b, c, ...) -> value  
    With two or more arguments, return the smallest argument.  
isinstance(object, class-or-type-or-tuple) -> bool  
    Return whether an object is an instance of a class or of a subclass thereof.  
    With a type as second argument, return whether that is the object's type.  
int(x) -> int  
    Convert a string or number to an integer, if possible. A floating point argument  
    will be truncated towards zero.  
str(x) -> str  
    Convert an object into a string representation.  
  
str:  
S.count(sub[, start[, end]]) -> int  
    Return the number of non-overlapping occurrences of substring sub in  
    string S[start:end]. Optional arguments start and end are  
    interpreted as in slice notation.  
S.find(sub[, i]) -> int  
    Return the lowest index in S (starting at S[i], if i is given) where the  
    string sub is found or -1 if sub does not occur in S.  
S.isalpha() --> bool  
    Return True if and only if all characters in S are alphabetic  
    and there is at least one character in S.  
S.isdigit() --> bool  
    Return True if and only if all characters in S are digits  
    and there is at least one character in S.  
S.islower() --> bool  
    Return True if and only if all cased characters in S are lowercase  
    and there is at least one cased character in S.  
S.isupper() --> bool  
    Return True if and only if all cased characters in S are uppercase  
    and there is at least one cased character in S.  
S.lower() --> str  
    Return a copy of S converted to lowercase.  
S.replace(old, new) -> str  
    Return a copy of string S with all occurrences of the string old replaced  
    with the string new.  
S.split([sep]) -> list of str  
    Return a list of the words in S, using string sep as the separator and  
    any whitespace string if sep is not specified.  
S.startswith(prefix) -> bool  
    Return True if S starts with the specified prefix and False otherwise.  
S.strip() --> str  
    Return a copy of S with leading and trailing whitespace removed.  
S.upper() --> str  
    Return a copy of S converted to uppercase.
```

```
list:  
    append(...)  
        L.append(object) -- append object to end  
    count(...)  
        L.count(value) -> integer -- return number of occurrences of value  
    index(...)  
        L.index(value, [start, [stop]]) -> integer -- return first index of value.  
        Raises ValueError if the value is not present.  
    insert(...)  
        L.insert(index, object) -- insert object before index  
    pop(...)  
        L.pop([index]) -> item -- remove and return item at index (default last).  
        Raises IndexError if list is empty or index is out of range.  
    remove(...)  
        L.remove(value) -- remove first occurrence of value.  
        Raises ValueError if the value is not present.  
  
set:  
    pop(...)  
        Remove and return an arbitrary set element.  
        Raises KeyError if the set is empty.  
  
dict:  
    keys(...)  
        D.keys() -> a set-like object containing all of D's keys  
    get(...)  
        D.get(k[,d]) -> returns D[k] if k is in D, otherwise returns d. d defaults to None.  
  
object:  
    __init__(...)  
        x.__init__(...) initializes x; called automatically when a new object is created  
    __str__(...)  
        x.__str__() <==> str(x)
```

```
class CustomerNode:
    def __init__(self, ticket_num, cust_name, server_name, next):
        '''(IntNode, int, str, str, CustomerNode) -> NoneType
        Create a new CustomerNode with ticket number ticket_num,
        customer name cust_name, server name server_name and
        pointing to next.
        REQ: server_name must be either "Anna" or "Brian".
        '''

        self._ticket_num = ticket_num
        self._cust_name = cust_name
        self._server_name = server_name
        self.next = next

    def __repr__(self):
        """(CustomerNode) -> str
        Return a string representing self.
        """

        return ("CustomerNode(" + repr(self._ticket_num) + ", " +
               repr(self._cust_name) + ", " + repr(self._server_name) +
               ", " + repr(self.next) + ")")

    def get_ticket_num(self):
        return self._ticket_num

    def get_cust_name(self):
        return self._cust_name

    def get_server_name(self):
        return self._server_name

def reclistmerge(L1, L2):
    ''' (CustomerNode, CustomerNode) -> CustomerNode
    '''

    #
    if L1 == None:
        return L2
    if L2 == None:
        return L1

    # debug printing
    print(L1.get_ticket_num(), L2.get_ticket_num())
    #
    if L1.get_ticket_num() < L2.get_ticket_num():
        #
        L1.next = reclistmerge(L1.next, L2)
        return L1
    else:
        #
        L2.next = reclistmerge(L1, L2.next)
        return L2
```

```

def listsplit(L):
    ''' (CustomerNode) -> (CustomerNode, CustomerNode)
    Split the linked list headed by L into 2 lists, first one with server
    "Anna" and second one with server "Brian".
    Within each list, the original order of nodes is preserved.
    Return the heads of the two lists.
    '''
    # This version uses recursion (and no loops).
    # if list is empty, then return empty lists
    # initialize lists to return
    # list is nonempty, so place first node in appropriate list, then
    # return the split lists
    # split the rest of the list, and make the first node be the head of
    # the corresponding split list

    (L_A.next, L_B) = listsplit(L.next)
    (L_A, L_B.next) = listsplit(L.next)
    else:
        if L == None:
            if L.get_server_name() == "Anna":
                L_A = L_B = None # A means Anna, B means Brian
                L_A = L
                L_B = L
                return (L_A, L_B)
            return (L_A, L_B)
    ****
class BTNode:
    def __init__(self, i):
        ''' (BTNode, int) -> None
        Create a new BTNode with integer i.
        '''
        self.my_int = i
        self.left = self.right = None # pointers to left and right subtrees

def hmmm(root):
    ''' (BTNode) -> BTNode
    '''
    return_node = None
    q = Queue() # create an empty queue
    if root != None:
        q.enqueue(root) # enqueue root of input binary tree

    while (not q.is_empty()): # while q not empty
        node = q.dequeue() # dequeue node from q
        print(node.my_int)
        if node.left != None:
            q.enqueue(node.left) # enqueue node.left into q
        if node.right != None:
            q.enqueue(node.right) # enqueue node.right into q
        if node.my_int % 2 == 0: # if my_int is even
            return_node = node

    return return_node

```