



# Mid-term presentation

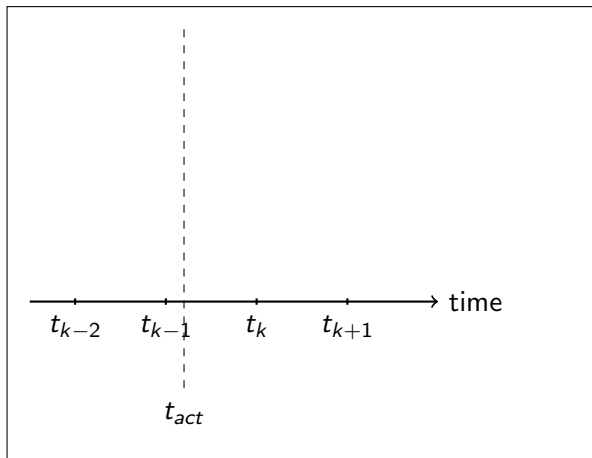
## The Quadrocopters

Technische Universität München

25. Mai 2015

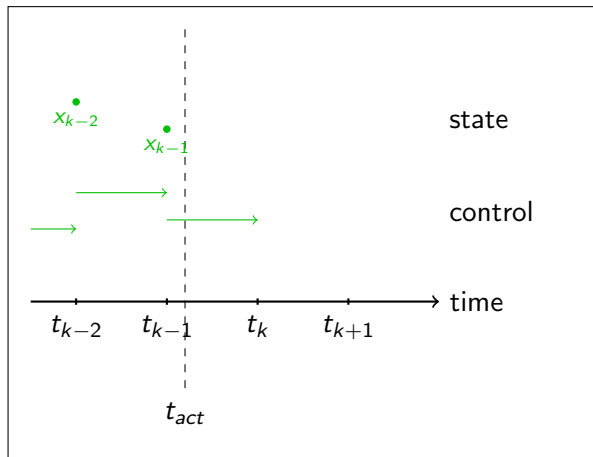
## 1 Realtime Optimization Approach

# Setting



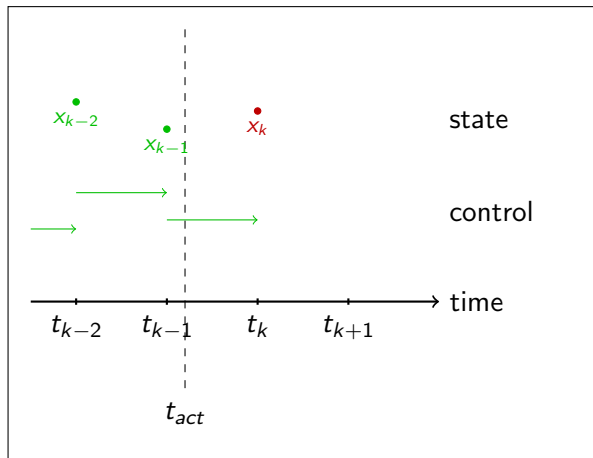
$y, s, q$  erklären

# Setting



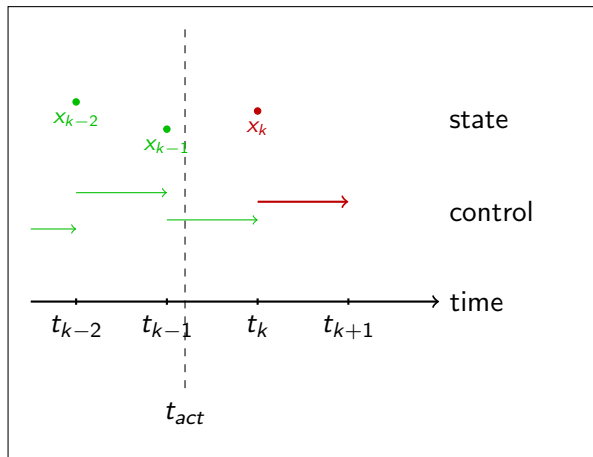
$y, s, q$  erklären

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$y, s, q$  erklären

# Minimization Problem

$$\min_{\substack{s_t, \dots, s_N \\ q_t, \dots, q_{N-1}}} \sum_{i=t}^{N-1} F_i(s_i, q_i) \quad s.t. \quad \begin{cases} x_t - s_t = 0 \\ h_i(s_i, q_i) - s_{i+1} = 0 \quad \forall i = t, \dots, N-1 \end{cases}$$

$F_i(s_i, q_i)$  discretized goal function

$x_t - s_t = 0$  expected state should be the real state at time  $t$

$h_i(s_i, q_i)$  solution of the ODE at time  $i$



# The Lagrangian

$$L^t(y) = \sum_{i=t}^{N-1} F_i(s_i, q_i) + \lambda_t^T (x_t - s_t) + \sum_{i=t}^{N-1} \lambda_{i+1}^T (h_i(s_i, q_i) - s_{i+1})$$

We are looking for  $y^*$  satisfying the KKT conditions.  
 $\Rightarrow \nabla_y L^t(y^*) = 0$

# The SQP method

How do we find  $y^*$ ?

$$y_{k+1} = y_k + \alpha_k \Delta y_k$$

$\Downarrow$

$$\min_{\Delta y} = \frac{1}{2} \Delta y^T A_k \Delta y + \nabla_y F(y_k)^T \Delta y$$

$\Downarrow$

$$A_k := \nabla_{y_k}^2 L(y_k).$$

# Newton-Raphson

$$y_{t+1} = y_t + \Delta y_t$$
$$\nabla_{y_t} L^t(y_t) + J^t(y_t) \Delta y_t = 0$$

$J^t(y_t)$     Approximated Hessian  $\nabla_{y_t}^2 L(y_t)$   
 $\alpha_t = 1$

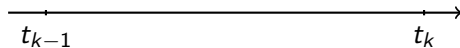
# Riccati Recursion

This formulation still depends on  $x_t \dots$

$$J^t(y^t) = \begin{pmatrix} -E & & & & & & & & & & \\ -E & Q_t^H & M_t^H & A_t^T & & & & & & & \\ & (M_t^T)^H & R_t^H & B_t^T & & & & & & & \\ & A_t & B_t & & -E & & & & & & \\ & & & -E & Q_{t+1}^H & M_{t+1}^H & A_{t+1}^T & & & & \\ & & & (M_{t+1}^T)^H & R_{t+1}^H & B_{t+1}^T & & & & & \\ & & & A_{t+1} & B_{t+1} & & & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & \ddots & & & \\ & & & & & & & & Q_{N-1}^H & M_{N-1}^H & A_{N-1}^T \\ & & & & & & & (M_{N-1}^T)^H & R_{N-1}^H & B_{N-1}^T & \\ & & & & & & & A_{N-1} & B_{N-1} & & -E \\ & & & & & & & & & -E & Q_N^H \end{pmatrix}$$

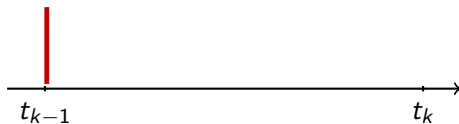
# Summary

What happens in interval  $[t_{k-1}, t_k]$  ?



# Summary

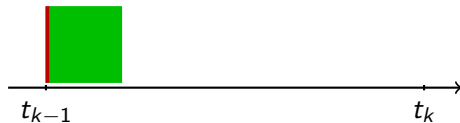
What happens in interval  $[t_{k-1}, t_k]$  ?



- 1 Calculate control  $u_{k-1}$

# Summary

What happens in interval  $[t_{k-1}, t_k]$  ?



- 1 Calculate control  $u_{k-1}$
- 2 Calculate  $y_k$  ( Riccati Part II)

# Summary

What happens in interval  $[t_{k-1}, t_k]$  ?



- ① Calculate control  $u_{k-1}$
- ② Calculate  $y_k$  ( Riccati Part II)
- ③ Prepare  $u_k$  (SQP & Riccati Part I)



# Finite Horizon

how to choose  $N$ ?

- $N = t_{end}$  problem gets smaller every time
- $N = t + n$  problem size is constant
- .
- .
- .