数列（2）数列概念及性质解答（2）

2024-03-23

（2009A）使不等式$\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n+1}<a-2007\frac{1}{3}$对一切正整数$n$都成立的最小正整数$a$的值为_____.

$2009Akey$：设$f(n)=\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n+1}$

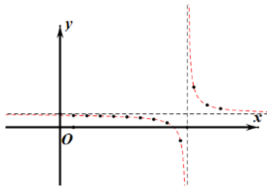
则$f(n+1)-f(n)=\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{2n+1}+\frac{1}{2n+2}+\frac{1}{2n+3}-(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n+1})$

$=\frac{1}{2n+2}+\frac{1}{2n+3}-\frac{2}{2n+2}<0$

$\therefore f(n)$递减,$\therefore f(n)_{\max}=f(1)=\frac{1}{2}+\frac{1}{3}<a-2007\frac{1}{3}$即$a>2007+\frac{2}{3}+\frac{1}{2}$,$\therefore$整数$a$的最小值为2009

（2021吉林）已知数列$\{a_n\}$的通项公式为$a_n=\frac{2n-17}{2n-19}(n=1,2,\cdots)$，则$\{a_n\}$的最大项是(　　)

$A.a_1$　　$B.a_9$　　$C.a_{10}$　　$D.a_{12}$



2021吉林key：$a_n=1+\frac{2}{2n-19}$的图像，得C

变式1（1）已知函数$f(x)=-x^2-ax+1$,数列$\{a_n\}$满足$a_n=f(n)$,且当$n\ge 8$时,$a_{n+1}<a_n$,则a的取值范围为____________.

$key: a_{n+1}-a_n=-(n+1)^2-a(n+1)+1+n^2+an-1$

$=-2n-1-a<0\Leftrightarrow a>-2n-1\le -17(n\ge 8),\therefore a\ge -17$

（2）已知函数$f(x)=\begin{cases}a^{x-6}, x\ge 7,\\ -x^2+(2a-50)x, x<7,\end{cases}$数列$\{a_n\}$满足$a_n=f(n)$,若数列$\{a_n\}$是递增数列,

则实数a的取值范围为_______________.

$key:\begin{cases}a>1\\ a-25\ge 6\\ a_7=a>a_6=-36+6(2a-50)\end{cases}$　　得$31\le a<\frac{336}{11}$

（3）若$\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{3n}>m$对$n\in N^*$恒成立，则整数$m$的最大值为______.

key：设$f(n)=\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{3n}$,则$f(n+1)-f(n)=\frac{1}{3n+3}+\frac{1}{3n+2}+\frac{1}{3n+1}-\frac{1}{n}<\frac{1}{3n}+\frac{1}{3n}+\frac{1}{3n}-\frac{1}{n}=0$,

$\therefore f(n)$递减

$key1$：$\because \frac{1}{n+i}+\frac{1}{3n-i}=\frac{4n}{(n+i)(3n+i)}=\frac{4n}{(\frac{n+i+3n-i}{2})}\ge\frac{1}{n}(i=0,1,\cdots,2n),\therefore f(n)\ge\frac{2n+1}{2}\cdot\frac{1}{n}>1$

$key2: f(n)=\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{3n}>\frac{(1+1+\cdots+1)^2}{n+(n+1)+\cdots+(3n)}=\frac{(2n+1)^2}{\frac{4n(2n+1)}{2}}>1$

而$f(1)=1+\frac{1}{2}+\frac{1}{3}=\frac{11}{6}<2$,$\therefore$整数$k$的最大值为1

数列（2）数列概念及性质解答（2）

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(2013Ⅰ)12.设$\triangle A_nB_nC_n$的三边长分别为a_n,b_n,c_n,$\triangle A_nB_nC_n$的面积为$S_n,n=1,2,3,\cdots$,若$b_1>c_1,b_1+c_1=2a_1$,

$a_{n+1}=a_n,b_{n+1}=\dfrac{c_n+a_n}{2},c_{n+1}=\dfrac{b_n+a_n}{2}$,则（ ）A.$\{S_n\}$为递减数列　　B.$\{S_n\}$为递增数列

C.$\{S_{2n-1}\}$为递增数列,$\{S_{2n}\}$为递减数列 D.$\{S_{2n-1}\}$为递增数列,$\{S_{2n}\}$为递增数列

2013Ⅰ key:由$b_1+c_1=2a_1$成立，若$b_k+c_k=2a_k$成立，则$b_{k+1}+c_{k+1}=\dfrac{b_k+c_k+2a_k}{2}=2a_k=2a_{k+1}=2a_1$,

$\therefore b_n+c_n=2a_n=2a_1$,由$b_1>c_1=2a_1-b_1$得$b_1>a_1,b_2<c_2=2a_1-b_2$得$b_2<a_1,\therefore b_{2n-1}>a_1,b_{2n}<a_1$,

$$S_n=\sqrt{\frac{3a_1}{2}\cdot\frac{a_1}{2}\cdot(\frac{3a_1}{2}-b_n)(\frac{3a_1}{2}-c_n)}=\sqrt{\frac{3}{4}a_1^2(-\frac{3}{4}a_1^2+b_nc_n)}=\sqrt{\frac{3}{4}a_1^2(-\frac{3}{4}a_1^2+2a_1b_n-b_n^2)}$$

$$S_{n+1}=\sqrt{\frac{3}{4}a_1^2(-\frac{3}{4}a_1^2+b_{n+1}c_{n+1})}=\sqrt{\frac{3}{4}a_1^2(-\frac{3}{4}a_1^2+\frac{3a_1-b_n}{2}\cdot\frac{b_n+a_1}{2})}=\sqrt{\frac{3}{4}a_1^2(a_1b_n-\frac{1}{4}b_n^2)}$$

$\therefore S_{n+1}^2-S_n^2=\dfrac{3}{4}a_1^2(\dfrac{3}{4}a_1^2-a_1b_n+\dfrac{3}{4}b_n^2)>0$,选$B$

(2014湖南)20.已知数列$\{a_n\}$满足$a_1=1,|a_{n+1}-a_n|=p^n,n\in N^*$.

(1) 若$\{a_n\}$为递增数列，且$a_1,2a_2,3a_3$成等差数列，求p的值；

(2) 若$p=\dfrac{1}{2}$，且$\{a_{2n-1}\}$是递增数列,$\{a_{2n}\}$是递减数列，求数列$\{a_n\}$的通项公式.

(2014湖南）解：(1)$\because\{a_n\}$是递增数列,$\therefore a_{n+1}-a_n>0,\therefore|a_{n+1}-a_n|=a_{n+1}-a_n=p^n>0$,

$\because a_1=1,a_2=1+p,a_3=p^2+p+1$

$\because a_1,2a_2,3a_3$成等差数列,$\therefore 4a_2-a_1-3a_3=4(1+p)-1-3(p^2+p+1)=-3p^2+p=0,\therefore p=\dfrac{1}{3}$

(2) 由已知得$|a_{2n+1}-a_{2n}|=\dfrac{1}{2^{2n}}$,且$|a_{2n}-a_{2n-1}|=\dfrac{1}{2^{2n-1}}$

$\therefore a_{2n+1}-a_{2n}=\pm\dfrac{1}{2^{2n}}$,且$a_{2n}-a_{2n-1}=\pm\dfrac{1}{2^{2n-1}},\therefore a_{2n+1}-a_{2n-1}=\pm\dfrac{1}{2^{2n}}\pm\dfrac{1}{2^{2n-1}}>0$,

若$\begin{cases}a_{2n+1}-a_{2n}=\dfrac{1}{2^{2n}}\\a_{2n}-a_{2n-1}=\dfrac{1}{2^{2n-1}}\end{cases}$得$a_{2n+1}-a_{2n-1}=\dfrac{1}{2^{2n}}+\dfrac{1}{2^{2n-1}}=\dfrac{3}{2^{2n}}$,

$\therefore a_{2n+1}=(a_{2n+1}-a_{2n-1})+\cdots+(a_3-a_1)+a_1=2-\dfrac{1}{2^{2n}},a_{2n}=a_{2n+1}-\dfrac{1}{2^{2n}}=2-\dfrac{2}{2^{2n}}$递增，不合

$\therefore\begin{cases}a_{2n+1}-a_{2n}=-\dfrac{1}{2^{2n}}\\a_{2n}-a_{2n-1}=\dfrac{1}{2^{2n-1}}\end{cases}$得$a_{2n+1}-a_{2n-1}=\dfrac{1}{2^{2n}}$

$\therefore a_{2n+1}=(a_{2n+1}-a_{2n-1})+\cdots+(a_3-a_1)+a_1=\dfrac{4}{3}-\dfrac{1}{3\cdot2^{2n}},a_{2n}=a_{2n+1}+\dfrac{1}{2^{2n}}=\dfrac{4}{3}+\dfrac{1}{3\cdot2^{2n-1}},a_{2n-1}=\dfrac{4}{3}-\dfrac{1}{3\cdot2^{2n-2}}$,

$$\therefore a_n=\begin{cases}\dfrac{4}{3}+\dfrac{1}{3\cdot2^{n-1}},n\text{为偶数},\\\dfrac{4}{3}-\dfrac{1}{3\cdot2^{n-1}},n\text{为奇数}\end{cases}=\frac{4}{3}+\frac{(-1)^n}{3\cdot2^{n-1}},n\in N^*.$$

数列（2）数列概念及性质解答（2）

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(2022乙)4.嫦娥二号卫星在完成探月任务后，继续进行深空探测，成为我国第一颗环绕太阳飞行的人造行星，为研究嫦娥二号绕日周期与地球绕日周期的比值，用到数列$\{b_n\}:b_1=1+\dfrac{1}{\alpha_1},b_2=1+\dfrac{1}{\alpha_1+\dfrac{1}{\alpha_2}}$,

$b_3=1+\dfrac{1}{\alpha_1+\dfrac{1}{\alpha_2+\dfrac{1}{\alpha_3}}},\cdots$,依此类推，其中$\alpha_k\in N^*(k=1,2,\cdots)$.则（ ）

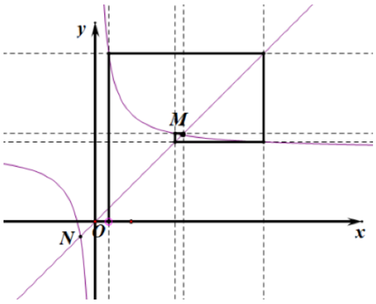
$A.b_1<b_5$　　$B.b_3<b_8$　　$C.b_6<b_2$　　$D.b_4<b_7$

2022乙key:（利用单调性）$b_1>b_5,b_3>b_8,b_2<b_6,b_4<b_7$,选$D$

(2005福建）已知函数$f(x)=2+\dfrac{1}{x}$，$g(x)=\dfrac{1}{x-2}$,求数列$\{a_n\}$中，$a_1=a$,且$a_{n+1}=f(a_n)(n\in N^*)$,当a取不同的值时，得到不同的数列$\{a_n\}$.(1）求a的值，使得$a_3=0$;

(2）求a的取值范围，使得当$n\ge 2,n\in N^*$时，都有$\dfrac{7}{3}<a_n<3$;

(3）设数列$\{b_n\}$满足$b_1=-\dfrac{1}{2},b_{n+1}=g(b_n)(n\in N^*)$,求证：不论$a$取$\{b_n\}$中的任何数，都可以得到一个有穷数列.



2005福建（1）解:由$a_{n+1}=f(a_n)=2+\dfrac{1}{a_n}$得$a_n=\dfrac{1}{a_{n+1}-2}$

$\therefore a_2=-\dfrac{1}{2},a=a_1=\dfrac{1}{-\dfrac{1}{2}-2}=-\dfrac{2}{5}$

(2）解：由$a_2=\dfrac{2a+1}{a}\in(\dfrac{7}{3},3)$得$1<a<3$,

若$\dfrac{7}{3}<a_n<3(n\ge 2)$成立，则$a_{n+1}=2+\dfrac{1}{a_n}\in(2+\dfrac{1}{3},2+\dfrac{3}{7}\subseteq(\dfrac{7}{3},3)$即$\dfrac{7}{3}<a_{n+1}<3$也成立

$\therefore a_n\in(\dfrac{7}{3},3),n\ge 2,\therefore a$的取值范围为$(1,3)$

(3）证明：设$a_1=b_N(N\in N^*)$,由$f(g(x))=x$,

得$a_2=f(b_N)=f(g(b_{N-1}))=b_{N-1},a_3=f(b_{N-1})=f(g(b_{N-2}))=b_{N-2},\cdots,a_N=b_1,a_{N+1}=2+\dfrac{1}{-\dfrac{1}{2}}=0$,

$\therefore\{a_n\}$是一个有$N+1$项的有穷数列

(2022北京)21.已知$Q:a_1,a_2,\cdots,a_k$为有穷整数数列，给定正整数m，若对任意的$n\in\{1,2,\cdots,m\}$，在Q中存在$a_i,a_{i+1},a_{i+2},\cdots,a_{i+j}(j\ge 0)$，使得$a_i+a_{i+1}+a_{i+2}+\cdots+a_{i+j}=n$，则称$Q$为$m-$连续可表数列.

(Ⅰ)判断$Q:2,1,4$是否为$5-$连续可表数列？是否为$6-$连续可表数列？说明理由；

(Ⅱ)若$Q:a_1,a_2,\cdots,a_k$为$8-$连续可表数列，求证：k的最小值为4;

(Ⅲ)若$Q:a_1,a_2,\cdots,a_k$为$20-$连续可表数列，且$a_1+a_2+\cdots+a_k<20$，求证：$k\ge 7$.

数列（2）数列概念及性质解答（2）

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(2022北京） (1) 解：$\because 1=1,2=2,3=2+1,4=4,5=1+4,\therefore Q$是$5-$连续可表数列

$6=2+4$,而2与4不连续,$\therefore Q$不是$6-$连续可表数列

(2) 证明：当$k=3$时，$C_3^1+C_3^2+C_3^3=7<8,\therefore k\ge 4$

取$Q:2,3,3,1$有$1,2,3,3+1=4,2+3=5,3+3=6,3+3+1=7,2+3+3=8,$

$\therefore Q$是$8-$连续可表数列,$\therefore k$的最小值为4

(3) 证明：由$a_1+a_2+\cdots+a_k<20$,而$a_i+a_{i+1}+\cdots+a_{i+j}=20,\therefore Q$中有负数，

则k个数最有$k-1+(k-1)+(k-2)+\cdots+2+1=\dfrac{k^2+k-2}{2}$个$Q$的非负连续项的和，

若至少有2个负数，则$\dfrac{k^2+k-2}{2}-1\ge 20$得$k\ge\dfrac{-1+\sqrt{177}}{2}>6,\therefore k\ge 7,$

若$a_1<0$或$a_k<0$,不妨设$a_1<0$,则$a_2+\cdots+a_k>20$

$\therefore\dfrac{k^2+k-2}{2}-1\ge 20$得$k\ge\dfrac{-1+\sqrt{177}}{2}>6,\therefore k\ge 7,$

若$a_m<0(1<m<k)$，则$a_i+\cdots+a_m+\cdots+a_{i+j}<20(i<m<i+j)$

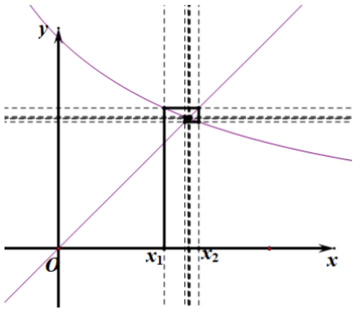
$\therefore Q$中的连续项的和小于$\dfrac{k^2+k-2}{2}-2\ge 20,\therefore k\ge 7$.证毕

(2009陕西)22.已知数列$\{x_n\}$满足$:x_1=\dfrac{1}{2},x_{n+1}=\dfrac{1}{1+x_n},n\in N^*$.

(1) 猜想数列$\{x_{2n}\}$的单调性，并证明你的结论；（2）证明$:|x_{n+1}-x_n|\le\dfrac{1}{6}(\dfrac{2}{5})^{n-1}$.

2009陕西 (1) 解:设$f(x)=\dfrac{1}{1+x}(x>0)$,有$f(x)$在$x>0$上递减，且$f(f(x))=\dfrac{x+1}{x+2}$在$x>0$上递增,且$x_{2n}=f(x_{2n-1})=f(f(x_{2n-2}))$

$\because x_1=\dfrac{1}{2},\therefore x_2=\dfrac{1}{1+\frac{1}{2}}=\dfrac{2}{3},x_3=\dfrac{3}{5},x_4=\dfrac{5}{8},\therefore x_2>x_4>0$



若$\dfrac{-1+\sqrt{5}}{2}<x_{2k}<x_{2k-2}\le\dfrac{2}{3}(k\ge 1)$，则$f(\dfrac{-1+\sqrt{5}}{2})<f(f(x_{2k}))<f(f(x_{2k-2}))$

而$f(\dfrac{-1+\sqrt{5}}{2})=\dfrac{-1+\sqrt{5}}{2},f(f(x_{2k}))=x_{2k+2},f(f(x_{2k-2}))=x_{2k},f(f(\dfrac{2}{3}))=\dfrac{5}{8}<\dfrac{2}{3}$

$\therefore\dfrac{-1+\sqrt{5}}{2}<x_{2k+2}<x_{2k}\le\dfrac{2}{3}$也成立,$\therefore\dfrac{-1+\sqrt{5}}{2}<x_{2n}<x_{2n-2}\le\dfrac{2}{3},\therefore\{x_{2n}\}$递减

(2) 由 (1) 得$:\dfrac{-1+\sqrt{5}}{2}<x_{2n+2}<x_{2n}\le\dfrac{2}{3},$

由$x_{2n-1}=f(x_{2n-2})$得$\dfrac{1}{2}\le x_{2n-1}<x_{2n+1}<\dfrac{-1+\sqrt{5}}{2},\therefore x_n\ge\dfrac{1}{2},\therefore\dfrac{1}{2+x_{n-1}}\le\dfrac{2}{5},$

$\therefore|x_{n+1}-x_n|=|\dfrac{1}{1+x_n}-\dfrac{1}{1+x_{n-1}}|=\dfrac{|x_n-x_{n-1}|}{1+x_n+x_{n-1}+x_nx_{n-1}}=\dfrac{|x_n-x_{n-1}|}{2+x_{n-1}}\le\dfrac{2}{5}|x_n-x_{n-1}|$,而$|x_2-x_1|=|\dfrac{2}{3}-\dfrac{1}{2}|=\dfrac{1}{6}$

$\therefore|x_n-x_{n-1}|=\dfrac{|x_n-x_{n-1}|}{|x_{n-1}-x_{n-2}|}\cdots\dfrac{|x_3-x_2|}{|x_2-x_1|}\cdot|x_2-x_1|\le(\dfrac{2}{5})^{n-1}\cdot\dfrac{1}{6}(n\in N^*)$,证毕

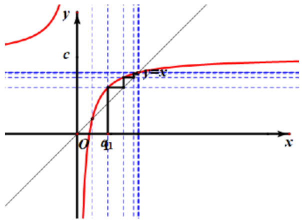
(2010Ⅰ) 已知数列$\{a_n\}$中，$a_1=1,a_{n+1}=c-\dfrac{1}{a_n}$.(1) 设$c=\dfrac{5}{2},b_n=\dfrac{1}{a_n-2}$,求数列$\{b_n\}$的通项公式；

(2) 求使不等式$a_n<a_{n+1}<3$成立的c的取值范围.

数列（2）数列概念及性质解答（2）

2024-03-23

2010I 解：（1）由$a_{n+1}-2=\frac{5}{2}-\frac{1}{a_n}-2=\frac{a_n-2}{2a_n}$得$\frac{1}{a_{n+1}-2}=\frac{2a_n-4+4}{a_n-2}=2+\frac{4}{a_n-2}$



$\therefore b_{n+1}=2b_n+2$即$b_{n+1}+2=2(b_n+2)$, $\therefore\{b_n+2\}$是首项为1, 公比为2的等比数列,

$\therefore b_n+2=2^{n-1}$即$b_n=2^{n-1}-2$

（2）由$a_2=c-1>a_1=1$得$c>2$, 且$a_2=c-1<3$得$2<c<4$,

由$\begin{cases} a_3=c-\frac{1}{a_2}=c-\frac{1}{c-1}>a_2=c-1 \\ a_3=c-\frac{1}{a_2}=c-\frac{1}{c-1}<3 \end{cases}$ 得$2<c<\frac{10}{3}$

设$f(x)=c-\frac{1}{x}(2<c<4)$, 则$f(x)$在$x>0$上递增，

由$1\le a_1<a_2<3$成立，若$1\le a_k<a_{k+1}<3$，则$f(1)\le f(a_k)<f(a_{k+1})<f(3)$

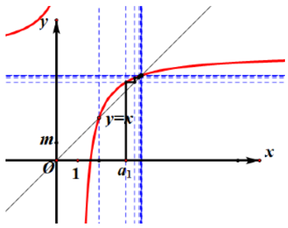
而$f(1)=c-1>1$, $f(3)=c-\frac{1}{3}<3$, $f(a_k)=a_{k+1}$, $f(a_{k+1})=a_{k+2}$

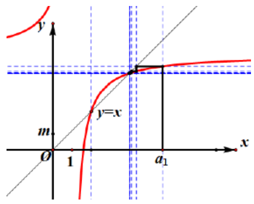
$\therefore 1\le a_{k+1}<a_{k+2}<3.\therefore 1\le a_n<a_{n+1}<3,\therefore c$的取值范围为$(2,\frac{10}{3})$

变式 1（1）① 已知数列$\{a_n\}$满足：$a_1=a$，$a_{n+1}=\frac{5a_n-8}{a_n-1}(n\in N^*)$，若对任意的正整数 n，都有 $a_n>3$，则实数 a 的取值范围（　B　）A. $(0,3)$　　B. $(3,+\infty)$　C. $[3,4)$　D. $[4,+\infty)$

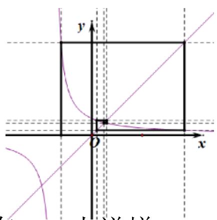
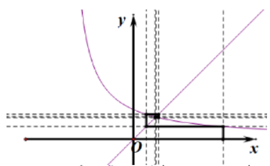
$key: a_1=a>3$, 且$a_2=\frac{5a-8}{a-1}>3$得$a>3$;

设$f(x)=\frac{5x-8}{x-1}$，则$f(x)=x\Leftrightarrow x=2, or, 4$, 如图

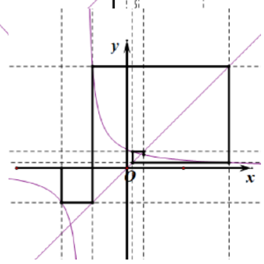




② 数列$\{a_n\}$满足$a_n=\frac{1}{4a_{n+1}}-\frac{3}{4}(n\in N^*)$.若存在实数$c$，使不等式$a_{2n}<c<a_{2n-1}$对任意$n\in N^*$恒成立，当$a_1=1$时，$c=$_______.

key: 递推函数$f(x)=\frac{1}{4x+3}$的不动点为$x=\frac{1}{4}, or, -1$



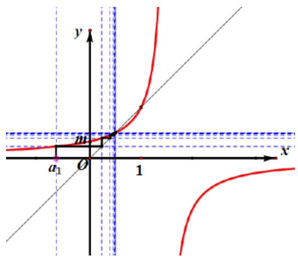
由$a_1=1>0,\therefore a_{n+1}=f(a_n)>0$, 记$g(x)=f(f(x))$, 则$g(\frac{1}{4})=\frac{1}{4}$, 且$g(x)$在$x>0$上递增

则$a_{n+2}=f(a_{n+1})=f(f(a_n))$,

若$0<a_{2n}<\frac{1}{4}<a_{2n-1}$，则$g(0)<g(a_{2n})<g(\frac{1}{4})<g(a_{2n-1})$即$0<a_{2n+2}<\frac{1}{4}<a_{2n+1}$

（2）① 若数列$\{a_n\}$满足$a_{n+1}=\frac{1}{3-2a_n}$, 且对任意$n\in N^*$, 有$a_{n+1}>a_n$，则a_1的取值范围为____.

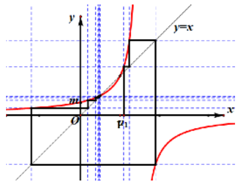
$key1$:（不动点及蛛网图）$a_2=\frac{1}{3-2a_1}>a_1$得$a_1\in(-\infty,\frac{1}{2})\cup(1,\frac{3}{2})$,



数列（2）数列概念及性质解答（2）

2024-03-23

$a_3=\dfrac{1}{3-2a_2}>a_2$得$a_2=\dfrac{1}{3-2a_1}\in(-\infty,\dfrac{1}{2})\cup(1,\dfrac{3}{2})$, 得$a_1\in(-\infty,\dfrac{1}{2})\cup(1,\dfrac{7}{6})\cup(\dfrac{3}{2},+\infty)$,



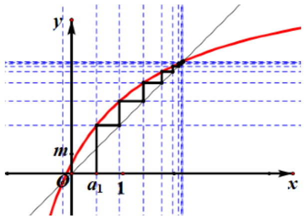
$\therefore a_1\in(-\infty,\dfrac{1}{2})\cup(1,\dfrac{7}{6})$,

设$f(x)=\dfrac{1}{3-2x}$，则$a_{n+1}=f(a_n)$, 且$f(x)=x\Leftrightarrow x=1, or, \dfrac{1}{2}$, 如图，$f(x)$在$(-\infty,\dfrac{1}{2})$上递增，

如图$a_1<\dfrac{1}{2}$，符合；若$a_1\in(1,\dfrac{7}{6})$, 不合, 综上：$a_1\in(-\infty,\dfrac{1}{2})$

② 已知数列$\{a_n\}$满足$0<a_1<1$，$a_{n+1}=\dfrac{4a_n+t}{a_n+2}(t\in R)$，若对于任意$n\in N^*$，都有$0<a_n<a_{n+1}<3$，则

t的取值范围是 （ B）A. $(-1,3]$　　B. $[0,3]$　　C. $(3,8)$　　D. $(8,+\infty)$



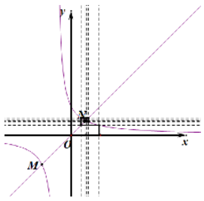
key: 由$a_2=\dfrac{4a_1+t}{a_1+2}\in(a_1,3)$得$\begin{cases}a_1^2-2a_1<t\\ t<6-a_1\end{cases}(0<a_1<1)$, $\therefore t\in[0,5]$

设$f(x)=\dfrac{4x+t}{x+2}$, 则$f(x)=x\Leftrightarrow x^2-2x-t=0(\Delta=4+4t>0)\Leftrightarrow x=1\pm\sqrt{1+t}$

$\therefore 1+\sqrt{1+t}\le 3$得$t\in[0,3]$, 选$B$

（3）已知数列$\{a_n\}$满足: $a_1=1, a_{n+1}=\dfrac{1}{2a_n+1}(n\in N^*)$. ① 数列$\{a_n\}$是单调递减数列；

② 对任意的$n\in N^*$, 都有$a_n\ge\dfrac{1}{3}$；③ 数列$\{|a_n-\dfrac{1}{2}|\}$是单调递减数列;



④ 对任意的$n\in N^*$, 都有$|a_{n+1}-a_n|\le\dfrac{2}{3}\cdot(\dfrac{6}{11})^{n-1}$. 则上述结论正确的个数是 （ C ）

A.1　　B.2　　C.3　　D.4

key: ④ $|a_{n+1}-a_n|=|\dfrac{1}{2a_n+1}-\dfrac{1}{2a_{n-1}+1}|=\dfrac{2|a_n-a_{n-1}|}{(2a_n+1)(2a_{n-1}+1)}$

$=\dfrac{2a_n}{2a_n+1}|a_n-a_{n-1}|\le\dfrac{2}{5}|a_n-a_{n-1}|\le\dfrac{6}{11}|a_n-a_{n-1}|$