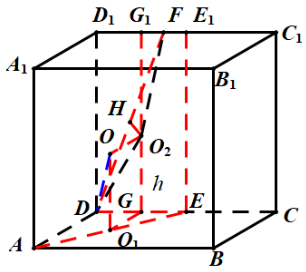
几何体结构特征解答（3）

2023-04-16

（3）已知棱长为a的正方体$ABCD-A_1B_1C_1D_1$中，E为DC的中点，F为线段D_1C_1上运动，则$F-ADE$的外接球表面积的最小值为_____.

key：要使$R=\sqrt{\frac{5}{16}a^2+O_2G^2}$最小，只要$O_2G$最小，

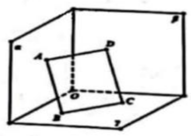
而$\tan\angle DO_2G=\frac{\frac{1}{4}a}{O_2G}$，只要$\angle DO_2G=\angle DFE$最大，只要$D_1F=\frac{1}{4}a$，

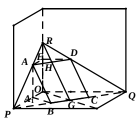


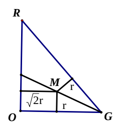
$$\therefore R_{\min}=\sqrt{\frac{5}{16}a^2+\left(\frac{\frac{a}{4}}{\tan(\angle DFE)_{\max}}\right)^2}=\sqrt{\frac{5}{16}a^2+\left(\frac{a}{4\cdot\frac{2\cdot\frac{1}{4}}{1-\frac{1}{16}}}\right)^2}=\frac{\sqrt{545}}{32}a$$

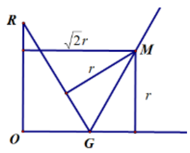
$$\therefore S_{表}=4\pi R^2=4\pi\left(\frac{5a^2}{16}+h^2\right)\ge\frac{545\pi a^2}{256}$$

（4）如图，已知三个两两相互垂直的半平面α,β,γ交于点O，矩形$ABCD$的边BC在半平面γ内，顶点A,D分别在半平面α,β内，$AD=2,AB=3,AD$与平面α所成角为$\frac{\pi}{4}$，二面角$A-BC-O$的余弦值为$\frac{1}{3}$，则同时与半平面α,β,γ和平面$ABCD$都相切的球的半径为___.









key：作$AE\perp\alpha$与β的交线c于E，连ED，

$\because\alpha\perp\gamma,\beta\perp\gamma,\therefore c\perp\gamma,\therefore c\perp BD\because AD//BC,\therefore c\perp AD,$

$\therefore c\perp$平面$AED,\therefore\angle AED=90°,\angle EAD=45°,$

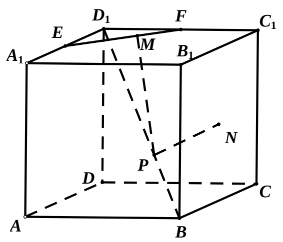
作$OG\perp BC$于G，连RG交AD于H，

则$\angle RGO=\theta(\cos\theta=\frac{1}{3},\tan\frac{\theta}{2}=\frac{\sqrt{2}}{2})$，且$\frac{EH}{RH}=\frac{1}{RH}=\frac{1}{3}$得$RH=3,\therefore RG=RH+HG=6,OG=2,OR=4\sqrt{2},$

由已知得球心在$\angle RGO$的平分线或外角平分线上，且到$\alpha,\beta,\gamma,ABCD$的距离相等

$$\therefore\left(2-\frac{r}{\tan\frac{\theta}{2}}\right)\cdot\frac{\sqrt{2}}{2}=r,或\left(2+\frac{r}{\tan\frac{\pi-\theta}{2}}\right)\cdot\frac{\sqrt{2}}{2}=r得r=\frac{\sqrt{2}}{2},or,2\sqrt{2}$$

(1407会考）在棱长为1的$ABCD-A_1B_1C_1D_1$中.若E,F分别是棱A_1D,C_1D_1的中点，N为线段B_1C的中点，若点P,M分别为线段D_1B,EF上的动点，则$PM+PN$的最小值为（　　）$A.1$　$B.\frac{3\sqrt{2}}{4}$　$C.\frac{2\sqrt{6}+\sqrt{2}}{4}$　$D.\frac{\sqrt{3}+1}{2}$　B



1407key:$\because P$在对角线上，

$\therefore P$面BCC_1B_1、面$ABCD$等距离,$\therefore PN=PO$

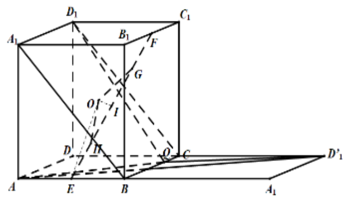
$$\therefore PM+PN=PM+PO\ge OM'=\sqrt{1+\left(\frac{\sqrt{2}}{4}\right)^2}=\frac{3\sqrt{2}}{4}，选B$$

变式2（1）已知在正方体$ABCD-A_1B_1C_1D_1$中，点E是AB中点，点F是B_1C_1中点，

几何体结构特征解答（3）

2023-04-16

若正方体$ABCD-A_1B_1C_1D_1$的内切球与直线EF交于点G,H，且$GH=3$，若点Q是棱BC上一个动点，则$AQ+D_1Q$的最小值为______.



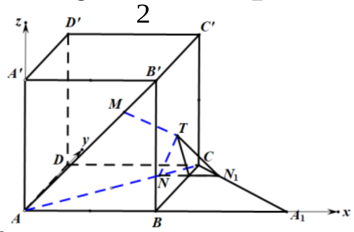
key:如图，$OE=OF, IG=IH=\dfrac{3}{2}$（$I$为$EF$的中点），

设$AB=a$,则$\sqrt{\dfrac{a^2}{4}-\dfrac{9}{4}}=OI=\sqrt{OE^2-\dfrac{1}{4}EF^2}=\sqrt{\dfrac{1}{2}a^2-\dfrac{3}{8}a^2}$,$\therefore a=3\sqrt{2}$

将矩形A_1BCD_1展开到正方形$ABCD$上，如图，

$\therefore AQ+QD_1=AQ+QD_1'\ge AD_1'=\sqrt{(1+\sqrt{2})^2+1^2}\,a=6\sqrt{2+\sqrt{2}}$

（2）已知正方体$ABCD-A'B'C'D'$的棱长为 1，点M，N分别为线段AB'，AC上的动点，点T在平面$BCC'B'$内，则$|MT|+|NT|$的最小值是（　B　）A．$\sqrt{2}$　　B．$\dfrac{2\sqrt{3}}{3}$　　C．$\dfrac{\sqrt{6}}{2}$　　D．1



设N关于BC的对称点为N_1，则$TM+TN=TM+TN_1\ge MN_1$

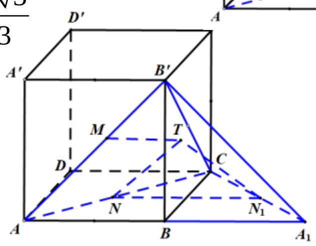
$key1$:设$M(m,0,m), N_1(n,2-n,0)$

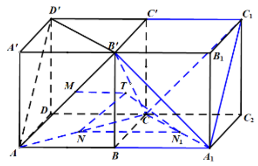
则$MN_1=\sqrt{(m-n)^2+(2-n)^2+m^2}=\sqrt{2m^2-2nm+2n^2-4n+4}$

$=\sqrt{2(m-\dfrac{n}{2})^2+\dfrac{3}{2}n^2-4n+4}\ge\sqrt{\dfrac{3}{2}(n-\dfrac{4}{3})^2+\dfrac{4}{3}}\ge\dfrac{2\sqrt{3}}{3}$

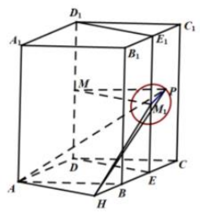
$key2$:MN_1的最小值就是异面直线AB'与CA_1的距离，

如图，即为B'到平面A_1CC_1的距离$\dfrac{2}{3}\cdot\sqrt{3}$





（3）已知直四棱柱$ABCD-A_1B_1C_1D_1$的高为4，底面边长均为2，且$\angle BAD=60°$,P是侧面BCC_1B_1内的一点，若$DP\perp D_1P$，则AP的最小值为_______.



key:E,E_1分别为BC,B_1C_1的中点，M_1为EE_1的中点,则$MM_1\perp$面BCC_1B_1,且$MM_1=\sqrt{3}$

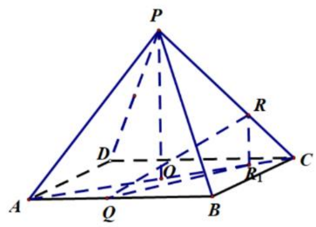
$\because DP\perp D_1P$,$\therefore P$在以M为球心，半径为2的球面与面BCC_1B_1的交线

（圆心为M_1,半径为1的圆）上,$\therefore$作$AH\perp BC$于H,则$MH=2\sqrt{2}$,

$\therefore AP\ge\sqrt{3+(2\sqrt{2}-1)^2}=2\sqrt{3-\sqrt{2}}$, $AP\le\sqrt{3+(2\sqrt{2}+1)^2}=2\sqrt{3+\sqrt{2}}$

（4）在正四棱锥$P-ABCD$中，$PA=PB=PC=PD=AB=1$，点Q，R分别在棱AB,PC上运动，当$|QR|$达到最小值时,$\dfrac{|PQ|}{|CQ|}$的值为（　）$A.\dfrac{\sqrt{70}}{10}$　$B.\dfrac{\sqrt{35}}{5}$　$C.\dfrac{\sqrt{35}}{10}$　$D.\dfrac{\sqrt{70}}{5}$

key:设$AQ=x\in[0,1], CR_1=y\in[0,\dfrac{\sqrt{2}}{2}]$,则



$QR=\sqrt{y^2+(\sqrt{2}-y)^2+x^2-2(\sqrt{2}-y)x\cdot\dfrac{\sqrt{2}}{2}}$

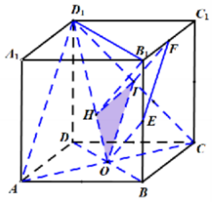
$=\sqrt{x^2-(2-\sqrt{2}y)x+2y^2-2\sqrt{2}y+2}=\sqrt{(x-\dfrac{2-\sqrt{2}y}{2})^2+\dfrac{3}{2}(y-\dfrac{\sqrt{2}}{3})^2+\dfrac{7}{6}}$

$\ge\sqrt{\dfrac{7}{6}}$（当且仅当$y=\dfrac{\sqrt{2}}{3},x=\dfrac{2}{3}$时，取$=$），$\therefore\dfrac{|PQ|}{|CQ|}=\dfrac{\sqrt{1+\frac{4}{9}-2\cdot1\cdot\frac{2}{3}\cdot\frac{1}{2}}}{\sqrt{\frac{1}{9}+1}}=\dfrac{\sqrt{70}}{10}$

（09竞赛）在棱长为1的正方体$ABCD-A_1B_1C_1D_1$中，E、F分别为AA_1、CC_1上的点，且$AE=C_1F$，则四边形$EBFD_1$的面积最小值为_______.

09$key:S=2.\frac{1}{2}\cdot\sqrt{3}\cdot d_{F\to BD_1}\ge\sqrt{3}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{6}}{2}$(异面直线$CC_1$于$BD_1$的距离为$\frac{\sqrt{2}}{2}$)

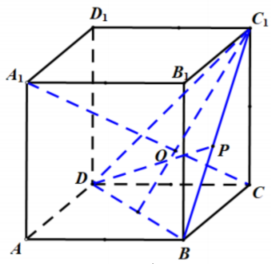
变式 1（1）在棱长为$\sqrt{2}$的正方体$ABCD-A_1B_1C_1D_1$中，棱BB_1，B_1C_1的中点分别为E,F，点P在平面BCC_1B_1内，作$PQ\perp$平面ACD_1，垂足为Q．当点P在$\triangle EFB_1$内（包含边界）运动时，点Q的轨迹所组成的图形的面积等于_____.



$key:S=S_{\triangle B_1EF}\cos\theta=\frac{1}{4}\cdot\frac{1}{\frac{\sqrt{3}}{4}\cdot 4}=\frac{1}{4\sqrt{3}}$

（2）在棱长为 1 的正方体$ABCD-A_1B_1C_1D_1$中，P是线段BC_1上的点，过A_1的平面α与直线PD垂直当P在线段BC_1上运动时，平面α截正方体$ABCD-A_1B_1C_1D_1$所得的截面面积的最小值是

A．1　　B．$\frac{5}{4}$　　C．$\frac{\sqrt{6}}{2}$　　D．$\sqrt{2}$



$key:\because A_1C\perp$平面$BDC_1,PD\subset$平面BDC_1
$\therefore A_1C\subset\alpha,\therefore\alpha$就是过$A_1C$的平面，
$\therefore\alpha$截正方体所得截面面积的最小值为$2\cdot\frac{1}{2}\cdot\sqrt{3}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{6}}{2}$

(1610）如图，在四面体$ABCD$中，$AB=CD=2,AD=BD=3,AC=BC=4,$点E,F,G,H分别在棱AD,BD,BC,AC上，若直线AB,CD都平行于平面$EFGH$,四边形$EFGH$面积的最大值是（ ）

$A.\frac{1}{2}$　$B.\frac{\sqrt{2}}{2}$　$C.1$　$D.2$　　　C

(18全国Ⅰ）已知正方体的棱长为1,每条棱所在直线与平面α所成角都相等，则α截此正方体所得截面面积的最大值为（ ）$A.\frac{3\sqrt{3}}{4}$　$B.\frac{2\sqrt{3}}{3}$　$C.\frac{3\sqrt{2}}{4}$　$D.\frac{\sqrt{3}}{2}$　　A