Hohete Tibeb Share Company
Education and Technology Team
Ethio-Parents' Schools: Addis Ababa (Gerji/Gullele) and Hawassa.
Work Sheet For Grade 12

1. Show that the following function is a bijective or not.
   a) $f: \mathbb{Z} \to \mathbb{Z}$, defined by $f(n) = 9n + 1 \ \ for\ all\ n \in \mathbb{Z}$.
   b) $f: \mathbb{Z} \to \mathbb{Z}$, defined by $f(n) = 3^n \ \ for\ all\ n \in \mathbb{Z}$.
2. Find the inverse of the given functions, if they exist.
   a) $\ln(x + 3)$
   b) $\frac{1+e^x}{1-e^x}$
3. Let $f(x) = 3 + x^2 + \tan\left(\frac{\pi}{2}x\right)$, where $-1 < x < 1$, find
   a) $f^{-1}(3)$
   b) $f(f^{-1}(5))$
4. Solve the following
   a) $(x^2 - 9)e = (x^2 - 9)e^{7-x}$
   b) $2\ln(\sqrt{x}) - \ln(1 - x) = 1$
5. Prove the following trigonometric identities.
   a) $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$
   b) $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$
6. Use the $\varepsilon - \delta$ definition to prove that :
   a) $\lim_{x\to 4}(x^2 + x - 11) = 9$
   b) $\lim_{x\to 9}\sqrt{x} = 3$
7. Evaluate the following limit
   a) $\lim_{x\to 0}\frac{1-\cos x}{x}$
   b) $\lim_{x\to\infty}\frac{\ln x}{\sqrt[3]{x}}$

8. Evaluate the following integral

   a) $\int (x^2+1)^4(2x)dx$

   b) $\int \frac{\ln x + x^2 e^x}{x} dx$

   c) $\int \frac{x^2+2x-1}{2x^3+3x^2-2x} dx$

   d) $\int \sin^5 x \cos^2 x \ dx$

9. Find the vertical asymptotes (if any) of the following functions.

   a) $f(x) = \sec \pi x$

   b) $f(x) = \frac{x^2}{x^2-4}$

10. Determine the intervals on which the function is continuous.

   a) $f(x) = \begin{cases} 5-x, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$

   b) $f(x) = \csc \frac{\pi x}{2}$

11. Find the *x – values* (if any) at which a function is not continuous. Which of the discontinuous are removable?

   a) $f(x) = \frac{x}{x^2-1}$

   b) $f(x) = 3x - \cos x$

12. Explain why the function has a zero (or a root) in the given interval.

   a) $f(x) = x^2 - 2 - \cos x \qquad [0, \ \pi]$

   b) $f(x) = \frac{-5}{x} + \tan(\frac{\pi x}{10}) \qquad [1, \ 4]$

13. Use the formal definition of derivatives to prove that :

   c) $\frac{d}{dx}[Cx^n] = C(nx^{n-1}), \ where \ C \ is \ a \ constant.$

   d) $\frac{d}{dx}[\cot x] = -\csc^2 x$

14. Prove that if f is differentiable at$x = c$, then f is continuous at $x = c$.

15. Identify whether the statement describes inferential statistics or descriptive statistics:

   a) The average age of the students in a mathematics class is 18 years.
   b) The chances of winning the national Lottery are one chance in twenty-two million.
   c) There is a relationship between smoking cigarettes and getting lung cancer.

16. Let $S = \{1,2,4,5,6,7,9\}$ and suppose a 4-digit number is formed by selecting digits, without replacement for S.
    a) How many different such numbers can be formed?
    b) How many different such numbers can be formed if both even & odd digits must be used?
17. A club has 8-girls & 6-boys. A basket ball team of 5-members is to be selected.
    a) In how many ways can the selection be made?
    b) In how many ways can the selection be made both so that team contains both boys & girls?
18. Consider the binomial expansion$(2a + b)^5$.
    a) The first two terms of the expansion
    b) The last two terms of the expansion
    c) The third & fourth terms of the expansion
    d) The coefficient of a^5 is the expansion

19. A bag contains seven black & five white balls. Four balls are drawn at random. What is the probability that
    a) all of them are black?
    b) two of them are black & two are white?