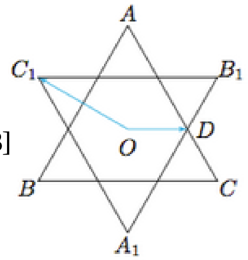
2023-02-19

②如图，两个正三角形$ABC, A_1B_1C_1$组成“六芒星”，O为“六芒星”的中心，P为“六芒星”图案上一点（包括边界），且$\overrightarrow{OP}=x\overrightarrow{OD}+y\overrightarrow{OC_1}$.

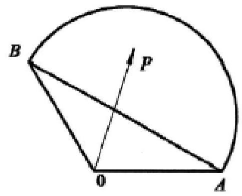
则$x\in$_____,$x+y\in$_______,$2x+y\in$_______.



$key:\overrightarrow{OP}\cdot\overrightarrow{OD}=x-\frac{3}{2}y,\overrightarrow{OP}\cdot\overrightarrow{OC_1}=-\frac{3}{2}x+3y,\therefore\begin{cases}x=2\overrightarrow{OP}\cdot(2\overrightarrow{OD}+\overrightarrow{OC_1})\in[-3,3]\\ y=\frac{4}{3}\overrightarrow{OP}\cdot(\frac{3}{2}\overrightarrow{OD}+\overrightarrow{OC_1})\end{cases}$

$x+y=\overrightarrow{OP}\cdot(6\overrightarrow{OD}+\frac{10}{3}\overrightarrow{OC_1})\in[-5,5],2x+y=\overrightarrow{OP}\cdot(10\overrightarrow{OD}+\frac{16}{3}\overrightarrow{OC_1})\in[-8,8]$

③如图,$\triangle ABO$是以$\angle O=120°$为顶点的等腰三角形，点P在以AB 为直径的半圆内（包括边界），若$\overrightarrow{OP}=x\overrightarrow{OA}+y\overrightarrow{OB}(x,y\in R)$，则$x+y\in$_______,$x^2+y^2\in$_______.

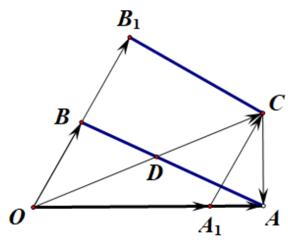


$key:$令$|\overrightarrow{OA}|=1$,则$|\overrightarrow{OP}|\le|\overrightarrow{OC}|+|\overrightarrow{CP}|\le\frac{1+\sqrt{3}}{2}$

$\overrightarrow{OP}=(x+y)(\frac{x}{x+y}\overrightarrow{OA}+\frac{y}{x+y}\overrightarrow{OB})=(x+y)\overrightarrow{OQ},\therefore x+y=\frac{|\overrightarrow{OP}|}{|\overrightarrow{OQ}|}\in[1,\sqrt{3}+1]$

$\frac{2+\sqrt{3}}{2}\ge\overrightarrow{OP}^2=x^2+y^2-xy\ge x^2+y^2-\frac{x^2+y^2}{2}=\frac{x^2+y^2}{2}\ge\frac{1}{2}\cdot\frac{(x+y)^2}{1+1}\ge\frac{1}{2},\therefore x^2+y^2\in[\frac{1}{2},2+\sqrt{3}]$

（3）①$\vec{a},\vec{b},\vec{c}$为平面内三个向量，满足$<\vec{a},\vec{b}>=\frac{\pi}{3}$，$\vec{a}\perp(\vec{a}-\vec{c})$且$\vec{b}\perp(2\vec{b}-\vec{c})$，若$\vec{c}=\lambda\vec{a}+\mu\vec{b}(\mu<2)$，则$\lambda+\mu$的最大值为______. $4-\frac{4\sqrt{2}}{3}$



$key:$令$\vec{a}=(1,0),\vec{b}=(\frac{1}{2}b,\frac{\sqrt{3}}{2}b)(b>0),\because\vec{a}\perp(\vec{a}-\vec{c})$,则$\vec{c}=(1,c)$

$\because\vec{b}\perp(2\vec{b}-\vec{c}),\therefore(\frac{1}{2}b,\frac{\sqrt{3}}{2}b)\cdot(b-1,\sqrt{3}b-c)=0$即$b-1+\sqrt{3}(\sqrt{3}b-c)=0$

$\therefore\vec{c}=(1,\frac{4b-1}{\sqrt{3}})=\lambda(1,0)+\mu(\frac{1}{2}b,\frac{\sqrt{3}}{2}b)=(\lambda+\frac{\mu}{2}b,\frac{\sqrt{3}}{2}\mu b)$

$\therefore\begin{cases}1=\lambda+\frac{\mu}{2}b\\ \frac{4b-1}{\sqrt{3}}=\frac{\sqrt{3}}{2}\mu b\end{cases}$得$\begin{cases}\lambda=\frac{4-4b}{3}\\ \mu=\frac{2(4b-1)}{3b}<2\text{即}b<1\end{cases},\therefore\lambda+\mu=\frac{4-4b}{3}+\frac{8b-2}{3b}=4-\frac{2}{3}(2b+\frac{1}{b})\le4-\frac{4\sqrt{2}}{3}$

②已知平面向量$\vec{a},\vec{b},\vec{c}$满足$|\vec{a}|=|\vec{b}|,\vec{c}=\lambda\vec{a}+\mu\vec{b},|\vec{c}|=1+\vec{a}\cdot\vec{b}=(\vec{a}+\vec{b})\cdot\vec{c}=1$,则$\frac{|\vec{a}-\vec{c}|}{|1+\mu-\lambda|}$的最小值为_____.

$key:$由已知设$\vec{a}=(a,0),\vec{b}=(0,a),\vec{c}=(\cos\theta,\sin\theta)$

则$\lambda=\frac{\cos\theta}{a},\mu=\frac{\sin\theta}{a}$,且$a\cos\theta+a\sin\theta=1,\therefore\lambda=\cos^2\theta+\sin\theta\cos\theta,\mu=\sin^2\theta+\sin\theta\cos\theta$

$\therefore\frac{|\vec{a}-\vec{c}|}{|1+\mu-\lambda|}=\frac{\sqrt{(a-\cos\theta)^2+\sin^2\theta}}{2\sin^2\theta}=\frac{\sqrt{2}}{2}\cdot\frac{1}{|\sin\theta\cos\theta+\sin^2\theta|}=\frac{\sqrt{2}}{2}\cdot\frac{1}{|\frac{1}{2}+\frac{\sqrt{2}}{2}\sin(2\theta-\frac{\pi}{4})|}\ge2-\sqrt{2}$

（19贵州）在$\triangle ABC$中,$\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0},\overrightarrow{GA}\cdot\overrightarrow{GB}=0$,则$\frac{(\tan A+\tan B)\tan C}{\tan A\cdot\tan B}=$______.$\frac{1}{2}$

$key:G$为$\triangle ABC$的重心,$\therefore\overrightarrow{GA}\cdot\overrightarrow{GB}=\frac{1}{3}(\overrightarrow{AB}+\overrightarrow{AC})\cdot\frac{1}{3}(\overrightarrow{BA}+\overrightarrow{BC})=\frac{1}{9}(\overrightarrow{AB}+\overrightarrow{AC})\cdot(-2\overrightarrow{AB}+\overrightarrow{AC})$

$=\frac{1}{9}(-2c^2+b^2-\frac{c^2+b^2-a^2}{2})=0$即$a^2+b^2=5c^2$

$\therefore \frac{(\tan A+\tan B)\tan C}{\tan A\tan B}=\frac{(\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B})\frac{\sin C}{\cos C}}{\frac{\sin A\sin B}{\cos A\cos B}}=\frac{\sin^2 C}{\sin A\sin B\cos C}=\frac{c^2}{ab\cos C}=\frac{c^2}{\frac{a^2+b^2-c^2}{2}}=\frac{1}{2}$

（20 贵州）（多选题）下列命题中，正确的是（　）

$A.$点O在$\triangle ABC$内部，且$m\overrightarrow{OA}+n\overrightarrow{OB}+p\overrightarrow{OC}=\vec{0}$，则$S_{\triangle BOC}:S_{\triangle COA}:S_{\triangle AOB}=m:n:p$

$B.$点O是$\triangle ABC$的重心，则$\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\vec{0}$

$C.$点O是$\triangle ABC$的内心，则$a\overrightarrow{OA}+b\overrightarrow{OB}+c\overrightarrow{OC}=\vec{0}$

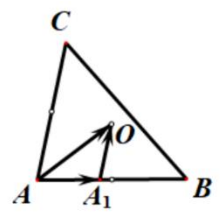
$D.$点O是$\triangle ABC$的外心，则$(\sin 2A)\overrightarrow{OA}+(\sin 2B)\overrightarrow{OB}+(\sin 2C)\overrightarrow{OC}=\vec{0}$

$key:\because O$在$\triangle ABC$内部$,\therefore m,n,p$同号$,\therefore A$对；BC都对；

过O作$OA_1//AC$交AB于A_1，则$\overrightarrow{AO}=\overrightarrow{AA_1}+\overrightarrow{A_1O}$，且$\frac{R}{\sin A}=\frac{|\overrightarrow{AA_1}|}{\cos B}=\frac{|\overrightarrow{A_1O}|}{\cos C}$

$(\because \angle OAA_1=\frac{\pi-2C}{2},\angle AA_1O=\pi-A,\angle AOA_1=\frac{\pi-2B}{2})$

$\therefore \overrightarrow{AO}=\frac{R\cos B}{\sin A}(\frac{\overrightarrow{AB}}{c})+\frac{R\cos C}{\sin A}(\frac{\overrightarrow{AC}}{b})=\frac{\cos B}{2\sin A\sin C}(\overrightarrow{OB}-\overrightarrow{OA})+\frac{\cos C}{2\sin A\sin B}(\overrightarrow{OC}-\overrightarrow{OA})$



$\therefore (4\sin A\sin B\sin C-\sin 2B-\sin 2C)\overrightarrow{OA}+(\sin 2B)\overrightarrow{OB}+(\sin 2C)\overrightarrow{OC}$

$=(\sin 2A)\overrightarrow{OA}+(\sin 2B)\overrightarrow{OB}+(\sin 2C)\overrightarrow{OC}=\vec{0}$

$(\because 4\sin A\sin B\sin C-\sin 2B-\sin 2C=4\sin A\sin B\sin C-2\sin(B+C)\cos(B-C)$

$=2\sin A(2\sin B\sin C-\cos B\cos C-\sin B\sin C)=\sin 2A)$

（19 中科大）4.设O为$\triangle ABC$的外心，且$4\overrightarrow{OA}+5\overrightarrow{OB}+6\overrightarrow{OC}=\vec{0}$，则$\tan A=$______.$\sqrt{7}$

$key:$可得$\triangle ABC$为锐角三角形，

则$(5\overrightarrow{OB}+6\overrightarrow{OC})^2=(-4\overrightarrow{OA})^2$得$\cos 2A=-\frac{3}{4},\therefore \tan A=\sqrt{7}$

(18A）设O为$\triangle ABC$的外心，若$\overrightarrow{AO}=\overrightarrow{AB}+2\overrightarrow{AC}$,则$\sin\angle BAC=$______.$\frac{\sqrt{10}}{4}$

$key:$(投影)$\overrightarrow{AO}\cdot\overrightarrow{AB}=\frac{\overrightarrow{AO}\cdot\overrightarrow{AB}}{|\overrightarrow{AB}|}\cdot|\overrightarrow{AB}|=\frac{1}{2}\overrightarrow{AB}^2=\overrightarrow{AB}^2+2\overrightarrow{AB}\cdot\overrightarrow{AC}$得$\frac{1}{2}c^2+2bc\cos A=0$

$\overrightarrow{AO}\cdot\overrightarrow{AC}=\frac{1}{2}\overrightarrow{AC}^2=\overrightarrow{AC}^2+\overrightarrow{AC}\cdot\overrightarrow{AB}$得$\frac{1}{2}b^2+2bc\cos A=0,\therefore b=c,\cos A=-\frac{1}{4},\sin A=\frac{\sqrt{10}}{4}$

（19 甘肃）2.$\triangle ABC$ 的三边分别为 a,b,c,O 为$\triangle ABC$ 的外心，已知$b^2-2b+c^2=0$，则 $\overrightarrow{BC}\cdot\overrightarrow{AO}$ 的取值范围为__________.$[-\frac{1}{4},2)$

$key:\overrightarrow{BC}\cdot\overrightarrow{AO}=\overrightarrow{BC}\cdot(\overrightarrow{AD}+\overrightarrow{DO})=(\overrightarrow{AC}-\overrightarrow{AB})\cdot\frac{1}{2}(\overrightarrow{AC}+\overrightarrow{AB})$

$=\frac{1}{2}(b^2-c^2)=b^2-b\in[-\frac{1}{4},2)(c^2=2b-b^2>0)$

（19A）8.已知I为$\triangle ABC$的内心，且$5\overrightarrow{IA}=4(\overrightarrow{IB}+\overrightarrow{IC})$.记$R,r$分别为$\triangle ABC$的外接圆、内切圆半径.

若$r=15$，则$R=$__________ .32

$key:\overrightarrow{IA}=-\frac{8}{5}\overrightarrow{ID},\therefore|\overrightarrow{AB}|=|\overrightarrow{AC}|,$且$b=c=\frac{4}{5}a,\therefore c^2-\frac{a^2}{4}=\frac{39}{100}a^2=39^2,\therefore a=\sqrt{3900}$

而$\sin\frac{A}{2}=\frac{r}{\frac{8}{5}r}=\frac{5}{8},\therefore 2R=\frac{10\sqrt{39}}{2\cdot\frac{5}{8}\cdot\sqrt{1-\frac{25}{64}}}=64$

（2018安徽）设H是$\triangle ABC$的垂心，且$3\overrightarrow{HA}+4\overrightarrow{HB}+5\overrightarrow{HC}=\vec{0}$，则$\cos\angle AHB=$_____.

18安徽$key:0=\overrightarrow{AB}\cdot(3\overrightarrow{HA}+4\overrightarrow{HB}+5\overrightarrow{HC})=-3\overrightarrow{AB}\cdot\overrightarrow{AC}+4\overrightarrow{BA}\cdot\overrightarrow{BC}$

$=-3\cdot\frac{\overrightarrow{AB}^2+\overrightarrow{AC}^2-(\overrightarrow{AB}-\overrightarrow{AC})^2}{2}+4\cdot\frac{\overrightarrow{BA}^2+\overrightarrow{BC}^2-(\overrightarrow{BA}-\overrightarrow{BC})^2}{2}$得$c^2+7a^2=7b^2$

$0=\overrightarrow{AC}\cdot(3\overrightarrow{HA}+4\overrightarrow{HB}+5\overrightarrow{HC})=-3\overrightarrow{AB}\cdot\overrightarrow{AC}+5\overrightarrow{CB}\cdot\overrightarrow{CA}$

$=-3\cdot\frac{\overrightarrow{AB}^2+\overrightarrow{AC}^2-(\overrightarrow{AB}-\overrightarrow{AC})^2}{2}+5\cdot\frac{\overrightarrow{CB}^2+\overrightarrow{CA}^2-(\overrightarrow{CB}-\overrightarrow{CA})^2}{2}$得$4c^2-4a^2=b^2,\therefore c^2=\frac{35}{32}b^2,a^2=\frac{27}{32}b^2,$

$\therefore\cos\angle AHB=-\cos C=-\frac{\frac{\overrightarrow{CA}^2+\overrightarrow{CB}^2-(\overrightarrow{CA}-\overrightarrow{CB})^2}{2}}{ab}=-\frac{\sqrt{6}}{6}$

变式 1（1）①O是$\triangle ABC$的外心$,\overrightarrow{AO}=x\overrightarrow{AB}+y\overrightarrow{AC},$且$2x+y=1,|\overrightarrow{AB}|=5,|\overrightarrow{AC}|=3,$则$\tan\angle BAC=$______.

$key:\overrightarrow{AO}=2x(\frac{1}{2}\overrightarrow{AB})+y\overrightarrow{AC},\therefore|\overrightarrow{CA}|=|\overrightarrow{CB}|,$或，$\angle ACB=90°,\therefore\tan\angle BAC=\frac{\sqrt{11}}{5}$,或$\frac{3}{4}$

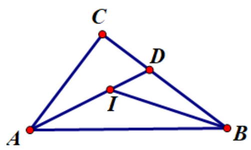
②已知O为锐角$\triangle ABC$的外心$,|\overrightarrow{AB}|=3,|\overrightarrow{AC}|=2\sqrt{3},$且$\overrightarrow{AO}=x\overrightarrow{AB}+y\overrightarrow{AC},9x+12y=8,$则$\overrightarrow{OB}\cdot\overrightarrow{OC}=$_______.

$key:\overrightarrow{AO}^2=x\overrightarrow{AB}\cdot\overrightarrow{AO}+y\overrightarrow{AC}\cdot\overrightarrow{AO}=x\cdot\frac{1}{2}\overrightarrow{AB}^2+y\cdot\frac{1}{2}\overrightarrow{AC}^2=\frac{9}{2}x+\frac{12}{2}y=4,$

$\therefore R=|\overrightarrow{AO}|=2,\sin C=\frac{\sqrt{3}}{2},\sin B=\frac{3}{4},\therefore\sin A=\frac{3+\sqrt{21}}{8},\therefore|\overrightarrow{BC}|=\frac{3+\sqrt{21}}{2}$

$\therefore\overrightarrow{OC}\cdot\overrightarrow{OB}=\frac{\overrightarrow{OC}^2+\overrightarrow{OB}^2-(\overrightarrow{OC}-\overrightarrow{OB})^2}{2}=\frac{\overrightarrow{OC}^2+\overrightarrow{OB}^2-\overrightarrow{BC}^2}{2}=-\frac{1+3\sqrt{21}}{2},$

（2）已知 I 为$\triangle ABC$ 的内心，$\cos A=\frac{7}{8}$，若$\overrightarrow{AI}=x\overrightarrow{AB}+y\overrightarrow{AC}$，则 $x+y$ 的最大值为（ D ）



A. $\frac{3}{4}$　　B. $\frac{1}{2}$　　C. $\frac{5}{6}$　　D. $\frac{4}{5}$

$key:\overrightarrow{AI}=(x+y)(\frac{x}{x+y}\overrightarrow{AB}+\frac{y}{x+y}\overrightarrow{AC})=(x+y)\overrightarrow{AD},$

（而$\frac{BD}{DC}=\frac{AB}{AC}=\frac{c}{b},\therefore BD=\frac{ac}{b+c},$而$\frac{AI}{ID}=\frac{AB}{BD}=\frac{c}{\frac{ac}{b+c}}=\frac{b+c}{a},\therefore AI=\frac{b+c}{a+b+c}AD$

$\therefore x+y=\frac{|\overrightarrow{AI}|}{|\overrightarrow{AD}|}=\frac{b+c}{a+b+c}=\frac{1}{1+\frac{a}{b+c}}\le\frac{4}{5}$(而$a^2=b^2+c^2-\frac{7}{4}bc=\frac{1}{16}(b+c)^2+\frac{15}{16}(b-c)^2\ge\frac{1}{16}(b+c)^2,\therefore\frac{a}{b+c}\ge\frac{1}{4}$)

（3）①已知$\triangle ABC$中,O为$\triangle ABC$所在平面内一点.若$\overrightarrow{OA}^2+\overrightarrow{BC}^2=\overrightarrow{OB}^2+\overrightarrow{CA}^2=\overrightarrow{OC}^2+\overrightarrow{AB}^2$，则点$O$是

$\triangle ABC$的____心. 垂心

②在$\triangle ABC$中，$AB=4,AC=3,BC=2$.若O是$\triangle ABC$的垂心，则$\overrightarrow{AO}\cdot\overrightarrow{AB}=$_____,$\overrightarrow{AO}\cdot\overrightarrow{BC}=$____.

$key:\overrightarrow{AO}\cdot\overrightarrow{AB}=\dfrac{\overrightarrow{AO}\cdot\overrightarrow{AB}}{|\overrightarrow{AB}|}\cdot|\overrightarrow{AB}|=\overrightarrow{AC}\cdot\overrightarrow{AB}=\dfrac{\overrightarrow{AB}^2+\overrightarrow{AC}^2-(\overrightarrow{AB}-\overrightarrow{AC})^2}{2}=\dfrac{21}{2},\overrightarrow{AO}\cdot\overrightarrow{BC}=0$

二.数量积:$\vec a\cdot\vec b=|\vec a||\vec b|\cos<\vec a,\vec b>$.①模:$\vec a^2=|\vec a|^2$ (即$|\vec a|=\sqrt{\vec a^2}$)，三角形不等式:$||\vec a|-|\vec b||\le|\vec a\pm\vec b|\le|\vec a|+|\vec b|$

②$\vec b$在$\vec a$上的投影:$\dfrac{\vec a\cdot\vec b}{|\vec a|}$；③向量夹角:$\cos<\vec a,\vec b>=\dfrac{\vec a\cdot\vec b}{|\vec a||\vec b|}$；④垂直充要条件:$\vec a\perp\vec b\Leftrightarrow\vec a\cdot\vec b=0\Leftrightarrow|\vec a+\vec b|=|\vec a-\vec b|$

⑤$|\vec a\cdot\vec b|=|\vec a|\cdot|\vec b|\cdot|\cos<\vec a,\vec b>|\le|\vec a|\cdot|\vec b|$（柯西不等式:$(x_1x_2+y_1y_2+z_1z_2)^2\le(x_1^2+y_1^2+z_1^2)(x_2^2+y_2^2+z_2^2)$）

向量数量积的坐标表示:$\vec a=(x_1,y_1),\vec b=(x_2,y_2)$.模:$|\vec a|=\sqrt{x_1^2+y_1^2}$

向量夹角:$\cos<\vec a,\vec b>=\dfrac{\vec a\cdot\vec b}{|\vec a||\vec b|}=\dfrac{x_1x_2+y_1y_2}{\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}}$;投影:$\dfrac{\vec a\cdot\vec b}{|\vec a|}=\dfrac{x_1x_2+y_1y_2}{\sqrt{x_1^2+y_1^2}}$

平行:$\vec a//\vec b\Leftarrow\dfrac{x_1}{x_2}=\dfrac{y_1}{y_2}$;垂直:$\vec a\perp\vec b\Leftrightarrow\vec a\cdot\vec b=0\Leftrightarrow x_1x_2+y_1y_2=0$.

（19 强基）8.已知$\triangle ABC$为斜边$AB=\sqrt{2019}$的直角三角形，则$\overrightarrow{AB}\cdot\overrightarrow{AC}+\overrightarrow{BA}\cdot\overrightarrow{BC}+\overrightarrow{CA}\cdot\overrightarrow{CB}=$____. -2019

（2021II）15. 已知向量$\vec a+\vec b+\vec c=\vec 0,|\vec a|=1,|\vec b|=|\vec c|=2$，则$\vec a\cdot\vec b+\vec b\cdot\vec c+\vec c\cdot\vec a=$_______. $-\dfrac{9}{2}$

变式:在$\triangle ABC$中，则$\overrightarrow{AB}\cdot\overrightarrow{BC}+\overrightarrow{BC}\cdot\overrightarrow{CA}+\overrightarrow{CA}\cdot\overrightarrow{AB}$的值为（ ）$A$.正数 B.负数 C.0 D.以上说法都有可能

$key:\overrightarrow{AB}\cdot\overrightarrow{BC}=-\overrightarrow{BA}\cdot\overrightarrow{BC}=-ca\cos B=-\dfrac{c^2+a^2-b^2}{2}$

$\therefore\overrightarrow{AB}\cdot\overrightarrow{BC}+\overrightarrow{BC}\cdot\overrightarrow{CA}+\overrightarrow{CA}\cdot\overrightarrow{AB}=-\dfrac{a^2+b^2+c^2}{2}<0$

$key2:\overrightarrow{AB}\cdot\overrightarrow{BC}=-\overrightarrow{BA}\cdot\overrightarrow{BC}=-\dfrac{\overrightarrow{BA}^2+\overrightarrow{BC}^2-(\overrightarrow{BA}+\overrightarrow{BC})^2}{2}=-\dfrac{c^2+a^2-b^2}{2}$

$key3:\overrightarrow{AB}\cdot\overrightarrow{BC}+\overrightarrow{BC}\cdot\overrightarrow{CA}=\overrightarrow{BC}\cdot(\overrightarrow{AB}+\overrightarrow{CA})=-\overrightarrow{BC}^2$

$key4:\overrightarrow{AB}\cdot\overrightarrow{BC}+\overrightarrow{BC}\cdot\overrightarrow{CA}+\overrightarrow{CA}\cdot\overrightarrow{AB}=\dfrac{(\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA})^2-\overrightarrow{AB}^2-\overrightarrow{BC}^2-\overrightarrow{CA}^2}{2}=-\dfrac{a^2+b^2+c^2}{2}<0$

(16A）在$\triangle ABC$中，已知$\overrightarrow{AB}\cdot\overrightarrow{AC}+2\overrightarrow{BA}\cdot\overrightarrow{BC}=3\overrightarrow{CA}\cdot\overrightarrow{CB}$，则$\sin C$的最大值为______. $\dfrac{\sqrt7}{3}$

$key1$:（定义，余弦定理）;$key2$:（极化）$\overrightarrow{AB}\cdot\overrightarrow{AC}=\dfrac{\overrightarrow{AB}^2+\overrightarrow{AC}^2-(\overrightarrow{AB}-\overrightarrow{AC})^2}{2}=\dfrac{1}{2}(b^2+c^2-a^2)$

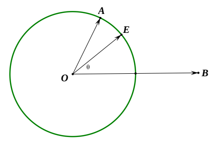
同理$\overrightarrow{BA}\cdot\overrightarrow{BC}=\dfrac{1}{2}(a^2+c^2-b^2),\overrightarrow{CA}\cdot\overrightarrow{CB}=\dfrac{1}{2}(a^2+b^2-c^2),\therefore a^2+2b^2=3c^2$

$\therefore\cos C=\dfrac{a^2+b^2-c^2}{2ab}=\dfrac{1}{6}(\dfrac{2a}{b}+\dfrac{b}{a})\ge\dfrac{\sqrt2}{3},\therefore(\sin C)_{\max}=\dfrac{\sqrt7}{3}$

(16理）已知平面向量$\vec a,\vec b$满足$|\vec a|=1,|\vec b|=2$,对任意的单位向量$\vec e$,有$|\vec a\cdot\vec e|+|\vec b\cdot\vec e|\le\sqrt6$，则$\vec a\cdot\vec b$的最大值为______.

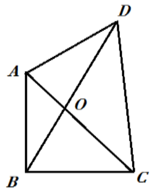
16理$key:key:$ 设$<\vec{a},\vec{b}>=\theta,<\vec{e},\vec{a}>=\alpha,$ 则$\sqrt{6}\geq\cos\alpha+2\cos(\theta-\alpha)=(1+2\cos\theta)\cos\alpha+2\sin\theta\sin\alpha$

$\therefore\sqrt{(1+2\cos\theta)^2+(2\sin\theta)^2}\leq\sqrt{6},\therefore\vec{a}\cdot\vec{b}_{\max}=\dfrac{1}{2}$



$key2:\sqrt{6}\geq|\vec{e}\cdot\vec{a}|+|\vec{e}\cdot\vec{b}|\geq|\vec{e}\cdot(\vec{a}+\vec{b})|,\therefore|\vec{a}+\vec{b}|\leq\sqrt{6},\therefore\vec{a}\cdot\vec{b}=\dfrac{(\vec{a}+\vec{b})^2-\vec{a}^2-\vec{b}^2}{2}\leq\dfrac{1}{2}$

（17高考09）如图，已知平面四边形$ABCD$，$AB\perp BC,AB=BC=AD=2,$
$CD=3,AC$与BD交于点$O,$ 记$I_1=\overrightarrow{OA}\cdot\overrightarrow{OB},I_2=\overrightarrow{OB}\cdot\overrightarrow{OC},I_3=\overrightarrow{OC}\cdot\overrightarrow{OD},$ 则
（　）$A.I_1<I_2<I_3$ $B.I_1<I_3<I_2$ $C.I_3<I_1<I_2$ $D.I_2<I_1<I_3$　　C



$2017key:I_2=|\overrightarrow{OB}|\cdot|\overrightarrow{OC}|\cos\angle BOC>0,$
$I_1=|\overrightarrow{OA}|\cdot|\overrightarrow{OB}|\cos\angle AOB<0,I_3=|\overrightarrow{OC}|\cdot|\overrightarrow{OD}|\cos\angle COD<0$
而$|\overrightarrow{OA}|<|\overrightarrow{OC}|,|\overrightarrow{OB}|<|\overrightarrow{OD}|,\therefore I_1>I_3$