# Midterm presentation

The Quadrocopters

Technische Universität München

26. Mai 2015

# Overview

1. Motivation
2. Model
3. Realtime Optimization Approach

# Optimal Control Problem

$$\min_{x,u} J(x,u) \qquad \dot{x} = f(x,u)$$

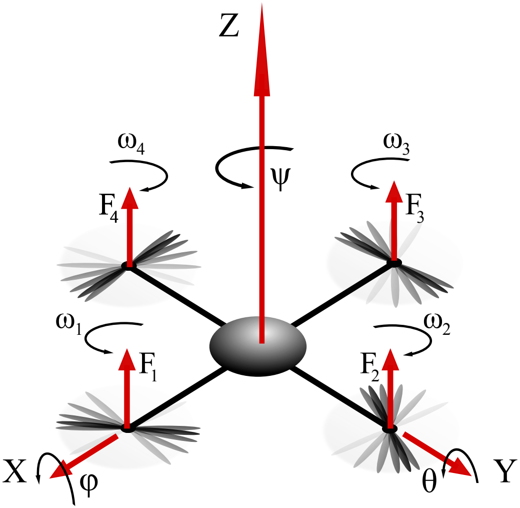
$$x: \ \text{state}$$

$$u: \ \text{control}$$

$\rightarrow$ additional difficulty: realtime approach

# Model

# Forces and Torques



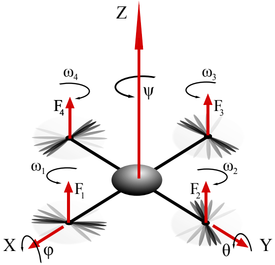
# Newton-Euler Equations

Forces

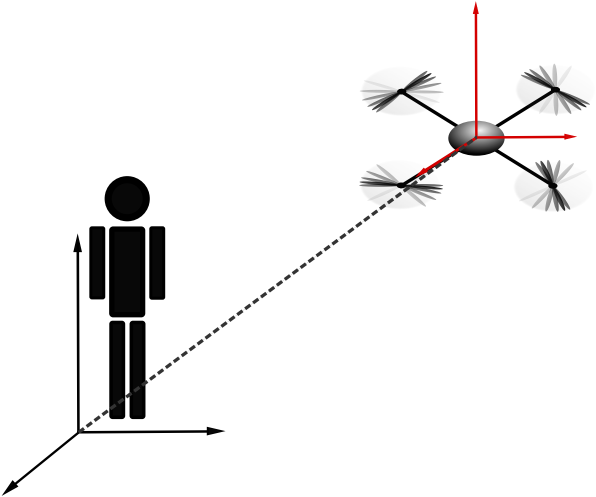
$$F_{ext} = F_g + \sum_{i=1}^{4} F_i$$

Torques

$$\tau_{ext} = \sum_{i=1}^{4} \tau_i + (\tau_\varphi + \tau_\theta)$$



# Coordinate Systems



# Quaternions

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

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problem $\rightarrow$ $\|q\| = 1$ additional constraint

# Dynamics

Equations representing dynamics...

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$$T(x,u) = M \cdot \begin{pmatrix} \dot{x}_8 \\ \vdots \\ \dot{x}_{13} \end{pmatrix} + \Theta(x)$$

...expressed as system of differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} x_1 \\ \vdots \\ x_7 \\ x_8 \\ \vdots \\ x_{13} \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_7 \\ M^{-1}(T(x,u) - \Theta(x)) \end{pmatrix}$$

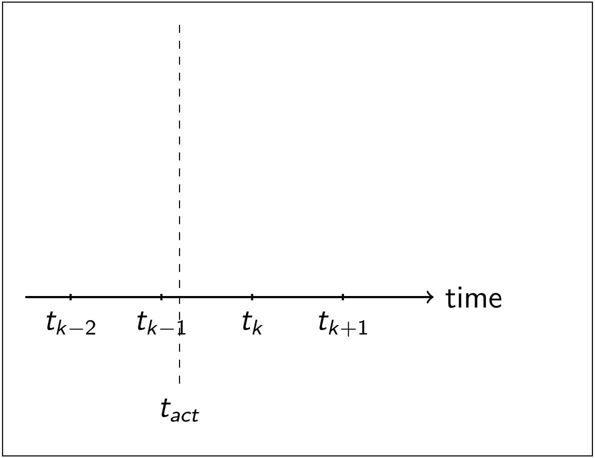
# Prospect

Refinement of the model $\rightarrow$ wind

$\rightarrow$ aerodynamical forces

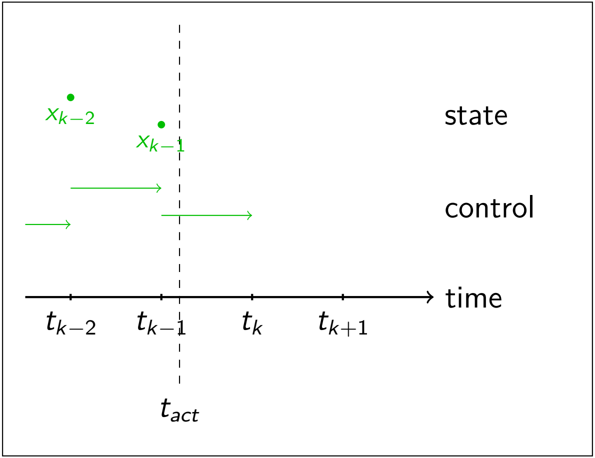
# Realtime Optimization Approach

# Setting




$y, s, q$ erklären

# Setting

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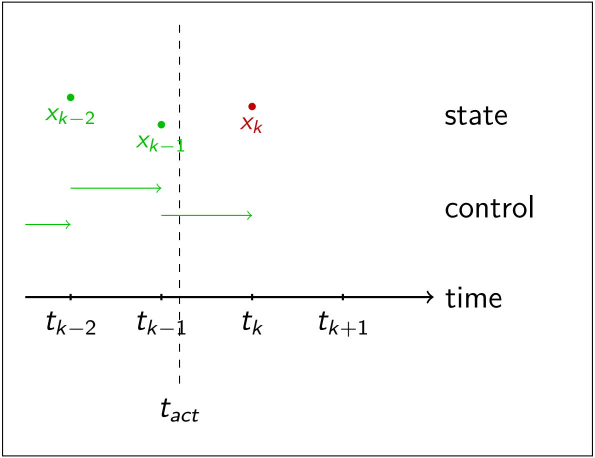


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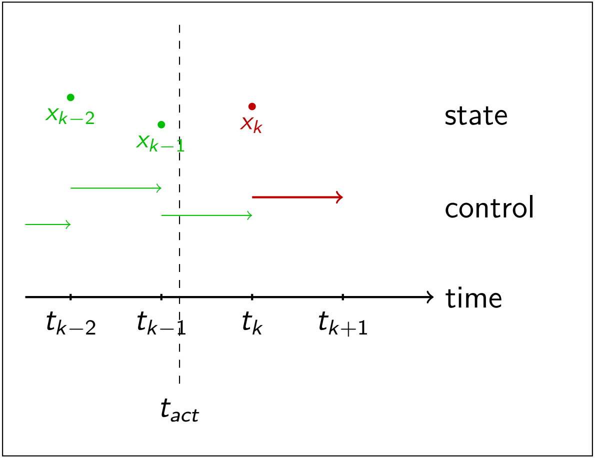


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# Minimization Problem

$$\min_{\substack{s_t,\dots,s_N\\ q_t,\dots,q_{N-1}}} \sum_{i=t}^{N-1} J_i(s_i, q_i) \quad s.t. \left\{ \begin{array}{c} x_t - s_t = 0 \\ h_i(s_i, q_i) - s_{i+1} = 0 \quad \forall i = t, \dots, N-1 \end{array} \right.$$

| | |
|---|---|
| $J_i(s_i, q_i)$ | discretized goal function |
| $x_t - s_t = 0$ | expected state should be the real state at time $t$ |
| $h_i(s_i, q_i)$ | solution of the ODE at time $i$ |

# The Lagrangian

$$L^t(y) = \sum_{i=t}^{N-1} J_i(s_i, q_i) + \lambda_t^T(x_t - s_t) + \sum_{i=t}^{N-1} \lambda_{i+1}^T(h_i(s_i, q_i) - s_{i+1})$$

We are looking for $y^*$ satisfying the KKT conditions:

$$\Rightarrow \nabla_y L^t(y^*) = 0$$

# The SQP Method

How to find $y^*$?

$$y_{k+1} = y_k + \alpha_k \Delta y_k$$

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$$\Downarrow$$

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$$\Downarrow$$

$$A_k := \nabla^2_{y_k} L(y_k).$$

# Newton-Raphson

$$y_{t+1} = y_t + \Delta y_t$$
$$\nabla_{y_t} L^t(y_t) + H^t(y_t)\Delta y_t = 0$$

$H^t(y_t)$ approximated Hessian $\nabla^2_{y_t} L(y_t)$

$\alpha_t = 1$

# Riccati Recursion

Approximated Hessian:

$$
H^t(y^t)=\begin{pmatrix}
 & -E & & & & & & & & & \\
-E & Q_t^H & M_t^H & A_t^T & & & & & & & \\
 & (M_t^T)^H & R_t^H & B_t^T & & & & & & & \\
 & A_t & B_t & & -E & & & & & & \\
 & & & -E & Q_{t+1}^H & M_{t+1}^H & A_{t+1}^T & & & & \\
 & & & & (M_{t+1}^T)^H & R_{t+1}^H & B_{t+1}^T & & & & \\
 & & & & A_{t+1} & B_{t+1} & & & & & \\
 & & & & & & & \ddots & & & \\
 & & & & & & \ddots & Q_{N-1}^H & M_{N-1}^H & A_{N-1}^T & \\
 & & & & & & & (M_{N-1}^T)^H & R_{N-1}^H & B_{N-1}^T & \\
 & & & & & & & A_{N-1} & B_{N-1} & & -E \\
 & & & & & & & & & -E & Q_N^H
\end{pmatrix}
$$

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What happens in interval $[t_{k-1}, t_k]$ ?

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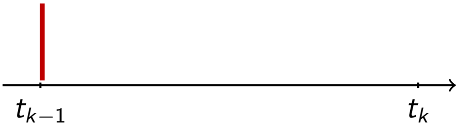


1. calculate control $u_{k-1}$ (Riccati Part II)

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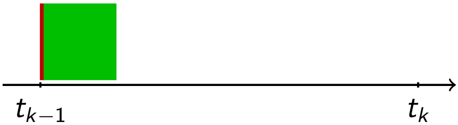


1. calculate control $u_{k-1}$ (Riccati Part II)
2. calculate $y_k$ (Riccati Part II)

# Summary

What happens in interval $[t_{k-1}, t_k]$ ?


1. calculate control $u_{k-1}$ (Riccati Part II)
2. calculate $y_k$ (Riccati Part II)
3. prepare $u_k$ (Newton & Riccati Part I)

# Finite Horizon

How to choose $N$?

- $N = t_{end} \rightarrow$ problem gets smaller every time
- $N = t + n \rightarrow$ problem size is constant
- .
- .
- .