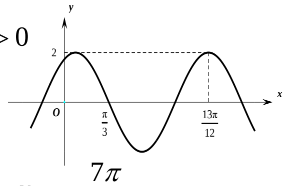
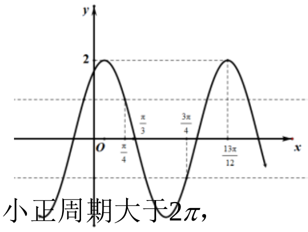
高一期末选讲（1）三角函数解答（2）

2023-06-10

（2021甲）若$f(x)$的部分图像如图所示，则满足条件$(f(x)-f(-\frac{7\pi}{4}))(f(x)-f(\frac{3\pi}{4}))>0$的最小正整数$x$为____.



（2021甲）key:由图知$\begin{cases}\omega\cdot\frac{\pi}{3}+\theta=\frac{\pi}{2}\\ \omega\cdot\frac{13}{12}\pi+\theta=2\pi\end{cases}$ 得$\begin{cases}\omega=2\\ \theta=-\frac{\pi}{6}\end{cases}$,$\therefore f(x)=2\cos(2x-\frac{\pi}{6})$,$\therefore x_{\max}=\frac{\pi}{12},x_{\min}=\frac{7\pi}{12}$,



而$f(-\frac{7\pi}{4})=f(\frac{\pi}{4})=f(-\frac{\pi}{12}),f(\frac{3\pi}{4})=f(\frac{5\pi}{12})$,

如图，得原不等式$\Leftrightarrow k\pi-\frac{\pi}{12}<x<k\pi+\frac{\pi}{4},or,k\pi+\frac{5\pi}{12}<x<k\pi+\frac{3\pi}{4},k\in Z,\therefore x_{\min}=2$

（2020福建）已知$f(x)=3\cos(\omega x+\varphi)(\omega>0,|\varphi|<\pi)$，若$f(\frac{5\pi}{8})=0,f(\frac{11\pi}{8})=3$,且$f(x)$的最小正周期大于$2\pi$，则$\varphi=$______.

（2020福建）key:$\begin{cases}\omega\cdot\frac{5\pi}{8}+\varphi=k_1\pi+\frac{\pi}{2}\\ \omega\cdot\frac{11\pi}{8}+\varphi=2k_2\pi\\ \frac{2\pi}{\omega}>2\pi\end{cases}$ $(|\varphi|<\pi)$得$\omega=\frac{2}{3},\varphi=-\frac{11\pi}{12}$

（2018安徽）函数$f(x)=|\sin(2x)+\sin(3x)+\sin(4x)|$的最小正周期为_____.

（2018安徽）key:（公倍数）$\pi,\frac{2\pi}{3},\frac{\pi}{2}$的公倍数为$2\pi$,

$f(x+\pi)=|\sin 2(x+\pi)+\sin 3(\pi+x)+\sin 4(\pi+x)|=|\sin 2x-\sin 3x+\sin 4x|,\therefore T=2\pi$

（2021福建）若5π是函数$f(x)=\cos nx\cdot\sin\frac{80}{n^2}x$的一个周期，则正整数$n$的所有可能取值为____.

（2021福建）key:$k_1\cdot\frac{2\pi}{n}=5\pi$,且$k_2\cdot\frac{n^2\pi}{40}=5\pi\Leftrightarrow\begin{cases}5n=2k_1\\ k_2n^2=200\end{cases}$

$\therefore n=2,k_1=5,k_2=50;n=10,k_2=2,k_1=25,\therefore n=2$,或10

变式：函数$y=\sin x(1+\tan x\cdot\tan\frac{x}{2})$的周期为_________.

变式：由$\begin{cases}x\neq k\pi+\frac{\pi}{2}\\ \frac{x}{2}\neq k\pi+\frac{\pi}{2}\end{cases}$得函数的定义域为$\{x|x\neq k\pi+\frac{\pi}{2}$,且$x\neq 2k\pi+\pi,k\in Z,x\in R\}$,$\therefore$定义域的周期为$2\pi$

而$y=\sin x\cdot(1+\frac{\sin x\sin\frac{x}{2}}{\cos x\cos\frac{x}{2}})=\frac{\sin x\cos\frac{x}{2}}{\cos x\cos\frac{x}{2}}=\tan x$,$\therefore$周期为$2\pi$

（2007A）设函数$f(x)$对所有的实数x都满足$f(x+2\pi)=f(x)$,求证：存在4个函数$f_i(x)(i=1,2,3,4)$满足：

①对$i=1,2,3,4$, $f_i(x)$是偶函数，且对任意的实数x，有$f_i(x+\pi)=f_i(x)$;

②对任意的实数x,有$f(x)=f_1(x)+f_2(x)\cos x+f_3(x)\sin x+f_4(x)\sin 2x$.

2007$Akey$:设$g(x)=\frac{f(x)+f(-x)}{2},h(x)=\frac{f(x)-f(-x)}{2}$，则有$g(-x)=g(x),h(-x)=-h(x)$,

$g(x+2\pi)=g(x),h(x+2\pi)=h(x)$,

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设$f_1(x)=\dfrac{g(x)+g(x+\pi)}{2}$, $f_2(x)=\begin{cases}\dfrac{g(x)-g(x+\pi)}{2\cos x}, x\neq k\pi+\dfrac{\pi}{2},\\ 0, x=k\pi+\dfrac{\pi}{2},\end{cases}$

$f_3(x)=\begin{cases}\dfrac{h(x)-h(x+\pi)}{2\sin x}, & x\neq k\pi,\\ 0, & x=k\pi,\end{cases}$ $f_4(x)=\begin{cases}\dfrac{h(x)+h(x+\pi)}{2\sin 2x}, & x\neq\dfrac{k\pi}{2}\\ 0, & x=\dfrac{k\pi}{2}\end{cases}$

则有$f_1(-x)=\dfrac{g(-x)+g(-x+\pi)}{2}=\dfrac{g(x)+g(x-\pi)}{2}=\dfrac{g(x)+g(x+\pi)}{2}=f_1(x)$,

$f_1(x+\pi)=\dfrac{g(x+\pi)+g(-x-\pi+\pi)}{2}=\dfrac{g(x+\pi)+g(x)}{2}=f_1(x)$;

$$f_2(x+\pi)=\begin{cases}\dfrac{g(x+\pi)-g(-x-\pi+\pi)}{2\cos(-x)}=\dfrac{g(x)-g(x+\pi)}{2\cos x}, x\neq k\pi+\dfrac{\pi}{2}\\ 0, -x=k\pi+\dfrac{\pi}{2}\end{cases}=f_2(x),$$

$$f_2(-x)=\begin{cases}\dfrac{g(-x)-g(-x+\pi)}{2\cos(-x)}=\dfrac{g(x)-g(x+\pi)}{2\cos x}, x\neq k\pi+\dfrac{\pi}{2}\\ 0, -x=k\pi+\dfrac{\pi}{2}\end{cases}=f_2(x);$$

$$f_3(x+\pi)=\begin{cases}\dfrac{h(x+\pi)-h(x+2\pi)}{2\sin(x+\pi)}=\dfrac{h(x)-h(x+\pi)}{2\sin x}, x\neq k\pi\\ 0, x+\pi=k\pi\end{cases}=f_3(x),$$

$$f_3(-x)=\begin{cases}\dfrac{h(-x)-h(-x+\pi)}{2\sin(-x)}=\dfrac{h(x)-h(x+\pi)}{2\sin x}, x\neq k\pi\\ 0, -x=k\pi\end{cases}=f_3(x);$$

$$f_4(x+\pi)=\begin{cases}\dfrac{h(x+\pi)-h(x+2\pi)}{2\sin 2(x+\pi)}=\dfrac{h(x)-h(x+\pi)}{2\sin 2x}, x\neq\dfrac{k\pi}{2}\\ 0, x+\pi=\dfrac{k\pi}{2}\end{cases}=f_4(x),$$

$$f_4(-x)=\begin{cases}\dfrac{h(-x)-h(-x+\pi)}{2\sin 2(-x)}=\dfrac{h(x)-h(x+\pi)}{2\sin 2x}, x\neq\dfrac{k\pi}{2}\\ 0, -x=\dfrac{k\pi}{2}\end{cases}=f_3(x)$$

且有$f(x)=f_1(x)+f_2(x)\cos x+f_3(x)\sin x+f_4(x)\sin 2x$.得证

（2008湖南）设$a=\sin(\sin 2008°), b=\sin(\cos 2008°), c=\cos(\sin 2008°), d=\cos(\cos 2008°)$,

则a,b,c,d的大小关系是（ ）$A\quad a<b<c<d$, $B\quad b<a<d<c$,

$C\quad c<d<b<a$, $D\quad d<c<a<b$.

2008湖南$key: 0>\sin 2008°=-\sin 28°>-\cos 28°=\cos 2008°>-1>-\dfrac{\pi}{2}$,选$B$

（2022甲）已知$a=\dfrac{31}{32}, b=\cos\dfrac{1}{4}, c=4\sin\dfrac{1}{4}$,则（ ）$A.c>b>a$ $B.b>a>c$ $C.a>b>c$ $D.a>c>b$

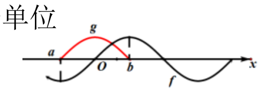
（2022甲）$key1: a-b=\dfrac{31}{32}-(1-2\sin^2\dfrac{1}{8})=-\dfrac{1}{32}+2\sin^2\dfrac{1}{8}<0\Leftrightarrow\sin\dfrac{1}{8}<\dfrac{1}{8}, \therefore a<b$

$\dfrac{c}{b}=4\tan\dfrac{1}{4}>4\cdot\tan\dfrac{1}{4}=1, \therefore c>b, \therefore c>b>a$,选$A$

（1999）（4）函数$f(x)=M\sin(\omega x+\varphi)(\omega>0)$在区间$[a,b]$上是增函数，且$f(a)=-M$，$f(b)=M$，则函数$g(x)=M\cos(\omega x+\varphi)$在$[a,b]$上（　）

$A.$是增函数　$B.$是减函数　$C.$可以取得最大值M　$D.$可以取得最小值$-M$

$1999: g(x)=M\sin(\omega x+\varphi+\frac{\pi}{2})=M\sin(\omega(x+\frac{\pi}{2\omega})+\varphi)$得$f(x)$的图象向左平移$\frac{\pi}{2\omega}(=\frac{T}{4})$个单位得$g(x)$的图象，如图，选$C$



（2015A）设ω是正实数，若存在$a,b(\pi\le a<b\le 2\pi)$，使得$\sin\omega a+\sin\omega b=2$，则$\omega$的取值范围是_____.

$2015Akey$:由已知得$\begin{cases}\omega a=2k_1\pi+\frac{\pi}{2}\\ \omega b=2k_2\pi+\frac{\pi}{2}\end{cases},\therefore \pi\le\frac{2k_1\pi+\frac{\pi}{2}}{\omega}<\frac{2k_2\pi+\frac{\pi}{2}}{\omega}\le 2\pi$

即$\frac{2k_2+\frac{1}{2}}{2}\le\omega\le 2k_1+\frac{1}{2}(k_2>k_1,k_1,k_2\in Z),\therefore \omega\in[\frac{9}{4},\frac{5}{2})\cup[\frac{13}{4},+\infty)$

（2016全国Ⅰ）（12）已知函数$f(x)=\sin(\omega x+\varphi)(\omega>0,|\varphi|\le\frac{\pi}{2}),x=-\frac{\pi}{4}$为$f(x)$的零点，$x=\frac{\pi}{4}$为$y=f(x)$图象的对称轴，且$f(x)$在$(\frac{\pi}{18},\frac{5\pi}{36})$单调，则$\omega$的最大值为（ ）$A.11$　$B.9$　$C.7$　$D.5$

（2016Ⅰ）$\begin{cases}\omega\cdot(-\frac{\pi}{4})+\varphi=k_1\pi\\ \omega\cdot\frac{\pi}{4}+\varphi=k_2\pi+\frac{\pi}{2}\end{cases},\therefore\begin{cases}\omega=2(k_2-k_1)+1\\ \varphi=\frac{k_1+k_2}{2}\pi+\frac{\pi}{4}\in[-\frac{\pi}{2},\frac{\pi}{2}]\end{cases},\therefore\begin{cases}k_1+k_2=0\\ \varphi=\frac{\pi}{4}\\ \omega=4k_2+1\end{cases},or,\begin{cases}k_1+k_2=-1\\ \varphi=-\frac{\pi}{4}\\ \omega=4k_2+3\end{cases},$

$key1$:由$T=\frac{2\pi}{\omega},\omega x+\varphi=k\pi+\frac{\pi}{2}$得$x=\frac{k\pi+\frac{\pi}{2}-\varphi}{\omega},\therefore$单调区间为$[\frac{k\pi+\frac{\pi}{2}-\varphi}{\omega},\frac{(k+1)\pi+\frac{\pi}{2}-\varphi}{\omega}],$

$\therefore\frac{k\pi+\frac{\pi}{2}-\varphi}{\omega}\le\frac{\pi}{18}<\frac{5\pi}{36}\le\frac{(k+1)\pi+\frac{\pi}{2}-\varphi}{\omega}$即$18(k+\frac{1}{2}-\frac{\varphi}{\pi})\le\omega\le\frac{36}{5}(k+\frac{3}{2}-\frac{\varphi}{\pi})$且$k\le\frac{1}{2}+\frac{3\varphi}{\pi}$

当$\varphi=\frac{\pi}{4}$时$,18(k+\frac{1}{4})\le\omega\le\frac{36}{5}(k+\frac{5}{4})$，且$\omega=4k_2+1,\therefore\omega_{\max}=9$

当$\varphi=-\frac{\pi}{4}$时$,18(k+\frac{3}{4})\le\omega\le\frac{36}{5}(k+\frac{7}{4})$，且$\omega=4k_2+3,\therefore\omega_{\max}=3$

$key2$:（排除法）

（2016A）设函数$f(x)=\sin^4\frac{kx}{10}+\cos^4\frac{kx}{10}(k\in N^*)$.若对任意实数$a$，均有$\{f(x)|a<x<a+1\}=\{f(x)|x\in R\}$，则$k$的最小值为______.

$2016Akey: f(x)=1-2\sin^2\frac{kx}{10}\cos^2\frac{kx}{10}=1-\frac{1}{2}\cdot\frac{1-\cos\frac{2kx}{5}}{2}=\frac{3}{4}+\frac{1}{4}\cos\frac{2kx}{5}$

且由已知得在$(a,a+1)$内，$f(x)$有极大及极小值点$,\therefore 1>\frac{2\pi}{\frac{2k}{5}}=\frac{5\pi}{k}$即$k>5\pi,\therefore k_{\min}=16$

变式：已知函数$f(x)=3\sin(\omega x+\frac{\pi}{6})(\omega>0)$.若在区间$[0,2]$至少有6个最值点，则$\omega$的取值范围为____；若在区间$[a,a+2](a\in R)$至少有6个最值点，则$\omega$的取值范围为____.

$key:\omega\ge\frac{8\pi}{3}$；$key:3\cdot\frac{2\pi}{\omega}\le 2$即$\omega\ge 3\pi$

（2019A）对任意闭区间I，用M_I表示函数$y=\sin x$在I上的最大值.若正数a满足$M_{[0,a]}=2M_{[a,2a]}$，则a的值为__.

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$2019Akey$：若$0<a<\frac{\pi}{2}$，则若$2a\ge\frac{\pi}{2}$，不合；若$2a\le\frac{\pi}{2}$，也不合；$\therefore a\ge\frac{\pi}{2}$，则$M_{[0,a]}=1$

$\therefore \max\{\sin a,\sin 2a\}=\frac{1}{2}$，且$2a<\frac{5\pi}{2}$，得$a=\frac{5\pi}{6}, or, \frac{13\pi}{12}$

（2019III）（12）（多选题）设函数$f(x)=\sin(\omega x+\frac{\pi}{5})(\omega>0)$，已知$f(x)$在$[0,2\pi]$有且仅有5个零点则（　）$A.f(x)$在$(0,2\pi)$有且仅有3个极大值点　$B.f(x)$在$(0,2\pi)$有且仅有2个极小值点

$C.f(x)$在$(0,\frac{\pi}{10})$单调递增　　　　$D.\omega$的取值范围是$[\frac{12}{5},\frac{29}{10})$

2019IIIkey：由$T=\frac{2\pi}{\omega}$，$\omega x+\frac{\pi}{5}=\frac{\pi}{2}$得$x=\frac{3\pi}{10\omega}$，

则$\frac{3\pi}{10\omega}+\frac{9}{4}\cdot\frac{2\pi}{\omega}\le 2\pi<\frac{3\pi}{10\omega}+\frac{11}{4}\cdot\frac{2\pi}{\omega}$即$\frac{12}{5}\le\omega<\frac{29}{10}$，有$\frac{3\pi}{10\omega}\in(\frac{3}{29}\pi,\frac{5}{40}\pi)$，$\therefore$ 选ACD

（2020贵州）（多选题）已知函数$f(x)=\sin x|\cos x|$，则以下叙述正确的是（　）

$A.$若$|f(x_1)|=|f(x_2)|$，则$x_1=x_2+k\pi(k\in Z)$　$B.f(x)$的最小正周期为π

$C.f(x)$在$[-\frac{\pi}{4},\frac{\pi}{4}]$上为增函数　　$D.f(x)$的图象关于$x=k\pi+\frac{\pi}{2}(k\in Z)$对称

2020贵州：$f(x)$是奇函数，$f(x+\pi)=-\sin x|\cos x|$，$\therefore$ 选CD

（2009新疆）若$f(x)=\sin(\omega x+\frac{\pi}{3})+\frac{1}{2}(\omega>0)$在$[\pi,\frac{3\pi}{2}]$内无零点，则$\omega$的取值范围为______.

2009新疆key：有$\sin(\omega x+\frac{\pi}{3})=-\frac{1}{2}$得$\omega x+\frac{\pi}{3}=k\pi+(-1)^k\cdot(-\frac{\pi}{6})$

由$\pi\le\dfrac{k\pi+(-1)^k\cdot(-\frac{\pi}{6})-\frac{\pi}{3}}{\omega}\le\frac{3\pi}{2}$得$\frac{2}{3}[k+(-1)^k\cdot(-\frac{1}{6})-\frac{1}{3}]\le\omega\le k+(-1)^k\cdot(-\frac{1}{6})-\frac{1}{3}$

$\therefore \underset{k=1}{\omega}\in[\frac{5}{9},\frac{5}{6}]$; $\underset{k=2}{\omega}\in[1,\frac{3}{2}]$; $\underset{k=3}{\omega}\in[\frac{17}{9},\frac{17}{6}]$; $\underset{k=4}{\omega}\in[\frac{7}{3},\frac{7}{2}],\cdots,\therefore \omega\in(0,\frac{5}{9})\cup(\frac{5}{6},1)\cup(\frac{3}{2},\frac{17}{9})$

（2020新疆）已知函数$f(x)=\sin(\omega x-\frac{\pi}{6})(\omega>0)$，若$f(0)=-f(\frac{\pi}{2})$，且$(0,\frac{\pi}{2})$上有且仅有三个零点，则$\omega=$______.

2020新疆key：由$\omega x-\frac{\pi}{6}=0$得$x=\frac{\pi}{6\omega}$，且$T=\frac{2\pi}{\omega}$，$\therefore$ 零点为$x=\frac{k\pi}{\omega}+\frac{\pi}{6\omega}\in(0,\frac{\pi}{2})$得$-\frac{1}{6}<k<\frac{3\omega-1}{6}>3$得$\omega>\frac{13}{3}$

由$f(0)=-f(\frac{\pi}{2})$得$f(\frac{\pi}{2})=\frac{1}{2}$，$\therefore \omega\cdot\frac{\pi}{2}-\frac{\pi}{6}=2k_1\pi+\frac{\pi}{6}, or, 2k_1\pi+\frac{5\pi}{6}$，即$\omega=4k_1+\frac{2}{3}, or, 4k_1+2$，$\therefore \omega=\frac{14}{3}, or, 6$

（2022乙）记函数$f(x)=\cos(\omega x+\varphi)(\omega>0,0<\varphi<\pi)$的最小正周期为$T$，若$f(T)=\frac{\sqrt{3}}{2}$，$x=\frac{\pi}{9}$为$f(x)$的零点，则$\omega$的最小值为______.

2022乙key：$\begin{cases}\omega\cdot\frac{\pi}{9}+\varphi=k_1\pi+\frac{\pi}{2}\\ \omega\cdot\frac{2\pi}{\omega}+\varphi=2k_2\pi\pm\frac{\pi}{6}(0<\varphi<\pi)\end{cases}$ 得$\varphi=\frac{\pi}{6}$，$\omega=9k_1+3$，$\therefore \omega_{\min}=3$

三、解三角形、

（2001A）如果满足$\angle ABC=60°, AC=12, BC=k$的$\triangle ABC$恰有一个，那么$k$的取值范围为（　）　D

$A.k=8\sqrt{3}$　$B.0<k\le12$　$C.k\ge12$　$D.0<k\le12$，或$k=8\sqrt{3}$

（2017吉林）在$\triangle ABC$中，$AB=1, BC=2$，则$\angle C$的取值范围为（　）

$A.(0,\frac{\pi}{6}]$　　$B.(\frac{\pi}{4},\frac{\pi}{2})$　　$C.(\frac{\pi}{6},\frac{\pi}{3})$　　$D.(0,\frac{\pi}{2})$

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（2017吉林）$key1:\dfrac{1}{\sin C}=\dfrac{2}{\sin A}$得$2\sin C=\sin A\le 1(C\in(0,\dfrac{\pi}{2})$得$C\in(0,\dfrac{\pi}{6}]$

$key2:\cos C=\dfrac{4+b^2-1}{2\times 2b}=\dfrac{3}{4b}+\dfrac{b}{4}\ge\dfrac{\sqrt{3}}{2},\therefore$ 选A

（2006江苏）在$\triangle ABC$中，角A,B,C所对的边分别是$a,b,c,\tan A=\dfrac{1}{2},\cos B=\dfrac{3\sqrt{10}}{10}$.若$\triangle ABC$最长的边为1，则最短边的长为（　　）$A.\dfrac{2\sqrt{5}}{5}$　$B.\dfrac{3\sqrt{5}}{5}$　$C.\dfrac{4\sqrt{5}}{5}$　$D.\dfrac{\sqrt{5}}{5}$

2006江苏key:由$\tan A=\dfrac{1}{2}<\dfrac{1}{\sqrt{3}}$,且$A\in(0,\pi)$得$A\in(0,\dfrac{\pi}{6})$,

$\cos B=\dfrac{3}{\sqrt{10}}>\dfrac{2}{\sqrt{5}}=\cos A$，且$B\in(0,\pi)$得$0<B<A<\dfrac{\pi}{6},\therefore C>\dfrac{2\pi}{3}$最大,$\therefore\dfrac{1}{\sin(A+B)}=\dfrac{b}{\sin B}$得$b=\dfrac{\sqrt{5}}{5}$,选$D$

（2019福建）在$\triangle ABC$中，若$AC=\sqrt{2},AB=2$,且$\dfrac{\sqrt{3}\sin A+\cos A}{\sqrt{3}\cos A-\sin A}=\tan\dfrac{5\pi}{12}$,则$BC=$_____.

（2019福建）key:由已知得$\dfrac{\sqrt{3}\tan A+1}{\sqrt{3}-\tan A}=\dfrac{\tan A+\tan\dfrac{\pi}{6}}{1-\tan\dfrac{\pi}{6}\tan A}=\tan(A+\dfrac{\pi}{6})=\tan\dfrac{5\pi}{12}$得$A=\dfrac{\pi}{4},\therefore BC=\sqrt{2}$

（2020A）在$\triangle ABC$中，$BC=4,CA=5,AB=6$,则$\sin^6\dfrac{A}{2}+\cos^6\dfrac{A}{2}=$______.

2020$Akey$:由$\sin^6\dfrac{A}{2}+\cos^6\dfrac{A}{2}=(\sin^2\dfrac{A}{2}+\cos^2\dfrac{A}{2})(\sin^4\dfrac{A}{2}+\sin^2\dfrac{A}{2}\cos^2\dfrac{A}{2}+\cos^4\dfrac{A}{2})=1-\dfrac{3}{4}\sin^2 A=\dfrac{43}{64}$

$(\because\cos A=\dfrac{5^2+6^2-4^2}{2\times5\times6}=\dfrac{3}{4})$

（1999A）在$\triangle ABC$中，记$BC=a,CA=b,AB=c$，若$9a^2+9b^2-19c^2=0$,则$\dfrac{\tan(\dfrac{\pi}{2}-C)}{\tan(\dfrac{\pi}{2}-A)+\tan(\dfrac{\pi}{2}-B)}=$____.

1999$Akey$:原式$=\dfrac{1}{\tan C(\dfrac{\cos A}{\sin A}+\dfrac{\cos B}{\sin B})}=\dfrac{\sin A\sin B\cos C}{\sin^2 C}=\dfrac{ab\cos C}{c^2}=\dfrac{a^2+b^2-c^2}{2c^2}=\dfrac{5}{9}$

（2021江西）$\triangle ABC$中，$AB=c,BC=a,AC=b$,且$a^4+b^4+c^4=2c^2(a^2+b^2)$,若$\angle A=72°$,则$\angle B=$____.

2021江西key:由已知得:$(a^2+b^2-c^2)^2=2a^2b^2,\therefore 2ab\cos C=a^2+b^2-c^2=\pm\sqrt{2}ab$

$\therefore\cos C=\pm\dfrac{\sqrt{2}}{2},\because A=72°,\therefore C=45°,\therefore B=63°$

（2021Ⅰ）记$\triangle ABC$是内角A,B,C的对边分别为a,b,c.已知$b^2=ac$，点D在边AC上，$BD\sin\angle ABC=a\sin C$.
⑴ 证明：$BD=b$;　（2）若$AD=2DC$,求$\cos\angle ABC$.

2021Ⅰ　⑴ 证明：由已知得及正弦定理得$BD=\dfrac{a\sin C}{\sin\angle ABC}=\dfrac{ac}{b}=b(\because b^2=ac)$得证

（2）解：由$BD=2DC$得$\overrightarrow{BD}=\dfrac{2}{3}\overrightarrow{BC}+\dfrac{1}{3}\overrightarrow{BA},\therefore b^2=\dfrac{4}{9}a^2+\dfrac{4}{9}ac\cos\angle ABC+\dfrac{1}{9}c^2$

即$\cos\angle ABC=\dfrac{9ac-4a^2-c^2}{4ac}=\dfrac{a^2+c^2-ac}{2ac}$得$a=\dfrac{3}{2}c$,或$a=\dfrac{c}{3}$（舍），$\therefore\cos\angle ABC=\dfrac{7}{12}$