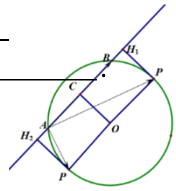
# 平面向量解答（5）

(1401学考）设$P$是半径为1的圆上一动点，若该圆的弦$AB=\sqrt{3}$，则$\overrightarrow{AP}\cdot\overrightarrow{AB}$的取值范围为________

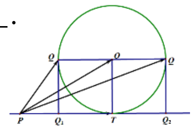


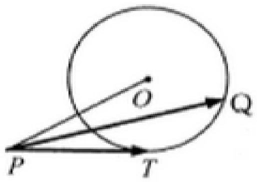
$1401key$:（投影：动向量·定向量）

$$\overrightarrow{AP}\cdot\overrightarrow{AB}=\frac{\overrightarrow{AP}\cdot\overrightarrow{AB}}{|\overrightarrow{AB}|}\cdot|\overrightarrow{AB}|\in[\frac{3}{2}-\sqrt{3},\frac{3}{2}+\sqrt{3}]$$

(1407会考）如图，点$P$是半径为1的圆$O$外一点，$OP=2$,过$P$作圆$O$的切线$PT$,

$T$为切点.若点$O$为圆$O$上一动点，则$\overrightarrow{PQ}\cdot\overrightarrow{PT}$的取值范围为________.



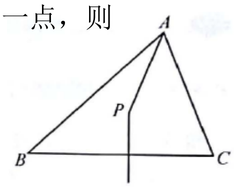




$$1407key:\overrightarrow{PQ}\cdot\overrightarrow{PT}=\frac{\overrightarrow{PQ}\cdot\overrightarrow{PT}}{|\overrightarrow{PT}|}\cdot|\overrightarrow{PT}|\in[3-\sqrt{3},3+\sqrt{3}]$$

（18 甘肃）2.在$\triangle ABC$中，已知$AB=4,AC=3$．如图所示，$P$是边$BC$的垂直平分线上一点，则

$\overrightarrow{BC}\cdot\overrightarrow{AP}=$______. $-\frac{7}{2}$





(202001)17.设点$A,B$的坐标分别为$(0,1),(1,0)$, $P,Q$分别是曲线$y=2^x$和$y=\log_2 x$上的动点，

记$I_1=\overrightarrow{AQ}\cdot\overrightarrow{AB},I_2=\overrightarrow{BP}\cdot\overrightarrow{BA}$.(　　)C

$A$.若$I_1=I_2$,则$\overrightarrow{PQ}=\lambda\overrightarrow{AB}(\lambda\in R)$　　$B$.若$I_1=I_2$,则$|\overrightarrow{AP}|=|\overrightarrow{BQ}|$

$C$.若$\overrightarrow{PQ}=\lambda\overrightarrow{AB}(\lambda\in R)$,则$I_1=I_2$　　$D$.若$|\overrightarrow{AP}|=|\overrightarrow{BQ}|$,则$I_1=I_2$

（202106 高考）17. 已知平面向量$\vec{a},\vec{b},\vec{c}(\vec{c}\neq\vec{0})$,满足$|\vec{a}|=1,|\vec{b}|=2,\vec{a}\cdot\vec{b}=0,(\vec{a}-\vec{b})\cdot\vec{c}=0$.记向量$\vec{d}$在$\vec{a},\vec{b}$方向上的投影分别为$x$，$y$，$\vec{d}-\vec{a}$在$\vec{c}$方向上的投影为$z$，则$x^2+y^2+z^2$的最小值为___________.

$key$:令$\vec{a}=(1,0),\vec{b}=(0,2),\vec{c}=(2,1)$,设$\vec{d}=(u,v)$，则$x=\vec{d}\cdot\vec{a}=u,y=\frac{\vec{d}\cdot\vec{b}}{|\vec{b}|}=v,z=\frac{\vec{d}-\vec{a}}{|\vec{c}|}=\frac{2u+v-2}{\sqrt{5}}$,

$key1$:（主元思想）$x^2+y^2+z^2=u^2+v^2+\frac{1}{5}(4u^2+v^2+4+4uv-8u-4v)=\frac{9}{5}u^2+\frac{1}{5}(4v-8)u+\frac{6}{5}v^2-\frac{4}{5}v+\frac{4}{5}$

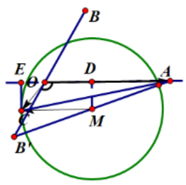
$$\geq\frac{4\cdot\frac{9}{5}\cdot(\frac{6}{5}v^2-\frac{4}{5}v+\frac{4}{5})-\frac{16}{25}(v-2)^2}{4\cdot\frac{9}{5}}=\frac{2}{9}(5v^2-2v+2)\geq\frac{2}{9}\cdot\frac{4\times5\times2-4}{4\times5}=\frac{2}{5}$$

$key2$:（柯西不等式）得$2x+y-\sqrt{5}z=-2$

$$\therefore(-2)^2=(2\cdot x+1\cdot y+(-\sqrt{5})\cdot z)^2\leq(4+1+5)(x^2+y^2+z^2),\therefore x^2+y^2+z^2\geq\frac{2}{5}$$

变式 1 （1）已知平面向量$\vec{a},\vec{b},\vec{c}$满足$|\vec{a}|=3,|\vec{b}|=2$.①已知$\vec{a}\cdot\vec{b}=3$.

（ⅰ）若$(\vec{a}-\vec{c})\cdot(\vec{b}+2\vec{c})=\frac{1}{2}$,则$|\vec{a}-\vec{c}|_{\min}=$\_\_\_\_\_,$\vec{a}\cdot\vec{c}\in$________;

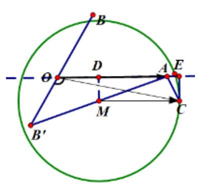




（ⅰ）$(\vec{a}-\vec{c})\cdot(\vec{b}+2\vec{c})=-2(\vec{c}-\vec{a})\cdot(\vec{c}-(-\frac{1}{2}\vec{b}))=-2\overrightarrow{CA}\cdot\overrightarrow{CB'}=-2(\overrightarrow{CM}^2-\frac{13}{4})=-2\overrightarrow{CM}^2+\frac{13}{2}=\frac{1}{2}$

得$|\overrightarrow{CM}|=\sqrt{3},\therefore|\vec{a}-\vec{c}|_{\min}=\frac{\sqrt{13}}{2}-\sqrt{3}$,

$$\vec{a}\cdot\vec{c}=\overrightarrow{OC}\cdot\overrightarrow{OA}=\frac{\overrightarrow{OC}\cdot\overrightarrow{OA}}{|\overrightarrow{OA}|}\cdot|\overrightarrow{OA}|\in[3(\frac{5}{4}-\sqrt{3}),3(\frac{5}{4}+\sqrt{3})](|\overrightarrow{OD}|=\frac{(\frac{1}{2}\vec{a}-\frac{1}{4}\vec{b})\cdot\vec{a}}{|\vec{a}|}=\frac{5}{4})$$



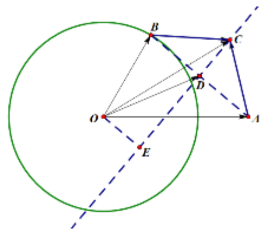


（ⅱ）若$(\vec{a}-\vec{c})\cdot(\vec{b}+2\vec{c})=-\frac{1}{2}$,则$|\vec{a}-\vec{c}|_{\min}=$\_\_\_\_\_,$\vec{a}\cdot\vec{c}\in$________.

（ii）$(\vec{a}-\vec{c})\cdot(\vec{b}+2\vec{c})=-2(\vec{c}-\vec{a})\cdot(\vec{c}-(-\frac{1}{2}\vec{b}))=-2\overrightarrow{CA}\cdot\overrightarrow{CB'}=-2(\overrightarrow{CM}^2-\frac{13}{4})=-2\overrightarrow{CM}^2+\frac{13}{2}=-\frac{1}{2}$

得$|\overrightarrow{CM}|=\sqrt{\frac{7}{2}},\therefore|\vec{a}-\vec{c}|_{\min}=\frac{\sqrt{14}-\sqrt{13}}{2}$,

$\vec{a}\cdot\vec{c}=\overrightarrow{OC}\cdot\overrightarrow{OA}=\frac{\overrightarrow{OC}\cdot\overrightarrow{OA}}{|\overrightarrow{OA}|}\cdot|\overrightarrow{OA}|\in[3(\frac{5}{4}-\frac{\sqrt{14}}{2}),3(\frac{5}{4}+\frac{\sqrt{14}}{2})]$

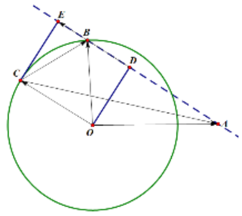


②（i）若$|\vec{c}-\vec{a}|=|\vec{c}-\vec{b}|$,则$|\vec{c}|_{\min}=$______________;

（i）$key1:$如图,$|\vec{c}|_{\min}=|\overrightarrow{OE}|=\frac{|\overrightarrow{OD}\cdot\overrightarrow{BA}|}{|\overrightarrow{BA}|}=\frac{|\frac{\vec{a}+\vec{b}}{2}\cdot(\vec{a}-\vec{b})|}{|\vec{a}-\vec{b}|}=\frac{5}{2|\vec{a}-\vec{b}|}\geq\frac{1}{2}$

$key2:|\vec{c}-\vec{a}|=|\vec{c}-\vec{b}|\Leftrightarrow(\vec{a}-\vec{b})\cdot\vec{c}=\frac{5}{2}=|\vec{a}-\vec{b}|\cdot|\vec{c}|\cos\theta\leq5|\vec{c}|,\therefore|\vec{c}|_{\min}=\frac{1}{2}$

（ii）若$|\vec{c}|=2$,则$(\vec{c}-\vec{b})\cdot(\vec{a}-\vec{b})\in$__________.

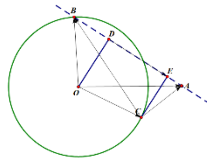


（ii）$key1:(\vec{c}-\vec{b})\cdot(\vec{a}-\vec{b})=\overrightarrow{BC}\cdot\overrightarrow{BA}\leq4\cdot5\cdot1=20$

$key2:$如图,$(\vec{c}-\vec{b})\cdot(\vec{a}-\vec{b})=\overrightarrow{BC}\cdot\overrightarrow{BA}=\frac{\overrightarrow{BC}\cdot\overrightarrow{BA}}{|\overrightarrow{BA}|}\cdot|\overrightarrow{BA}|=BE\cdot|\overrightarrow{BA}|$

(其中$x=|\overrightarrow{BD}|,|\overrightarrow{DA}|=y$,且$4-x^2=9-y^2$即$(y^2-x^2)=(y+x)(y-x)=5$,

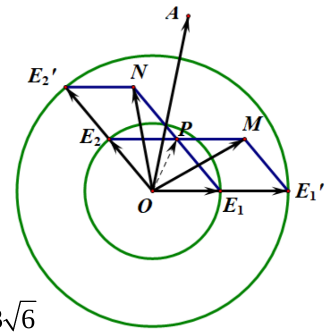
令$y+x=t,y-x=\frac{5}{t}$则$x=\frac{1}{2}(t-\frac{5}{t})$



$(\vec{c}-\vec{b})\cdot(\vec{a}-\vec{b})\geq(x-2)\cdot(x+y)=(\frac{1}{2}(t-\frac{5}{t})-2)\cdot t=\frac{1}{2}t^2-2t-\frac{5}{2}=\frac{1}{2}(t-2)^2-\frac{9}{2}\geq-\frac{9}{2}$

$(\vec{c}-\vec{b})\cdot(\vec{a}-\vec{b})\leq(x+2)\cdot(x+y)=(\frac{1}{2}(t-\frac{5}{t})+2)\cdot t=\frac{1}{2}t^2+2t-\frac{5}{2}\leq\frac{25}{2}+10-\frac{5}{2}=20$

（2）已知$\vec{e_1},\vec{e_2}$为单位向量，且$|\vec{e_1}+2\vec{e_2}|\leq2$，若非零向量$\vec{a}$满足$\vec{a}\cdot\vec{e_1}\leq\vec{a}\cdot\vec{e_2}$，则$\frac{\vec{a}\cdot(2\vec{e_1}+\vec{e_2})}{|\vec{a}|}$的最大值是（ D ） A. $\frac{3\sqrt{3}}{4}$ B. $\frac{3\sqrt{3}}{2}$ C. $\frac{3\sqrt{6}}{2}$ D. $\frac{3\sqrt{6}}{4}$



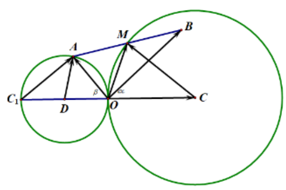

$key:$设$<\vec{e_1},\vec{e_2}>=\theta\in[0,\pi]$, 则$|\vec{e_1}+2\vec{e_2}|=\sqrt{5+4\cos\theta}\leq2$即$\cos\theta\leq-\frac{1}{4}$

$\because\vec{a}\cdot\vec{e_1}\leq\vec{a}\cdot\vec{e_2}$,如图,$\therefore\frac{\vec{a}\cdot(2\vec{e_1}+\vec{e_2})}{|\vec{a}|}=\frac{\vec{a}\cdot(2\vec{e_1}+\vec{e_2})}{|\vec{a}|\cdot|2\vec{e_1}+\vec{e_2}|}\cdot|2\vec{e_1}+\vec{e_2}|$

$\leq\frac{|\overrightarrow{OP}\cdot\overrightarrow{OM}|}{|\overrightarrow{OP}|\cdot|\overrightarrow{OM}|}\cdot|\overrightarrow{OM}|=\frac{\frac{1}{2}(\vec{e_1}+\vec{e_2})\cdot(2\vec{e_1}+\vec{e_2})}{\frac{1}{2}|\vec{e_1}+\vec{e_2}|}=\frac{3+3\cos\theta}{\sqrt{2+2\cos\theta}}=\frac{3}{\sqrt{2}}\sqrt{1+\cos\theta}\leq\frac{3\sqrt{6}}{4}$

（3）若平面向量$\vec{a},\vec{b},\vec{c}$满足$\vec{a}\cdot(\vec{a}+\vec{c})=0$，$|\vec{c}|=1$，$|\vec{a}+\vec{b}-2\vec{c}|=2$，则$\vec{a}\cdot\vec{b}$的最大值为______.

$key:$设$\angle MOC=\alpha,\angle AOD=\beta$,则$|\overrightarrow{OM}|=\cos\alpha,|\overrightarrow{OA}|=\frac{1}{2}\cos\beta$




$\vec{a}\cdot\vec{b}=\overrightarrow{OM}^2-\overrightarrow{AM}^2=\cos^2\alpha-[\cos^2\alpha+\frac{1}{4}\cos^2\beta+2\cos\alpha\cdot\frac{1}{2}\cos\beta\cdot\cos(\alpha+\beta)]$

$=-\frac{1}{4}\cos^2\beta-\cos\alpha\cos\beta\cos(\alpha+\beta)=-\frac{1}{4}\cos^2\beta-\frac{1}{2}\cos\beta\cdot[\cos(2\alpha+\beta)+\cos\beta]$

$\leq-\frac{3}{4}\cos^2\beta+\frac{1}{2}|\cos\beta|\leq\frac{-\frac{1}{4}}{4\cdot(-\frac{3}{4})}=\frac{1}{3}$