

Computational Social Science

Social Network Analysis

Fariba Karimi

Prof.

Computer Science Department, TU Graz

e-mail: karimi@tugraz.at

web: www.networkinequality.com

TU Graz, 22.10.2024

Recap

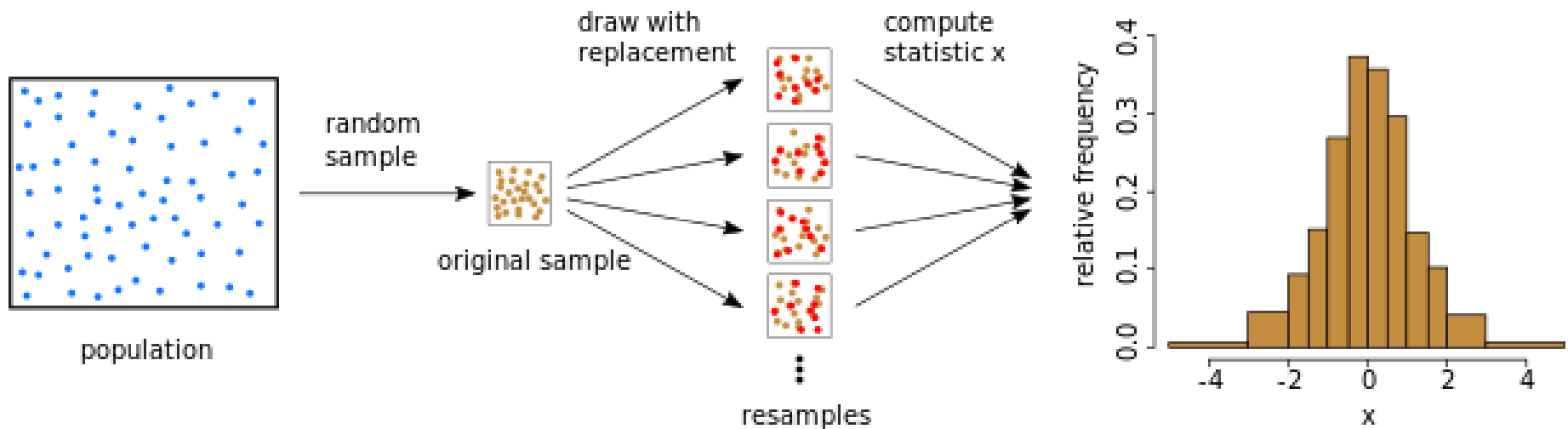
History of Social psychology Social Impact Theory

and magnetism) operating in a social force field or social structure. As an example of what I mean by a social force field, Figure 1 depicts the plight of a hapless striped target beset by a variety of spotted sources, all having some impact.

Principle 1: Social Forces, $I = f(SIN)$

As a first principle, I suggest that when some number of social sources are acting on a target individual, the amount of impact experienced by the target should be a multiplicative function of the

Recap Bootstrapping



Source: Wikipedia

Network analysis


Network science & Graph theory have common roots

- Network elements: Nodes (vertex, actor), Links (edges, connections)

Different ways we use network analysis:

- Mapping empirical data into a network
- Generating/modeling synthetic networks with different mechanism
- Inferring mechanisms from real data : network inference

Network Datasets

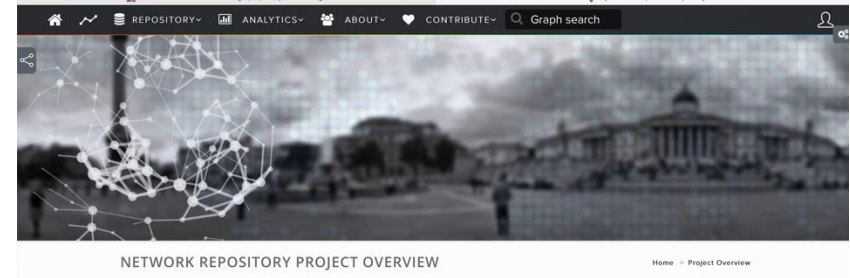
 **Netzschleuder** network catalogue, repository and centrifuge

Tip: click on the table header to sort the list. Hover your mouse over it to obtain a legend.

Multiple regexp terms separated by '&'

Name	Title	Nodes	Edges	$\langle k \rangle$	σ_k	λ_k	τ	r	c	ϕ	S	Kind	Mode	n	Tags
7th_graders	Vickers 7th Graders (1981)	29	740	25.52	20.34	17.73	1.71	-0.01	0.76	2	1.00	Directed	Unipartite	1	Social Offline Multilayer Unweighted Metadata
academia_edu	Academia.edu (2011)	200,169	1,398,063	6.98	46.24	109.99	78.34	-0.02	0.04	16	1.00	Directed	Unipartite	1	Social Online Unweighted
add_health	Adolescent health (ADD HEALTH) (1994)	2,587	12,969	5.01	5.65	11.92	29.03	0.29	0.17	10	0.98	Directed	Unipartite	84	Social Offline Weighted
adjnoun	Word adjacencies of David Copperfield	112	425	7.59	6.85	11.54	2.27	-0.13	0.16	5	1.00	Undirected	Unipartite	1	Informational Language Unweighted

<https://networks.skewed.de>



<https://networkrepository.com/platform.php>

By Jure Leskovec

 **Stanford Large Network Dataset Collection**

 **SNAP**

- **Social networks** : online social networks, edges represent interactions between people
- **Networks with ground-truth communities** : ground-truth network communities in social and information networks
- **Communication networks** : email communication networks with edges representing communication
- **Citation networks** : nodes represent papers, edges represent citations
- **Collaboration networks** : nodes represent scientists, edges represent collaborations (co-authoring a paper)
- **Web graphs** : nodes represent webpages and edges are hyperlinks

<https://snap.stanford.edu/data/>

Generating synthetic networks (network modeling)

- Understand the logic of algorithms
- **Write** your own code!!!
- Double-check your own code with the source code.

Example of a source code:

- https://networkx.org/documentation/stable/_modules/networkx/generators/random_graphs.html#gnp_random_graph

Overview of today's lecture

Theme: Properties of social networks and why they are important to study them.

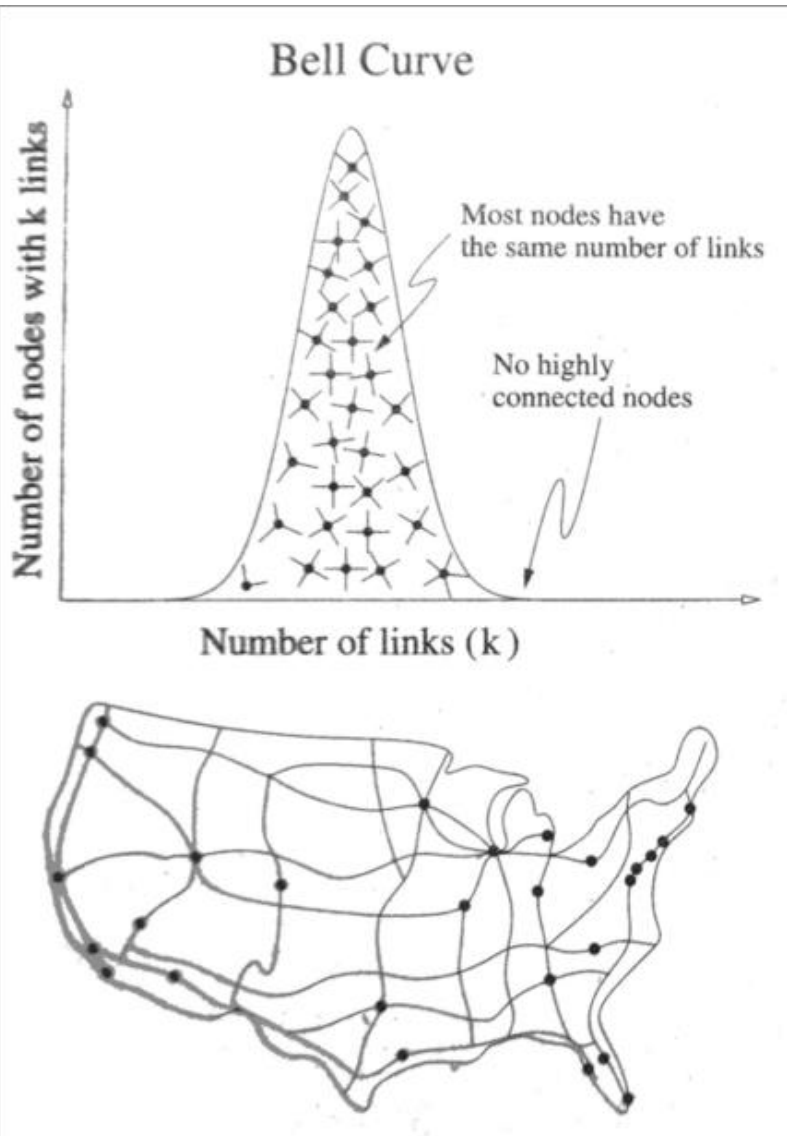
- Small world
- Popularity in social networks
- Many clusters
- Assortative

Part 1

SHAPE OF OUR SOCIAL NETWORKS: NON- RANDOMNESS

**POPULARITY OR RICH GET
RICHER EFFECT**

Classical view to networks



Classical view to social network assumed that connections follow a normal distribution.

In this view, people in the network on average have a certain number of connections.

Researchers used random graph models as models of social networks. Examples of widely used random graph models are [Erdős–Rényi model](#) or [ERGM](#).

Contemporary network science

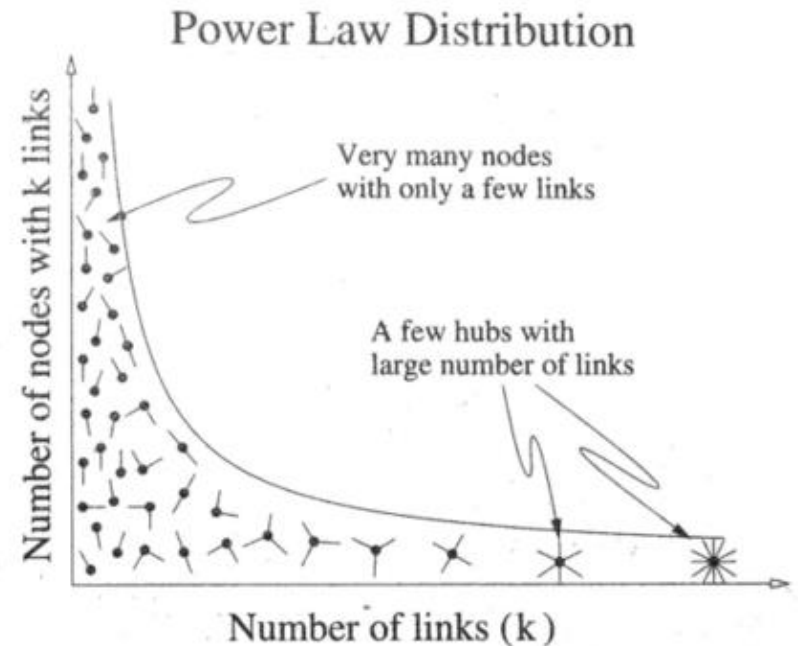
In reality, many large-scale social and technical systems follow a different kinds of distribution.

Many social and technical networks follow a power-law degree distribution.

Power-law networks are more heterogeneous.

We can no longer talk about an “average man”.

Average degree is not well-defined. We need to revisit the mathematical tools.

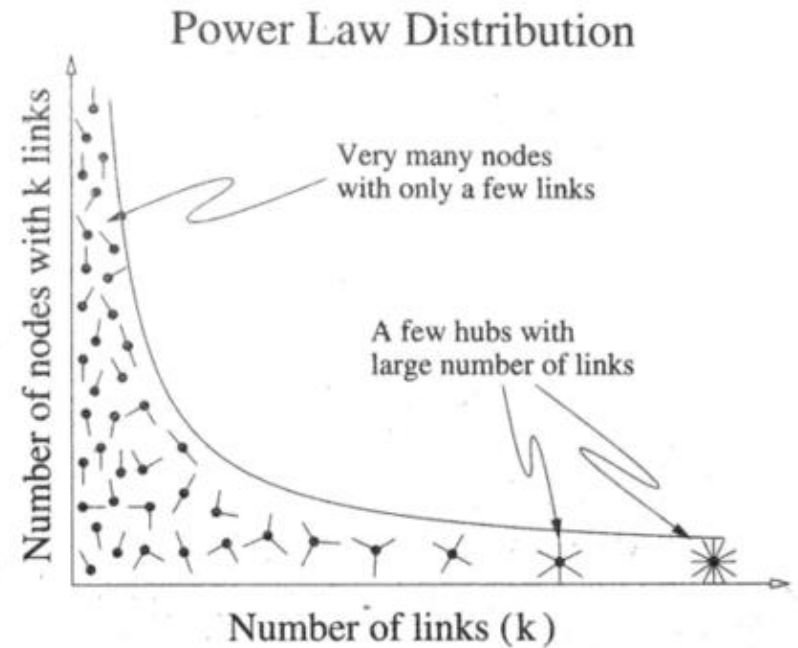


Contemporary network science

Plotting power-law networks

$$P(k) \sim k^{-a} \sim 1/k^a$$

$$\text{Log}(p(k)) = ?$$

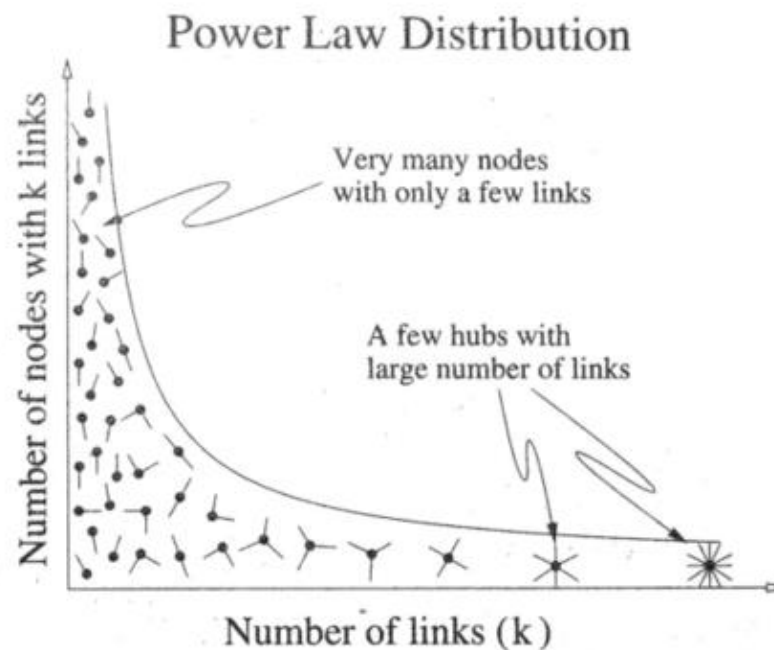
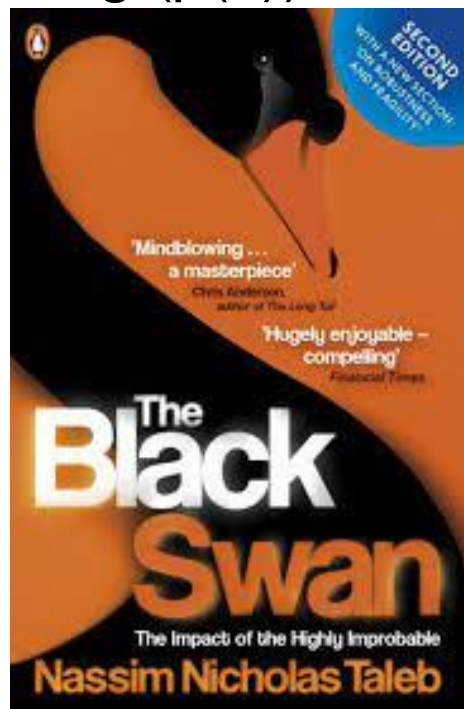


Contemporary network science

Plotting power-law (scale-free) networks

$$P(k) \sim k^{-a} \sim 1/k^a$$

$$\log(p(k)) = -a \log(k)$$



Power law distributions are observed in many socio-economic systems

Contents hide

(Top)

Empirical examples

- > Properties
- > Power-law functions
- > Power-law probability distributions
- Validating power laws
- See also
- References
- External links

Power law

Article Talk

Read Edit View history Tools

From Wikipedia, the free encyclopedia

Not to be confused with [Force \(law\)](#). For other uses, see [Power \(disambiguation\)](#).

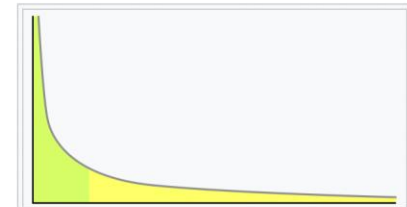
"Scaling law" redirects here. For statistical laws of scaling deep learning models, see [Neural scaling law](#).

In [statistics](#), a **power law** is a [functional relationship](#) between two quantities, where a [relative change](#) in one quantity results in a relative change in the other quantity proportional to the change raised to a constant [exponent](#): one quantity varies as a power of another. The change is independent of the initial size of those quantities.

For instance, the area of a square has a power law relationship with the length of its side, since if the length is doubled, the area is multiplied by 2^2 , while if the length is tripled, the area is multiplied by 3^2 , and so on.^[1]

Empirical examples [[edit](#)]

The distributions of a wide variety of physical, biological, and human-made phenomena approximately follow a power law over a wide range of magnitudes: these include the sizes of craters on the [moon](#) and of [solar flares](#),^[2] cloud sizes,^[3] the foraging pattern of various species,^[4] the sizes of activity patterns of neuronal populations,^[5] the frequencies of [words](#) in most languages, frequencies of [family names](#), the [species richness](#) in [clades](#) of organisms,^[6] the sizes of [power outages](#), volcanic eruptions,^[7] human judgments of stimulus intensity^{[8][9]} and many other quantities.^[10] Empirical distributions can only fit a power law for a limited range of values, because a pure power law would allow for arbitrarily large or small values. [Acoustic attenuation](#) follows frequency power-laws within wide frequency bands for many complex media. [Allometric scaling laws](#) for relationships between biological variables are among the best known power-law functions in nature.



An example power-law graph that demonstrates ranking of popularity. To the right is the [long tail](#), and to the left are the few that dominate (also known as the [80–20 rule](#)).

Question:

What mechanism generates power law networks, and how do we model it?

Preferential Attachment as a mechanism for generating scale-free networks

[Barabasi 1999]

„The rich get richer“ effect or Matthew effect in sociology.

For whoever has will be given more, and they will have an abundance. Whoever does not have, even what they have will be taken from them. (Matthew 25:29)

In the sociology of science, "Matthew effect" was a term coined by Robert K. Merton to describe how, among other things, eminent scientists will often get more credit than a comparatively unknown researcher, even if their work is similar; it also means that credit will usually be given to researchers who are already famous

Preferential Attachment as a mechanism for generating scale-free networks

[Barabasi 1999]

Barabasi and Albert defined a similar mechanism in networks:

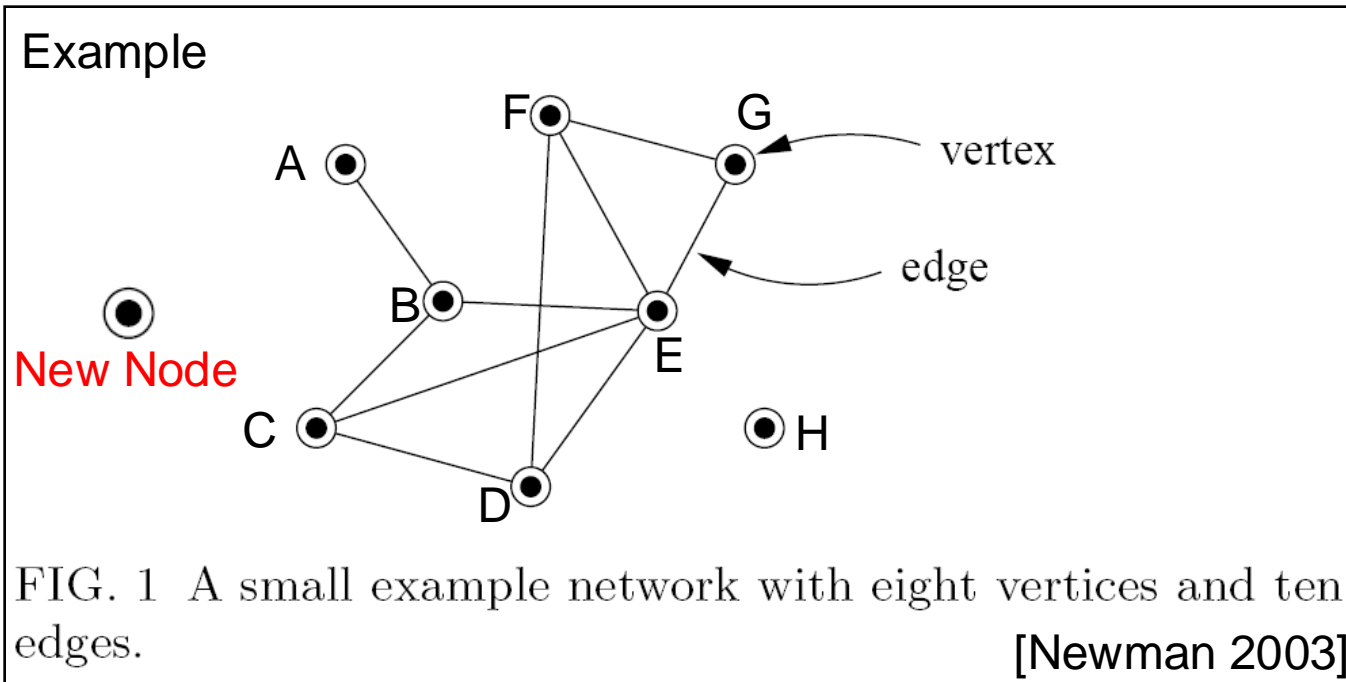
Preferential Attachment refers to the high probability of a new vertex to connect to a vertex that already has a large number of connections

Example:

1. a new website linking to more established ones
2. a new individual linking to well-known individuals in a social network

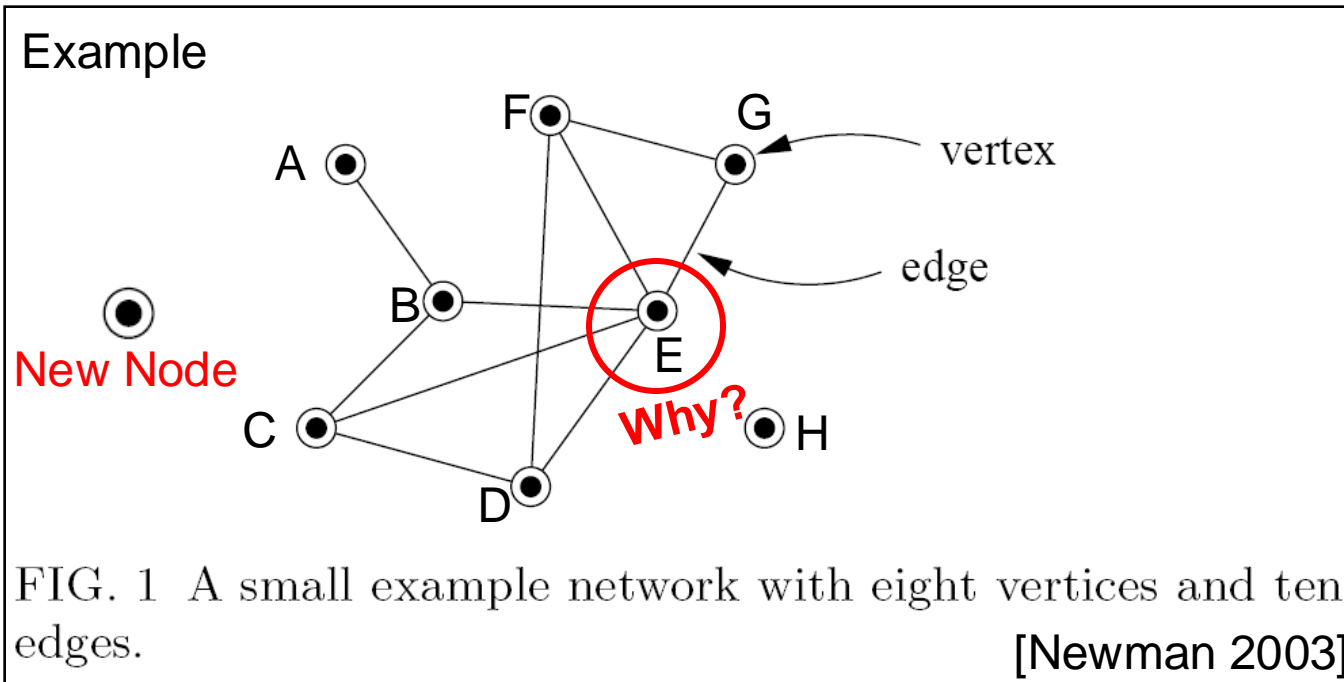
Preferential Attachment Example

Which node has the highest probability of being linked by a new node in a network that exhibits traits of preferential attachment?



Preferential Attachment Example

Which node has the highest probability of being linked by a new node in a network that exhibits traits of preferential attachment?



Preferential attachment model

Demo:

<http://estebanmoro.org/2012/11/preferential-attachment-be-first/>

Preferential attachment mechanism produce the scale-free property in networks.

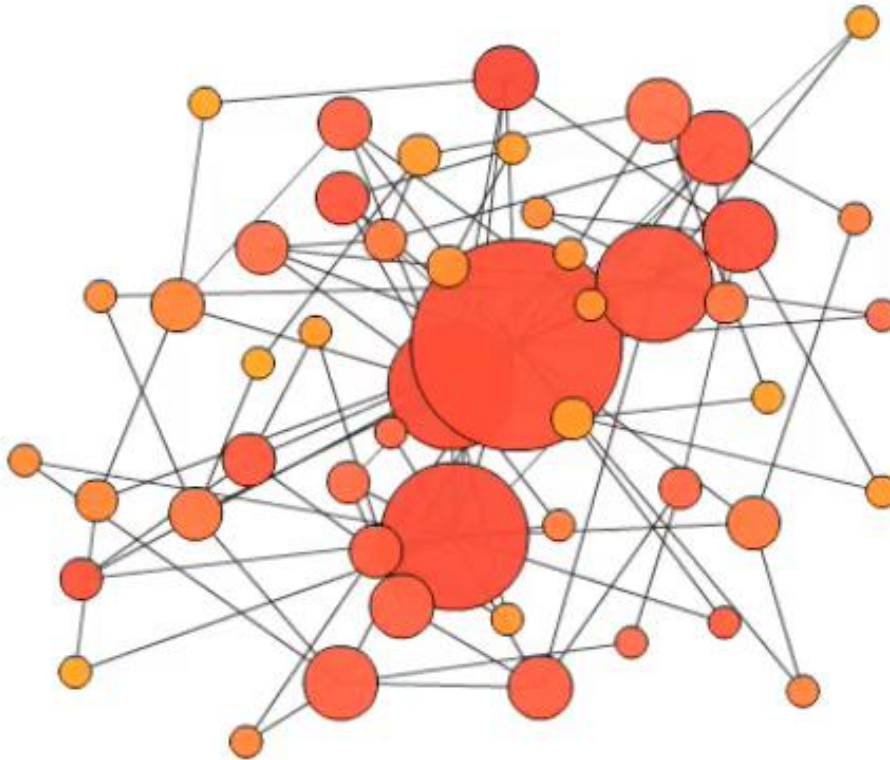
Barabasi&Albert.
Science 1999.

Why should we care about the shape of our social networks?

The structure of networks impacts its **resilience** to attack, the capacity to spread ideas and information, and other dynamical processes

Network resilience

Scale-free networks are robust against random attacks



Network resilience

Scale-free networks are vulnerable against targeted attacks

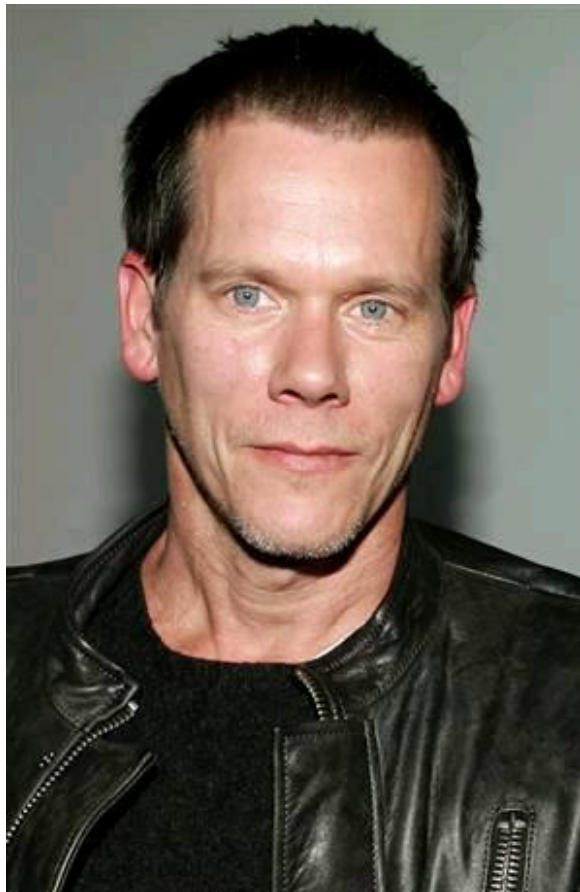
Demo: <http://networksciencebook.com/images/ch-08/video-8-2.webm>

Part 2

SHAPE OF OUR SOCIAL WORLD: SMALL WORLD

The Bacon Number

www.oracleofbacon.org



A screenshot of the IMDb (Internet Movie Database) page for actor Kevin Bacon. The page layout includes a top navigation bar with links like 'NOW PLAYING', 'MOVIE/TV NEWS', 'MY MOVIES', 'NEW ON DVD', 'IMDb TV', 'MESSAGE BOARDS', 'SHOWTIMES & TICKETS', and 'IMDb pro'. Below this is a search bar and a list of links: 'Home', 'Top Movies', 'Photos', 'Independent Film', 'GameBase', 'Browse', and 'Help'. The main header for Kevin Bacon's page features his name, a 'photos' link, a 'board' link, a 'contact' link, and an 'IMDb PRO details' link. There is a 'Shop Kevin Bacon' banner for Amazon.com. The 'Overview' section shows his 'Date of Birth' as 8 July 1958 in Philadelphia, Pennsylvania, USA, with a 'more' link. The 'Mini Biography' section states that Kevin Bacon's early training as an actor came from The Manning Street. The 'Trivia' section mentions his line, "I am a G-damn genius," is quoted in both "Hollow Man" (2000) and "more". The 'Awards' section notes he was nominated for a Golden Globe and has 8 wins and 7 nominations, with a 'more' link. The 'Alternate Names' section lists 'The Bacon Brothers / Kevin Bacon III / the Bacon Brothers'. The 'Filmography' section includes a 'Jump to filmography as:' list with links for Actor, Director, Producer, Soundtrack, Thanks, Self, and Archive Footage. A list of films follows, including 'Taking Chance' (2008), 'Frost/Nixon' (2008), 'Saving Angelo' (2007), 'Rails & Ties' (2007), 'Death Sentence' (2007), 'The Air I Breathe' (2007), 'Where the Truth Lies' (2005), and 'Beauty Shop' (2005). On the left side of the page, there are sections for 'Quicklinks' (categorized), 'Top Links' (biography, by votes, awards, news articles, message board), 'Filmographies' (categorized, by type, by year, by ratings, by votes, by TV series, awards, titles for sale, by genre, by keyword, power search, credited with, tv schedule), and 'Biographical' (biography, other works, publicity, contact information).

The Bacon Number

[Watts 2002]

TABLE 3.1 DISTRIBUTION OF ACTORS ACCORDING
TO BACON NUMBER

BACON NUMBER	NUMBER OF ACTORS	CUMULATIVE TOTAL NUMBER OF ACTORS
0	1	1
1	1,550	1,551
2	121,661	123,212
3	310,365	433,577
4	71,516	504,733
5	5,314	510,047
6	652	510,699
7	90	510,789
8	38	510,827
9	1	510,828
10	1	510,829

The Erdős Number

Paul Erdős was a famous Hungarian Mathematician, 1913-1996.

Erdős posed and solved problems in number theory and other areas and founded the field of discrete mathematics.

- 511 co-authors (Erdős number 1)
- ~ 1500 Publications

The Erdős Number

The Erdős Number:

Through how many research collaboration links is an arbitrary scientist connected to Paul Erdős?

More generally, how many handshakes away are two researchers? There is a link if they write a paper together (co-authorship link)

What is Erdős Number of your favorite professor to Tim Berners-Lee? <https://www.csauthors.net/distance/>

me -> -> P. Erdős ?

Stanley Milgram coined the term Small World

- A social psychologist
- Yale and Harvard University
- Study on the Small World Problem,
**beyond well defined communities
and relations**
(such as actors, scientists, ...)

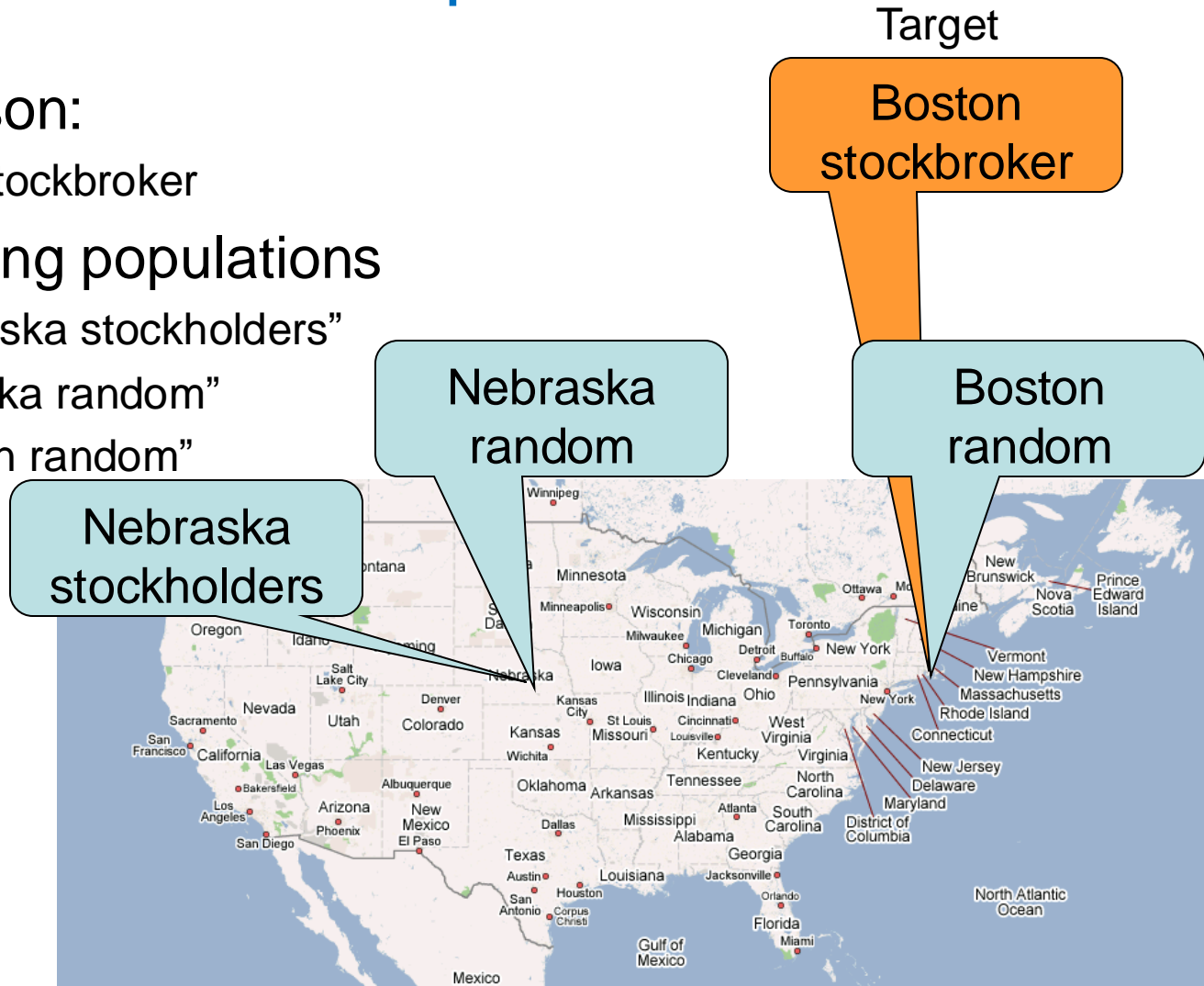


1933-1984

- Milgram is famous for the obedience Study
- What we will discuss today:
„An Experimental Study of the Small World Problem”

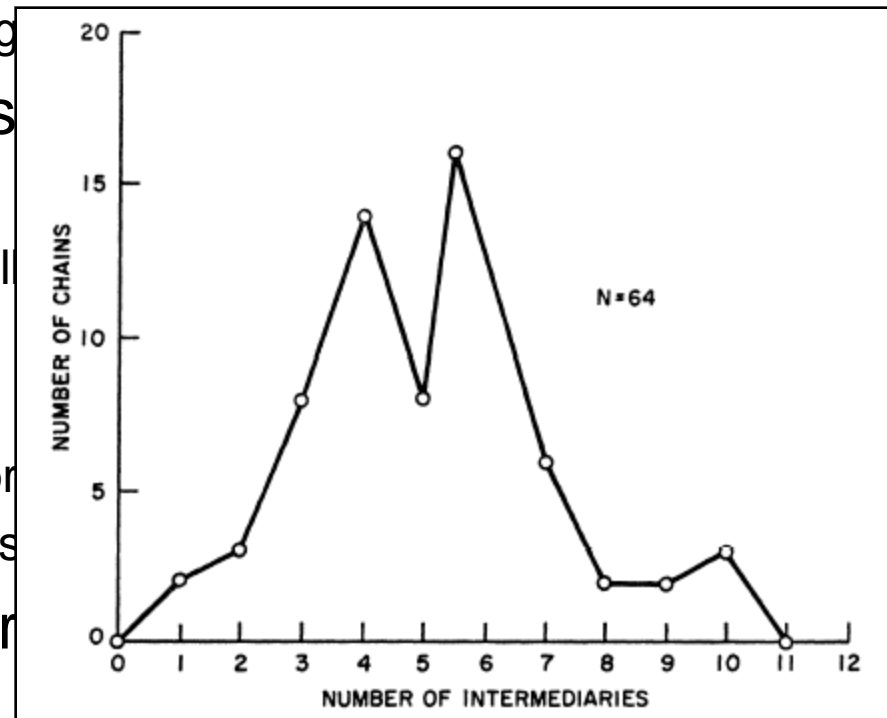
Set Up

- Target person:
 - A Boston stockbroker
- Three starting populations
 - 100 “Nebraska stockholders”
 - 96 “Nebraska random”
 - 100 “Boston random”



Results I

- How many of the starters would be able to establish contact with the target?
 - 64 out of 296 reached the target
- How many intermediaries starters with the target?
 - Well, that depends: the overall
 - Through hometown: 6.1 links
 - Through business: 4.6 links
 - Boston group faster than Nebraska
 - Nebraska stockholders not fast
- What form would the distribution take?



What does it mean?

- Imagine you know on average 100 people very well with their first name. everyone on average know 100 persons very well. How many people are you connected to with 3 handshakes?

Small Worlds

<http://www.infosci.cornell.edu/courses/info204/2007sp/>

- Every pair of nodes in a graph is connected by a path with an extremely small number of steps
Social networks have a **low diameter**.
- Question 1: Why social networks are small worlds?
What **social mechanisms** create such small-world phenomena?

Small Worlds

<http://www.infosci.cornell.edu/courses/info204/2007sp/>

- Every pair of nodes in a graph is connected by a path with an extremely small number of steps (low diameter)
- Question 1: Why social networks are small worlds? What **social mechanisms** create such small world phenomena?
- Question 2: **Why is it important** to understand the Small World properties of our social networks? What does it mean?

Formalizing the Small World Problem

[Watts and Strogatz 1998]

The small-world phenomenon is assumed to be present when local clustering is high and average path lengths are relatively small

average path length: L

clustering coefficient: C

Examples for Small World Networks

[Watts and Strogatz 1998]

Table 1 Empirical examples of small-world networks

$L > L_{\text{random}}$ but $C \gg C_{\text{random}}$	L_{actual}	L_{random}	C_{actual}	C_{random}
Film actors	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.080	0.005
<i>C. elegans</i>	2.65	2.25	0.28	0.05

Characteristic path length L and clustering coefficient C for three real networks, compared to random graphs with the same number of vertices (n) and average number of edges per vertex (k). (Actors: $n = 225,226$, $k = 61$. Power grid: $n = 4,941$, $k = 2.67$. *C. elegans*: $n = 282$, $k = 14$.) The graphs are defined as follows. Two actors are joined by an edge if they have acted in a film together. We restrict attention to the giant connected component¹⁶ of this graph, which includes $\sim 90\%$ of all actors listed in the Internet Movie Database (available at <http://us.imdb.com>), as of April 1997. For the power grid, vertices represent generators, transformers and substations, and edges represent high-voltage transmission lines between them. For *C. elegans*, an edge joins two neurons if they are connected by either a synapse or a gap junction. We treat all edges as undirected and unweighted, and all vertices as identical, recognizing that these are crude approximations. **All three networks show the small-world phenomenon: $L \gtrsim L_{\text{random}}$ but $C \gg C_{\text{random}}$.**

Formalizing the Small World Problem

[Watts and Strogatz 1998]

The small-world phenomenon is assumed to be present when

$$L \sim L_{\text{random}} \text{ but } C \gg C_{\text{random}}$$

Or in other words: We are looking for networks where local clustering is high and global path lengths are small

What's the rationale for the above formalism?

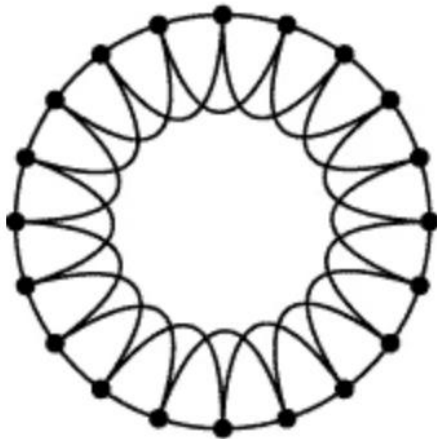
One potential answer:

Cavemen and Solaris Worlds

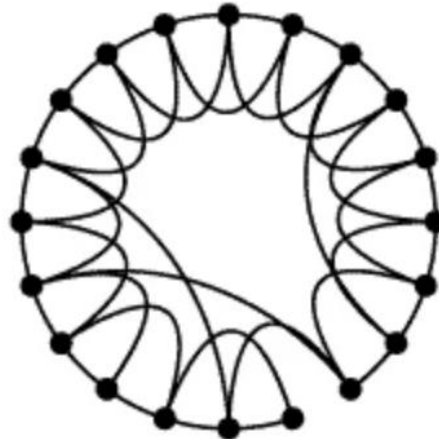
Formalizing the Small World Problem

[Watts and Strogatz, Nature 1998]

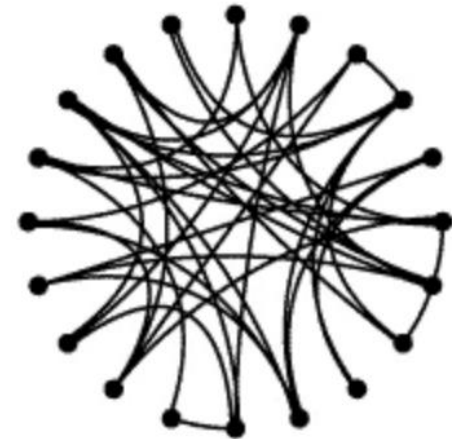
Regular



Small-world



Random



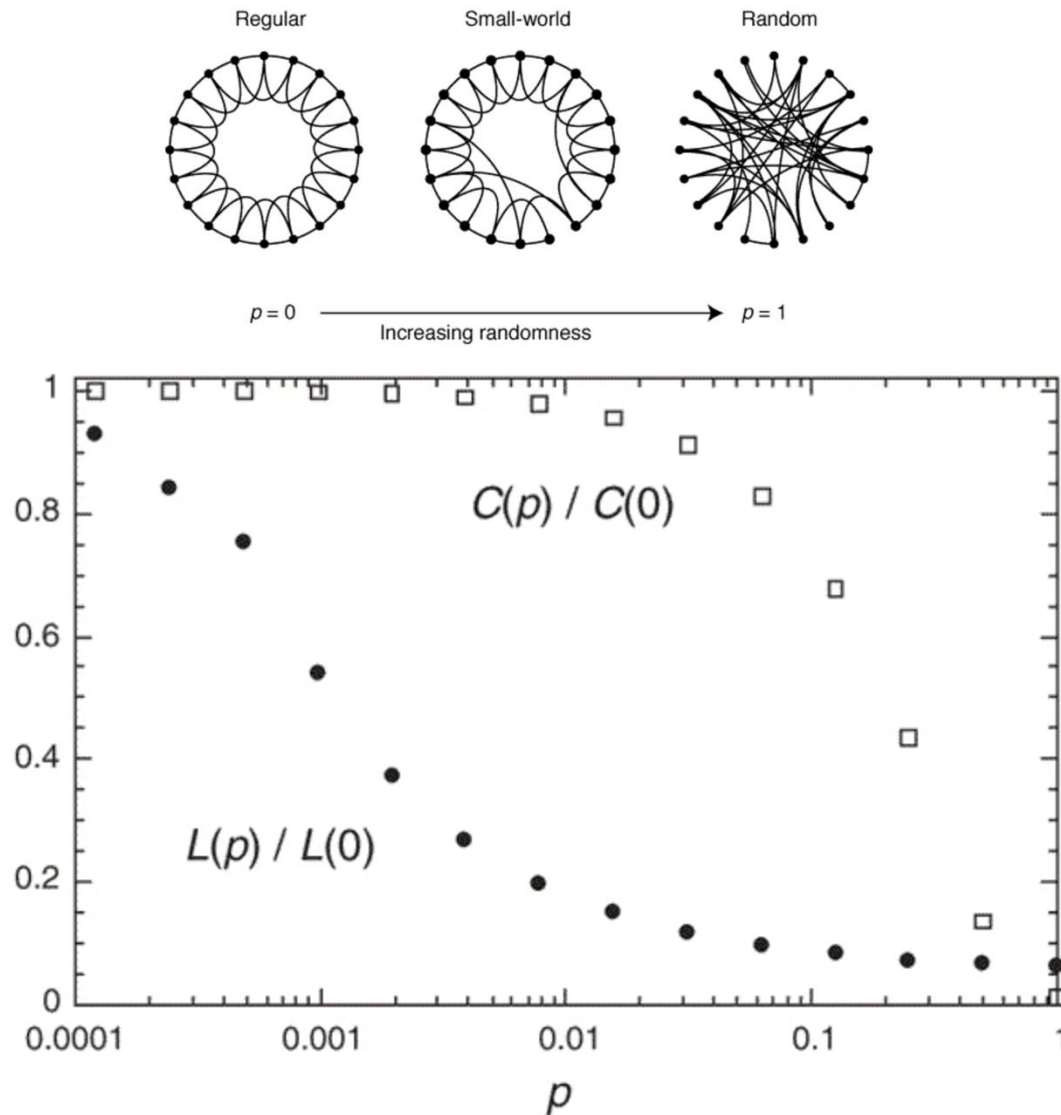
$p = 0$

Increasing randomness

$p = 1$

Formalizing the Small World Problem

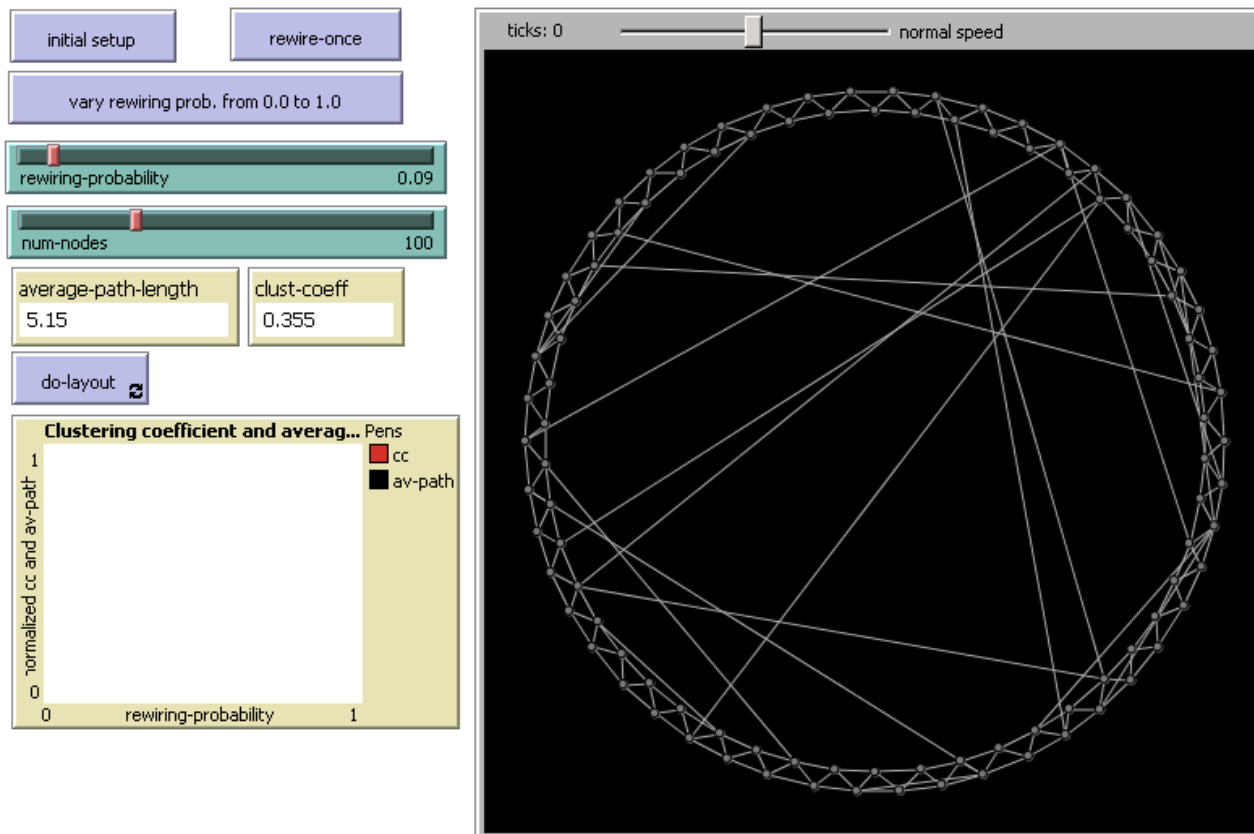
[Watts and Strogatz, Nature 1998]



Demo – Small Worlds

<http://www.netlogoweb.org/launch>

Watts Strogatz Small World Model



Contemporary Software

- Where does the small-world phenomenon come into play in contemporary software, in organizations, ..?
- Xing, LinkedIn, Facebook, ...
- Business Processes, Information and Knowledge Flow

Part 3

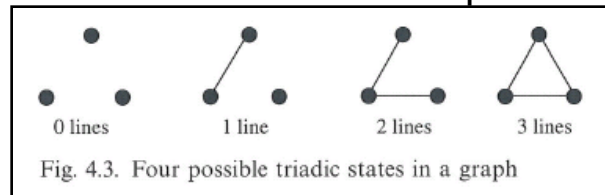
SHAPE OF OUR SOCIAL WORLD: MANY CLUSTERS

Fundamental Concepts in SNA

[Wassermann and Faust 1994]

- Triad

- Def: A subgroup of three actors and the possible ties among them



- **Transitivity**

- If actor i „likes“ j, and j „likes“ k, then i also „likes“ k

- **Balance**

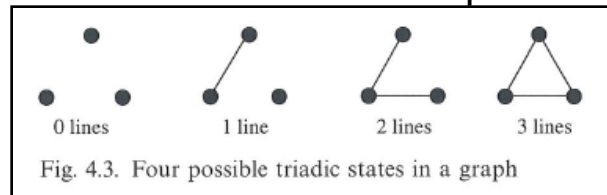
- If actor i and j like each other, they should be similar in their evaluation of some k
- If actor i and j dislike each other, they should evaluate k differently

Fundamental Concepts in SNA

[Wassermann and Faust 1994]

- Triad

- Def: A subgroup of three actors and the possible ties among them

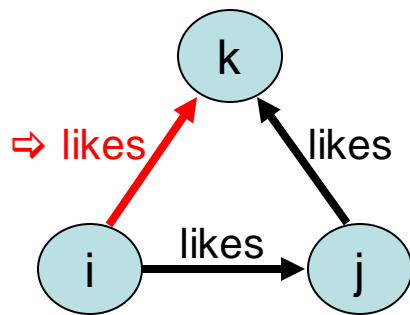


- **Transitivity**

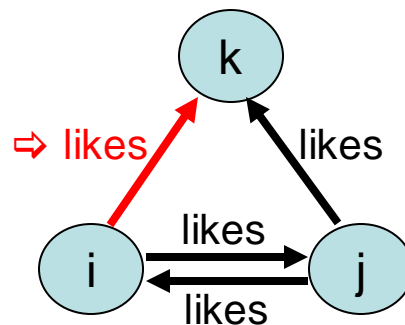
- If actor i „likes“ j, and j „likes“ k, **then i also „likes“ k**

- **Balance**

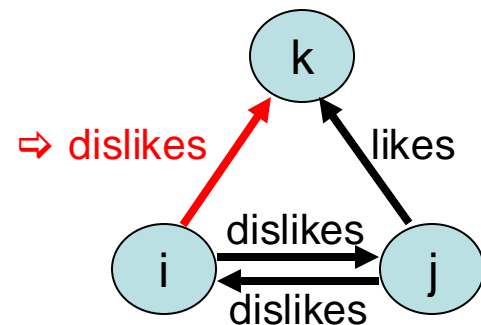
- If actor i and j like each other, they should be similar in their evaluation of some k
- If actor i and j dislike each other, they should evaluate k differently



Example 1: Transitivity



Example 2: Balance

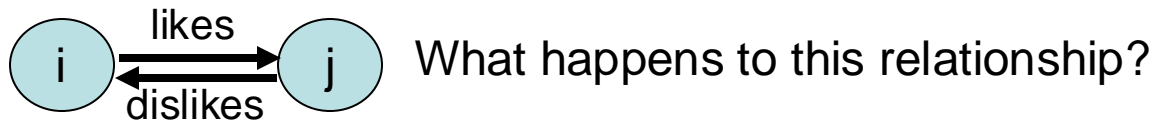


Example 3: Balance

Fundamental Concepts in SNA

[Wassermann and Faust 1994]

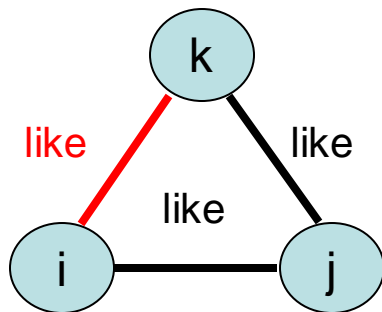
- **Maintaining social ties**



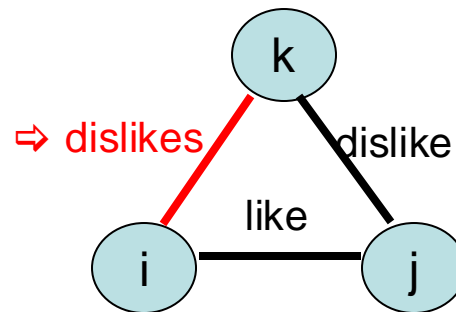
Fundamental Concepts in SNA

[Wassermann and Faust 1994]

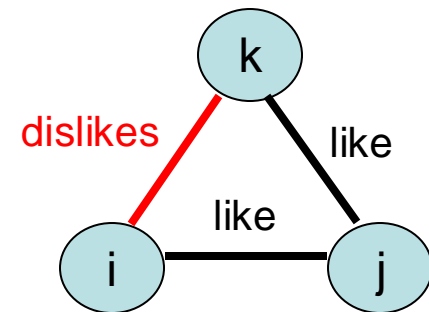
- **What happens to these relationships?**
- **Balance theory**
 - A social triangle is balanced when the cognitive capacity to maintain it is low



Example 1: balance?

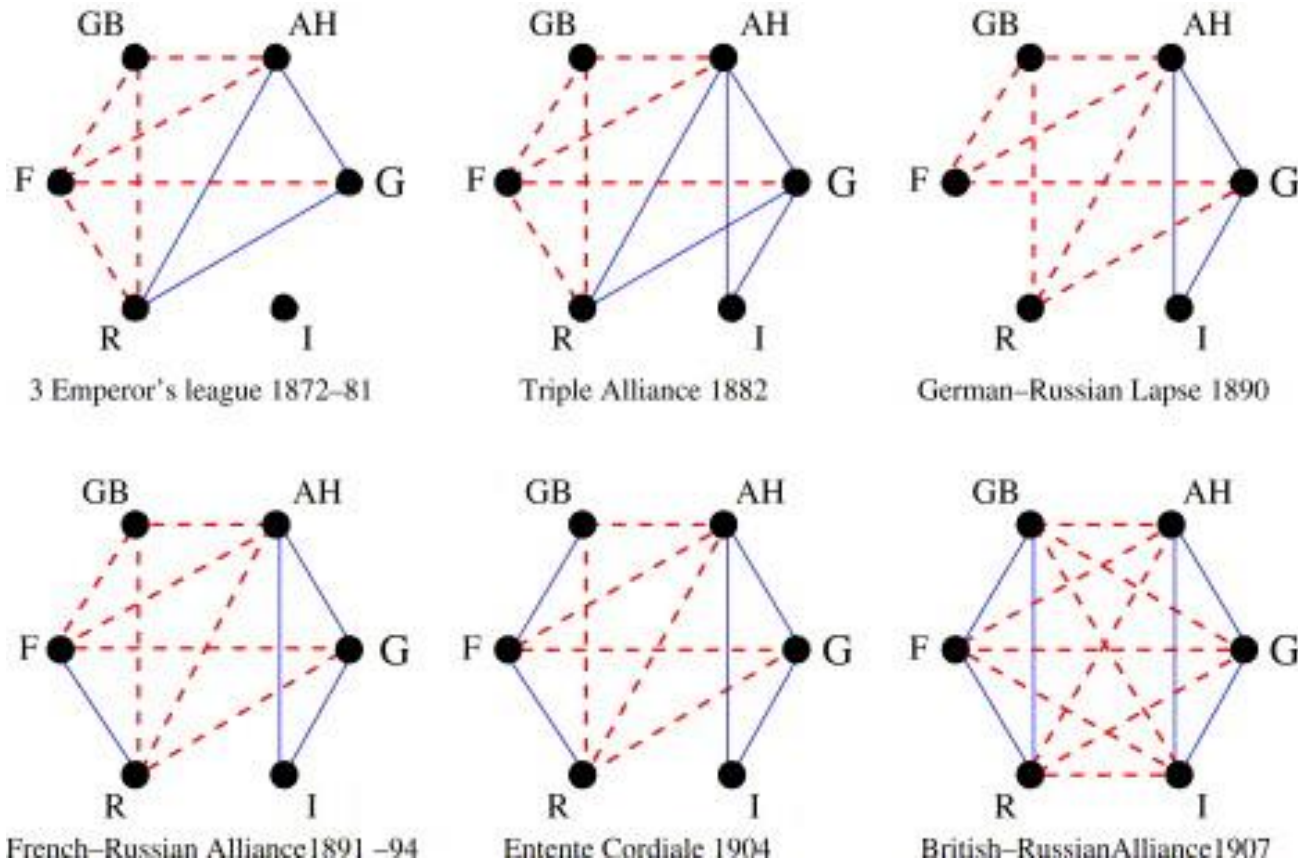


Example 2: balance?



Example 3: balance?

Balance theory and conflict



Evolution of the major relationship changes between the protagonists of World War I from 1872–1907. Here GB=Great Britain, AH=Austria–Hungary, G=Germany, I=Italy, R=Russia, and F=France. Antal et al, Physica D (2006).

Part 4

SHAPE OF OUR SOCIAL NETWORKS: ASSORTATIVE

Assortative Mixing

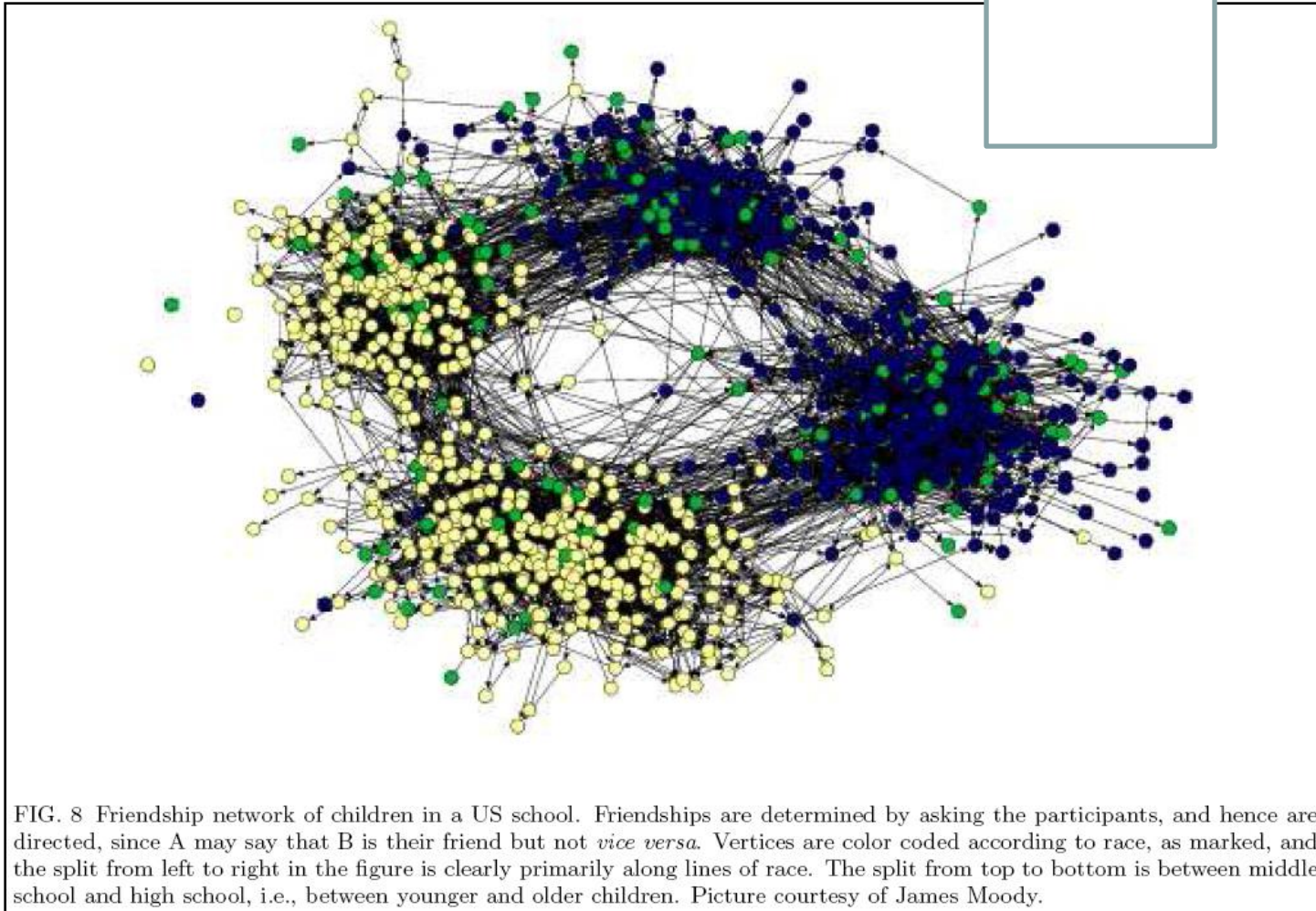
[Newman 2003]

Assortative Mixing refers to selective linking of nodes to other nodes who share some common property

- Topological example: **Degree assortativity**
high degree nodes in a network associate preferentially with other high-degree nodes
- Social example: **Homophily**
nodes of a certain type tend to associate with the same type of nodes (e.g. by race)

Homophily and friendship in high school

[Moody 2001]



Why understanding assortativity/homophily is important in social networks?

- Degree assortativity can affect the spreading dynamic
- Homophily affects the adoption of norms and culture

Part 5

MECHANISTIC MODELS: BA-HOMOPHILY MODEL EXAMPLE

Why modeling networks?

Why simple rules in modeling?

Occam's razor

is the problem-solving principle that recommends searching for explanations constructed with the smallest possible set of elements. It is also known as the **principle of parsimony**.

Mechanistic models of Networks

Quantitative social science is not only about regression analysis or, in general, data inference. Computer simulations of social mechanisms have an over 60 years long history. They have been used for many different purposes—to test scenarios, to test the consistency of descriptive theories (proof-of-concept models), to explore emergent phenomena, for forecasting, etc... (Holme and Liljeros. "Mechanistic models in computational social science." *Frontiers in Physics* (2015))

Mechanistic models of Networks

Quantitative social science is not only about regression analysis or, in general, data inference. Computer simulations of social mechanisms have an over 60 years long history. They have been used for many different purposes—to test scenarios, to test the consistency of descriptive theories (proof-of-concept models), to explore emergent phenomena, for forecasting, etc... (Holme and Liljeros. "Mechanistic models in computational social science." *Frontiers in Physics* (2015))

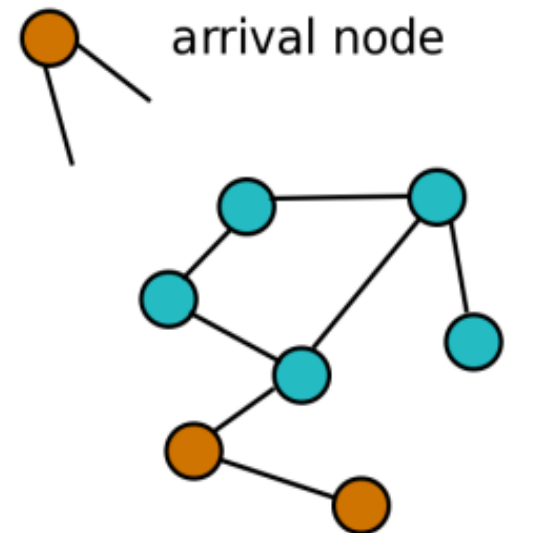
Examples of Mechanistic models in social networks:

Preferential attachment models, fitness model, homophily models are mechanistic models in which they use mechanisms to generate networks in order to:

- Understand causality,
- micro to macro behaviour,
- analytically tractable

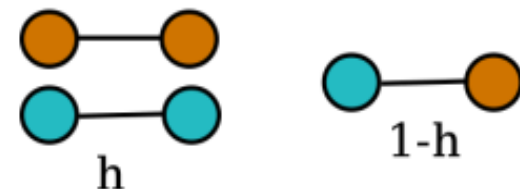
Network growth model with Preferential Attachment (BA) and tunable homophily

- 2 group of nodes with unequal size
- Arrival node connects to existing nodes based on preferential attachment homophily



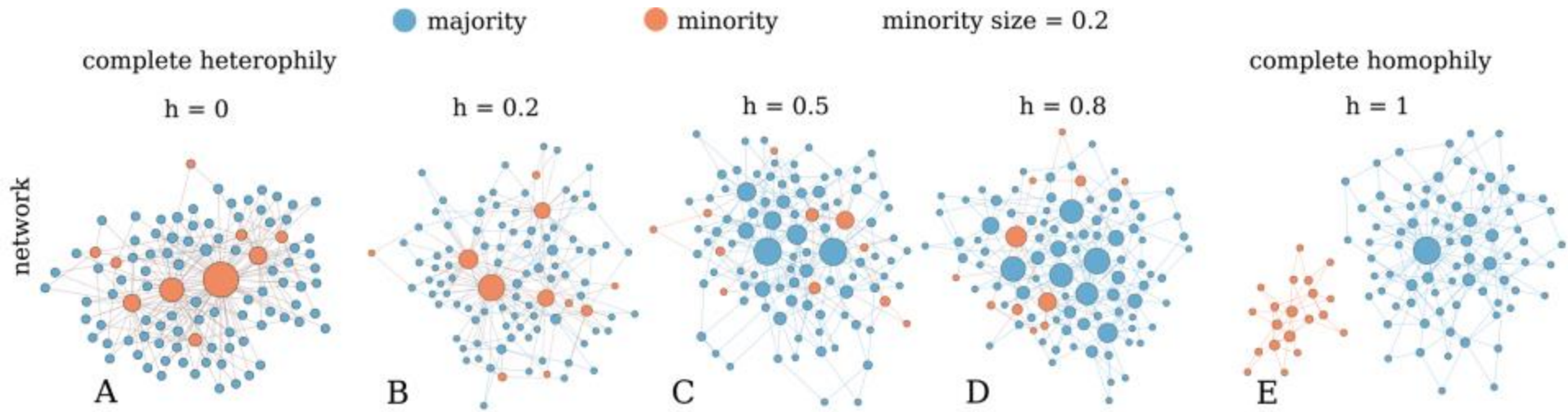
$$p_{\text{connect}} \sim h \cdot k$$

$$0 \leq \text{homophily } (h) \leq 1$$

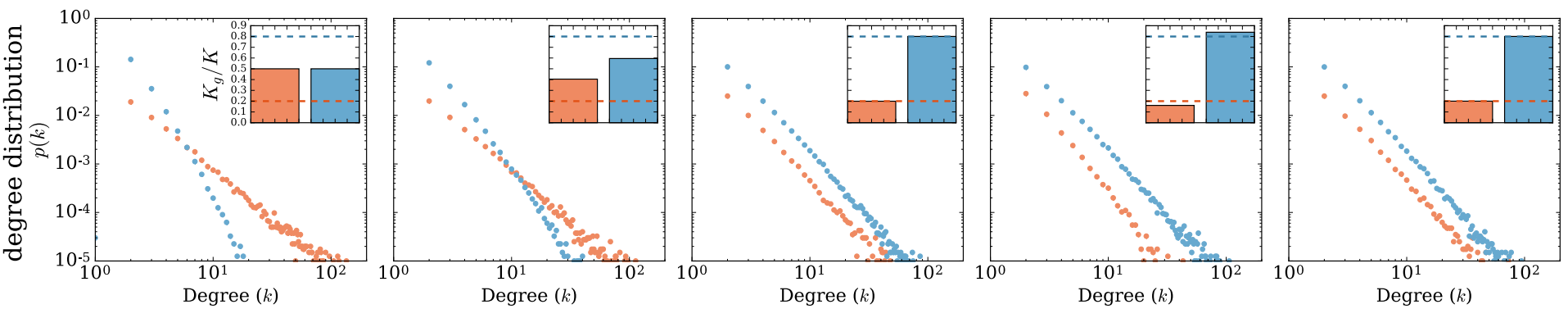


BA-Homophily network model

Karimi (2018)



Degree distribution



Part 6

SOCIAL TIES AND THEIR MEANINGS

In reality ...

Isn't all of this an over simplification of the world of social systems?

- Ties/relationships vary in intensity
- People who have strong ties tend to share a similiar set of acquaintances
- Ties change over time
- Nodes (people) have different characteristics, and they are *actors*
- ...

The Strength of Weak Ties

[Granovetter 1973]

The strength of an interpersonal tie is a

- (probably linear) combination of the amount of time
- The emotional intensity
- The intimacy
- The reciprocal services which characterize the tie



Mark Granovetter,
Stanford University

Can you give examples of strong / weak ties?

The Strength of Weak Ties and Mutual Acquaintances [Granovetter 1973]

Consider:

Two arbitrarily selected individuals A and B and

The set $S = [C, D, E, \dots]$ of all persons with ties to either or both of them

Hypothesis:

The stronger the tie between A and B, what happens to the set S?

The Strength of Weak Ties and Mutual Acquaintances [Granovetter 1973]

Consider:

Two arbitrarily selected individuals A and B and

The set $S = [C, D, E, \dots]$ of all persons with ties to either or both of them

Hypothesis:

The stronger the tie between A and B, the larger the proportion of individuals in S to whom they will both be tied.

Theoretical corroboration:

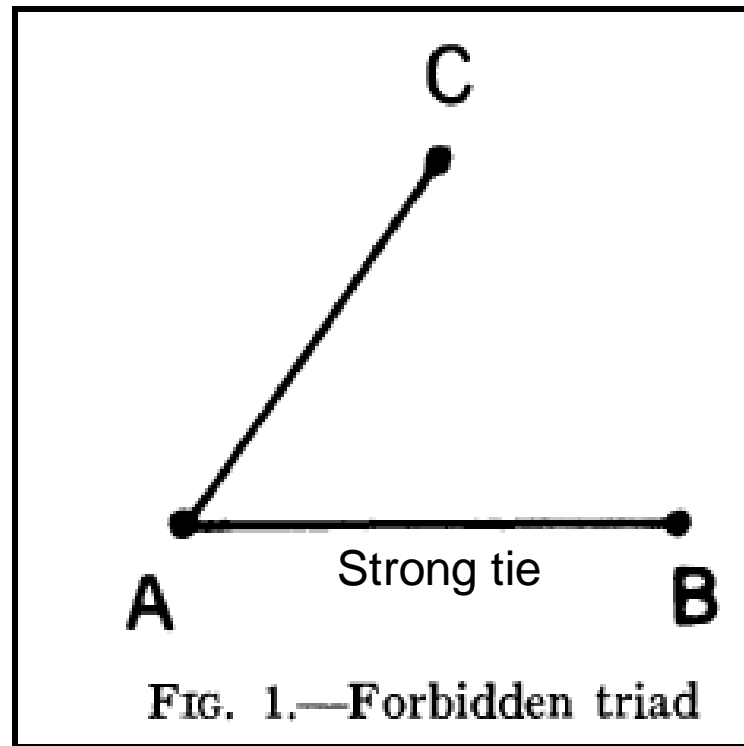
Stronger ties involve larger time commitments – probability of B meeting with some friend of A (who B does not know yet) is increased

The stronger a tie connecting two individuals, the more similar they are

The Strength of Weak Ties

[Granovetter 1973]

The forbidden triad



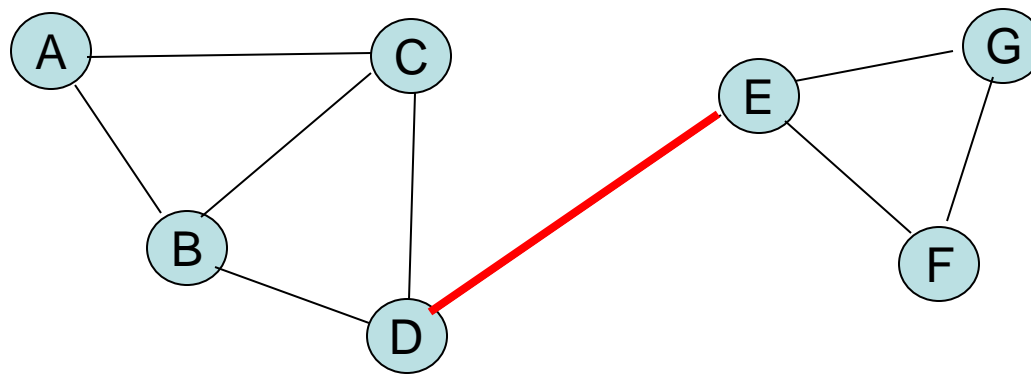
Why is it called the forbidden triad?

Bridges

[Granovetter 1973]

A bridge is a line in a network which provides **the only path** between two points.

In social networks, a bridge between A and B provides the only route along which information or influence can flow from any contact of A to any contact of B



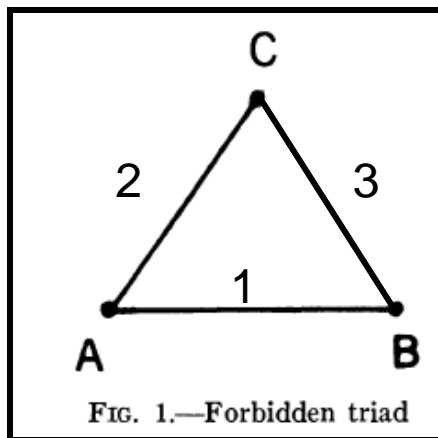
Which edge
represents a
bridge?
Why?

Bridges and Strong Ties

[Granovetter 1973]

Example:

1. Imagine the strong tie between A and B
 2. Imagine the strong tie between A and C
 3. Then, the forbidden triad **implies** that a tie **exists** between C and B
(it forbids that a tie between C and B does not exist)
1. From that follows, that A-B is not a bridge (because there is another path A-B that goes through C)

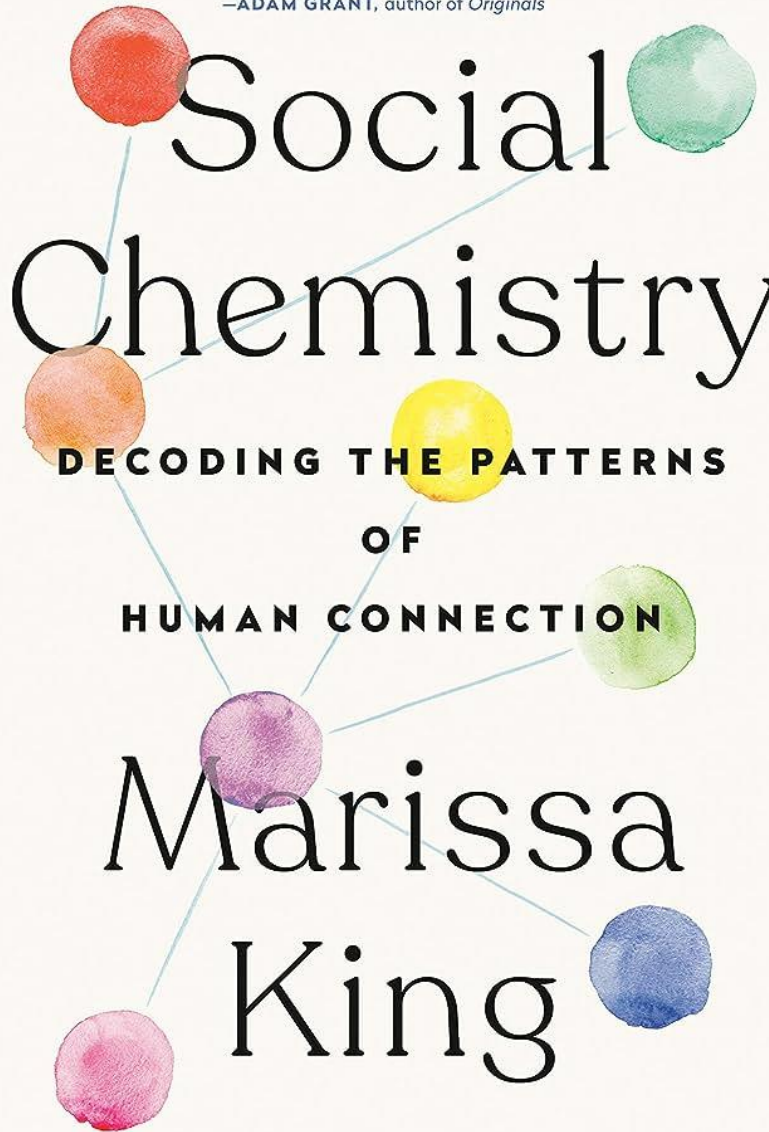


Why is this interesting?

- ⇒ Strong ties can be a bridge ONLY IF neither party to it has any other strong ties
- ⇒ Highly unlikely in a social network of any size
- ⇒ Weak ties suffer no such restriction, though they are not automatically bridges
- ⇒ But, **all bridges are weak ties**

"One of the most interesting and useful books ever written on networking."

—ADAM GRANT, author of *Originals*



Social Chemistry

DECODING THE PATTERNS
OF
HUMAN CONNECTION

Marissa
King

In Reality

[Granovetter 1973]

it probably happens only rarely, that a specific tie provides the only path between two points

Local bridges: the shortest path between its two points (other than itself)

- Bridges are efficient paths
- Alternatives are more costly
- Local bridges of degree n
- A local bridge is more significant as its degree increases

What's the degree of a bridge in an absolute sense?

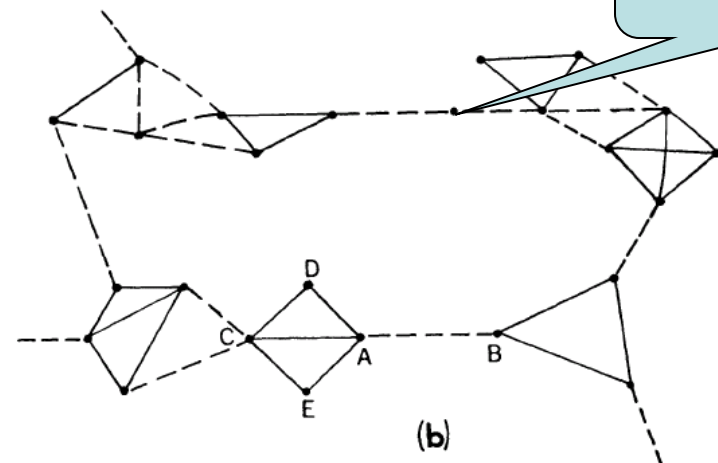
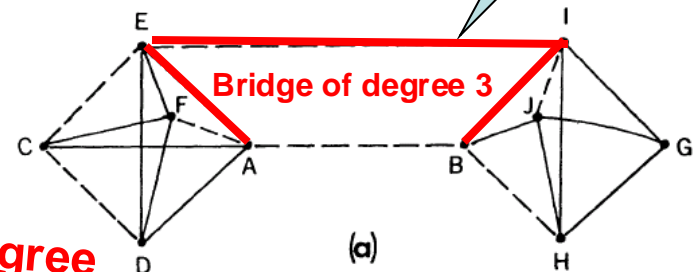
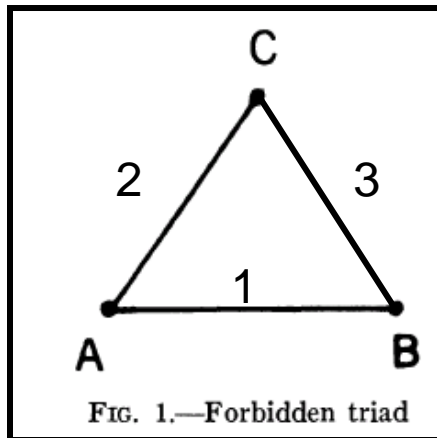


FIG. 2.—Local bridges. *a*, Degree 3; *b*, Degree 13. — = strong tie; --- = weak tie.

In Reality ...

Strong ties can represent *local* bridges BUT
They are weak (i.e. they have a low degree)

Why?



What's the degree of the local bridge
A-B?

Implications of Weak Ties

[Granovetter 1973]

- Those weak ties, that are local bridges, create more, and shorter paths.
- The removal of the average weak tie would do more damage to transmission probabilities than would that of the average strong one
- **Paradox:** While *weak ties* have been denounced as generative of alienation, *strong ties*, breeding local cohesion, lead to overall fragmentation

What are sources
of weak
ties/bridges?

Can you identify some
implications for social
networks on the web / for
search in these networks?

How does this relate to
Milgram's experiment?

Completion rates in Milgram's experiment were reported higher for acquaintance than friend relationships [Granovetter 1973]

Implications of Weak Ties

[Granovetter 1973]

- Example: Spread of information/rumors in social networks
 - Studies have shown that people rarely act on mass-media information unless it is also transmitted through personal ties [Granovetter 2003, p 1274]
 - Information/rumors moving through strong ties is much more likely to be limited to a few cliques than that going via weak ones, bridges will not be crossed

**How does information spread
through weak ties?**

Any questions?

See you next week!