高一期末选讲（1）三角函数

三角函数解答（1）2023-06-07

一、三角变换

1.同角三角函数关系: $\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha},\sin^2\alpha+\cos^2\alpha=1$

2.诱导公式:（以下$k\in Z$） 周期性: $\sin(2k\pi+\alpha)=\sin\alpha,\cos(2k\pi+\alpha)=\cos\alpha,\tan(k\pi+\alpha)=\tan\alpha,$

奇偶性: $\sin(-\alpha)=-\sin\alpha,\cos(-\alpha)=\cos\alpha,\tan(-\alpha)=-\tan\alpha$

$\sin(\pi-\alpha)=\sin\alpha,\cos(\pi-\alpha)=-\cos\alpha,\tan(\pi-\alpha)=-\tan\alpha,\sin(\pi+\alpha)=\sin\alpha,\cos(\pi+\alpha)=-\cos\alpha,$

$\sin(\dfrac{\pi}{2}-\alpha)=\cos\alpha,\cos(\dfrac{\pi}{2}-\alpha)=\sin\alpha,\tan(\dfrac{\pi}{2}-\alpha)=\dfrac{1}{\tan\alpha};\sin(\dfrac{\pi}{2}+\alpha)=\cos\alpha,\cos(\dfrac{\pi}{2}+\alpha)=-\sin\alpha,$

$\tan(\dfrac{\pi}{2}+\alpha)=-\dfrac{1}{\tan\alpha};\sin(\dfrac{3\pi}{2}-\alpha)=-\cos\alpha,\cos(\dfrac{3\pi}{2}-\alpha)=-\sin\alpha,\tan(\dfrac{3\pi}{2}-\alpha)=\dfrac{1}{\tan\alpha};$

$\sin(\dfrac{3\pi}{2}+\alpha)=-\cos\alpha,\cos(\dfrac{3\pi}{2}+\alpha)=\sin\alpha,\tan(\dfrac{3\pi}{2}+\alpha)=-\dfrac{1}{\tan\alpha}$

3.和差倍角公式： $\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta;\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

$\sin2\alpha=2\sin\alpha\cos\alpha\Rightarrow\sin\alpha\cos\alpha=\dfrac{1}{2}\sin2\alpha$

$\cos2\alpha=\cos^2\alpha-\sin^2\alpha=2\cos^2\alpha-1=1-2\sin^2\alpha,$

万能公式: $\tan2\alpha=\dfrac{2\tan\alpha}{1-\tan^2\alpha},\sin2\alpha=\dfrac{2\tan\alpha}{1+\tan^2\alpha},\cos2\alpha=\dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$

升幂公式: $1+\cos\alpha=2\cos^2\dfrac{\alpha}{2},1-\cos\alpha=2\sin^2\dfrac{\alpha}{2}$

降幂公式: $\cos^2\alpha=\dfrac{1+\cos2\alpha}{2},\sin^2\alpha=\dfrac{1-\cos2\alpha}{2},$

$\tan(\alpha\pm\beta)=\dfrac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}\Rightarrow\tan\alpha\pm\tan\beta=\tan(\alpha\pm\beta)(1\mp\tan\alpha\tan\beta)$

三倍角: $\sin\alpha\sin(60^\circ-\alpha)\sin(60^\circ+\alpha)=\dfrac{1}{4}\sin3\alpha,\cos\alpha\cos(60^\circ-\alpha)\cos(60^\circ+\alpha)=\dfrac{1}{4}\cos3\alpha,$

$\tan\alpha\tan(60^\circ-\alpha)\tan(60^\circ+\alpha)=\tan3\alpha.$

半角公式: $\sin\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1-\cos\alpha}{2}},\cos\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1+\cos\alpha}{2}},\tan\dfrac{\alpha}{2}=\dfrac{1-\cos\alpha}{\sin\alpha}=\dfrac{\sin\alpha}{1+\cos\alpha}$

和差化积: $\sin\alpha+\sin\beta=2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2},\sin\alpha-\sin\beta=2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2},$

$\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2},\cos\alpha-\cos\beta=-2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}.$

积化和差: $\sin\alpha\cos\beta=\dfrac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)];$

$\cos\alpha\cos\beta=\dfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)],\sin\alpha\sin\beta=-\dfrac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)].$

（2018河北） 已知$\dfrac{\sin\theta}{\sqrt{3}\cos\theta+1}>1$, 则$\tan\theta$的取值范围是________.

（2018河北）$key:\sin\theta>\sqrt{3}\cos\theta+1>0,or,\sin\theta<\sqrt{3}\cos\theta+1<0$

$$\Leftrightarrow\begin{cases}\sin(\theta-\dfrac{\pi}{3})>\dfrac{1}{2}\text{即}2k\pi+\dfrac{\pi}{2}<\theta<2k\pi+\dfrac{7\pi}{6}\\ \cos\theta>-\dfrac{1}{\sqrt{3}}\end{cases},or,\begin{cases}\sin(\theta-\dfrac{\pi}{3})<\dfrac{1}{2}\text{即}2k\pi+\dfrac{7\pi}{6}<\theta<2k\pi+\dfrac{5\pi}{2}\\ \cos\theta<-\dfrac{1}{\sqrt{3}}\end{cases}$$

$\therefore\tan\theta\in(-\infty,-\sqrt{2}]\cup(\dfrac{\sqrt{3}}{3},\sqrt{2}]$

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（2008四川）设$\alpha \in (0,\dfrac{\pi}{2})$，则$\dfrac{\sin^3\alpha}{\cos\alpha}+\dfrac{\cos^3\alpha}{\sin\alpha}$的最小值为（ ）$A.\dfrac{27}{64}$ $B.\dfrac{3\sqrt{2}}{5}$ $C.1$ $D.\dfrac{5\sqrt{3}}{6}$

2008四川key:原式$=\dfrac{\sin^4\alpha+\cos^4\alpha}{\sin\alpha\cos\alpha}=\dfrac{1-2\sin^2\alpha\cos^2\alpha}{\sin\alpha\cos\alpha}=\dfrac{2}{\sin 2\alpha}-\sin 2\alpha \ge 2-1=1$,选$C$

（2018湖南）若$3\sin^3 x+\cos^3 x=3$，则$\sin^{2018} x+\cos^{2018} x$的值为______.

2008湖南key: $\cos x(1-\sin x)(1+\sin x)=\cos^3 x=3(1-\sin^3 x)=3(1-\sin x)(1+\sin x+\sin^2 x)$

$\therefore \sin x=1, or, (1+\sin x)(3-\cos x)+3\sin^2 x=0$无解,$\therefore$ 原式$=1$

（2008重庆2017山东）函数$f(x)=\dfrac{\sin x-1}{\sqrt{3-2\cos x-2\sin x}}(0\le x\le 2\pi)$的值域为_____.

（2012重庆2017山东）$key1$: $f(x)=\dfrac{(1-\cos x,1-\sin x)\cdot(0,-1)}{\sqrt{(1-\cos x)^2+(1-\sin x)^2}\cdot 1}\in[-1,0]$

$$key2: f(x)=\begin{cases}0, x=\dfrac{\pi}{2},\\ -\dfrac{1}{\sqrt{(\dfrac{1-\cos x}{1-\sin x})^2+1}}\in[-1,0), x\ne\dfrac{\pi}{2}\end{cases},\therefore 值域为[-1,0]$$

（2006浙江）设a、b是非零实数,$x\in R$,若$\dfrac{\sin^4 x}{a^2}+\dfrac{\cos^4 x}{b^2}=\dfrac{1}{a^2+b^2}$,则$\dfrac{\sin^{2008} x}{a^{2006}}+\dfrac{\cos^{2008} x}{b^{2006}}=$____.

（2006浙江）$key1$:由已知得$(\dfrac{\sqrt{a^2+b^2}\sin^2 x}{a})^2+(\dfrac{\sqrt{a^2+b^2}\cos x}{b})^2=1$令$\dfrac{\sqrt{a^2+b^2}\sin^2 x}{a}=\cos\theta,\dfrac{\sqrt{a^2+b^2}\cos^2 x}{b}=\sin\theta$

$\therefore \sin^2 x=\dfrac{a\cos\theta}{\sqrt{a^2+b^2}},\cos^2 x=\dfrac{b\sin\theta}{\sqrt{a^2+b^2}},\therefore 1=\dfrac{a}{\sqrt{a^2+b^2}}\cos\theta+\dfrac{b}{\sqrt{a^2+b^2}}\sin\theta=\sin(\theta+\varphi)(\cos\varphi=\dfrac{b}{\sqrt{a^2+b^2}},\sin\varphi=\dfrac{a}{\sqrt{a^2+b^2}})$

$\therefore \theta+\varphi=2k\pi+\dfrac{\pi}{2}(k\in Z),\therefore \cos\theta=\sin\varphi=\dfrac{a}{\sqrt{a^2+b^2}},\sin\theta=\cos\varphi=\dfrac{b}{\sqrt{a^2+b^2}}$

$\therefore \sin^2 x=\dfrac{a^2}{a^2+b^2},\cos^2 x=\dfrac{b^2}{a^2+b^2},\therefore \dfrac{\sin^{2008} x}{a^{2006}}+\dfrac{\cos^{2008} x}{b^{2006}}=\dfrac{(\dfrac{a^2}{a^2+b^2})^{1004}}{a^{2006}}+\dfrac{(\dfrac{b^2}{a^2+b^2})^{1004}}{b^{2006}}=\dfrac{1}{(a^2+b^2)^{1003}}$

$key:\dfrac{a^2+b^2}{a^2}\sin^4 x+\dfrac{a^2}{a^2+b^2}\ge 2\sin^2 x,\dfrac{a^2+b^2}{b^2}\cos^4 x+\dfrac{b^2}{a^2+b^2}\ge 2\cos^2 x$

$\therefore \dfrac{a^2+b^2}{a^2}\sin^4 x+\dfrac{a^2}{a^2+b^2}+\dfrac{a^2+b^2}{b^2}\cos^4 x+\dfrac{b^2}{a^2+b^2}\ge 2$得$\dfrac{1}{a^2}\sin^4 x+\dfrac{1}{b^2}\cos^4 x\ge\dfrac{1}{a^2+b^2}$,

$\therefore \sin^2 x=\dfrac{a^2}{a^2+b^2},\cos^2 x=\dfrac{b^2}{a^2+b^2}$

$\therefore \dfrac{\sin^{2008} x}{a^{2006}}+\dfrac{\cos^{2008} x}{b^{2006}}=\dfrac{a^{2008}}{a^{2006}(a^2+b^2)^{1004}}+\dfrac{b^{2008}}{b^{2006}(a^2+b^2)^{1004}}=\dfrac{1}{(a^2+b^2)^{1003}}$

（2018陕西）若$0<x<\dfrac{\pi}{2}$，且$\dfrac{\sin^4 x}{9}+\dfrac{\cos^4 x}{4}=\dfrac{1}{13}$，则$\tan x=$（ ）$A.\dfrac{1}{2}$ $B.\dfrac{2}{3}$ $C.1$ $D.\dfrac{3}{2}$

2018陕西key: $\dfrac{1}{13}=\dfrac{\sin^4 x}{9}+\dfrac{\cos^4 x}{4}\ge\dfrac{(\sin^2 x+\cos^2 x)^2}{13}=\dfrac{1}{13},\therefore \dfrac{\sin^2 x}{9}=\dfrac{\cos^2 x}{4},\therefore \tan x=\dfrac{3}{2}$,选$D$

（2020广西）设θ_1,θ_2为锐角，且$\dfrac{\sin^{2020}\theta_1}{\cos^{2018}\theta_2}+\dfrac{\cos^{2020}\theta_1}{\sin^{2018}\theta_2}=1$,则$\theta_1+\theta_2=$_____.

（2020广西)key:由$\dfrac{\sin^{2020}\theta_1}{\cos^{2018}\theta_2}+\underbrace{\cos^2\theta_2+\cdots+\cos^2\theta_2}_{1009}\ge 1010\sin^2\theta_1$;

$\dfrac{\cos^{2020}\theta_1}{\sin^{2018}\theta_2}+\underbrace{\sin^2\theta_2+\cdots+\sin^2\theta_2}_{1009}\ge 1010\cos^2\theta_1$

$\therefore 1010=\dfrac{\sin^{2020}\theta_1}{\cos^{2018}\theta_2}+\dfrac{\cos^{2020}\theta_1}{\sin^{2018}\theta_2}+1009\ge 1010,\therefore \sin\theta_1=\cos\theta_2,\sin\theta_2=\cos\theta_1,\therefore \theta_1+\theta_2=\dfrac{\pi}{2}$

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（2009陕西）设$0<\alpha<\pi<\beta<2\pi$，若对任意的$x\in R$，等式$\cos(x+\alpha)+\sin(x+\beta)+\sqrt{2}\cos x=0$恒成立，试求$\alpha,\beta$的值.

2009陕西$key:(\cos\alpha+\sin\beta+\sqrt{2})\cos x+(-\sin\alpha+\cos\beta)\sin x=0,\therefore\begin{cases}\cos\alpha=-\sin\beta-\sqrt{2}\\ \sin\alpha=\cos\beta\end{cases}$

$\therefore 1=3+2\sqrt{2}\sin\beta$得$\sin\beta=-\dfrac{\sqrt{2}}{2},\cos\alpha=-\dfrac{\sqrt{2}}{2}(\because 0<\alpha<\pi<\beta<2\pi)$得$,\therefore\alpha=\dfrac{3\pi}{4}$,

$\therefore\sin\alpha=\dfrac{\sqrt{2}}{2}=\cos\beta,\therefore\beta=\dfrac{7\pi}{4}$

（2017浙江）设$x,y\in R$,且$\dfrac{\sin^2x-\cos^2x+\cos^2x\cos^2y-\sin^2x\sin^2y}{\sin(x+y)}=1$，则$x-y=$____.

2017浙江$key:1=\dfrac{\sin^2x\cos^2y-\cos^2x\sin^2y}{\sin(x+y)}=\sin(x-y),\therefore x-y=2k\pi+\dfrac{\pi}{2},k\in Z$

（2017湖北）求实数a的取值范围，是不等式$\sin2\theta-2\sqrt{2}a\cos(\theta-\dfrac{\pi}{4})-\dfrac{\sqrt{2}a}{\sin(\theta+\dfrac{\pi}{4})}>-3-a^2$对$\theta\in[0,\dfrac{\pi}{2}]$恒成立.

2017湖北key:令$t=\sin(\theta+\dfrac{\pi}{4})\in[\dfrac{\sqrt{2}}{2},1]$，则$\sin2\theta=-\cos(\dfrac{\pi}{2}+2\theta)=2\sin^2(\theta+\dfrac{\pi}{4})-1=2t^2-1,\cos(\theta-\dfrac{\pi}{4})=\sin(\theta+\dfrac{\pi}{4})=t$

$\therefore 2t^2-1-2\sqrt{2}at-\dfrac{\sqrt{2}a}{t}>-3-a^2\Leftrightarrow 2t^2-2\sqrt{2}at+a^2+2-\dfrac{\sqrt{2}a}{t}=(\sqrt{2}t-a)^2+\dfrac{\sqrt{2}(\sqrt{2}t-a)}{t}>0$

$\Leftrightarrow(\sqrt{2}t-a)(\sqrt{2}t^2-at+\sqrt{2})>0\Leftrightarrow\begin{cases}a<\sqrt{2}t\geq1\\ a<\sqrt{2}(t+\dfrac{1}{t})\geq2\sqrt{2}\end{cases}$,或$\begin{cases}a>\sqrt{2}t\leq\sqrt{2}\\ a>\sqrt{2}(t+\dfrac{1}{t})\leq3\end{cases}$

$\therefore a\in(-\infty,1)\cup(3,+\infty)$

（2020III）（9）已知$2\tan\theta-\tan(\theta+\dfrac{\pi}{4})=7$，则$\tan\theta=$（ ） $A.-2$ $B.-1$ $C.1$ $D.2$

2020III:$7=2\tan\theta-\dfrac{\tan\theta+1}{1-\tan\theta}$得$\tan\theta=2$,选$D$

（2018浙江）已知$\alpha,\beta\in(\dfrac{3\pi}{4},\pi),\cos(\alpha+\beta)=\dfrac{4}{5},\cos(\alpha-\dfrac{\pi}{4})=-\dfrac{5}{13}$,则$\cos(\beta+\dfrac{\pi}{4})=$____.

2018浙江$key:\alpha+\beta\in(\dfrac{3\pi}{2},2\pi),\sin(\alpha+\beta)=-\dfrac{3}{5},\alpha-\dfrac{\pi}{4}\in(\dfrac{\pi}{2},\dfrac{3\pi}{4}),\sin(\alpha-\dfrac{\pi}{4})=\dfrac{12}{13}$

$\therefore\cos(\beta+\dfrac{\pi}{4})=\cos(\alpha+\beta-(\alpha-\dfrac{\pi}{4}))=-\dfrac{56}{65}$

（2019内蒙古）已知$\sin2(\alpha+\beta)=n\sin2\gamma$，则$\dfrac{\tan(\alpha+\beta+\gamma)}{\tan(\alpha+\beta-\gamma)}=$______.

2019内蒙古$key:\sin(\alpha+\beta+\gamma+\alpha+\beta-\gamma)=n\sin(\alpha+\beta+\gamma-(\alpha+\beta-\gamma))$得$\dfrac{\tan(\alpha+\beta+\gamma)}{\tan(\alpha+\beta-\gamma)}=\dfrac{n+1}{n-1}$

（2017湖南）设$0\leq x\leq\pi$，且$3\sin\dfrac{x}{2}=\sqrt{1+\sin x}-\sqrt{1-\sin x}$，则$\tan x=$____.

2017湖南$key:3\sin\dfrac{x}{2}=\cos\dfrac{x}{2}+\sin\dfrac{x}{2}-|\sin\dfrac{x}{2}-\cos\dfrac{x}{2}|=\begin{cases}2\cos\dfrac{x}{2},\dfrac{\pi}{4}\leq\dfrac{x}{2}\leq\dfrac{\pi}{2}\\ 2\sin\dfrac{x}{2},0\leq\dfrac{x}{2}\leq\dfrac{\pi}{4}\end{cases},\therefore\sin\dfrac{x}{2}=0,\therefore\tan x=0$

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(2021甲)9.若$\alpha\in(0,\dfrac{\pi}{2}),\tan 2\alpha=\dfrac{\cos\alpha}{2-\sin\alpha}$，则$\tan\alpha=$（ ）A.$\dfrac{\sqrt{15}}{15}$ B.$\dfrac{\sqrt5}{5}$ C.$\dfrac{\sqrt5}{3}$ D.$\dfrac{\sqrt{15}}{3}$

2021甲：$\tan 2\alpha=\dfrac{2\sin\alpha\cos\alpha}{1-2\sin^2\alpha}=\dfrac{\cos\alpha}{2-\sin\alpha}$得$\sin=\dfrac{1}{4}$,∴$\tan\alpha=\dfrac{\sqrt{15}}{15}$,∴选$A$

变式：已知$\cos\theta-\sin\theta=\dfrac{7\sqrt2}{25},\theta\in(\pi,2\pi)$,则$\sin(\dfrac{\theta}{2}+\dfrac{\pi}{8})$的值为______.

变式key∵$\cos(\theta+\dfrac{\pi}{4})=1-2\sin^2(\dfrac{\theta}{2}+\dfrac{\pi}{8})=\dfrac{7}{25}$,∴$\sin(\dfrac{\theta}{2}+\dfrac{\pi}{8})=\pm\dfrac{3}{5}$

∵$\pi<\theta<2\pi$,∴$\dfrac{\theta}{2}+\dfrac{\pi}{8}\in(\dfrac{5\pi}{8},\dfrac{9\pi}{8})\subseteq(\dfrac{\pi}{2},\dfrac{7\pi}{6})$,∴$\sin(\dfrac{\theta}{2}+\dfrac{\pi}{8})\in(-\dfrac{1}{2},1)$,∴$\sin(\dfrac{\theta}{2}+\dfrac{\pi}{8})=\dfrac{3}{5}$

(2018Ⅰ)求值:$\sin^2(\alpha-\dfrac{\pi}{3})+\sin^2(\alpha+\dfrac{\pi}{3})-\cos^2\alpha=$( )A.$-\dfrac{1}{2}$ B.$\dfrac{1}{2}$ C.0 D.-1

2018Ⅰ key：原式$=\dfrac{1-\cos(2\alpha-\frac{2\pi}{3})}{2}+\dfrac{1-\cos(2\alpha+\frac{2\pi}{3})}{2}-\dfrac{1+\cos 2\alpha}{2}$

$=\dfrac{1}{2}-\dfrac{1}{2}\cdot 2\cos\dfrac{2\alpha-\frac{2\pi}{3}+2\alpha+\frac{2\pi}{3}}{2}\cos\dfrac{2\alpha-\frac{2\pi}{3}-2\alpha-\frac{2\pi}{3}}{2}-\dfrac{1}{2}\cos 2\alpha=\dfrac{1}{2}$,∴选$B$

变式1（1）①已知$\dfrac{1-\cos 2\alpha}{\sin\alpha\cos\alpha}=1,\tan(\beta-\alpha)=-\dfrac{1}{3}$,则$\tan(\beta-2\alpha)=$______.

$key:1=\dfrac{2\sin^2\alpha}{\sin\alpha\cos\alpha}=2\tan\alpha$即$\tan\alpha=\dfrac{1}{2}$,∴$\tan(\beta-\alpha-\alpha)=\dfrac{-\frac{1}{3}-\frac{1}{2}}{1+(-\frac{1}{3})\cdot\frac{1}{2}}=-1$

②已知$\tan\alpha\tan\beta=\dfrac{7}{3},\tan\dfrac{\alpha+\beta}{2}=\dfrac{\sqrt2}{2}$,则$\cos(\alpha-\beta)=$________.

$key:\cos(\alpha+\beta)=\dfrac{1-\tan^2\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}}=\dfrac{1}{3}$,而$\dfrac{7}{3}=\tan\alpha\tan\beta=\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$,

∴$\dfrac{3+7}{3-7}=\dfrac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}=\dfrac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$,∴$\cos(\alpha-\beta)=-\dfrac{5}{6}$

③已知α、β为锐角，且$\dfrac{1+\sin\alpha-\cos\alpha}{\sin\alpha}\cdot\dfrac{1+\sin\beta-\cos\beta}{\sin\beta}=2$，则$\tan\alpha\tan\beta=$_____.

$key:\dfrac{2\sin^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\cdot\dfrac{2\sin^2\frac{\beta}{2}+2\sin\frac{\beta}{2}\cos\frac{\beta}{2}}{2\sin\frac{\beta}{2}\cos\frac{\beta}{2}}=(1+\tan\dfrac{\alpha}{2})(1+\tan\dfrac{\beta}{2})=2$

即$\tan\dfrac{\alpha}{2}+\tan\dfrac{\beta}{2}=1-\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}$,∴$\tan\dfrac{\alpha+\beta}{2}=1$,∴$\alpha+\beta=\dfrac{\pi}{2}$,∴$\tan\alpha\tan\beta=1$

(2005浙江）若$\sin x+\sin y=1$,则$\cos x+\cos y$的取值范围是（ ）A.$[-2,2]$ B.$[-1,1]$ C.$[0,\sqrt3]$ D.$[-\sqrt3,\sqrt3]$

(2005浙江)key:设$t=\cos x+\cos y$,则$t^2+1=2+2\cos(x-y)\in[0,4]$,∴$t\in[-\sqrt3,\sqrt3]$,选$D$

变式：若$\sin\alpha\cos\beta=\dfrac{1}{3}$,则$\cos\alpha\sin\beta$的取值范围为______.

key:设$t=\cos\alpha\sin\beta$,则$t+\dfrac{1}{3}=\sin(\alpha+\beta)\in[-1,1]$,∴$t\in[-\dfrac{4}{3},\dfrac{2}{3}]$

$t-\dfrac{1}{3}=\sin(\beta-\alpha)\in[-1,1]$,∴$t\in[-\dfrac{2}{3},\dfrac{4}{3}]$,∴$t\in[-\dfrac{2}{3},\dfrac{2}{3}]$

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（2018河南）已知$\cos(\alpha+\beta)=\cos\alpha+\cos\beta$,则$\cos\alpha$的取值范围为______.

2018河南$key:(\cos\alpha-1)\cos\beta-\sin\alpha\sin\beta=\cos\alpha,,\therefore\dfrac{|\cos\alpha|}{\sqrt{(\cos\alpha-1)^2+\sin^2\alpha}}\le 1$得$\cos\alpha\in[-1,-1+\sqrt{2}]$

（2018河北）设$\alpha,\beta\in(0,\dfrac{\pi}{2})$，证明:$\cos\alpha+\cos\beta+\sqrt{2}\sin\alpha\sin\beta\le\dfrac{3\sqrt{2}}{2}$.

2018河北$key:\cos\alpha+\sqrt{2}\sin\beta\sin\alpha+\cos\beta\le\sqrt{1+2\sin^2\beta}+\dfrac{1}{\sqrt{2}}\cdot\sqrt{2}\cos\beta$

$\le\sqrt{(1+\dfrac{1}{2})(1+2\sin^2\beta+2\cos^2\beta)}=\dfrac{3\sqrt{2}}{2}$得证

变式（1）已知$\alpha,\beta,\gamma\in[0,\dfrac{\pi}{2}]$,则$\cos\alpha+2\cos\beta+\cos\gamma-\cos(\alpha+\gamma)-2\cos(\beta+\gamma)$的最大值为____

$key:$原式$=\cos(\dfrac{\alpha+\alpha+\gamma}{2}+\dfrac{\alpha-(\alpha+\gamma)}{2})-\cos(\dfrac{\alpha+\alpha+\gamma}{2}-\dfrac{\alpha-(\alpha+\gamma)}{2})+2(\cos\beta-\cos(\beta+\gamma))+\cos\gamma$

$=2\sin\dfrac{2\alpha+\gamma}{2}\sin\dfrac{\gamma}{2}+4\sin\dfrac{2\beta+\gamma}{2}\sin\dfrac{\gamma}{2}+\cos(\dfrac{\gamma}{2}+\dfrac{\gamma}{2})(\because\alpha,\beta,\gamma\in[0,\dfrac{\pi}{2}],\therefore\dfrac{2\alpha+\gamma}{2},\dfrac{2\beta+\gamma}{2}\in[0,\dfrac{3\pi}{4}])$

$\le 2\sin\dfrac{\gamma}{2}+4\sin\dfrac{\gamma}{2}+\cos^2\dfrac{\gamma}{2}-\sin^2\dfrac{\gamma}{2}=-2\sin^2\dfrac{\gamma}{2}+6\sin\dfrac{\gamma}{2}+1(\because\sin\dfrac{\gamma}{2}\in[0,\dfrac{\sqrt{2}}{2}])=-2(\sin\gamma-\dfrac{3}{2})^2+\dfrac{11}{2}\le 3\sqrt{2}$

（2）设$\alpha,\beta\in[0,\pi]$，则$(\sin\alpha+\sin(\alpha+\beta))\sin\beta$的最大值为______.

$key:$原式$=2\sin\dfrac{\alpha+\alpha+\beta}{2}\cos\dfrac{\alpha-\alpha-\beta}{2}\sin\beta=2\sin\dfrac{2\alpha+\beta}{2}\cos\dfrac{\beta}{2}\sin\beta\le 4\cos^2\dfrac{\beta}{2}\cdot\sin\dfrac{\beta}{2}$

$=4\sqrt{\cos^2\dfrac{\beta}{2}\cdot\cos^2\dfrac{\beta}{2}\cdot 2\sin^2\dfrac{\beta}{2}\cdot\dfrac{1}{2}}\le 4\sqrt{\dfrac{1}{2}(\dfrac{2}{3})^3}=\dfrac{8\sqrt{3}}{9}$

（2021广西）设$\sin\alpha+\sin\beta=\dfrac{4\sqrt{2}}{5},\cos\alpha+\cos\beta=\dfrac{4\sqrt{3}}{5}$,则$\tan\alpha+\tan\beta=$_______.

2021广西$key:\dfrac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}=\dfrac{2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}}{2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}}=\tan\dfrac{\alpha+\beta}{2}=\sqrt{\dfrac{2}{3}},\therefore\sin(\alpha+\beta)=\dfrac{2\sqrt{6}}{5},\cos(\alpha+\beta)=\dfrac{1}{5},$

$(\sin\alpha+\sin\beta)^2+(\cos\alpha+\cos\beta)^2=2+2\cos(\alpha-\beta)=\dfrac{16}{5}$得$\cos(\alpha-\beta)=\dfrac{3}{5}$

$\therefore\tan\alpha+\tan\beta=\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\sin\beta}{\cos\beta}=\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}=\dfrac{\dfrac{2\sqrt{6}}{5}}{\dfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]}=\sqrt{6}$

（1997A）设$\dfrac{\pi}{12}\le z\le y\le x$,且$x+y+z=\dfrac{\pi}{2}$，则$(\cos x\sin y\cos z)_{\max}=$____,$(\cos x\sin y\cos z)_{\min}=$____.

（1997A）$key:\because x\ge y\ge z\ge\dfrac{\pi}{12}$,且$3z\le\dfrac{\pi}{6}+x\le x+y+z=\dfrac{\pi}{2}\le 3x$,且$\dfrac{\pi}{2}\ge x+2\cdot\dfrac{\pi}{12},\therefore x\in[\dfrac{\pi}{6},\dfrac{\pi}{3}],z\in[\dfrac{\pi}{12},\dfrac{\pi}{6}],$

$\therefore\cos x\sin y\cos z=\cos x\cdot\dfrac{1}{2}[\sin(y+z)+\sin(y-z)]\ge\dfrac{1}{2}\cos^2 x\ge\dfrac{1}{8}$

$\cos x\sin y\cos z=\cos z\cdot\dfrac{1}{2}[\sin(y+x)+\sin(y-x)]\le\dfrac{1}{2}\cos^2 z\le\dfrac{1}{2}\cdot(\dfrac{\sqrt{6}+\sqrt{2}}{4})^2=\dfrac{2+\sqrt{3}}{8}$

（2021江苏）已知$\dfrac{\sin a}{\sin b}=\dfrac{\sin c}{\sin d}=\dfrac{\sin(a-c)}{\sin(b-d)},a,b,c,d\in(0,\pi)$,证明：$a=b,c=d$.

（2021江苏）$key:\because a,b,c,d\in(0,\pi),\therefore\dfrac{\sin a}{\sin b}=\dfrac{\sin c}{\sin d}=\dfrac{\sin(a-c)}{\sin(b-d)}>0$,且$\dfrac{a-c}{2},\dfrac{b-d}{2}\in(-\dfrac{\pi}{2},\dfrac{\pi}{2}),a-c\ne 0,b-d\ne 0,$

$\therefore\cos\dfrac{a-c}{2},\cos\dfrac{b-d}{2}\ne 0,\sin\dfrac{a-c}{2}\ne 0,\sin\dfrac{b-d}{2}\ne 0,$

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由 $\dfrac{2\sin\frac{a+c}{2}\cos\frac{a-c}{2}}{2\sin\frac{b+d}{2}\cos\frac{b-d}{2}}=\dfrac{\sin a+\sin c}{\sin b+\sin d}=\dfrac{\sin a}{\sin b}=\dfrac{\sin c}{\sin d}=\dfrac{\sin(a-c)}{\sin(b-d)}=\dfrac{2\sin\frac{a-c}{2}\cos\frac{a-c}{2}}{2\sin\frac{b-d}{2}\cos\frac{b-d}{2}}$

$\therefore\dfrac{\sin\frac{a+c}{2}}{\sin\frac{b+d}{2}}=\dfrac{\sin\frac{a-c}{2}}{\sin\frac{b-d}{2}}=\dfrac{\sin\frac{a+c}{2}+\sin\frac{a-c}{2}}{\sin\frac{b+d}{2}+\sin\frac{b-d}{2}}=\dfrac{2\sin\frac{a}{2}\cos\frac{c}{2}}{2\sin\frac{b}{2}\cos\frac{d}{2}}=\dfrac{\sin\frac{a+c}{2}-\sin\frac{a-c}{2}}{\sin\frac{b+d}{2}-\sin\frac{b-d}{2}}=\dfrac{2\cos\frac{a}{2}\sin\frac{c}{2}}{2\cos\frac{b}{2}\sin\frac{d}{2}},\therefore\tan\frac{a}{2}\cdot\tan\frac{d}{2}=\tan\frac{b}{2}\cdot\tan\frac{c}{2}$

由 $\dfrac{2\cos\frac{a+c}{2}\sin\frac{a-c}{2}}{2\cos\frac{b+d}{2}\sin\frac{b-d}{2}}=\dfrac{\sin a-\sin c}{\sin b-\sin d}=\dfrac{\sin a}{\sin b}=\dfrac{\sin c}{\sin d}=\dfrac{\sin(a-c)}{\sin(b-d)}=\dfrac{2\sin\frac{a-c}{2}\cos\frac{a-c}{2}}{2\sin\frac{b-d}{2}\cos\frac{b-d}{2}}$

$\therefore\dfrac{\cos\frac{a+c}{2}}{\cos\frac{b+d}{2}}=\dfrac{\cos\frac{a-c}{2}}{\cos\frac{b-d}{2}}=\dfrac{\cos\frac{a+c}{2}+\cos\frac{a-c}{2}}{\cos\frac{b+d}{2}+\cos\frac{b-d}{2}}=\dfrac{2\cos\frac{a}{2}\cos\frac{c}{2}}{2\cos\frac{b}{2}\cos\frac{d}{2}}=\dfrac{\cos\frac{a+c}{2}-\cos\frac{a-c}{2}}{\cos\frac{b+d}{2}-\cos\frac{b-d}{2}}=\dfrac{-2\sin\frac{a}{2}\sin\frac{c}{2}}{-2\sin\frac{b}{2}\sin\frac{d}{2}},\therefore\tan\frac{a}{2}\tan\frac{c}{2}=\tan\frac{b}{2}\tan\frac{d}{2}$

$\therefore\tan^2\frac{a}{2}=\tan^2\frac{b}{2},\therefore a=b,\therefore\tan\frac{c}{2}=\tan\frac{d}{2},\therefore c=d$ 得证

$(1991A)\cos^2 10^\circ+\cos^2 50^\circ-\sin 40^\circ\sin 80^\circ=$ ______.

$1991Akey1$: 原式 $=\dfrac{1+\cos 20^\circ}{2}+\dfrac{1+\cos 100^\circ}{2}-\dfrac{1}{2}(\cos 40^\circ-\cos 120^\circ)$

$=\dfrac{3}{4}+\cos 60^\circ\cos 40^\circ-\dfrac{1}{2}\cos 40^\circ=\dfrac{3}{4}$

$key2$: 设 $A=\cos^2 10^\circ+\cos^2 50^\circ-\sin 40^\circ\sin 80^\circ,B=\sin^2 10^\circ+\sin^2 50^\circ-\cos 40^\circ\cos 80^\circ,$

$\therefore\begin{cases}A+B=2-\cos 40^\circ\\ A-B=\cos 20^\circ+\cos 100^\circ+\cos 120^\circ=\cos 40^\circ-\dfrac{1}{2}\end{cases},\therefore A=\dfrac{3}{4}$

$key3$: $A=\sin^2 80^\circ+\sin^2 40^\circ-\sin 40^\circ\sin 80^\circ=(\sin 60^\circ)^2=\dfrac{3}{4}$

变式：（1）$\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ=$ ______. $\dfrac{1}{16}$

（2）$\sin 20^\circ\sin 40^\circ\sin 60^\circ\sin 80^\circ=$ ______. 原式 $=\dfrac{\sqrt{3}}{2}\cdot\dfrac{1}{4}\sin 60^\circ=\dfrac{3}{16}$

（2015湖北）已知顶角为20°的等腰三角形的底边长为a，腰长为b，则$\dfrac{a^3+b^3}{ab^2}$的值为______.

2015湖北key: $\dfrac{a^3+b^3}{ab^2}=\dfrac{\sin^3 20^\circ+\sin^3 80^\circ}{\sin 20^\circ\sin^2 80^\circ}=\dfrac{(\sin 20^\circ+\sin 80^\circ)(\sin^2 20^\circ-\sin 20^\circ\sin 80^\circ+\sin^2 80^\circ)}{\sin 20^\circ\sin^2 80^\circ}$

$=\dfrac{2\sin 50^\circ\cos 30^\circ[\frac{1-\cos 40^\circ}{2}-\frac{1}{2}(\cos 60^\circ-\cos 100^\circ)+\frac{1-\cos 160^\circ}{2}]}{\sin 20^\circ\sin^2 80^\circ}$

$=\dfrac{\sqrt{3}\cos 40^\circ(\frac{3}{4}-\frac{1}{2}\cos 40^\circ-\frac{1}{2}\cos 80^\circ+\frac{1}{2}\cos 20^\circ)}{\sin 20^\circ\sin 80^\circ\cdot 2\sin 40^\circ\cos 40^\circ}=\dfrac{\sqrt{3}(\frac{3}{4}-\cos 60^\circ\cos 20^\circ+\frac{1}{2}\cos 20^\circ)}{\frac{1}{2}\sin 60^\circ}=3$

$(2013重庆)4\cos 50^\circ-\tan 40^\circ=(\quad)$ $A.\sqrt{2}$ $B.\dfrac{\sqrt{3}+\sqrt{2}}{2}$ $C.\sqrt{3}$ $D.2\sqrt{2}-1$

2013重庆key: 原式 $=4\sin 40^\circ-\dfrac{\sin 40^\circ}{\cos 40^\circ}=\dfrac{2\sin 80^\circ-\sin 40^\circ}{\cos 40^\circ}=\dfrac{\sin 80^\circ+2\cos 60^\circ\sin 20^\circ}{\cos 40^\circ}$

$=\dfrac{2\sin 50^\circ\cos 30^\circ}{\sin 50^\circ}=\sqrt{3}$, 选$C$

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（2015福建）若$\sin\frac{\pi}{9}+\sin\frac{2\pi}{9}+\cdots+\sin\frac{n\pi}{9}=\frac{1}{2}\tan\frac{4\pi}{9}$，则正整数$n$的最小值为______.

$$2015福建key:\sin\frac{\pi}{9}+\sin\frac{2\pi}{9}+\cdots+\sin\frac{n\pi}{9}=\frac{1}{2\sin\frac{\pi}{18}}(\cos\frac{\pi}{18}-\cos\frac{3\pi}{18}+\cos\frac{3\pi}{18}-\cos\frac{5\pi}{18}+\cdots+\cos\frac{2n-1}{18}\pi-\cos\frac{2n+1}{18}\pi)$$

$$=\frac{1}{2\sin\frac{\pi}{18}}(\cos\frac{\pi}{18}-\cos\frac{2n+1}{18}\pi)=\frac{1}{2}(\tan\frac{4\pi}{9}-\frac{\cos\frac{2n+1}{18}\pi}{\sin\frac{\pi}{18}})=\frac{1}{2}\tan\frac{4\pi}{9},\therefore n_{\min}=4$$

（2017广东）设m,n均为正整数，则$\sum\limits_{k=0}^{m-1}\cos\frac{2k\pi}{m}+\sum\limits_{k=0}^{n-1}\sin\frac{2k\pi}{n}=$______.

$$2017广东key:\sum_{k=0}^{m-1}\cos\frac{2k\pi}{m}=\frac{1}{2\sin\frac{\pi}{m}}\sum_{k=0}^{m-1}(\sin\frac{2k+1}{m}\pi-\sin\frac{2k-1}{m}\pi)=\frac{\sin\frac{2m-1}{m}\pi+\sin\frac{1}{m}\pi}{2\sin\frac{\pi}{m}}=0$$

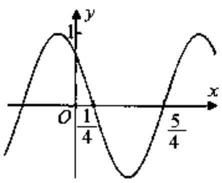
$$\sum_{k=0}^{n-1}\sin\frac{2k\pi}{n}=\frac{1}{2\sin\frac{\pi}{n}}\sum_{k=0}^{n-1}(\cos\frac{2k-1}{n}\pi-\cos\frac{2k+1}{n}\pi)=\frac{\cos\frac{1}{n}\pi-\cos\frac{2n-1}{n}\pi}{2\sin\frac{\pi}{n}}=0,\therefore 原式=0$$

二、三角函数图象性质及应用

（2015Ⅰ）（8）函数$f(x)=\cos(\omega x+\varphi)$的部分图像如图所示，则$f(x)$的单调递减区间为（ ）

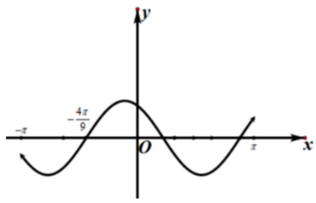
$A.(k\pi-\frac{1}{4},k\pi+\frac{3}{4}),k\in Z$ $B.(2k\pi-\frac{1}{4},2k\pi+\frac{3}{4}),k\in Z$

$C.(k-\frac{1}{4},k+\frac{3}{4}),k\in Z$ $D.(2k-\frac{1}{4},2k+\frac{3}{4}),k\in Z$



（2015Ⅰ）$key:\begin{cases}\omega\cdot\frac{1}{4}+\varphi=\frac{\pi}{2}\\ \omega\cdot\frac{5}{4}+\varphi=\frac{3\pi}{2}\end{cases}$ 得$\omega=\pi,\varphi=\frac{\pi}{4},\therefore T=2$,递减区间为$[2k-\frac{1}{4},2k+\frac{3}{4}]$,选$D$

（2020Ⅰ）（7）设函数$f(x)=\cos(\omega x+\frac{\pi}{6})$在$[-\pi,\pi]$的图像大致如下图，



则$f(x)$的最小正周期为（ ）$A.\frac{10\pi}{9}$ $B.\frac{7\pi}{6}$ $C.\frac{4\pi}{3}$ $D.\frac{3\pi}{2}$

（2020Ⅰ）$\omega\cdot(-\frac{4\pi}{9})+\frac{\pi}{6}=-\frac{\pi}{2}$得$\omega=\frac{3}{2},\therefore T=\frac{4\pi}{3}$,选$C$