Reading notes

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Nomologically Valid: no counter examples that do not violate the laws of nature
Conceptually Valid: no counter examples that do not violate the conceptual connections between words

Formally valid: when validity is dependent on Form rather than context of the premises

A is either an X or a Y .
A isn't a Y .
∴ A is an X .

Can we find a counterexample?

Validity:

- Rules out the possibility of the premises and conclusions not being true at once
- It is possible for the conclusion to be false but the argument valid
- If the conclusion is false then one of the premises must be false
- Not about the actual truth or falsity of sentences, it is about whether it is possible for all premises to be true and conclusion to not be true

Soundness:

- Argument is sound if it is both valid and all its premises are true

Inductive arguments: generalises observations about past events to a conclusion about future cases

- It is still possible fro that conclusion to be refuted with a counter example

Entailment:

- The realtionship between premises and conclusion

Practice Questions:
A:

1. Valid
2. Valid
3. Invalid
4. Valid
5. Invalid
6. Invalid

B:

1. True

Joint possibility:

- Joint impossibility: Sentences that are not possible to be true at once
  - If the sentences are joint impossible, you cannot add another sentence to fix it
  - If the premises are join impossible –> the argument is valid
- Joint Possible: all sentences must be possible to exist at the same time

Necessary Truths, Necessary Falsehoods and contingencies:

- Contingent: a sentence that is capable of being true and capable of being false
- Truth: Something that is just true in general
- Falsehood: A sentence that is impossible

Equivalence: when two sentences have the same truth-value in every case

- These sentences are typically similar to each other, if one is false then the other also has to be false (vice versa)

John went to the store after he washed the dishes.
John washed the dishes before he went to the store.

A
If *A*, then *C*
∴C

Conceptually Valid: the concepts carry through

*Arguments are not valid based **solely** on their form*

TFL: Truth Functional logic

Atomic Sentences:

- Add subscripts to letters to represent subsentences and use a symbolization key such as:
  - *A*: It is raining outside
    C : Jenny is miserable

Logical Connectives:

symbol	what it is called	rough meaning
¬	negation	'It is not the case that...'
∧	conjunction	'Both... and ...'

$\vee$	disjunction	'Either. . . or . . .'
$\rightarrow$	conditional	'If . . . then . . .'
$\leftrightarrow$	biconditional	'. . . if and only if . . .'

- Negation:
    - "it is not the case that" or "not" the sentence – you are negating the sentence
    - It is possible to have a double negation given the conditions
      7. Jane is happy.
      8. Jane is unhappy.
    - Sentence 8 is its own sentence because it could be that Jane is neither happy or unhappy – and it is not the same thing as "It is not the case that Jane is happy"
- Conjunction:
    - We use it to deal with "and", it is used to connect two subsentences together. The subsentences are the conjuncts of the conjunction.
    - It does not matter if you have multiple conjunctions
    - Even if you are referring to one subject, you still must define the two subtences
    - A sentence can be symbolized as $(A \wedge B)$ if it can be paraphrased in English as 'Both..., and...', or as '..., but ...', or as 'although ..., ...'.
    - You can ignore "but"
- Disjunction:
    - subsentences are the disjuncts of the disjunction
    - A sentence can be symbolized as $(A \vee B)$ if it can be paraphrased in English as 'Either. . . , or. . . .'
    - *Exclusive or*: Excludes the possibility of both of them being true
    - *Inclusive or:* Allows for the possibility of both
- Conditional:
    - 'If P, then F' use '$\rightarrow$' to symbolize 'if…then…' format when P and F are subsentences
        - P is the Antecedent of the conditional
        - F is the Consequent of the conditions
    - Be careful as the sentence may be different if the subsentences are switched
    - A sentence can be symbolized as $(A \rightarrow B)$ if it can be paraphrased in English as 'If A, then B' or 'A only if B'.
        - Can have many forms – look out for this
        - Tells us that only if the antecedent is true, then the consequent is true
        - Does not discuss a casual connection between two events
- Biconditional:
    - Amounts to stating things both directions of the conditional
    - '$(D \rightarrow M) \wedge (M \rightarrow D)$' = $(D \leftrightarrow M)$
    - New Terminology:
        - *'if'* – the English conditional
        - *'iff'* – the English biconditional
    - A sentence can be symbolized as $(A \leftrightarrow B)$ if it can be paraphrased in English as 'A iff B'; that is, as 'A if and only if B'.

Unless: A special case

***J* : You will wear a jacket**

***D*: You will catch a cold**

- **Unless you wear a jacket, you will catch a cold.** $(\neg J \rightarrow D)$
- **You will catch a cold unless you wear a jacket.** $(\neg D \rightarrow J)$
- OR $(J \vee D)$ – Both sentences are equal in their meanings as such this symbolization works too, it is not possible for these two happen at the same time
- All Three of these are considered to be equal

- If a sentence can be paraphrased as 'Unless *A*, *B*,' then it can be symbolized as '$(A \vee B)$'.
- This can get complicated when interpreting speech in the english language because unless can be used as a biconditional in regular speech

Expression of TFL: Any string of symbols of TFL

Sentences of TFL: An Inductive definition

- Every Sentence letter is a sentence
- If A is a sentence then -A is also a sentence
- If A and B are sentences, then (A <any connective> B) is a sentence
- Nothing else is a sentence.

Main Logical Operation:

- A sentential connective that is introduced last
- We can find it by:
    - If the first symbol in the sentence is ¬ then that is the main logical operator
    - Otherwise, count brackets. For each open bracket, add 1, for each closing one, subtract 1, when the counter is at exactly 1, the first operator you hit (apart from ¬) is the main logical operator

Scope of Negation: The scope of '¬' is the subsentence for which '¬' is the main logical operator

Scope of a connective: the subsentence for which that connective is the main logical operator

Bracket Conventions:

- Allowed to omit the outermost bracket of the sentence
- In more complex sentences, we can use '[…]' for faster reading

Ambiguity

Lexical Ambiguity: when a sentence contains a word[s] that can have multiple meanings

Structural Ambiguity: a sentence can be interpreted in different ways.

Vagueness: when something is not specified yet rather described, (i.e. "tall" but in what context?)

Sentences in TFL cannot be structurally ambigous

Scope ambiguities: depend on whether or not a connective is in the scope of another

Quotation Conventions:
- Indicate that you are moving from talking about an object to talk about a name of that object

Object Language and metalanguage:
- Object Language: TFL
- Metalanguage: English
- Difference between these two is in their exactness

Metavariables:
- Augmented metalanguage that is used to talk about any expression of TFL
- i.e. Cursive A is used to describe the expressions of TFL while A is a specific sentence letter of TFL

Quotation conventions for arguments:
- TFL cannot identify cert
- ain sentences as premises and conclusions
- Notation:
    - $\mathscr{A}_1, \ldots, \mathscr{A}_n \therefore \mathscr{C}$
- "therefore" is not part of object language, is part of metalanguage

Truth Tables:
- Negation:

$\mathscr{A}$	$\neg\mathscr{A}$
T	F
F	T

- Conjunction: If A and B are then True

$\mathscr{A}$	$\mathscr{B}$	$\mathscr{A} \wedge \mathscr{B}$
T	T	T
T	F	F
F	T	F
F	F	F

- Disjunction: IF A or B is true then True (inclusive or)

$\mathscr{A}$	$\mathscr{B}$	$\mathscr{A} \vee \mathscr{B}$
T	T	T
T	F	T
F	T	T
F	F	F

- Conditional: If A is true then B is True

$\mathscr{A}$	$\mathscr{B}$	$\mathscr{A} \rightarrow \mathscr{B}$
T	T	T
T	F	F
F	T	T
F	F	T

- Biconditional: Both must be true for the expression to evaluate to true

$\mathscr{A}$	$\mathscr{B}$	$\mathscr{A} \leftrightarrow \mathscr{B}$
T	T	T
T	F	F
F	T	F
F	F	T

The idea of Truth Functionality:
- A connective is Truth-functional iff the truth value of a sebtebce with that connective as its main logical operator is uniquely determined by the truth value(s) of the constituent sentence(s)
- Every connective in TFL is truth-functional.
- To determine the turth value of a TFL sentence, we only need to know the truth value of its componenets.
- "It is necessarily the case" is not truth-functional

Symbolizing vs. Translating:
- All connectives of TFL map us between truth values

- All connectives of TFL map as between truth values
- TFL ignores the subtleties of of the original text which enhance it beyond just truth values
- It also cannot capture the subtle differences between sentences.
- TFL cannot work with meaning as well
- When we treat a TFL sentence as symbolizing an English sentence, we are stipulating that the TFL sentence is to take the same truth value as that English sentence.

Indicative vs. Subjunctive conditionals:
- The truth value of the sentence is not uniquely determined by the truth value of the parts
- Subjunctive conditionals ask us to imagine something contrary to fact, and then ask us to evaluate what would have happened in that case

Complete truth tables:
- We can assign truth values directly: A valuation
- A valuation is any assignment of truth values to particular sentence letters of TFL.
- Each row of the truth table represents a possible valuation, the complete table represents all the possible valuations – It allows us to calculate the truth value of complex sentences
- The values under the main logical operator tell us the value of the sentence
-

H	I	$(H \wedge I) \rightarrow H$
T	T	T T T **T** T
T	F	T F F **T** T
F	T	F F T **T** F
F	F	F F F **T** F

- A *complete truth table* has a line for every possible assignment of True and False to the relevant sentence letters. Each line is a *valuation* and the truth able has a line for all the different valuations
- The size of said truth table can vary based on the number of sentence letters in the table.
  - A sentence that only needs 1 sentence letter requires only two rows (for T and F)
  - This is true even if the same letter is repeated multiple times
  -

C	$[(C \leftrightarrow C) \rightarrow C] \wedge \neg(C \rightarrow C)$
T	T T T T T **F**F T T T
F	F T F F F **F**F F T F

  - We can continue this trend inifinitely based on the number of sentence letters we have.
    For example, with 3 letters:

M	N	P	$M \wedge (N \vee P)$
T	T	T	T **T** T T T
T	T	F	T **T** T T F
T	F	T	T **T** F T T
T	F	F	T **F** F F F
F	T	T	F **F** T T T
F	T	F	F **F** T T F
F	F	T	F **F** F T T
F	F	F	F **F** F F F

  - **A rule for this is that if we have n different sentence letters, then the table must have 2^n rows/lines**
- To fill the columns, start from right to left
  1. At the right most, alternate between T and F
  2. In the next column to its left, write two T's and two F's and alternate/repeat
  3. In the next left column, write 4 T's and 4 F's and alternate/repeat
  4. Continue this trend…
  - **RULE:**

Column Index(i)	**3**	**2**	**1**	**0**
2^i T & F	**8 T & F**	**4 T & F**	**2 T & F**	**1 T & F**

Brackets:
- If the same connective is used (conjuction or disjunction) throughout a sentence, then associative property applies.
  Example: Logically equivalent sentences
  -
    $((A \vee B) \vee C)$

    $(A \vee (B \vee C))$
  - We should not write: A v B v C
- Associative property ***does not*** apply to conditionals, or if there is variation of the conjunctions and disjunctions in the sentence, then the order of the brackets is important and there is no logical equivalence
  Example of conditionals:

  $((A \rightarrow B) \rightarrow C)$

  $(A \rightarrow (B \rightarrow C))$

  Example of Variation:

  $((A \vee B) \wedge C)$

  $(A \vee (B \wedge C))$

  It is then needless to say that we cannot write these without their brackets, it would be far to ambiguous

Semantic Concepts:

The surrogate for a necessary truth is a ***tautology***: A is a tautology iff it is true on every valuation. We can use truth

The surrogate for a necessary truth is a ***tautology***: A is a tautology iff it is true on every valuation. We can use truth tables to determine this and further concepts.

- There are some necessary truths that we cannot adequately symbolize in TFL, an example of this "2+2 = 4". This must be true however we cannot represent it in TFL since it would only be a sentence letter and *no sentence letter can be a tautology by itself*

As such, the opposite to this is a ***contradiction***: A is a contradiction iff it is false on every valuation

Another variation of this is *equivalance*: A and B are equivalent iff for every valuation, their truth values are the same.

- An example of this:

P	Q	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	**F** T T T	F T **F** F T
T	F	**F** T T F	F T **F** T F
F	T	**F** F T T	T F **F** F T
F	F	**T** F F F	T F **T** T F

The surrogate for jointly possible is *jointly satisfiable*: $A_1, A_2, \ldots, A_n$ are jointly satisfiable iff there is some valuation which makes them all true.

Similar to this, we can develop the idea of entailment: The $A_1, A_2, \ldots, A_n$ entail (in TFL) the sentence C iff no valuation of the relevant sentence letters makes all of $A_1, A_2, \ldots, A_n$ true and C false.

An example of this is:

J	L	$\neg L \to (J \vee L)$	$\neg L$	J
T	T	F T **T** T T T	**F** T	**T**
T	F	T F **T** T T F	**T** F	**T**
F	T	F T **T** F T T	**F** T	**F**
F	F	T F **F** F F F	**T** F	**F**

If $A_1, A_2, \ldots, A_n$ entail C in TFL, then $A_1, A_2, \ldots, A_n \therefore C$ is valid.

- This is because if we assume that $A_1, A_2, \ldots, A_n$ entails C then there is also no valuation that makes $A_1, A_2, \ldots, A_n$ true and also makes C false
- The test for this is to first symbolize them in the TFL and then use the truth tables to test their entailment.

The double turnstile shall be defined as the symbol for entailment: $A_1, A_2, \ldots, A_n \vDash C$

- This is not a symbol of TFL rather it is a symbol of metalanguage
- $P, P \to Q \vDash Q$ is the same thing as The TFL sentences 'P ' and 'P →Q ' entail 'Q'

Some special cases of the turnstil are as follows:

- $\vDash C$
  - Since there are no sentences on the left side of the turnstile, this simply means that there is no valuation that makes C false, as such ever valuation makes C true.
    Thus **C is a tautology**
- $A \vDash$
  - Similarly, for this case: we can say that **A is a contradiction**

Furthermore, to say: it is not the case that $A\ 1, \ldots, A\ n \vDash C$, we can say $A\ 1, A\ 2, \ldots, A\ n \nvDash C$

- This means that some valuation makes all of $A_1, A_2, \ldots, A_n$ true whilst making C false, this is not the same is $A_1, A_2, \ldots, A_n \vDash \neg C$ since it is possible that some other valuation makes $A_1, A_2, \ldots, A_n$ true and makes C true. As such: $P \nvDash Q$ but also $P \nvDash \neg Q$.

'⊨' versus '→'

- $A \vDash C$ iff no valuation of the sentence letters makes A true and C false.
- $A \to C$ is a tautology iff no valuation of the sentence letters makes $A \to C$ false.
- $A \to C$ is a tautology iff $A \vDash C$
- '→' is a sentential connective of TFL.
- '⊨' is a symbol of augmented English.
- When we use the turnstile we are forming a metalinguistic sentence that mentions the surrounding TFL sentences

Limitations of TFL

- TFL is not expressive enough to symbolize everything we might want to so that we can use the skills we have
- Sentence letters cannot be allowed to take truth values other than True or False, it cannot capture the intended truth conditions
- Some logical conventions in the English language cannot be properly captured by the TFL connectives, namely the material conditional, these are called the *paradoxes of the material conditional.*

The very idea of natural selection:

- Truth tables cannot give us much insight in certain cases where different forms of reasoning are present
- The aim of a natural deduction system is to show that particular arguments are valid in such a way that we understand the reasoning the arguments involved.

- understand the reasoning the arguments involved.
- Complicated arguments can become untraceable in TFL if we choose to use truth tables, there must be a more efficient way to

Conditional:

-

m	$\mathscr{A} \to \mathscr{B}$	
n	$\mathscr{A}$	
	$\mathscr{B}$	$\to$E, m, n

- To get B, you must first satisfy A
- When citing a the reason you must write the line of the conditional first and then the antecedent after that
- We can further make an additional assumption for the sake of the argument to show that we are no longer dealing with just our original premise
- **We are not claiming**, we are **assuming**. We must mark this on our sheet
-

i	$\quad \mathscr{A}$	AS
j	$\quad \mathscr{B}$	
	$\mathscr{A} \to \mathscr{B}$	$\to$I, i–j

- Example:

1	$P \to Q$	PR
2	$Q \to R$	PR
3	$\quad P$	AS
4	$\quad Q$	$\to$E, 1, 3
5	$\quad R$	$\to$E, 2, 4
6	$P \to R$	$\to$I, 3–5

Additional Assumption and Subproofs:

- Once a subproof is closed, we cannot use anything within that proof
- To cite an individual line when applying a rule:
  1. the line must come before the line where the rule is applied, but
  2. not occur within a subproof that has been closed before the line where the rule is applied.
- By closing a subproof, we are discharging the assumptions of the subproof, we cannot use anything that was derived using the discharged assumption
- Any assumption works to start a subproof with

Biconditional:

- Must prove both A and B

i	$\quad \mathscr{A}$	AS
j	$\quad \mathscr{B}$	
k	$\quad \mathscr{B}$	AS
l	$\quad \mathscr{A}$	
	$\mathscr{A} \leftrightarrow \mathscr{B}$	$\leftrightarrow$I, i–j, k–l

Disjunction:

-

m	$\mathscr{A} \vee \mathscr{B}$	
i	$\quad \mathscr{A}$	AS
j	$\quad \mathscr{C}$	
k	$\quad \mathscr{B}$	AS
l	$\quad \mathscr{C}$	
	$\mathscr{C}$	$\vee$E, m, i–j, k–l

Negation:

-

m	$\neg\mathscr{A}$	
n	$\mathscr{A}$	
	$\perp$	$\neg$E, m, n

Disjunctive Syllogism

m	$\mathscr{A} \vee \mathscr{B}$	
n	$\neg\mathscr{A}$	

	$\mathscr{B}$	DS, m, n

Modus Tollens

m	$\mathscr{A} \to \mathscr{B}$	
n	$\neg\mathscr{B}$	
	$\neg\mathscr{A}$	MT, m, n

Double negation Elimination:

m	$\neg\neg\mathscr{A}$	
	$\mathscr{A}$	DNE, m

Excluded Middle:

i		$\mathscr{A}$	AS
j		$\mathscr{B}$	
k		$\neg\mathscr{A}$	AS
l		$\mathscr{B}$	
	$\mathscr{B}$		LEM, i–j, k–l

Soundness and Completeness:

Concept	Truth table (semantic) definition	Proof-theoretic (syntactic) definition
Tautology/ theorem	A sentence whose truth table only has Ts under the main connective	A sentence that can be derived without any premises.
Contradiction/ inconsistent sentence	A sentence whose truth table only has Fs under the main connective	A sentence whose negation can be derived without any premises
Contingent sentence	A sentence whose truth table contains both Ts and Fs under the main connective	A sentence that is not a theorem or contradiction
Equivalent sentences	The columns under the main connectives are identical.	The sentences can be derived from each other
Unsatisfiable/ inconsistent sentences	Sentences which do not have a single line in their truth table where they are all true.	Sentences from which one can derive the contradiction $\perp$
Satisfiable/ Consistent sentences	Sentences which have at least one line in their truth table where they are all true.	Sentences from which one cannot derive the contradiction $\perp$
Valid argument	An argument whose truth table has no lines where there are all Ts under main connectives for the premises and an F under the main connective for the conclusion.	An argument where one can derive the conclusion from the premises.

Logical property	To prove it present	To prove it absent
Being a theorem	Derive the sentence	Find a false line in the truth table for the sentence
Being a contradiction	Derive the negation of the sentence	Find a true line in the truth table for the sentence
Contingency	Find a false line and a true line in the truth table for the sentence	Prove the sentence or its negation
Equivalence	Derive each sentence from the other	Find a line in the truth tables for the sentence where they have different values
Consistency	Find a line in truth table for the sentence where they all are true	Derive a contradiction from the sentences
Validity	Derive the conclusion from the premises	Find a line in the truth table where the premises are true and the conclusion false.