COMP 361: Numerical Methods: Assignment 3

Duc Nguyen

Winter 2020

Contents

1 Newton's method to compute the cube root of 6

Problem: Show how to use Newtons method to compute the cube root of 5. Numerically carry out the first 10 iterations of Newtons method, using $x_0 = 1$. Analytically determine the fixed points of the Newton iteration and determine whether they are attracting or repelling. If a fixed point is attracting then determine analytically if the convergence is linear or quadratic. Draw the x_{k+1} versus x_k diagram, again taking $x_0 = 1$, and draw enough iterations in the diagram, so that the long time behavior is clearly visible. For which values of x_0 will Newtons method converge?
Solution:

1.1 10 first iterations using Newton's method

Cube root of 5 is a zero of the function g (x) such that:
$g(x) = x^3 - 5$

The first 10 iterations using Newton's method were carried out by using Python with Jupyter Notebook. Check out the source code and presentation in directory: program.Problem1.ipynb

```
# The main algorithm
def newton_raphson(f, diff, init_x, max_iter=1000):
    x = init_x
    estimates = []
    listX = [x]
    for i in range(max_iter):
        deltaX = -f(x)/diff(x)
        x = x + deltaX
        listX.append(x)
        estimates.append(x)
    return x, listX, estimates
```

The results after computing are:

Xi = 1 X(i+1) = 2.3333333333333333

Xi = 2.3333333333333333 X(i+1) = 1.8616780045351473

Xi = 1.8616780045351473 X(i+1) = 1.722001880058607

Xi = 1.722001880058607 X(i+1) = 1.7100597366002945

Xi = 1.7100597366002945 X(i+1) = 1.709975950782189

Xi = 1.709975950782189 X(i+1) = 1.709975946676697

Xi = 1.709975946676697 X(i+1) = 1.709975946676697

Xi = 1.709975946676697 X(i+1) = 1.709975946676697

Xi = 1.709975946676697 X(i+1) = 1.709975946676697

Xi = 1.709975946676697 X(i+1) = 1.709975946676697

1.2 Fixed point analytically

Newton's method: $x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$

Applying Newton's method

$$\begin{aligned} \implies f(x) = x - \frac{x^3 - 5}{3x^2} &= \frac{3x^3 - x^3 + 5}{3x^2} \\ &= \frac{2x^3 + 5}{3x^2} \end{aligned}$$

To find the fixed points, let x = f (x)

$$\begin{aligned} x &= \frac{2x^3 + 5}{3x^2} \\ 3x^3 &= 2x^3 + 5 \\ x^3 &= 5 \\ \implies x &= \sqrt[3]{5} \end{aligned}$$

The fixed point of the Newton iteration is $x = \sqrt[3]{5}$