2023-03-26

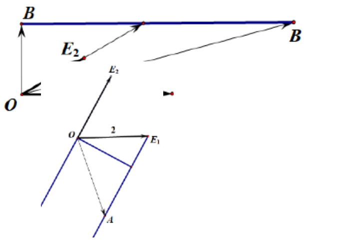
三、模（方法：几何意义（距离）；代数：$\sqrt{\vec{a}^2}$ ）

（05 竞赛）已知$\vec{a},\vec{b}$是两个互相垂直的单位向量，而$|\vec{c}|=13,\vec{c}\cdot\vec{a}=3,\vec{c}\cdot\vec{b}=4$.则对于任意的实数$t_1,t_2$，$|\vec{c}-t_1\vec{a}-t_2\vec{b}|$的最小值为（　）$A.5\quad B.7\quad C.12\quad D.13$　C

（07 文理）非零向量$\vec{a},\vec{b}$满足$|\vec{a}+\vec{b}|=|\vec{b}|$，则（　）C

$A.|2\vec{a}|>|2\vec{a}+\vec{b}|\qquad B.|2\vec{a}|<|2\vec{a}+\vec{b}|\qquad C.|2\vec{b}|>|\vec{a}+2\vec{b}|\qquad D.|2\vec{b}|<|\vec{a}+2\vec{b}|$

（13 文理）设$\vec{e_1},\vec{e_2}$为单位向量,非零向量$\vec{b}=x\vec{e_1}+y\vec{e_2},x,y\in R$,若$\vec{e_1},\vec{e_2}$的夹角为$\frac{\pi}{6}$,则$\frac{|y|}{|\vec{b}|}$的最大值等于________.2



$$key:\frac{|y|}{|\vec{b}|}=\sqrt{\frac{y^2}{x^2+y^2+\sqrt{3}xy}}=\frac{1}{\sqrt{t^2+\sqrt{3}t+1}}\le 2(t=\frac{x}{y})$$

$$key2:\frac{|y|}{|\vec{b}|}\le\frac{|y|}{\frac{1}{2}|y|}=2$$

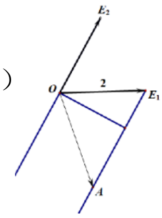
变式：已知单位向量$\vec{e_1},\vec{e_2}$的夹角为$\frac{\pi}{3}$,设$\vec{a}=2\vec{e_1}+\lambda\vec{e_2}$,则当$\lambda<0$时，$\lambda+|\vec{a}|$的取值范围为________.

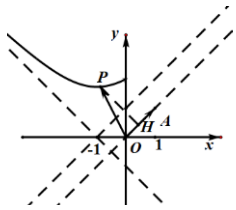
$$变式key1:\lambda+|\vec{a}|=\lambda+\sqrt{4+2\lambda+\lambda^2}=(1,1)\cdot(\lambda,\sqrt{(\lambda+1)^2+3})$$

$$=\overrightarrow{OA}\cdot\overrightarrow{OP}(点A(1,1),P(\lambda,\sqrt{(\lambda+1)^2+3})在曲线y^2-(x+1)^2=3(x<0)上)$$

$$=\frac{\overrightarrow{OP}\cdot\overrightarrow{OA}}{|\overrightarrow{OA}|}\cdot|\overrightarrow{OA}|\in(-1,2)$$

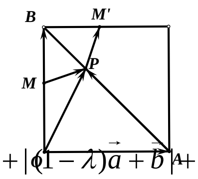
$$key2:\lambda+|\vec{a}|=|\overrightarrow{OA}|-|\overrightarrow{AE_1}|\in(-1,2)$$





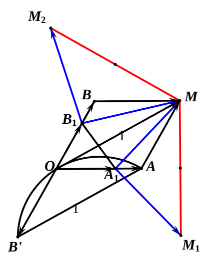
(16竞赛）已知向量$|\overrightarrow{OA}|=|\overrightarrow{OB}|=24,\overrightarrow{OA}\perp\overrightarrow{OB}$.若$t\in[0,1]$，则$|t\overrightarrow{AB}-\overrightarrow{AO}|+|\frac{5}{12}\overrightarrow{BO}-(1-t)\overrightarrow{BA}|$的最小值为______. 26

$$16竞赛key:|t\overrightarrow{AB}-\overrightarrow{AO}|+|\frac{5}{12}\overrightarrow{BO}-(1-t)\overrightarrow{BA}|=|\overrightarrow{OP}|+|\overrightarrow{PM'}|\ge|\overrightarrow{OM'}|=26$$



变式:已知非零平面向量$\vec{a},\vec{b}$夹角为$\frac{\pi}{3}$,且$|\vec{a}+\vec{b}|=1$,若$\lambda,\mu>0$,则$|\lambda\vec{a}-\mu\vec{b}|+|(1-\lambda)\vec{a}+\vec{b}|+|\vec{a}+(1-\mu)\vec{b}|$的最小值为____.

$key:$如图，设$\vec{a}=\overrightarrow{OA},\vec{b}=\overrightarrow{OB},\overrightarrow{OM}=\vec{a}+\vec{b},-\vec{b}=\overrightarrow{OB'}$,则$|\overrightarrow{AB'}|=1,\angle AOB'=\frac{2\pi}{3}$,



$$\therefore|\lambda\vec{a}-\mu\vec{b}|+|(1-\lambda)\vec{a}+\vec{b}|+|\vec{a}+(1-\mu)\vec{b}|(\overrightarrow{OA_1}=\lambda\vec{a},\overrightarrow{OB_1}=\mu\vec{b})$$

$$=|\overrightarrow{A_1B_1}|+|\overrightarrow{A_1M}|+|\overrightarrow{B_1M}|=|\overrightarrow{A_1B_1}|+|\overrightarrow{A_1M_1}|+|\overrightarrow{B_1M_2}|$$

$$\ge|\overrightarrow{M_1M_2}|=2\sqrt{p^2+q^2+pq}=\sqrt{3}(其中\alpha=\angle MOA,$$

$$p=\sin\alpha,q=\sin(60^\circ-\alpha),则p^2+q^2+pq=\sin^2\alpha+\sin^2(60^\circ-\alpha)+\sin\alpha\sin(60^\circ-\alpha)$$

$$=\sin^2\alpha+(\frac{\sqrt{3}}{2}\cos\alpha-\frac{1}{2}\sin\alpha)^2+\sin\alpha\cdot(\frac{\sqrt{3}}{2}\cos\alpha-\frac{1}{2}\sin\alpha)=\frac{3}{4})$$

（2017 高考）(15）已知向量$\vec{a},\vec{b}$满足$|\vec{a}|=1,|\vec{b}|=2$,则$|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$的最小值是_______,最大值是_______. $4,2\sqrt{5}$

变式 2（1）①函数 $y=2\sqrt{1-x}-\sqrt{3+x}$ 的值域为________；

$y=(2,-1)\cdot(\sqrt{1-x},\sqrt{3+x})\in[-2,4]$

②函数 $y=2x-\sqrt{4-x^2}$的值域为_________ ； $[-2\sqrt{5},4]$

③函数 $y=2\sin\alpha-3\cos\alpha(\alpha\in[-\frac{\pi}{6},\frac{2\pi}{3}])$的值域为___________ ； $[-\frac{3\sqrt{3}}{2}-1,\frac{3}{2}+\sqrt{3}]$

④ （19 福建）函数 $y=x+\sqrt{2x-x^2}$ 的值域为______.$[0,\sqrt{2}+1]$

⑤已知实数a,b,c,d满足$a+b+c+d=1,a^2+2b^2+3c^2+4d^2=1$，则$d$的取值范围为______.

$key:|1-d|=|1\cdot a+\frac{1}{\sqrt{2}}\cdot\sqrt{2}b+\frac{1}{\sqrt{3}}\cdot\sqrt{3}c|\le\sqrt{(1+\frac{1}{2}+\frac{1}{3})(a^2+2b^2+3c^2)}=\sqrt{\frac{11}{6}(1-4d^2)}$

$\therefore d\in[\frac{6-\sqrt{259}}{50},\frac{6+\sqrt{259}}{50}]$

（2）①函数 $y=2x-\sqrt{x^2+1}$ 的值域为____________；

$key:y=(2,-1)\cdot(x,\sqrt{x^2+1})\in(-\infty,+\infty)$

②已知$2x+y=4,x,y>0$.则$x+\sqrt{x^2+y^2}$的最小值为______, $\frac{16}{5}$

$\frac{2x-y}{\sqrt{x^2+y^2}}$的取值范围为_____. $(-1,2)$

（3）①（2008 重庆）函数 $y=\frac{\sin\alpha-1}{\sqrt{3-2\cos\alpha-2\sin\alpha}}$的值域为________. $[-1,0]$

$key:y=\frac{(1,1)\cdot(\sin\alpha-1)}{\sqrt{(\cos\alpha-1)^2+(\sin\alpha-1)^2}}$

②函数 $y=\frac{|(\cos\alpha+\sqrt{2}\sin\alpha)t-\sqrt{2}|}{\sqrt{t^2-2\sqrt{2}t\cos\alpha+2}}(t\in R,\alpha\in(0,\frac{\pi}{2}))$ 的最大值是（　　）B

A. $\sqrt{2}$　　B. $\sqrt{3}$　　C.2　　D. $\sqrt{5}$

$key:y=\frac{|(t\cos\alpha-\sqrt{2},t\sin\alpha)\cdot(1,\sqrt{2})|}{\sqrt{(t\cos\alpha-\sqrt{2})^2+(t\sin\alpha)^2}\cdot\sqrt{3}}\cdot\sqrt{3}$

④设$a_1,a_2,a_3,a_4\in R$,且$a_3a_4-a_1a_2=1$,记$f(a_1,a_2,a_3,a_4)=a_1^2+a_2^2+a_3^2+a_4^2+a_1a_3+a_2a_4$,则$f(a_1,a_2,a_3,a_4)$的最小值为（ ）$A.1$ $B.\sqrt{3}$ $C.2$ $D.2\sqrt{3}$　B

$key:\vec{a}=(a_1,a_4),\vec{b}=(a_3,a_2),\vec{c}=(a_4,-a_1)$,则$\vec{b}\cdot\vec{c}=a_3a_4-a_1a_2=1=ab\cos(\frac{\pi}{2}-\theta)=ab\sin\theta$

$\therefore f(a_1,a_2,a_3,a_4)=\vec{a}^2+\vec{b}^2+\vec{a}\cdot\vec{b}=a^2+b^2+ab\cos\theta\ge ab(2+\cos\theta)=\frac{2+\cos\theta}{\sin\theta}\ge\sqrt{3}$

(17竞赛）已知平面向量$\vec{a},\vec{b},\vec{c}$满足$|\vec{a}|=1,|\vec{b}|=2,|\vec{c}|=3$,若$\vec{b}\cdot\vec{c}=0$,则$|\vec{a}-\lambda\vec{b}-(1-\lambda)\vec{c}|(0<\lambda<1)$所有取不到的值的集合为____. $(-\infty,\frac{6}{\sqrt{13}}-1)\cup[4,+\infty)$

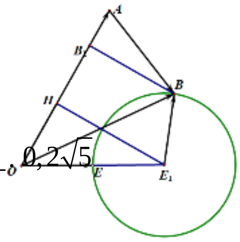
变式：已知平面向量$\vec{a},\vec{b},\vec{c}$满足$|\vec{a}|=3,|\vec{b}|=|\vec{c}|=5,0<\lambda<1$,若$\vec{b}\cdot\vec{c}=0$,则$|\vec{a}-\vec{b}+\lambda(\vec{b}-\vec{c})|+|\frac{3}{5}\vec{c}+(1-\lambda)(\vec{b}-\vec{c})|$的最小值为_____.

$key:$ 如图:$\overrightarrow{OC'}=-\vec{c},\overrightarrow{OE}=-\frac{2}{5}\vec{c}$, 则$\overrightarrow{OD}=\vec{b}-\vec{c},\overrightarrow{OF}=\vec{b}-\frac{2}{5}\vec{c}$

设$\overrightarrow{OP}=\lambda\overrightarrow{OD}$, 则$\vec{b}-\lambda(\vec{b}-\vec{c})=\overrightarrow{PB},\vec{b}-\frac{2}{5}\vec{c}-\lambda(\vec{b}-\vec{c})=\overrightarrow{PF}$

$\therefore|\vec{b}-\lambda(\vec{b}-\vec{c})|+|\vec{b}-\frac{2}{5}\vec{c}-\lambda(\vec{b}-\vec{c})|-3=|\overrightarrow{PB}|+|\overrightarrow{PF}|-3=|\overrightarrow{PC'}|+|\overrightarrow{PF}|-3\geq\sqrt{34}-3$

(18高考）已知$\vec{a},\vec{b},\vec{e}$是平面向量,$\vec{e}$是单位向量. 若非零向量$\vec{a}$与$\vec{e}$的夹角为$\frac{\pi}{3}$, 向量$\vec{b}$满足$\vec{b}^2-4\vec{e}\cdot\vec{b}+3=0$，则$|\vec{a}-\vec{b}|$的最小值为（　　）$A.\sqrt{3}-1$　$B.\sqrt{3}+1$　$C.2$　$D.2-\sqrt{3}$　　A



18高考$key:(\vec{b}-2\vec{e})^2=1,\therefore|\vec{a}-\vec{b}|_{\min}=\sqrt{3}-1$

(19高考）已知正方形$ABCD$的边长为1，当每个$\lambda_i(i=1,2,3,4,5,6)$取遍±1时，$|\lambda_1\overrightarrow{AB}+\lambda_2\overrightarrow{BC}+\lambda_3\overrightarrow{CD}+\lambda_4\overrightarrow{DA}+\lambda_5\overrightarrow{AC}+\lambda_6\overrightarrow{BD}|$的最小值是____, 最大值是________ $0,2\sqrt{5}$

$key:M=|\lambda_1\overrightarrow{AB}+\lambda_2\overrightarrow{BC}+\lambda_3\overrightarrow{CD}+\lambda_4\overrightarrow{DA}+\lambda_5\overrightarrow{AC}+\lambda_6\overrightarrow{BD}|$

$=|\lambda_1(1,0)+\lambda_2(0,1)+\lambda_3(-1,0)+\lambda_4(0,-1)+\lambda_5(1,1)+\lambda_6(-1,1)|$

$=|(\lambda_1-\lambda_3+\lambda_5-\lambda_6,\lambda_2-\lambda_4+\lambda_5+\lambda_6)|$

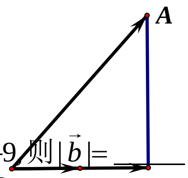
$=\sqrt{(\lambda_1-\lambda_3+\lambda_5-\lambda_6)^2+(\lambda_2-\lambda_4+\lambda_5+\lambda_6)^2}\,(a=\lambda_1-\lambda_3,b=\lambda_2-\lambda_4)$

$=\sqrt{a^2+b^2+2a(\lambda_5-\lambda_6)+2b(\lambda_5+\lambda_6)+4}$

当$\lambda_5-\lambda_6=0$时，$M=\sqrt{a^2+b^2\pm4b+4}=\sqrt{a^2+(b\pm2)^2}\in[0,2\sqrt{5}]$

当$\lambda_5+\lambda_6=0$时，$M=\sqrt{b^2+a^2\pm4a+4}=\sqrt{b^2+(a\pm2)^2}\in[0,2\sqrt{5}]$

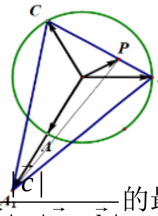
(19A)在平面直角坐标系中，$\vec{e}$是单位向量，向量$\vec{a}$满足$\vec{a}\cdot\vec{e}=2$, 且$|\vec{a}|^2\leq5|\vec{a}+t\vec{e}|$对任意实数$t$成立，则$|\vec{a}|$的取值范围是______.$[\sqrt{5},2\sqrt{5}]$



$key:\frac{1}{5}a^2\leq\sqrt{a^2-4}$得$|\vec{a}|\in[\sqrt{5},2\sqrt{5}]$

（20 重庆）1.已知向量$\vec{a},\vec{b}$满足$|\vec{a}-\vec{b}|=3,|\vec{a}+2\vec{b}|=6,\vec{a}^2+\vec{a}\cdot\vec{b}-2\vec{b}^2=-9$, 则$|\vec{b}|=$______.$\sqrt{7}$

(2020竞赛）设平面上三个不共线单位向量$\vec{a},\vec{b},\vec{c}$, 满足$\vec{a}+\vec{b}+\vec{c}=\vec{0}$.若$0\leq t\leq1$，则$|-2\vec{a}+t\vec{b}+(1-t)\vec{c}|$的取值范围为____________.



2020:$key:$由$(\vec{a}+\vec{b})^2=(-\vec{c})^2$得$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=-\frac{1}{2}$

$\therefore|-2\vec{a}+t\vec{b}+(1-t)\vec{c}|=|2\vec{a}-(t\vec{b}+(1-t)\vec{c})|\in[\frac{5}{2},\sqrt{7}]$

(2021北京)2.向量$\vec{a}$、$\vec{b}$、$\vec{c}$满足$\vec{a}\neq\vec{b},\vec{c}\neq\vec{0},(\vec{c}-\vec{a})\cdot(\vec{c}-\vec{b})=0$, 则$\frac{|\vec{c}|}{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}$的最大值为_____.

(2021北京）$key:\frac{|\vec{c}|}{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}=\frac{|\overrightarrow{OC}|}{2(|\overrightarrow{OD}|+|\overrightarrow{DC}|)}\leq\frac{1}{2}$

（2022 乙）3. 已知向量$\vec{a},\vec{b}$满足$|\vec{a}|=1,|\vec{b}|=\sqrt{3},|\vec{a}-2\vec{b}|=3$，则$\vec{a}\cdot\vec{b}=$（　C　）

A. -2　　　　B. -1　　　　C. 1　　　　D. 2

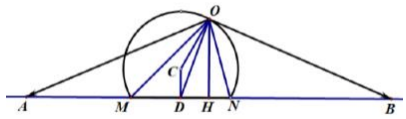
变式 1 （ 1 ）①已知平面向量$\vec{e_1},\vec{e_2}$满足$|2\vec{e_2}-\vec{e_1}|=2$，设$\vec{a}=\vec{e_1}+4\vec{e_2},\vec{b}=\vec{e_1}+\vec{e_2}$，若$1\leq\vec{a}\cdot\vec{b}\leq2$，则$|\vec{a}|$的取值范围为_______.

$key:$由$\begin{cases}\vec{e_1}+4\vec{e_2}=\vec{a}\\ \vec{e_1}+\vec{e_2}=\vec{b}\end{cases}$得$\begin{cases}\vec{e_1}=\dfrac{-\vec{a}+4\vec{b}}{3}\\ \vec{e_2}=\dfrac{\vec{a}-\vec{b}}{3}\end{cases}$，$\therefore 2=|2\vec{e_2}-\vec{e_1}|=|\vec{a}-2\vec{b}|,\therefore 4=a^2-4ab\cos\theta+4b^2$，且$ab\cos\theta\in[1,2]$

$\therefore 4ab\cos\theta=a^2+4b^2-4\in[4,8]$，且$a^2+4b^2-4\le 4ab$

$\therefore\begin{cases}8\le a^2+4b^2\le 12\\ -2\le a-2b\le 2\end{cases}\Leftrightarrow\begin{cases}8\le a^2+b_1^2\le 12\\ -2\le a-b_1\le 2\end{cases}(a>0,b_1=2b>0)$得$a\in[-1+\sqrt{3},1+\sqrt{5}]$

②已知$|\overrightarrow{OA}|=|\overrightarrow{OB}|=1$，若存在$m,n\in R$，使得$m\overrightarrow{AB}+\overrightarrow{OA}$与$n\overrightarrow{AB}+\overrightarrow{OB}$夹角为$60°$，且$|(m\overrightarrow{AB}+\overrightarrow{OA})-(n\overrightarrow{AB}+\overrightarrow{OB})|=\dfrac{1}{2}$，则$|\overrightarrow{AB}|$的最小值为________.

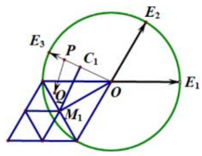


$key:\overrightarrow{OM}=m\overrightarrow{AB}+\overrightarrow{OA},\overrightarrow{ON}=n\overrightarrow{AB}+\overrightarrow{OB}$，则$|\overrightarrow{MN}|=\dfrac{1}{2}$，

$\therefore|\overrightarrow{AB}|=2\sqrt{1-d^2}\ge\dfrac{\sqrt{13}}{2}(d\le\dfrac{\sqrt{3}}{4})$

③已知平面向量$\vec{e_1},\vec{e_2},\vec{e_3}$，$|\vec{e_1}|=|\vec{e_2}|=|\vec{e_3}|=1$，$<\vec{e_1},\vec{e_2}>=60°$.若对区间$[\dfrac{1}{2},1]$内的三个任意的实数$\lambda_1,\lambda_2,\lambda_3$，都有$|\lambda_1\vec{e_1}+\lambda_2\vec{e_2}+\lambda_3\vec{e_3}|\ge\dfrac{1}{2}|\vec{e_1}+\vec{e_2}+\vec{e_3}|$，则向量$\vec{e_1}$与$\vec{e_3}$夹角的最大值的余弦值为（　　）

$A.-\dfrac{3+\sqrt{6}}{6}$　　$B.-\dfrac{3+\sqrt{5}}{6}$　　$C.-\dfrac{3-\sqrt{6}}{6}$　　$D.-\dfrac{3-\sqrt{5}}{6}$



$key:$如图，$|\lambda_1\vec{e_1}+\lambda_2\vec{e_2}+\lambda_3\vec{e_3}|=|\overrightarrow{PQ}|\ge\dfrac{1}{2}|\vec{e_1}+\vec{e_2}+\vec{e_3}|=|\overrightarrow{C_1M_1}|$

设$<\vec{e_1},\vec{e_3}>$的最大值为θ，

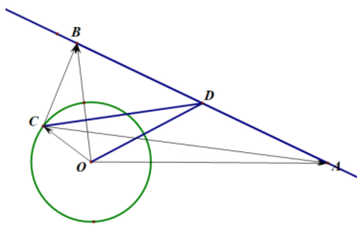
$\therefore\vec{e_3}\cdot\dfrac{1}{2}(-\vec{e_1}-\vec{e_2}-\vec{e_3})=0\Leftrightarrow\cos\theta+\cos(\theta-60°)+1=\dfrac{3}{2}\cos\theta+\dfrac{\sqrt{3}}{2}\sin\theta+1=0$得$\cos\theta=\dfrac{-3-\sqrt{6}}{6}$

（2）①已知非零平面向量$\vec{a},\vec{b},\vec{c}$，.满足$|\vec{a}|=4,|\vec{b}|=2|\vec{c}|$，且$(\vec{a}-\vec{c})\cdot(\vec{b}-\vec{c})=3$，则$|\vec{a}-\vec{b}|$的最小值是（ A ）

A.$\dfrac{2\sqrt{6}}{3}$　　B.$\dfrac{3\sqrt{5}}{5}$　　C.2　　D.3

$key:(\vec{a}-\vec{c})\cdot(\vec{b}-\vec{c})=\overrightarrow{CA}\cdot\overrightarrow{CB}=\overrightarrow{CD}^2-x^2=3,\therefore|\overrightarrow{CD}|=\sqrt{3+x^2}$(设$|\vec{c}|=c,x=\dfrac{1}{2}|\overrightarrow{AB}|$)

$|\overrightarrow{OD}|^2=\dfrac{2\overrightarrow{OA}^2+2\overrightarrow{OB}^2-(\overrightarrow{OA}-\overrightarrow{OB})^2}{4}=8+2c^2-x^2$



$\therefore\begin{cases}c+\sqrt{3+x^2}\ge\sqrt{8+2c^2-x^2}\\ c+\sqrt{8+2c^2-x^2}\ge\sqrt{3+x^2}\\ \sqrt{3+x^2}+\sqrt{8+2c^2-x^2}\ge c\end{cases}\Leftrightarrow|c-\sqrt{3+x^2}|\le\sqrt{8+2c^2-x^2}\le c+\sqrt{3+x^2}$

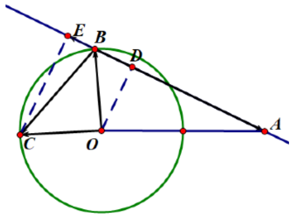
$\Leftrightarrow|5+c^2-2x^2|\le 2c\sqrt{3+x^2}\Leftrightarrow c^4-2(4x^2+1)c^2+(2x^2-5)^2\le 0$

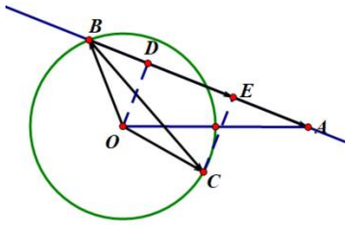
$\therefore\Delta=4(4x^2+1)^2-4(2x^2-5)^2\ge 0$即$x\ge\dfrac{\sqrt{6}}{3}$

②已知平面向量$\vec{a},\vec{b},\vec{c}$满足$|\vec{a}|=1,|\vec{b}|=2,|\vec{c}-\vec{a}|=|\vec{c}-\vec{b}|$，则$|\vec{c}|$的最小值为____，此时$\vec{a}\cdot\vec{b}=$______.

$key1:|\vec{c}|_{\min}=\dfrac{|\frac{\vec{a}+\vec{b}}{2}\cdot(\vec{b}-\vec{a})|}{|\vec{b}-\vec{a}|}=\dfrac{3}{2|\vec{b}-\vec{a}|}\ge\dfrac{1}{2}$

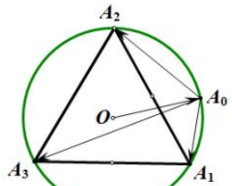
$key2:$由$|\vec{c}-\vec{a}|^2=|\vec{c}-\vec{b}|^2$ 得$\vec{c}\cdot(\vec{b}-\vec{a})=\dfrac{3}{2}$,如图:





$\therefore|\vec{c}|=\dfrac{3}{2|\vec{b}-\vec{a}|\cdot|\cos<\vec{b}-\vec{a},\vec{c}>|}\ge\dfrac{1}{2},\vec{a}\cdot\vec{b}=-2$

④平面向量$\vec{a_i}$满足:$|\vec{a_i}|=1(i=0,1,2,3)$，且$\sum_{i=1}^{3}\vec{a_i}=\vec{0}$,则$|\vec{a_0}+\vec{a_1}+\vec{a_2}|+|\vec{a_0}+\vec{a_1}+\vec{a_3}|+|\vec{a_0}+\vec{a_2}+\vec{a_3}|$的取值范围为_______.$[2\sqrt{3},4]$



$key:$设$\overrightarrow{OA_i}=\vec{a_i}(i=1,2,3),\because\vec{a_1}+\vec{a_2}+\vec{a_3}=\vec{0}$,且$|\vec{a_1}|=|\vec{a_2}|=|\vec{a_3}|=1$,

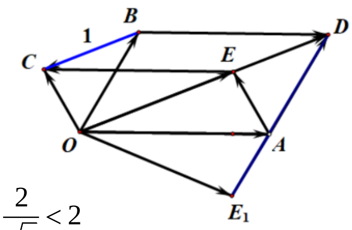
则$\triangle A_1A_2A_3$为正三角形，设$\overrightarrow{OA_0}=\vec{a_0}$,不妨设点$A_0$在圆弧$A_1A_2$上,记$\angle A_0A_3A_1=\theta\in[0,\dfrac{\pi}{3}]$,

则$|\vec{a_0}+\vec{a_1}+\vec{a_2}|+|\vec{a_0}+\vec{a_1}+\vec{a_3}|+|\vec{a_0}+\vec{a_2}+\vec{a_3}|=|\vec{a_0}-\vec{a_3}|+|\vec{a_0}-\vec{a_2}|+|\vec{a_0}-\vec{a_1}|=|\overrightarrow{A_0A_1}|+|\overrightarrow{A_0A_2}|+|\overrightarrow{A_0A_3}|$,

则$|\overrightarrow{A_0A_1}|+|\overrightarrow{A_0A_2}|+|\overrightarrow{A_0A_3}|=2\sin\theta+2\sin(\dfrac{\pi}{3}-\theta)+2\sin(\dfrac{\pi}{3}+\theta)=4\sin(\theta+\dfrac{\pi}{3})\in[2\sqrt{3},4]$

（3）已知向量$\vec{a}$与$\vec{b}$夹角为$\dfrac{\pi}{3}$，向量$\vec{c}$满足$|\vec{b}-\vec{c}|=1$且$\dfrac{\vec{a}+\vec{b}}{|\vec{b}|}=\dfrac{\vec{a}+\vec{c}}{|\vec{c}|}$，则下列说法正确的是（ ） A

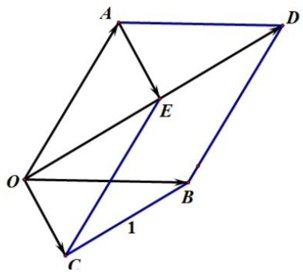
$A.|\vec{b}|+|\vec{c}|<2$　　$B.|\vec{a}|+|\vec{b}|>2$　　$C.|\vec{b}|<1$　　$D.|\vec{a}|>1$



$key:\overrightarrow{OE}=\vec{a}+\vec{c},\overrightarrow{OC}=\overrightarrow{AE}=\vec{c}$,作$EE_1\perp OA$，且$|\overrightarrow{OE}|=|\overrightarrow{OE_1}|$

则$\dfrac{|\overrightarrow{OD}|}{|\overrightarrow{OE_1}|}=\dfrac{|\vec{b}|}{|\vec{c}|},\therefore E_1,A,D$三点共线$,\therefore\angle BOC=60°$

$\therefore|\vec{b}|>|\overrightarrow{BC}|=1$,且$1=\vec{b}^2+\vec{c}^2-|\vec{b}|\cdot|\vec{c}|\ge\dfrac{3}{4}(|\vec{b}|+|\vec{c}|)^2,\therefore|\vec{b}|+|\vec{c}|\le\dfrac{2}{\sqrt{3}}<2$



$key:$设$\overrightarrow{OA}=\vec{a},\overrightarrow{OB}=\vec{b},\overrightarrow{OC}=\vec{c}=\overrightarrow{AE}$

$\vec{a}+\vec{b}=\overrightarrow{OD},\vec{a}+\vec{c}=\overrightarrow{OE},\because\dfrac{\vec{a}+\vec{b}}{|\vec{b}|}=\dfrac{\vec{a}+\vec{c}}{|\vec{c}|}$即$\dfrac{\overrightarrow{OD}}{|\overrightarrow{OB}|}=\dfrac{\overrightarrow{OE}}{|\overrightarrow{AE}|}$

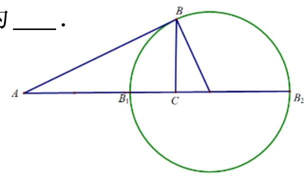
$\therefore O,E,D$共线，且$\dfrac{\sin\angle120°}{\sin\angle ODB}=\dfrac{|\overrightarrow{OD}|}{|\overrightarrow{OB}|}=\dfrac{|\overrightarrow{OE}|}{|\overrightarrow{AE}|}=\dfrac{\sin\angle OAE}{\sin\angle AOE}$

$\because\angle AOE=\angle BDO,\therefore\angle OAE=60°,\therefore\angle AOC=120°,\therefore\angle COB=60°$

$\therefore1=\vec{b}^2+\vec{c}^2-|\vec{b}|\cdot|\vec{c}|=\dfrac{3}{4}(|\vec{b}|+|\vec{c}|)^2+\dfrac{1}{4}(|\vec{b}|-|\vec{c}|)^2\ge\dfrac{3}{4}(|\vec{b}|+|\vec{c}|)^2,\therefore|\vec{b}|+|\vec{c}|<\dfrac{2}{\sqrt{3}}<2$

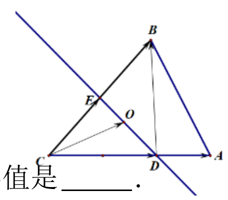
(2018河北）在$\triangle ABC$中，$AC=3,\sin C=k\sin A(k\ge2)$，则$\triangle ABC$的面积最大值为___.

2018河北$key:|\overrightarrow{AB}|=k|\overrightarrow{BC}|$,如图$B$的轨迹是阿波罗尼斯圆,



其直径$B_1B_2=\dfrac{3}{k+1}+\dfrac{3}{k-1}=\dfrac{6k}{k^2-1},\therefore S_{\triangle ABC}=\dfrac{1}{2}\cdot3\cdot\dfrac{3k}{k^2-1}=\dfrac{9}{2}\cdot\dfrac{1}{k-\frac{1}{k}}\le3$

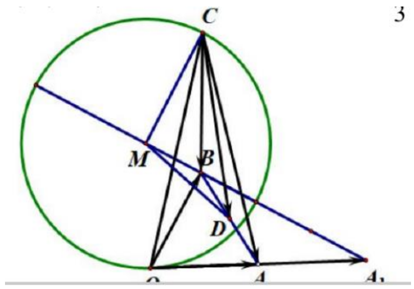
变式 2（1） 设O为$\triangle ABC$的外心，满足$\overrightarrow{CO}=t\overrightarrow{CA}+(\dfrac{1}{2}-\dfrac{3}{4}t)\overrightarrow{CB},t\in R$,若$|\overrightarrow{AB}|=3$,则$\triangle ABC$面积的最大值为________



$key:\overrightarrow{CO}=\dfrac{3}{2}t(\dfrac{2}{3}\overrightarrow{CA})+(1-\dfrac{3}{2}t)(\dfrac{1}{2}\overrightarrow{CB}),\therefore|\overrightarrow{DB}|=2|\overrightarrow{DA}|,\therefore S_{\triangle ABC}=3S_{\triangle ABD}\le9$

（2）①已知向量$\vec{a},\vec{b},|\vec{a}|=|\vec{b}|=2$,若$\vec{a}\cdot\vec{b}=2$,且$|\dfrac{1}{2}\vec{c}-\vec{a}|=|\vec{c}-\vec{b}|$，则$(\vec{c}-\vec{a})\cdot(\vec{c}-\vec{b})$的最小值是______.

key:由$|\frac{1}{2}\vec{c}-\vec{a}|=|\vec{c}-\vec{b}|$得$|\overrightarrow{CA_1}|=2|\overrightarrow{CB}|$,



$\therefore C$在以M为圆心半径为$\frac{4\sqrt{3}}{3}$的圆上，如图，

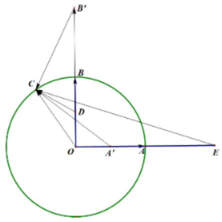
$\therefore(\vec{c}-\vec{a})\cdot(\vec{c}-\vec{b})=\overrightarrow{CA}\cdot\overrightarrow{CB}=\overrightarrow{CD}^2-1$($D$为$AB$的中点)

（而$\overrightarrow{MD}=\frac{1}{2}(\vec{a}+\vec{b})-(2\vec{a}+\frac{4}{3}(\vec{b}-2\vec{a}))=\frac{7}{6}\vec{a}-\frac{5}{6}\vec{b},\therefore|\overrightarrow{MD}|=\frac{\sqrt{39}}{3}$）

$\ge(\frac{4\sqrt{3}}{3}-\frac{\sqrt{39}}{3})^2-1=\frac{26-8\sqrt{3}}{3}$

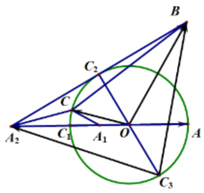
②已知平面向量$\vec{a},\vec{b},\vec{c}$满足：$|\vec{a}|=|\vec{b}|=|\vec{c}|=1$，$\vec{a}\cdot\vec{b}=0$，则$|2\vec{c}-\vec{a}|+|\frac{1}{2}\vec{c}-\vec{b}|$的最小值为（A ）

A. $\frac{\sqrt{17}}{2}$　　B．2　　C．$\frac{5}{2}$　　D．$\sqrt{5}$



key:$|2\vec{c}-\vec{a}|+|\frac{1}{2}\vec{c}-\vec{b}|=2|\vec{c}-\frac{1}{2}\vec{a}|+\frac{1}{2}|\vec{c}-2\vec{b}|=2|\overrightarrow{A'C}|+\frac{1}{2}|\overrightarrow{B'C}|=|\overrightarrow{CE}|+|\overrightarrow{CD}|\ge|\overrightarrow{DE}|=\frac{\sqrt{17}}{2}$

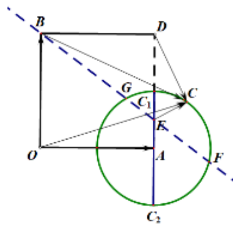
③已知向量$\vec{a},\vec{b},\vec{c}$满足$|\vec{a}|=\frac{1}{2}|\vec{b}|=|\vec{c}|=1,\vec{a}\cdot\vec{b}=1$，则$|c+\frac{1}{2}\vec{a}|+\frac{1}{2}|\vec{c}-\vec{b}|$的取值范围为_______.



$key1$:$|\vec{c}+\frac{1}{2}\vec{a}|+\frac{1}{2}|\vec{c}-\vec{b}|=|\overrightarrow{CA_1}|+\frac{1}{2}|\overrightarrow{CB}|=\frac{1}{2}(|\overrightarrow{CA_2}|+|\overrightarrow{CB}|)\ge\frac{1}{2}|\overrightarrow{A_2B}|=\sqrt{3}$

$|\vec{c}+\frac{1}{2}\vec{a}|+\frac{1}{2}|\vec{c}-\vec{b}|=|\overrightarrow{CA_1}|+\frac{1}{2}|\overrightarrow{CB}|=\frac{1}{2}(|\overrightarrow{CA_2}|+|\overrightarrow{CB}|)\le\frac{1}{2}(|\overrightarrow{A_2C_3}|+|\overrightarrow{BC_3}|)=\sqrt{7}$

④已知$\vec{a},\vec{b}$是平面内两个互相垂直的单位向量,若向量$\vec{c}$满足$|\vec{c}-\vec{a}|=\frac{1}{2}$,则$2|\vec{c}-\vec{b}|-|\vec{a}+\vec{b}-\vec{c}|$的最大值为_______.



key:如图,$2|\vec{c}-\vec{b}|-|\vec{a}+\vec{b}-\vec{c}|=2|\overrightarrow{BC}|-|\overrightarrow{DC}|=2|\overrightarrow{BC}|-2|\overrightarrow{CE}|$

$=2(|\overrightarrow{BC}|-|\overrightarrow{CE}|)\le2(|\overrightarrow{FB}|-|\overrightarrow{FE}|)=2|\overrightarrow{BE}|=\frac{5}{2}$