Midterm presentation

The Quadrocopters

Technische Universität München

29. Juni 2015

Overview

Optimal Control Problem

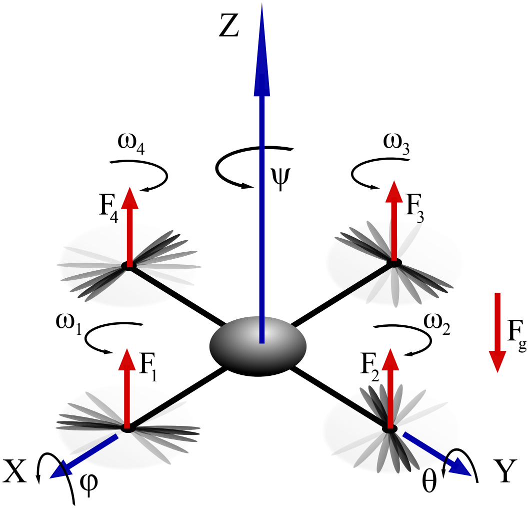
$$\min_{x,u} J(x,u) \qquad \dot{x} = f(x,u)$$

$$x: \ \text{state}$$
$$u: \ \text{control}$$

$\rightarrow$ additional difficulty: realtime approach

Model

Forces and Torques



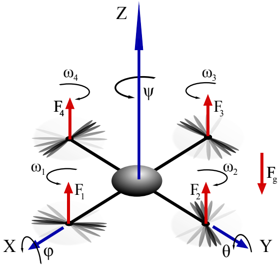
Newton-Euler Equations

Forces

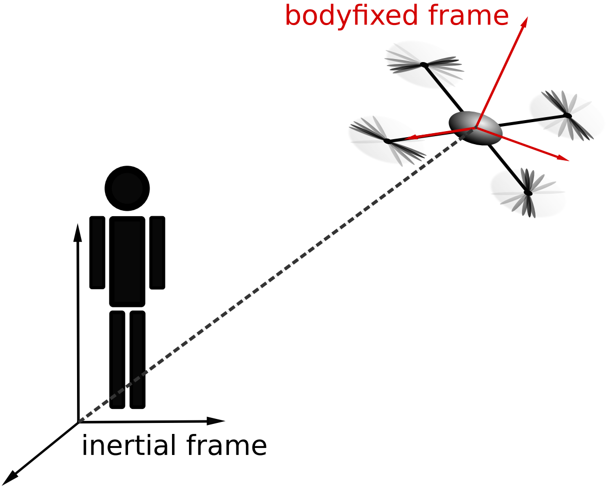
$$F_{ext} = F_g + \sum_{i=1}^{4} F_i$$

Torques

$$\tau_{ext} = \sum_{i=1}^{4} \tau_i + (\tau_\varphi + \tau_\theta)$$



Coordinate Systems



Quaternions

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

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representing rotation $\Leftrightarrow$ $\|q\| = 1$

advantage $\rightarrow$ no singularities

problem $\rightarrow$ $\|q\| = 1$ additional constraint

Dynamics

Equations representing dynamics...

$$T(x, u) = M \cdot \begin{pmatrix} \dot{x}_8 \\ \vdots \\ \dot{x}_{13} \end{pmatrix} + \Theta(x)$$

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$$T(x,u) = M \cdot \begin{pmatrix} \dot{x}_8 \\ \vdots \\ \dot{x}_{13} \end{pmatrix} + \Theta(x)$$

...expressed as system of differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} x_1 \\ \vdots \\ x_7 \\ x_8 \\ \vdots \\ x_{13} \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_7 \\ M^{-1}(T(x,u) - \Theta(x)) \end{pmatrix}$$

Prospect

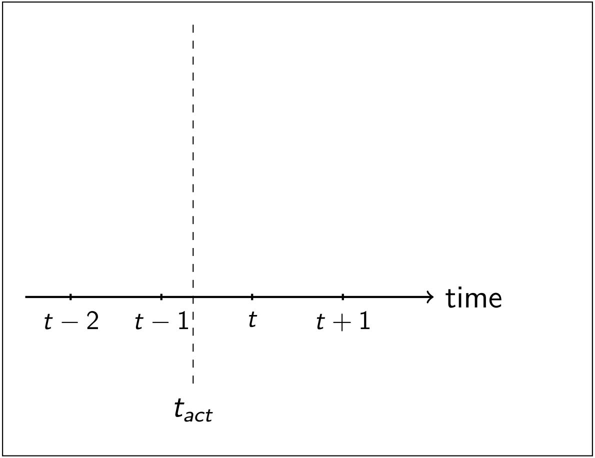
Refinement of the model

$\rightarrow$ wind

$\rightarrow$ aerodynamical forces

Realtime Optimization Approach

Setting



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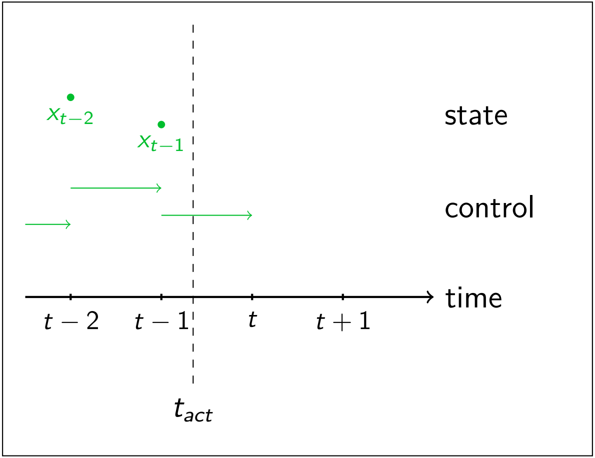

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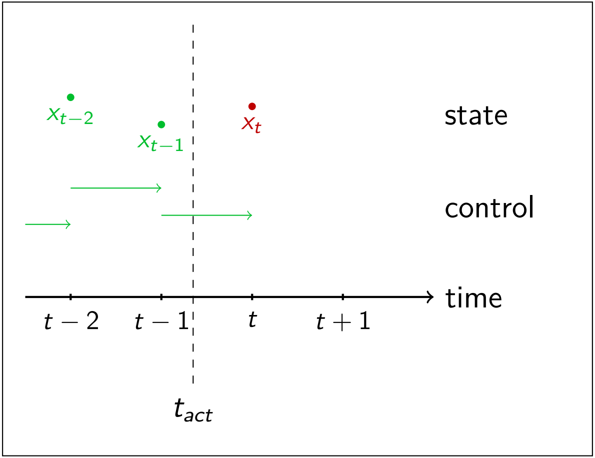

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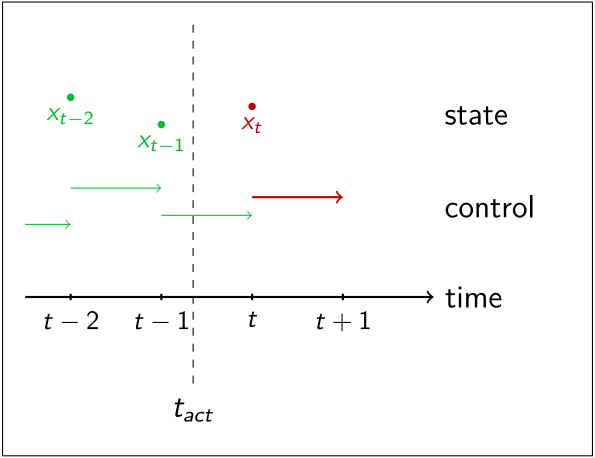

Discrete Problem

$$\min_{s,q} \sum_{i=t}^{N-1} J_i(s_i, q_i) \quad s.t. \quad \left\{ \begin{array}{r} x_t - s_t = 0 \\ h_i(s_i, q_i) - s_{i+1} = 0 \\ p_{A_i}(s_i, q_i) = 0 \end{array} \right. \quad \forall i = t, ..., N-1$$

$J_i(s_i, q_i)$	discretized goal function
$x_t - s_t = 0$	expected state $=$ real state
$h_i(s_i, q_i)$	solution of the ODE at time i
$p_{A_i}(s_i, q_i)$	activ inequality constraints

The Lagrangian

$$
\begin{aligned}
L^t(y) = \;& \sum_{i=t}^{N-1} J_i(s_i, q_i) + \lambda_t^T (x_t - s_t) \\
& + \sum_{i=t}^{N-1} \lambda_{i+1}^T \left(h_i(s_i, q_i) - s_{i+1}\right) \\
& + \sum_{i=t}^{N-1} \mu_i^T p_{A_i}(s_i, q_i)
\end{aligned}
$$

$$y := (\lambda, s, q, \mu)$$

Search for optimal y^* :

$$\Rightarrow \nabla_y L^t(y^*) = 0$$

The SQP Method

How to find y^*?

$$y_{k+1} = y_k + \alpha_k \Delta y_k$$
$$\min_{\Delta y_k} \frac{1}{2} \Delta y_k^T A_k \Delta y_k + \nabla_{y_k} J(y_k)^T \Delta y_k$$

The SQP Method

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Special case:

$A_k = H(y_k)$ approximated Hessian $\nabla^2_{y_k} L(y_k)$

$\alpha_k = 1$

Newton-Methode

Find Δy_k with:

$$\nabla_{y_k} L(y_k) + H(y_k)\Delta y_k = 0$$

Newton-Methode

Find Δy_k with:

$$\nabla_{y_k} L(y_k) + H(y_k)\Delta y_k = 0$$

1 iteration per timestep:

$$y_{t+1} = y_1 = y_0 + \Delta y_0$$

Riccati Recursion

Solves linear system fast
TODO: Vergleich verschiedener Horizon

Summary

What happens in interval $[t-1, t]$?

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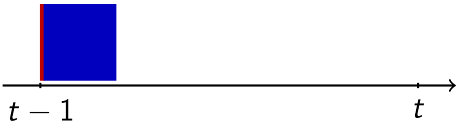

1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)

Summary

What happens in interval $[t-1, t]$?

1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)
3. prepare u_t (Newton & Riccati Part I)

Finite Horizon

$$\min_{s,q} \sum_{i=t}^{N-1} J_i(s_i, q_i) \quad s.t. \left\{ \begin{aligned} x_t - s_t &= 0 \\ h_i(s_i, q_i) - s_{i+1} &= 0 \\ p_{A_i}(s_i, q_i) &= 0 \end{aligned} \right. \quad \forall i = t, ..., N-1$$

How to choose N?

$N = t_{end}$ $\quad \rightarrow$ problem size decreasing

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How to choose N?

$N = t_{end} \quad \rightarrow$ problem size decreasing

$N = t + n \quad \rightarrow$ problem size constant

Results

Questions?