Real Time Control of a Quadcopter

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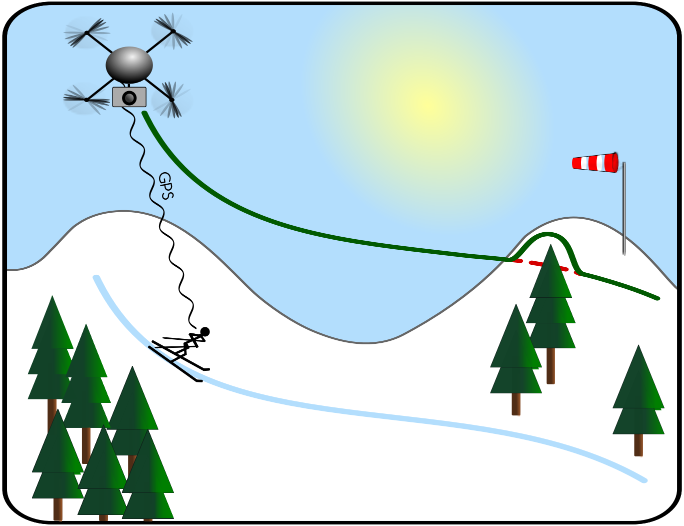
Motivation



Optimal Control Formulation

$$\min_{x,u} J(x,u) \qquad \text{s.t.} \qquad \begin{aligned} \tilde{h}(x,u) &= 0 \\ \dot{x} &= f(x,u) \end{aligned}$$

x : state

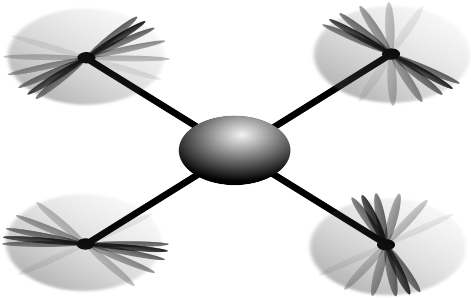
u : control

Optimal Control Formulation

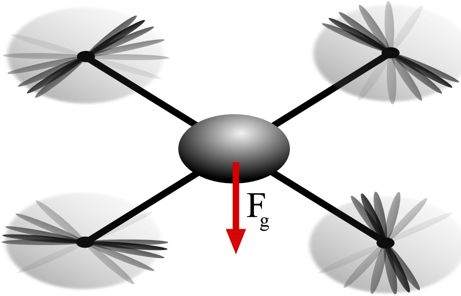
$$\min_{x,u} J(x,u) \qquad \text{s.t.} \qquad \left.\begin{array}{r} \tilde{h}(x,u) = 0 \\ \dot{x} = f(x,u) \end{array}\right\} \Rightarrow h(x,u) = 0$$

$$\begin{aligned} x &: \quad \text{state} \\ u &: \quad \text{control} \end{aligned}$$

Model



Forces



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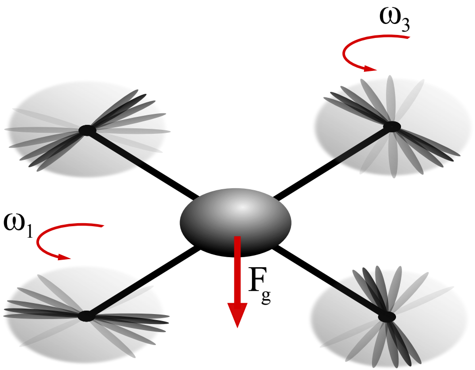

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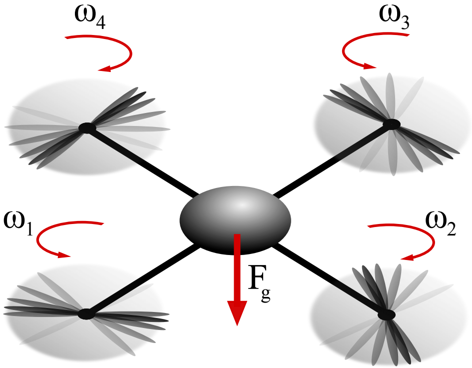

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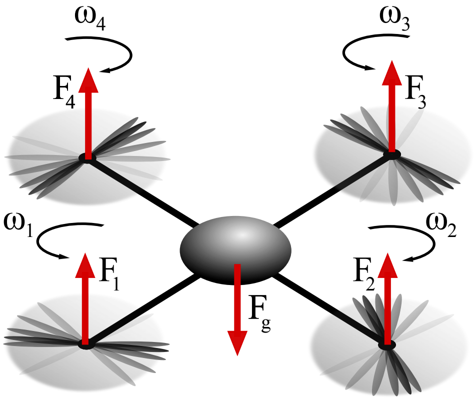

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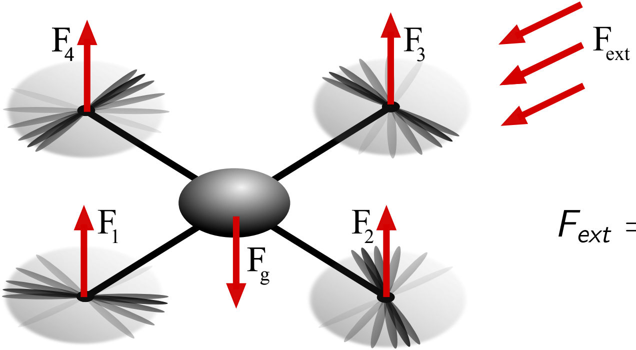
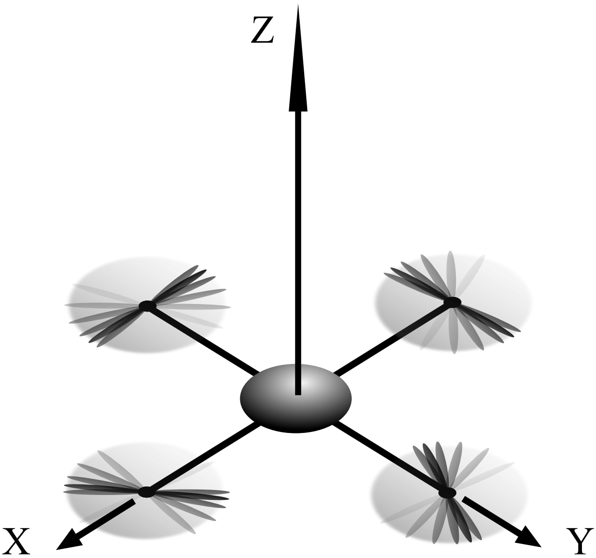
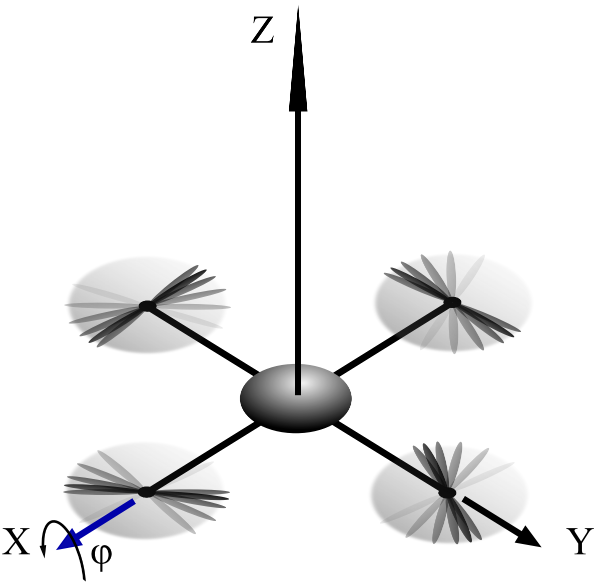


$$F_{ext} = F_g + \sum_{i=1}^{4} F_i$$

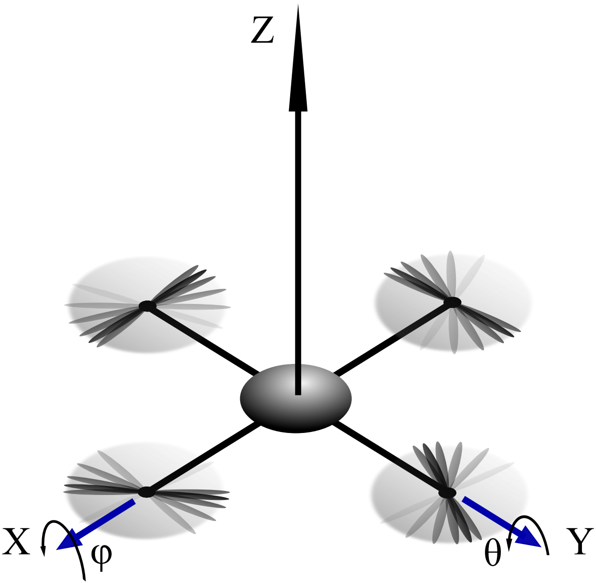
Torques



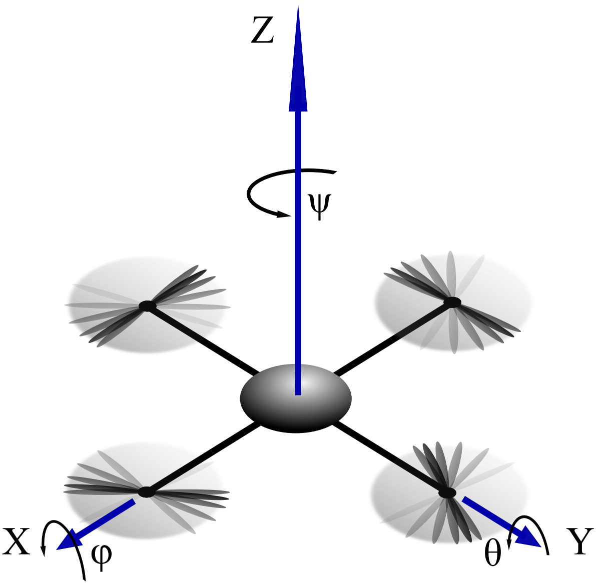
Torques



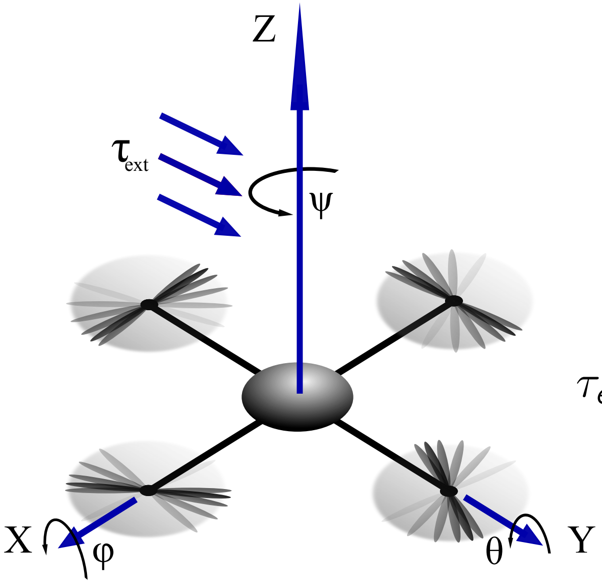
Torques



Torques



Torques

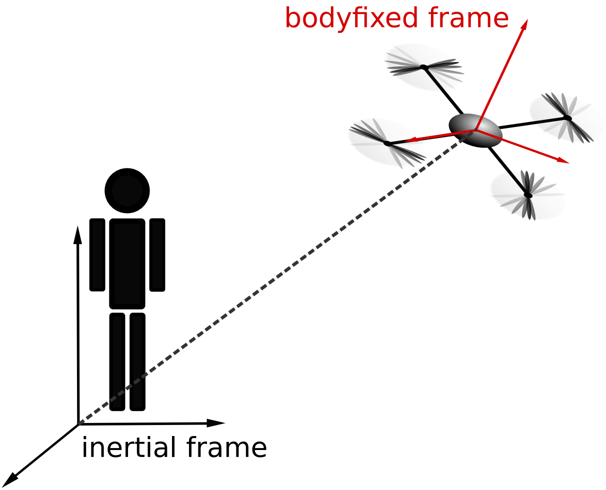


$$\tau_{ext} = \tau_{\psi} + \tau_{\varphi} + \tau_{\theta}$$

Obtain ODE

$$\left.\begin{array}{l} F_{ext} = F_g + \sum_{i=1}^{4} F_i \\ \tau_{ext} = \tau_\psi + \tau_\varphi + \tau_\theta \end{array}\right\} \quad \Rightarrow \quad \dot{x} = f(x, u)$$

Coordinate Systems



Quaternions

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

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$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

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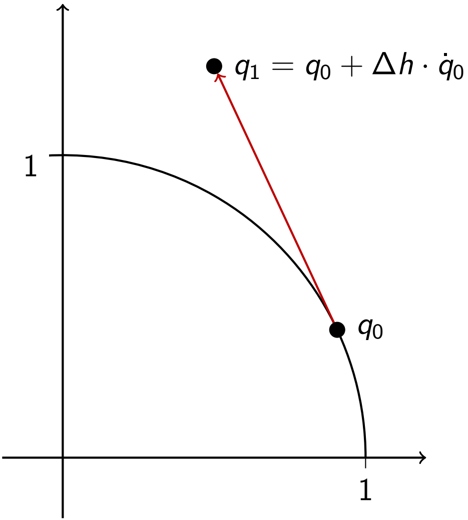
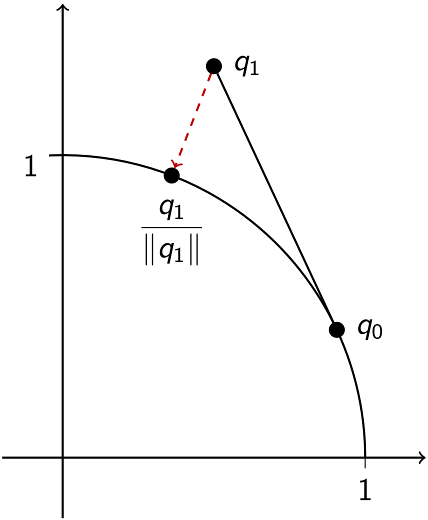
represent rotation $\quad \Leftrightarrow \quad \|q\| = 1 \quad \Leftrightarrow \quad q \in \mathcal{S}^3$

Drift Correction



$$\dot{q} = \tilde{f}(q)$$

Drift Correction



$$\dot{q} = \tilde{f}(q) - \lambda(q)$$

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