COMP 361: Numerical Methods: Assignment 3

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Contents

1 Newton's method to compute the cube root of 6

Problem: Show how to use Newtons method to compute the cube root of 5. Numerically carry out the first 10 iterations of Newtons method, using $x_0 = 1$. Analytically determine the fixed points of the Newton iteration and determine whether they are attracting or repelling. If a fixed point is attracting then determine analytically if the convergence is linear or quadratic. Draw the x_{k+1} versus x_k diagram, again taking $x_0 = 1$, and draw enough iterations in the diagram, so that the long time behavior is clearly visible. For which values of x_0 will Newtons method converge?
Solution:

1.1 10 first iterations using Newton's method

Cube root of 5 is a zero of the function g (x) such that:
$g(x) = x^3 - 5$

The first 10 iterations using Newton's method were carried out by using Python with Jupyter Notebook. Check out the source code and presentation in directory: program.Problem1.ipynb

```
# The main algorithm
def newton_raphson(f, diff, init_x, max_iter=1000):
    x = init_x
    estimates = []
    listX = [x]
    for i in range(max_iter):
        deltaX = -f(x)/diff(x)
        x = x + deltaX
        listX.append(x)
        estimates.append(x)
    return x, listX, estimates
```

The results after computing are:

Xi = 1 X(i+1) = 2.3333333333333333

Xi = 2.3333333333333333 X(i+1) = 1.8616780045351473

Xi = 1.8616780045351473 X(i+1) = 1.722001880058607

Xi = 1.722001880058607 X(i+1) = 1.7100597366002945

Xi = 1.7100597366002945 X(i+1) = 1.709975950782189

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1.2 Fixed point finding

Newton's method: $x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$
Applying Newton's method

$$\implies f(x) = x - \frac{x^3 - 5}{3x^2} = \frac{3x^3 - x^3 + 5}{3x^2}$$
$$= \frac{2x^3 + 5}{3x^2}$$

To find the fixed points, let x = f (x)

$$x = \frac{2x^3 + 5}{3x^2}$$
$$3x^3 = 2x^3 + 5$$
$$x^3 = 5$$
$$\implies x = \sqrt[3]{5}$$

The fixed point of the Newton iteration is $x = \sqrt[3]{5}$

1.3 Fixed point analytically

$$f'(x) = (\frac{2x^3 + 5}{3x^2})' = (\frac{2}{3}x + \frac{5}{3x^2})' = \frac{2}{3} - \frac{10}{3x^3}$$

$$\implies |f'(x^*)| = |f'(\sqrt[3]{5})| = |\frac{2}{3} - \frac{10}{3(\sqrt[3]{5})^3}| = |\frac{2}{3} - \frac{10}{15}| = 0$$

$|f'(x^*)| = 0 \implies$ the fixed point is **attracting** and it is converging **quadratically**.

1.4 Fixed point iteration diagram

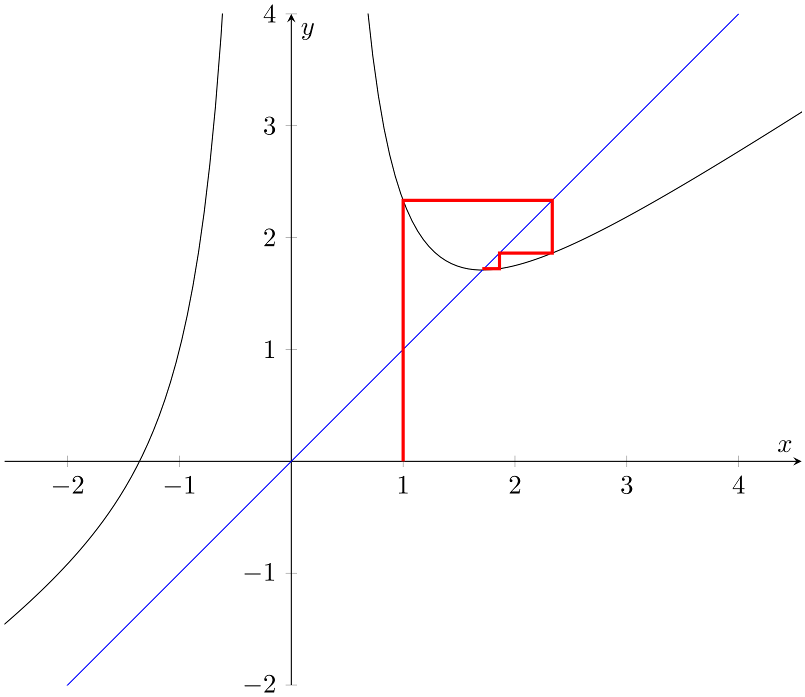

Figure 1: Fixed point iteration diagram