Real Time Control of a Quadcopter

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Technische Universität München

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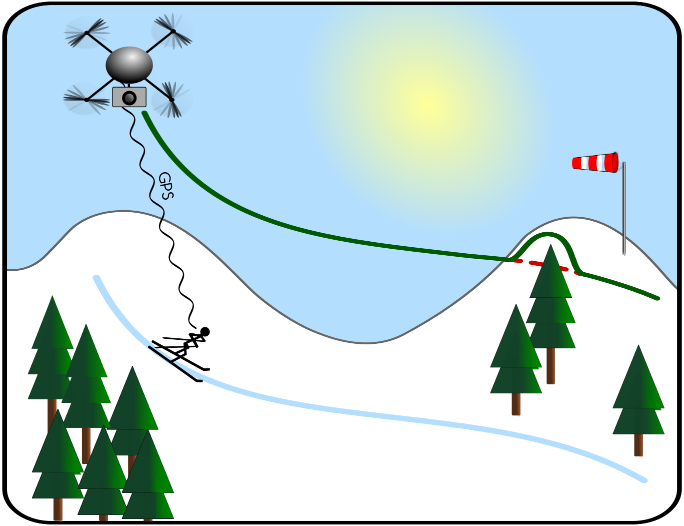
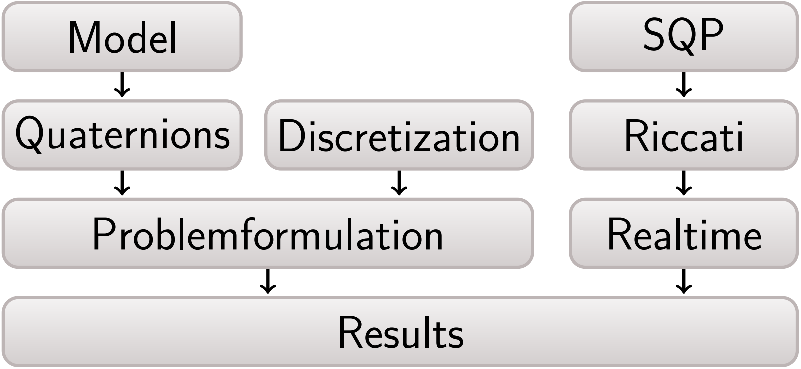
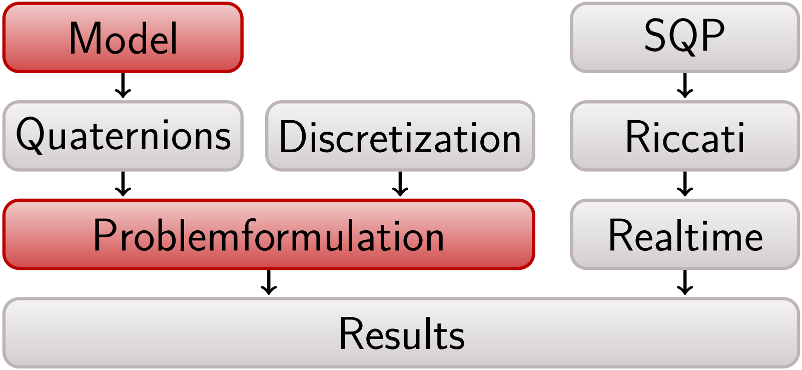
Motivation




Model
SQP
Quaternions
Discretization
Riccati
Problemformulation
Realtime
Results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Optimal Control Formulation

$$\min_{x,u} J(x,u) \quad \text{s.t.} \quad \begin{aligned} \tilde{h}(x,u) &= 0 \\ \dot{x}(t) &= f(x(t), u(t)) \end{aligned}$$

x : state

u : control

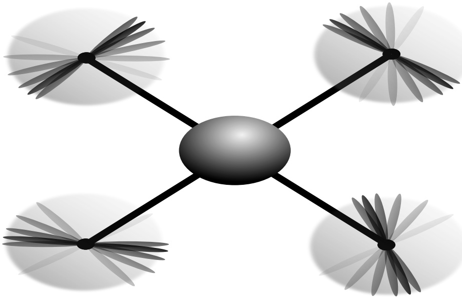
Optimal Control Formulation

$$\min_{x,u} J(x,u) \quad \text{s.t.} \quad \left.\begin{aligned} \tilde{h}(x,u) = 0 \\ \dot{x}(t) = f(x(t),u(t)) \end{aligned}\right\} \Rightarrow h(x,u) = 0$$

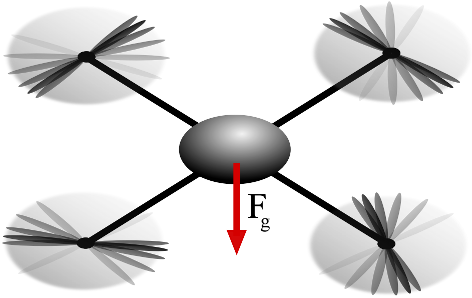
x : state

u : control

Model



Model



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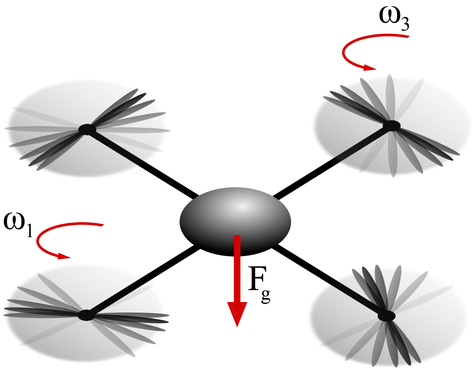
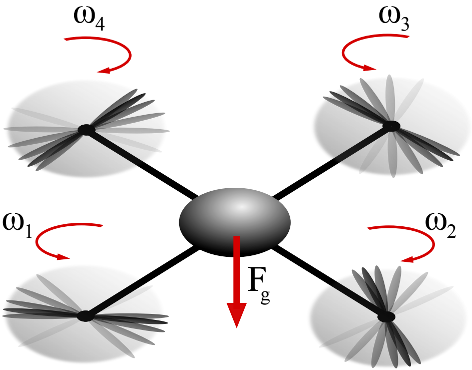
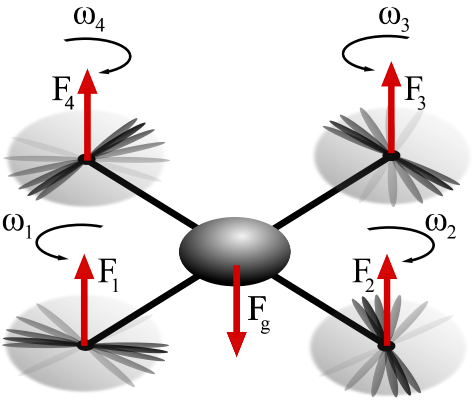


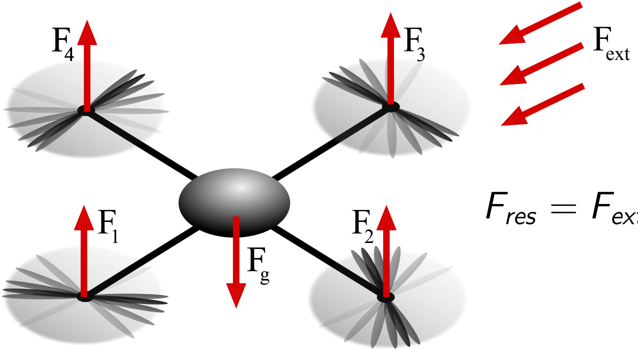
Model



Model

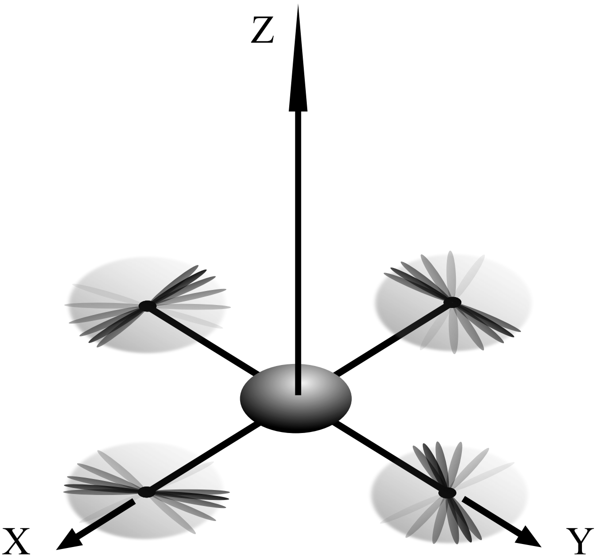


Forces

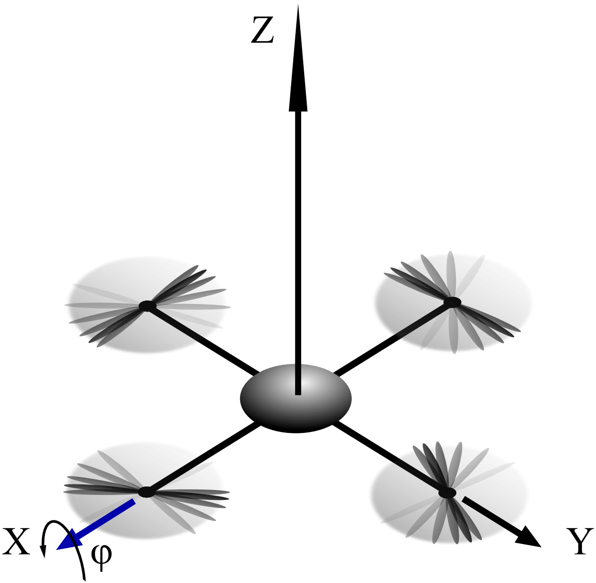


$$F_{res} = F_{ext} + F_g + \sum_{i=1}^{4} F_i$$

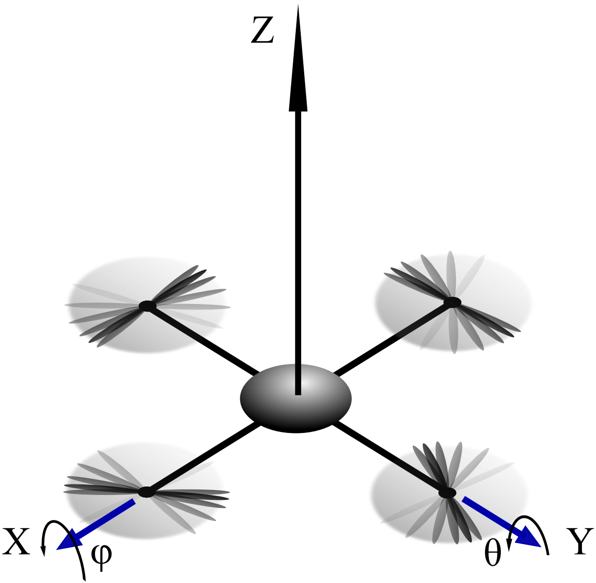
Torques



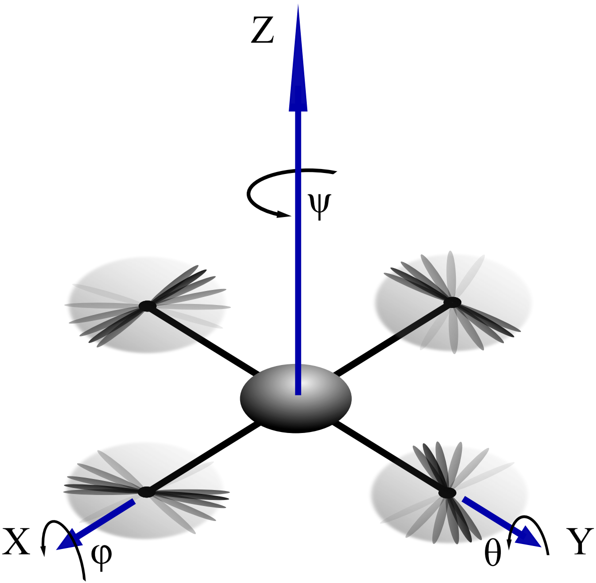
Torques



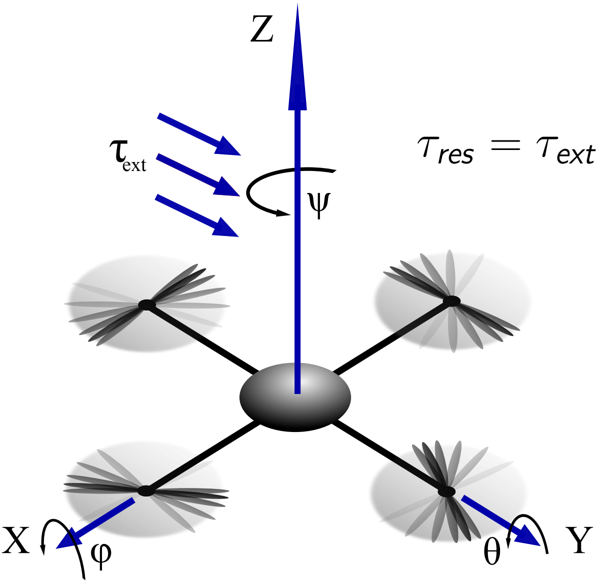
Torques



Torques



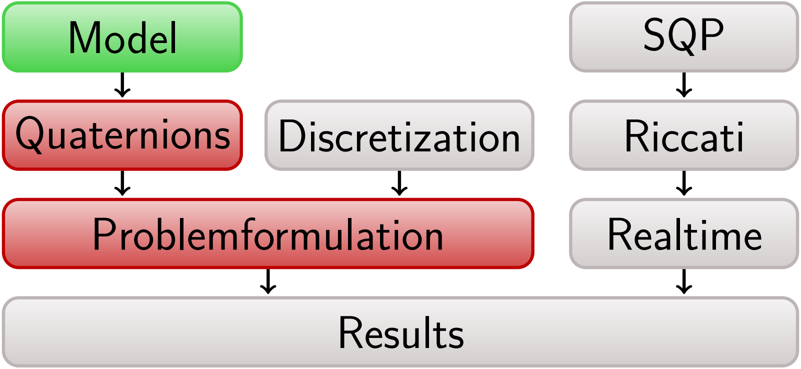
Torques



$$\tau_{res} = \tau_{ext} + \tau_{\psi} + \tau_{\varphi} + \tau_{\theta}$$

Obtain ODE

$$\left.\begin{aligned} F_{res} &= F_{ext} + F_g + \sum_{i=1}^{4} F_i \\ \tau_{res} &= \tau_{ext} + \tau_\psi + \tau_\varphi + \tau_\theta \end{aligned}\right\} \quad \Rightarrow \quad \dot{x}(t) = f(x(t), u(t))$$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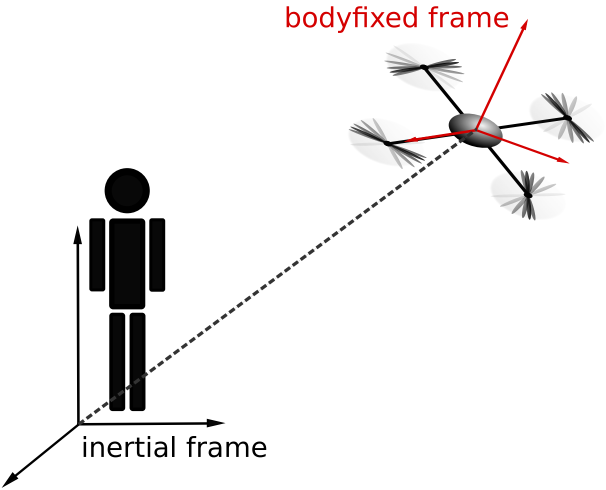
Coordinate Systems



Quaternions

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

$$\Leftrightarrow$$

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

Quaternions

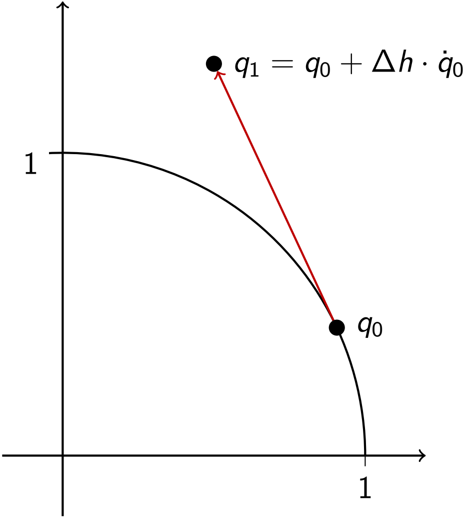
$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

$$\Leftrightarrow$$

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

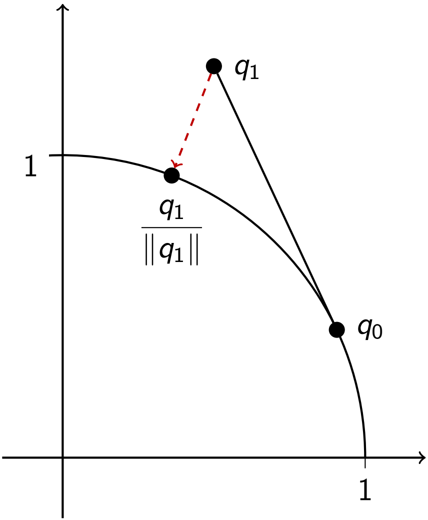
represent rotation $\quad \Leftrightarrow \quad \|q\| = 1 \quad \Leftrightarrow \quad q \in \mathcal{S}^3$

Drift Correction

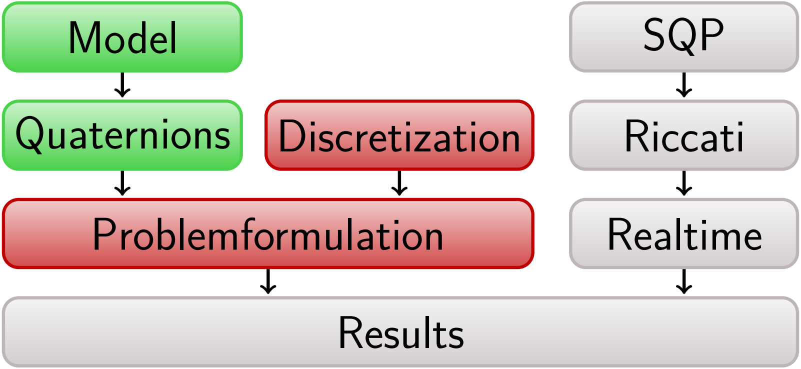


$$\dot{q}(t) = \tilde{f}(q(t))$$

Drift Correction



$$\dot{q}(t) = \tilde{f}(q(t)) - \lambda(q(t))$$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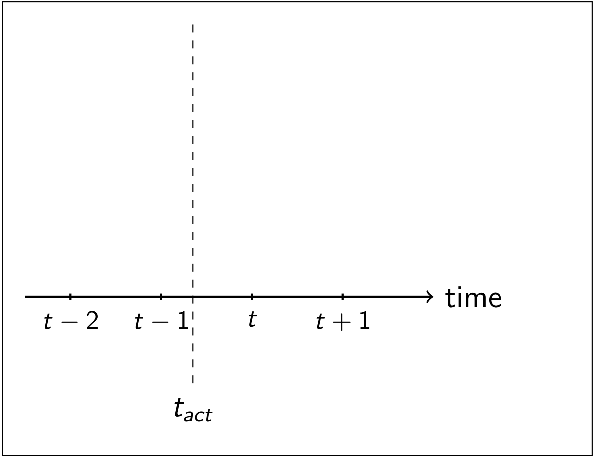
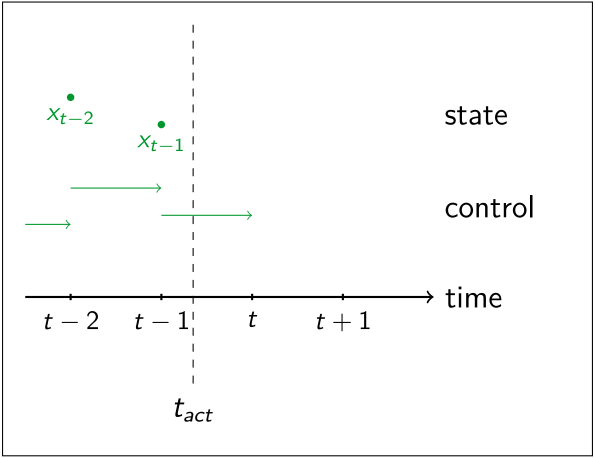
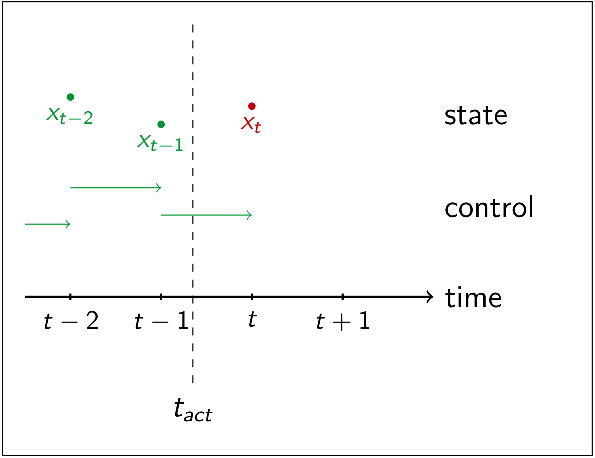
Setting



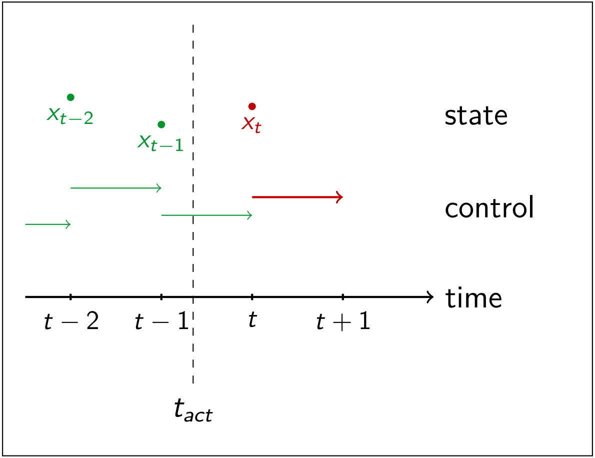
Setting



Setting



Setting

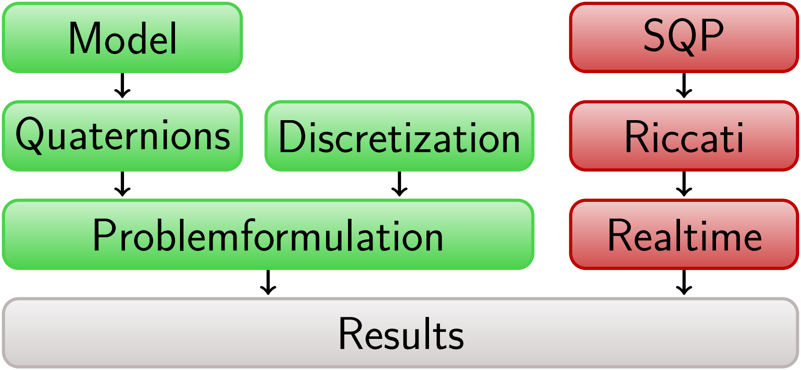


Discrete Problem

$$\min_{x,u} \sum_{i=t}^{N} J_i(x_i, u_i) \qquad \text{s.t.} \quad h_i(x_i, u_i) = 0 \quad i = t, \ldots, N$$

$J_i(x_i, u_i)$ discretized goal function

$h_i(x_i, u_i)$ equality constraints at time i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The Lagrangian

$$L(y) = \sum_{i=t}^{N} J_i(x_i, u_i) + \sum_{i=t}^{N} \lambda_i^T h_i(x_i, u_i)$$

$$y := (\lambda, x, u)$$

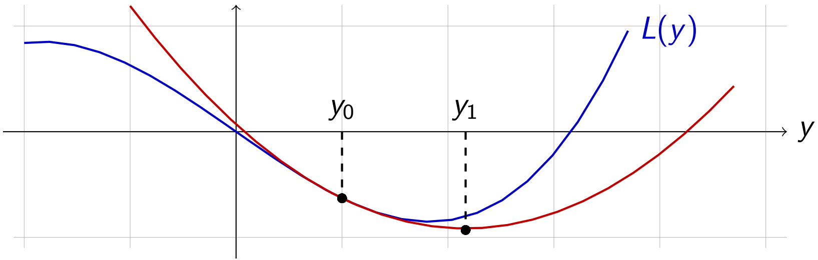
$$y^* \text{ optimal} \quad \Leftrightarrow \quad \nabla_y L(y^*) = 0$$

The SQP Method

Find y^*:

$$y_1 = y_0 + s$$

$$\min_s \frac{1}{2} s^T \nabla^2 L(y_0) s + \nabla L(y_0)^T s$$



Quasi Newton-Method

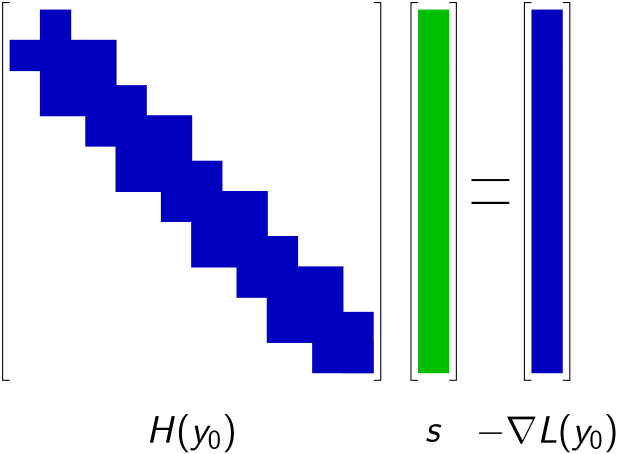
Find s with:

$$\nabla L(y_0) + \nabla^2 L(y_0)s = 0$$

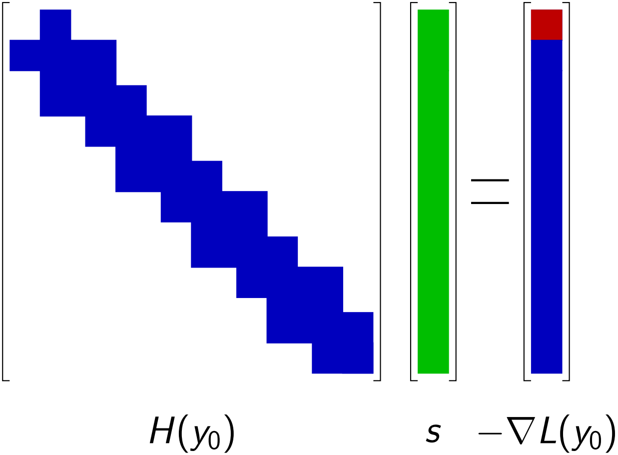
Approximate $\nabla^2 L(y_0)$ and solve:

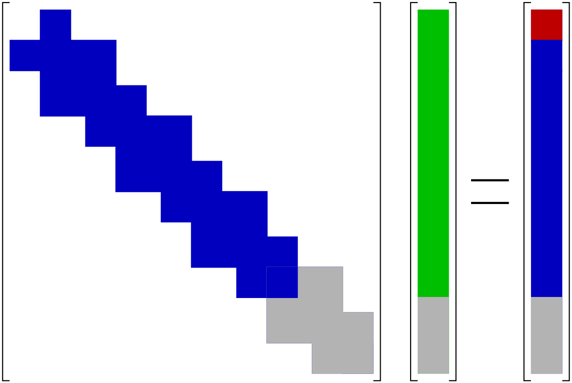
$$H(y_0)s = -\nabla L(y_0)$$

Riccati Recursion



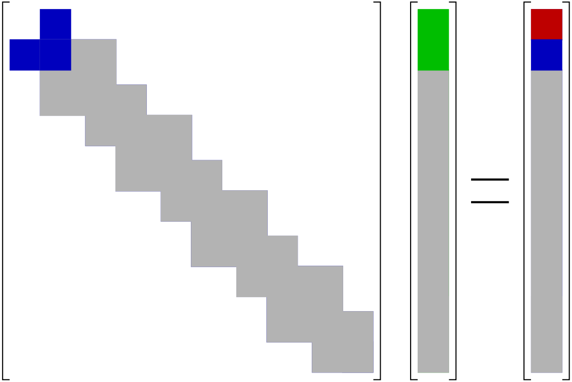
Riccati Recursion





$H(y_0) \quad s \quad -\nabla L(y_0)$

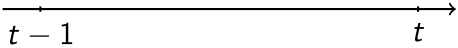
Riccati Recursion



$H(y_0)$ $\quad s \quad$ $-\nabla L(y_0)$

Summary

What happens in interval $[t-1, t]$?



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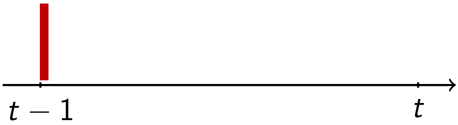
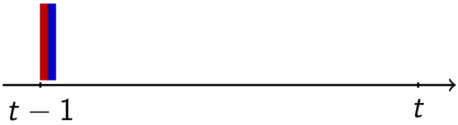


1. calculate control u_{t-1} (Riccati Part II)

Summary

What happens in interval $[t-1, t]$?



1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)

Summary

What happens in interval $[t-1, t]$?

1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)
3. prepare u_t (Newton & Riccati Part I)

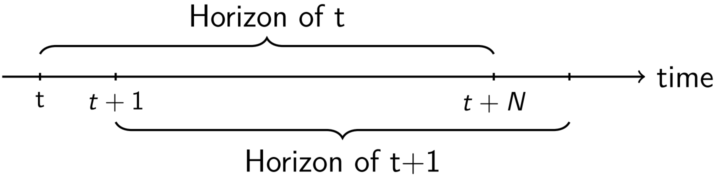
Summary

What happens in interval $[t-1, t]$?

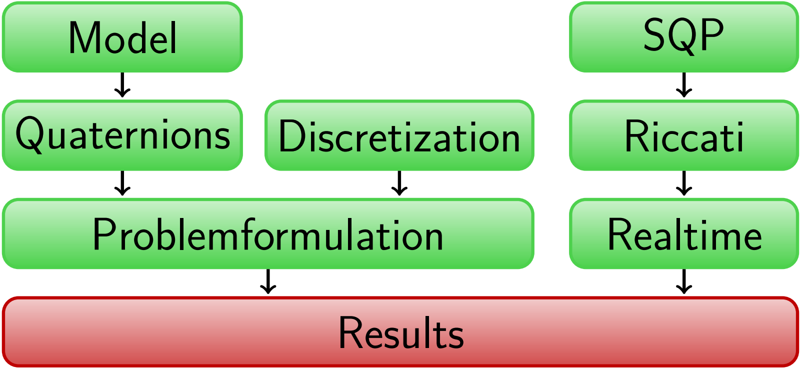
1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)
3. prepare u_t (Newton & Riccati Part I)

$\Rightarrow$ with horizon 15 this is 25% faster.

Finite Horizon



	runtime	error
$N = 20$		
$N = 50$		
$N = 100$		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References I

Stephen Boyd.
Solving the lqr problem by block elimination.
Lecture notes, 2009.

James Diebel.
Representing attitude: Euler angles, unit quaternions, and rotation vectors.
10 2006.

Moritz Diehl, Hans Georg Bock, and Johannes P. Schlöder.
A real-time iteration scheme for nonlinear optimization in optimal feedback control.
SIAM J. Control Optim., 2005.

References II

Moritz Diehl, Bock H. Georg, Johannes P. Schlöder, Rolf Findeisen, Zoltan Nagy, and Frank Allgöwer.
Real-time optimization and nonlinear model predictive control of processes governed by differential-algebraic equations.
Journal of Process Control, 2002.

Moritz Mathias Diehl.
Real-Time Optimization for Large Scale Nonlinear Processes.
PhD thesis, Ruprecht-Karls-Universität Heidelberg, 2001.

Luis Rodolfo Garcia Carrillo, Alejandro Enrique Dzul Lopez, Rogelio Lozano, and Claude Pegard.
Quad Rotorcraft Control.
Springer-Verlag London, 2013.