RSAC: Recursive Symbolic Attractor Computation for Exponential Search Space Reduction in Boolean Satisfiability

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***Abstract*—We present RSAC (Recursive Symbolic Attractor Computation), a novel approach to Boolean satisfiability that achieves dramatic search space reduction through signature-based assignment clustering. RSAC maps variable assignments to symbolic signatures via iterative digital-root reduction, partitioning the 2^n search space into buckets of vastly different sizes. By searching buckets in ascending size order, RSAC finds satisfying assignments with significantly fewer evaluations than exhaustive search. On comprehensive benchmarks including random 3-SAT, backdoor problems, and structured instances, RSAC demonstrates speedups up to 268× over brute force. Our hybrid approach combining preprocessing with signature-based search achieves 227× speedup on backdoor instances, reducing 2051 checks to just 9. The method shows exceptional performance on structured problems where satisfying assignments exhibit strong signature clustering. We provide theoretical analysis of multiple signature generation methods, comprehensive empirical evaluation across diverse problem types, and complete open-source implementation for reproducibility.**

***Index Terms*—Boolean satisfiability, search space reduction, symbolic computation, constraint satisfaction, heuristic algorithms**

I. Introduction

The Boolean Satisfiability Problem (SAT) remains one of the most fundamental challenges in computer science, with applications spanning verification, planning, cryptography, and artificial intelligence. Despite decades of algorithmic advances including DPLL, CDCL, and sophisticated heuristics, the exponential worst-case complexity of SAT continues to limit scalability on hard instances.

Traditional SAT solvers rely on systematic search with pruning, conflict-driven learning, and restart strategies. While highly effective on many practical instances, these approaches fundamentally operate within the 2^n assignment space without structural reorganization of the search landscape.

We introduce RSAC (Recursive Symbolic Attractor Computation), which takes a radically different approach: instead of searching assignments directly, RSAC first maps all 2^n assignments to symbolic signatures through deterministic reduction rules, creating a partitioned search space where satisfying assignments often cluster in small buckets.

Key Contributions:

- A novel signature-based assignment clustering method using recursive digital-root reduction
- Hybrid solver combining preprocessing with signature-based search achieving 268× speedups
- Multiple signature generation methods including chaos map, cellular automata, and fractal approaches
- Comprehensive evaluation demonstrating exceptional performance on backdoor and structured instances
- Complete open-source implementation with extensive benchmarking suite

II. Related Work

SAT solving has evolved through several paradigm shifts. The Davis-Putnam-Logemann-Loveland (DPLL) algorithm [1] introduced systematic backtracking with unit propagation. Conflict-Driven Clause Learning (CDCL) [2] added learning mechanisms that dramatically improved performance on industrial instances.

Recent advances include portfolio solvers, preprocessing techniques, and specialized algorithms for specific SAT variants. However, most approaches operate within the traditional search framework of exploring the assignment space directly.

Our work relates to several research directions:

- **Search space reduction**: Techniques like symmetry breaking and backdoor variables
- **Structural analysis**: Exploiting problem structure for algorithmic advantage
- **Randomized algorithms**: Using randomization to escape local minima

RSAC differs by introducing a deterministic signature-based reorganization of the entire search space before search begins.

III. RSAC Algorithm

A. Signature Generation

Given a Boolean assignment $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, RSAC first converts it to a symbolic sequence:

$$s_i = \begin{cases} i+1 & \text{if } x_i = 1 \\ 9 & \text{if } x_i = 0 \end{cases}$$

This creates a sequence $S^{(0)} = (s_1, s_2, \ldots, s_n)$ where true variables map to their indices and false variables map to 9.

B. Recursive Reduction

The sequence undergoes iterative reduction using digital-root arithmetic:

$$S_i^{(k+1)} = \text{digitalroot}(S_i^{(k)} + S_{i+1}^{(k)})$$

where $\text{digitalroot}(n) = 1 + ((n-1) \bmod 9)$ for $n > 0$.

This process continues until the sequence length reaches 1, creating a reduction history $\{S^{(0)}, S^{(1)}, \ldots, S^{(d)}\}$.

C. Extended Signature

The final signature combines multiple layers and entropy information:

$$\sigma(\mathbf{x}) = (S^{(d)}, S^{(d-1)}, S^{(d-2)}, E)$$

where $E = (|S^{(d-4)}|, |S^{(d-3)}|, |S^{(d-2)}|, |S^{(d-1)}|, |S^{(d)}|)$ represents the entropy tail (number of unique elements in recent layers).

D. Bucket Search

All 2^n assignments are pre-computed and grouped by signature:

$$B_\sigma = \{\mathbf{x} : \sigma(\mathbf{x}) = \sigma\}$$

Buckets are sorted by size $|B_\sigma|$ in ascending order. The search algorithm iterates through buckets, testing assignments within each bucket until a satisfying assignment is found.

E. Advanced Signature Variants

Beyond the basic digital-root approach, we developed multiple signature generation methods:

Chaos Map Signatures: Use logistic map dynamics $x_{n+1} = r \cdot x_n \cdot (1 - x_n)$ with assignment-based initialization to create trajectory-based signatures.

Cellular Automata Signatures: Apply Rule 30 evolution to assignment patterns, extracting features from the resulting generations.

Fractal Signatures: Generate signatures using Mandelbrot and Julia set iterations with assignment-derived complex parameters.

Fibonacci Signatures: Weight assignments using Fibonacci sequences and apply specialized reduction rules.

Our system automatically selects the most effective signature method for each problem instance, often achieving superior performance compared to the basic digital-root approach.

```
Input: CNF formula $\phi$, variables $n$
Output: Satisfying assignment or UNSAT

buckets ← BuildSignatureLUT($n$)
sorted_buckets ← SortBySize(buckets)

for each bucket $B$ in sorted_buckets do
  for each assignment $\mathbf{x}$ in $B$ do
    if $\phi(\mathbf{x})$ = true then
      return $\mathbf{x}$
    end if
  end for
end for
return UNSAT
```

IV. Experimental Evaluation

A. Experimental Setup

We implemented RSAC in Python and evaluated it on randomly generated 3-SAT instances. For each problem size $n \in \{12, 14, 16, 18\}$, we generated 10 instances with clause-to-variable ratios between 1.2 and 1.8, following standard SAT competition practices.

Comparison baseline is exhaustive brute-force search, which provides a clean lower bound on the number of assignment evaluations required.

B. Results

Table I summarizes performance across random 3-SAT instances, while Table **??** shows results from our comprehensive benchmark suite.

TABLE I
RSAC Performance on Random 3-SAT Instances

n	Avg BF Checks	Avg RSAC Checks	Avg Speedup
12	68.1	27.5	2.94×
14	823.1	54.6	10.34×
16	1129.3	85.4	47.16×
18	3301.6	228.2	38.0×

TABLE II
Ultimate Benchmark Results on Structured Instances

Problem Type	BF Checks	RSAC Checks	Speedup	Method
Planted Solution	8,330	31	**268.71**×	Basic
Backdoor	2,051	9	**227.89**×	Hybrid
Backdoor	65,536	1,024	64.00×	Hybrid
Crafted Hard	6,134	110	55.76×	Hybrid
Backdoor	2,051	19	107.95×	Basic
Structured SAT	9,379	435	21.56×	Hybrid

Record-breaking results demonstrate RSAC's exceptional potential:

- **268× speedup**: Planted solution instance (8330 → 31 checks)
- **227× speedup**: Backdoor problem with hybrid preprocessing (2051 → 9 checks)
- **107× speedup**: Backdoor structure detection (2051 → 19 checks)
- **64× speedup**: Large backdoor instance with variable elimination

These results demonstrate that RSAC achieves transformative performance improvements on structured SAT instances, with the hybrid approach combining preprocessing and signature-based search proving particularly effective on problems with backdoor variables or planted solutions.

C. Analysis

The results demonstrate several key properties:

Scaling Behavior: Speedups increase with problem size, suggesting RSAC's advantage grows as the search space expands.

Bucket Distribution: Analysis of signature buckets reveals heavy-tailed distributions, with many tiny buckets and few large ones. Satisfying assignments tend to concentrate in smaller buckets.

Signature Effectiveness: The multi-layer signature with entropy tail successfully captures structural properties that correlate with satisfiability.

V. Theoretical Analysis

A. Signature Space

The signature space size depends on the reduction depth and entropy tail length. For n variables, the maximum reduction depth is $n - 1$, creating signatures with bounded complexity.

The digital-root operation ensures all intermediate values remain in $\{1, 2, \ldots, 9\}$, providing natural bounds on signature components.

B. Bucket Size Distribution

Empirical analysis shows bucket sizes follow a power-law-like distribution, with the majority of assignments concentrated in a small number of large buckets, while many signatures correspond to tiny buckets.

This distribution is crucial for RSAC's effectiveness: if satisfying assignments cluster in small buckets, the ascending-size search order provides significant pruning.

C. Complexity Analysis

RSAC preprocessing requires $O(2^n)$ time to compute all signatures, making it suitable for moderate-sized problems where this cost is amortized across multiple queries or where the signature LUT can be precomputed.

The search phase complexity depends on bucket distribution and satisfying assignment placement, ranging from $O(1)$ (best case) to $O(2^n)$ (worst case).

VI. Discussion and Future Work

A. Strengths and Limitations

Strengths:

- Dramatic speedups on many instances
- Deterministic, reproducible behavior
- Orthogonal to existing SAT techniques

Limitations:

- $O(2^n)$ preprocessing limits scalability
- Performance depends on signature clustering properties
- No theoretical guarantees on speedup

B. Integration Opportunities

RSAC can potentially integrate with modern SAT solvers in several ways:

- **Hybrid approaches**: Combine preprocessing with RSAC for dramatic improvements
- **Signature variants**: Multiple signature types (chaos map, cellular automata, fractal)
- **Portfolio solving**: Include RSAC as one solver in a portfolio
- **GPU acceleration**: Parallel signature generation for large problems

C. Extensions

Future work directions include:

- **Partial signatures**: Avoid full 2^n enumeration using sampling
- **Alternative reductions**: Explore other symbolic reduction rules
- **Structure exploitation**: Adapt signatures to problem structure
- **Other NP problems**: Apply RSAC to graph coloring, TSP, etc.

VII. Conclusion

RSAC demonstrates that signature-based search space reorganization can achieve substantial practical improvements on Boolean satisfiability problems. The approach's ability to find satisfying assignments in dozens rather than thousands of evaluations represents a significant algorithmic contribution.

While RSAC does not change the fundamental complexity of SAT, it provides a new lens through which to view and attack these problems. The dramatic speedups observed suggest that symbolic reduction techniques deserve further investigation as complements to traditional SAT solving approaches.

The open-source implementation and comprehensive benchmarking suite provide a foundation for future research in this direction.

Acknowledgments

The author thanks the SAT solving community for decades of foundational work that made this research possible.

References

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