Real Time Control of a Quadcopter

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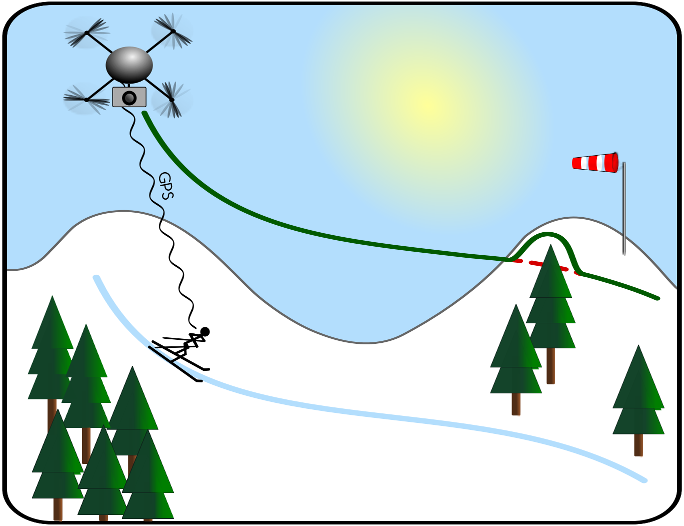
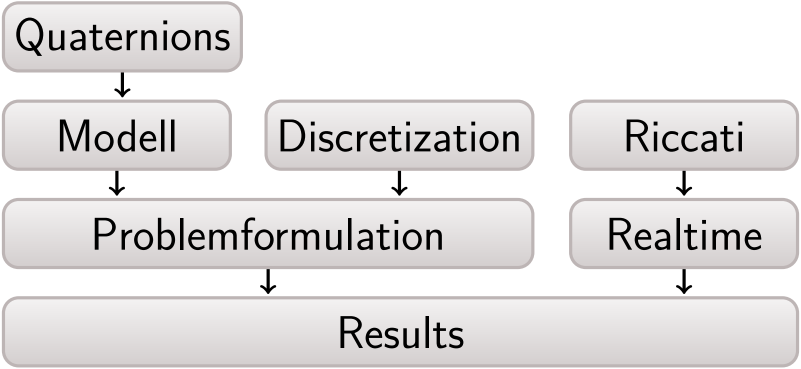
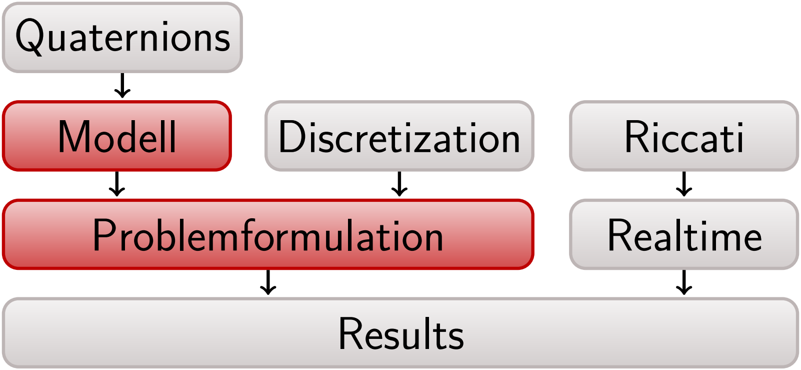
Technische Universität München

11 July 2015

Motivation




Quaternions
Modell
Discretization
Riccati
Problemformulation
Realtime
Results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Optimal Control Formulation

$$\min_{x,u} J(x,u) \qquad \text{s.t.} \qquad \begin{array}{l} \tilde{h}(x,u) = 0 \\ \dot{x} = f(x,u) \end{array}$$

$$\begin{array}{rl} x: & \text{state} \\ u: & \text{control} \end{array}$$

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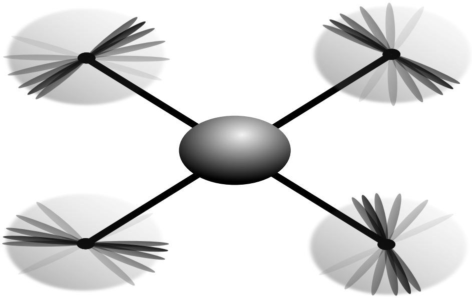
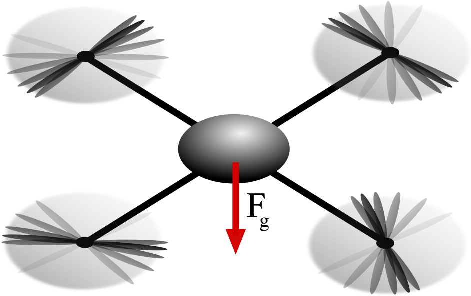
Model



Forces



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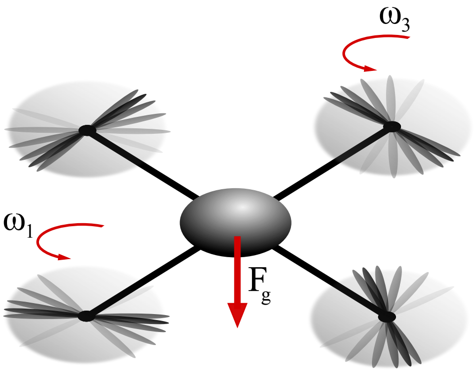

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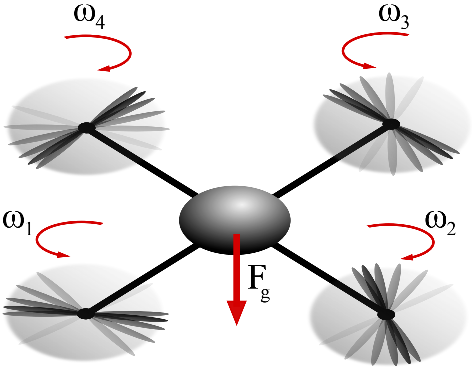

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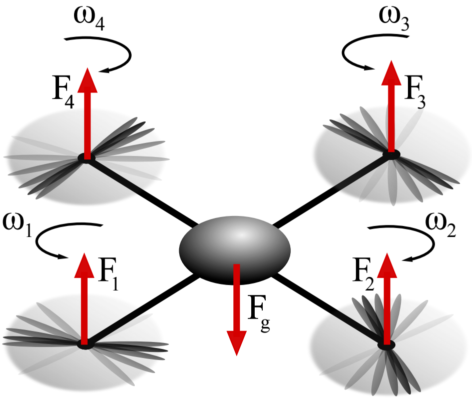

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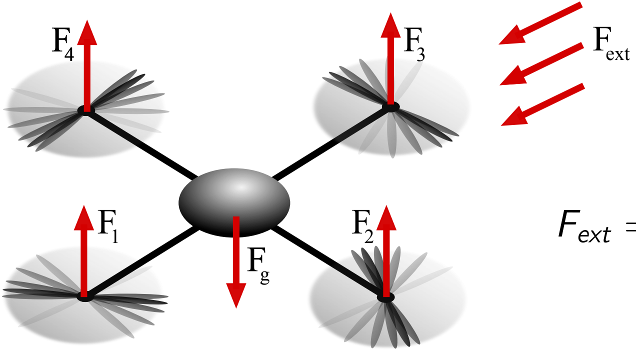
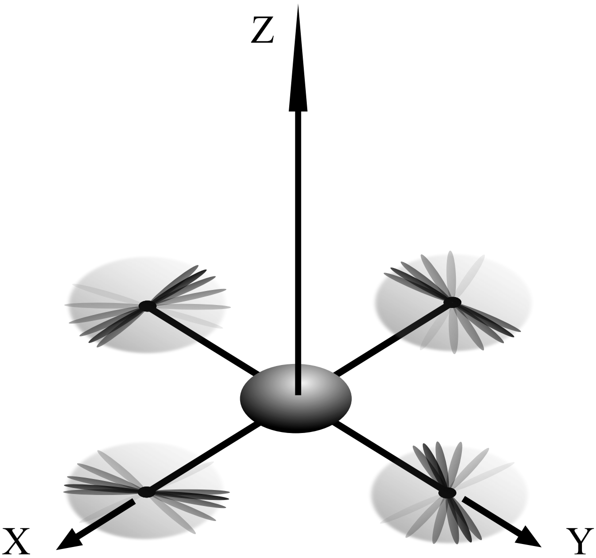


$$F_{ext} = F_g + \sum_{i=1}^{4} F_i$$

Torques



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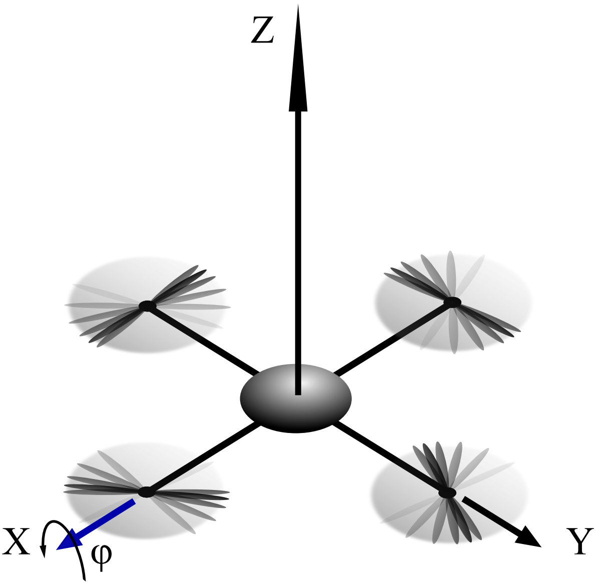

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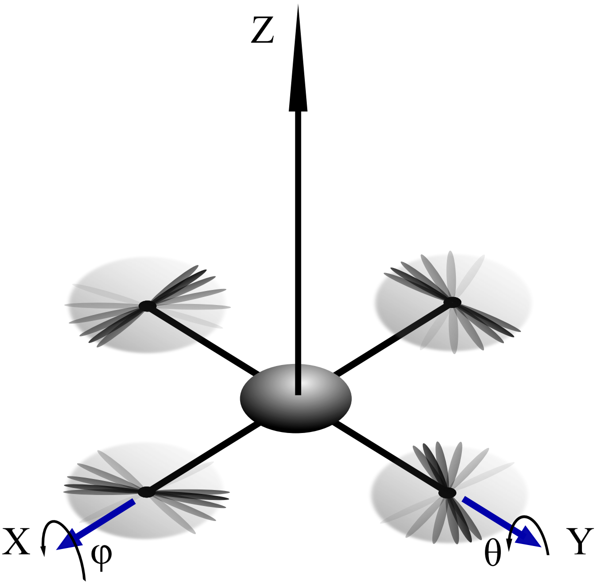
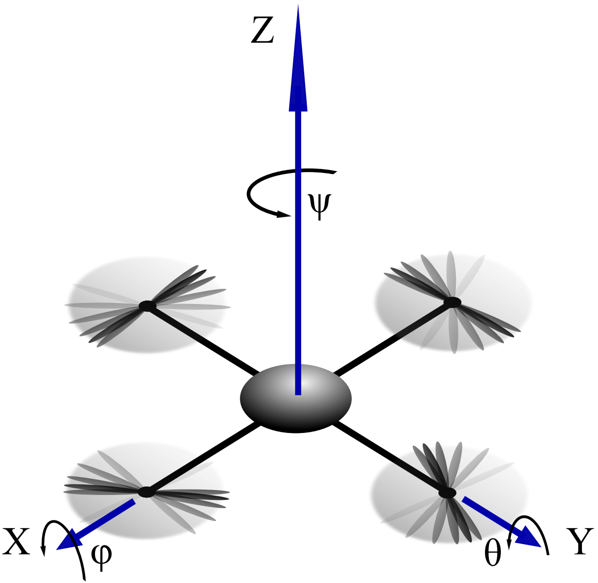


Torques



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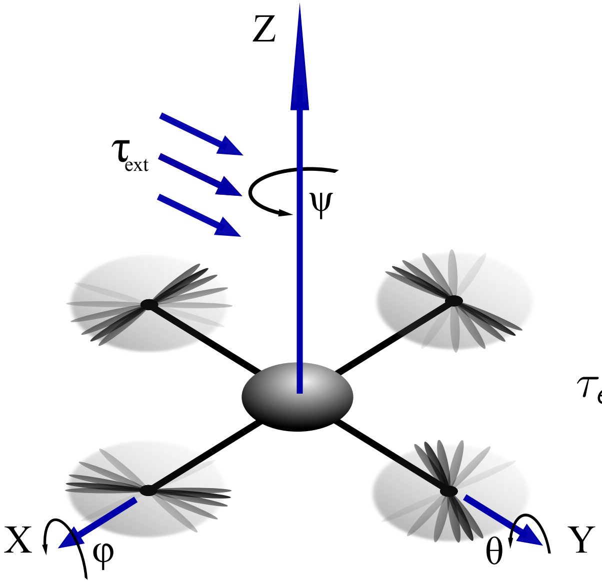
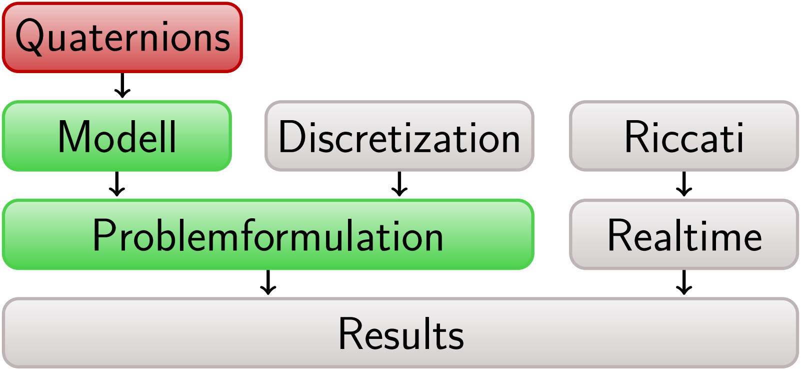


$$\tau_{ext} = \tau_\psi + \tau_\varphi + \tau_\theta$$

Obtain ODE

$$\left.\begin{array}{l} F_{ext} = F_g + \sum_{i=1}^{4} F_i \\ \tau_{ext} = \tau_\psi + \tau_\varphi + \tau_\theta \end{array}\right\} \quad \Rightarrow \quad \dot{x} = f(x, u)$$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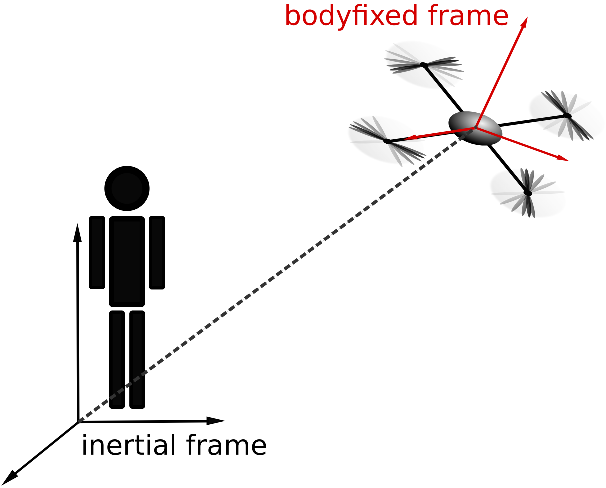
Coordinate Systems



Quaternions

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

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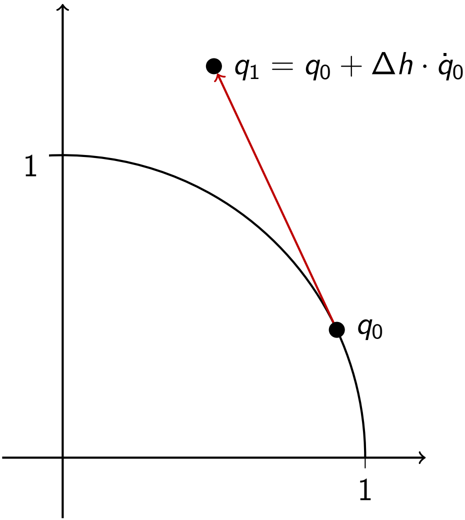
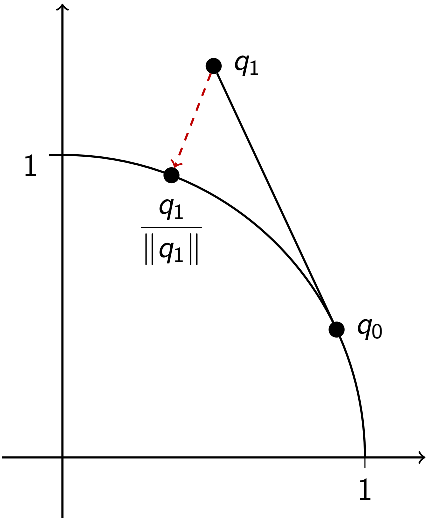
represent rotation $\quad \Leftrightarrow \quad \|q\| = 1 \quad \Leftrightarrow \quad q \in \mathcal{S}^3$

Drift Correction

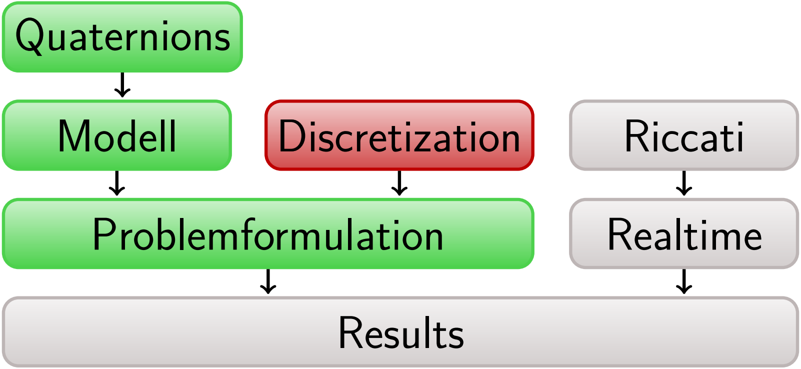


$$\dot{q} = \tilde{f}(q)$$

Drift Correction



$$\dot{q} = \tilde{f}(q) - \lambda(q)$$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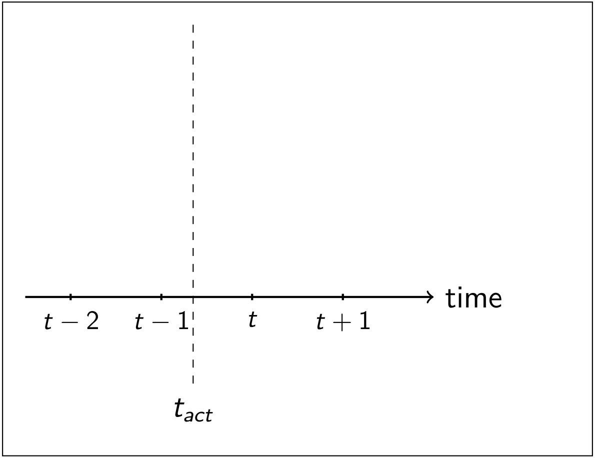
Setting



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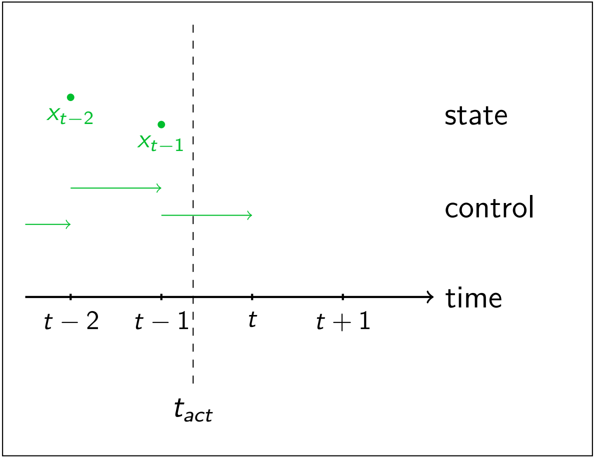

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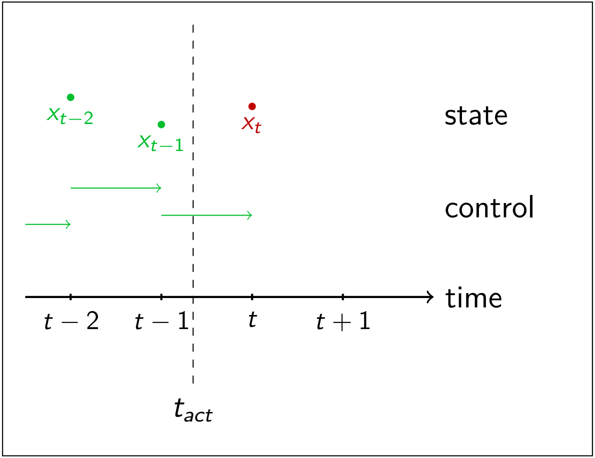
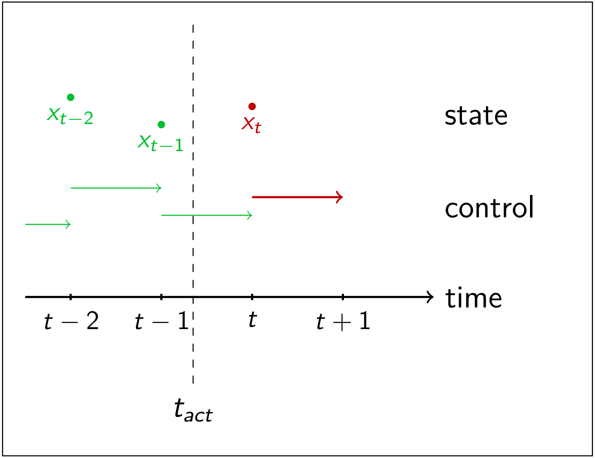


Setting



Discrete Problem

$$\min_{x,u} \sum_{i=t}^{N} J_i(x_i, u_i) \quad \text{s.t.} \quad h_i(x_i, u_i) = 0 \quad i = t, \ldots, N$$

$J_i(x_i, u_i)$ discretized goal function

$h_i(x_i, u_i)$ equality condition at time i

The Lagrangian

$$L^t(y) = \sum_{i=t}^{N} J_i(x_i, u_i) + \sum_{i=t}^{N} \lambda_i^T h_i(x_i, u_i)$$

$$y := (\lambda, x, u)$$

$$y^* \text{ optimal} \Leftrightarrow \nabla_y L^t(y^*) = 0$$

The SQP Method

Find y^*:

$$y_{k+1} = y_k + s_k$$

$$\min_{s_k} \frac{1}{2} s_k^T \nabla^2 L(y_k) s_k + \nabla L(y_k)^T s_k$$

Quasi Newton-Method

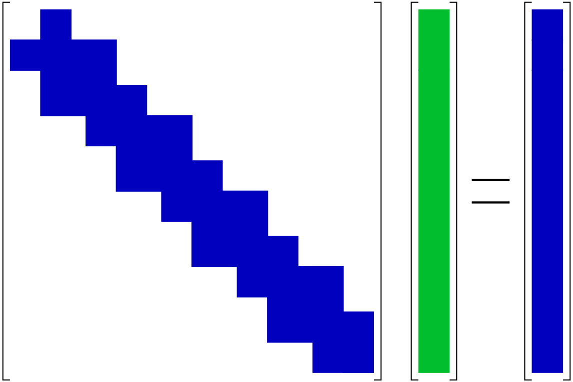
Find s_k with:

$$\nabla L(y_k) + \nabla^2 L(y_k) s_k = 0$$

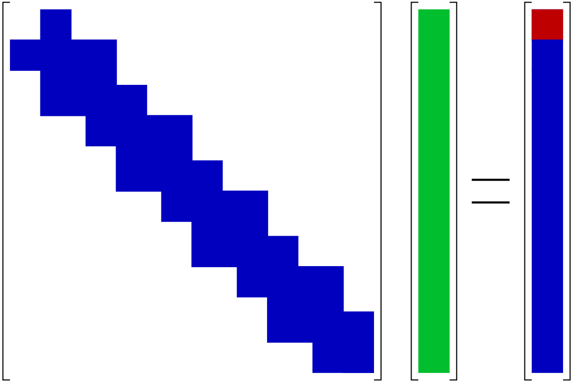
Approximate $\nabla^2 L(y_k)$ and solve:

$$H(y_k) s_k = -\nabla L(y_k)$$

Riccati Recursion



Riccati Recursion



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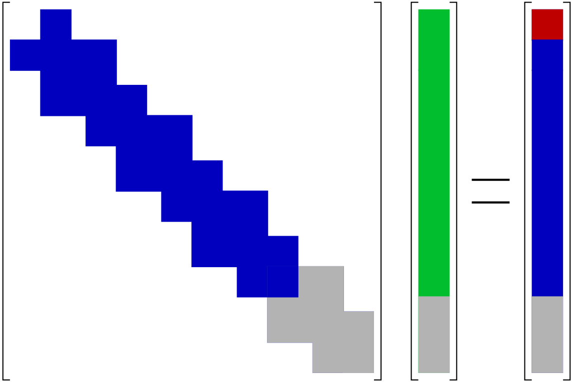

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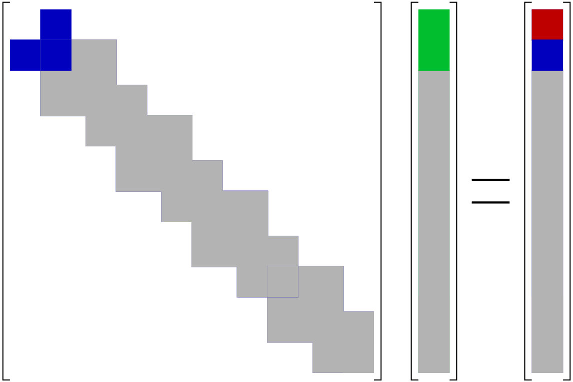

Summary

What happens in interval $[t-1, t]$?

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1. calculate control u_{t-1} (Riccati Part II)

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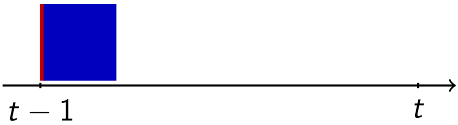


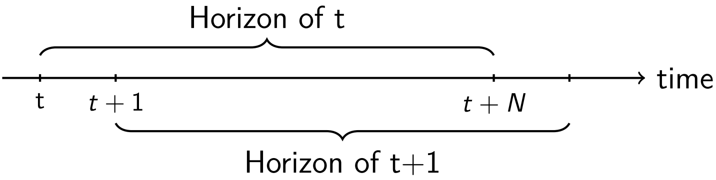
1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)

Summary

What happens in interval $[t-1, t]$?

1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)
3. prepare u_t (Newton & Riccati Part I)

Finite Horizon



	runtime	error
$N = 20$		
$N = 50$		
$N = 100$		

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