变式 1（1）已知 E,F 是四面体的棱 AB,CD 的中点，过 EF 的平面与棱 AD,BC 分别相交于 G,H,则（ C）

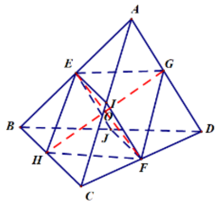
A．GH 平分 EF，$\dfrac{BH}{HC}=\dfrac{AG}{GD}$　　　B．EF 平分 GH，$\dfrac{BH}{HC}=\dfrac{GD}{AG}$

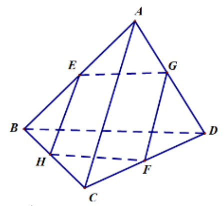
C．EF 平分 GH，$\dfrac{BH}{HC}=\dfrac{AG}{GD}$　　　D．GH 平分 EF，$\dfrac{BH}{HC}=\dfrac{GD}{AG}$

$key:\dfrac{OG}{OH}=\dfrac{d_{g\to IEJF}}{d_{H\to IEJF}}=\dfrac{d_{A\to IEJF}}{d_{B\to IEJF}}=1$

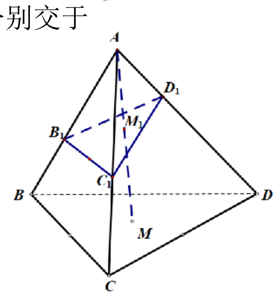
(其中I,J分别为AC,BD的中点，有$AD//$平面$IEFJ//BC$)

$\dfrac{BH}{HC}=\dfrac{d_{B\to EHFG}}{d_{C\to EHFG}}=\dfrac{d_{A\to EHFG}}{d_{D\to EHFG}}=\dfrac{AG}{GD}$





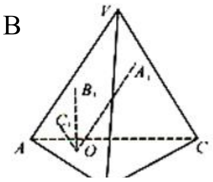
（2）在三棱锥$A-BCD$中,M为底面$\triangle BCD$的重心,任作一截面与侧棱AB、AC、AD分别交于点B_1、C_1、D_1,与AM交于点M_1,则$\dfrac{AB}{AB_1}+\dfrac{AC}{AC_1}+\dfrac{AD}{AD_1}-\dfrac{3AM}{AM_1}=$________.0



$key:\dfrac{AB}{AB_1}=\dfrac{AB_1+BB_1}{AB_1}=1+\dfrac{BB_1}{AB_1}=1+\dfrac{d_B}{d_A}$,同理$\dfrac{AC}{AC_1}=1+\dfrac{d_C}{d_A},\dfrac{AD}{AD_1}=1+\dfrac{d_D}{d_A}$

$\therefore\dfrac{AB}{AB_1}+\dfrac{AC}{AC_1}+\dfrac{AD}{AD_1}=3+\dfrac{d_B+d_C+d_D}{d_A}=3+\dfrac{3d_M}{d_A},\dfrac{3AM}{AM_1}=3(1+\dfrac{MM_1}{AM_1})=3+\dfrac{3d_M}{d_A}$

（3）如图，过四面体$V-ABC$的底面上任意一点O，分别作$OA_1//VA,OB_1//VB,OC_1//VC,A_1$、$B_1$、$C_1$分别是直线与侧面的交点，则$\dfrac{OA_1}{VA}+\dfrac{OB_1}{VB}+\dfrac{OC_1}{VC}=$(　　) $A.\dfrac{1}{3}$　　$B.1$　　$C.2$　　$D.3$　　B

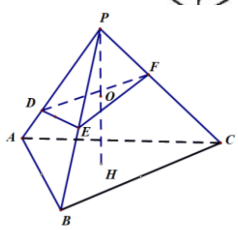


$key:\dfrac{OA_1}{OA}=\dfrac{d_{O\to VBC}}{d_{A\to VBC}}=\dfrac{V_{O-VBC}}{V_{A-VBC}}=\dfrac{V_{O-VBC}}{V_{VABC}}$

(4）如图，正四面体$P-ABC$的体积为V，底面积为S，O是高PH的中点，过O的平面α与棱PA,PB,PC分别交于D,E,F，设三棱锥$P-DEF$的体积为V_0，截面$\triangle DEF$的面积为S_0，则（　　）

$A.V\le 8V_0,S\le 4S_0$　　$B.V\le 8V_0,S\ge 4S_0$

$C.V\ge 8V_0,S\le 4S_0$　　$D.V\ge 8V_0,S\ge 4S_0$



$key:$设$\overrightarrow{PD}=x\overrightarrow{PA},\overrightarrow{PE}=y\overrightarrow{PB},\overrightarrow{PF}=z\overrightarrow{PC}$,

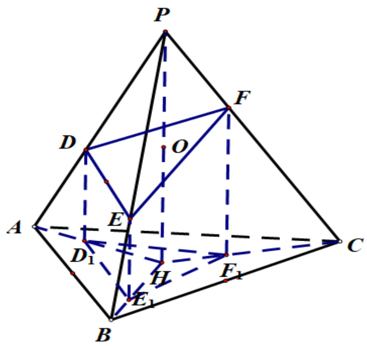
则$\dfrac{1}{x}=\dfrac{PA}{PD}=\dfrac{d_A}{d_P}+1,\dfrac{1}{y}=\dfrac{PB}{PE}=\dfrac{d_B}{d_P}+1,\dfrac{1}{z}=\dfrac{PC}{PF}=\dfrac{d_C}{d_P}+1,$

$\therefore\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=\dfrac{d_A+d_B+d_C}{d_P}+3=\dfrac{3d_H}{d_P}+3=6$即$xy+yz+zx=6xyz,$

$key2:$(共面充要条件应用)$\overrightarrow{PD}=x\overrightarrow{PA},\overrightarrow{PE}=y\overrightarrow{PB},\overrightarrow{PF}=z\overrightarrow{PC}$

则$\overrightarrow{PO}=\dfrac{1}{2}\overrightarrow{PH}=\dfrac{1}{2}(\dfrac{1}{3x}\overrightarrow{PD}+\dfrac{1}{3y}\overrightarrow{PE}+\dfrac{1}{3z}\overrightarrow{PF}),\therefore\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=6,\therefore xyz\ge\dfrac{1}{8}$

$\therefore V_0=V_{P-DEF}=xyzV_{P-ABC}\ge\dfrac{1}{8}V,S_0\ge S_{\triangle D_1E_1F_1}=(xy+yz+zx)\cdot\dfrac{S}{3}\ge\dfrac{1}{4}S$

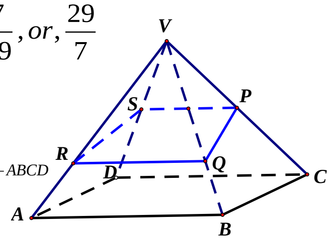


2（1）已知正四棱锥$V-ABCD$中，P是棱VC的中点，R、Q分别在VA、VB上.

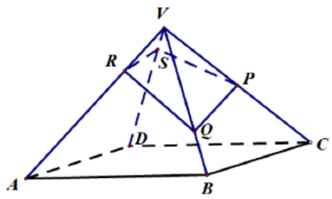
若$\dfrac{VR}{VA}=\dfrac{VQ}{VB}=\dfrac{2}{3}$, 则平面$PQR$将此四棱锥分成的两部分的体积之比为______.$\dfrac{7}{29},or,\dfrac{29}{7}$

$key:$由已知得$RQ//AB//CD//PS,S$为平面PQR与VD的交点，

$\therefore S$是VD的中点,$\therefore V_{V-PQRS}=V_{Q-VPS}+V_{Q-VRS}=\dfrac{2}{3}\cdot\dfrac{1}{4}V_{B-VCD}+\dfrac{2}{3}\cdot\dfrac{2}{3}\cdot\dfrac{1}{2}V_{B-VAD}=\dfrac{7}{36}V_{V-ABCD}$



若$\frac{VR}{VA}=\frac{1}{3},\frac{VQ}{VB}=\frac{2}{3}$，则平面$PQR$将此四棱锥分成的两部分的体积之比为______. $\frac{5}{58},or,\frac{58}{5}$



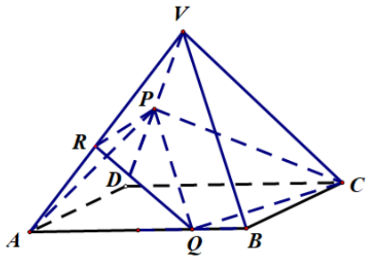
key：设VD与平面PQR交于S，且$x=\frac{VS}{SD}=\frac{d_V}{d_D}=\frac{d_C}{d_D}=\frac{\frac{1}{2}d_A}{d_D}=\frac{2d_B}{d_D}$

$\therefore d_A=2d_C,d_B=\frac{1}{2}d_C$，而$3d_C=d_A+d_C=d_B+d_D=\frac{1}{2}d_C+d_D,\therefore\frac{d_C}{d_D}=\frac{2}{5}=x$

$\therefore V_{V-PQRS}=V_{Q-VRS}+V_{Q-VPS}=\frac{2}{3}\cdot\frac{2}{7}\cdot\frac{1}{3}V_{B-VAD}+\frac{2}{3}\cdot\frac{2}{7}\cdot\frac{1}{2}V_{B-VCD}=\frac{2}{63}V_{V-ABCD}+\frac{1}{21}V_{V-BCD}=\frac{5}{63}V_{V-ABCD}$

（2）已知正四棱锥 V-$ABCD$，点 P 棱 VD 的中点，点 Q 在棱 AB 上，且 $BQ=\frac{1}{4}BA$，过 C、P、Q 的平面 α 截此四棱锥所得截面多边形的边数为 n，截面将四棱锥分成的两部分的体积之比为 $\lambda(0<\lambda<1)$ ，则(   )

$A.n=3,\lambda=\frac{7}{9}$ $B.n=4,\lambda=\frac{7}{9}$ $C.n=4,\lambda=\frac{27}{29}$ $D.n=4,\lambda=\frac{15}{17}$

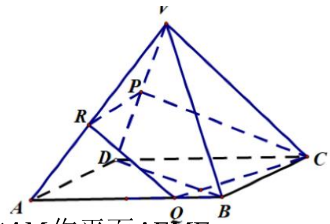


key：设$VA\cap\alpha=R,\overrightarrow{VR}=\lambda\overrightarrow{VA}$，则$\lambda\overrightarrow{VA}=\overrightarrow{VR}=x\overrightarrow{VP}+y\overrightarrow{VQ}+z\overrightarrow{VC}(x+y+z=1)$

$=\frac{x}{2}\overrightarrow{VD}+y(\overrightarrow{VB}+\frac{1}{4}(\overrightarrow{VD}-\overrightarrow{VC}))+z\overrightarrow{VC}=(\frac{x}{2}+\frac{y}{4})\overrightarrow{VD}+y\overrightarrow{VB}+(z-\frac{y}{4})\overrightarrow{VC}$

$=\lambda(\overrightarrow{VB}+\overrightarrow{VD}-\overrightarrow{VC}),\therefore\begin{cases}\frac{x}{2}+\frac{y}{4}=\lambda\\y=\lambda\\z-\frac{y}{4}=-\lambda\end{cases}$ 即$\begin{cases}x=\frac{3\lambda}{2}\\y=\lambda\\z=-\frac{3\lambda}{4}\end{cases}$，$\therefore x+y+z=\frac{7}{4}\lambda=1$即$\lambda=\frac{4}{7}$

$V_{PR-AQCD}=V_{P-ARQ}+V_{P-AQCD}=\frac{3}{7}\cdot\frac{3}{4}\cdot\frac{1}{2}V_{D-VAB}+\frac{7}{16}V_{V-ABCD}=\frac{29}{56}V_{V-ABCD}$



$key2$：设$\frac{VR}{RA}=x$，则$x=\frac{d_V}{d_A}=\frac{d_D}{d_A}=\frac{d_D}{3d_B}=\frac{4}{3}$

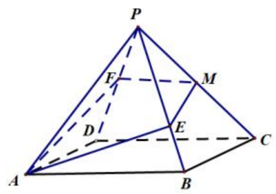
$\therefore V_{PR-AQCD}=V_{P-ARQ}+V_{P-AQCD}=\frac{3}{7}\cdot\frac{3}{4}\cdot\frac{1}{2}V_{D-VAB}+\frac{7}{16}V_{V-ABCD}=\frac{29}{56}V_{V-ABCD}$

（3） 设$P-ABCD$是一个高为3,底面边长为2的正四棱锥，M为PC中点，过AM作平面$AEMF$与线段PB,PD分别交于E,F（可以是线段端点），则三棱锥$P-AEMF$的体积的取值范围为

(   ) $A.[\frac{4}{3},2]$ $B.[\frac{4}{3},\frac{3}{2}]$ $C.[1,\frac{3}{2}]$ $D.[1,2]$

key：设$\overrightarrow{PE}=\lambda\overrightarrow{PB},\overrightarrow{PF}=\mu\overrightarrow{PD}(\lambda,\mu\in[0,1])$,

则$\mu\overrightarrow{PD}=\overrightarrow{PF}=x\overrightarrow{PA}+y\overrightarrow{PE}+z\overrightarrow{PM}(x+y+z=1)=x\overrightarrow{PA}+y\lambda\overrightarrow{PB}+\frac{1}{2}z\overrightarrow{PC}=\mu(\overrightarrow{PA}+\overrightarrow{PC}-\overrightarrow{PB})$



$\therefore\begin{cases}x=\mu\\y=\frac{-\mu}{\lambda}\\z=2\mu\end{cases},\therefore\mu=\frac{\lambda}{3\lambda-1}\in[0,1]$得$\lambda\in[\frac{1}{2},1]$,

$\therefore V_{P-AEMF}=V_{A-PEM}+V_{A-PFM}=\frac{1}{2}\cdot\lambda\cdot\frac{1}{2}V_{P-ABCD}+\frac{1}{2}\cdot\frac{\lambda}{3\lambda-1}\cdot\frac{1}{2}V_{P-ABCD}=\lambda+\frac{\lambda}{3\lambda-1}$

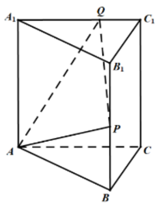
$=\lambda-\frac{1}{3}+\frac{\frac{1}{9}}{\lambda-\frac{1}{3}}+\frac{2}{3}\in[\frac{4}{3},\frac{3}{2}]$

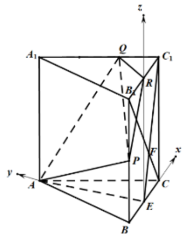
3（1）在底面边长为1的正三棱柱$ABC-A_1B_1C_1$中，$AC_1\perp B_1C$，P为BB_1的中点,$\overrightarrow{A_1Q}=2\overrightarrow{QC_1}$.

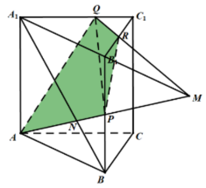
则点B到平面APQ的距离为____,

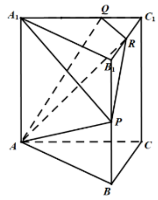
A_1到平面APQ的距离为______,C到平面APQ的距离为____;

平面APQ将此正三棱柱分成的两部分的体积之比为_______.









key:取BC的中点E,连AE，则$AE\perp$面BCC_1B_1,

连EC_1,$\because AC_1\perp B_1C,\therefore B_1C\perp C_1E$，解得$CC_1=\dfrac{\sqrt{2}}{2}$,

设平面APQ交B_1C_1于点R，则$\dfrac{C_1R}{RB_1}=\dfrac{d_{C_1}}{d_{B_1}}=\dfrac{\frac{1}{2}d_{A_1}}{d_B}=\dfrac{\frac{1}{2}\cdot 2d_B}{d_B}=1$,

$\therefore d_{B_1}=d_B=\dfrac{1}{2}d_{A_1}=d_{C_1},d_C=\dfrac{3}{2}d_{A_1}$,

建系如图，则$A(0,\dfrac{\sqrt{3}}{2},0),P(-\dfrac{1}{2},0,\dfrac{\sqrt{2}}{4}),R(0,0,\dfrac{\sqrt{2}}{2}),B_1(-\dfrac{1}{2},0,\dfrac{\sqrt{2}}{2})$

设平面APR的法向量$\vec{n}=(x,y,z)$,则$\begin{cases}\vec{n}\cdot\overrightarrow{AP}=-\dfrac{1}{2}x-\dfrac{\sqrt{3}}{2}y+\dfrac{\sqrt{2}}{4}z=0\\ \vec{n}\cdot\overrightarrow{AR}=-\dfrac{\sqrt{3}}{2}y+\dfrac{\sqrt{2}}{2}z=0\end{cases}$，令$y=\sqrt{2}$得$\vec{n}=(-\dfrac{\sqrt{6}}{2},\sqrt{2},\sqrt{3})$

$$\therefore d_{B_1}=\frac{|\overrightarrow{PB_1}\cdot\vec{n}|}{|\vec{n}|}=\frac{\frac{\sqrt{6}}{4}}{\sqrt{\frac{3}{2}+2+3}}=\frac{\sqrt{3}}{2\sqrt{13}}=\frac{\sqrt{39}}{26},\therefore d_B=\frac{\sqrt{39}}{26},d_{A_1}=\frac{\sqrt{39}}{13},d_C=\frac{3\sqrt{39}}{26}$$

$$V_{A_1-APRQ}=V_{A_1-ARQ}+V_{A_1-APR}=V_{R-AA_1Q}+V_{R-A_1AP}=\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{4}+\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\cdot 1\cdot\frac{\sqrt{3}}{4}=\frac{5\sqrt{6}}{144}$$

$$V_{ABC-A_1B_1C_1}=\frac{\sqrt{3}}{4}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{6}}{8},\therefore\frac{V_{上}}{V_{下}}=\frac{5}{13}$$

（2）已知正方体$ABCD-A_1B_1C_1D_1$中，P、Q分别为棱AB、CC_1的中点，R在棱A_1D_1上，且$A_1R=2RD_1$，若平面PQR与直线BC交于S，则$BS:BC=$______.

key:$\overrightarrow{DP}=(1,\dfrac{1}{2},0),\overrightarrow{DQ}=(0,0,\dfrac{1}{2}),\overrightarrow{DR}=(\dfrac{1}{3},0,1),\overrightarrow{DS}=(t,1,0)=x\overrightarrow{DP}+y\overrightarrow{DQ}+z\overrightarrow{DR}=(x+\dfrac{1}{3}z,\dfrac{x}{2},\dfrac{y}{2}+z)$

$$\therefore\begin{cases}t=x+\dfrac{z}{3}\\ 1=\dfrac{x}{2}\\ \dfrac{y}{2}+z=0\\ x+y+z=1\end{cases}\quad 得x=2,y=-2,z=1,t=\frac{7}{3},\therefore BS:BC=\frac{4}{3}$$