Real Time Control of a Quadcopter

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Technische Universität München

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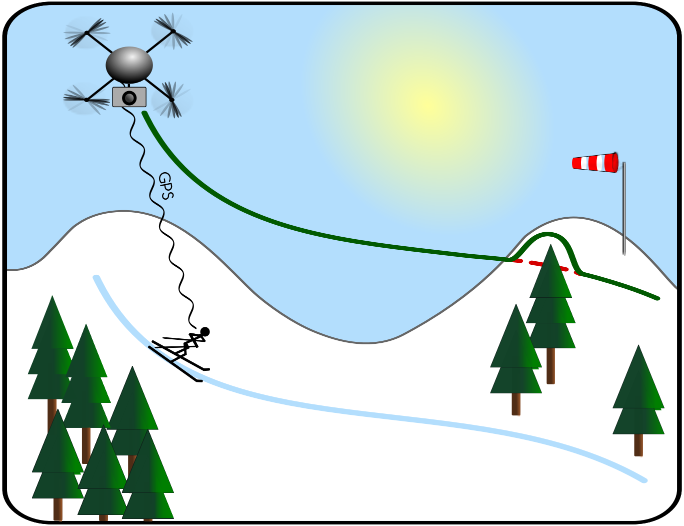
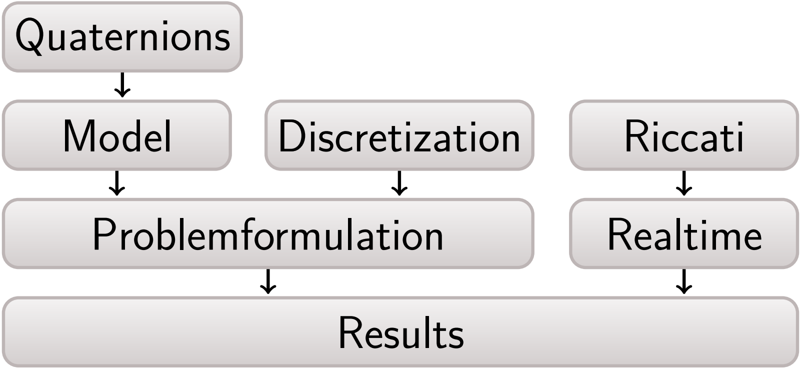
Motivation




Quaternions
Model
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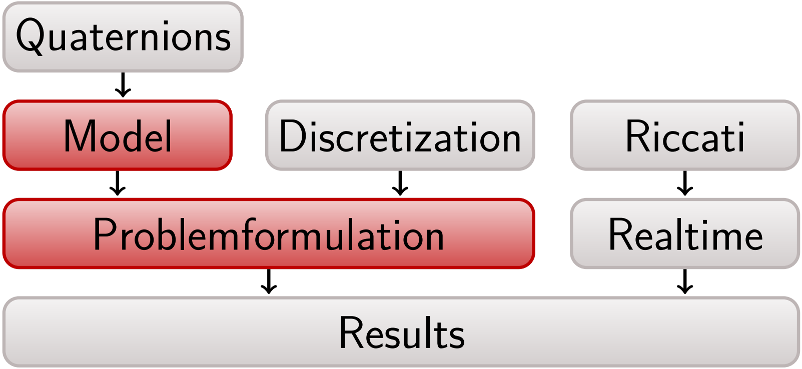
Problemformulation
Realtime
Results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Optimal Control Formulation

$$\min_{x,u} J(x,u) \quad \text{s.t.} \quad \begin{aligned} \tilde{h}(x,u) &= 0 \\ \dot{x}(t) &= f(x(t), u(t)) \end{aligned}$$

x : state

u : control

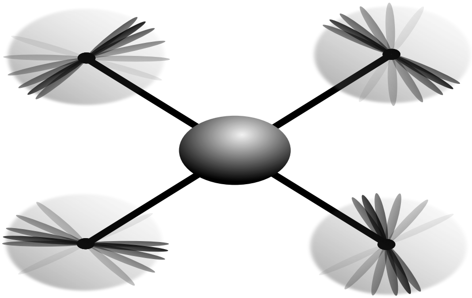
Optimal Control Formulation

$$\min_{x,u} J(x,u) \quad \text{s.t.} \quad \left.\begin{array}{c} \tilde{h}(x,u) = 0 \\ \dot{x}(t) = f(x(t), u(t)) \end{array}\right\} \Rightarrow h(x,u) = 0$$

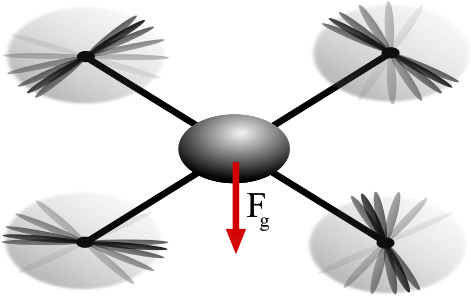
x : state

u : control

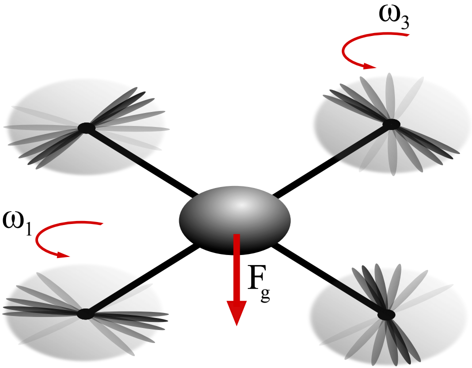
Model



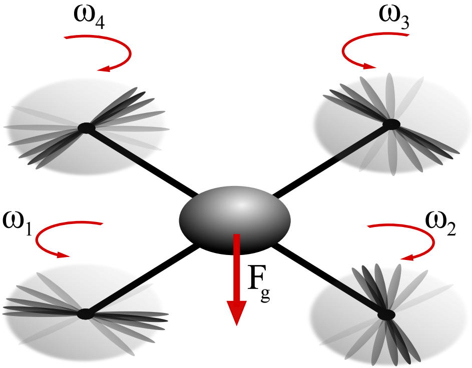
Model



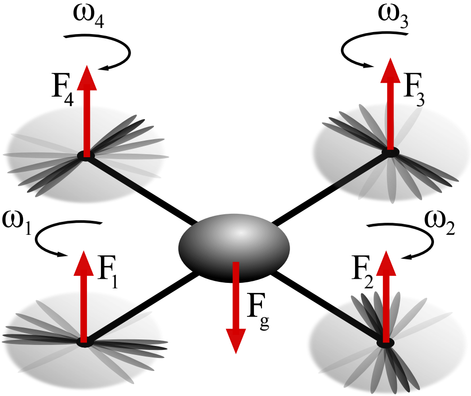
Model



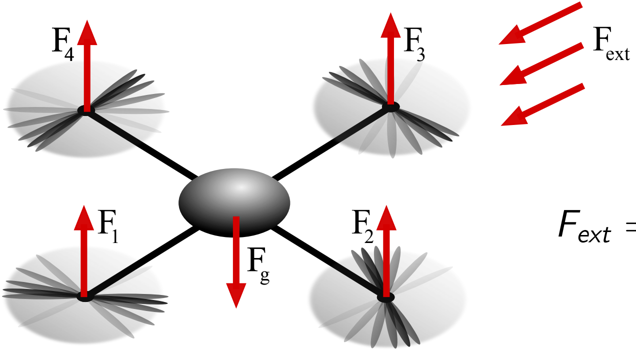
Model



Model

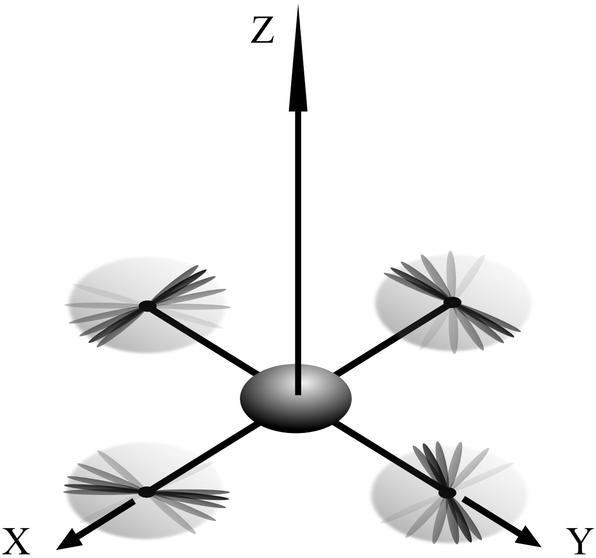


Forces

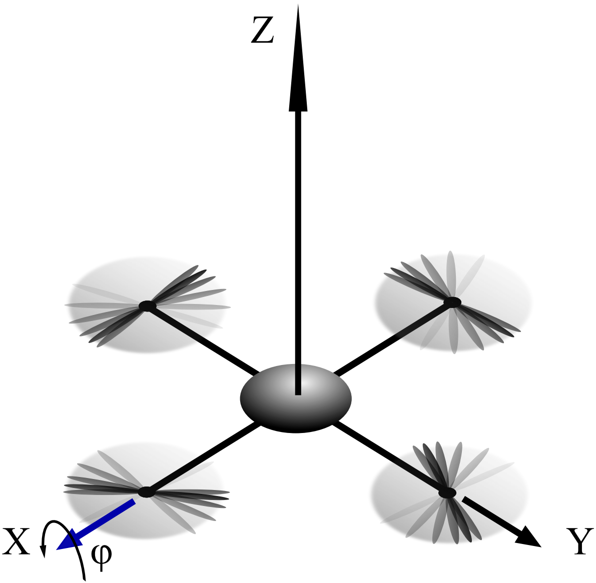


$$F_{ext} = F_g + \sum_{i=1}^{4} F_i$$

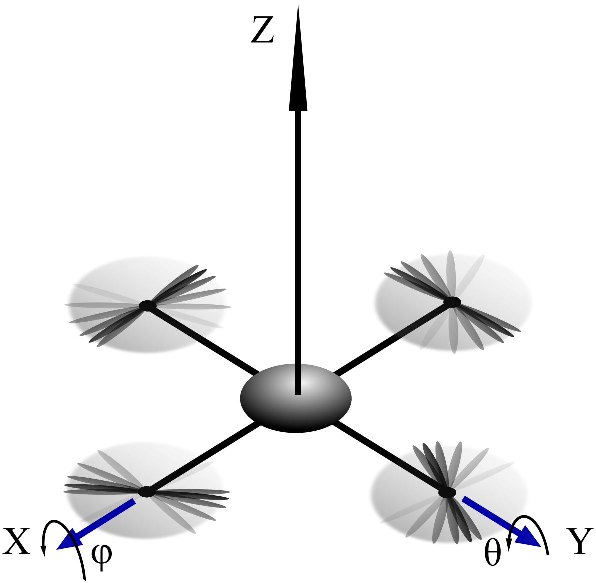
Torques



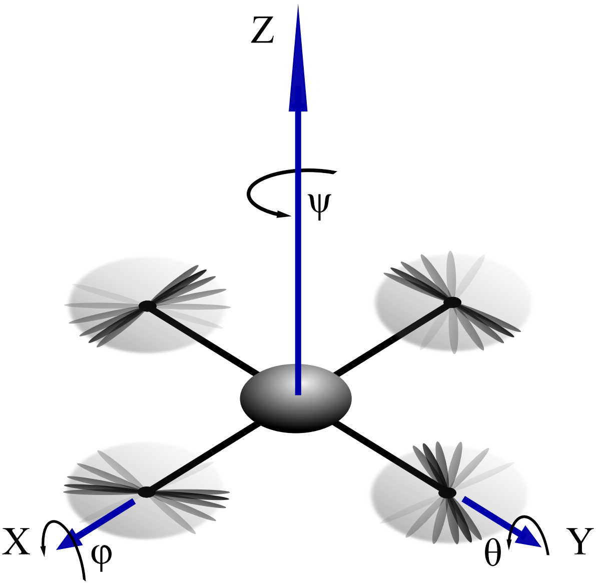
Torques



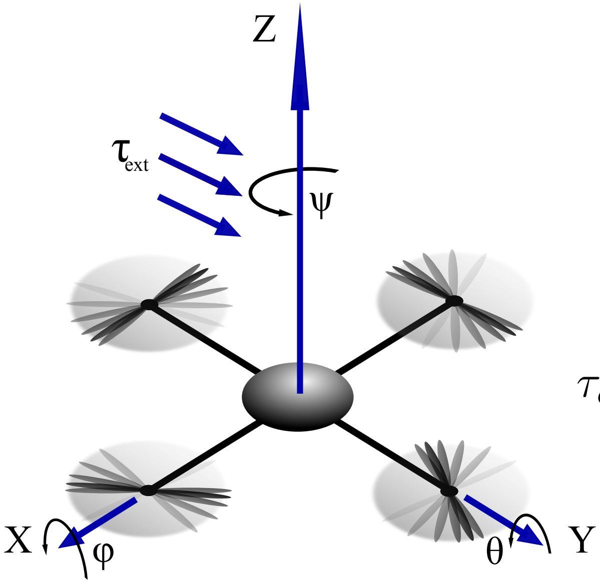
Torques



Torques



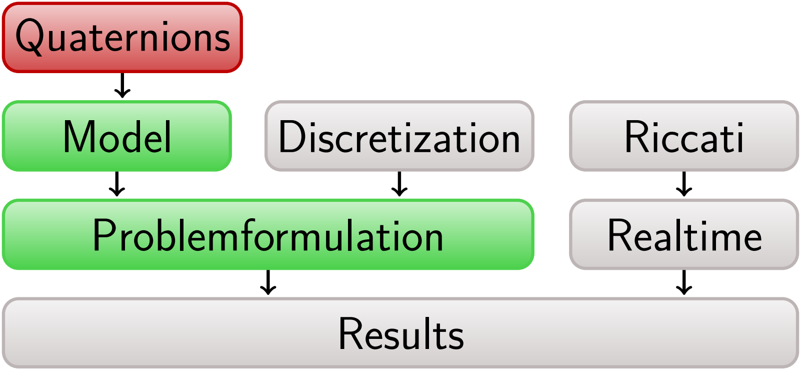
Torques



$$\tau_{ext} = \tau_{\psi} + \tau_{\varphi} + \tau_{\theta}$$

Obtain ODE

$$\left.\begin{array}{l} F_{ext} = F_g + \sum_{i=1}^{4} F_i \\ \tau_{ext} = \tau_\psi + \tau_\varphi + \tau_\theta \end{array}\right\} \quad \Rightarrow \quad \dot{x}(t) = f(x(t), u(t))$$



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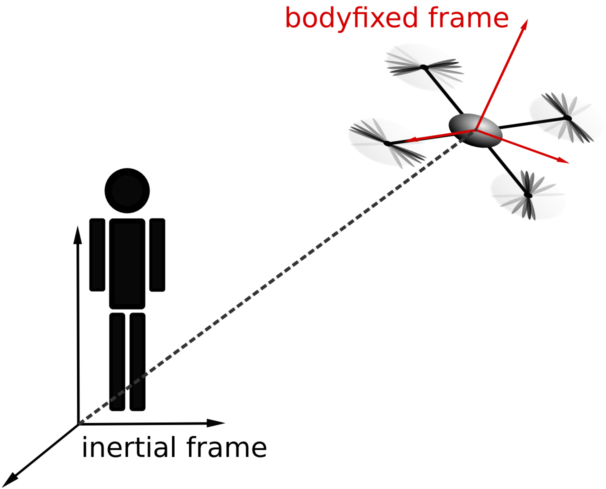
Coordinate Systems



Quaternions

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

$$\Leftrightarrow$$

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

Quaternions

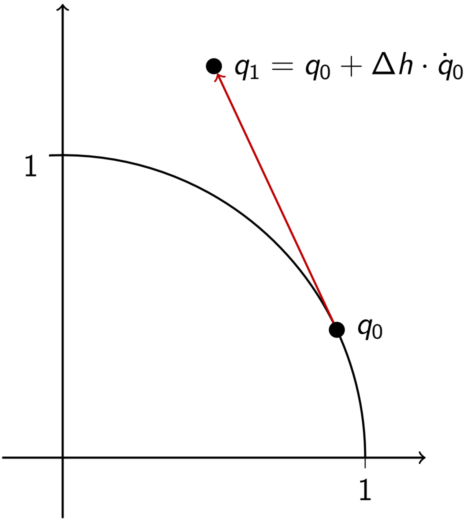
$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

$$\Leftrightarrow$$

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in \mathbb{R}^4$$

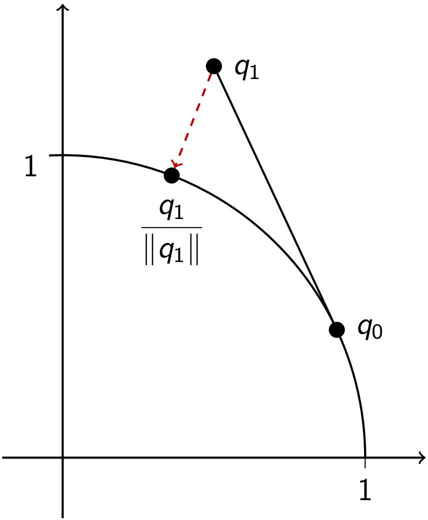
represent rotation $\quad \Leftrightarrow \quad \|q\| = 1 \quad \Leftrightarrow \quad q \in \mathcal{S}^3$

Drift Correction

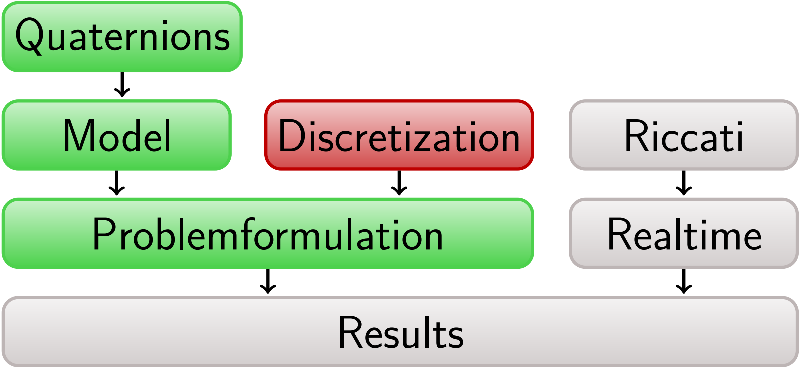


$$\dot{q}(t) = \tilde{f}(q(t))$$

Drift Correction



$$\dot{q}(t) = \tilde{f}(q(t)) - \lambda(q(t))$$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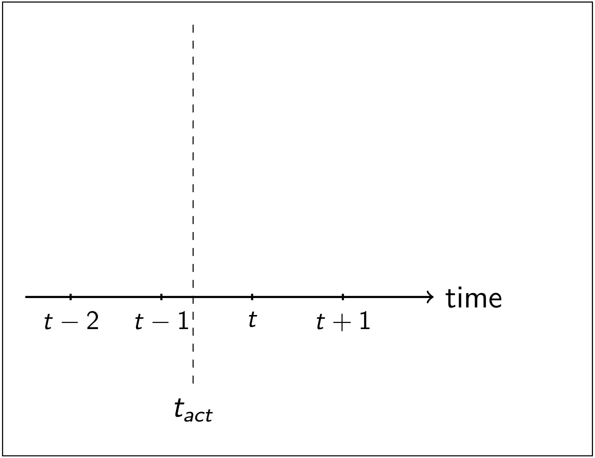
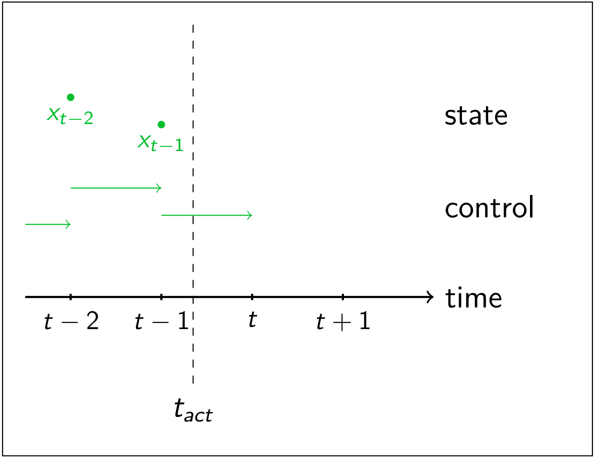
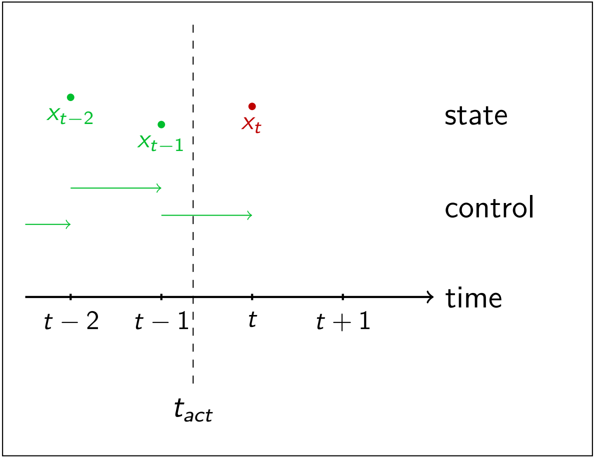
Setting



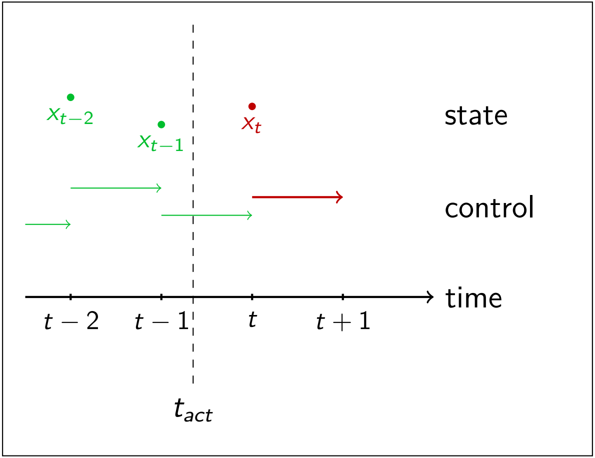
Setting



Setting



Setting

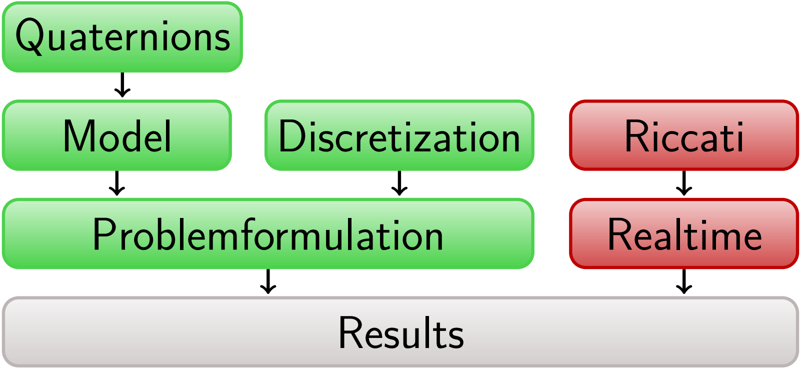


Discrete Problem

$$\min_{x,u} \sum_{i=t}^{N} J_i(x_i, u_i) \qquad \text{s.t.} \quad h_i(x_i, u_i) = 0 \quad i = t, \ldots, N$$

$J_i(x_i, u_i)$ discretized goal function

$h_i(x_i, u_i)$ equality constraints at time i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The Lagrangian

$$L(y) = \sum_{i=t}^{N} J_i(x_i, u_i) + \sum_{i=t}^{N} \lambda_i^T h_i(x_i, u_i)$$

$$y := (\lambda, x, u)$$

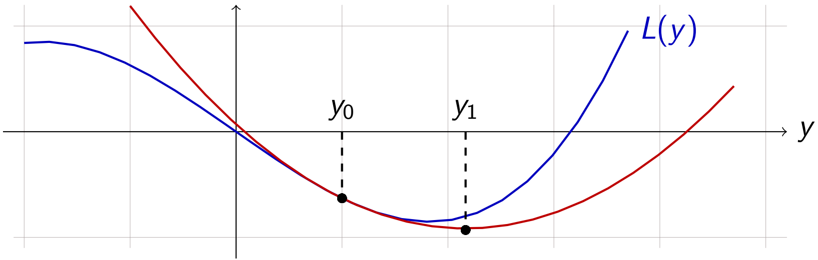
$$y^* \text{ optimal} \quad \Leftrightarrow \quad \nabla_y L(y^*) = 0$$

The SQP Method

Find y^*:

$$y_1 = y_0 + s$$

$$\min_s \frac{1}{2} s^T \nabla^2 L(y_0) s + \nabla L(y_0)^T s$$



Quasi Newton-Method

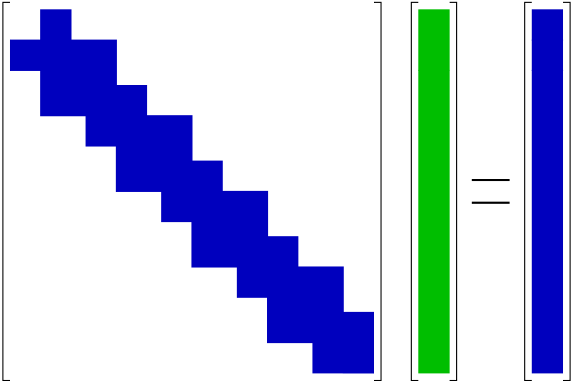
Find s with:

$$\nabla L(y_0) + \nabla^2 L(y_0)s = 0$$

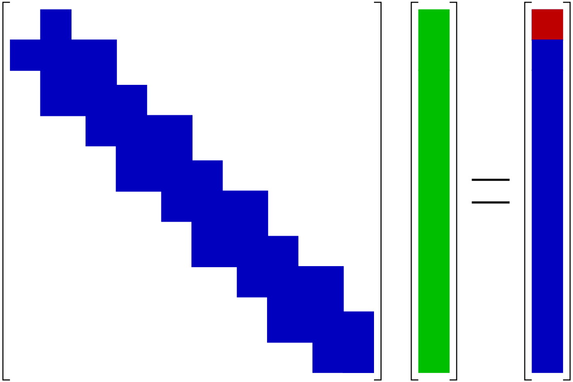
Approximate $\nabla^2 L(y_0)$ and solve:

$$H(y_0)s = -\nabla L(y_0)$$

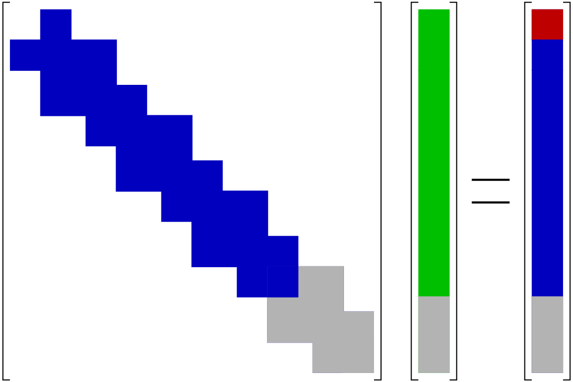
Riccati Recursion



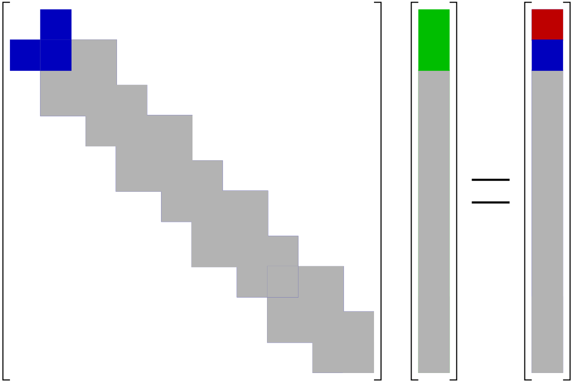
Riccati Recursion



Riccati Recursion

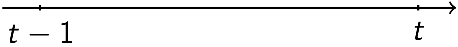


Riccati Recursion



Summary

What happens in interval $[t-1, t]$?



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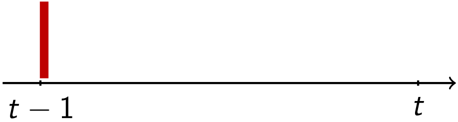
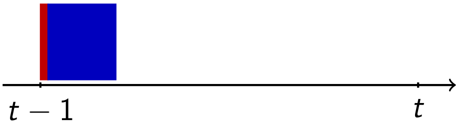


1. calculate control u_{t-1} (Riccati Part II)

Summary

What happens in interval $[t-1, t]$?



1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)

Summary

What happens in interval $[t-1, t]$?

1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)
3. prepare u_t (Newton & Riccati Part I)

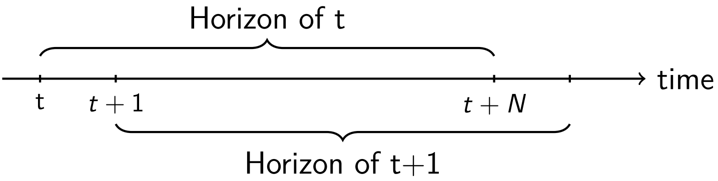
Summary

What happens in interval $[t-1, t]$?

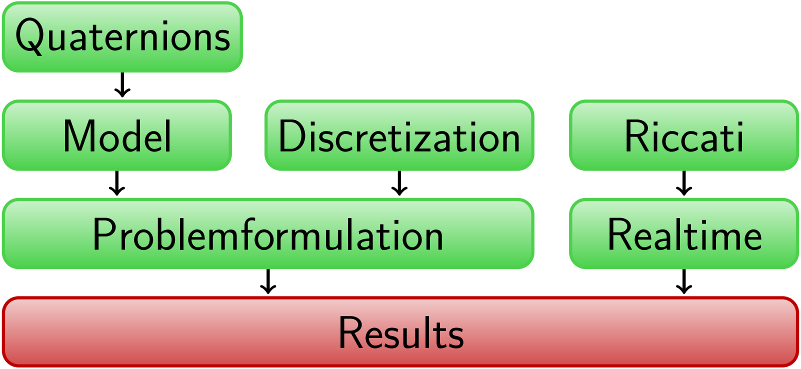
1. calculate control u_{t-1} (Riccati Part II)
2. calculate y (Riccati Part II)
3. prepare u_t (Newton & Riccati Part I)

$\Rightarrow$ with horizon 15 this is 25% faster.

Finite Horizon



	runtime	error
$N=20$		
$N=50$		
$N=100$		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