Project Evaluation Methods (PEM)

Notebook of exercises – Chapter 2

Academic year 2023/ 24

Exercise 1

Using the *Excel*, generate M=20 random observations from the Exponential distribution with parameter α=1 and α=3.

Exercise 2

Using the *Excel*, generate M=20 random observations from the Gamma distribution with parameters (k, α) = (3, 1) and (k, α) = (5, 3).

Exercise 3

Using the *Excel*, generate M=20 random observations from the Normal distribution with parameters (μ, σ) = (0, 1) and (μ, σ) = (2, 3).

Exercise 4

Using the *Excel*, generate M=20 random observations from the Chi-Square distribution with n=3 and n=5 degrees of freedom.

Exercise 5

Using the *Excel*, generate M=20 random observations from the Triangular distribution with the following probability density function:

$$f(x) = \begin{cases} x & \text{if } 0 \le x \le 1, \\ 2 - x & \text{if } 1 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

Exercise 6

Using the *Excel*, generate M=20 random observations from the Triangular distribution with parameters (1, 2, 5).

Exercise 7

Consider the random variable X, which follows the exponential distribution with parameter α=2.

a) Using the *Excel*, apply the Monte Carlo simulation to estimate the average value of X.

Hint: Adapt the procedure developed in slides 46, 47, and 48.

b) Using the *Excel*, apply the Monte Carlo simulation with the *Method of Complementary Random Numbers* to estimate the average value of X.

Hint: Adapt the procedure developed in slides 49 and 50.

Exercise 8

Using the *Excel*, generate M=20 random observations from the Binomial distribution with parameters $(n, p) = (10, 0.3)$.

Exercise 9

Using the *Excel*, generate M=20 random observations from the Poisson distribution with parameter $\lambda = 2$.

Exercise 10

Suppose a player has the opportunity to play the following game:

- Repeatedly flip an unbiased coin until obtain five heads.

The player has to pay €1 for each flip of the coin, but he receives €10 at the end of each session of the game. The player is not allowed to quit during a game session.
The player should participate in this game?

Apply Monte Carlo simulation to justify your answer.

Hint: Adapt the procedure developed in slides 23, 24, 25 and 26.

Exercise 11

A company from the computer area is planning to develop a new model of personal computer. For this reason, the management department of the company would like to know the likely financial performance and the extent of financial risk of this investment. Initial outlays of 50000€ are required in arranging contracts for the supply of components, tooling up and hiring computer technicians who would assemble the new PCs. Are also known overheads and marketing costs, which are estimated at 500€ per PC sold.
The management department of the computer firm has identified four uncertain variables which will be important in determining the performance of this project:

i. **PC sales in each year**;
ii. **Market price, per PC**;
iii. **Component cost, per PC**; and
iv. **Labour cost, per PC**.

The management department adopts a planning horizon of five years for the project (because, management considers that after five years the PC will have run its useful life) and a discount rate of 7%.
The management department decided that the Triangular distribution is an acceptable approximation for the four variables. The parameters of the Triangular distribution for each of the four uncertain variables are indicated in the following table:

Variables	Pessimistic Value	Modal Value	Optimistic Value
Sells (PC's a year)	50	100	130
Market Price (euros per PC)	2200	2500	3000
Component Cost (euros per PC)	1200	1000	900
Labour Cost (euros per PC)	350	300	200

Apply the Monte Carlo simulation to determine an estimate for the NPV of the investment project. Based on statistical analysis of results, help the manager to gather information about the likely financial performance and the extent of financial risk of this investment.

Resolution: The main steps of Monte Carlo simulation procedure are presented in the slides 53, …, 66.

Exercise 12

An airline company is planning to introduce a new country run, to provide two return services a week (104 services per year) to a rural city with a population of 30000 persons. The airline will use a forty-passenger aircraft, which it can purchase for 4.2 million of euros.

An airstrip owned by the local government in the rural area is available without charge, but the company will have to carry out restoration of the disused airstrip and terminal at a cost of €200000.

The company intends to develop a financial model to

i. simulate the net cash flows from this project over seven-year period; and
ii. provide an estimate of the net present value of the investment.

In order to develop the financial model, the company collected the relevant information about the project:

- The demand of passengers in year **t** (one-way flights) is estimated according to the expression $\mathbf{5000+300\times(t-1)+w}$, $\mathbf{t}=1,\ldots, 7$, where **w** denotes a random component that follows the Normal distribution with mean value of zero and standard deviation of 400 passengers.

- In addition to passenger services, the company has a contract to transport mail to the rural city. The estimated revenue from the mail service in year **t** is given by the expression $\mathbf{€200000+v}$, $\mathbf{t}=1,\ldots, 7$, where **v** denotes a random component that follows the Normal distribution with mean value of zero and standard deviation of €10000.

- The cost of fuel per flight in each year **t**, $\mathbf{t} = 1, ..., 7$, is a random variable that follows the Log-Normal distribution with mean value of €1500 and standard deviation of €100.

- The personnel outlay in year **t** is estimated according to the expression $\mathbf{€250000+q}$, $\mathbf{t}=1,\ldots,7$, where **q** denotes a random component that follows the Normal distribution with mean value of zero and standard deviation of €25000.

- The annual maintenance cost for the aircraft is estimated at 5% of the initial price (that is, is equal to 0,05×€4200000=€210000).

- The cost of booking and other services is €10 per passenger flight.

- The price of one-way airline ticket is a random variable that follows the Log-Normal distribution with mean value of €280 and standard deviation of €20.

- The appropriate discount rate is 8%.
- Company tax is 30% of the annual operating surplus.
- The aircraft and ground facilities can be depreciated at a rate of 10% per annum for taxation purposes.
- After seven years, the aircraft has a salvage value of 1 million of euros.

Develop the financial model required by the airline.

Resolution: The main steps of the financial model are presented in the slides 67, …,78.

Exercise 13

A company from the computer area is planning to develop a new model of personal computer. For this reason, the management department of the company would like to know the likely financial performance and the extent of financial risk of this investment. Initial outlays of 50000€ are required in arranging contracts for the supply of components, tooling up and hiring computer technicians who would assemble the new PCs. Are also known overheads and marketing costs, which are estimated at 800€ per PC sold.

The management department of the computer firm has identified four uncertain variables which will be important in determining the performance of this project:

i. **PC sales in each year**;
ii. **Market price, per PC**;
iii. **Component cost, per PC**; and
iv. **Labour cost, per PC**.

The management department adopts a planning horizon of three years for the project (because, management considers that after three years the PC will have run its useful life) and a discount rate of 10%.
The management department decided that the Triangular distribution is an acceptable approximation for the four variables. The parameters of the Triangular distribution for each of the four uncertain variables are indicated in the following table:

Variables	Pessimistic Value	Modal Value	Optimistic Value
Sells (PC's a year)	50	100	130
Market Price (euros per PC)	2200	2500	3000
Component Cost (euros per PC)	1200	1000	900
Labour Cost (euros per PC)	350	300	200

a) Develop the expression that translates the firm's net cash flow for each year of activity. Justify your answer.

b) Suppose that for each of the three years were generated random observations of the variables **Sells**, **Market Price**, **Component Cost** and **Labour Cost**. The values generated are shown in the following table:

Year [t]	**Sells** (PCs/ year)	**Market Price** (€/ PC)	**Component Cost** (€/ PC)	**Labour Cost** (€/ PC)	**Net Cash Flow** (€/ ano)
1	63	2564	1200	299	
2	91	2392	1037	277	
3	103	2236	974	315	

Complete the table and compute the NPV of the project underlying this combination.

c) Suppose that the firm has applied the Monte Carlo simulation to obtain a reliable estimate for the NPV of the project. The simulation results are summarized in Descriptive Statistics Table and in the Cumulative Relative Frequency Table:

Summary Table - Descriptive Statistics	
Average	27959,45192
Standard Error	1202,004087
Median	27680,09636
Standard Deviation	26877,62848
Sample Variance	722406912,6
Kurtosis	-0,161326903
Skewness	0,215022011
Interval	156091,7467
Minimum	-39023,13899
Maximum	117068,6077
Sum	13979725,96
Score	500

Block	*Frequency*	*% Cumulative*
-39023,13899	1	0,20%
-31928,0596	3	0,80%
-24832,98021	2	1,20%
-17737,90081	12	3,60%
-10642,82142	19	7,40%
-3547,742026	23	12,00%
3547,337368	35	19,00%
10642,41676	46	28,20%
17737,49615	47	37,60%
24832,57555	47	47,00%
31927,65494	41	55,20%
39022,73434	55	66,20%
46117,81373	50	76,20%
53212,89312	36	83,40%
60307,97252	22	87,80%
67403,05191	19	91,60%
74498,1313	18	95,20%
81593,2107	7	96,60%
88688,29009	9	98,40%
95783,36948	4	99,20%
102878,4489	3	99,80%
109973,5283	0	99,80%
More	1	100,00%

Based on the results of Monte Carlo simulation, prepare a short report with the main conclusions. Not forgetting to justify the viability or otherwise of the project.

Exercise 14

A Corporation is considering an investment proposal to expand one of its product lines. The economic life of the project is estimated to be 5 years. Capital outlay in the first year is 2 million of euros and, at the end of the year 2, another capital expenditure of €700000 is required for an upgrade. Due to the uncertainty associated with the cash flow in each year for the project, the management department estimated three possible scenarios:

	Estimated Cash flows (millions of euros)				
	Year 1	Year 2	Year 3	Year 4	Year 5
Scenario 1	1,9	0,3	0,7	1,3	0,21
Scenario 2	0,7	0,98	0,3	0,25	0,4
Scenario 3	0,45	2	2,1	1,45	1,3

Assuming the discount rate of 5% per annum, determine an estimate for the NPV of the project.

Exercise 15

A Corporation is considering an investment proposal to expand one of its product lines. The economic life of the project is estimated to be 5 years. Capital outlay in the first year is 2 million of euros and, at the end of the year 2, another capital expenditure of €700000 is required for an upgrade. Due to the uncertainty associated with the cash flow in each year for the project, the management department decided that the exponential distribution with parameter 0.7 is an acceptable approximation for the cash flow in each year.

Assuming the discount rate of 5% per annum, determine, using Monte Carlo simulation, an estimate for the NPV of the project.