Mid-term presentation

The Quadrocopters

Technische Universität München

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Newton-Euler Equations

Forces

$$F_{ext} = F_g + \sum_{i=1}^{4} F_i$$

Torques

$$\tau_{ext} = \sum_{i=1}^{4} \tau_i + (\tau_\phi + \tau_\theta)$$

Quaternions

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a, b, c, d \in \mathbb{R}$$

representing rotation $\Leftrightarrow \|q\| = 1$
Advantage $\rightarrow$ no singularities
Problem $\rightarrow \|q\| = 1$ additional coontraint

Dynamics

$$T(x,u) = M \cdot \begin{pmatrix} \dot{x}_8 \\ \vdots \\ \dot{x}_{13} \end{pmatrix} + \Theta(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} x_1 \\ \vdots \\ x_7 \\ x_8 \\ \vdots \\ x_{13} \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_7 \\ M^{-1}(T(x,u) - \Theta(x)) \end{pmatrix}$$

Prospect

Refinement of Model
wind
aerodynamical forces

Realtime

y, s, q erklären

Minimization Problem

$$\min_{\substack{s_t, \ldots, s_N \\ q_t, \ldots, q_{N-1}}} \sum_{i=t}^{N-1} F_i(s_i, q_i) \quad s.t. \left\{ \begin{array}{c} x_t - s_t = 0 \\ h_i(s_i, q_i) - s_{i+1} = 0 \ \ \forall i = t, \ldots, N-1 \end{array} \right.$$

$F_i(s_i, q_i)$	discretized goal function
$x_t - s_t = 0$	expected state should be the real state at time t
$h_i(s_i, q_i)$	solution of the ODE at time i

The Lagrangian

$$L^t(y) = \sum_{i=t}^{N-1} F_i(s_i, q_i) + \lambda_t^T(x_t - s_t) + \sum_{i=t}^{N-1} \lambda_{i+1}^T(h_i(s_i, q_i) - s_{i+1})$$

We are looking for y^* satisfying the KKT conditions.

$$\Rightarrow \nabla_y L^t(y^*) = 0$$

The SQP method

How do we find y^*?

$$y_{k+1} = y_k + \alpha_k \Delta y_k$$

$$\Downarrow$$

$$\min_{\Delta y} = \frac{1}{2} \Delta y^T A_k \Delta y + \nabla_y F(y_k)^T \Delta y$$

$$\Downarrow$$

$$A_k := \nabla^2_{y_k} L(y_k).$$

Newton-Raphson

$$y_{t+1} = y_t + \Delta y_t$$
$$\nabla_{y_t} L^t(y_t) + J^t(y_t)\Delta y_t = 0$$

$J^t(y_t)$ Approximated Hessian $\nabla^2_{y_t} L(y_t)$
$\alpha_t = 1$

Riccati Recursion

This formulation still depends on x_t ...

$$
J^t(y^t) = \begin{pmatrix}
 & -E & & & & & & & & & \\
-E & Q_t^H & M_t^H & A_t^T & & & & & & & \\
 & (M_t^T)^H & R_t^H & B_t^T & & & & & & & \\
 & A_t & B_t & & -E & & & & & & \\
 & & & -E & Q_{t+1}^H & M_{t+1}^H & A_{t+1}^T & & & & \\
 & & & & (M_{t+1}^T)^H & R_{t+1}^H & B_{t+1}^T & & & & \\
 & & & & A_{t+1} & B_{t+1} & & & & & \\
 & & & & & & & \ddots & & & \\
 & & & & & & \ddots & Q_{N-1}^H & M_{N-1}^H & A_{N-1}^T & \\
 & & & & & & & (M_{N-1}^T)^H & R_{N-1}^H & B_{N-1}^T & \\
 & & & & & & & A_{N-1} & B_{N-1} & & -E \\
 & & & & & & & & & -E & Q_N^H
\end{pmatrix}
$$

Finite Horizon

how to choose N?

- $N = t_{end}$ problem gets smaller every time
- $N = t + n$ problem size is constant
- .
- .
- .