Real Time Control of a Quadcopter

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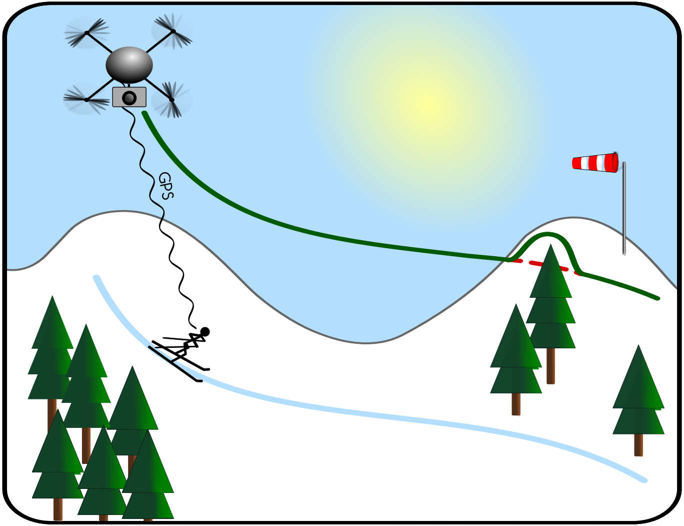
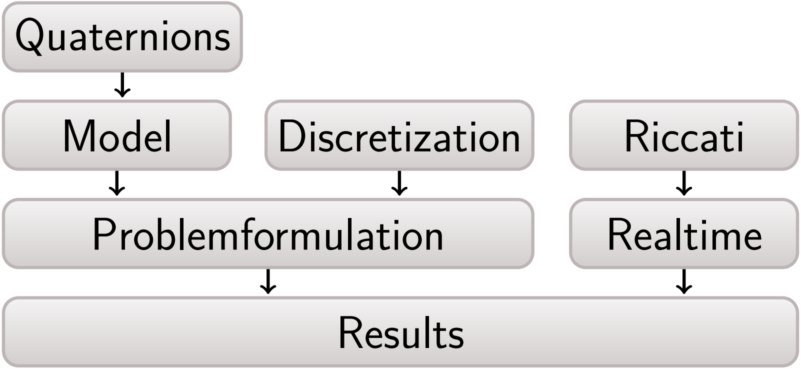
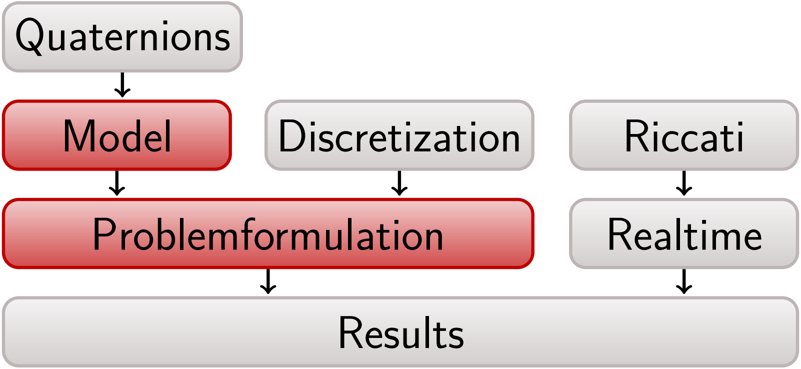
Technische Universität München

11 July 2015

Motivation




Quaternions
Model
Discretization
Riccati
Problemformulation
Realtime
Results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Optimal Control Formulation

$$\min_{x,u} J(x,u) \quad \text{s.t.} \quad \begin{aligned} \tilde{h}(x,u) &= 0 \\ \dot{x}(t) &= f(x(t), u(t)) \end{aligned}$$

x : state

u : control

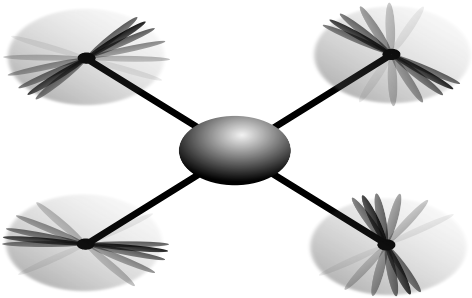
Optimal Control Formulation

$$\min_{x,u} J(x,u) \quad \text{s.t.} \quad \left.\begin{array}{c} \tilde{h}(x,u) = 0 \\ \dot{x}(t) = f(x(t), u(t)) \end{array}\right\} \Rightarrow h(x,u) = 0$$

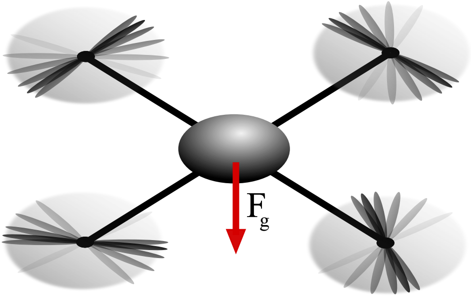
x : state

u : control

Model



Forces



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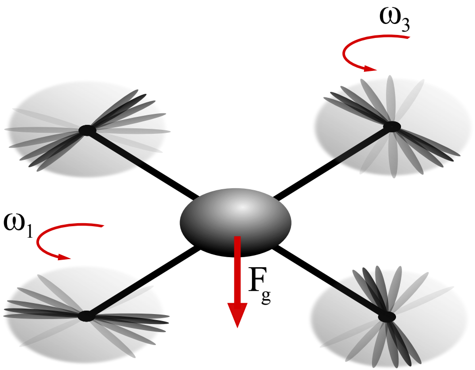

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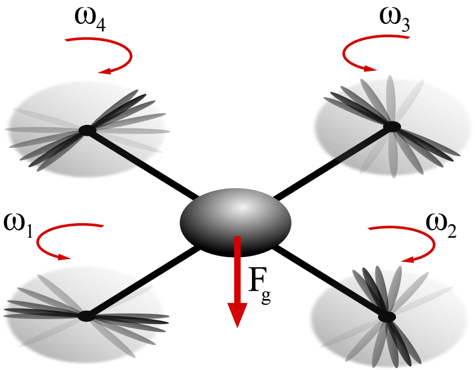

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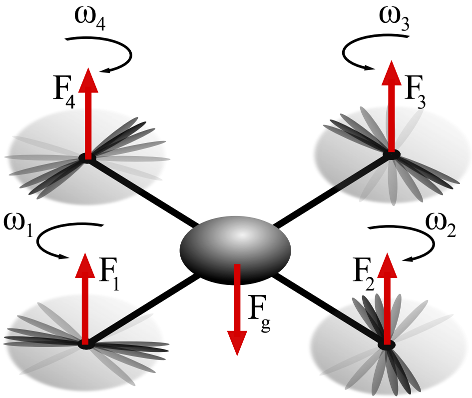

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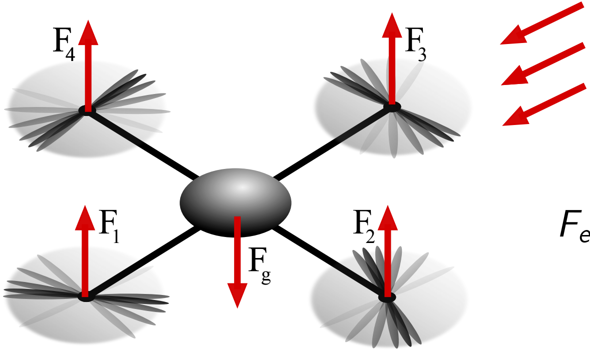
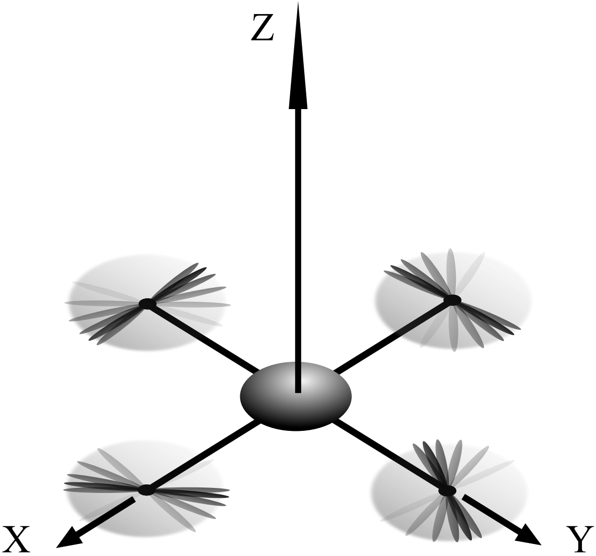
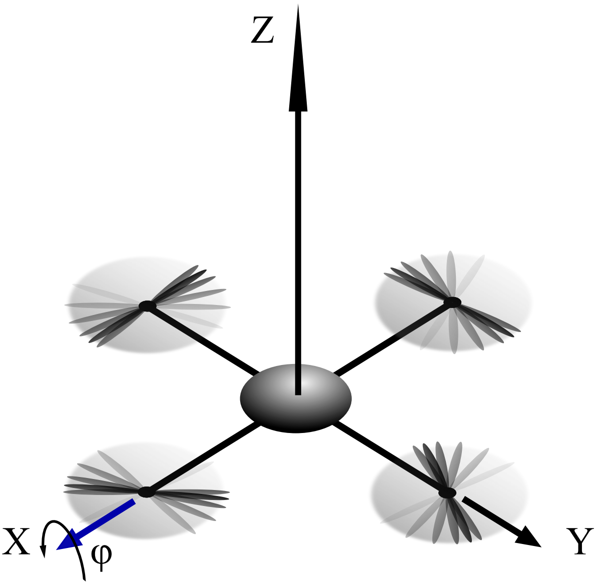


$$F_{ext} = F_g + \sum_{i=1}^{4} F_i$$

Torques



Torques



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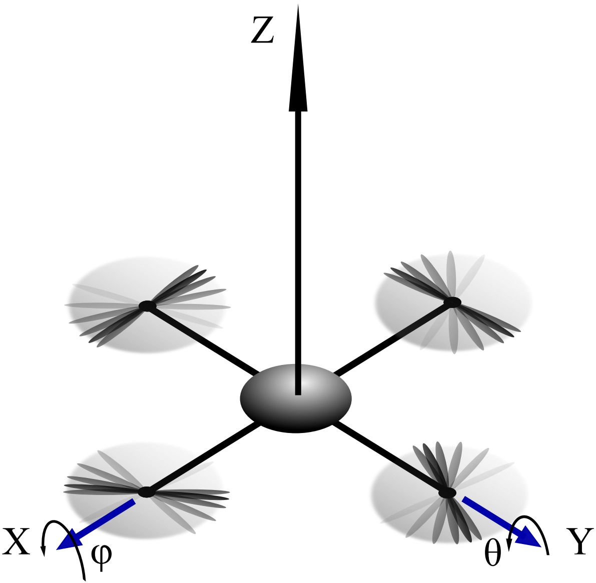
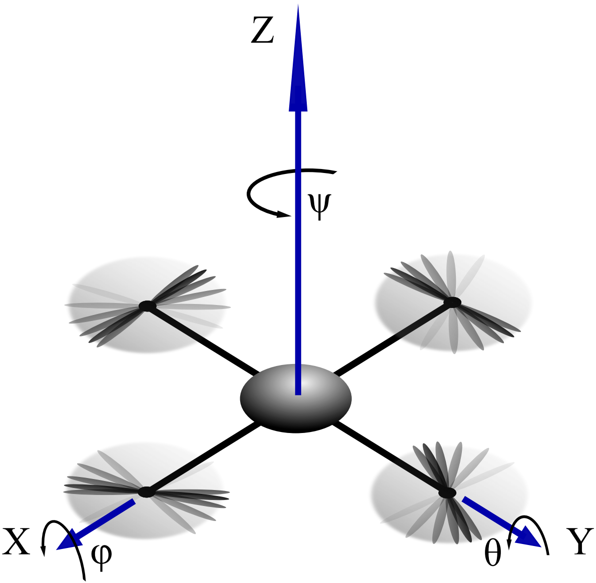
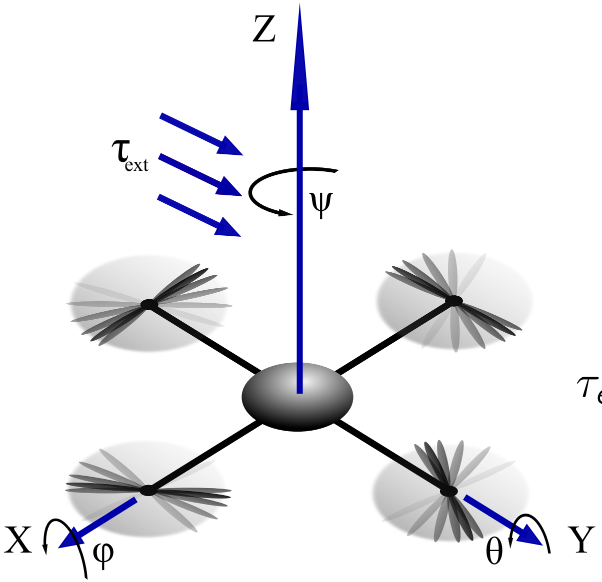


Torques



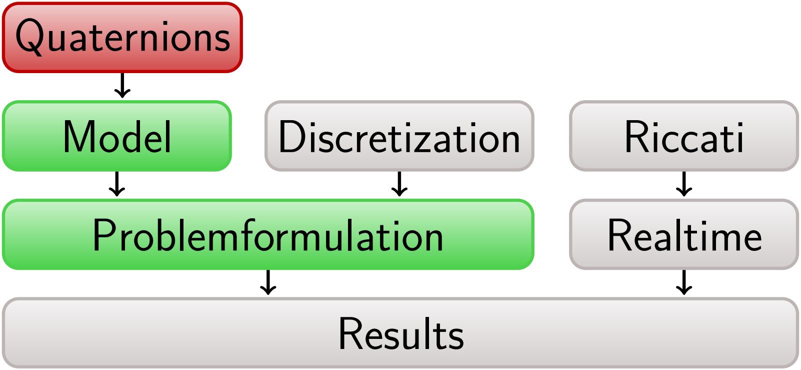
Torques



$$\tau_{ext} = \tau_{\psi} + \tau_{\varphi} + \tau_{\theta}$$

Obtain ODE

$$\left.\begin{aligned} F_{ext} &= F_g + \textstyle\sum_{i=1}^{4} F_i \\ \tau_{ext} &= \tau_\psi + \tau_\varphi + \tau_\theta \end{aligned}\right\} \quad \Rightarrow \quad \dot{x}(t) = f(x(t), u(t))$$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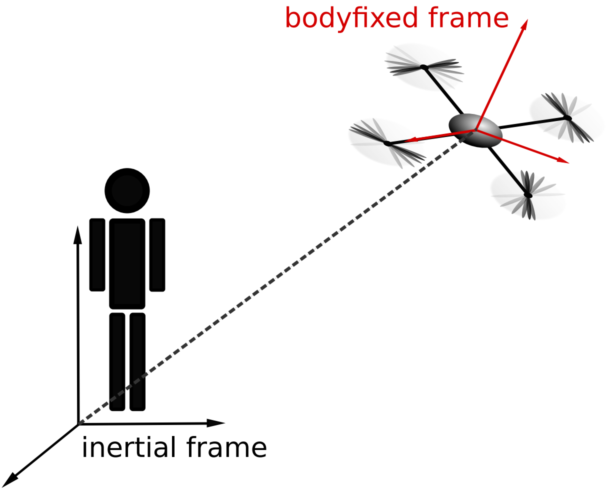
Coordinate Systems



Quaternions

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

$$\Leftrightarrow$$

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

Quaternions

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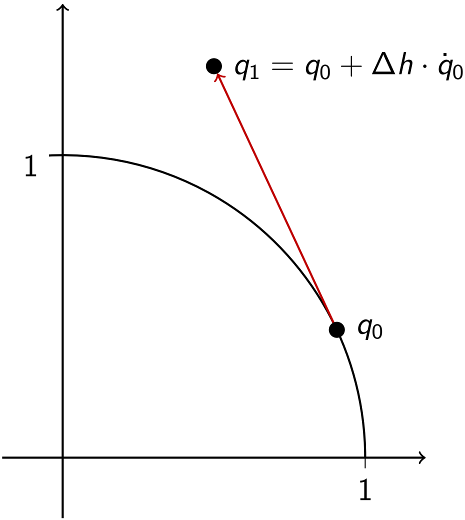
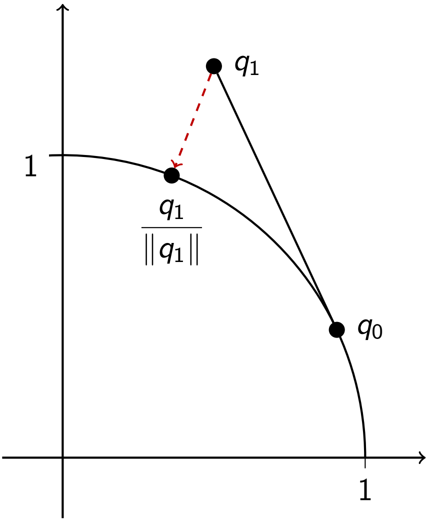
represent rotation $\quad \Leftrightarrow \quad \|q\| = 1 \quad \Leftrightarrow \quad q \in \mathcal{S}^3$

Drift Correction

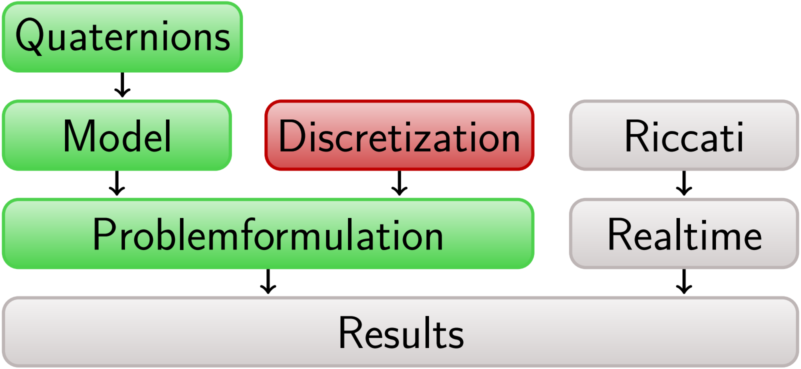


$$\dot{q}(t) = \tilde{f}(q(t))$$

Drift Correction



$$\dot{q}(t) = \tilde{f}(q(t)) - \lambda(q(t))$$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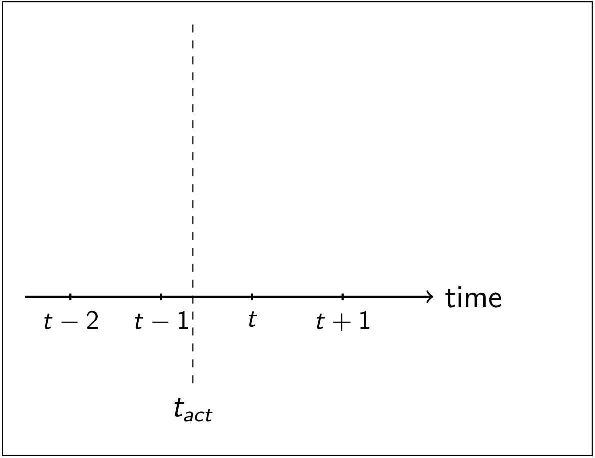
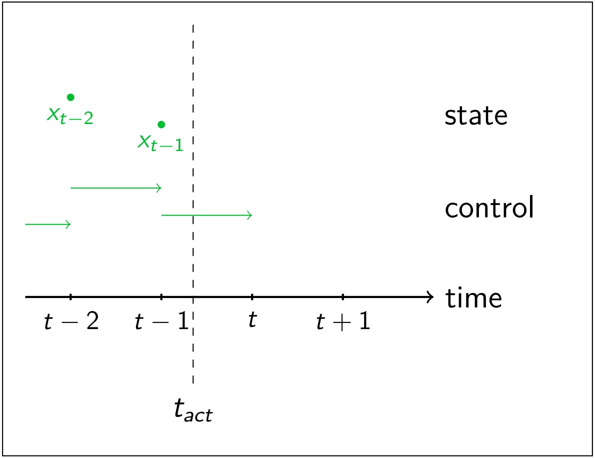
Setting



Setting



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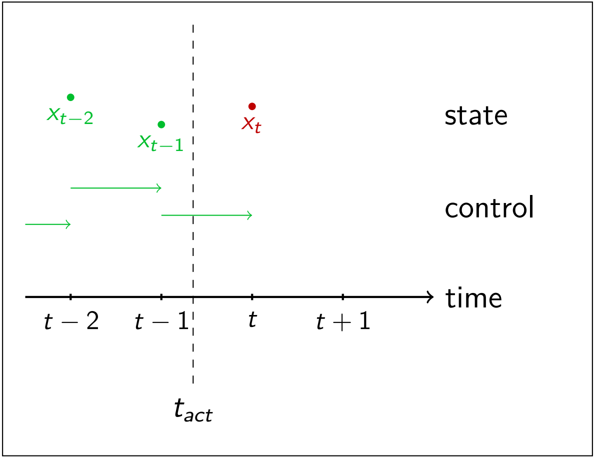

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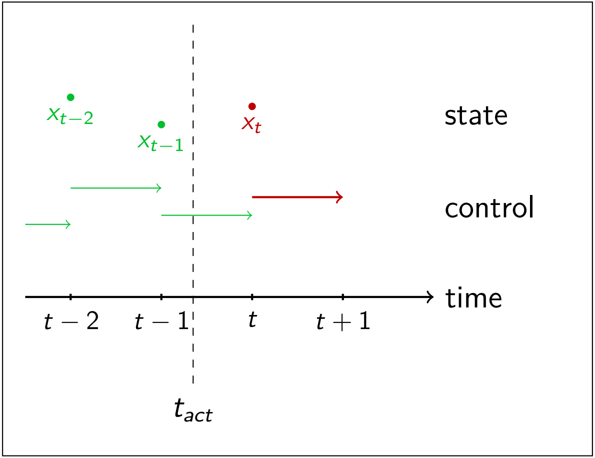
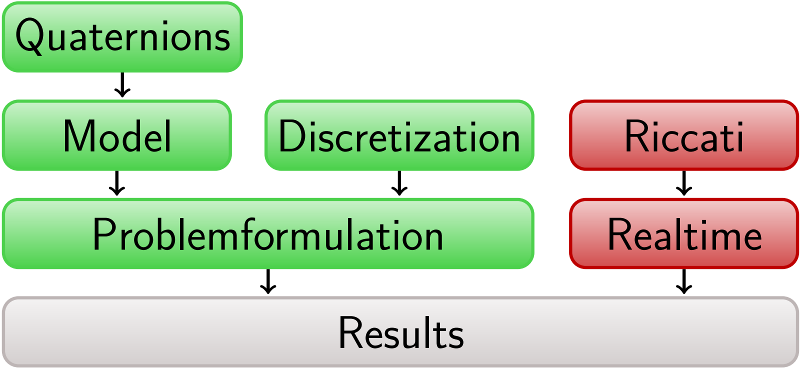


Discrete Problem

$$\min_{x,u} \sum_{i=t}^{N} J_i(x_i, u_i) \qquad \text{s.t.} \quad h_i(x_i, u_i) = 0 \quad i = t, \ldots, N$$

$J_i(x_i, u_i)$ discretized goal function

$h_i(x_i, u_i)$ equality constraints at time i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The Lagrangian

$$L(y) = \sum_{i=t}^{N} J_i(x_i, u_i) + \sum_{i=t}^{N} \lambda_i^T h_i(x_i, u_i)$$

$$y := (\lambda, x, u)$$

$$y^* \text{ optimal} \quad \Leftrightarrow \quad \nabla_y L(y^*) = 0$$

The SQP Method

Find y^*:

$$y_{k+1} = y_k + s_k$$
$$\min_{s_k} \frac{1}{2} s_k^T \nabla^2 L(y_k) s_k + \nabla L(y_k)^T s_k$$

Bild!!!

Quasi Newton-Method

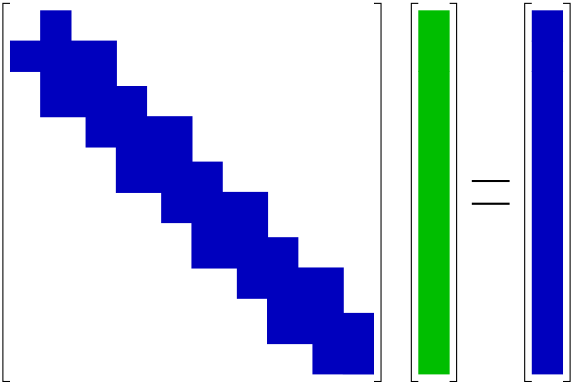
Find s_k with:

$$\nabla L(y_k) + \nabla^2 L(y_k) s_k = 0$$

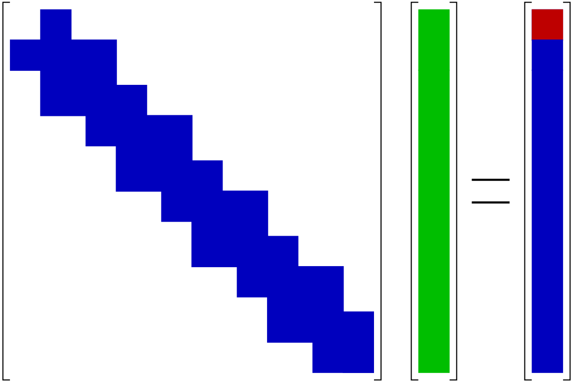
Approximate $\nabla^2 L(y_k)$ and solve:

$$H(y_k) s_k = -\nabla L(y_k)$$

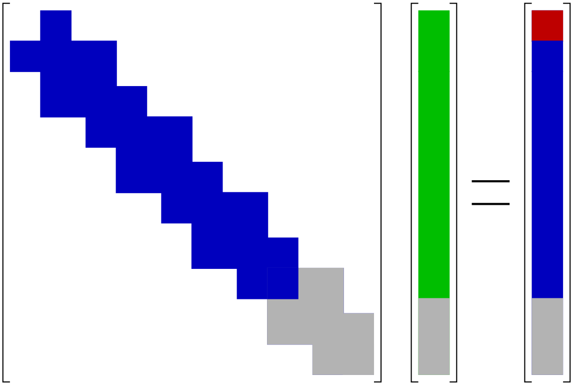
Riccati Recursion



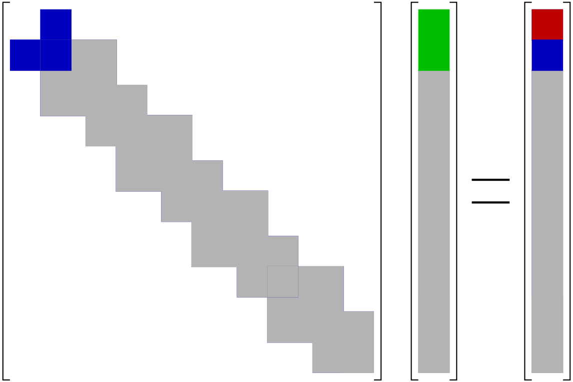
Riccati Recursion



Riccati Recursion



Riccati Recursion



Summary

What happens in interval $[t-1, t]$?

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1. calculate control u_{t-1} (Riccati Part II)

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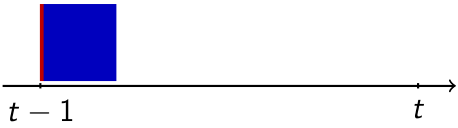


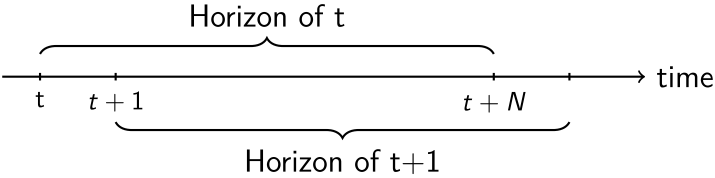
1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)

Summary

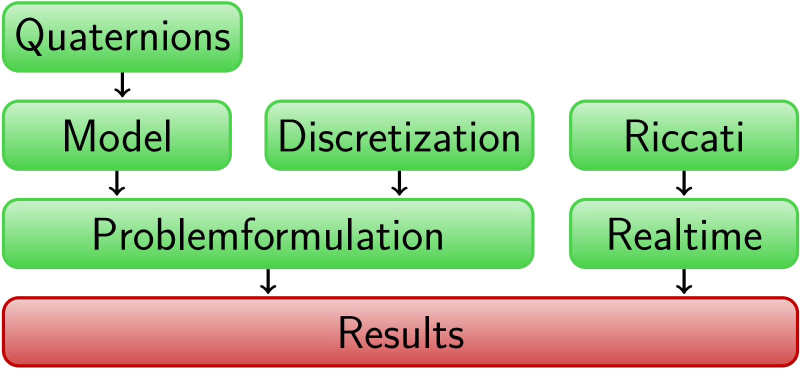
What happens in interval $[t-1, t]$?

1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)
3. prepare u_t (Newton & Riccati Part I)

Finite Horizon



	runtime	error
$N = 20$		
$N = 50$		
$N = 100$		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