Water Networks

Sai Krishna Kanth Hari

June 2017

Nomenclature

Sets

δ_i^+	Set of all incoming arcs at node i
δ_i^-	Set of all outgoing arcs at node i
$\mathcal{A}$	Set of all arcs, $\mathcal{S}_e \cup \mathcal{V}_e \cup \mathcal{P}_e$
$\mathcal{N}$	Set of all nodes, $\mathcal{J} \cup \mathcal{W}$
$\mathcal{P}_e$	Set of all pumps
$\mathcal{S}_e$	Set of all pipes
$\mathcal{V}_e$	Set of all valves

Parameters

β_a	Pump scaling factor representing the characteristics of pump(arc) a
λ_a	Friction factor in pipe(arc) a
$\overline{h_i}$	Upper bound on potential at a node i
a_{ij}	Arc from node i to j
d_i	Demand at node $i \in \mathcal{N}$
k_a	Roughness coefficient of pipe(arc) a
L_a	Length of pipe(arc) a
v_a^{max}	Maximum flow velocity allowed in a pipe(arc) a
$\underline{h_i}$	Lower bound on potential at a node i

Discrete variables

D_a	Diameter of pipe(arc) a
s_a	On/off status of a valve a

Continuous variables

h_i	Potential at a node i
hp_a	Non-negative variable for modeling pump(arc) a
q_a	Flow in a pipe (arc) a
w_a	Operating speed of pump(arc) a

1 Description

Assumptions :
Steady state.
Flow along the length of the pipe is constant (Potential flow coupling equation).
Friction factor doesn't depend on flow.
Pumps operate at a constant speed.

2 MINLP Formulation

Flow conservation at nodes :

Regular junctions :

$$\sum_{a\in\delta_i^+} q_a - \sum_{b\in\delta_i^-} q_b = d_i, \quad \forall i \in \mathcal{J}$$

Reservoirs/Tanks :

$$D_w^{min} - d_w^{current} \leq \sum_{a\in\delta_w^+} q_a - \sum_{b\in\delta_w^-} q_b \leq D_w^{max} - d_w^{current}, \quad \forall w \in \mathcal{W}$$

$$D^{min} \leq d_w^{current} \leq D^{max}, \quad \forall w \in \mathcal{W}$$

Flow bounds :

$$-\frac{\pi}{4} v_a^{max} D_a^2 \leq q_a \leq \frac{\pi}{4} v_a^{max} D_a^2, \quad \forall a \in \mathcal{S}_e$$

Potential bounds :

Regular junctions :

$$h_i \geq H_i \quad \forall i \in \mathcal{J}$$

Water Sources :

$$h_w = H_w, \quad \forall w \in \mathcal{W}$$

Potential-Flow Coupling :

$$(y_a^+ - y_a^-)(h_i - h_j) = \lambda_a . q_a^2, \quad \forall i \in \mathcal{N}, a = a_{ij} \in \delta_i^+$$

$$\lambda_a = \frac{8.L_a}{\pi^2.g.D_a^5}.f_a$$

Bi-directional flow :

$$(y_a^+ - 1)\sum_{k\in\mathcal{I}} d_k \leq q_a \leq (1 - y_a^-)\sum_{k\in\mathcal{I}} d_k$$

$$(1 - y_a^+)(\underline{h_i} - \overline{h_j}) \leq h_i - h_j \leq (1 - y_a^-)(\overline{h_i} - \underline{h_j})$$

$$y_a^+ + y_a^- = 1$$

Gate Valves (Bi-directional) :

$$-s_a.\frac{\pi}{4} v_a^{max} D_a^2 \leq q_a \leq s_a.\frac{\pi}{4} v_a^{max} D_a^2, \quad \forall a \in \mathcal{V}_e$$

$$(\underline{h_i} - \overline{h_j})(1 - s_a) \leq h_i - h_j \leq (\overline{h_i} - \underline{h_j})(1 - s_a)$$

Uni-directional pump (constant speed) :

$$y_a^+ Q_p^{min} \leq q_a \leq y_a^+ Q_p^{max} \quad \forall$$

$$y_a^+(h_i - h_j) = \alpha_a q_a |q_a| - \beta_a hp_a$$

$$hp_a \geq 0 \quad \forall a \in \mathcal{P}_e$$

3 Convex Relaxation

$$\gamma_a = (y_a^+ - y_a^-)(h_i - h_j) \quad \forall a \in \mathcal{S}_e$$
$$\zeta_a = y_a^+(h_i - h_j) \quad \forall a \in \mathcal{P}_e$$

<u>*McCormick relaxation*</u> :

$$\gamma_a \geq h_j - h_i + (\underline{h_i} - \overline{h_j})(y_a^+ - y_a^- + 1)$$
$$\gamma_a \geq h_i - h_j + (\overline{h_i} - \underline{h_j})(y_a^+ - y_a^- - 1)$$
$$\gamma_a \leq h_j - h_i + (\overline{h_i} - \underline{h_j})(y_a^+ - y_a^- + 1)$$
$$\gamma_a \leq h_i - h_j + (\underline{h_i} - \overline{h_j})(y_a^+ - y_a^- - 1)$$

$$\zeta_a \geq h_j - h_i + (\underline{h_i} - \overline{h_j})(y_a^+ + 1)$$
$$\zeta_a \geq h_i - h_j + (\overline{h_i} - \underline{h_j})(y_a^+ - 1)$$
$$\zeta_a \leq h_j - h_i + (\overline{h_i} - \underline{h_j})(y_a^+ + 1)$$
$$\zeta_a \leq h_i - h_j + (\underline{h_i} - \overline{h_j})(y_a^+ - 1)$$

<u>**Flow conservation at nodes**</u> :

Regular junctions :

$$\sum_{a \in \delta_i^+} q_a - \sum_{b \in \delta_i^-} q_b = d_i, \quad \forall i \in \mathcal{J}$$

Reservoirs/Tanks :

$$D_w^{min} \leq \sum_{a \in \delta_w^+} q_a - \sum_{b \in \delta_w^-} q_b \leq D_w^{max}, \quad \forall w \in \mathcal{W}$$

<u>**Flow bounds**</u> :

$$-\frac{\pi}{4} v_a^{max} D_a^2 \leq q_a \leq \frac{\pi}{4} v_a^{max} D_a^2, \quad \forall a \in \mathcal{S}_e$$

<u>**Potential bounds**</u> :

Regular junctions :

$$h_i \geq H_i \quad \forall i \in \mathcal{J}$$

Water Sources :

$$h_w = H_w, \quad \forall w \in \mathcal{W}$$

<u>**Potential-Flow Coupling**</u> :

$$\gamma_a \geq \lambda_a . q_a^2, \quad \forall i \in \mathcal{N}, a = a_{ij} \in \delta_i^+$$
$$\lambda_a = \frac{8.L_a}{\pi^2 . g . D_a^5} . f_a$$

<u>**Bi-directional flow**</u> :

$$(y_a^+ - 1) \sum_{k \in \mathcal{I}} d_k \leq q_a \leq (1 - y_a^-) \sum_{k \in \mathcal{I}} d_k$$
$$(1 - y_a^+)(\underline{h_i} - \overline{h_j}) \leq h_i - h_j \leq (1 - y_a^-)(\overline{h_i} - \underline{h_j})$$
$$y_a^+ + y_a^- = 1$$

Gate Valves (Bi-directional) :

$$-s_a.\frac{\pi}{4}v_a^{max}D_a^2 \leq q_a \leq s_a.\frac{\pi}{4}v_a^{max}D_a^2, \quad \forall a \in \mathcal{V}_e$$

$$(\underline{h_i} - \overline{h_j})(1 - s_a) \leq h_i - h_j \leq (\overline{h_i} - \underline{h_j})(1 - s_a)$$

Uni-directional pump (constant speed) :

$$y_a^+ Q_p^{min} \leq q_a \leq y_a^+ Q_p^{max} \quad \forall a \in \mathcal{P}_e$$

$$\zeta \geq \alpha_a q_a |q_a|^{\eta_a} - \beta_a hp_a$$

$$hp_a \geq 0 \quad \forall a \in \mathcal{P}_e$$