

Mid-term presentation

The Quadrocopters

Technische Universität München

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1 Model

2 Realtime Optimization Approach

Newton-Euler Equations

Forces

$$F_{\text{ext}} = F_g + \sum_{i=1}^4 F_i$$

Torques

$$\tau_{\text{ext}} = \sum_{i=1}^4 \tau_i + (\tau_\phi + \tau_\theta)$$

Quaternions

$$q = a + ib + jc + kd \quad a, b, c, d \in \mathbb{R}$$

representing rotation $\Leftrightarrow \|q\| = 1$

Advantage \rightarrow no singularities

Problem $\rightarrow \|q\| = 1$ additional constraint

$$T(x, u) = M \cdot \begin{pmatrix} \dot{x}_8 \\ \vdots \\ \dot{x}_{13} \end{pmatrix} + \Theta(x)$$

$$\frac{d}{dt} \begin{pmatrix} x_1 \\ \vdots \\ x_7 \\ x_8 \\ \vdots \\ x_{13} \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_7 \\ M^{-1}(T(x, u) - \Theta(x)) \end{pmatrix}$$

Refinement of Model
wind
aerodynamical forces

y, s, q erklären

Minimization Problem

$$\min_{\substack{s_k, \dots, s_N \\ q_k, \dots, q_{N-1}}} \sum_{i=k}^{N-1} F_i(s_i, q_i) \quad \text{s.t.} \quad \begin{cases} x_k - s_k = 0 \\ h_i(s_i, q_i) - s_{i+1} = 0 \quad \forall i = k, \dots, N-1 \end{cases}$$

$F_i(s_i, q_i)$ discretized goal function

$x_k - s_k = 0$ expected state should be the real state at time k

$h_i(s_i, q_i)$ solution of the ODE at time i

The Lagrangian

$$L^k(y) = \sum_{i=k}^{N-1} F_i(s_i, q_i) + \lambda_k^T (x_k - s_k) + \sum_{i=k}^{N-1} \lambda_{i+1}^T (h_i(s_i, q_i) - s_{i+1})$$

We are looking for y^* satisfying the KKT conditions.

$$\Rightarrow \nabla_y L^k(y^*) = 0$$

The SQP method

How do we find y^* ?

$$y_{k+1} = y_k + \alpha_k \Delta y_k$$

\Downarrow

$$\min_{\Delta y} = \frac{1}{2} \Delta y^T A_k \Delta y + \nabla_y F(y_k)^T \Delta y$$

\Downarrow

$$A_k := \nabla_{y_k}^2 L(y_k).$$

Newton-Raphson

$$y_{k+1} = y_k + \Delta y_k$$
$$\nabla_y L^k(y_k) + J^k(y_k) \Delta y_k = 0$$

$$J^k(y_k) \quad \text{Approximated Hessian } \nabla_{y_k}^2 L(y_k)$$
$$\alpha_k = 1$$

Riccati Recursion

This formulation still depends on $x_t \dots$

$$J^k(y^k) = \begin{pmatrix} -E & & & & & & & & & \\ -E & Q_k^H & M_k^H & A_k^T & & & & & & \\ & (M_k^T)^H & R_k^H & B_k^T & & & & & & \\ & A_k & B_k & & -E & & & & & \\ & & & -E & Q_{k+1}^H & M_{k+1}^H & A_{k+1}^T & & & \\ & & & (M_{k+1}^T)^H & R_{k+1}^H & B_{k+1}^T & & & & \\ & & & A_{k+1} & B_{k+1} & & & & & \\ & & & & & & \ddots & & & \\ & & & & & & & \ddots & & \\ & & & & & & & & Q_{N-1}^H & M_{N-1}^H & A_{N-1}^T \\ & & & & & & & & (M_{N-1}^T)^H & R_{N-1}^H & B_{N-1}^T \\ & & & & & & & & A_{N-1} & B_{N-1} & -E \\ & & & & & & & & & -E & Q_N^H \end{pmatrix}$$

Finite Horizon

how to choose N ?

- $N = t_{end}$ problem gets smaller every time
- $N = t + n$ problem size is constant
- .
- .
- .