

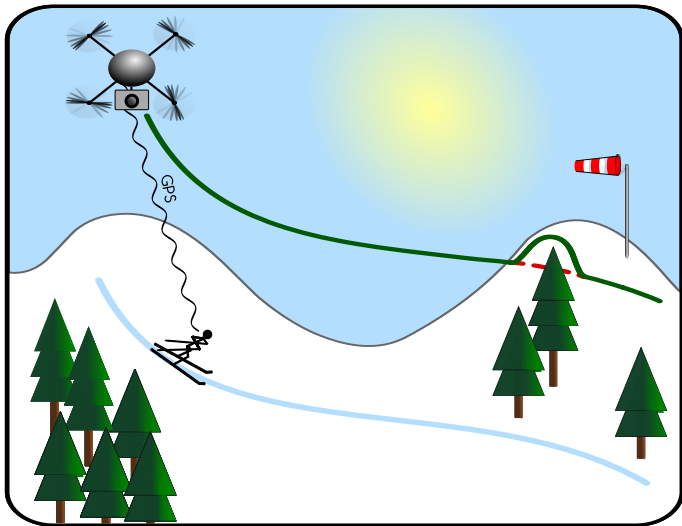
# Real Time Control of a Quadcopter

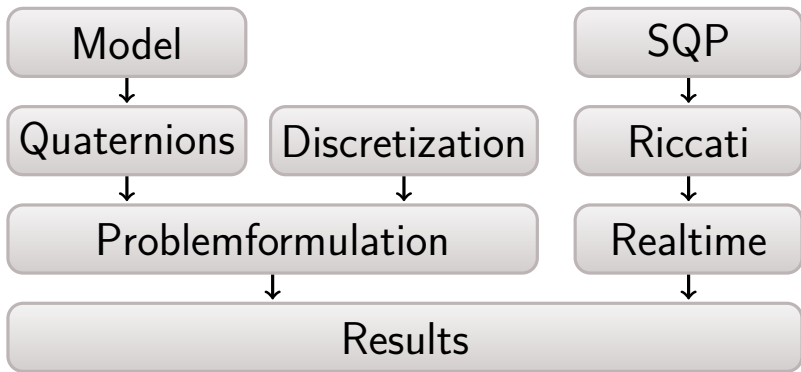
Simon Kick, Philipp Fröhlich, Benedikt König, Annika Stegie

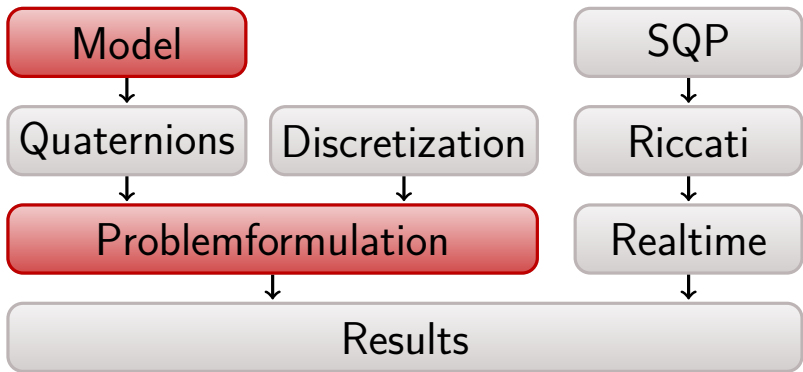
Technische Universität München

11 July 2015

# Motivation







# Optimal Control Formulation

$$\min_{x,u} J(x, u) \quad \text{s.t.} \quad \begin{aligned} \tilde{h}(x, u) &= 0 \\ \dot{x}(t) &= f(x(t), u(t)) \end{aligned}$$

$x$  : state

$u$  : control

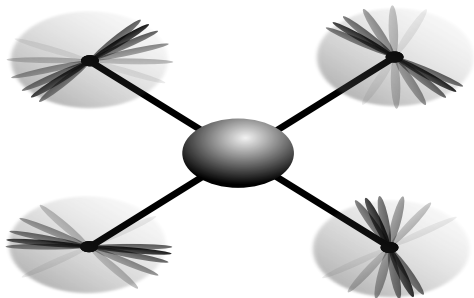
# Optimal Control Formulation

$$\min_{x,u} J(x,u) \quad \text{s.t.} \quad \left. \begin{array}{l} \tilde{h}(x,u) = 0 \\ \dot{x}(t) = f(x(t), u(t)) \end{array} \right\} \Rightarrow h(x,u) = 0$$

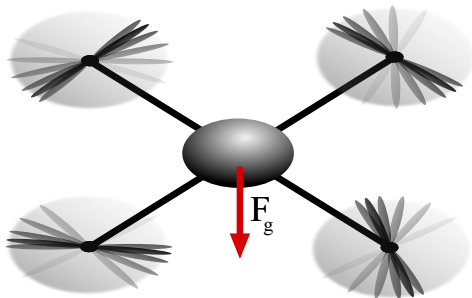
$x$  : state

$u$  : control

# Model

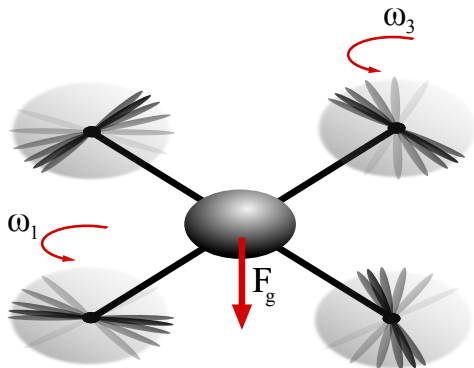


# Model

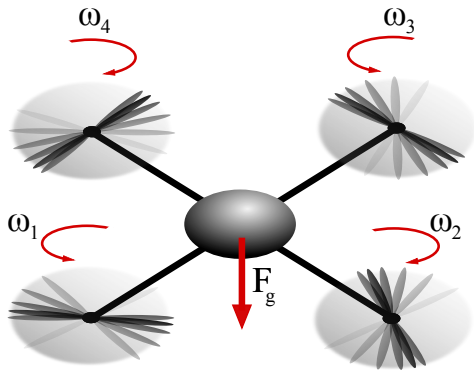




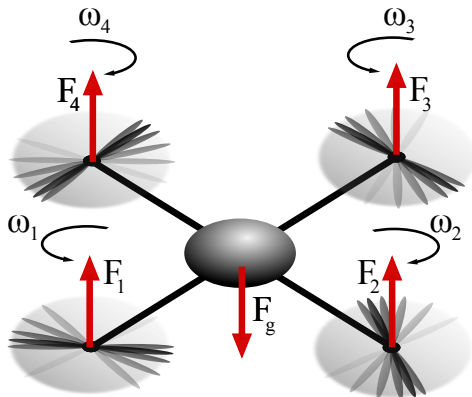
# Model



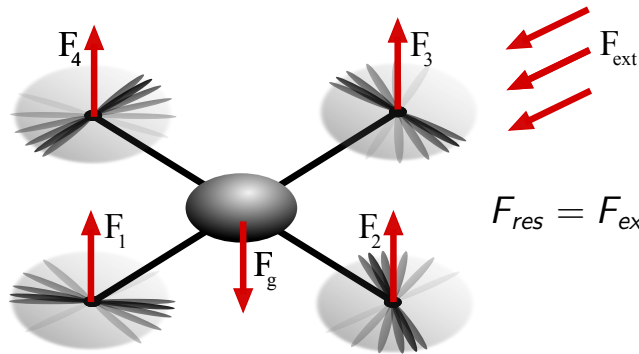
# Model



# Model

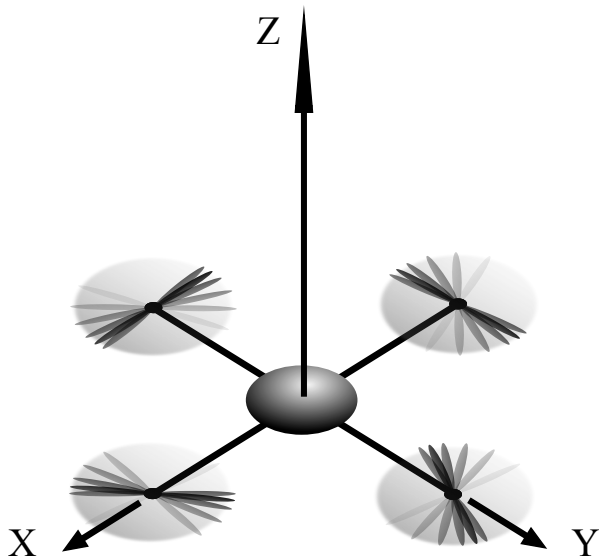


# Forces

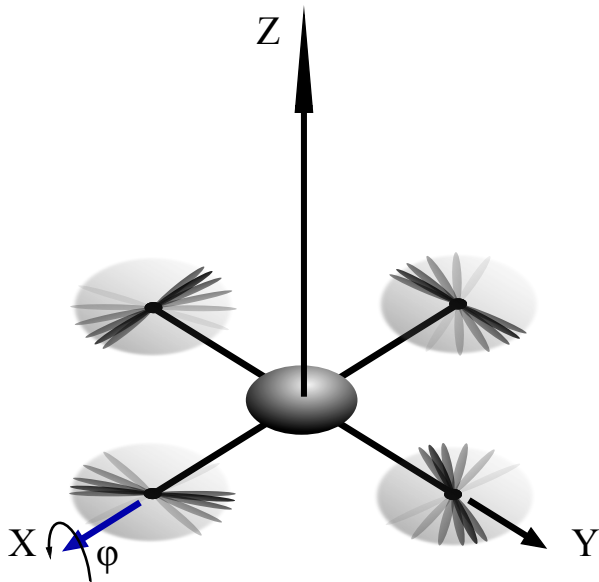


$$F_{res} = F_{ext} + F_g + \sum_{i=1}^4 F_i$$

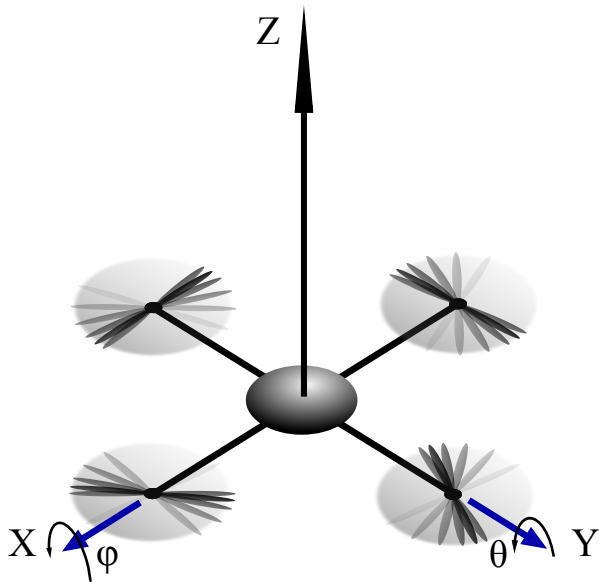
# Torques



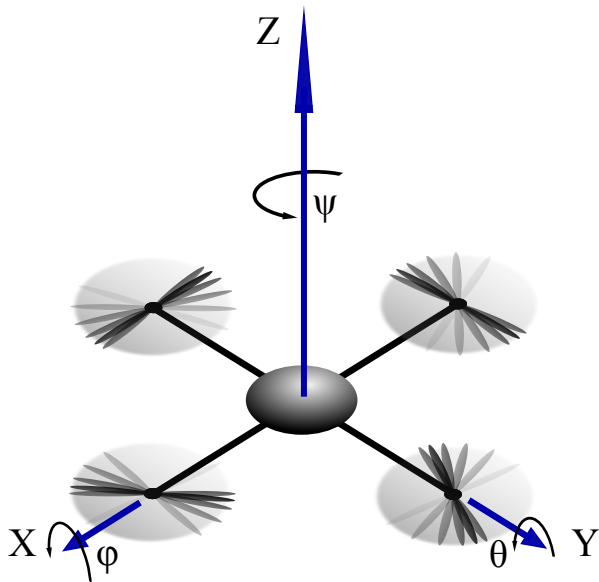
# Torques



# Torques

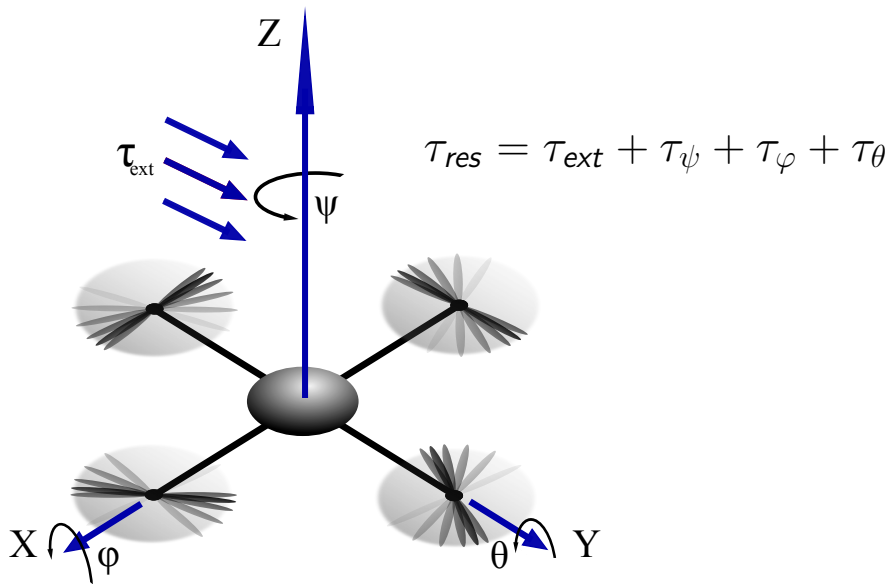


# Torques



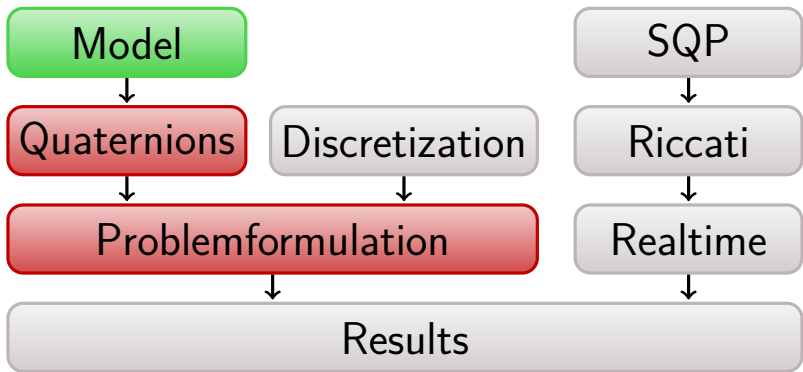


# Torques

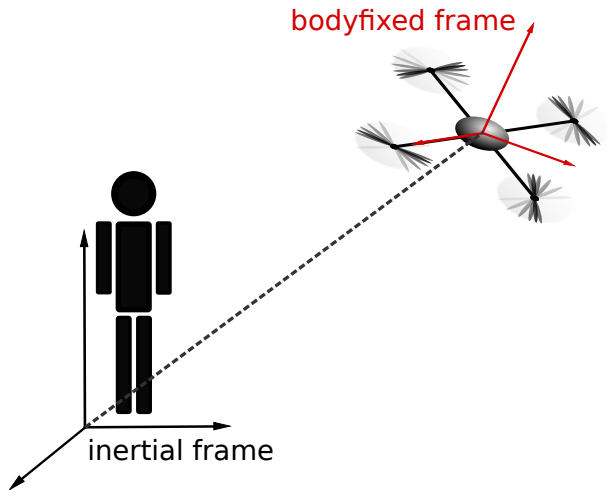


# Obtain ODE

$$\left. \begin{aligned} F_{res} &= F_{ext} + F_g + \sum_{i=1}^4 F_i \\ \tau_{res} &= \tau_{ext} + \tau_{\psi} + \tau_{\varphi} + \tau_{\theta} \end{aligned} \right\} \Rightarrow \dot{x}(t) = f(x(t), u(t))$$



# Coordinate Systems



# Quaternions

$$q = a + ib + jc + kd \quad a, b, c, d \in \mathbb{R}$$

$$\Leftrightarrow$$

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

# Quaternions

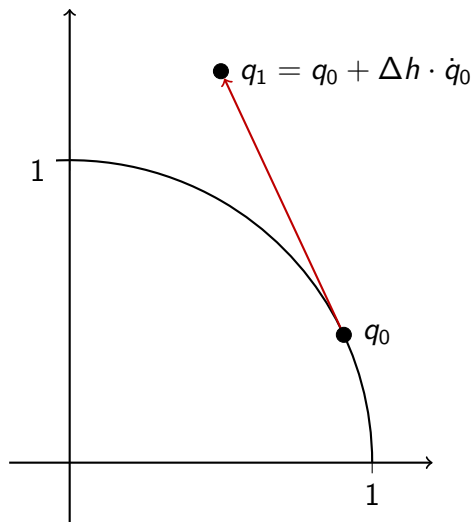
$$q = a + ib + jc + kd \quad a, b, c, d \in \mathbb{R}$$

$$\Leftrightarrow$$

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

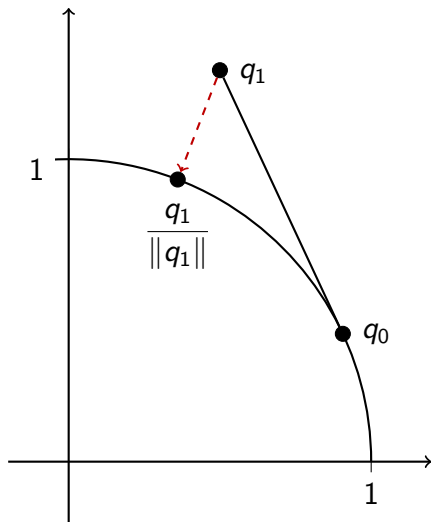
$$\text{represent rotation} \quad \Leftrightarrow \quad \|q\| = 1 \quad \Leftrightarrow \quad q \in \mathcal{S}^3$$

# Drift Correction



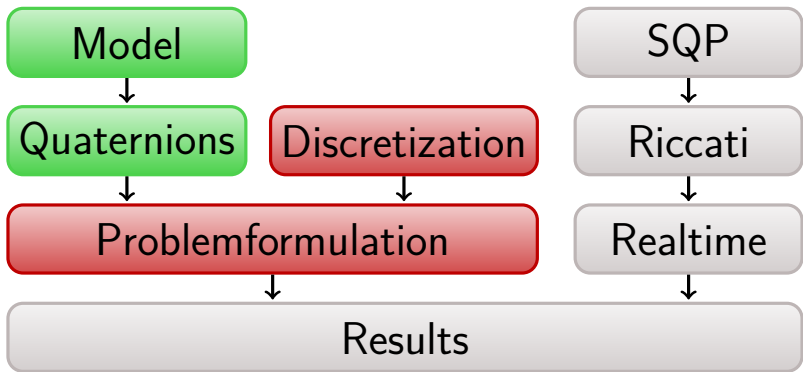
$$\dot{q}(t) = \tilde{f}(q(t))$$

# Drift Correction

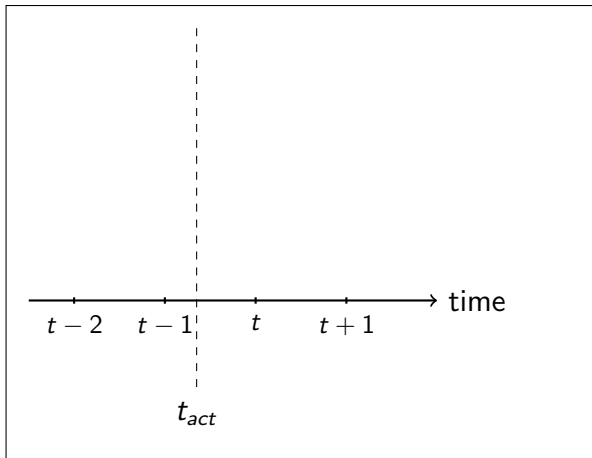


$$\dot{q}(t) = \tilde{f}(q(t)) - \lambda(q(t))$$

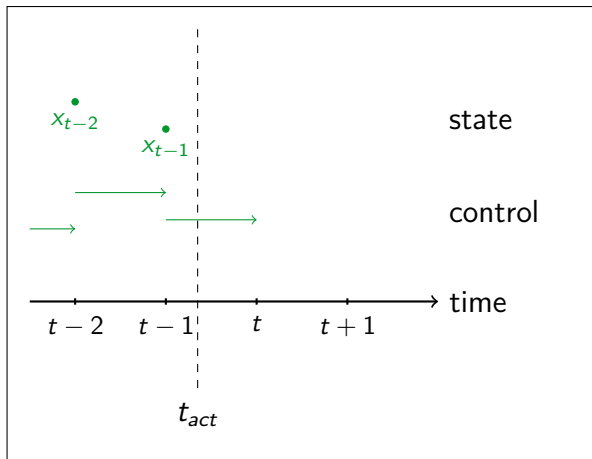




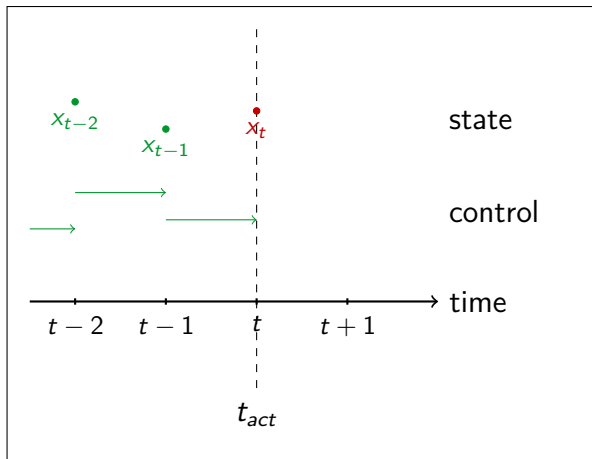
# Setting



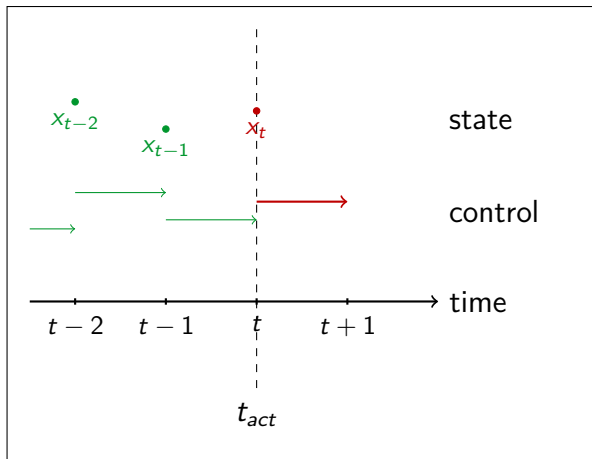
# Setting



# Setting



# Setting

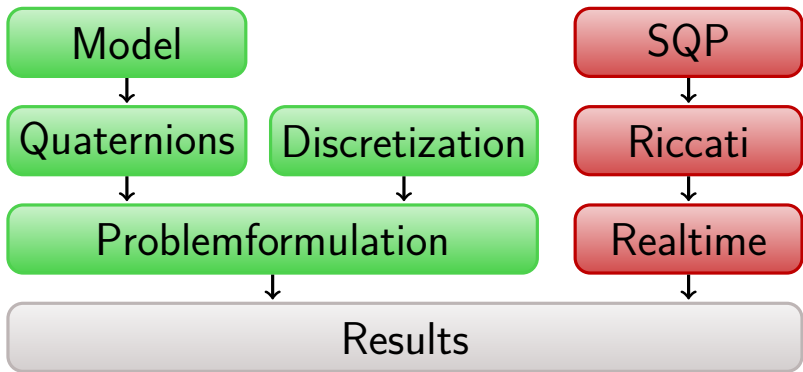


# Discrete Problem

$$\min_{x,u} \sum_{i=t}^{t+N} J_i(x_i, u_i) \quad \text{s.t.} \quad h_i(x_i, u_i) = 0 \quad i = t, \dots, t + N$$

$J_i(x_i, u_i)$  discretized goal function

$h_i(x_i, u_i)$  equality constraints at time  $i$



# The Lagrangian

$$L(y) = \sum_{i=t}^{t+N} J_i(x_i, u_i) + \sum_{i=t}^{t+N} \lambda_i^T h_i(x_i, u_i)$$

$$y := (\lambda, x, u)$$

$$y^* \text{ optimal} \quad \Leftrightarrow \quad \nabla_y L(y^*) = 0$$

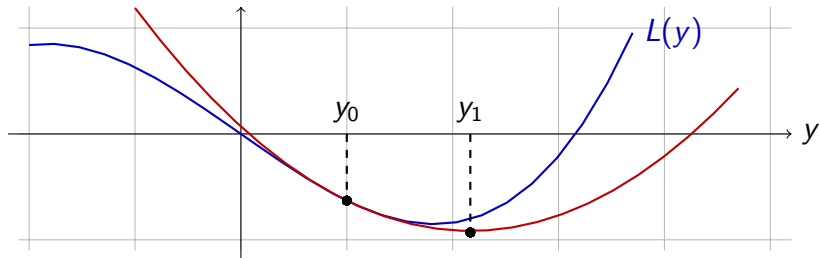


# The SQP Method

Find  $y^*$ :

$$y_1 = y_0 + s$$

$$\min_s \frac{1}{2} s^T \nabla^2 L(y_0) s + \nabla L(y_0)^T s$$



# Quasi Newton-Method

Find  $s$  with:

$$\nabla L(y_0) + \nabla^2 L(y_0)s = 0$$

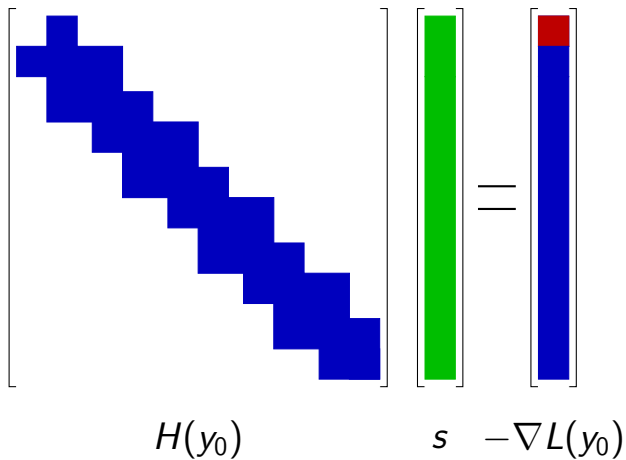
Approximate  $\nabla^2 L(y_0)$  and solve:

$$H(y_0)s = -\nabla L(y_0)$$

# Riccati Recursion

$$H(y_0) \quad s \quad -\nabla L(y_0)$$

# Riccati Recursion

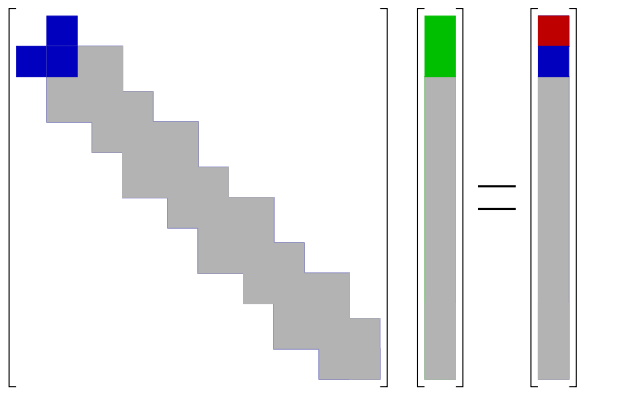

$$\begin{bmatrix} \text{Blue Matrix} \end{bmatrix} = \begin{bmatrix} \text{Green Vector} \end{bmatrix} - \begin{bmatrix} \text{Blue Vector} \end{bmatrix}$$

$H(y_0)$        $s$        $-\nabla L(y_0)$

# Riccati Recursion

$$H(y_0) \quad s \quad = \quad -\nabla L(y_0)$$

# Riccati Recursion


$$H(y_0) \quad s \quad -\nabla L(y_0)$$

# Summary

What happens in interval  $[t - 1, t]$  ?



# Summary

What happens in interval  $[t - 1, t]$  ?



- 1 calculate control  $u_{t-1}$  (Riccati Part II)



# Summary

What happens in interval  $[t - 1, t]$  ?



- 1 calculate control  $u_{t-1}$  (Riccati Part II)
- 2 calculate  $y$  (Riccati Part II)

# Summary

What happens in interval  $[t - 1, t]$  ?



- 1 calculate control  $u_{t-1}$  (Riccati Part II)
- 2 calculate  $y$  (Riccati Part II)
- 3 prepare  $u_t$  (Newton & Riccati Part I)

# Summary

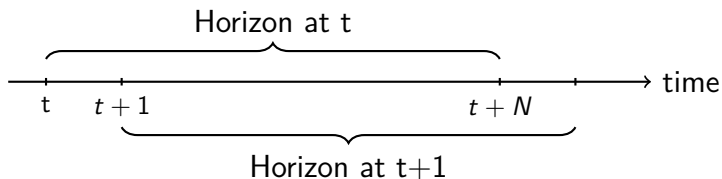
What happens in interval  $[t - 1, t]$  ?



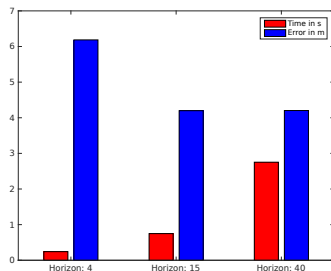
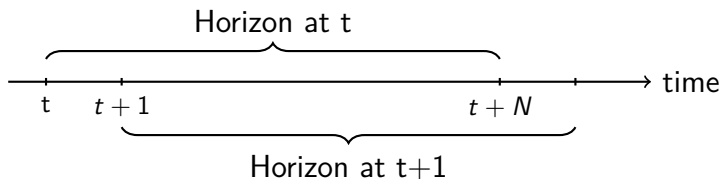
- 1 calculate control  $u_{t-1}$  (Riccati Part II)
- 2 calculate  $y$  (Riccati Part II)
- 3 prepare  $u_t$  (Newton & Riccati Part I)

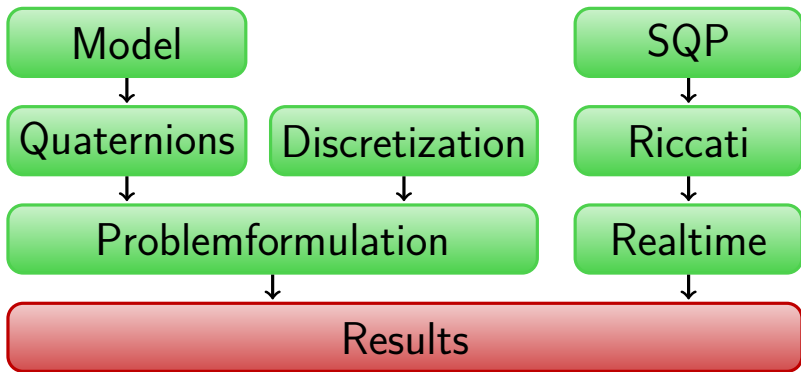
$\Rightarrow$  with horizon 18 this is 28% faster.

# Finite Horizon

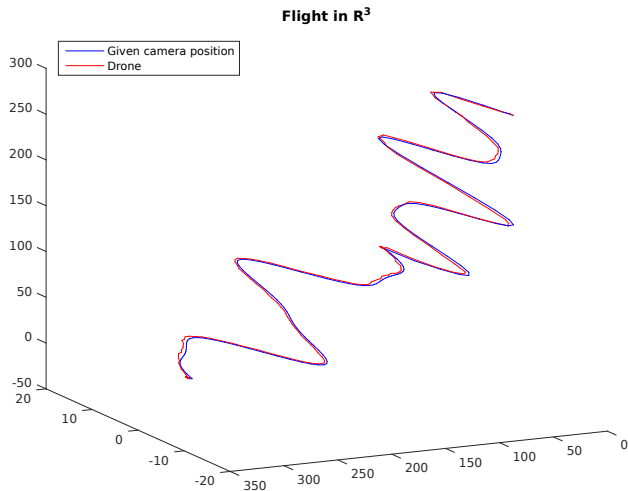


# Finite Horizon

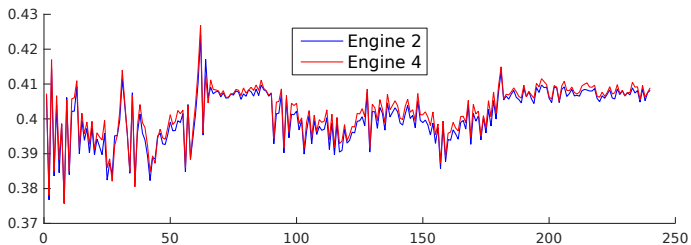
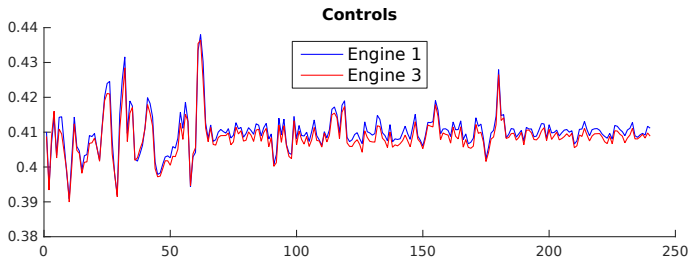




# Following a Skier

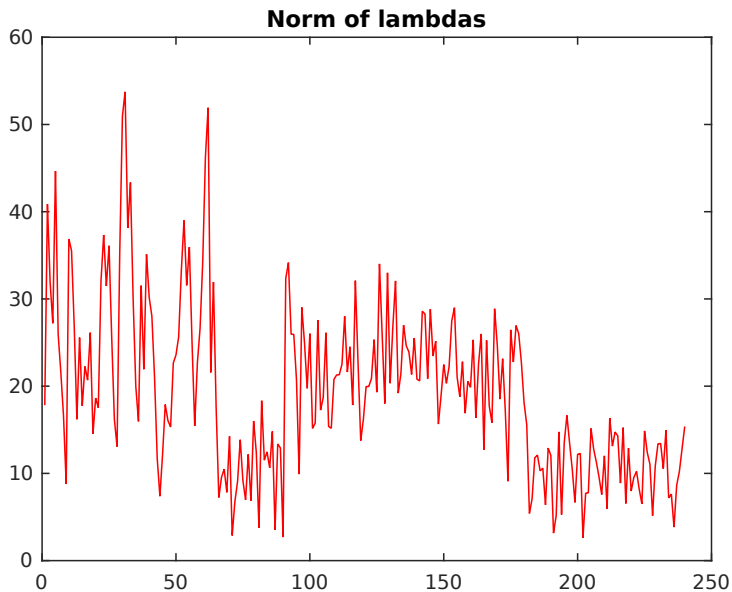


# Following a Skier





# Following a Skier



# References I



Stephen Boyd.

Solving the lqr problem by block elimination.

Lecture notes, 2009.



James Diebel.

Representing attitude: Euler angles, unit quaternions, and rotation vectors.

10 2006.



Moritz Diehl, Hans Georg Bock, and Johannes P. Schlöder.

A real-time iteration scheme for nonlinear optimization in optimal feedback control.

*SIAM J. Control Optim.*, 2005.

# References II



Moritz Diehl, Bock H. Georg, Johannes P. Schlöder, Rolf Findeisen, Zoltan Nagy, and Frank Allgöwer.

Real-time optimization and nonlinear model predictive control of processes governed by differential-algebraic equations.

*Journal of Process Control*, 2002.



Moritz Mathias Diehl.

*Real-Time Optimization for Large Scale Nonlinear Processes.*

PhD thesis, Ruprecht-Karls-Universität Heidelberg, 2001.



Luis Rodolfo Garcia Carrillo, Alejandro Enrique Dzul Lopez, Rogelio Lozano, and Claude Pegard.

*Quad Rotorcraft Control.*

Springer-Verlag London, 2013.