Real Time Control of a Quadcopter

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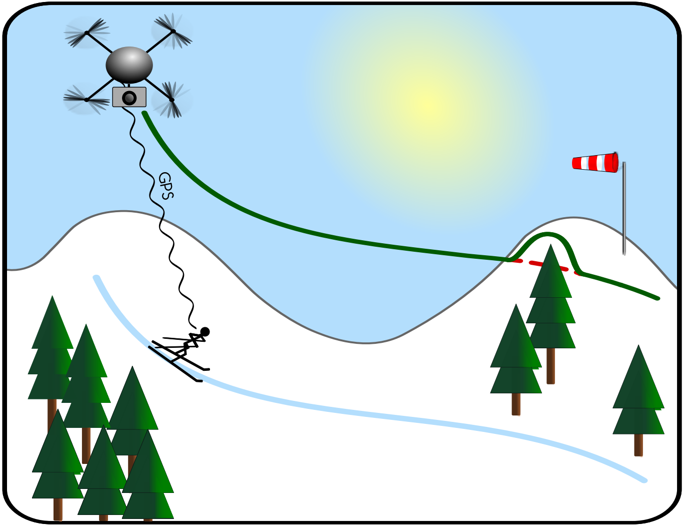
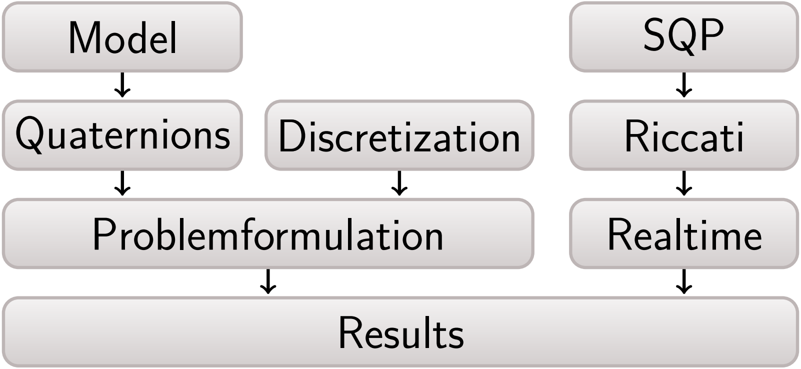
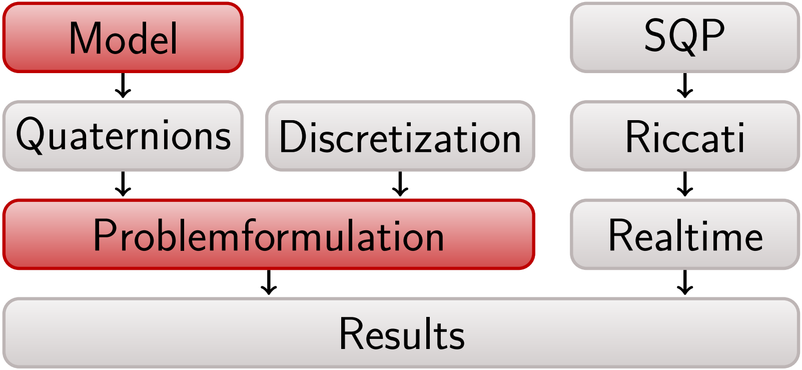
Technische Universität München

11 July 2015

Motivation




Model
SQP
Quaternions
Discretization
Riccati
Problemformulation
Realtime
Results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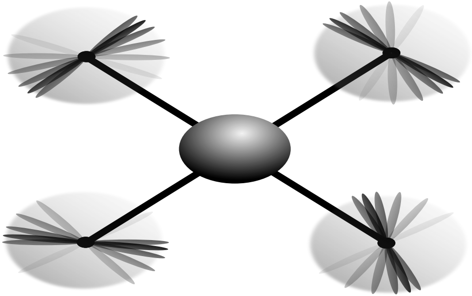
Optimal Control Formulation

$$\min_{x,u} J(x,u) \quad \text{s.t.} \quad \begin{aligned} \tilde{h}(x,u) &= 0 \\ \dot{x}(t) &= f(x(t), u(t)) \end{aligned}$$

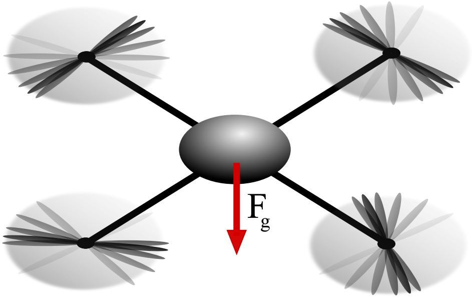
x : state

u : control

Model



Model



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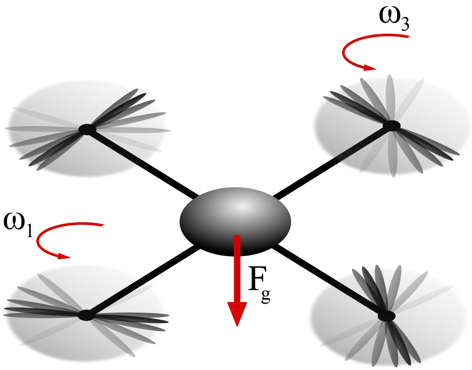
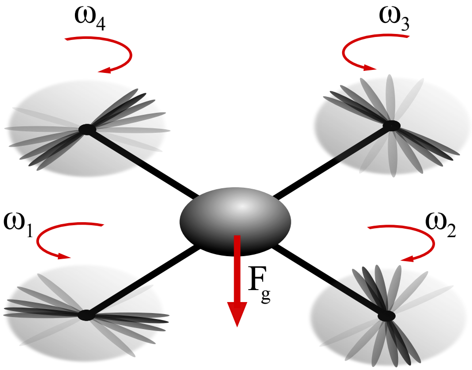
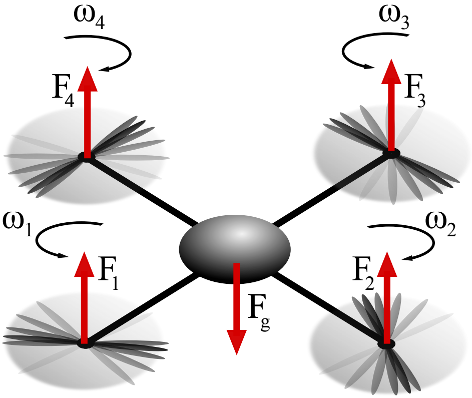


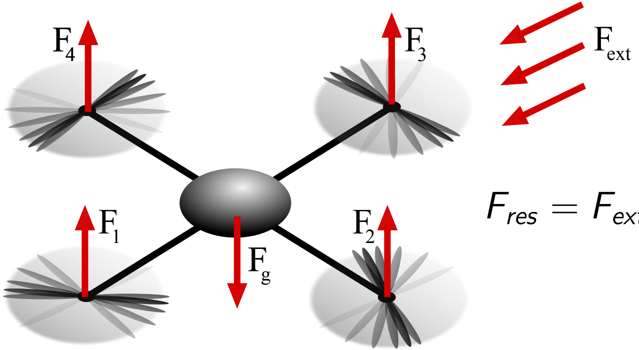
Model



Model

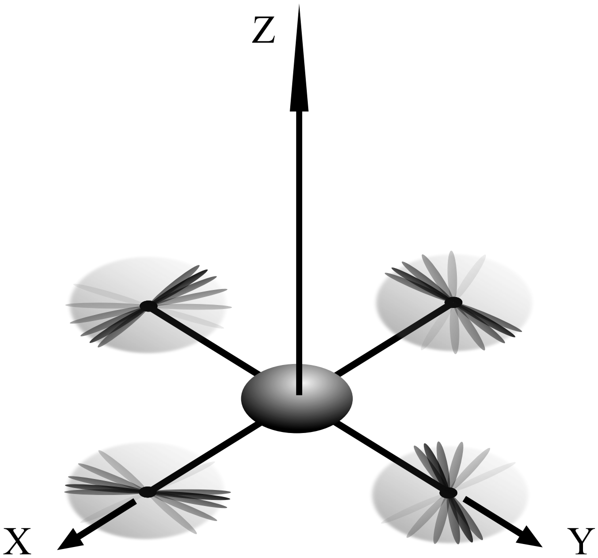


Forces

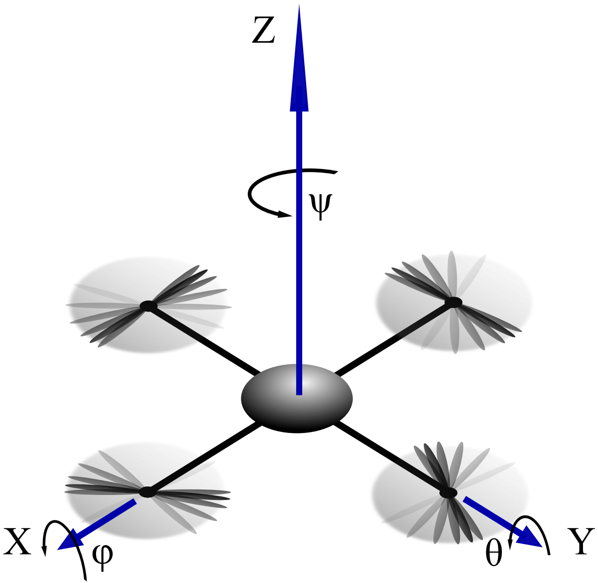


$$F_{res} = F_{ext} + F_g + \sum_{i=1}^{4} F_i$$

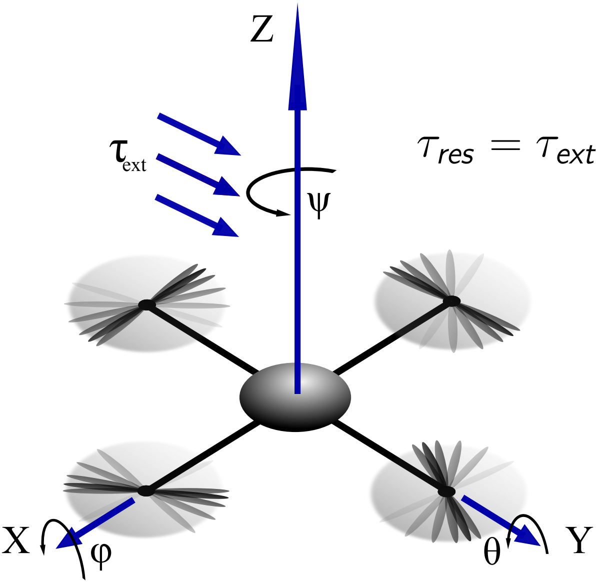
Torques



Torques



Torques



$$\tau_{res} = \tau_{ext} + \tau_{\psi} + \tau_{\varphi} + \tau_{\theta}$$

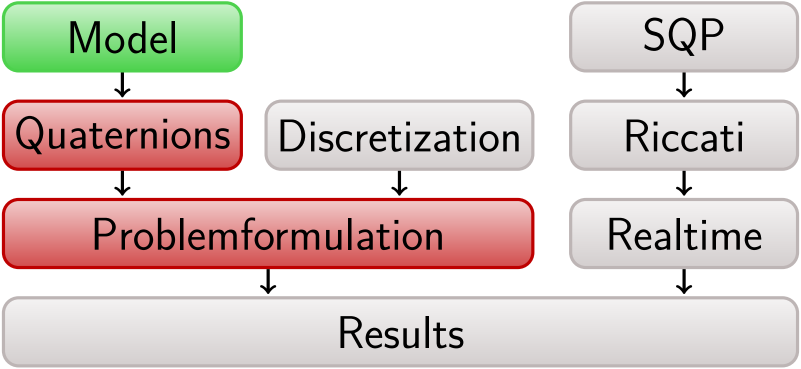
Obtain ODE

$$\left.\begin{aligned} F_{res} &= F_{ext} + F_g + \sum_{i=1}^{4} F_i \\ \tau_{res} &= \tau_{ext} + \tau_\psi + \tau_\varphi + \tau_\theta \end{aligned}\right\} \quad \Rightarrow \quad \dot{x}(t) = f(x(t), u(t))$$

Obtain ODE

$$\left.\begin{aligned} F_{res} &= F_{ext} + F_g + \textstyle\sum_{i=1}^{4} F_i \\ \tau_{res} &= \tau_{ext} + \tau_{\psi} + \tau_{\varphi} + \tau_{\theta} \end{aligned}\right\} \quad \Rightarrow \quad \dot{x}(t) = f(x(t), u(t))$$

$$\left.\begin{aligned} \tilde{h}(x, u) = 0 \\ \dot{x}(t) = f(x(t), u(t)) \end{aligned}\right\} \Rightarrow h(x, u) = 0$$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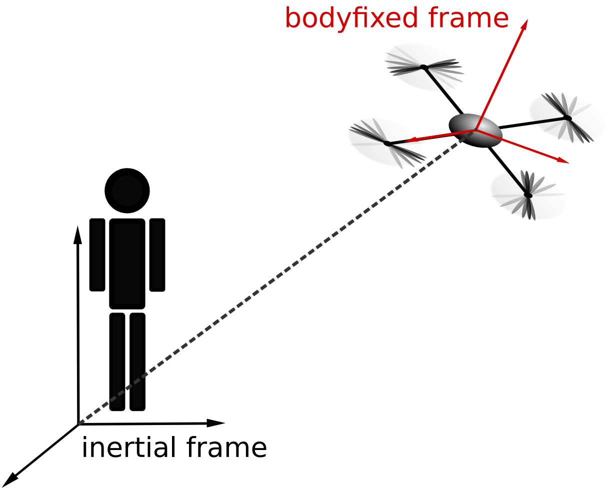
Copter Rotation



Quaternions

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

$$\Leftrightarrow$$

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

Quaternions

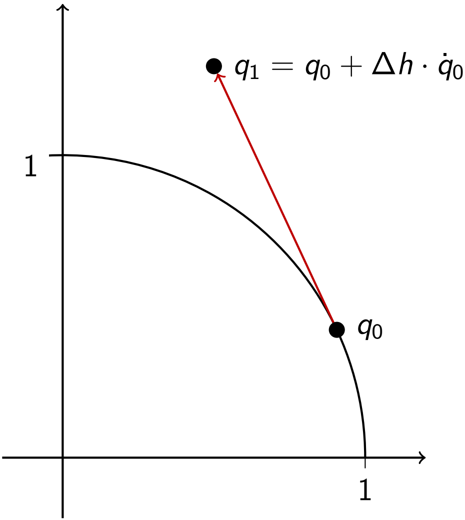
$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

$$\Leftrightarrow$$

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

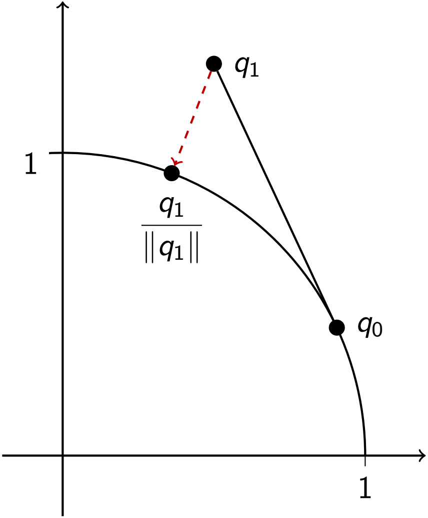
represent rotation $\quad \Leftrightarrow \quad \|q\| = 1 \quad \Leftrightarrow \quad q \in \mathcal{S}^3$

Drift Correction

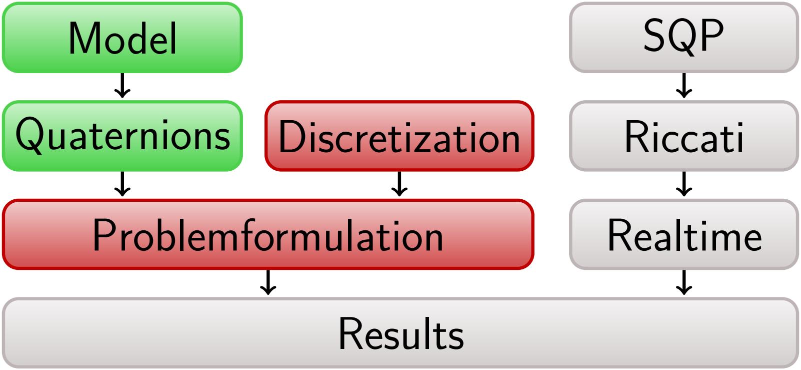


$$\dot{q}(t) = \tilde{f}(q(t))$$

Drift Correction



$$\dot{q}(t) = \tilde{f}(q(t)) - \lambda(q(t))$$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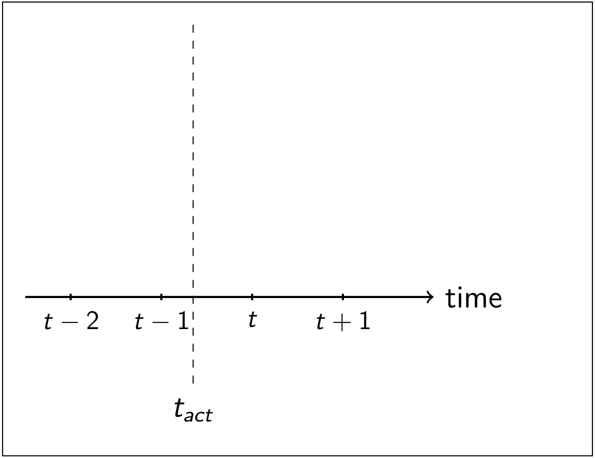
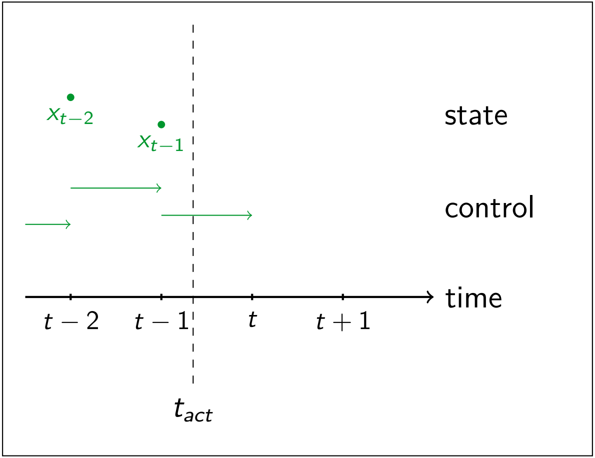
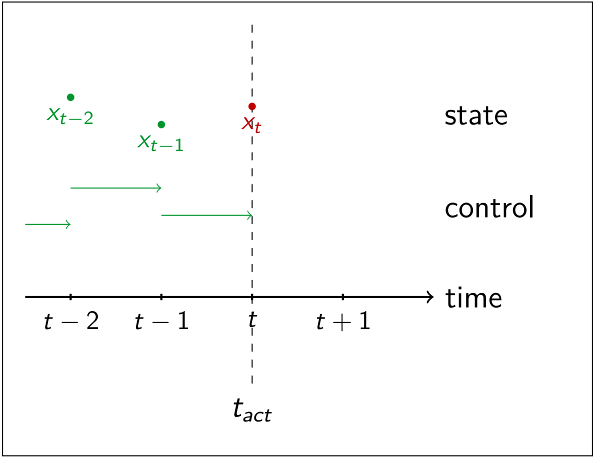
Setting



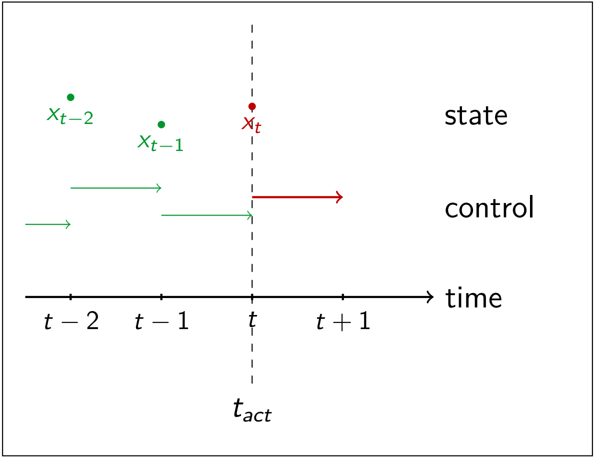
Setting



Setting



Setting

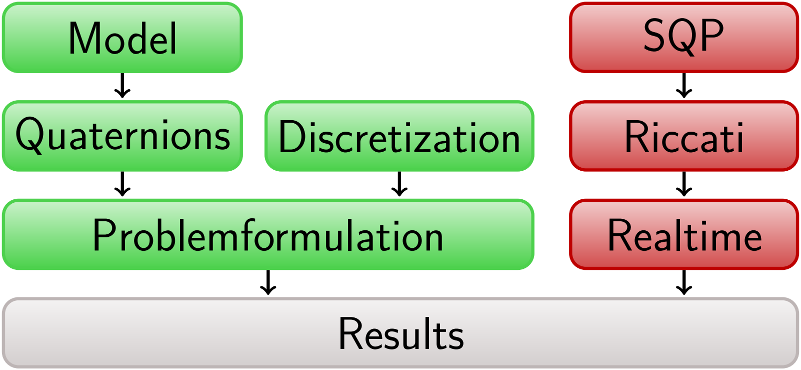


Discrete Problem

$$\min_{x,u} \sum_{i=t}^{t+N} J_i(x_i, u_i) \qquad \text{s.t.} \quad h_i(x_i, u_i) = 0 \quad i = t, ..., t+N$$

$J_i(x_i, u_i)$ discretized goal function

$h_i(x_i, u_i)$ equality constraints at time i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The Lagrangian

$$L(y) = \sum_{i=t}^{t+N} J_i(x_i, u_i) + \sum_{i=t}^{t+N} \lambda_i^T h_i(x_i, u_i)$$

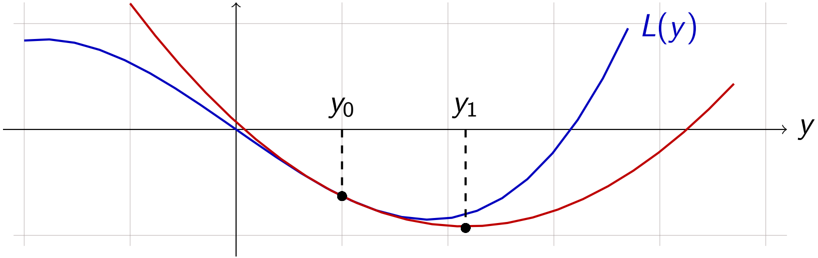
$$y := (\lambda, x, u)$$
$$y^* \text{ optimal} \quad \Leftrightarrow \quad \nabla_y L(y^*) = 0$$

The SQP Method

Find y^*:

$$y_1 = y_0 + s$$

$$\min_s \frac{1}{2} s^T \nabla^2 L(y_0) s + \nabla L(y_0)^T s$$



Quasi Newton-Method

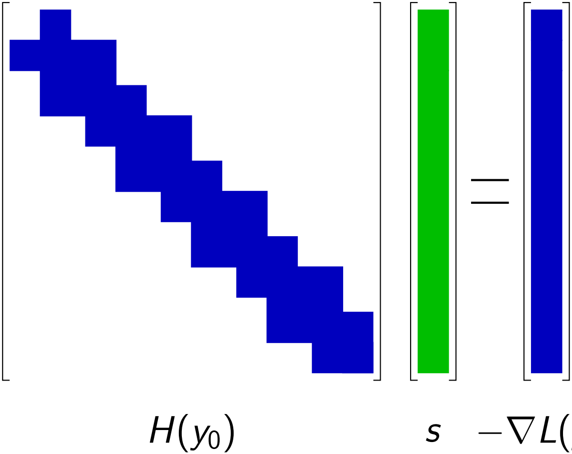
Find s with:

$$\nabla L(y_0) + \nabla^2 L(y_0) s = 0$$

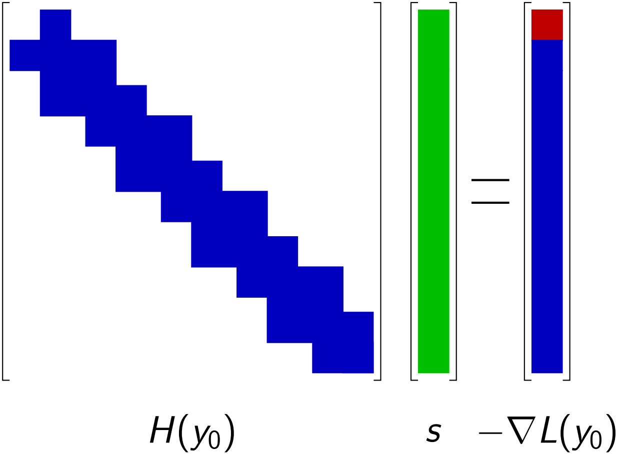
Approximate $\nabla^2 L(y_0)$ and solve:

$$H(y_0) s = -\nabla L(y_0)$$

Riccati Recursion



Riccati Recursion



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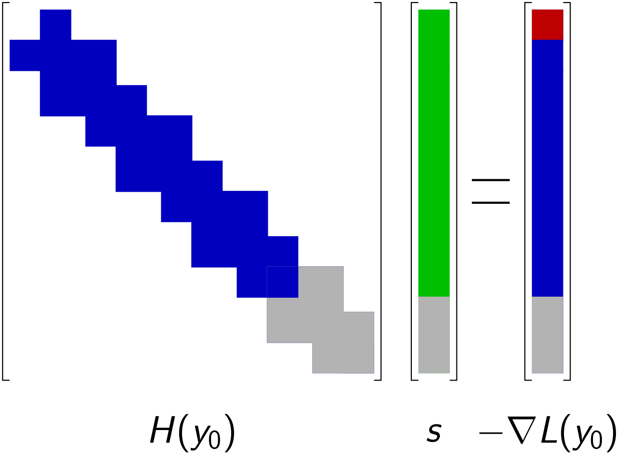
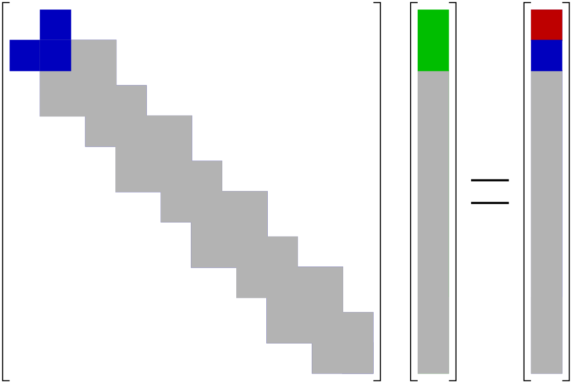


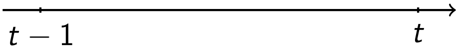
Riccati Recursion



$H(y_0)$ $\quad s \quad$ $-\nabla L(y_0)$

Summary

What happens in interval $[t-1, t]$?



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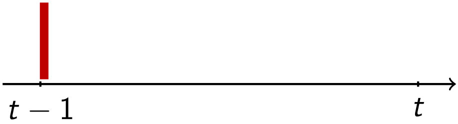
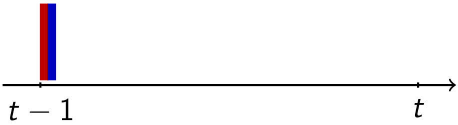


1. calculate control u_{t-1} (Riccati Part II)

Summary

What happens in interval $[t-1, t]$?



1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)

Summary

What happens in interval $[t-1, t]$?

1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)
3. prepare u_t (Newton & Riccati Part I)

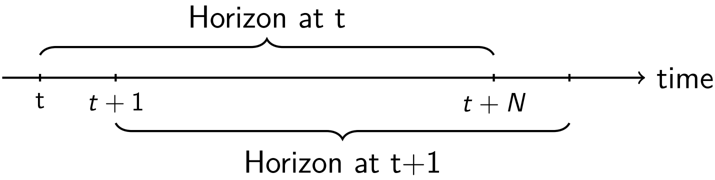
Summary

What happens in interval $[t-1, t]$?

1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)
3. prepare u_t (Newton & Riccati Part I)

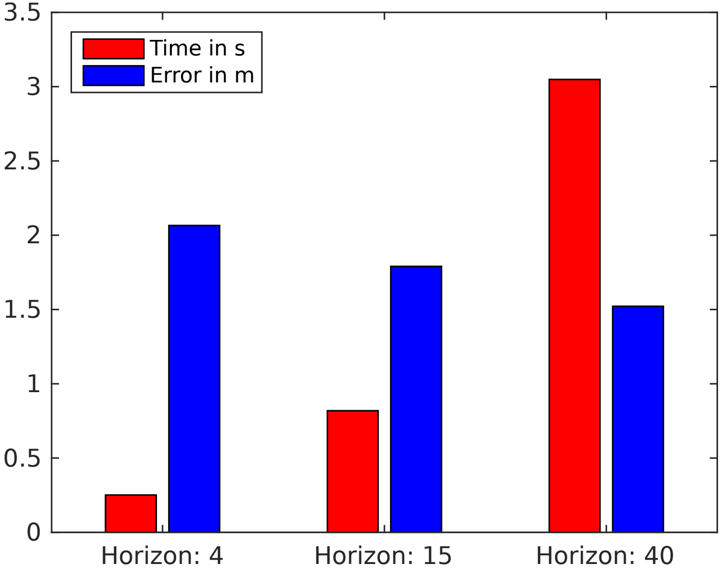
$\Rightarrow$ with horizon 18 this is 28% faster

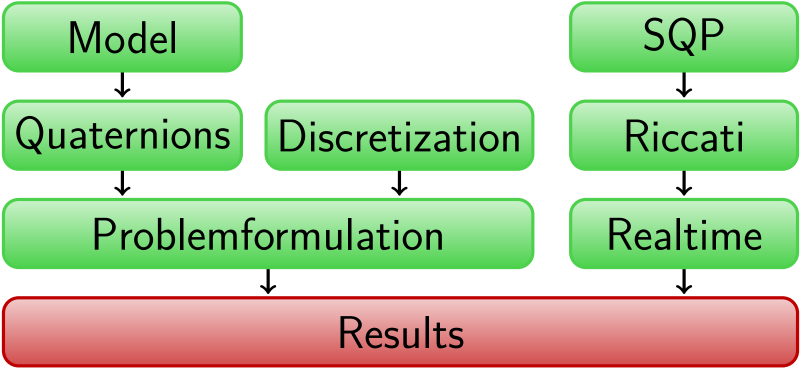
Finite Horizon



$\Rightarrow$ compromise between quality and speed

Finite Horizon

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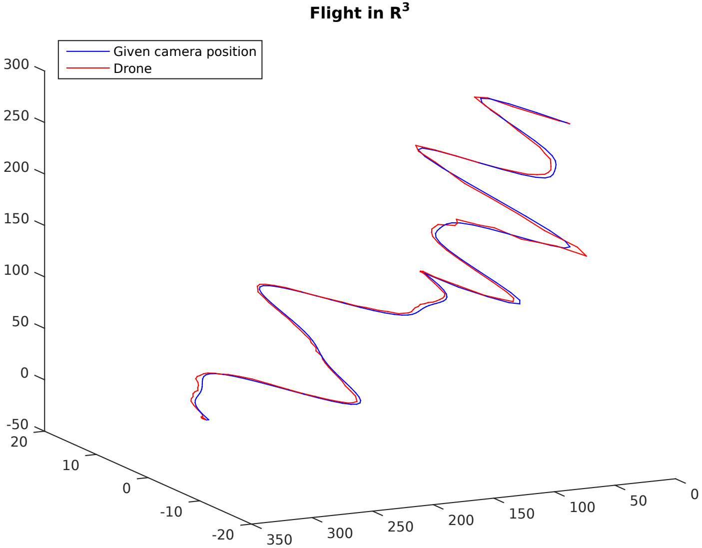
Following a Skier



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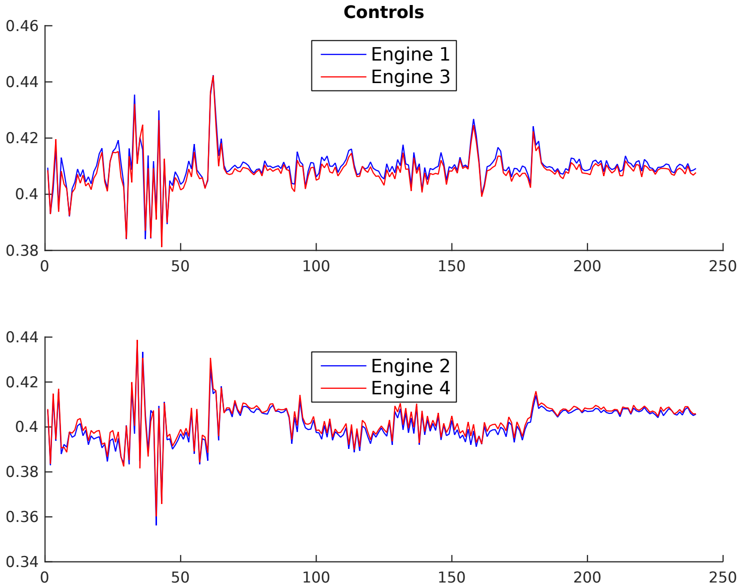

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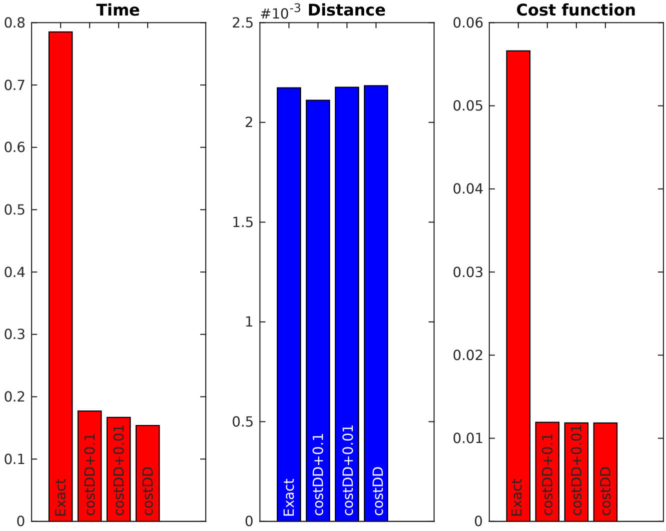

References I

Stephen Boyd.
Solving the lqr problem by block elimination.
Lecture notes, 2009.

James Diebel.
Representing attitude: Euler angles, unit quaternions, and rotation vectors.
10 2006.

Moritz Diehl, Hans Georg Bock, and Johannes P. Schlöder.
A real-time iteration scheme for nonlinear optimization in optimal feedback control.
SIAM J. Control Optim., 2005.

References II

- Moritz Diehl, Bock H. Georg, Johannes P. Schlöder, Rolf Findeisen, Zoltan Nagy, and Frank Allgöwer.
  Real-time optimization and nonlinear model predictive control of processes governed by differential-algebraic equations.
  Journal of Process Control, 2002.
- Moritz Mathias Diehl.
  Real-Time Optimization for Large Scale Nonlinear Processes.
  PhD thesis, Ruprecht-Karls-Universität Heidelberg, 2001.
- Luis Rodolfo Garcia Carrillo, Alejandro Enrique Dzul Lopez, Rogelio Lozano, and Claude Pegard.
  Quad Rotorcraft Control.
  Springer-Verlag London, 2013.

References III

D. Hartmann, K. Landis, M. Mehrer, S. Moreno, and J. Kim.
Quadcopter dynamic modeling and simulation (quad-sim) v1.00.
Git, 2014.

Elias Reyes-Valeria, Rogerio Enriquez-Caldera, Sergio Camacho-Lara, and Jose Guichard.
Lqr control for a quadrotor using unit quaternions: Modeling and simulation.
IEEE Xplore, 2013.

Jürgen Richter-Gebert and Thorsten Orendt.
Geometriekalküle.
Springer: Berlin, Heidelberg, 2009.

What we have learned from the Project:

- you have to know your plan to ignore it
- MATLAB® is special
- tests are helpful - or drive you crazy
- loopings can be cheap, too
- keep your colorscheme
- keep calm and do case studies

Any Questions?