期末选讲（1）三角函数解答（3）

2023-06-22

三、解三角形、

（2001A）如果满足$\angle ABC=60^\circ, AC=12, BC=k$的$\triangle ABC$恰有一个，那么$k$的取值范围为（　）

D

$A.k=8\sqrt{3}$　$B.0<k\le 12$　$C.k\ge 12$　$D.0<k\le 12$,或$k=8\sqrt{3}$

（2017吉林）在$\triangle ABC$中，$AB=1, BC=2$，则$\angle C$的取值范围为（　）

$A.(0,\dfrac{\pi}{6}]$　$B.(\dfrac{\pi}{4},\dfrac{\pi}{2})$　$C.(\dfrac{\pi}{6},\dfrac{\pi}{3})$　$D.(0,\dfrac{\pi}{2})$

（2017吉林）$key1:\dfrac{1}{\sin C}=\dfrac{2}{\sin A}$得$2\sin C=\sin A\le 1(C\in(0,\dfrac{\pi}{2})$得$C\in(0,\dfrac{\pi}{6}]$

$key2:\cos C=\dfrac{4+b^2-1}{2\times 2b}=\dfrac{3}{4b}+\dfrac{b}{4}\ge\dfrac{\sqrt{3}}{2},\therefore$选$A$

（2006江苏）在$\triangle ABC$中，角A,B,C所对的边分别是$a,b,c,\tan A=\dfrac{1}{2},\cos B=\dfrac{3\sqrt{10}}{10}$.若$\triangle ABC$最长的边为1，则最短边的长为（　　）$A.\dfrac{2\sqrt{5}}{5}$　$B.\dfrac{3\sqrt{5}}{5}$　$C.\dfrac{4\sqrt{5}}{5}$　$D.\dfrac{\sqrt{5}}{5}$

2006江苏$key:$由$\tan A=\dfrac{1}{2}<\dfrac{1}{\sqrt{3}}$,且$A\in(0,\pi)$得$A\in(0,\dfrac{\pi}{6})$,

$\cos B=\dfrac{3}{\sqrt{10}}>\dfrac{2}{\sqrt{5}}=\cos A$，且$B\in(0,\pi)$得$0<B<A<\dfrac{\pi}{6},\therefore C>\dfrac{2\pi}{3}$最大,$\therefore\dfrac{1}{\sin(A+B)}=\dfrac{b}{\sin B}$得$b=\dfrac{\sqrt{5}}{5}$,选$D$

（2019福建）在$\triangle ABC$中，若$AC=\sqrt{2}, AB=2$,且$\dfrac{\sqrt{3}\sin A+\cos A}{\sqrt{3}\cos A-\sin A}=\tan\dfrac{5\pi}{12}$,则$BC=$____.

（2019福建）$key:$由已知得$\dfrac{\sqrt{3}\tan A+1}{\sqrt{3}-\tan A}=\dfrac{\tan A+\tan\dfrac{\pi}{6}}{1-\tan\dfrac{\pi}{6}\tan A}=\tan(A+\dfrac{\pi}{6})=\tan\dfrac{5\pi}{12}$得$A=\dfrac{\pi}{4},\therefore BC=\sqrt{2}$

（2020A）在$\triangle ABC$中，$BC=4, CA=5, AB=6$,则$\sin^6\dfrac{A}{2}+\cos^6\dfrac{A}{2}=$_____.

2020$Akey:$由$\sin^6\dfrac{A}{2}+\cos^6\dfrac{A}{2}=(\sin^2\dfrac{A}{2}+\cos^2\dfrac{A}{2})(\sin^4\dfrac{A}{2}+\sin^2\dfrac{A}{2}\cos^2\dfrac{A}{2}+\cos^4\dfrac{A}{2})=1-\dfrac{3}{4}\sin^2 A=\dfrac{43}{64}$

$(\because\cos A=\dfrac{5^2+6^2-4^2}{2\times5\times6}=\dfrac{3}{4})$

（1999A）在$\triangle ABC$中，记$BC=a, CA=b, AB=c$，若$9a^2+9b^2-19c^2=0$,则$\dfrac{\tan(\dfrac{\pi}{2}-C)}{\tan(\dfrac{\pi}{2}-A)+\tan(\dfrac{\pi}{2}-B)}=$____.

1999$Akey:$原式$=\dfrac{1}{\tan C(\dfrac{\cos A}{\sin A}+\dfrac{\cos B}{\sin B})}=\dfrac{\sin A\sin B\cos C}{\sin^2 C}=\dfrac{ab\cos C}{c^2}=\dfrac{a^2+b^2-c^2}{2c^2}=\dfrac{5}{9}$

（2021江西）$\triangle ABC$中，$AB=c, BC=a, AC=b$,且$a^4+b^4+c^4=2c^2(a^2+b^2)$,若$\angle A=72^\circ$,则$\angle B=$____.

2021江西$key:$由已知得$:(a^2+b^2-c^2)^2=2a^2b^2,\therefore 2ab\cos C=a^2+b^2-c^2=\pm\sqrt{2}ab$

$\therefore\cos C=\pm\dfrac{\sqrt{2}}{2},\because A=72^\circ,\therefore C=45^\circ,\therefore B=63^\circ$

（2021Ⅰ）记$\triangle ABC$是内角A,B,C的对边分别为a,b,c.已知$b^2=ac$，点D在边AC上，$BD\sin\angle ABC=a\sin C$.

（1）证明：$BD=b$；（2）若$AD=2DC$,求$\cos\angle ABC$.

2021Ⅰ（1）证明：由已知得及正弦定理得$BD=\dfrac{a\sin C}{\sin\angle ABC}=\dfrac{ac}{b}=b(\because b^2=ac)$得证

（2）解：由$BD=2DC$得$\overrightarrow{BD}=\dfrac{2}{3}\overrightarrow{BC}+\dfrac{1}{3}\overrightarrow{BA},\therefore b^2=\dfrac{4}{9}a^2+\dfrac{4}{9}ac\cos\angle ABC+\dfrac{1}{9}c^2$

2023-06-22

即$\cos\angle ABC=\dfrac{9ac-4a^2-c^2}{4ac}=\dfrac{a^2+c^2-ac}{2ac}$得$a=\dfrac{3}{2}c$,或$a=\dfrac{c}{3}$（舍），$\therefore\cos\angle ABC=\dfrac{7}{12}$

（2022 乙） 记$\triangle ABC$的内角A,B,C的对边分别为a,b,c,已知$\sin C\sin(A-B)=\sin B\sin(C-A)$.

(1）证明$:2a^2=b^2+c^2$;(2）若$a=5,\cos A=\dfrac{25}{31}$,求$\triangle ABC$的周长.

⑴ 证明$:\because$ 在$\triangle ABC$中$,\sin C\sin(A-B)=\sin C(\sin A\cos B-\cos A\sin B)=\sin A\sin C\cos B-\cos A\sin B\sin C$

$=\dfrac{1}{4R^2}(ac\cdot\dfrac{a^2+c^2-b^2}{2ac}-bc\cdot\dfrac{b^2+c^2-a^2}{2bc})=\dfrac{1}{8R^2}(2a^2-2b^2)$

$\sin B\sin(C-A)=\sin B\sin C\cos A-\sin B\cos C\sin A=\dfrac{1}{4R^2}(bc\cdot\dfrac{b^2+c^2-a^2}{2bc}-ab\cdot\dfrac{a^2+b^2-c^2}{2ab})=\dfrac{1}{8R^2}(2c^2-2a^2)$

$\therefore a^2-b^2=c^2-a^2$即$2a^2=b^2+c^2$

$key2:\sin C\sin(A-B)=\sin(A+B)\sin(A-B)=\sin^2A\cos^2B-\cos^2A\sin^2B=\sin^2A(1-\sin^2B)-(1-\sin^2A)\sin^2B$
$=\sin^2A-\sin^2B$

$\sin B\sin(C-A)=\sin(C+A)\sin(C-A)=\sin^2C(1-\sin^2A)-(1-\sin^2C)\sin^2A=\sin^2C-\sin^2A$

$\therefore 2\sin^2A=\sin^2B+\sin^2C,\therefore 2a^2=b^2+c^2$

$key3$: （积化和差）$\sin C\sin(A-B)=\sin(A+B)\sin(A-B)=-\dfrac{1}{2}(\cos(A+B+A-B)-\cos((A+B)-(A-B))]$

$=-\dfrac{1}{2}(1-2\sin^2A-1+2\sin^2B)=\sin^2B-\sin^2A$

同理$\sin B\sin(C-A)=\sin(C+A)\sin(C-A)=\sin^2A-\sin^2C,\therefore 2\sin^2A=\sin^2B+\sin^2C,\therefore 2a^2=b^2+c^2$

（2）由$25=a^2=b^2+c^2-2bc\cos A=b^2+c^2-\dfrac{50}{31}bc$,且$b^2+c^2=50$,

$\therefore 2bc=31,\therefore(b+c)^2=81,\therefore b+c=9,\therefore\triangle ABC$周长为14

(1998Ⅰ）在$\triangle ABC$中，a,b,c分别是角A,B,C的对边，设$a+c=2b,A-C=\dfrac{\pi}{3}$,求$\sin B$的值.

1998Ⅰ $key1:\because$ 在$\triangle ABC$中，由$a+c=2b$,且$A-C=\dfrac{\pi}{3}$得$\sin A+\sin C=2\sin\dfrac{A+C}{2}\cos\dfrac{A-C}{2}$

$=2\cos\dfrac{B}{2}\cdot\dfrac{\sqrt{3}}{2}=2\sin B=4\sin\dfrac{B}{2}\cos\dfrac{B}{2}$得$\sin\dfrac{B}{2}=\dfrac{\sqrt{3}}{4},\therefore\sin B=2\cdot\dfrac{\sqrt{3}}{4}\cdot\dfrac{\sqrt{13}}{4}=\dfrac{\sqrt{39}}{8}$

$key2:$设由已知得$a>b>c$，设$a=b+d,c=b-d(b>2d>0)$,

作$AD\perp AB$交AC于D，则$\angle DAC=\angle C$,设$CD=x$,

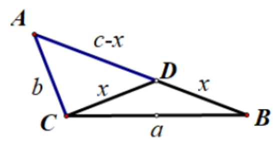
则$x^2+(b-d)^2-(b-d)x=(b+d-x)^2$得$x=\dfrac{4bd}{b+3d}$

$\therefore\cos C=\dfrac{b}{2x}=\dfrac{b+3d}{8d}=\dfrac{b^2+(b+d)^2-(b-d)^2}{2b(b+d)}=\dfrac{b+4d}{2(b+d)}$得$b=\sqrt{13}d,a=(\sqrt{13}+1)d,c=(\sqrt{13}-1)d$

$\therefore\cos B=\dfrac{5}{8},\therefore\sin B=\dfrac{\sqrt{39}}{8}$

(2022Ⅰ)18.记$\triangle ABC$的内角A,B,C的对边分别为a,b,c,已知$\dfrac{\cos A}{1+\sin A}=\dfrac{\sin 2B}{1+\cos 2B}$.

⑴ 若$C=\dfrac{2\pi}{3}$,求B; （2）求$\dfrac{a^2+b^2}{c^2}$的最小值.



2022Ⅰ 解： ⑴$\because$ 在$\triangle ABC$中，由$\dfrac{\cos A}{1+\sin A}=\dfrac{2\sin B\cos B}{2\cos^2B}=\dfrac{\sin B}{\cos B}$

$\Leftrightarrow\cos A\cos B=\sin B+\sin A\sin B\Leftrightarrow\cos(A+B)=\sin B=-\cos C=\sin(C-\dfrac{\pi}{2}),\therefore B=C-\dfrac{\pi}{2},A=\dfrac{3\pi}{2}-2C$

期末选讲（1）三角函数解答（3）

2023-06-22

$key2:\tan(\frac{\pi}{4}-\frac{A}{2})=\frac{\cos^2\frac{A}{2}-\sin^2\frac{A}{2}}{(\sin\frac{A}{2}+\cos\frac{A}{2})^2}=\frac{2\sin B\cos B}{2\cos^2 B}=\tan B,$

$\therefore\frac{\pi}{4}-\frac{A}{2}=B=\frac{\pi}{4}-\frac{\pi-B-C}{2}=\frac{B+C}{2}-\frac{\pi}{4}$即$B=C-\frac{\pi}{2},\because C=\frac{2\pi}{3},\therefore B=\frac{\pi}{6}$

（2）$key1:\frac{a^2+b^2}{c^2}=\frac{\sin^2 A+\sin^2 B}{\sin^2 C}=\frac{\cos^2 2C+\cos^2 C}{\sin^2 C}=4\sin^2 C+\frac{2}{\sin^2 C}-5\geq 4\sqrt{2}-5$（当且仅当$\sin^2 C=\frac{\sqrt{2}}{2}$时，取$=$）

$key2:x^2+b^2=(c-x)^2$得$x=\frac{c^2-b^2}{2c},\therefore\cos B=\frac{a}{2x}=\frac{ac}{c^2-b^2}=\frac{a^2+c^2-b^2}{2ac}$得$a^2=\frac{(b^2-c^2)^2}{b^2+c^2}$

$\therefore\frac{a^2+b^2}{c^2}=\frac{\frac{(b^2-c^2)^2}{b^2+c^2}+b^2}{c^2}=2(t^2+1)+\frac{4}{t^2+1}-5\geq 4\sqrt{2}-5$(其中$t=\frac{b}{c}>0$)

（2017吉林）已知锐角$\triangle ABC$中，$\sin(A+B)=\frac{3}{5},\sin(A-B)=\frac{1}{5},AB=3$，则$\triangle ABC$的面积为___.

2017吉林$key1$:如图，$\frac{a-x}{\frac{1}{5}}=\frac{x}{\frac{3}{5}}$得$x=\frac{3}{4}a,\therefore\frac{1}{16}a^2+b^2-2\cdot\frac{1}{4}a\cdot b\cdot\frac{4}{5}=\frac{9}{16}a^2$得$b=\frac{2+3\sqrt{6}}{10}a$

而$a^2+b^2-2ab\cdot\frac{4}{5}=9$得$a^2=\frac{50}{7-2\sqrt{6}}$

$\therefore S_{\triangle ABC}=\frac{1}{2}\cdot a\cdot\frac{2+3\sqrt{6}}{10}a\cdot\frac{3}{5}=\frac{3(2+3\sqrt{6})}{100}\cdot\frac{50}{7-2\sqrt{6}}=\frac{3(\sqrt{6}+2)}{2}$

$key2:\begin{cases}\sin A\cos B+\cos A\sin B=\frac{3}{5}\\ \sin A\cos B-\cos A\sin B=\frac{1}{5}\end{cases}$得$\sin A\cos B=\frac{2}{5},\cos A\sin B=\frac{1}{5},\therefore\tan A=2\tan B$

$\therefore-\frac{3}{4}=\tan(A+B)=\frac{3\tan B}{1-2\tan^2 B}$得$\tan B=1+\frac{\sqrt{6}}{2},\tan A=2+\sqrt{6}$

$\therefore 5=\frac{3}{\frac{3}{5}}=\frac{a}{\sin A}$得$a=\frac{5(2+\sqrt{6})}{\sqrt{11+4\sqrt{6}}},\therefore S=\frac{1}{2}\cdot\frac{5(2+\sqrt{6})}{\sqrt{11+4\sqrt{6}}}\cdot 3\cdot\frac{2+\sqrt{6}}{\sqrt{14+4\sqrt{6}}}=\frac{3(2+\sqrt{6})}{2}$

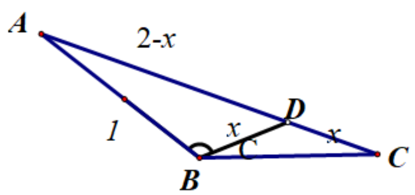
（2019江苏）已知$\triangle ABC$中，$AC=8,BC=10,32\cos(A-B)=31$，则$\triangle ABC$的面积为______.

2019江苏$key:(10-x)^2=64+x^2-16x\cdot\frac{31}{32}$得$x=8,$

$\therefore\frac{8}{\sin C}=\frac{2}{\sin(B-C)}=\frac{2}{\sqrt{1-(\frac{31}{32})^2}}$得$\sin C=\frac{3\sqrt{7}}{8},\therefore S_{\triangle ABC}=15\sqrt{7}$

（2021A）在$\triangle ABC$中，$AB=1,AC=2,B-C=\frac{2\pi}{3}$，则$\triangle ABC$的面积为_____.

（2021A）key:如图，有$1+x^2+x=(2-x)^2$得$x=\frac{3}{5},$



$\therefore\frac{\frac{3}{5}}{\sin A}=\frac{\frac{7}{5}}{\frac{\sqrt{3}}{2}}$得$\sin A=\frac{3\sqrt{3}}{14},\therefore S_{\triangle ABC}=\frac{3\sqrt{3}}{14}$

（2018河北）在$\triangle ABC$中，$AC=3,\sin C=k\sin A(k\geq 2)$，则$\triangle ABC$的面积最大值为_____.

期末选讲（1）三角函数解答（3）

2023-06-22

2018河北$key:$由$\sin C=k\sin A\Leftrightarrow BA=kBC$

$\therefore B$的轨迹是阿氏圆，且$2R=\frac{3}{k+1}+\frac{3}{k-1}=\frac{6k}{k^2-1},\therefore S_{\triangle ABC}\le\frac{1}{2}\cdot 3\cdot\frac{63}{k^2-1}=9\cdot\frac{1}{k-\frac{1}{k}}\le 3$

变式：在$\triangle ABC$中，$b=\sqrt{3}$，$B=\frac{\pi}{3}$.①则$a+c\in$_____;$ac\in$______,$a^2+c^2\in$_______.

②若$\triangle ABC$为锐角三角形，则$a+c\in$_____;$ac\in$______;$a^2+c^2\in$___;

$2a+c\in$______;$2a^2+c^2\in$______.

变式:$3=b^2=a^2+c^2-ac,S=\frac{1}{2}ac\cdot\frac{\sqrt{3}}{2}\in(0,\frac{3\sqrt{3}}{4}]$

①$\therefore ac=\frac{4}{\sqrt{3}}S\in(0,3]$，$a+c=\sqrt{3+4\sqrt{3}S}\in(\sqrt{3},2\sqrt{3}],a^2+c^2=3+\frac{4}{\sqrt{3}}S\in(3,\frac{25}{3}]$

②$\because\triangle ABC$是锐角三角形,$\therefore S\in(\frac{\sqrt{3}}{2},\frac{3\sqrt{3}}{4}]$,

$\therefore ac=\frac{4}{\sqrt{3}}S\in(2,3]$，$a+c=\sqrt{3+4\sqrt{3}S}\in(3,2\sqrt{3}],a^2+c^2=3+\frac{4}{\sqrt{3}}S\in(5,\frac{25}{3}]$

由$2a+c=2(2\sin A+\sin(A+\frac{\pi}{3}))(\begin{cases}A\in(0,\frac{\pi}{2})\\A+\frac{\pi}{3}\in(\frac{\pi}{2},\pi)\end{cases}$ 得$A\in(\frac{\pi}{6},\frac{\pi}{2})$

$=\sqrt{3}\cos A+5\sin A=(\sqrt{3},5)\cdot(\cos A,\sin A)\in(4,2\sqrt{7}]$

$2a^2+c^2=4(2\sin^2A+\sin^2(A+\frac{\pi}{3}))=8\cdot\frac{1-\cos 2A}{2}+4\cdot\frac{1-\cos(2A+\frac{2\pi}{3})}{2}$

$=6+(-3,\sqrt{3})\cdot(\cos 2A,\sin 2A)\in(6,6+2\sqrt{3}]$

（2018浙江）在$\triangle ABC$在，$AB+AC=7$,且$\triangle ABC$的面积为4，则$\sin A$的最小值为_____.

2018浙江$key:S=\frac{1}{2}cb\sin A=4$,且$c+b=7,\therefore\sin A=\frac{8}{bc}\ge\frac{8}{(\frac{b+c}{2})^2}=\frac{32}{49}$

（2022新高考Ⅱ）记$\triangle ABC$的内角A,B,C的对边分别为a,b,c,分别以a,b,c为边长的正三角形的面积依次为S_1,S_2,S_3,已知$S_1-S_2+S_3=\frac{\sqrt{3}}{2},\sin B=\frac{1}{3}$.(1）求$\triangle ABC$的面积；（2）若$\sin A\sin C=\frac{\sqrt{2}}{3}$,求$b$.

2022Ⅱ：（1）由$S_1-S_2+S_3=\frac{\sqrt{3}}{4}(a^2-b^2+c^2)=\frac{\sqrt{3}}{2}$即$2=a^2+c^2-b^2=2ac\cos B,\therefore\cos B>0$

$\therefore S_{\triangle ABC}=\frac{1}{2}ac\sin B=\frac{1}{2}\tan B=\frac{\sqrt{2}}{8}$

（2）由（1）得$4R^2\sin A\sin C\cos B=4R^2\cdot\frac{\sqrt{2}}{3}\cdot\frac{2\sqrt{2}}{3}=1$得$R=\frac{3}{4},\therefore b=\frac{3}{2}\cdot\frac{1}{3}=\frac{1}{2}$

(1993A）在$\triangle ABC$中，角A,B,C的对边长分别为a,b,c.若$c-a$等于AC边上的高h，则$\sin\frac{C-A}{2}+\cos\frac{C+A}{2}$的值是（ ）$A.1$ $B.\frac{1}{2}$ $C.\frac{1}{3}$ $D.-1$

期末选讲（1）三角函数解答（3）

2023-06-22

1993$Akey$:由已知的$c-a=a\sin C$即$\sin C-\sin A=\sin A\sin C$

$\Leftrightarrow 2\cos\frac{C+A}{2}\sin\frac{C-A}{2}=\frac{1}{2}[\cos(C-A)-\cos(A+C)]$

$=\frac{1}{2}(1-2\sin^2\frac{C-A}{2}-2\cos^2\frac{A+C}{2}+1)\therefore \sin^2\frac{C-A}{2}+2\cos\frac{C+A}{2}\sin\frac{C-A}{2}+\cos^2\frac{A+C}{2}=1,\therefore$ 选A

变式1（1）若$h_a=a$,则$\frac{c}{b}+\frac{b}{c}$的取值范围为________;$\frac{c}{b}+\frac{b}{c}+\frac{a^2}{bc}$的最大值为____.

key:由$a=h_a=c\sin B=b\sin C$即$\sin A=\sin B\sin C,\therefore a^2=bc\sin B\sin C=bc\sin A,$

$\therefore\frac{c}{b}+\frac{b}{c}=\frac{b^2+c^2}{bc}=\frac{a^2+2bc\cos A}{bc}=\sin A+2\cos A\in[2,2\sqrt{5}]$(由几何意义得$A\in(0,\arctan\frac{4}{3}]$)

$\therefore\frac{c}{b}+\frac{b}{c}+\frac{a^2}{bc}=2\sin A+2\cos A=2\sqrt{2}\sin(A+\frac{\pi}{4})\le 2\sqrt{2}$

（2020A）在$\triangle ABC$中，$BC=4,AB=6,$边AC上的中线长为$\sqrt{10}$，则$\sin^6\frac{A}{2}+\cos^6\frac{A}{2}$的值为______.

2020$Akey$:$\overrightarrow{BA}^2+\overrightarrow{BC}^2=2\times10+\frac{1}{2}\overrightarrow{AC}^2$得$|\overrightarrow{AC}|=8,\therefore\cos A=\frac{7}{8}$

$\therefore\sin^6\frac{A}{2}+\cos^6\frac{A}{2}=\sin^4\frac{A}{2}-\sin^2\frac{A}{2}\cos^2\frac{A}{2}+\cos^4\frac{A}{2}=1-\frac{3}{4}\sin^2 A=\frac{211}{256}$

（2006天津）在$Rt\triangle ABC$中，c,r,S分别表示它的斜边长，内切圆半径和面积，则$\frac{cr}{S}$的取值范围是_______.

2006天津key:$\frac{cr}{S}=\frac{c\cdot\frac{a+b-c}{2}}{\frac{1}{2}ab}=\frac{\sin A+\cos A-1}{\sin A\cos A}=\frac{\sin A+\cos A-1}{\frac{(\sin A+\cos A)^2-1}{2}}=\frac{2}{\cos A+\sin A+1}$

$=\frac{2}{\sqrt{2}\sin(A+\frac{\pi}{4})+1}\in[2\sqrt{2}-2,1)$

（2017贵州）已知$\triangle ABC$中，$A=\frac{\pi}{3},\frac{AB}{AC}=\frac{5}{8},$内切圆半径$r=2\sqrt{3}.$则$\triangle ABC$的面积为______.

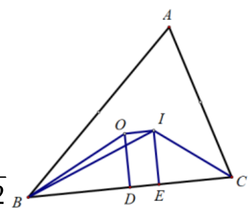
2017贵州key:$\begin{cases}\frac{c}{b}=\frac{5}{8}\\ \frac{b+c-a}{2}\tan\frac{A}{2}=r\text{即}\frac{b+c-a}{2}\cdot\frac{\sqrt{3}}{3}=2\sqrt{3}\text{得}b+c-a=12\text{得}b=16,c=10\\ a^2=b^2+c^2-bc\end{cases}$

$\therefore S_{\triangle ABC}=40\sqrt{3}$

（2015江苏）设$\triangle ABC$的外心为O，内心为1，$\angle B=45°.$若$OI//BC$，则$\cos C$的值为____.

2015江苏key:$OD=R\cos A=IE=\frac{a+c-b}{2}\tan\frac{b}{2}$得$\cos A=(\sin A+\sin C-\sin B)\tan\frac{B}{2}$

$=(2\sin\frac{A+C}{2}\cos\frac{A-C}{2}-2\sin\frac{A+C}{2}\cos\frac{A+C}{2})\frac{\cos\frac{A+C}{2}}{\sin\frac{A+C}{2}}$



$=2\cos\frac{A+C}{2}(\cos\frac{A-C}{2}-\cos\frac{A+C}{2})=\cos A+\cos C-(1-\cos B),\therefore\cos C=1-\frac{\sqrt{2}}{2}$

期末选讲（1）三角函数解答（3）

2023-06-22

(1996Ⅰ）已知$\triangle ABC$的三个内角A,B,C满足$A+C=2B,\dfrac{1}{\cos A}+\dfrac{1}{\cos C}=-\dfrac{\sqrt2}{\cos B}$，求$\cos\dfrac{A-C}{2}$的值.

1996Ⅰ key:由$A+C=\dfrac{2\pi}{3},B=\dfrac{\pi}{3}$,且$\cos A+\cos C=-2\sqrt2\cos A\cos C$

即$\cos\dfrac{A-C}{2}=2\cos\dfrac{A+C}{2}\cos\dfrac{A-C}{2}=-\sqrt2(\cos(A+C)+\cos(A-C))=\dfrac{\sqrt2}{2}-\sqrt2(2\cos^2\dfrac{A-C}{2}-1)$

即$(2\sqrt2\cos\dfrac{A-C}{2}+3)(\cos\dfrac{A-C}{2}-\dfrac{\sqrt2}{2})=0,\therefore\cos\dfrac{A-C}{2}=\dfrac{\sqrt2}{2}$

(2006湖南）已知在$\triangle ABC$中,$\sin A(\sin B+\cos B)-\sin C=0,\sin B+\cos 2C=0$,则角$A,B,C$的大小关系为（　）$A.C>B>A$　$B.A>B>C$　$C.B>C>A$　$D.C>A>B$

2006湖南key:$\sin A\sin B-\sin A\cos B=\sin)A+B)=\sin A\cos B+\cos A\sin B$得$A=\dfrac{\pi}{4}$

由$\sin B=-\cos 2C=\sin(2C-\dfrac{\pi}{2})$,而$B\in(0,\pi),2C-\dfrac{\pi}{2}\in(-\dfrac{\pi}{2},\dfrac{3\pi}{2})$,

$\therefore B=2C-\dfrac{\pi}{2},or,B+2C-\dfrac{\pi}{2}=\pi$(舍)$,\therefore C=\dfrac{5\pi}{12},B=\dfrac{\pi}{3},\therefore$选$A$

(2015山东)$\triangle ABC$中，$\angle A<\angle B<\angle C,\dfrac{\sin A+\sin B+\sin C}{\cos A+\cos B+\cos C}=\sqrt3$.则$\angle B=$_____.

2015山东key:由已知得$\sin A-\sqrt3\cos A+\sin B-\sqrt3\cos B+\sin C-\sqrt3\cos C$

$=2\sin(A-\dfrac{\pi}{3})+2\sin(B-\dfrac{\pi}{3})+2\sin(C-\dfrac{\pi}{3})=4\sin\dfrac{A+C-\frac{2\pi}{3}}{2}\cos\dfrac{A-C}{2}+2\sin(B-\dfrac{\pi}{3})$

$=-4\sin\dfrac{B-\frac{\pi}{3}}{2}\cos\dfrac{A-C}{2}+4\sin\dfrac{B-\frac{\pi}{3}}{2}\cos\dfrac{B-\frac{\pi}{3}}{2}=0(\because A<B<C),\therefore B=\dfrac{\pi}{3}$

(2015陕西)$\triangle ABC$中，若$\tan\dfrac{A}{2}+\tan\dfrac{B}{2}=1$，则$\tan\dfrac{C}{2}$的最小值为______.

2015陕西key:$\tan\dfrac{C}{2}=\dfrac{1}{\tan\dfrac{A+B}{2}}=1-\tan\dfrac{A}{2}\tan\dfrac{B}{2}\ge1-\dfrac{1}{4}=\dfrac{3}{4}$

(2015辽宁）已知$\triangle ABC$的三边长a,b,c成等比数列，边a,b,c所对的角依次为A,B,C，且

$\sin A\sin B+\sin B\sin C+\cos 2B=1$，则$B=$（　）$A.\dfrac{\pi}{4}$　$B.\dfrac{\pi}{3}$　$C.\dfrac{\pi}{2}$　$D.\dfrac{2\pi}{3}$

2015辽宁key:$\sin B(\sin A+\sin C)=2\sin^2 B$得$2b=a+c$，而$b^2=ac,\therefore a=b=c,\therefore$选$B$

(2017新疆）已知在$\triangle ABC$中,$\tan A+\tan C=2(1+\sqrt2)\tan B$.则$\angle B$的最小值为______.

2017新疆$key1$:$\tan A\tan B\tan C=\tan A+\tan B+\tan C=(3+2\sqrt2)\tan B$,

$\therefore\dfrac{3+2\sqrt2}{1}=\tan A\tan C=\dfrac{\sin A\sin C}{\cos A\cos C}\Leftrightarrow\dfrac{4+2\sqrt2}{2+2\sqrt2}=\dfrac{\cos(A-C)}{-\cos(A+C)}$,

$\therefore\cos B=\dfrac{\sqrt2}{2}\cos(A-C)\le\dfrac{\sqrt2}{2},\therefore B\ge\dfrac{\pi}{4}$

$key2$:$2(1+\sqrt2)\tan B=\tan(A+C)\cdot(1-\tan A\tan C)=-\tan B+\tan A\tan B\tan C$

$\therefore\tan A\tan C=3+2\sqrt2$

(2017内蒙古）锐角三角形的内角A,B满足$\tan A-\dfrac{1}{\sin 2A}=\tan B$，且$\cos^2\dfrac{B}{2}=\dfrac{\sqrt6}{3}$，则$\sin 2A=$______.

期末选讲（1）三角函数解答（3）

2023-06-22

（2017内蒙古）$key:\cos^2\frac{B}{2}=\frac{1+\cos B}{2}=\frac{\sqrt{6}}{3}(B\in(0,\frac{\pi}{2}))$得$\cos B=\frac{2\sqrt{6}}{3}-1,$

$\tan A-\frac{1}{\sin 2A}=\frac{\sin A}{\cos A}-\frac{1}{\sin 2A}=\frac{2\sin^2 A-1}{\sin 2A}=-\frac{\cos 2A}{\sin 2A}=\frac{\sin B}{\cos B}$得$\cos(2A-B)=0$

而$2A-B\in(-\frac{\pi}{2},\pi),\therefore 2A-B=\frac{\pi}{2},\therefore \sin 2A=\sin(\frac{\pi}{2}+B)=\cos B=\frac{2\sqrt{6}-3}{3}$

$key2:\tan A-\frac{1}{\sin 2A}=\tan A-\frac{1+\tan^2 A}{2\tan A}=\frac{\tan^2 A-1}{2\tan A}=-\tan(\frac{\pi}{2}-2A)=\tan(2A-\frac{\pi}{2})=\tan B$

$\because 2A-\frac{\pi}{2}=(-\frac{\pi}{2},\frac{\pi}{2}),B\in(0,\frac{\pi}{2}),\therefore 2A-\frac{\pi}{2}=B,\therefore \sin 2A=\sin(\frac{\pi}{2}+B)=\cos B=\frac{2\sqrt{6}-3}{3}$

$key3:\tan A-\tan B=\frac{\sin(A-B)}{\cos A\cos B}=\frac{1}{\sin 2A}=\frac{1}{2\sin A\cos A}\Leftrightarrow \cos B=2\sin A\sin(A-B)$

$=\cos B-\cos(2A-B)$得$\cos(2A-B)=0$，而$2A-B\in(-\frac{\pi}{2},\pi),\therefore 2A-B=\frac{\pi}{2}$

（2021山东）设A,B,C是$\triangle ABC$的三个内角，则使得$\frac{1}{\sin A}+\frac{1}{\sin B}\ge\frac{\lambda}{3+2\cos C}$恒成立的实数$\lambda$的最大值是__.

（2021山东）$\Leftrightarrow \lambda\le\frac{(3+2\cos C)(\sin A+\sin B)}{\sin A\sin B}=\frac{(3+2\cos C)\cdot 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{\frac{1}{2}(\cos(A-B)-\cos(A+B))}$

$=4(3+2\cos C)\cdot\frac{\cos\frac{C}{2}\cos\frac{A-B}{2}}{2\cos^2\frac{A-B}{2}-1+\cos C}=4(3+2\cos C)\cdot\frac{\cos\frac{C}{2}}{2\cos\frac{A-B}{2}-\frac{2\sin^2\frac{C}{2}}{\cos\frac{A-B}{2}}}(\because 0<\cos\frac{A-B}{2}\le 1)$

$\ge\frac{4(3+2\cos C)\cos\frac{C}{2}}{2\cos^2\frac{C}{2}}=\frac{2(4\cos^2\frac{C}{2}+1)}{\cos\frac{C}{2}}\ge 8,\therefore \lambda_{\max}=8$

（2021江西）锐角$\triangle ABC$在，若$\cos^2 A,\cos^2 B,\cos^2 C$的和等于$\sin^2 A,\sin^2 B,\sin^2 C$中的某个值，证明：

$\tan A,\tan B,\tan C$必可按某顺序组成一个等差数列.

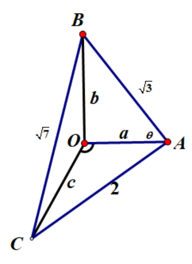
（2021江西）$key:\frac{1+\cos 2A}{2}+\frac{1+\cos 2C}{2}+\cos 2B=1+\cos(A+C)\cos(A-C)+\cos 2(A+C)=0$

即$\cos(A+C)(2\cos(A+C)+\cos(A-C))=0(\because A+C\ne\frac{\pi}{2})$

$\therefore 2(\cos A\cos C-\sin A\sin C)+\cos A\cos C+\sin A\sin C=0,\therefore \tan A\tan C=3$

$\tan A+\tan C=\tan(A+C)(1-\tan A\tan C)=2\tan B,\therefore \tan A,\tan B,\tan C$成等差数列

（2014浙江竞赛）设正实数a,b,c满足$\begin{cases}a^2+b^2=3,\\ a^2+c^2+ac=4,\\ b^2+c^2+\sqrt{3}bc=7,\end{cases}$ 则$a=$___$,b=$____$,c=$____.



期末选讲（1）三角函数解答（3）

2023-06-22

（2014浙江）$key:\dfrac{2}{\sin120^\circ}=\dfrac{c}{\sin\theta}$即$c=\dfrac{4}{\sqrt{3}}\sin\theta, a=\sqrt{3}\sin\theta, b=\sqrt{3}\cos\theta$

$\therefore a^2+c^2+ac=3\sin^2\theta+\dfrac{16}{3}\sin^2\theta+4\sin^2\theta=\dfrac{37}{3}\sin^2\theta=4,$

$\therefore \sin\theta=\dfrac{2\sqrt{3}}{\sqrt{37}}, \cos\theta=\dfrac{5}{\sqrt{37}}, \therefore a=\dfrac{6\sqrt{37}}{37}, b=\dfrac{5\sqrt{111}}{37}, c=\dfrac{8\sqrt{37}}{37}$

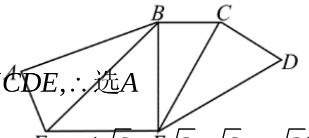
（2017安徽）设圆内接四边形$ABCD$的边长分别为$AB=3, BC=4, CD=5, DA=6,$则四边形$ABCD$的面积是____.

2017安徽$key:$设$\angle BAD=\theta,$则$9+36-2\cdot3\cdot6\cos\theta=16+25+2\cdot4\cdot5\cos\theta$得$\cos\theta=\dfrac{1}{19}$

$\therefore S_{ABCD}=\dfrac{1}{2}\cdot(3\times6+4\times5)\sqrt{1-\dfrac{1}{19^2}}=6\sqrt{10}$

（2019北京）如图，$\angle BAF=\angle FEB=\angle EBC=\angle ECD=90^\circ, \angle ABF=30^\circ, \angle BFE=45^\circ, \angle BCE=60^\circ,$

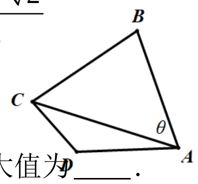
$AB=2CD,$则$\tan\angle CDE$等于（　）$A.\dfrac{4\sqrt{2}}{3}$　　$B.\dfrac{3\sqrt{2}}{8}$　　$C.\dfrac{8\sqrt{6}}{3}$　　$D.\dfrac{5\sqrt{2}}{6}$



（2019北京）$key:2=\dfrac{AB}{CD}=\dfrac{AB}{BF}\cdot\dfrac{BF}{BE}\cdot\dfrac{BE}{EC}\cdot\dfrac{EC}{CD}=\cos30^\circ\cdot\dfrac{1}{\sin45^\circ}\cdot\sin60^\circ\cdot\tan\angle CDE,\therefore$选$A$

变式（1）在平面四边形$ABCD$中,$A=B=C=75^\circ, BC=2,$则AB的取值范围为____. $(\sqrt{6}-\sqrt{2},\sqrt{6}+\sqrt{2})$

$key:$如图,$\dfrac{AB}{\sin(75^\circ+\theta)}=\dfrac{2}{\sin\theta}(\theta=\angle BAC\in(30^\circ,75^\circ))$得$AB=\dfrac{2\sin(75^\circ+\theta)}{\sin\theta}=\dfrac{\sqrt{6}+\sqrt{2}}{2\tan\theta}+\dfrac{\sqrt{6}-\sqrt{2}}{2}$



$\in(\sqrt{6}-\sqrt{2},\sqrt{6}+\sqrt{2})(\because \tan\theta\in(\dfrac{\sqrt{3}}{3},2+\sqrt{3}))$

（2）已知凸四边形$ABCD$中，$AB=2, BC=4, CD=5, DA=3,$则四边形$ABCD$的面积S的最大值为____.

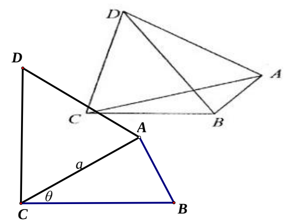
$key:$由$4+9-12\cos A=BD^2=16+25-40\cos D$得$10\cos D-3\cos A=7$

$\therefore S=S_{ABCD}=3\sin A+10\sin D$

$\therefore 49+S^2=109-60\cos(A+D)\in[49,169],\therefore S^2\in[0,120],\therefore S_{\max}=2\sqrt{30}$

（3）在平面四边形$ABCD$中，$AB=1, BC=2, \triangle ACD$为正三角形，则$\triangle BCD$的面积的最大值为（　）

$A.2\sqrt{3}+2$　$B.\dfrac{\sqrt{3}+1}{2}$　$C.\dfrac{\sqrt{3}}{2}+2$　$D.\sqrt{3}+1$



$key:$设$AC=a, \angle ACB=\theta\in(0,\pi)$，则$a^2+4-4a\cos\theta=1$

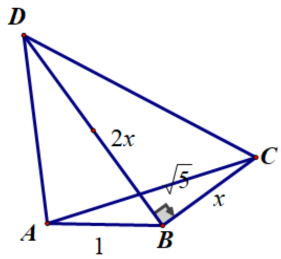
即$(a\cos\theta-2)^2+(a\sin\theta)^2=1$，令$\begin{cases}a\cos\theta-2=\cos\alpha\\ a\sin\theta=\sin\alpha\end{cases}$

则$S_{\triangle BCD}=\dfrac{1}{2}\cdot a\cdot2\sin(60^\circ+\theta)=\dfrac{1}{2}(\sqrt{3}a\cos\theta+a\sin\theta)=\dfrac{1}{2}(2\sqrt{3}+\sin\alpha+\sqrt{3}\cos\alpha)\le\sqrt{3}+1,$选$D$

（4）在平面四边形$ABCD$中，$AB=1, AC=\sqrt{5}, BD\perp BC, BD=2BC,$则$AD$的最小值为______.

2023-06-22

$key1:$ 设$BC=x$，则$AD=\sqrt{1+4x^2-2\cdot 1\cdot 2x\cdot\cos(\angle ABC-\frac{\pi}{2})}$



$=\sqrt{1+4x^2-4x\sqrt{1-(\frac{1+x^2-5}{2x})^2}}=\sqrt{1+4x^2-2\sqrt{20-(x^2-6)^2}}$

（由$\sqrt{5}-1<x<\sqrt{5}+1$，令$t=x^2-6\in(-2\sqrt{5},2\sqrt{5})$）

$=\sqrt{4t+25-2\sqrt{20-t^2}}=\sqrt{(4,-2)\cdot(t,\sqrt{20-t^2})+25}\geq\sqrt{5}$

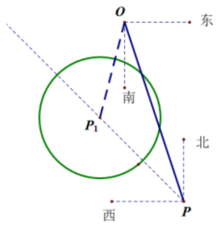
$key2:$ 设$\angle DBA=\theta, BC=x$，则$BD=2x$，且$AC^2=1+x^2-2\cdot 1\cdot x\cos(\frac{\pi}{2}+\theta)=x^2+2x\sin\theta+1=5$

即$(x\sin\theta+1)^2+(x\cos\theta)^2=5$，令$\begin{cases}x\sin\theta+1=\sqrt{5}\sin\alpha\\ x\cos\theta=\sqrt{5}\cos\alpha\end{cases}$

$\therefore AD^2=1+4x^2-4x\cos\theta=1+4[(\sqrt{5}\sin\alpha-1)^2+(\sqrt{5}\cos\alpha)^2]-4\sqrt{5}\cos\alpha$

$=25-4\sqrt{5}(\cos\alpha+2\sin\alpha)\geq 5$

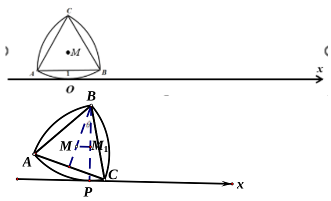
（2003全国）在某海滨城市附近海面有一台风，据监测，当前台风位于城市O的东偏南$\theta(\cos\theta=\frac{\sqrt{2}}{10})$方向$300km$的海面$P$处，并以$20km/h$的速度向西偏北$45°$方向移动，台风侵袭的范围为圆形区域，当前半径为$60km$，并以$10km/h$的速度不断增大，问几小时后该城市开始受到台风的侵袭？持续约多少时间？

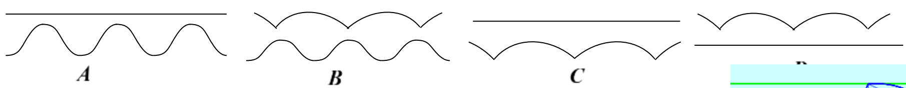


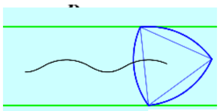
$key: OP_1^2=300^2+(20t)^2-2\cdot 300\cdot 20t\cos(\theta-\frac{\pi}{4})\leq(60+10t)^2$

即$t^2-36t+288\leq 0$得$12\leq t\leq 24$

（11年江西文科）如图，一个“凸轮”放置于直角坐标系x轴上方，其“底端”落在原点O处，一顶点及中心M在y轴正半轴上，它的外围由以正三角形的顶点为圆心，以正三角形的边长为半径的三段等弧组成．今使“凸轮”沿x轴正向滚动前进，在滚动过程中“凸轮”每时每刻都有一个“最高点”．其中心也在不断移动位置，则在“凸轮”滚动一周的过程中，将其“最高点”和“中心点”所形成的图形按上、下放置，应大致为（ ）

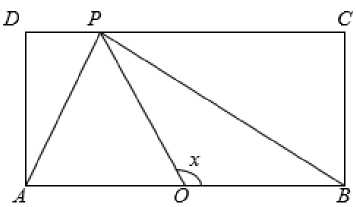


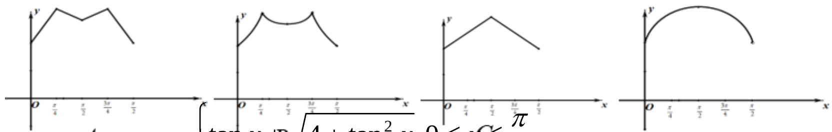




$key: d_{M\to l}=1-\frac{\sqrt{3}}{3}\cos\theta$在$\theta=0$附近递增

（2015II）如图，长方形$ABCD$的边$AB=2, BC=1, O$是AB的中点，点P沿着边BC, CD与DA运动，记$\angle BOP=x$，将动点P到A, B两点距离之和表示为x的函数$f(x)$，则$y=f(x)$的图象大致是（ ）





2015II：$f(x)=\begin{cases}\tan x+\sqrt{4+\tan^2 x}, 0\leq x\leq\frac{\pi}{4},\\ \sqrt{\frac{\sin^2 x-\sin 2x+1}{\sin^2 x}}+\sqrt{\frac{\sin^2 x+\sin 2x+1}{\sin^2 x}}, \frac{\pi}{4}\leq x<\frac{\pi}{2}.\end{cases}$ 且$f(x)$的图象关于$x=\frac{\pi}{2}$对称，

$f(\frac{\pi}{4})=1+\sqrt{5}>f(\frac{\pi}{2})=2\sqrt{2}$，故选$B$