Chapter 1-3

# 计算机图形学基础

## Fundamentals of Computer Graphics

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|---|---|
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Chapter 1-3 目录

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3/ 图像的线性变换
4/ 图像的仿射变换
5/ OpenCV仿射变换

# 第三部分

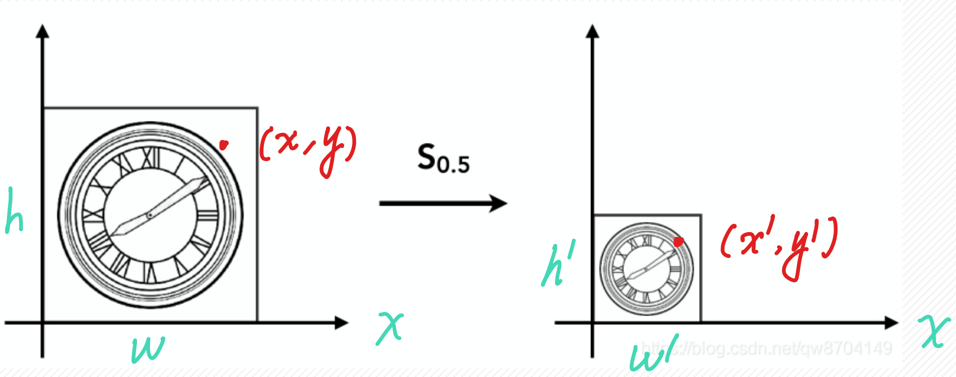
## Linear Trans 图像的线性变换

- ✓ 缩放变换
- ✓ 旋转变换
- ✓ 剪切变换

(板书)

# 缩放变换

[1]




$$\Delta\ f_x = \frac{w'}{w},\quad f_y = \frac{h'}{h}$$

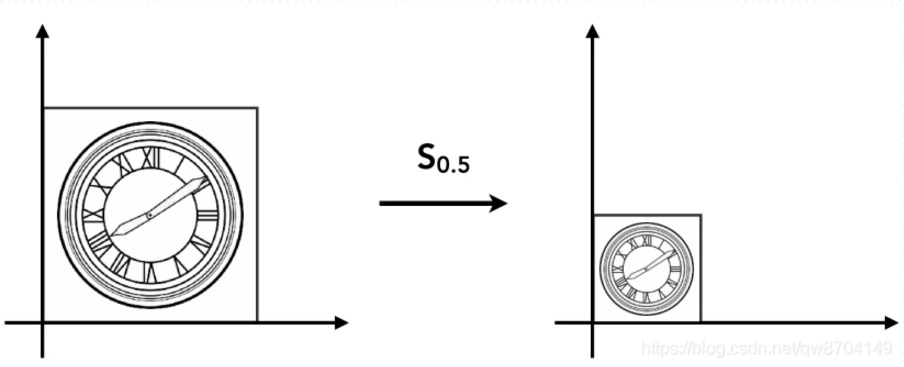
$$\begin{cases} x' = f_x \cdot x \\ y' = f_y \cdot y \end{cases}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Reference:
[1] https://www.aiuai.cn/aifarm1946.html

# 缩放变换

[1]




新图形与旧图形各像素点坐标存在如下关系：

$$\begin{cases} x' = f_x \cdot x \\ y' = f_y \cdot y \end{cases}$$

可以表示成如下形式：

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Reference:
[1] https://www.aiuai.cn/aifarm1946.html

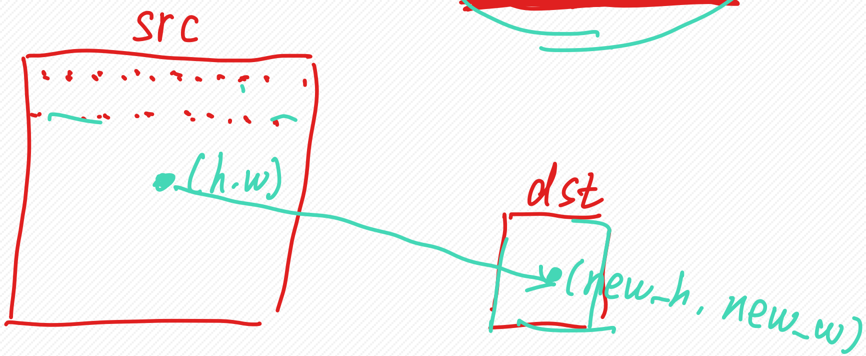
# 设计图像缩放算法（伪代码）

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

(200,400)

```
Input: src_image, dst_shape
src_shape := src_image.shape
fx := dst_shape[0] / src_shape[0]
fy := dst_shape[1] / src_shape[1]
dst_image := zeros(shape=dst_shape)
For h from 0 to src_shape[0]:
    For w from 0 to src_shape[1]:
        trans_matrix := [[fx, 0], [0, fy]]
        [new_h, new_w].T = trans_matrix * [h, w].T
        dst_image[new_h, new_w] = src_image[h, w]
Return dst_image
```

(h,w)



# 设计图像缩放算法（伪代码）

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

**Input:** *src_image*, *dst_shape*

*src_shape* := *src_image*.shape

*fx* := *dst_shape*[0] / *src_shape*[0]

*fy* := *dst_shape*[1] / *src_shape*[1]

*dst_image* := zeros(*shape*=*dst_shape*)

**For** *h* **from** 0 **to** *src_shape*[0]:

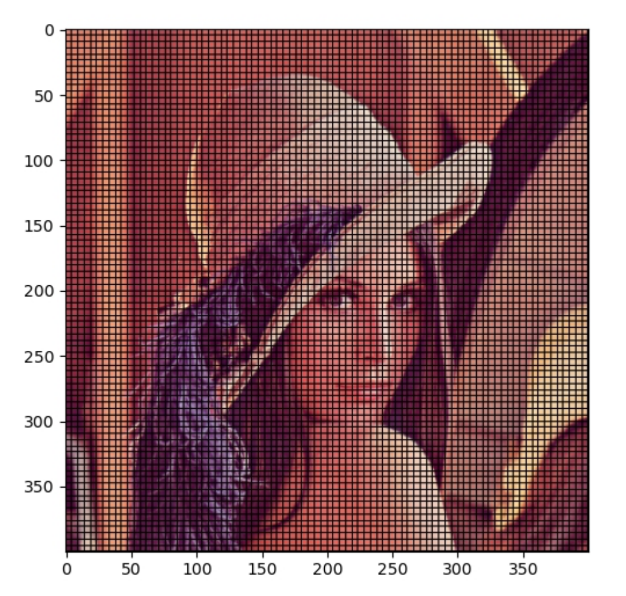
    **For** *w* **from** 0 **to** *src_shape*[1]:

        *trans_matrix* := [[*fx*, 0], [0, *fy*]]

        [*new_h*, *new_w*].T = *trans_matrix* * [*h*, *w*].T

        *dst_image*[*new_h*, *new_w*] = *src_image*[*h*, *w*]

**Return** *dst_image*



# 设计图像缩放算法（伪代码）

dst

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

src

**Input:** *src_image*, *dst_shape*

*src_shape* := *src_image*.shape

*fx* := *dst_shape*[0] / *src_shape*[0]

*fy* := *dst_shape*[1] / *src_shape*[1]

*dst_image* := zeros(*shape*=*dst_shape*)

**For** *h* **from** 0 **to** *src_shape*[0]:

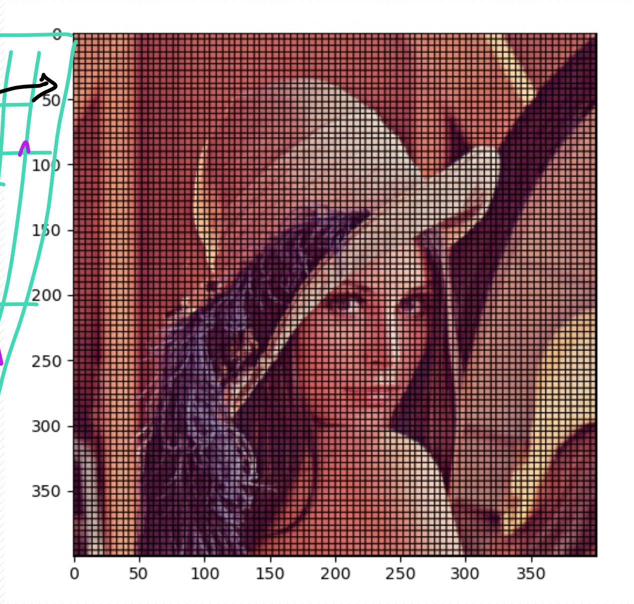
    **For** *w* **from** 0 **to** *src_shape*[1]:

        *trans_matrix* := [[*fx*, 0], [0, *fy*]]

        [*new_h*, *new_w*].T = *trans_matrix* * [*h*, *w*].T

        *dst_image*[*new_h*, *new_w*] = *src_image*[*h*, *w*]

**Return** *dst_image*



# 缩放变换

原变换为:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

求其逆变换:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$y = ax$

$x = \frac{1}{a} y = a^{-1} y$

$$\begin{cases} x' = f_x \cdot x \\ y' = f_y \cdot y \end{cases} \Leftrightarrow \begin{cases} x = f_x^{-1} \cdot x' \\ y = f_y^{-1} \cdot y' \end{cases}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f_x^{-1} & 0 \\ 0 & f_y^{-1} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

# 设计图像缩放算法（伪代码）

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

**Input:** *src_image*, *dst_shape*
*src_shape* := *src_image*.shape
*fx* := *dst_shape*[0] / *src_shape*[0]
*fy* := *dst_shape*[1] / *src_shape*[1]
*dst_image* := zeros(*shape*=*dst_shape*)
**For** *h* **from** 0 **to** *src_shape*[0]:
  **For** *w* **from** 0 **to** *src_shape*[1]:
    *trans_matrix* := [[*fx*, 0], [0, *fy*]]
    [*new_h*, *new_w*].T = *trans_matrix* * [*h*, *w*].T
    *dst_image*[*new_h*, *new_w*] = *src_image*[*h*, *w*]
**Return** *dst_image*

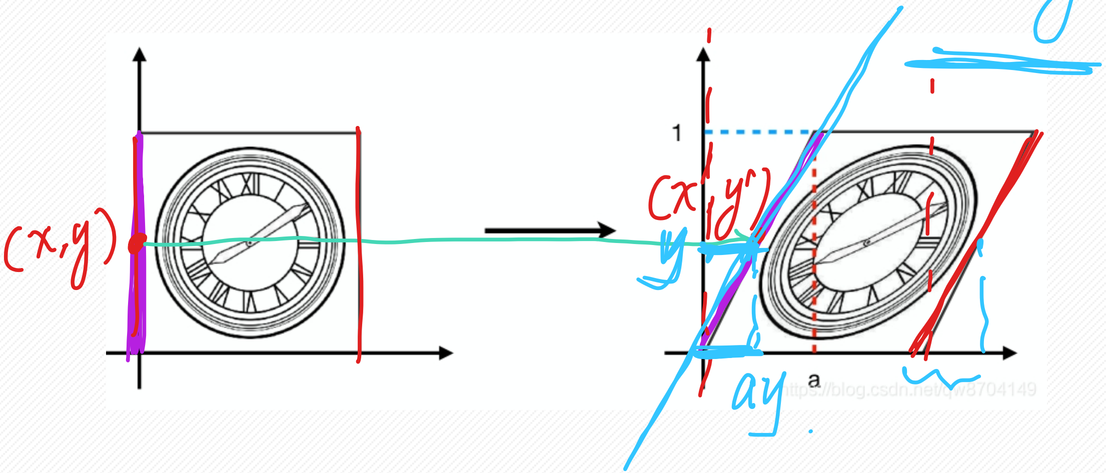
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

**Input:** *src_image*, *dst_shape*
*src_shape* := *src_image*.shape
*fx* := *dst_shape*[0] / *src_shape*[0]
*fy* := *dst_shape*[1] / *src_shape*[1]
*dst_image* := zeros(*shape*=*dst_shape*)
**For** *h* **from** 0 **to** *dst_shape*[0]:
  **For** *w* **from** 0 **to** *dst_shape*[1]:
    *trans_matrix* := inv([[*fx*, 0], [0, *fy*]])
    [*new_h*, *new_w*].T := *trans_matrix* * [*h*, *w*].T  x , y
    *dst_image*[*h*, *w*] := interpolation(*src_image*, *new_h*, *new_w*)
**Return** *dst_image*

# 剪切变换

(板书)

$x = ay$ [1]



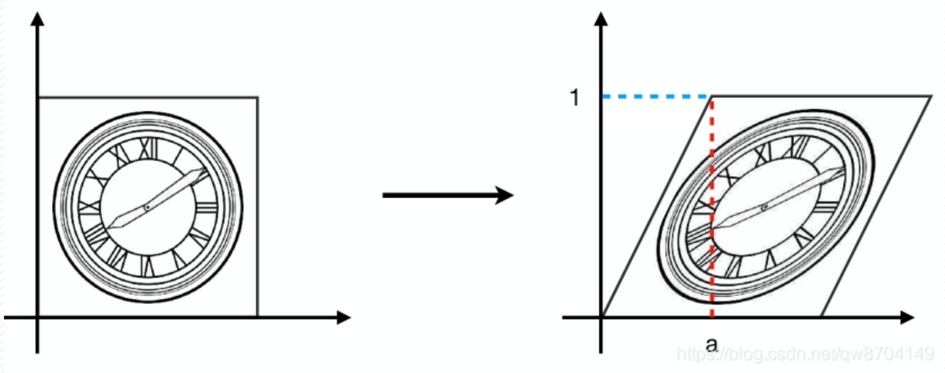

$$\begin{cases} x' = x + ay \\ y' = y \end{cases}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Reference:
[1] https://www.aiuai.cn/aifarm1946.html

# 剪切变换

[1]




新图形与旧图形各像素点坐标存在如下关系：

$$\begin{cases} x' = x + ay \\ y' = y \end{cases}$$

可以表示成如下形式：

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

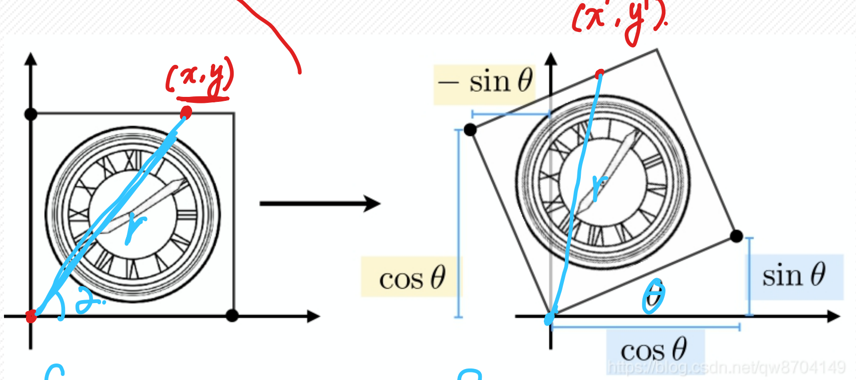
逆变换：

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

Reference:
[1] https://www.aiuai.cn/aifarm1946.html

旋转.

# ~~剪切~~变换




[1]

(板书)

$$\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta. \end{cases}$$

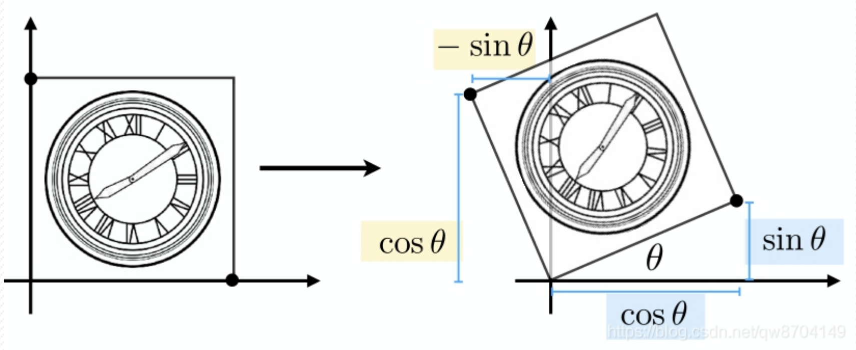
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{cases} x = r\cos\alpha. \\ y = r\sin\alpha. \end{cases}$$

$$\star \begin{cases} x' = r\cos(\alpha+\theta) = \overset{x}{\underline{r\cos\alpha}}\cos\theta - \overset{y}{\underline{r\sin\alpha}}\sin\theta \\ y' = r\sin(\alpha+\theta) = \underset{y}{\underline{r\sin\alpha}}\cos\theta + \underset{x}{\underline{r\cos\alpha}}\sin\theta. \end{cases}$$

Reference:
[1] https://www.aiuai.cn/aifarm1946.html

# 剪切变换




[1]

新图形与旧图形各像素点坐标存在如下关系：

$$\begin{cases} x' = x\cdot\cos\theta - y\cdot\sin\theta \\ y' = x\cdot\sin\theta + y\cdot\cos\theta \end{cases}$$

可以表示成如下形式：

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

逆变换：

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

Reference:
[1] https://www.aiuai.cn/aifarm1946.html

# 小结

缩放变换

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f_x & 0 \\ 0 & f_y \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

剪切变换

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

旋转变换

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

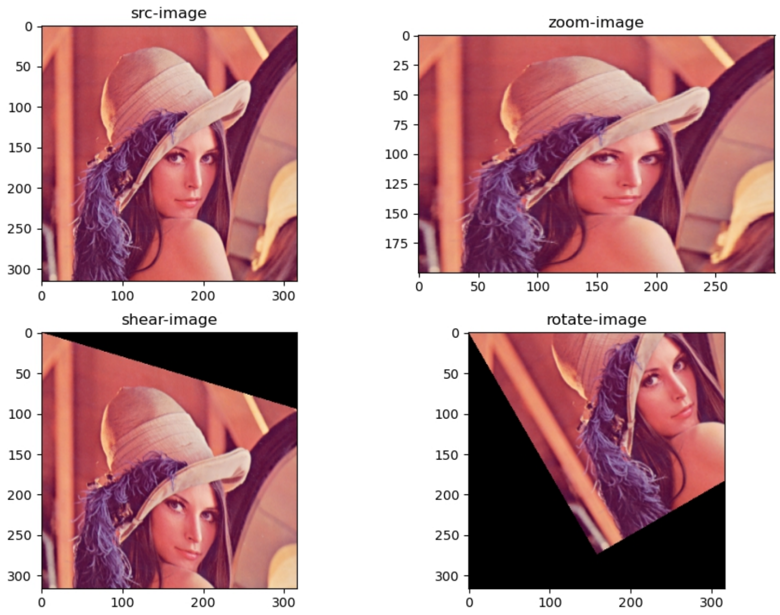
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

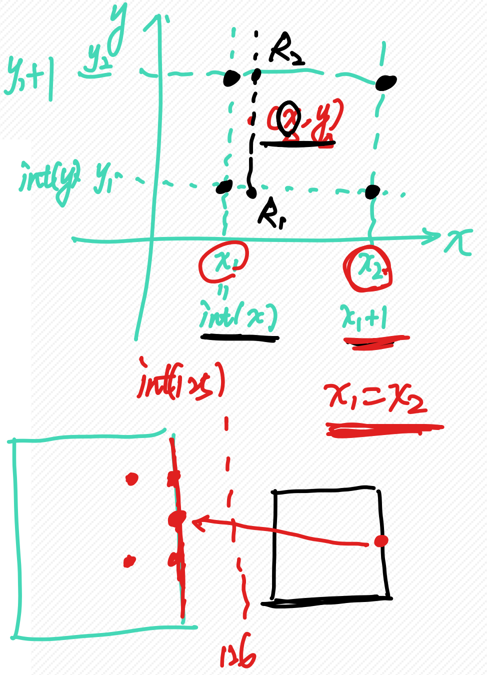
线性变换

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

# 实验1：图像的线性变换




$$\lambda = \frac{x_2 - x}{x_2 - x_1}$$

# 第四部分

## Linear Trans 图像的仿射变换

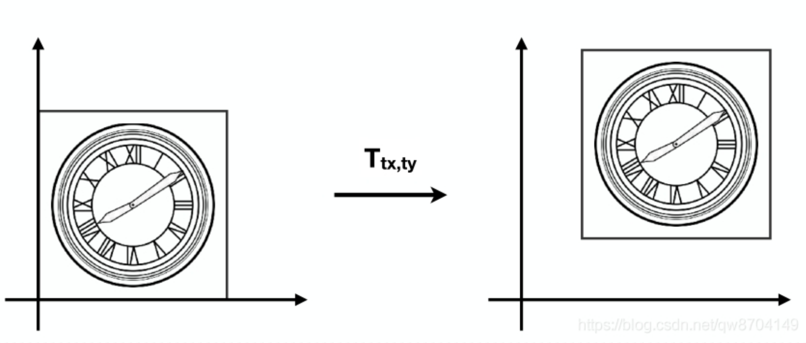
✓ 平移变换

(板书)

# 剪切变换

[1]

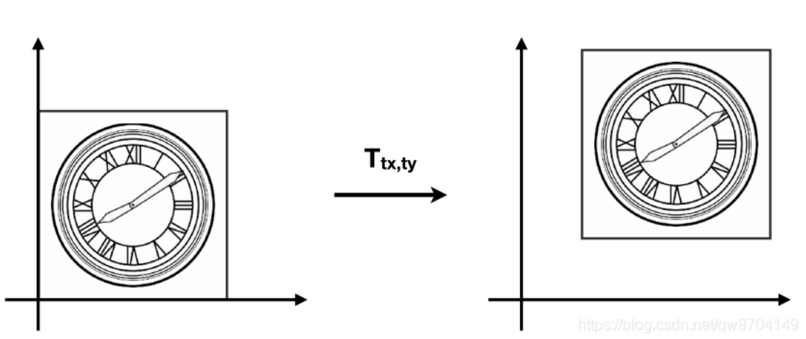
$$\begin{cases} x' = x + t_x \\ y' = y + t_y \end{cases}$$




$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

Reference:
[1] https://www.aiuai.cn/aifarm1946.html

# 剪切变换




[1] 新图形与旧图形各像素点坐标存在如下关系：

$$\begin{cases} x' = x + t_x \\ y' = y + t_y \end{cases}$$

可以表示成如下形式：

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

Reference:
[1] https://www.aiuai.cn/aifarm1946.html

# 仿射变换

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

# 仿射变换

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

这种形式，与之前的线性变换的形式不一致，这不完美。
经过前辈的尝试，发现可以用齐次坐标解决形式一致性问题。

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

则逆变换可以表示为：

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

# 小结

**缩放变换**

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

**旋转变换**

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

**平移变换**

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

**仿射变换**

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

# 第五部分

## 使用OpenCV实现仿射变换

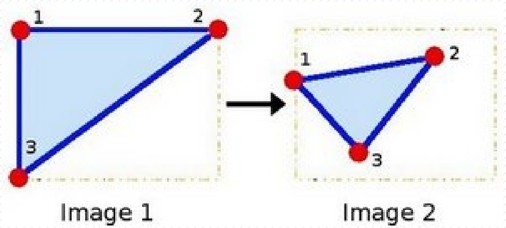
✓ OpenCV仿射变换

# OpenCV实现图像仿射变换

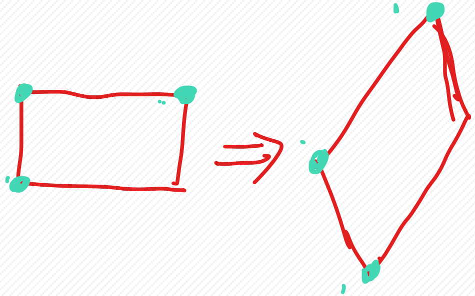
$2\times3$

$$\begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

- getAffineTransform 函数
  - 输入原始图像三个关键点坐标，和对应目标图像的三个关键点坐标
  - 返回仿射变换矩阵（前两行）



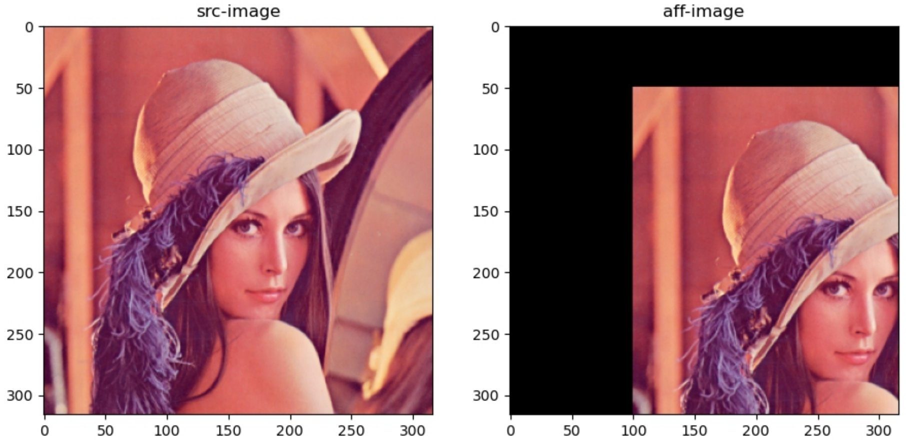

[1]



- warpAffine函数
  - 输入原始图像，仿射变换矩阵，目标图像大小
  - 返回经变换后的图像

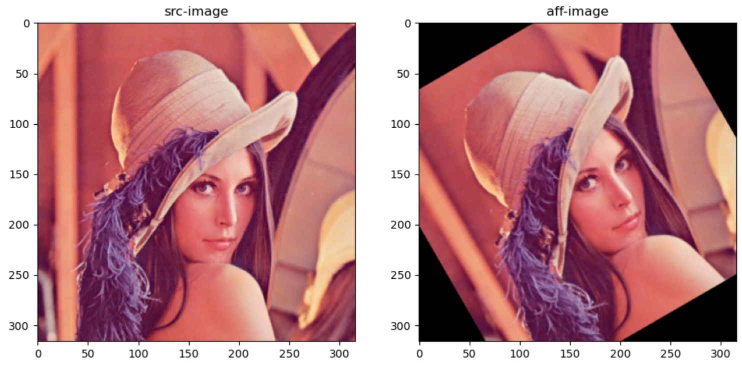
Reference:
[1] https://www.cnblogs.com/xiaoniu-666/p/13322556.html

# 实验2：OpenCV仿射变换



# 实验3：OpenCV实现图像旋转

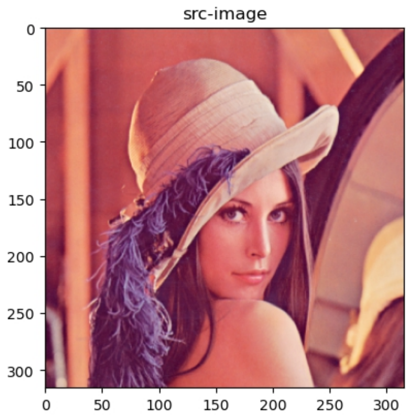
- getRotationMatrix2D 函数
  - 输入旋转中心、旋转角度（角度制）、缩放比例
  - 返回仿射变换矩阵（前两行），可配合warpAffine函数使用。

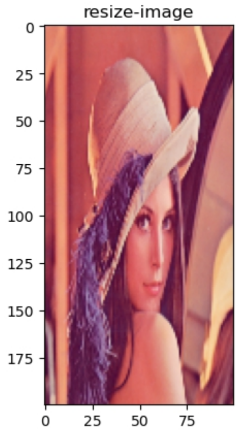



Reference:
[1] https://www.cnblogs.com/xiaoniu-666/p/13322556.html

# 实验4：OpenCV实现图像旋转

- resize 函数
  - 输入图像、目标大小
  - 返回目标图像




Reference:
[1] https://www.cnblogs.com/xiaoniu-666/p/13322556.html

# THANKS!

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