数列（1）等差等比数列解答（1）

2024-01-13

一、等差数列

(1) 定义：$a_n-a_{n-1}=d$（d为常数）

$\Leftrightarrow 2a_n=a_{n+1}+a_{n-1}$

$\Leftrightarrow a_n=a_1+(n-1)d=pn+q \Leftrightarrow a_n=a_m+(n-m)d$

$\Leftrightarrow S_n=An^2+Bn \Leftrightarrow S_n=\dfrac{n(a_1+a_n)}{2}$

（2）性质：若$\{a_n\}$是等差数列，则

①若$\{k_n\}$是等差数列，且$k_n\in N^*$，则$\{a_{k_n}\}$是等差数列

②若$p_1+p_2+\cdots+p_m=q_1+q_2+\cdots+q_m, p_i,q_i\in N^*$,则$a_{p_1}+a_{p_2}+\cdots+a_{p_m}=a_{q_1}+a_{q_2}+\cdots+a_{q_m}$.

(1993I ）已知等差数列$\{a_n\}$的公差$d>0$，首项$a_1>0, S_n=\sum\limits_{i=1}^{n}\dfrac{1}{a_ia_{i+1}}$，则$\lim\limits_{n\to\infty}S_n=$____.

1993I $key: S_n=\dfrac{1}{d}(\dfrac{1}{a_1}-\dfrac{1}{a_1+nd}),\therefore \lim\limits_{n\to\infty}S_n=\dfrac{1}{a_1d}$

(2005II)11.如果$a_1,a_2,\cdots,a_8$为各项都大于零的等差数列，公差$d\neq 0$，则（　　）

$A.a_1a_8>a_4a_5$　$B.a_1a_8<a_4a_5$　$C.a_1+a_8>a_4+a_5$　$D.a_1a_8=a_4a_5$

2005II $key: a_1a_8-a_4a_5=a_1(a_1+7d)-(a_1+3d)(a_1+4d)=-12d^2<0$, 选$B$

（2006江苏）设数列$\{a_n\},\{b_n\},\{c_n\}$满足：$b_n=a_n-a_{n+2}, c_n=a_n+2a_{n+1}+3a_{n+2}(n=1,2,3,\cdots)$,
证明:$\{a_n\}$为等差数列的充要条件是$\{c_n\}$为等差数列且$b_n\le b_{n+1}(n=1,2,3\cdots)$.

2006江苏证明：①必要性:$\because \{a\}$是等差数列，设其公差为d_a,

则$c_{n+1}-c_n=(a_{n+1}+2a_{n+2}+3a_{n+3})-(a_n+2a_{n+1}+3a_{n+2})$

$=a_{n+1}-a_n+2(a_{n+2}-a_{n+1})+3(a_{n+3}-a_{n+2})=d_a+2d_a+3d_a=6d_a$为常数,$\therefore \{c_n\}$是等差数列，

且$b_n=-2d_a=b_{n+1}, \therefore b_n\le b_{n+1}$

②充分性:$\because \{c_n\}$为等差数列（设其公差为d_c),

则$-2d_c=c_n-c_{n+2}=a_n-a_{n+2}+2(a_{n+1}-a_{n+3})+3(a_{n+2}-a_{n+4})=b_n+2b_{n+1}+3b_{n+2}$

$\therefore 0=(b_n+2b_{n+1}+3b_{n+2})-(b_{n+1}+2b_{n+2}+3b_{n+3})=(b_n-b_{n+1})+2(b_{n+1}-b_{n+2})+3(b_{n+2}-b_{n+3})\le 0$

$(\because b_n\le b_{n+1}, \therefore b_n-b_{n+1}\le 0)$

$\therefore b_n-b_{n+1}=b_{n+1}-b_{n+2}=b_{n+2}-b_{n+3}=0$

$\therefore -2d_c=6b_n=6(a_n-a_{n+2})$即$a_{n+2}-a_n=\dfrac{1}{3}d_c$

$\therefore c_n=a_n+2a_{n+1}+3(a_n+\dfrac{1}{3}d_c)=4a_n+2a_{n+1}+d_c$

$\therefore c_{n+1}=4a_{n+1}+2a_{n+2}+d_c, \therefore d_c=2a_{n+2}+2a_{n+1}-4a_n=2(a_n+\dfrac{1}{3}d_c)+2a_{n+1}-4a_n=2a_{n+1}-2a_n+\dfrac{2}{3}d_c$

$\therefore a_{n+1}-a_n=\dfrac{1}{6}d_c$为常数,$\therefore \{a_n\}$为等差数列

由①②可知:$\{a_n\}$为等差数列的充要条件是$\{c_n\}$为等差数列且$b_n\le b_{n+1}(n=1,2,3\cdots)$.

（2009 江苏）设$\{a_n\}$是公差不为零的等差数列，S_n为其前 n 项和，满足$a_2^2+a_3^2=a_4^2+a_5^2, S_7=7$.

（1）求数列$\{a_n\}$的通项公式及前 n 项和S_n；（2）试求所有的正整数 m，使得$\dfrac{a_ma_{m+1}}{a_{m+2}}$为数列$\{a_n\}$中的项.

数列（1）等差等比数列解答（1）

2024-01-13

解：（1）由$\begin{cases} a_4^2 - a_2^2 + a_5^2 - a_3^2 = 2d(2a_1 + 4d) + 2d(2a_1 + 6d) = 0(d \neq 0) \\ S_7 = 7a_1 + \dfrac{6\times 7}{2}d = 7 \end{cases}$ 得$a_1 = -5, d = 2$

$\therefore a_n = 2n - 7, S_n = n^2 - 6n$

（2）由（1）得$:\dfrac{a_m a_{m+1}}{a_{2m+3}} = \dfrac{(2m-7)(2m-5)}{2m-3} = 2m - 3 - 6 + \dfrac{8}{2m-3} \in \{a_n\}$

$\therefore \dfrac{8}{2m-3} = \pm 1,$得$m = 2$