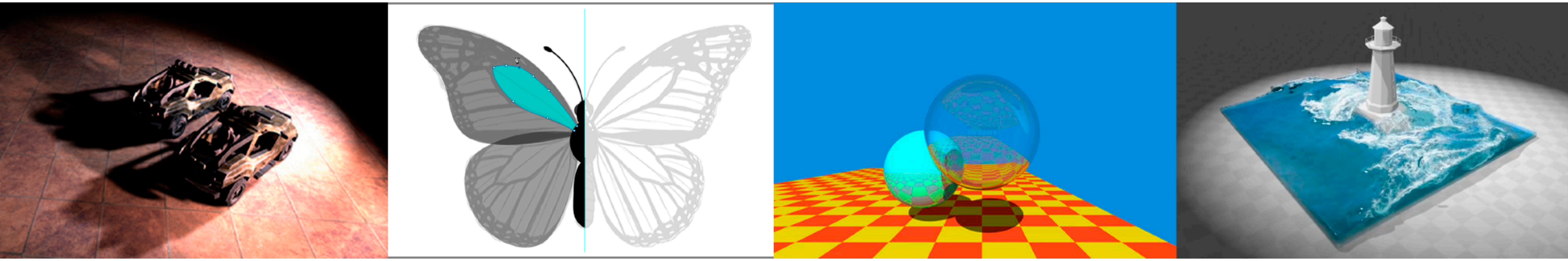
Introduction to Computer Graphics

GAMES101, Lingqi Yan, UC Santa Barbara

Lecture 6:
Rasterization 2
(Antialiasing and Z-Buffering)

反走样　　　　深度缓冲



http://www.cs.ucsb.edu/~lingqi/teaching/games101.html

Announcements

- Homework 1
  - Already 49 submissions so far!
  - In general, start early
- Today's topics are not easy
  - Having knowledge on Signal Processing is appreciated
  - But no worries if you don't

Last Lectures

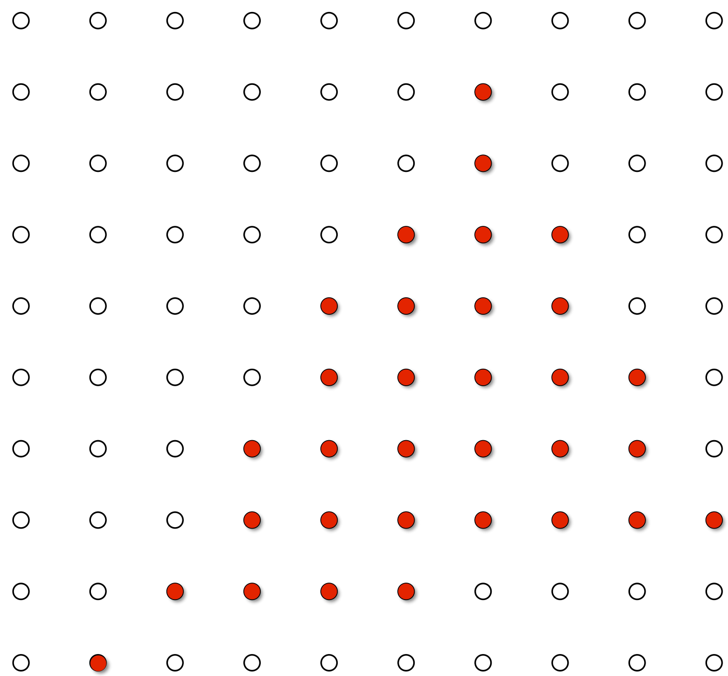
- Viewing
  - View + Projection + Viewport

    视图　投影　视口.
- Rasterizing triangles
  - Point-in-triangle test
  - Aliasing

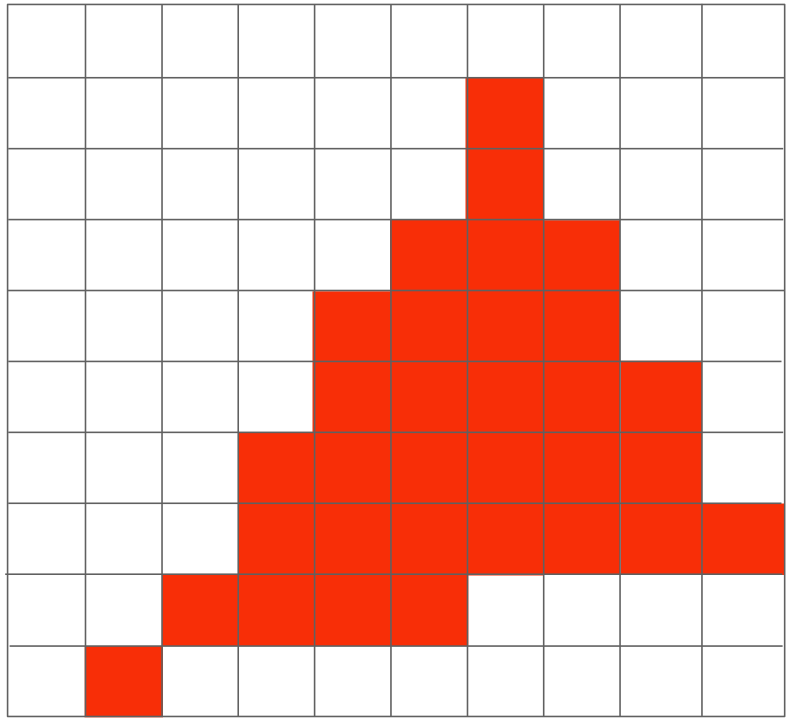
Today

- Antialiasing 反走样 反锯齿.
    - Sampling theory
    - Antialiasing in practice
- Visibility / occlusion
    - Z-buffering

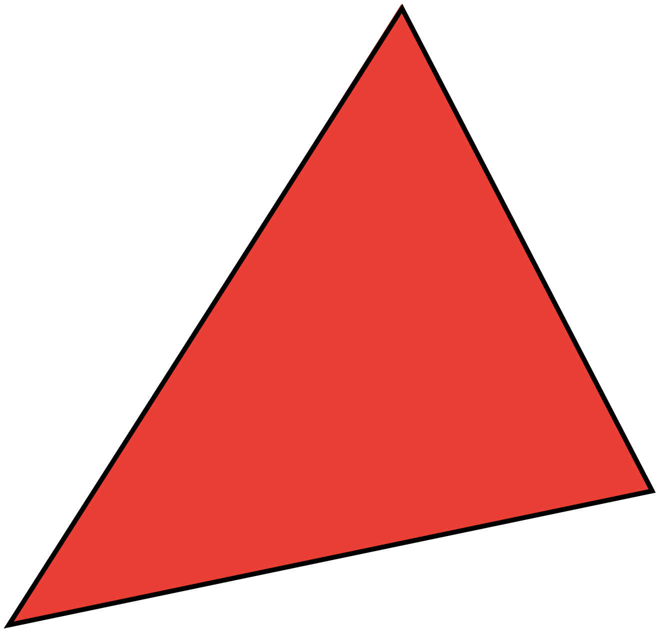
Recap: Testing in/out △ at pixels' centers



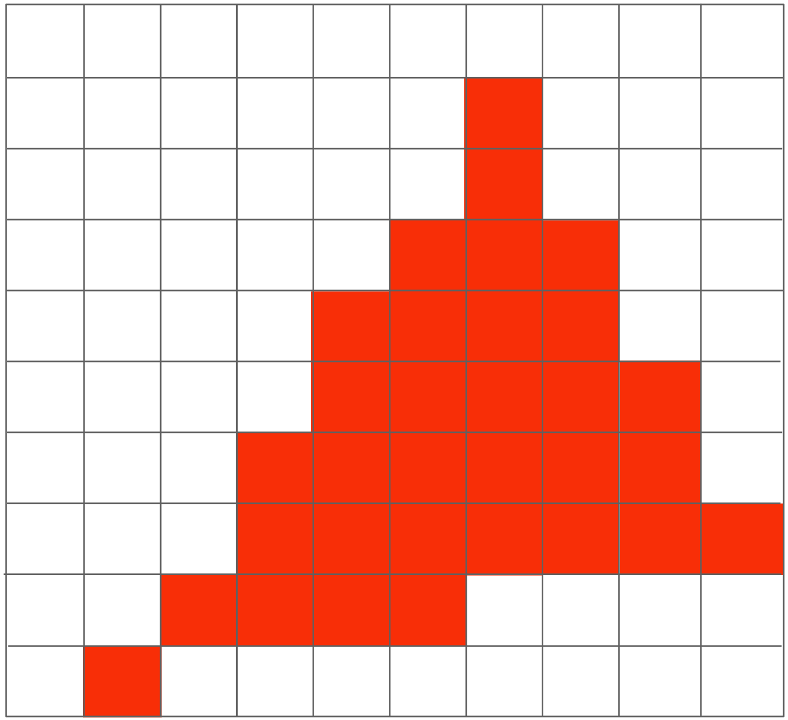
Pixels are uniformly-colored squares



Compare: The Continuous Triangle Function



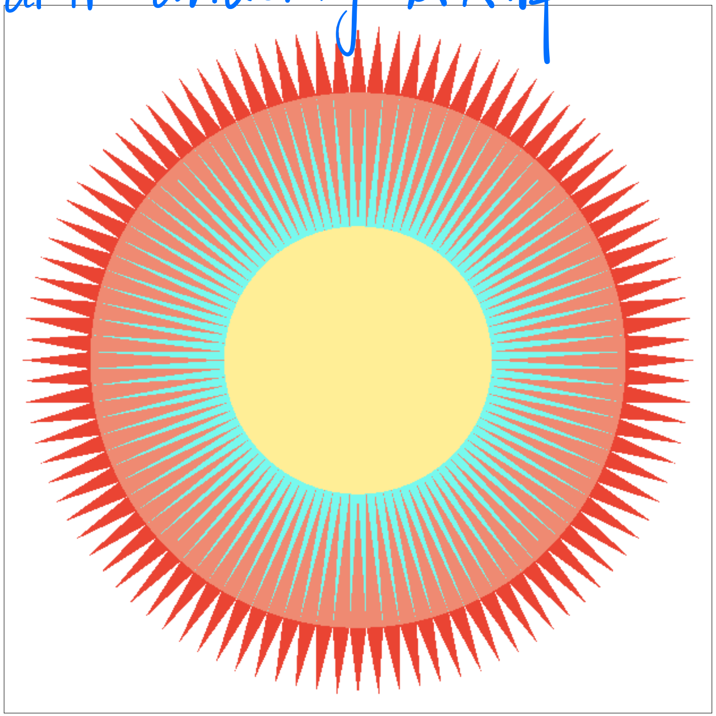
What's Wrong With This Picture?

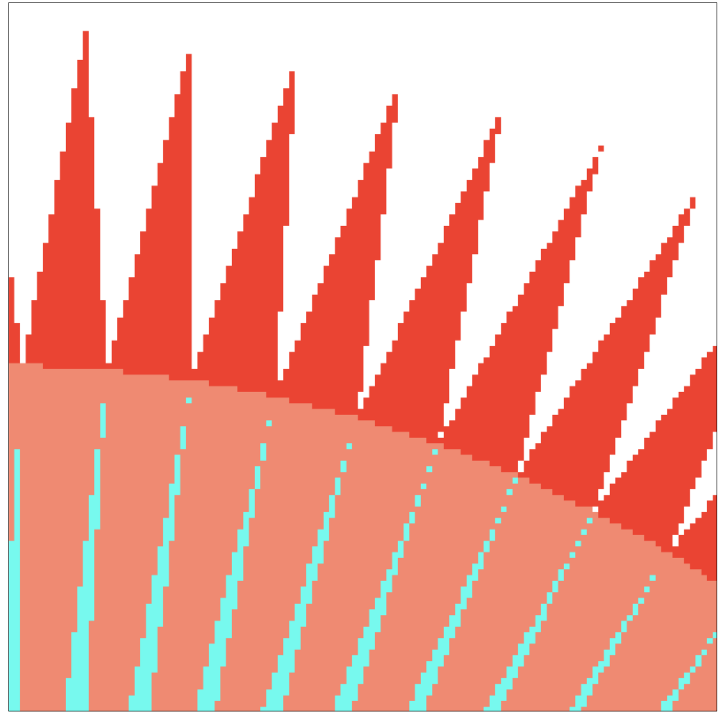


Jaggies!

Aliasing 走样（锯齿）

anti-aliasing 反走样



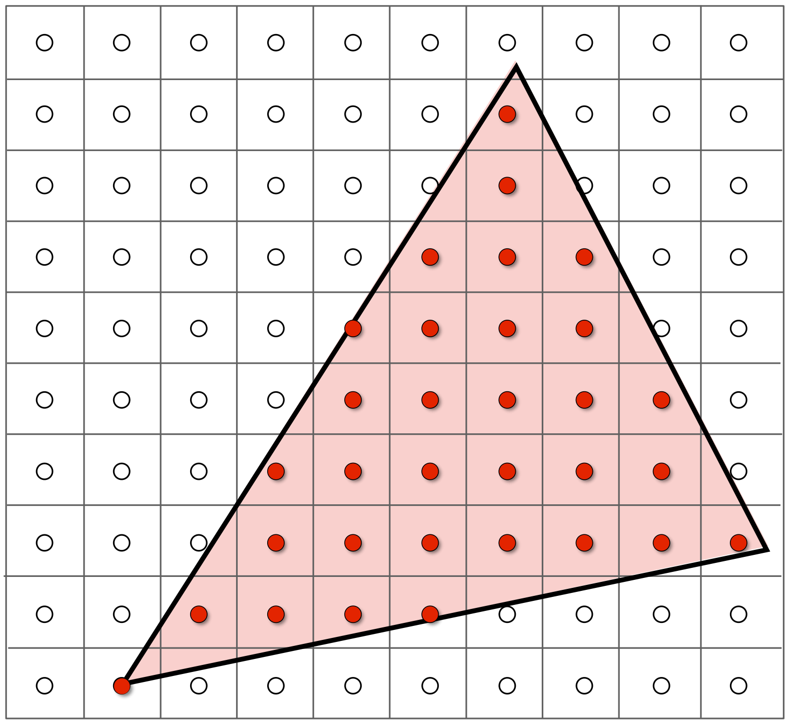


Is this the best we can do?

Slide courtesy of Prof. Ren Ng, UC Berkeley

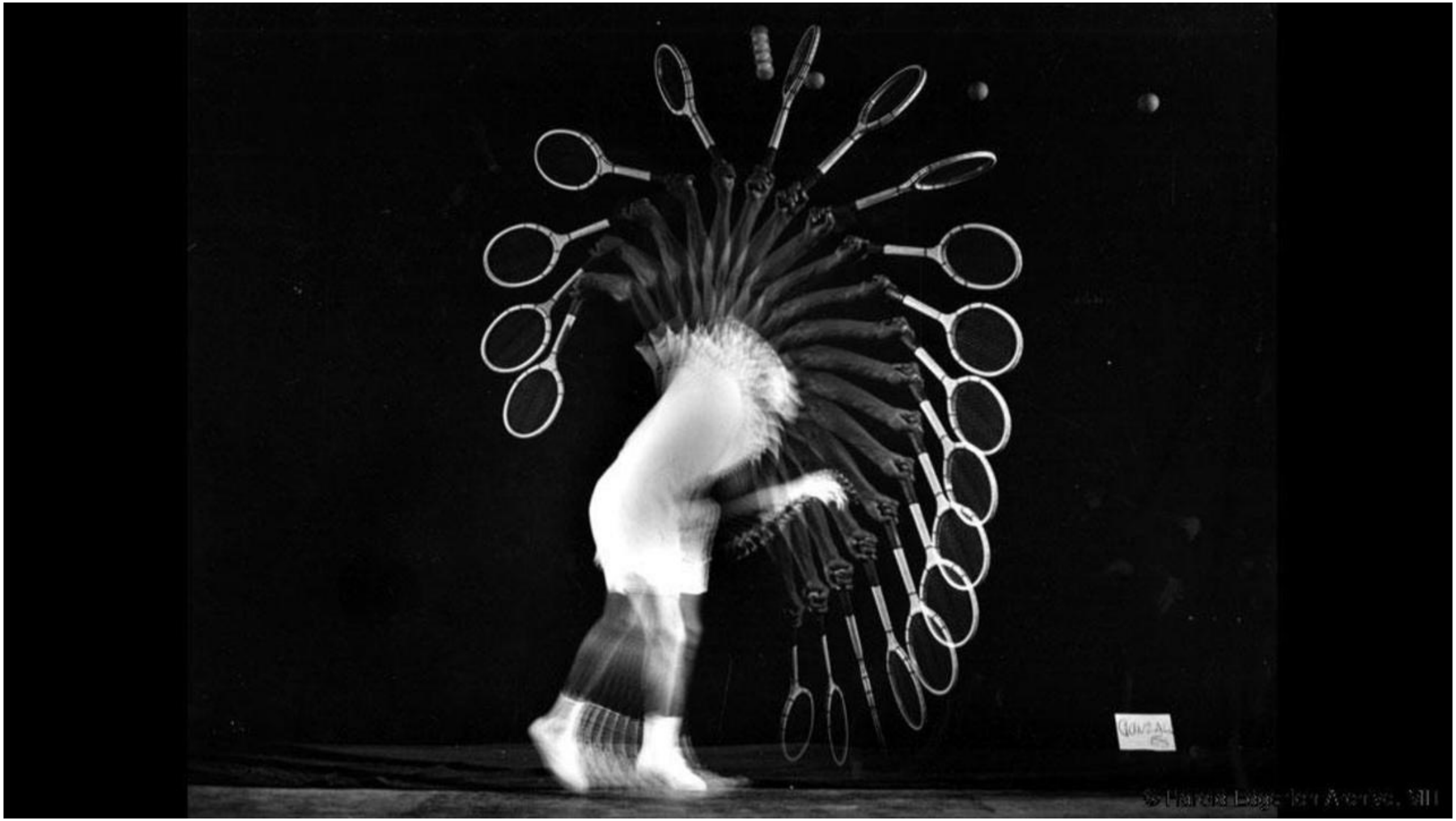
Sampling is Ubiquitous in Computer Graphics

Rasterization = Sample 2D Positions



Photograph = Sample Image Sensor Plane

Video = Sample Time

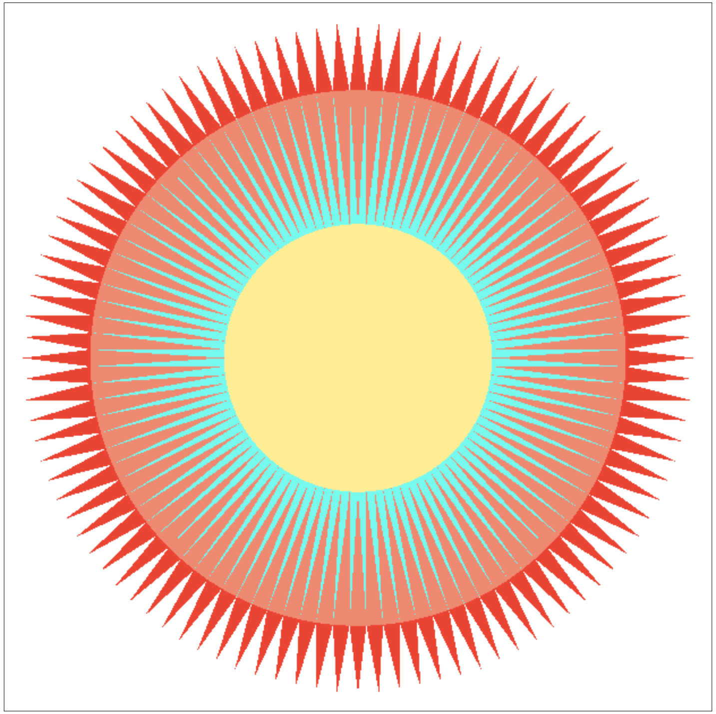


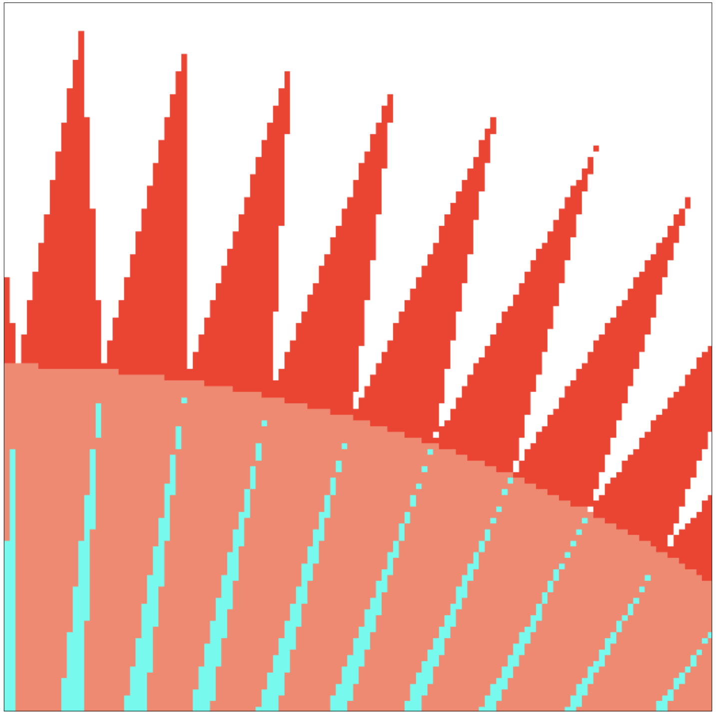
Harold Edgerton Archive, MIT

瑕疵

Sampling Artifacts
(Errors / Mistakes / Inaccuracies) in Computer Graphics

Jaggies (Staircase Pattern)

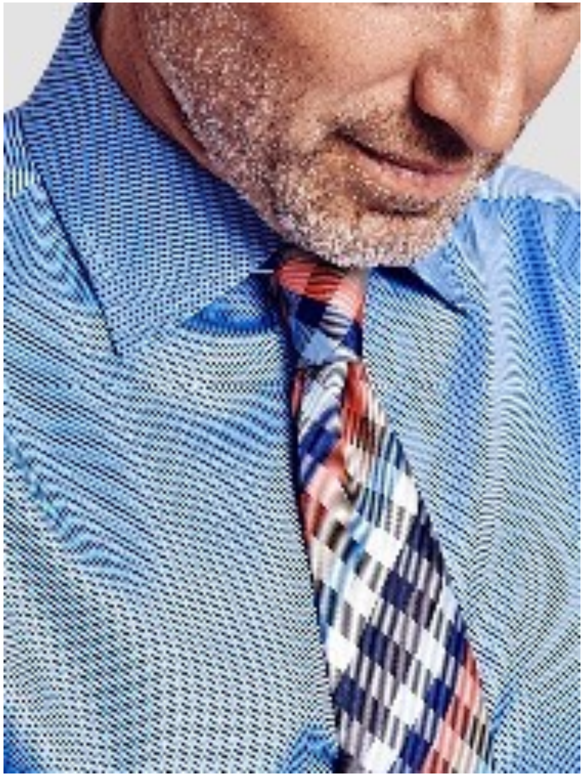




This is also an example of “aliasing” – a sampling error

Moiré Patterns in Imaging

[mwa:]



lystit.com

Skip odd rows and columns

Wagon Wheel Illusion (False Motion)

Sampling Artifacts in Computer Graphics

Artifacts due to sampling - "Aliasing"

- Jaggies – sampling in space
- Moire – undersampling images
- Wagon wheel effect – sampling in time "时间"
- [Many more] …

"空间"采样

Behind the Aliasing Artifacts

- Signals are changing too fast (high frequency), but sampled too slowly

频率高

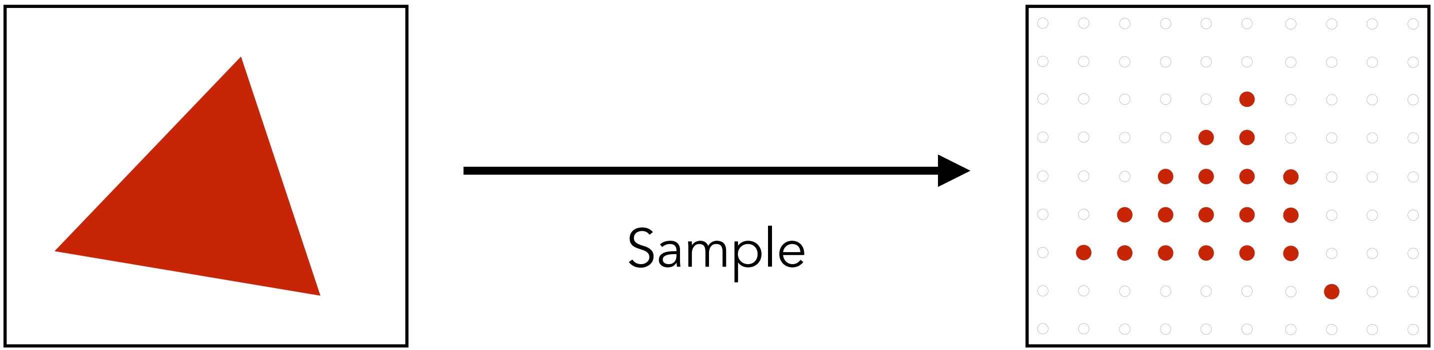
采样太慢

反走样 idea

Antialiasing Idea: Blurring (Pre-Filtering) Before Sampling

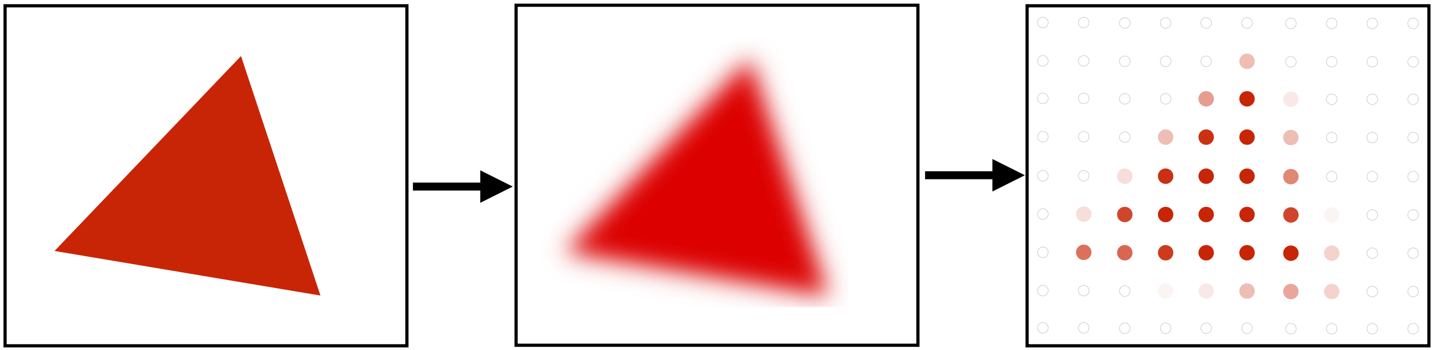
采样之前做滤波(模糊)

Rasterization: Point Sampling in Space



Note jaggies in rasterized triangle
where pixel values are **pure red or white**

Rasterization: Antialiased Sampling



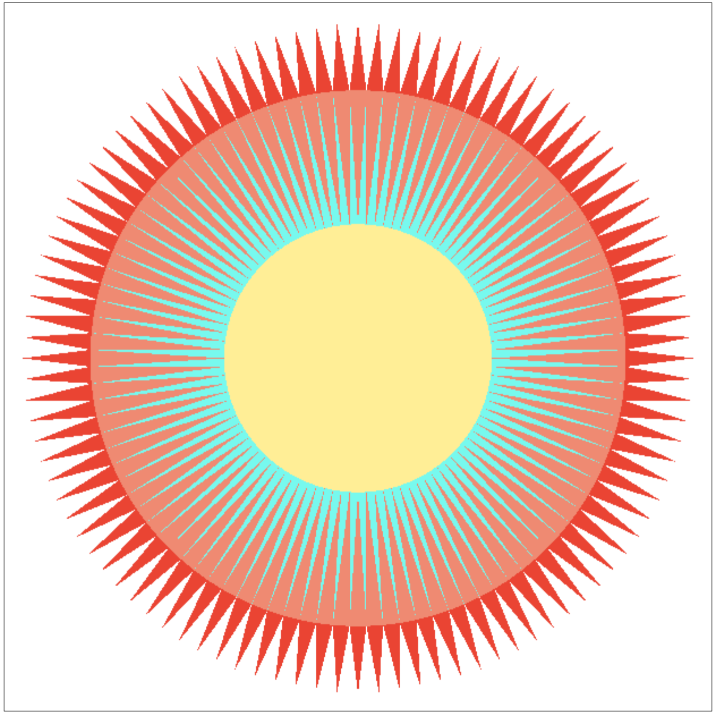
Pre-Filter

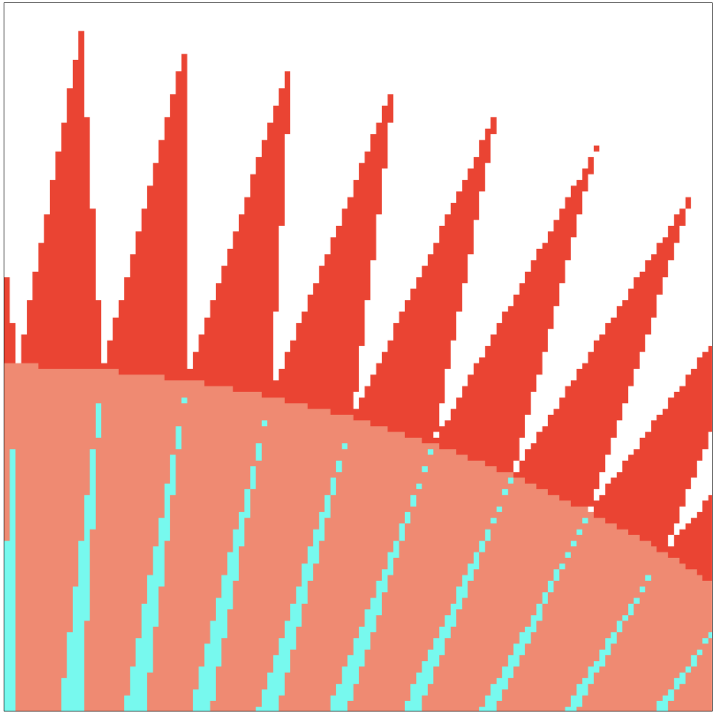
(remove frequencies above Nyquist) (?)

Sample

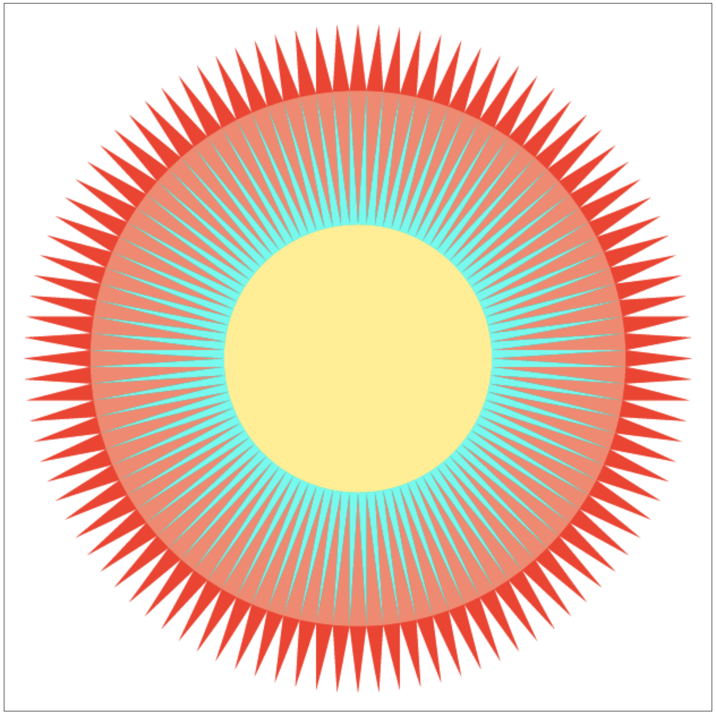
Note antialiased edges in rasterized triangle
where pixel values take intermediate values

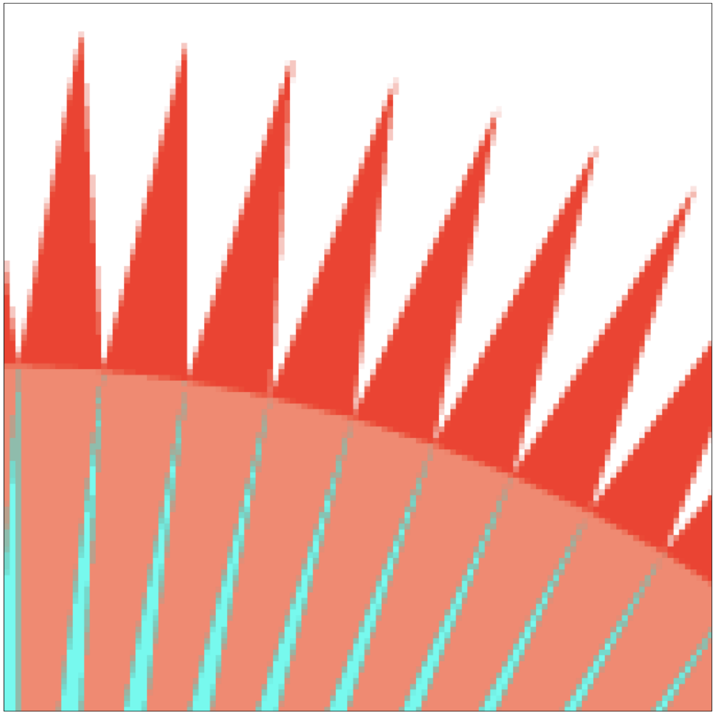
Point Sampling



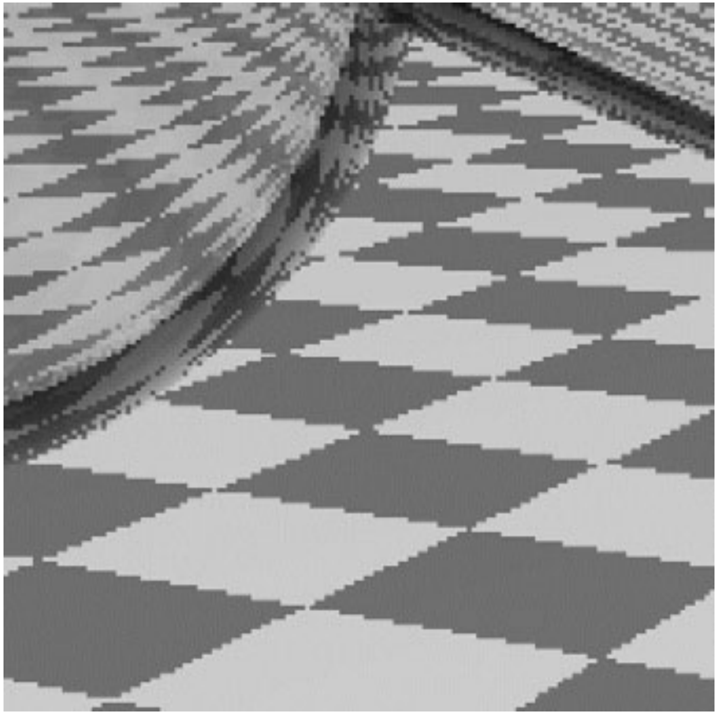


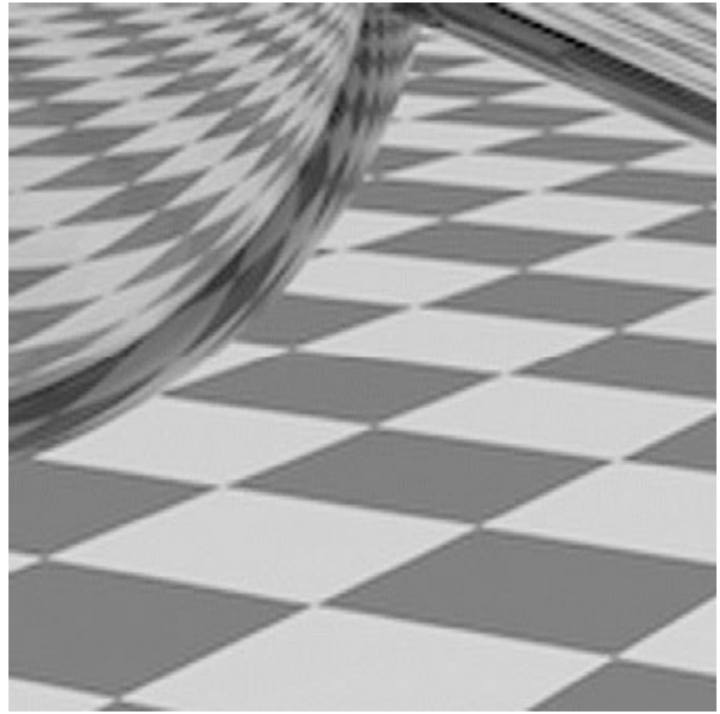
Antialiasing



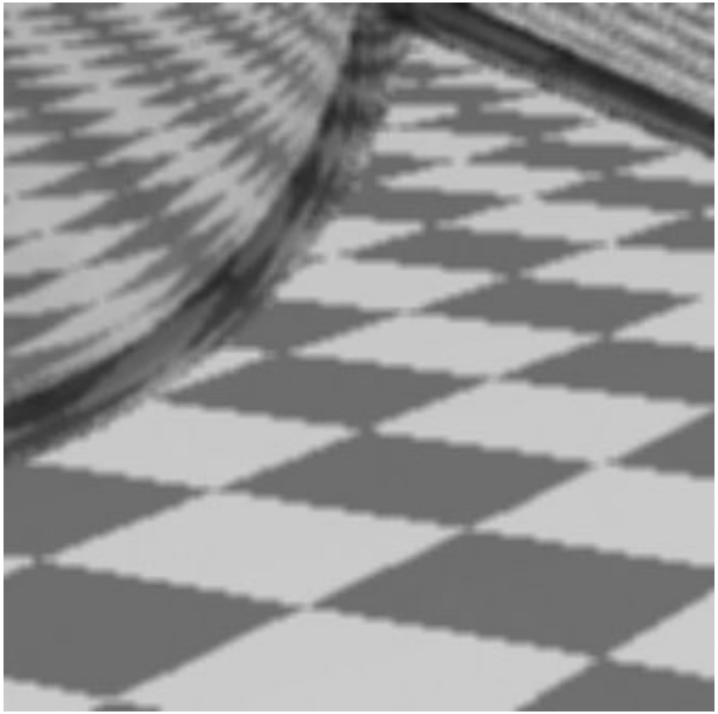


Point Sampling vs Antialiasing

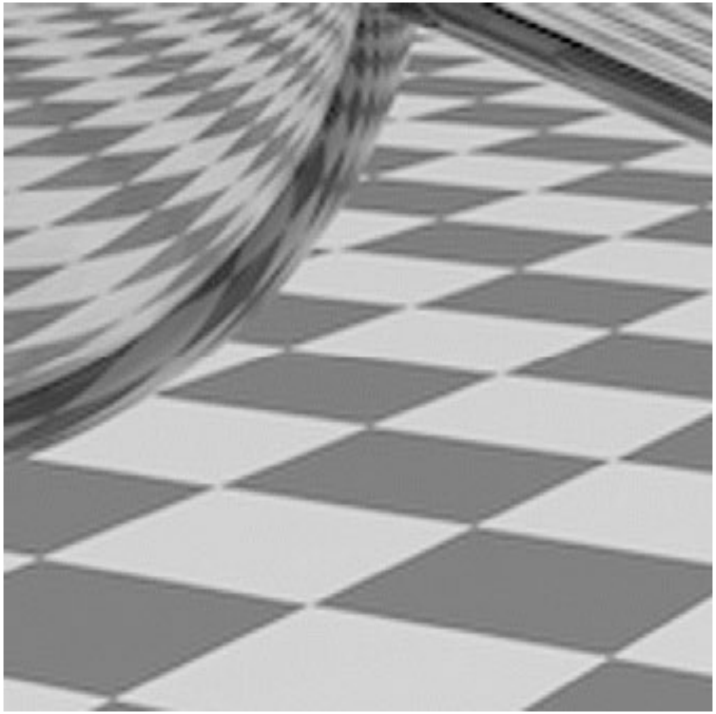




Antialiasing vs Blurred Aliasing



(Sample then filter, WRONG!)



(Filter then sample)

But why?

1. Why undersampling introduces aliasing?
2. Why pre-filtering then sampling can do antialiasing?

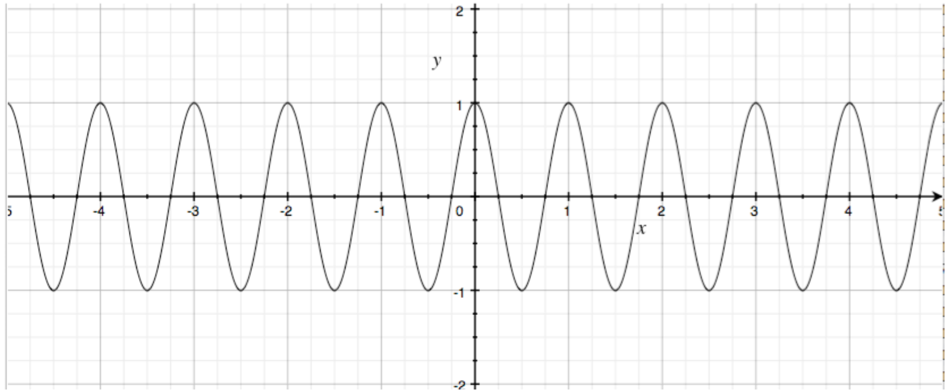
Let's dig into fundamental reasons

And look at how to implement antialiased rasterization

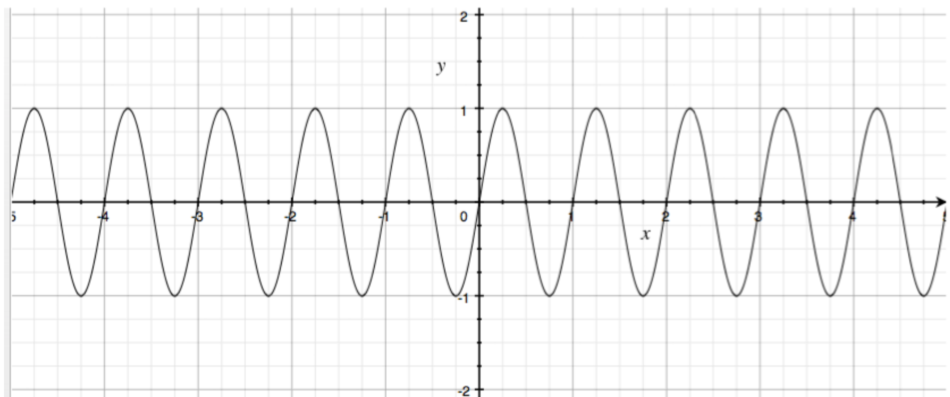
频域

Frequency Domain

Sines and Cosines



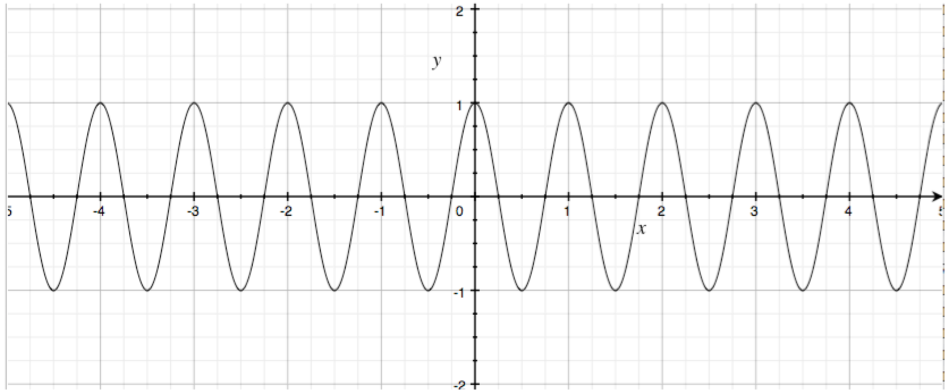
$$\cos 2\pi x$$



$$\sin 2\pi x$$

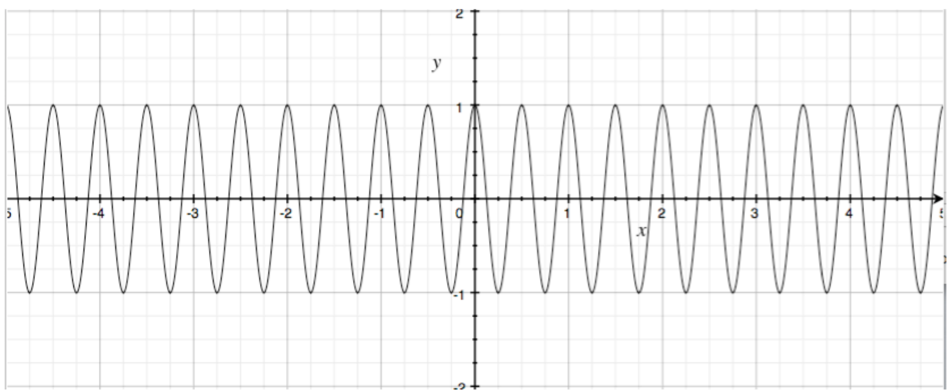
Frequencies $\cos \overset{w}{\underline{2\pi f}} x$

$$f = \frac{1}{T}$$



$$f = 1$$

$$\cos 2\pi x$$



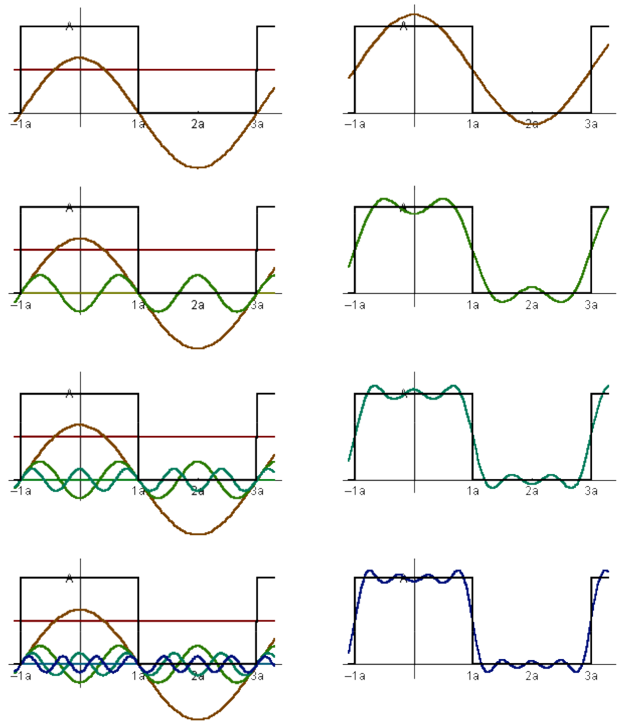
$$f = 2$$

$$\cos 4\pi x$$

Fourier Transform

Represent a function as a weighted sum of sines and cosines

Joseph Fourier 1768 - 1830



$$f(x) = \frac{A}{2} + \frac{2A\cos(t\omega)}{\pi} - \frac{2A\cos(3t\omega)}{3\pi} + \frac{2A\cos(5t\omega)}{5\pi} - \frac{2A\cos(7t\omega)}{7\pi} + \cdots$$

Fourier Transform Decomposes A Signal Into Frequencies

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i \omega x} dx$$

$f(x)$ — 空间域 **spatial domain** → **Fourier transform** → **frequency domain** 频域 — $F(\omega)$

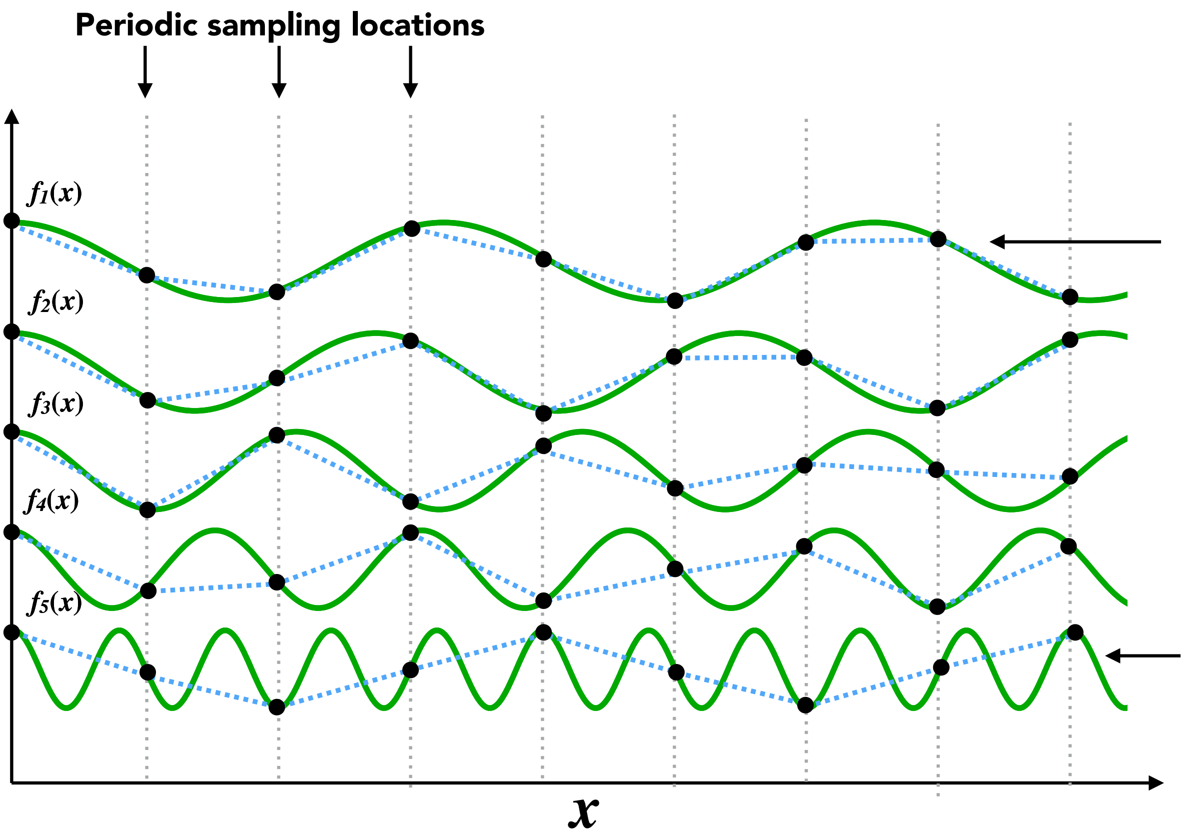
spatial domain ← **Inverse transform** ← **frequency domain**

$$f(x) = \int_{-\infty}^{\infty} F(\omega) e^{2\pi i \omega x} d\omega$$

Recall $e^{ix} = \cos x + i \sin x$

更高的频率需要更快的采样

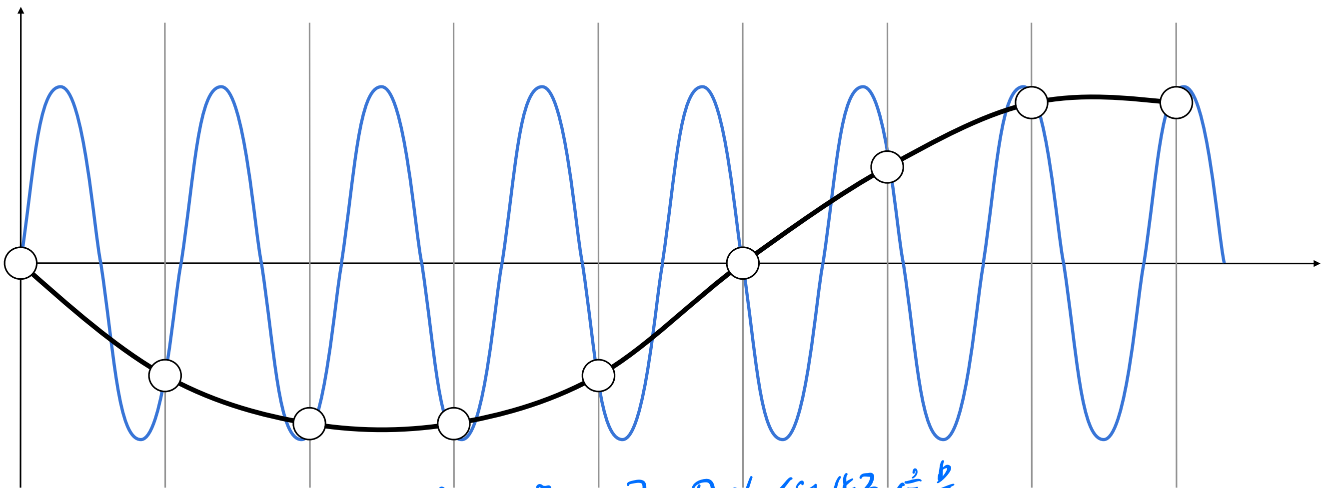
Higher Frequencies Need Faster Sampling



Low-frequency signal: sampled adequately for reasonable reconstruction

High-frequency signal is insufficiently sampled: reconstruction incorrectly appears to be from a low frequency signal

Undersampling Creates Frequency Aliases



高频信号采样不足：采样错误地表现为低频信号

High-frequency signal is insufficiently sampled: samples erroneously appear to be from a low-frequency signal

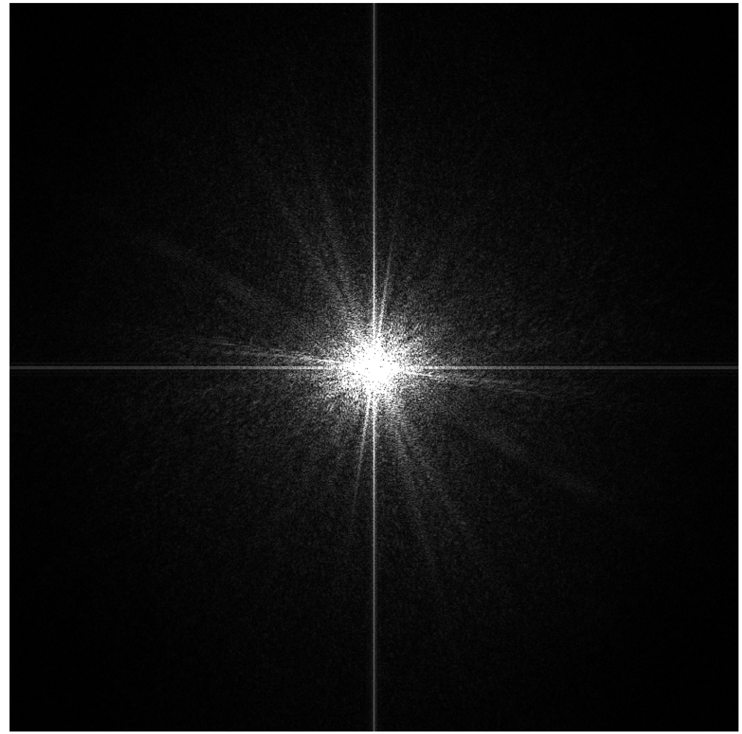
Two frequencies that are indistinguishable at a given sampling rate are called "aliases"

在给定的采样率下无法区分的两个频率 aliases

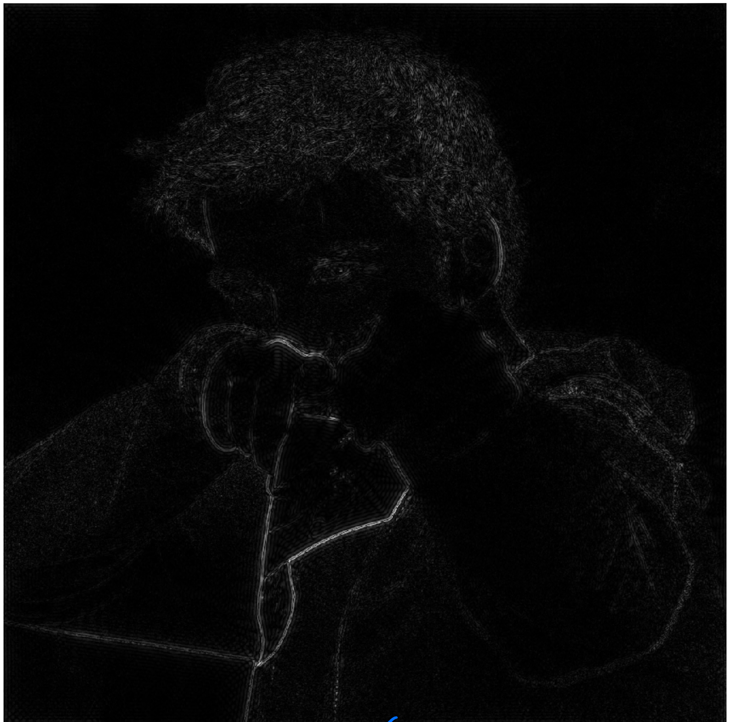
Filtering = Getting rid of certain frequency contents

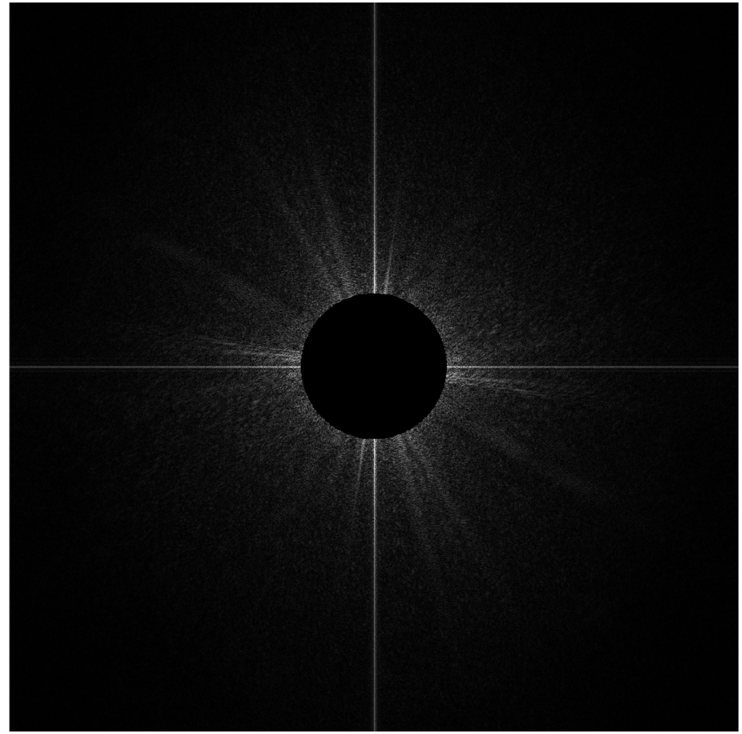
滤波：去掉某些特定频率

Visualizing Image Frequency Content



Filter Out Low Frequencies Only (Edges)



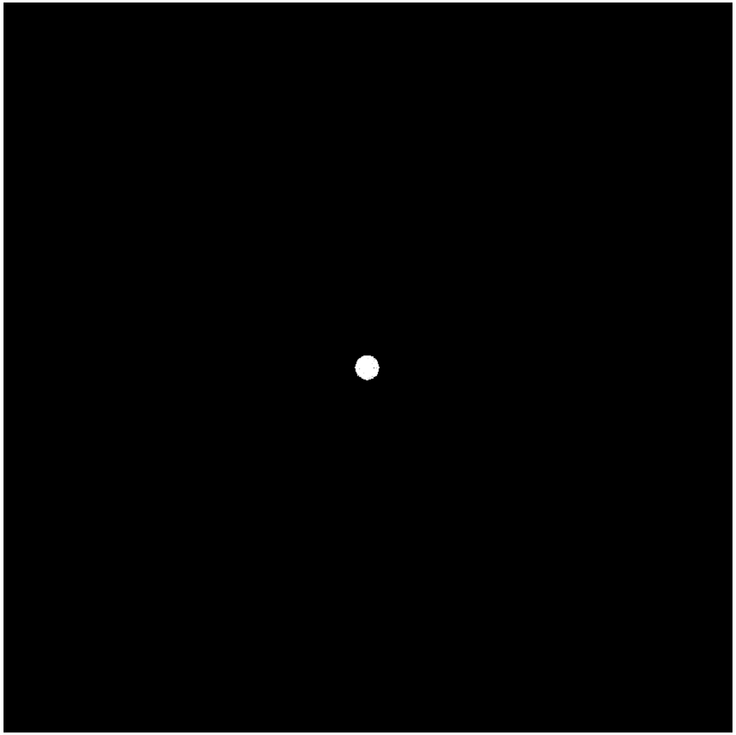


边缘信息
(高频信号)

High-pass filter

高通滤波器

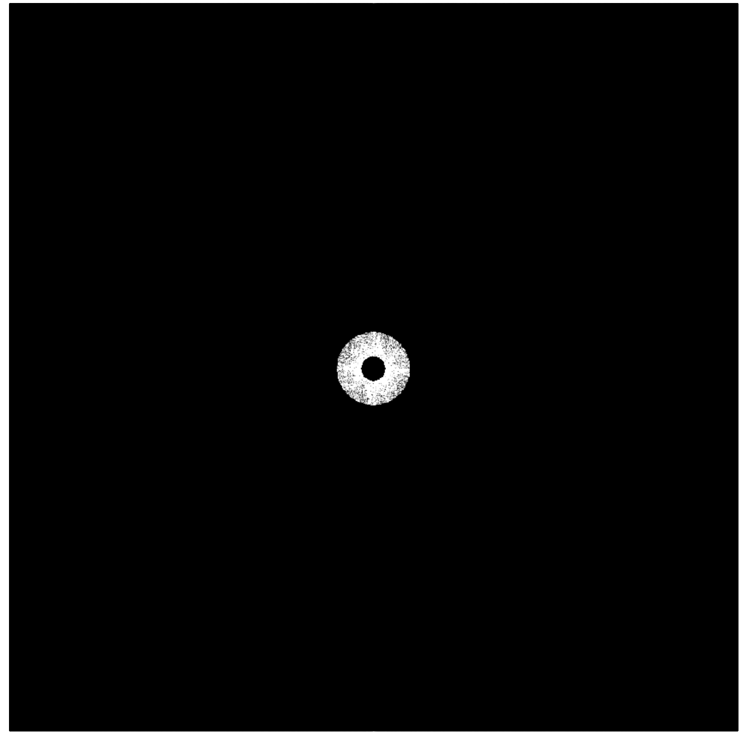
Filter Out High Frequencies (Blur)



Low-pass filter

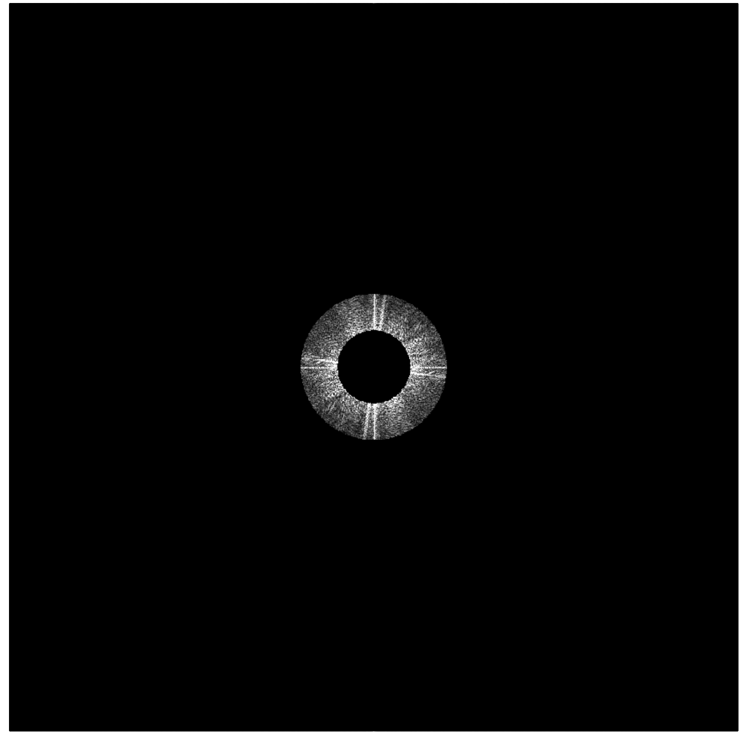
低通滤波器

Filter Out Low and High Frequencies

带通滤波器

Filter Out Low and High Frequencies



滤波　　　　　卷积

Filtering = Convolution

(= Averaging)

Convolution

Signal	1	3	5	3	7	1	3	8	6	4
Filter	1/4	1/2	1/4							

Point-wise local averaging in a “sliding window”

Convolution

Signal

1	3	5	3	7	1	3	8	6	4

Filter

1/4	1/2	1/4

1 x (1/4) + 3 x (1/2) + 5 x (1/4) = 3

Result

	3								

Convolution

Signal	1	3	5	3	7	1	3	8	6	4
Filter		1/4	1/2	1/4						
Result		3	4							

3 x (1/4) + 5 x (1/2) + 3 x (1/4) = 4

Convolution Theorem

空间域卷积 = 频域的乘积.

Convolution in the spatial domain is equal to multiplication in the frequency domain, and vice versa

Option 1:

- Filter by convolution in the spatial domain

Option 2:

- Transform to frequency domain (Fourier transform)
- Multiply by Fourier transform of convolution kernel
- Transform back to spatial domain (inverse Fourier)

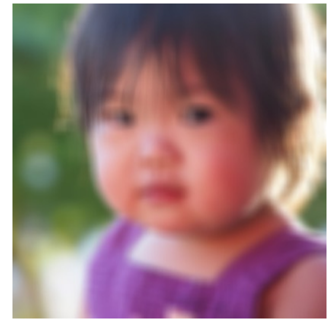
Convolution Theorem

空间域

Spatial Domain

卷积

$$* \ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} =$$

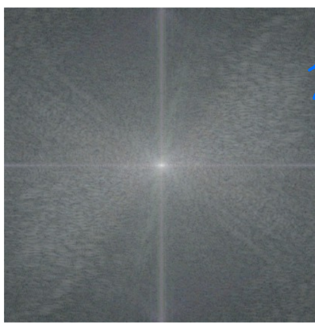


Fourier Transform

Inv. Fourier Transform

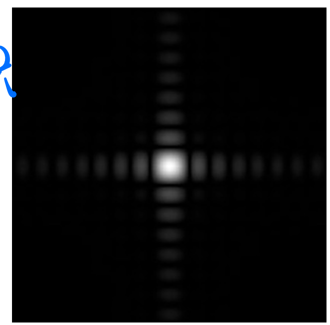
频域

Frequency Domain



乘积

$\times$



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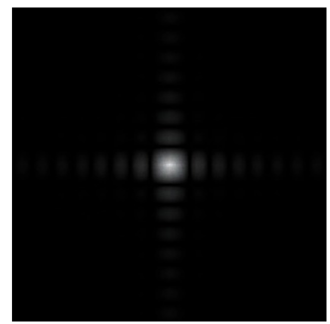
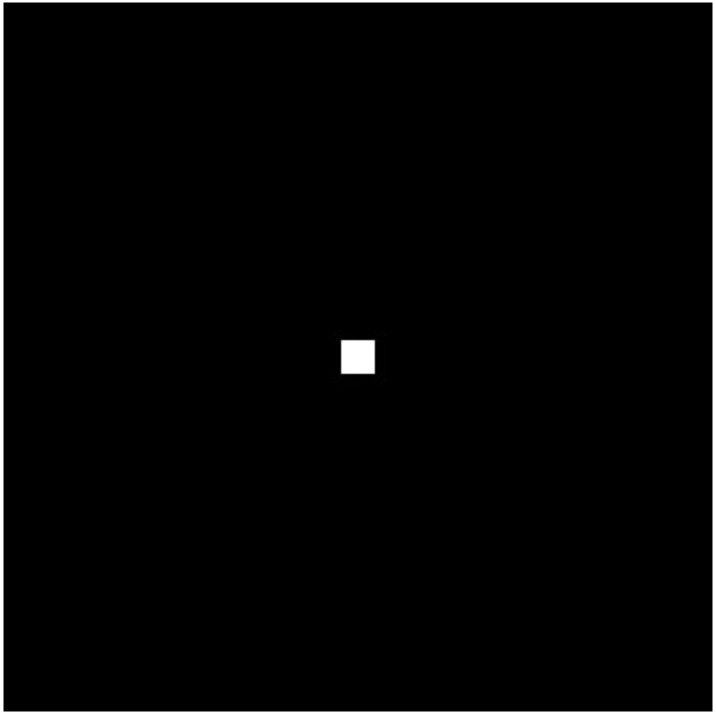


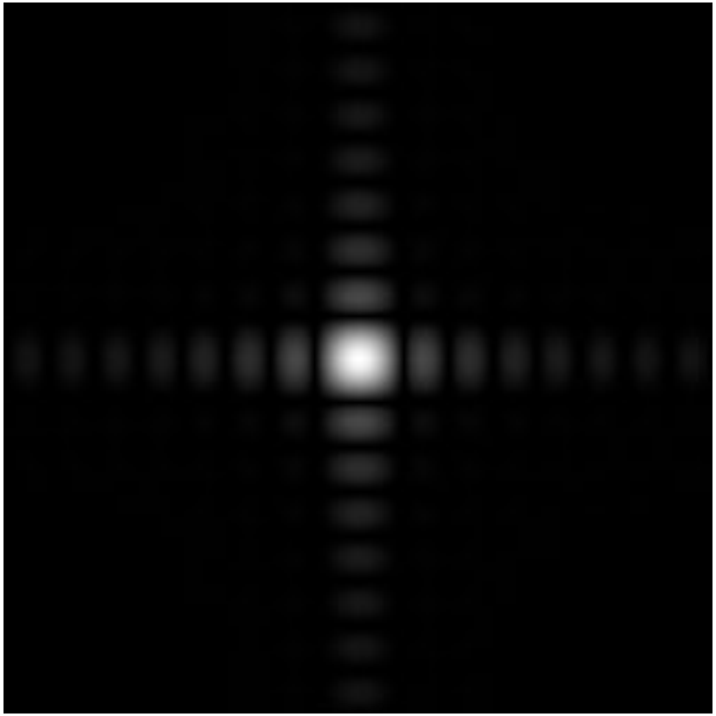
Box Filter <低通滤波器>

$$\frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}$$

Example: 3x3 box filter

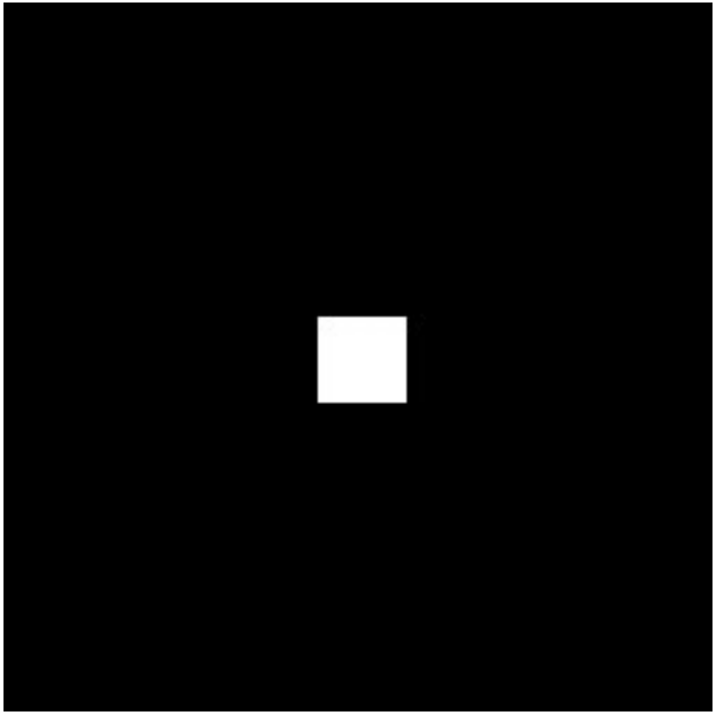
Box Function = “Low Pass” Filter

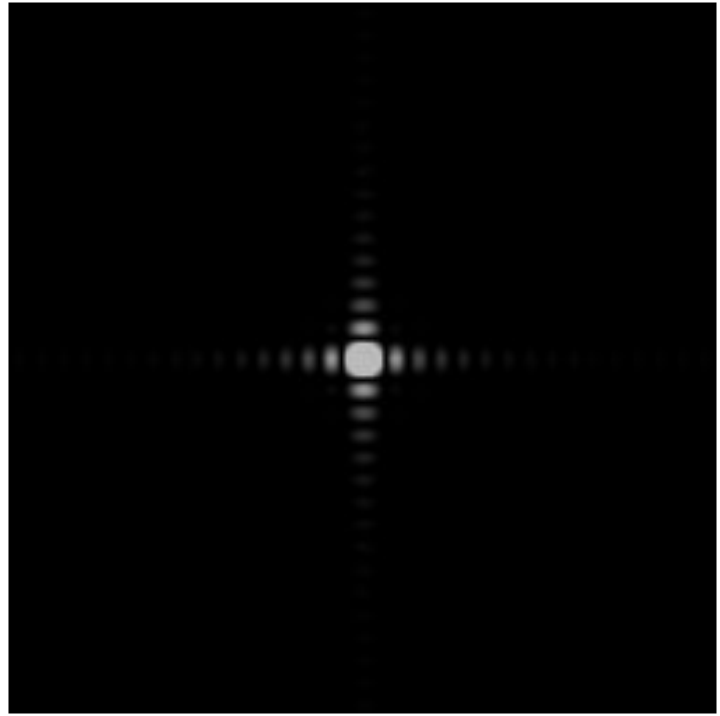




Wider Filter Kernel = Lower Frequencies

更宽的滤波核 = 更低的频率



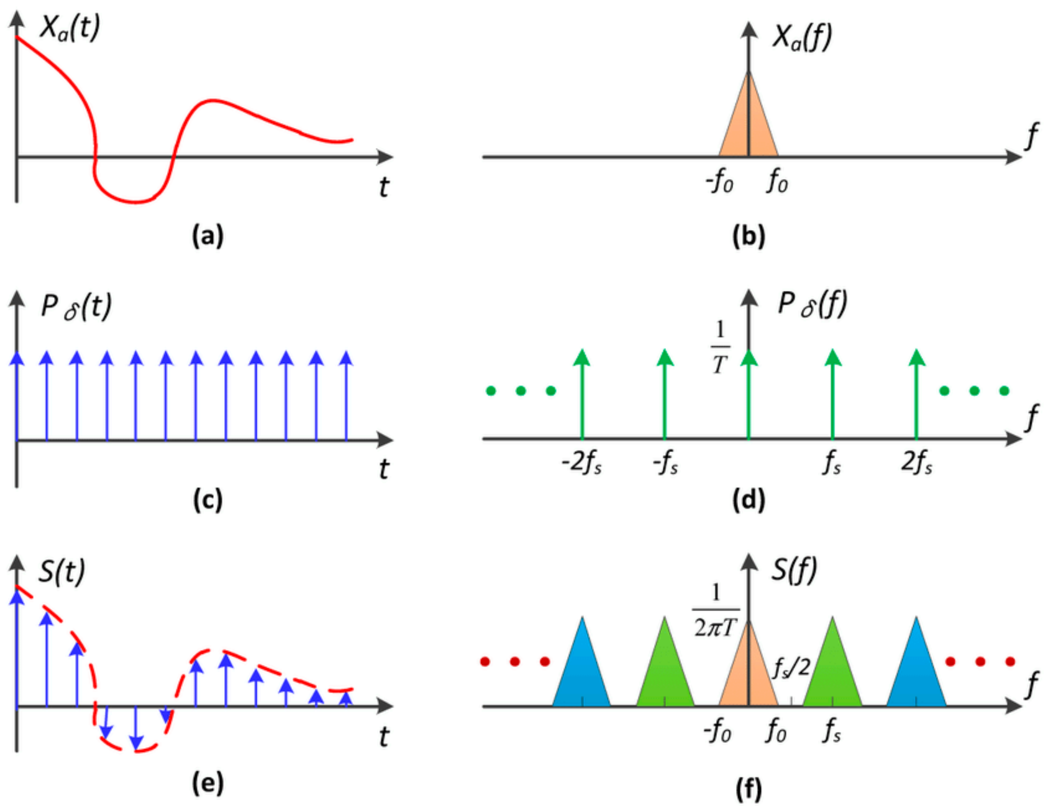


Sampling = Repeating Frequency Contents

采样 = 重复频率内容

Sampling = Repeating Frequency Contents

重复原始信号频谱



https://www.researchgate.net/figure/The-evolution-of-sampling-theorem-a-The-time-domain-of-the-band-limited-signal-and-b_fig5_301556095

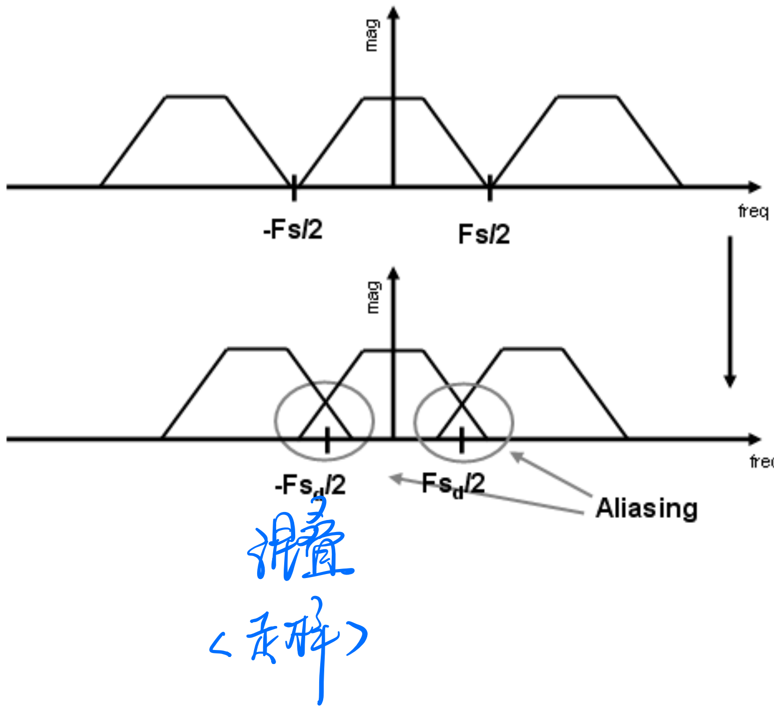
Aliasing = Mixed Frequency Contents

密集采样

Dense sampling:

稀疏采样

Sparse sampling:



Antialiasing

How Can We Reduce Aliasing Error?

① 增加采样率

Option 1: Increase sampling rate

- Essentially increasing the distance between replicas in the Fourier domain
- Higher resolution displays, sensors, framebuffers…
- But: costly & may need very high resolution

② 反走样

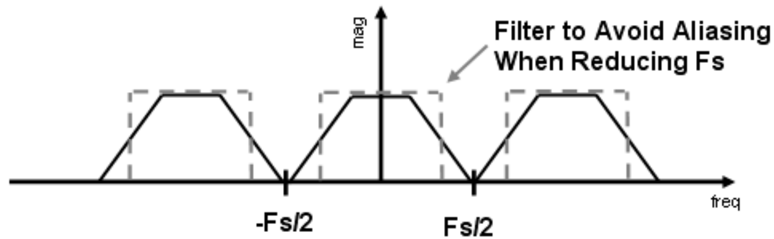
Option 2: Antialiasing “滤波”使信号频谱更窄

- Making Fourier contents “narrower” before repeating
- i.e. Filtering out high frequencies before sampling

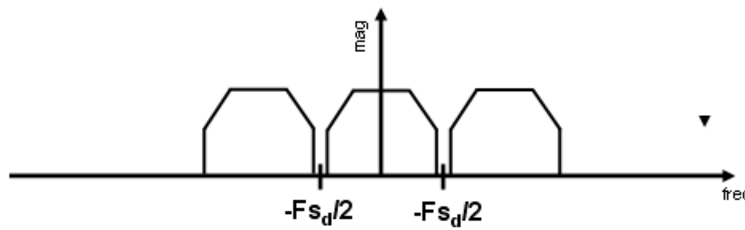
采样前过滤掉高频信号

Antialiasing = Limiting, then repeating

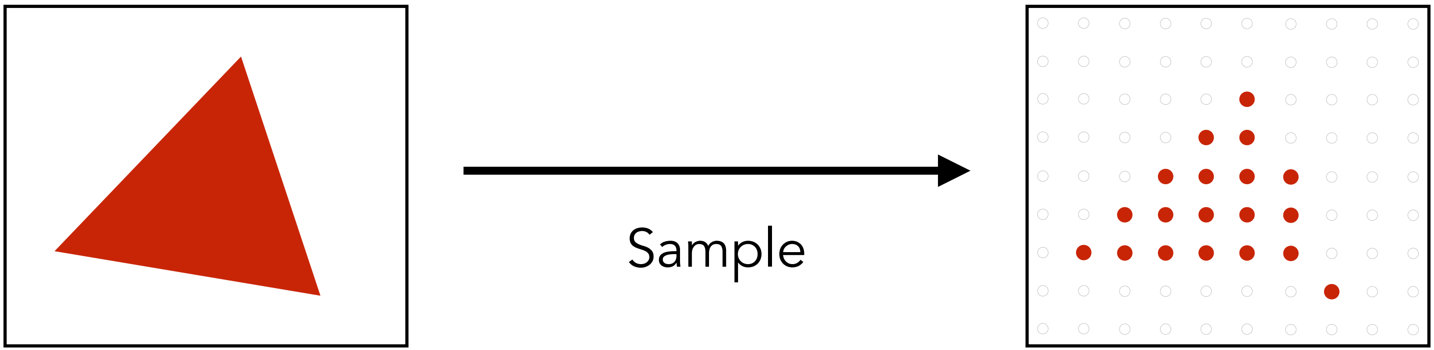
Filtering



Then sparse sampling

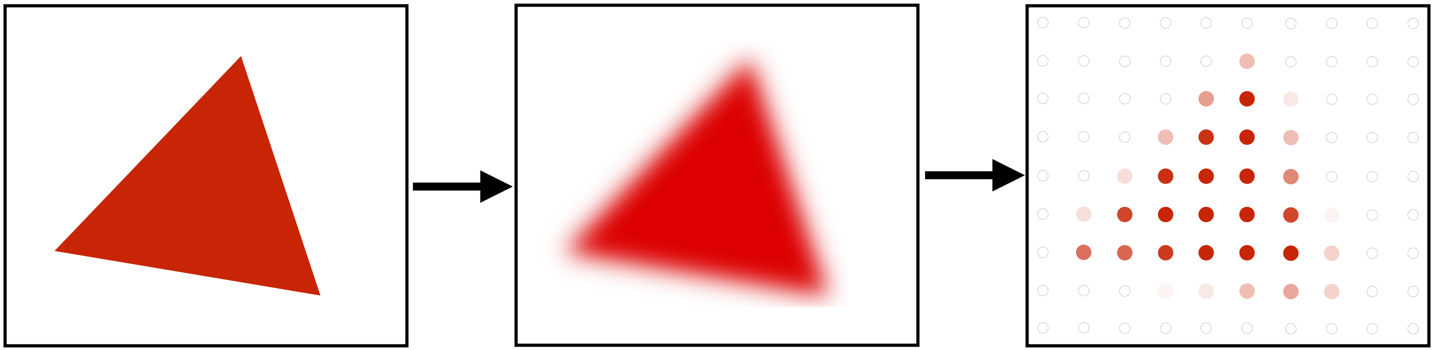


Regular Sampling



Note jaggies in rasterized triangle
where pixel values are pure red or white

Antialiased Sampling



Pre-Filter

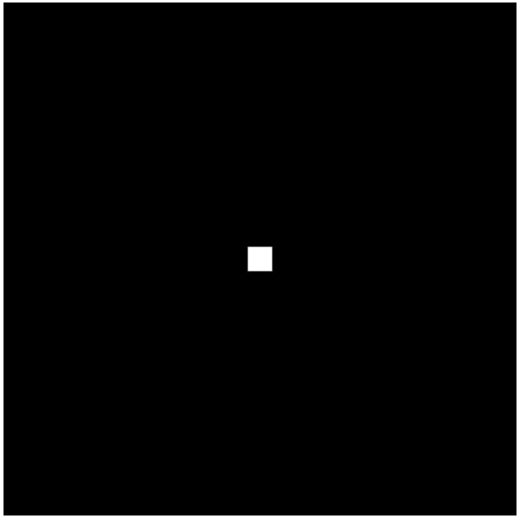
(remove frequencies above Nyquist)

Sample

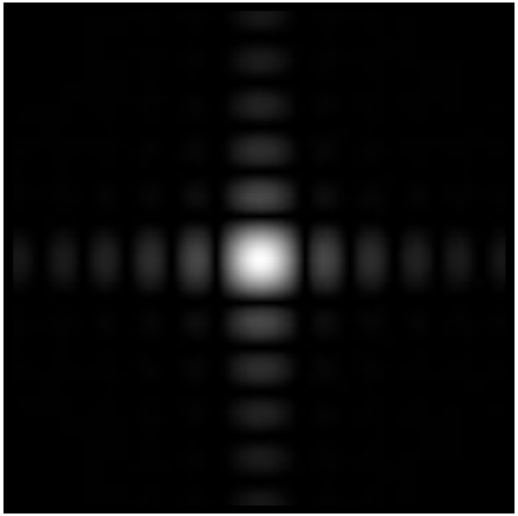
Note antialiased edges in rasterized triangle
where pixel values take intermediate values

A Practical Pre-Filter

A 1 pixel-width box filter (low pass, blurring)



Spatial Domain



Frequency Domain

Antialiasing By Averaging Values in Pixel Area

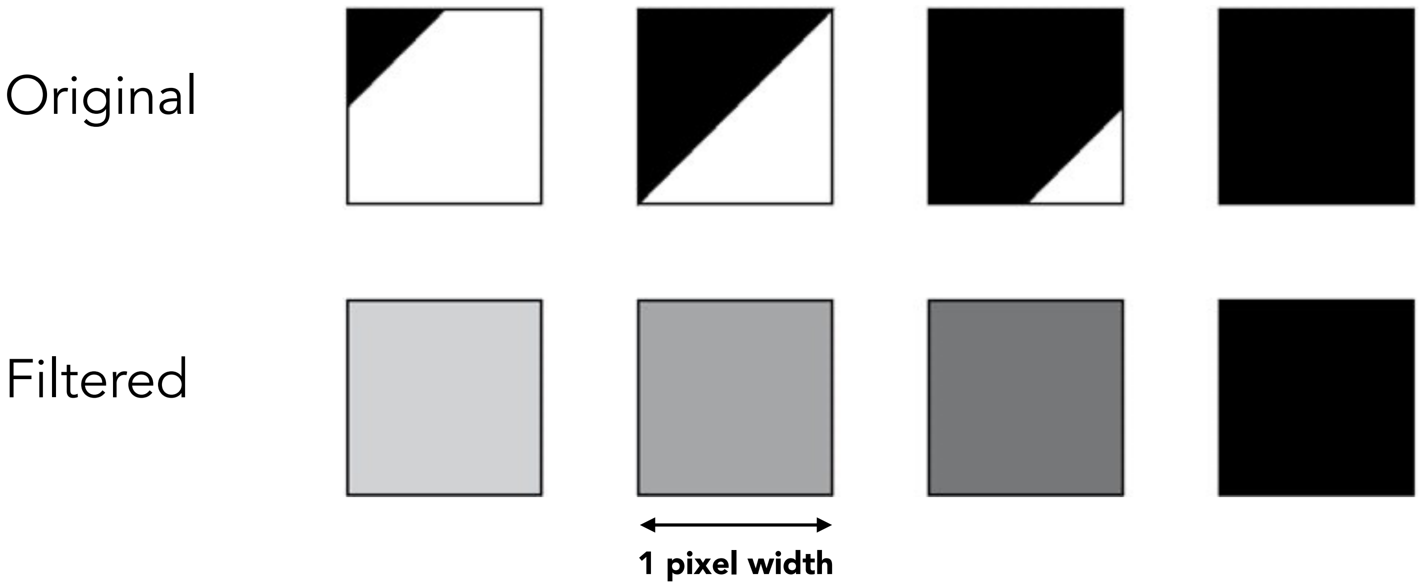
Solution: 用1像素的框模糊卷积

- **Convolve** f(x,y) by a 1-pixel box-blur
  - Recall: convolving = filtering = averaging
- **Then sample** at every pixel's center

然后去每个像素的中心采样.

Antialiasing by Computing Average Pixel Value

In rasterizing one triangle, the average value inside a pixel area of f(x,y) = inside(triangle,x,y) is equal to the area of the pixel covered by the triangle.



MSAA 多重采样抗锯齿(采样)
Multisampling Anti-Aliasing

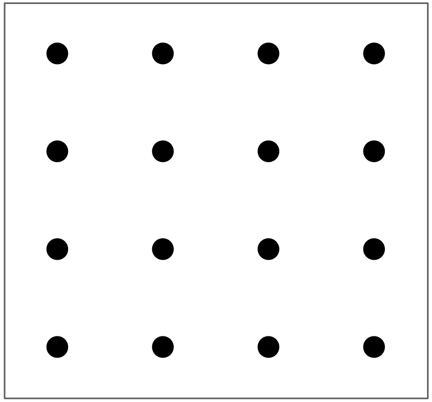
Antialiasing By Supersampling (MSAA)

完成对信号的模糊操作

Supersampling

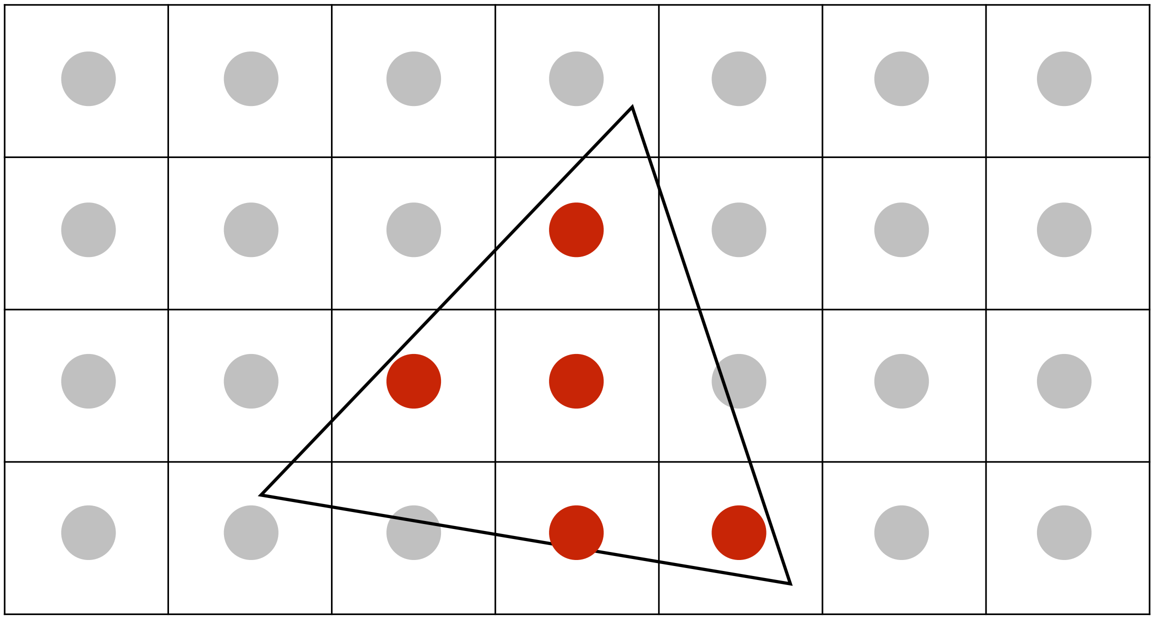
Approximate the effect of the 1-pixel box filter by sampling multiple locations within a pixel and averaging their values:

通过采样1个像素内的多个位置并平均它们
近似1像素盒滤波器的效果



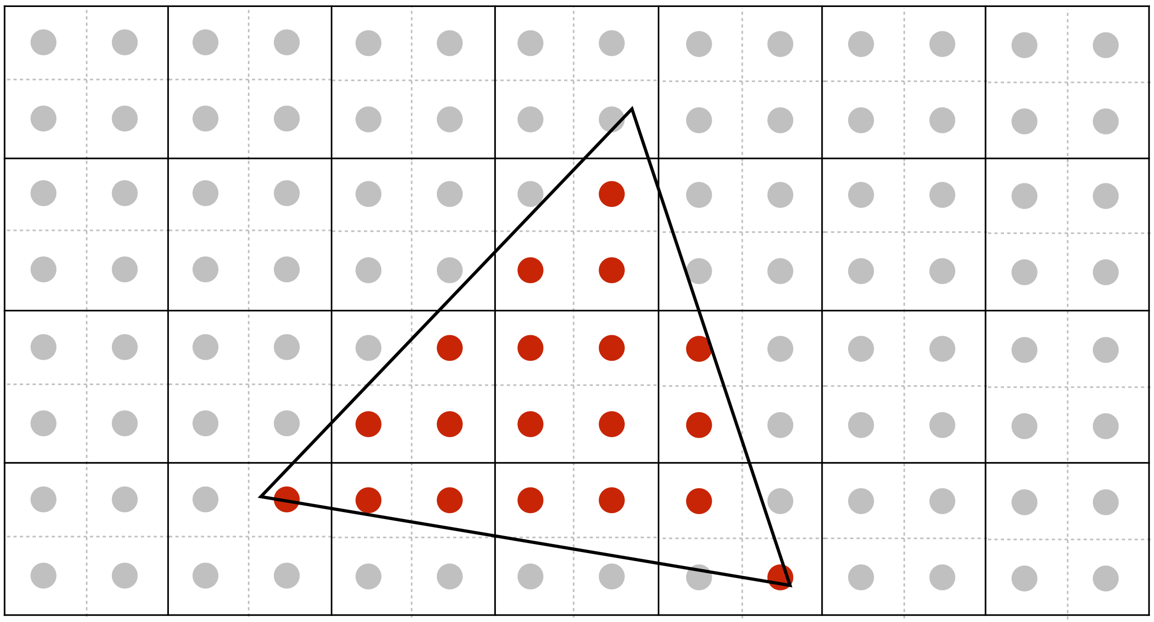
4x4 supersampling

Point Sampling: One Sample Per Pixel



Supersampling: Step 1

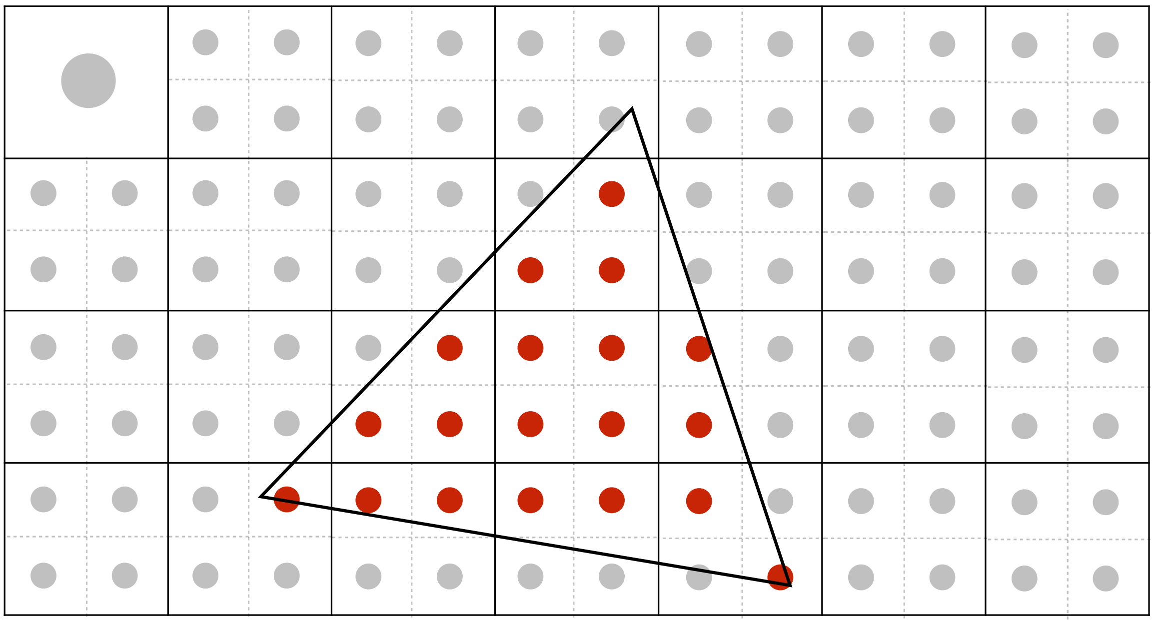
Take NxN samples in each pixel.



2x2 supersampling

Supersampling: Step 2

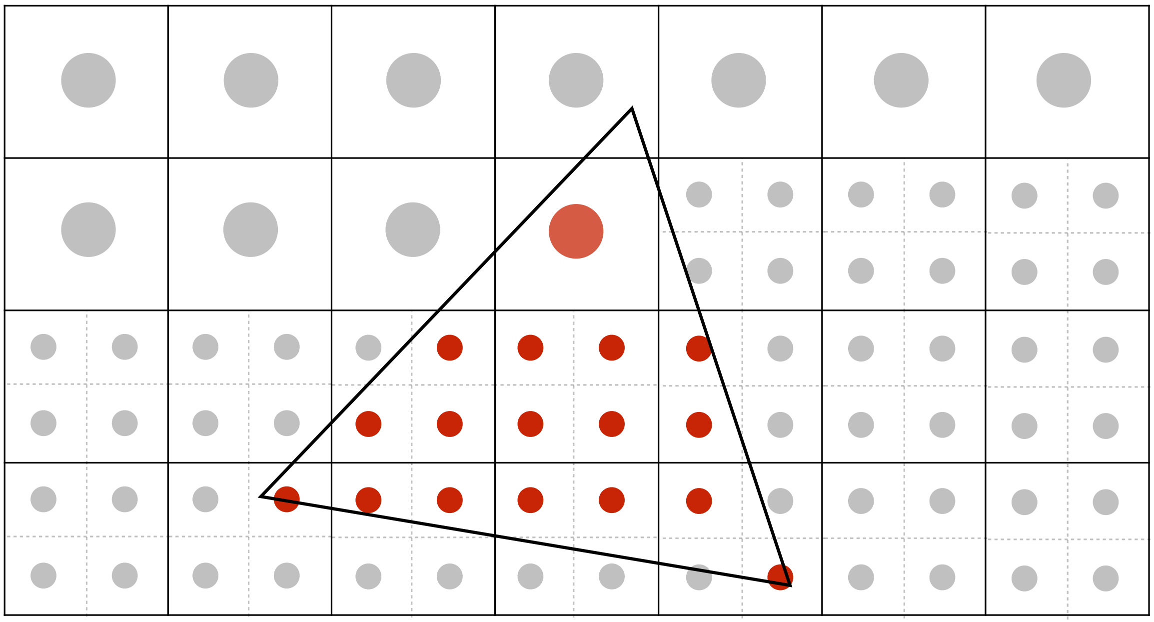
Average the NxN samples "inside" each pixel.



Averaging down

Supersampling: Step 2

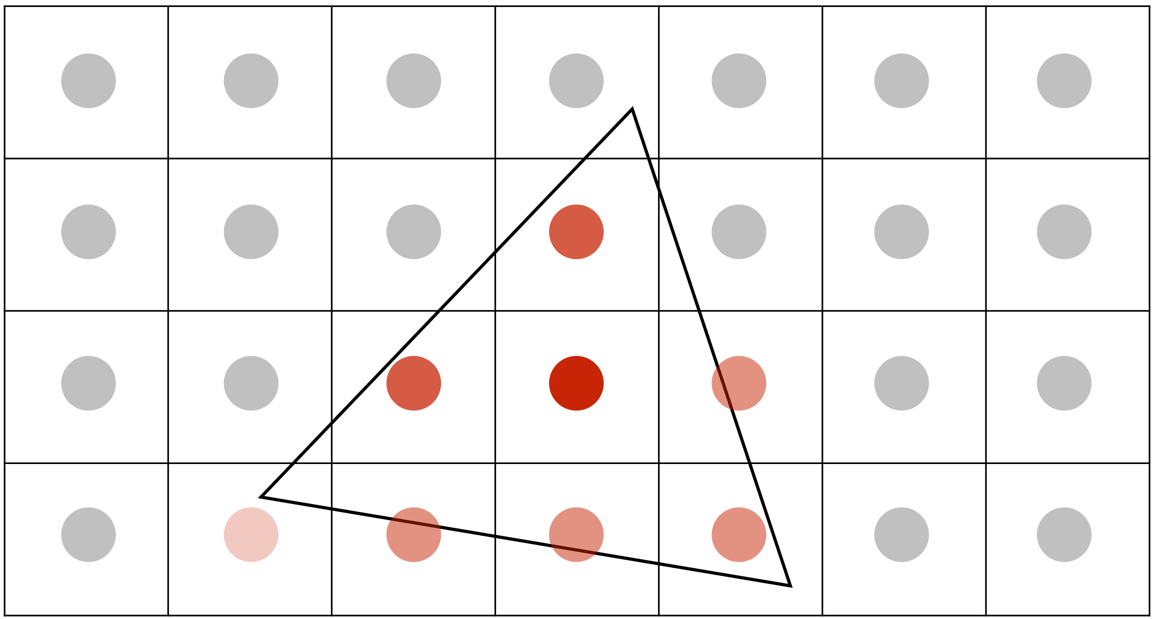
Average the NxN samples “inside” each pixel.



Averaging down

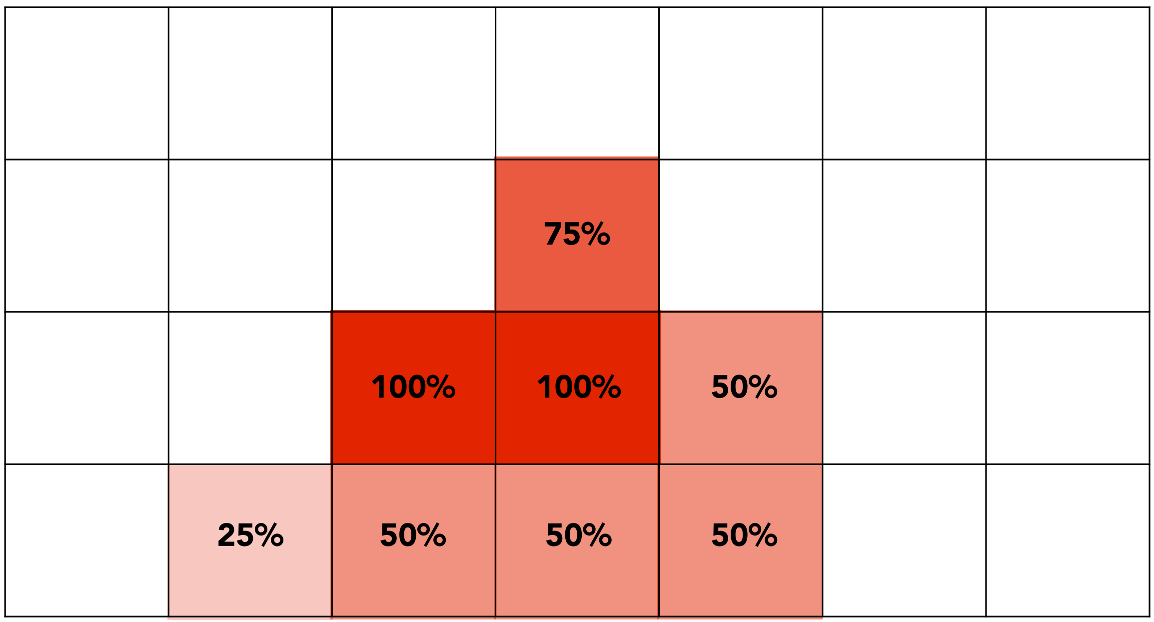
Supersampling: Step 2

Average the NxN samples "inside" each pixel.

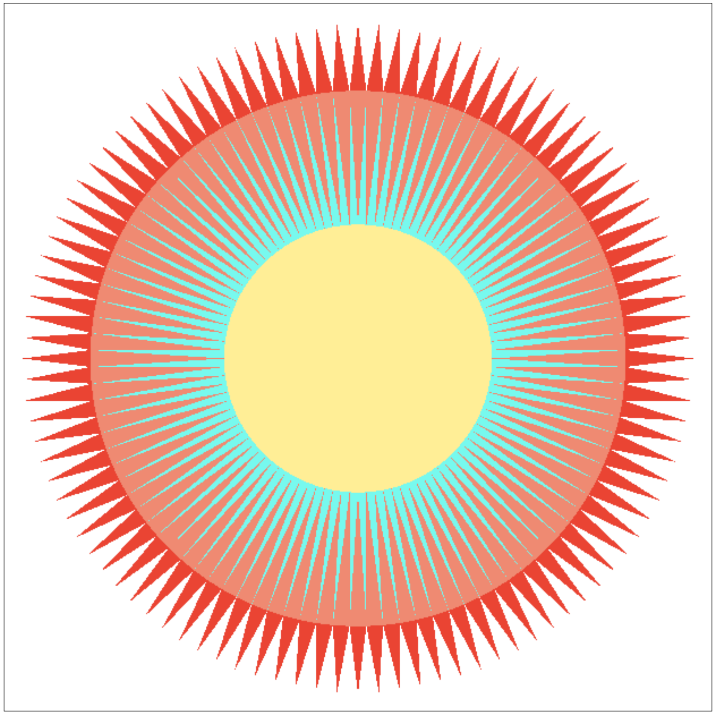


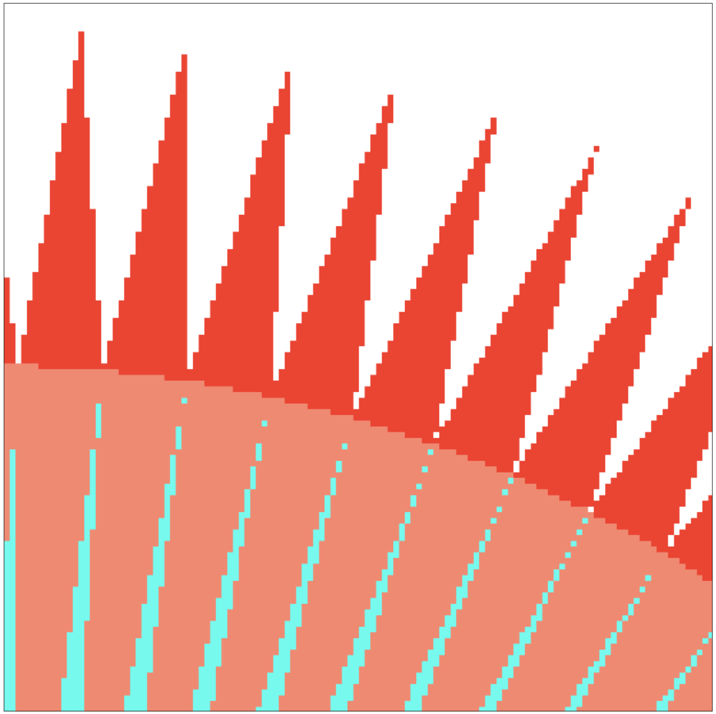
Supersampling: Result

This is the corresponding signal emitted by the display

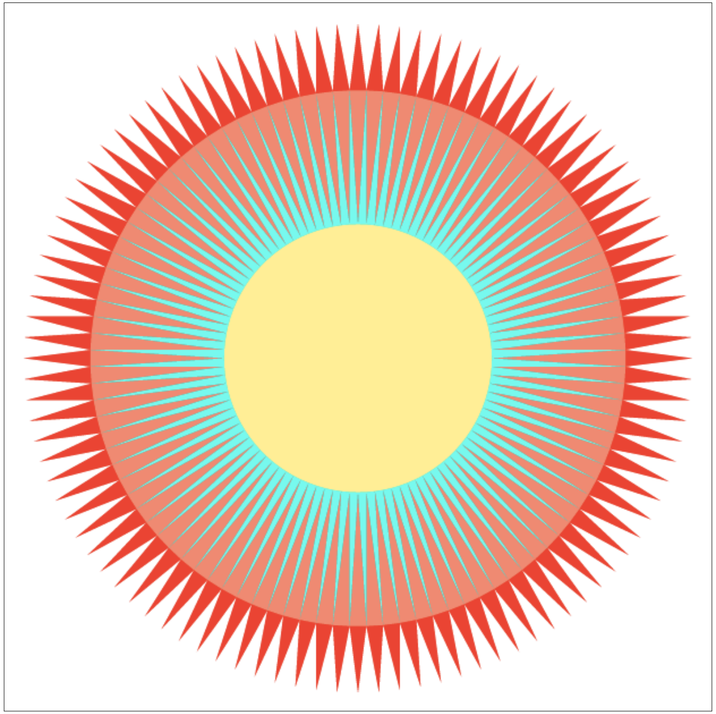


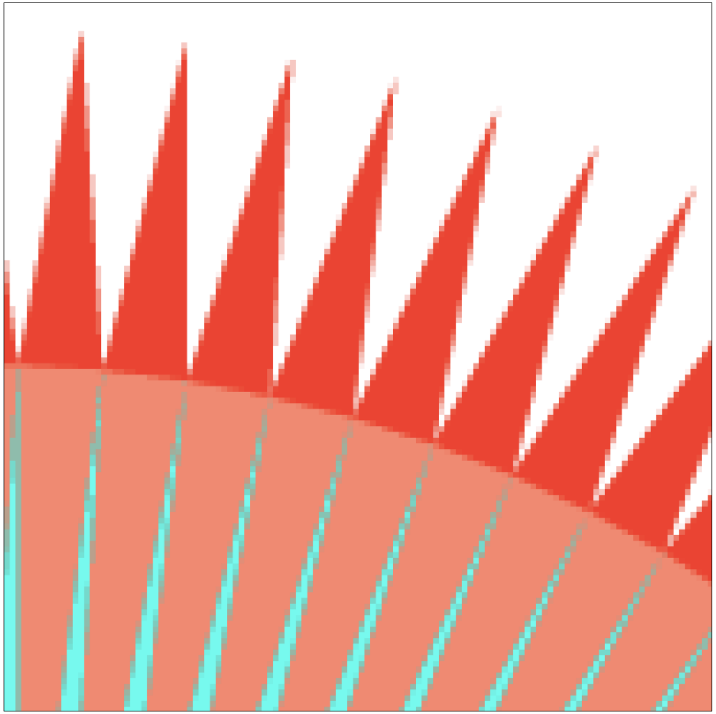
Point Sampling





4x4 Supersampling





Antialiasing Today

No free lunch!

多重采样抗锯齿<采样有关>

- What's the cost of MSAA?

Milestones (personal idea)

- FXAA (Fast Approximate AA) 快速近似抗锯齿.<图像处理有关>
- TAA (Temporal AA) 时间相关抗锯齿<找相邻帧信息>
  <复用结果>

超分辨率

Super resolution / super sampling <MSAA分布到时间上>

- From low resolution to high resolution
- Essentially still "not enough samples" problem
- DLSS (Deep Learning Super Sampling)

Thank you!