

Midterm presentation

The Quadrocopters

Technische Universität München

26. Mai 2015

Overview

- 1 Motivation
- 2 Model
- 3 Realtime Optimization Approach
- 4 Results

Optimal Control Problem

$$\min_{x,u} J(x, u) \quad \dot{x} = f(x, u)$$

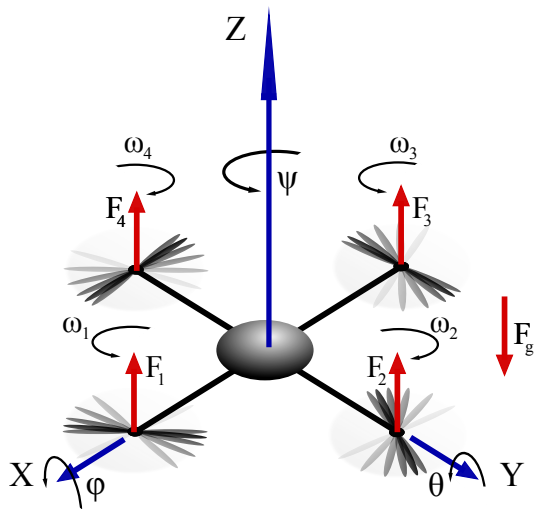
x : state

u : control

→ additional difficulty: realtime approach

Model

Forces and Torques



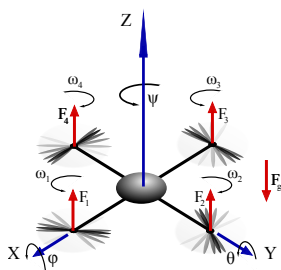
Newton-Euler Equations

Forces

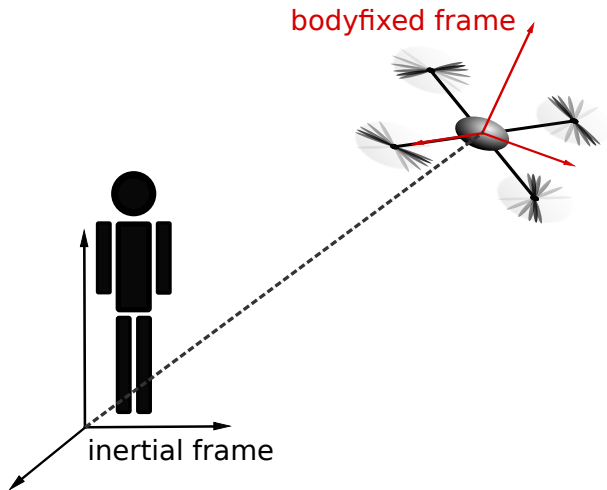
$$F_{\text{ext}} = F_g + \sum_{i=1}^4 F_i$$

Torques

$$\tau_{\text{ext}} = \sum_{i=1}^4 \tau_i + (\tau_\varphi + \tau_\theta)$$



Coordinate Systems



Quaternions

$$q = a + ib + jc + kd \quad a, b, c, d \in \mathbb{R}$$

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problem $\rightarrow \|q\| = 1$ additional constraint

Dynamics

Equations representing dynamics...

$$T(x, u) = M \cdot \begin{pmatrix} \dot{x}_8 \\ \vdots \\ \dot{x}_{13} \end{pmatrix} + \Theta(x)$$

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$$T(x, u) = M \cdot \begin{pmatrix} \dot{x}_8 \\ \vdots \\ \dot{x}_{13} \end{pmatrix} + \Theta(x)$$

...expressed as system of differential equations:

$$\frac{d}{dt} \begin{pmatrix} x_1 \\ \vdots \\ x_7 \\ x_8 \\ \vdots \\ x_{13} \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_7 \\ M^{-1}(T(x, u) - \Theta(x)) \end{pmatrix}$$

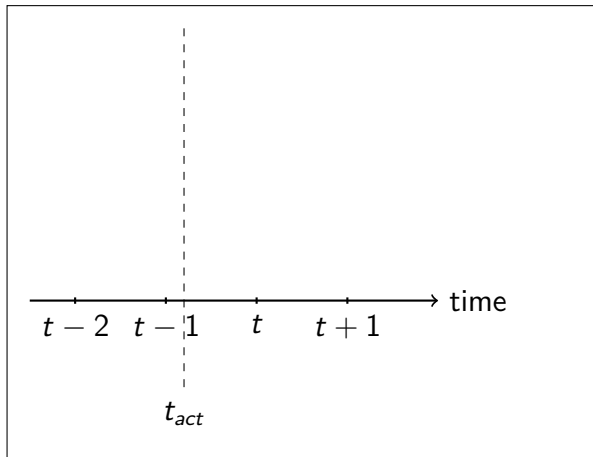
Refinement of the model

→ wind

→ aerodynamical forces

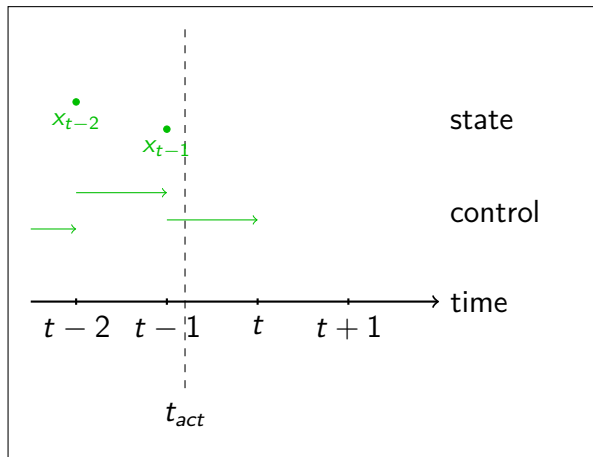
Realtime Optimization Approach

Setting



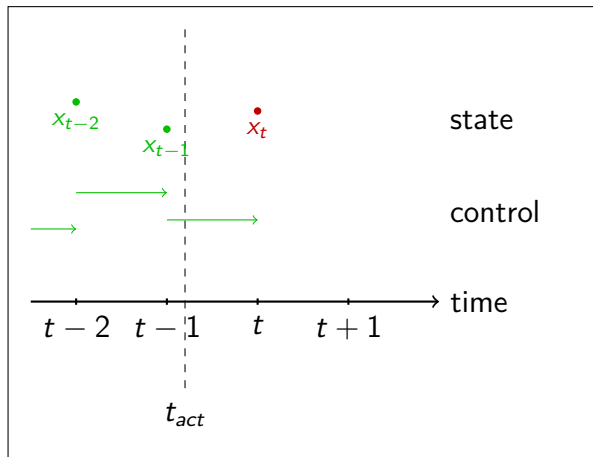
y, s, q erkl \tilde{A} ren

Setting



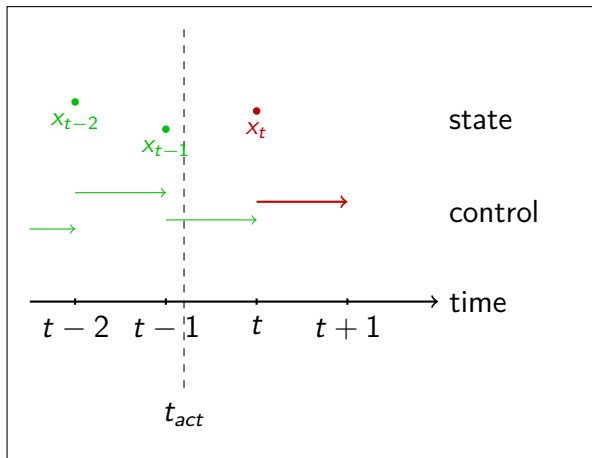
y, s, q erklärten

Setting



y, s, q erklärten

Setting



y, s, q erklärten

Minimization Problem

$$\min_{\substack{s_t, \dots, s_N \\ q_t, \dots, q_{N-1}}} \sum_{i=t}^{N-1} J_i(s_i, q_i) \quad s.t. \quad \begin{cases} x_t - s_t = 0 \\ h_i(s_i, q_i) - s_{i+1} = 0 \quad \forall i = t, \dots, N-1 \end{cases}$$

$J_i(s_i, q_i)$ discretized goal function

$x_t - s_t = 0$ expected state should be the real state at time t

$h_i(s_i, q_i)$ solution of the ODE at time i

The Lagrangian

$$L^t(y) = \sum_{i=t}^{N-1} J_i(s_i, q_i) + \lambda_t^T (x_t - s_t) + \sum_{i=t}^{N-1} \lambda_{i+1}^T (h_i(s_i, q_i) - s_{i+1})$$

We are looking for y^* satisfying the KKT conditions:

$$\Rightarrow \nabla_y L^t(y^*) = 0$$

The SQP Method

How to find y^* ?

$$y_{k+1} = y_k + \alpha_k \Delta y_k$$
$$\min_{\Delta y_k} = \frac{1}{2} \Delta y_k^T A_k \Delta y_k + \nabla_{y_k} J(y_k)^T \Delta y_k$$

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Special case:

$A_k = H(y_k)$ approximated Hessian $\nabla_{y_k}^2 L(y_k)$

$\alpha_k = 1$

Newton-Raphson

Find Δy_k with:

$$\nabla_{y_k} L(y_k) + H(y_k) \Delta y_k = 0$$

Newton-Raphson

Find Δy_k with:

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There is only time for 1 SQP step:

$$y_{t+1} = y_1 = y_0 + \Delta y_0$$

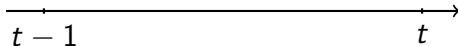
Riccati Recursion

Approximated Hessian:

$$H^t(y^t) = \begin{pmatrix} & -E & & & & & & & \\ -E & Q_t^H & M_t^H & A_t^T & & & & & \\ & (M_t^T)^H & R_t^H & B_t^T & & & & & \\ & A_t & B_t & & -E & & & & \\ & & & -E & Q_{t+1}^H & M_{t+1}^H & A_{t+1}^T & & \\ & & & (M_{t+1}^T)^H & R_{t+1}^H & B_{t+1}^T & & & \\ & & & A_{t+1} & B_{t+1} & & & & \\ & & & & & & \ddots & & \\ & & & & & & & \ddots & \\ & & & & & & & & Q_{N-1}^H & M_{N-1}^H & A_{N-1}^T \\ & & & & & & & & (M_{N-1}^T)^H & R_{N-1}^H & B_{N-1}^T \\ & & & & & & & & A_{N-1} & B_{N-1} & -E \\ & & & & & & & & & -E & Q_N^H \end{pmatrix}$$

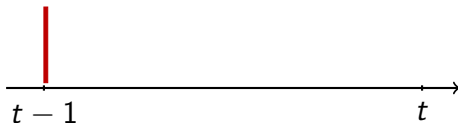
Summary

What happens in interval $[t_{k-1}, t_k]$?



Summary

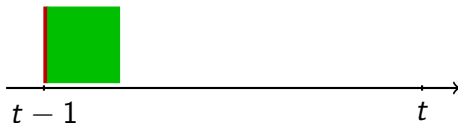
What happens in interval $[t_{k-1}, t_k]$?



- 1 calculate control u_{k-1} (Riccati Part II)

Summary

What happens in interval $[t_{k-1}, t_k]$?



- 1 calculate control u_{k-1} (Riccati Part II)
- 2 calculate y_k (Riccati Part II)

Summary

What happens in interval $[t_{k-1}, t_k]$?



- 1 calculate control u_{k-1} (Riccati Part II)
- 2 calculate y_k (Riccati Part II)
- 3 prepare u_k (Newton & Riccati Part I)

Finite Horizon

$$\min_{\substack{s_t, \dots, s_N \\ q_t, \dots, q_{N-1}}} \sum_{i=t}^{N-1} J_i(s_i, q_i) \quad s.t. \quad \begin{cases} x_t - s_t = 0 \\ h_i(s_i, q_i) - s_{i+1} = 0 \quad \forall i = t, \dots, N-1 \end{cases}$$

How to choose N ?

$N = t_{end} \rightarrow$ problem size decreasing

Finite Horizon

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How to choose N ?

$N = t_{end} \quad \rightarrow$ problem size decreasing

$N = t + n \quad \rightarrow$ problem size constant

Results

Questions?



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Solving the LQR Problem by Block Elimination.

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Lecture notes



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Representing Attitude: Euler Angles, Unit Quaternions, and Rotation Vectors.

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and Simulation.

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