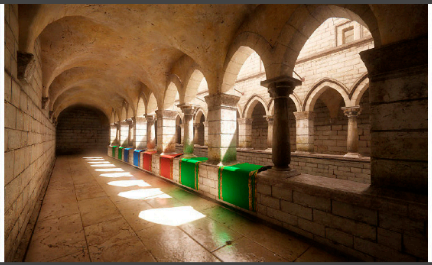
Real-Time High Quality Rendering

GAMES202, Lingqi Yan, UC Santa Barbara

Lecture 6:
Real-Time Environment Mapping
(Precomputed Radiance Transfer)

Announcements

- No class next week (traveling)
- Some changes to the pace
  - One lecture per week
  - Only on Saturdays (GMT+8)
  - Lecture 7 will be on Apr 17
- Some changes to the order of lectures
  - Will talk about Precomputed Radiance Transfer (PRT) first

Last Lecture

- Distance field soft shadows
- Shading from environment lighting
  - The split sum approximation

Today

- Finishing up
  - Shadow from environment lighting
- Background knowledge
  - Frequency and filtering
  - Basis functions
- Real-time environment lighting (& global illumination)
  - Spherical Harmonics (SH)
  - Prefiltered env. lighting
  - Precomputed Radiance Transfer (PRT)

Shadow from Environment Lighting

- In general, very difficult for real-time rendering
- Different perspectives of view
  - As a many-light problem:
    Cost of SM is linearly to #light
  - As a sampling problem:
    Visibility term V can be arbitrarily complex
    And V cannot be easily separated from the environment

Visibility term 从多个方向来看可能截然不同. 很复杂.
很难从环境中分离出来.

Shadow from Environment Lighting

- Industrial solution
  - Generate one (or a little bit more) shadows from the brightest light sources
    从最亮的光源生成一个(或多一点)阴影.
- Related research
  1. Imperfect shadow maps <用于全局光照中阴影生成>
  2. Light cuts 用于 offline rendering 中多光源问题.
  3. RTRT (might be the ultimate solution)
     Real Time Ray Tracing.
  4. Precomputed radiance transfer 预计算 radience 传递
     "PRT"

Questions?

Today

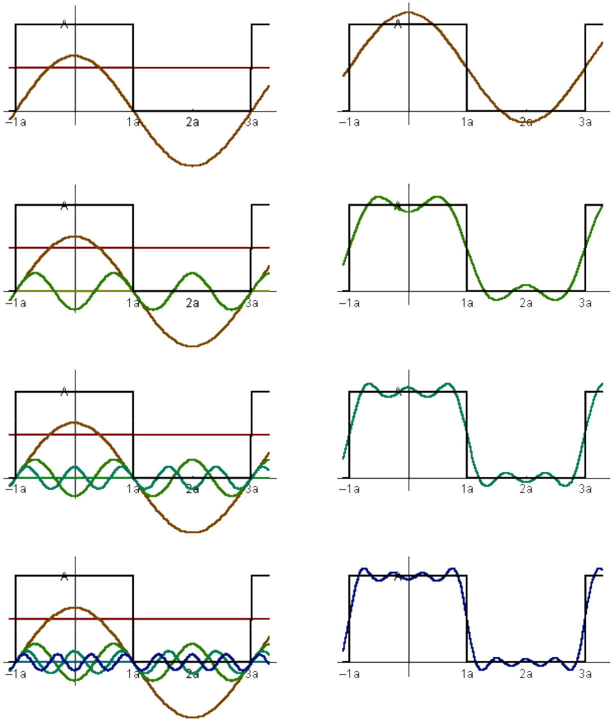
- Finishing up
  - Shadow from environment lighting
- Background knowledge
  - Frequency and filtering 频域/滤波
  - Basis functions 基函数
- Real-time environment lighting (& global illumination)
  - Spherical Harmonics (SH)
  - Prefiltered env. lighting
  - Precomputed Radiance Transfer (PRT)

Fourier Transform

Represent a function as a weighted sum of sines and cosines

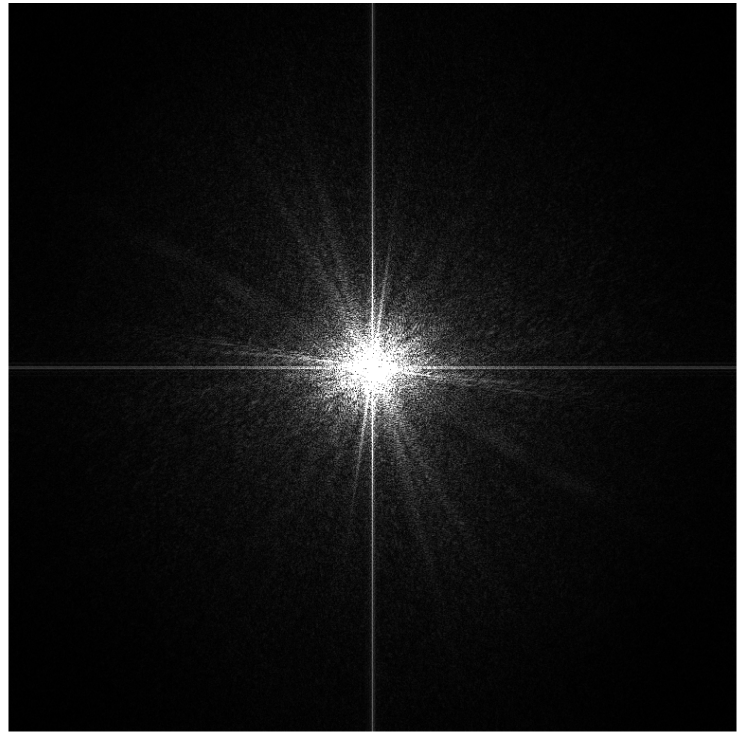
将函数表示为正弦和余弦的加权和

Joseph Fourier 1768 - 1830



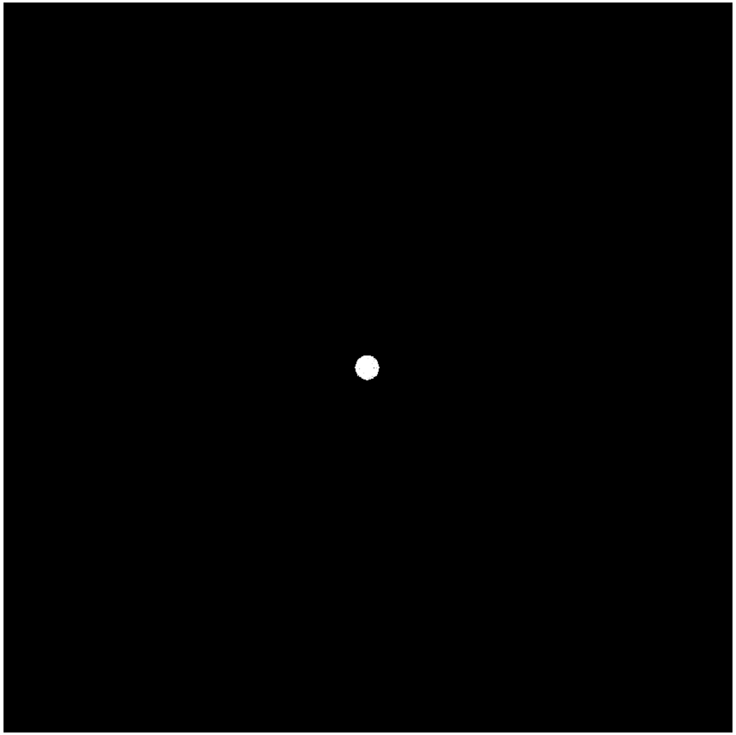
$$f(x) = \frac{A}{2} + \frac{2A\cos(t\omega)}{\pi} - \frac{2A\cos(3t\omega)}{3\pi} + \frac{2A\cos(5t\omega)}{5\pi} - \frac{2A\cos(7t\omega)}{7\pi} + \cdots$$

Visualizing Image Frequency Content



Filtering = Getting rid of certain frequency contents

去除某些频率内容



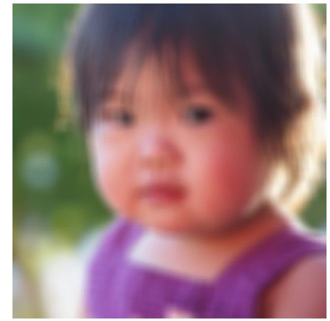
Low-pass filter

Convolution Theorem

Spatial Domain

时域<空间域>卷积

$$* \;\; \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \;=$$

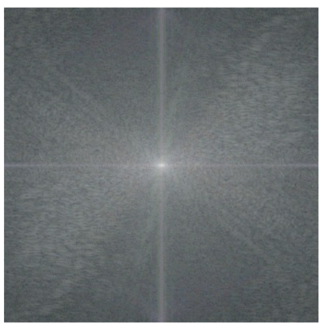


Fourier Transform ↓ ↓

Inv. Fourier Transform ↑

频域乘积

Frequency Domain



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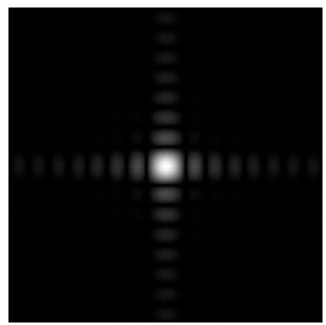

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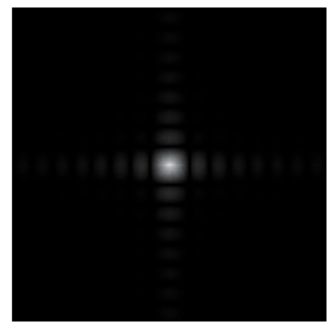


A general understanding

任何 乘积积分可被认为 滤波.

- Any product integral can be considered as filtering

$$\int_{\Omega} f(x)g(x)\,\mathrm{d}x$$

低频. 平滑 缓慢变化.

- Low frequency == smooth function / slow changes / etc.

- The frequency of the integral is the lowest of any individual's

积分的频率是所有个体中最低的.

基函数

Basis Functions

可用于表示其他函数的一组函数

- A set of functions that can be used to represent other functions in general

$$f(x) = \sum_i c_i \cdot \underline{\underline{B_i(x)}}$$

基函数

傅里叶级数是一组基函数

- The Fourier series is a set of basis functions

- The polynomial series can also be a set of basis functions

多项式级数也可以是一组基函数

Questions?

Today

- Finishing up
  - Shadow from environment lighting
- Background knowledge
  - Frequency and filtering
  - Basis functions
- Real-time environment lighting (& global illumination)
  - Spherical Harmonics (SH)
  - Prefiltered env. lighting
  - Precomputed Radiance Transfer (PRT)

球面谐波函数

Spherical Harmonics

定义在球面上的一组二维基函数 $B_i(\omega)$

- A set of 2D basis functions $B_i(\omega)$ defined on the sphere

类似于1维傅里叶级数

- Analogous to Fourier series in 1D

前n阶共有 n^2 个SH函数

$l = 0$

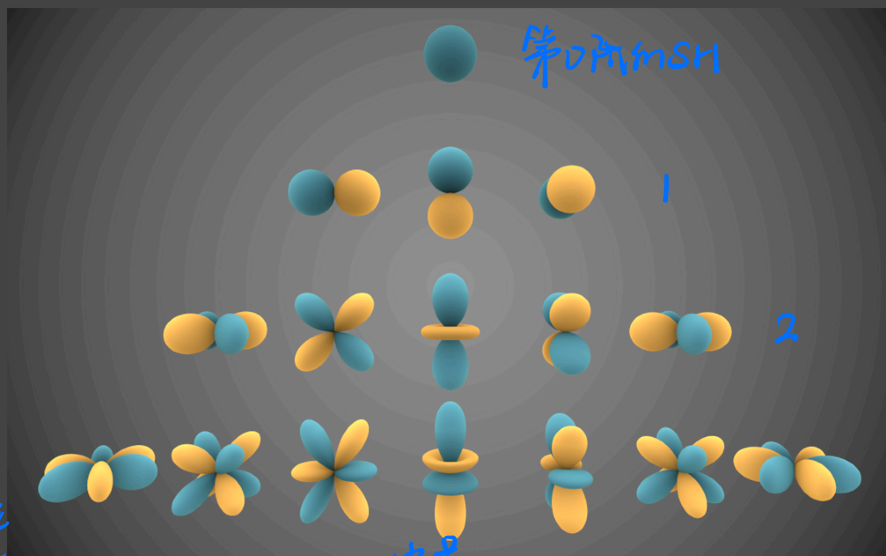
$l = 1$

$l = 2$

2l+1 个函数

$l = 3$

频率越高



第0阶的SH

1

2

频率每层都相同

l=0.1.2....

编号

… -2 -1 $m = 0$ 1 2 …

Spherical Harmonics

- Each SH basis function $B_i(\omega)$ is associated with a (Legendre) polynomial

每个SH基函数$B_i(\omega)$都与 Legendre 多项式相关联.
勒让德

- Projection: obtaining the coefficients of each SH basis function

求每个SH基函数的系数：投影

二维函数
比如从四面八方来的环境光照
可用以SH为基函数的一组函数进行恢复.

$$c_i = \int_\Omega f(\omega) B_i(\omega)\, \mathrm{d}\omega$$

求系数

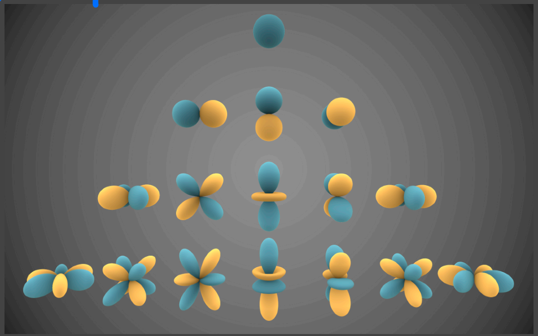
原函数

（乘积积分）

对原函数和基函数做 product integral

- Reconstruction: restoring the original function using (truncated) coefficients and basis functions

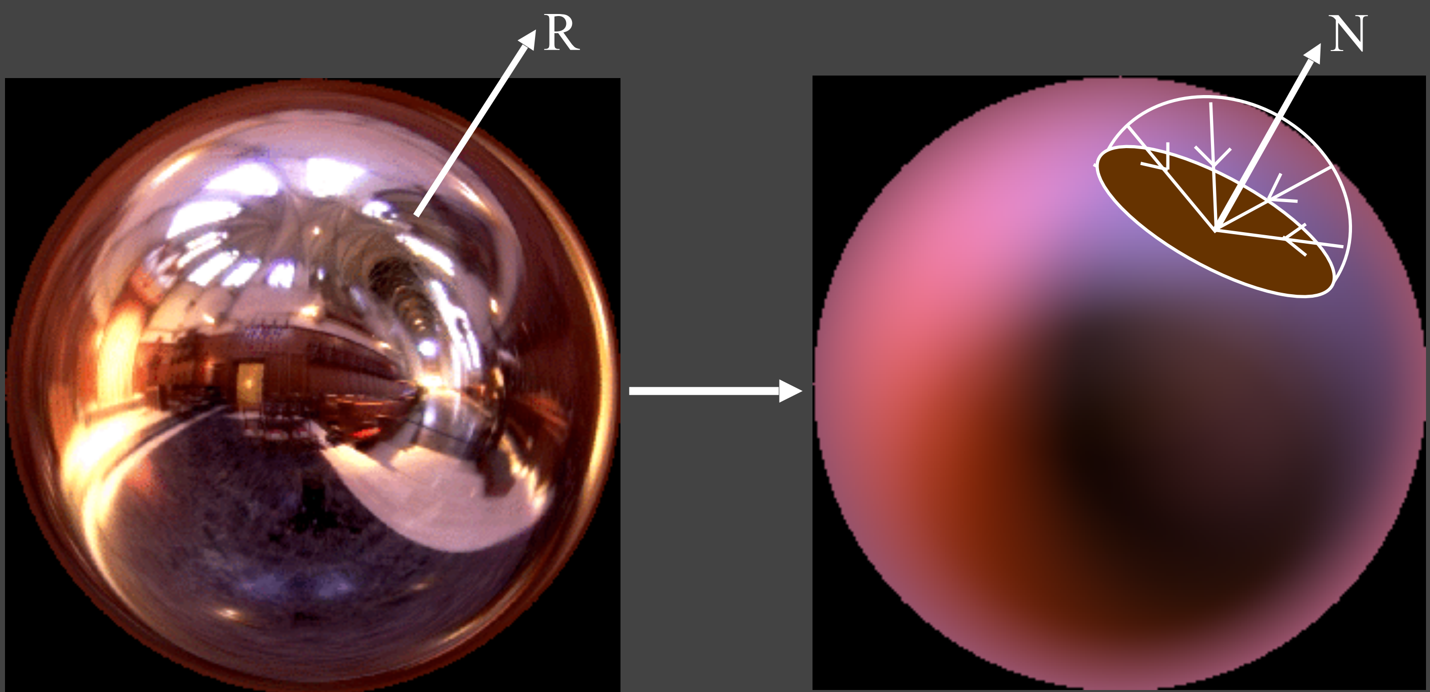
使用（截断）系数和基函数
恢复原始函数



Next Slides Courtesy of Prof. Ravi Ramamoorthi from UC San Diego

Recall: Prefiltering

- Prefiltering + single query = no filtering + multiple queries



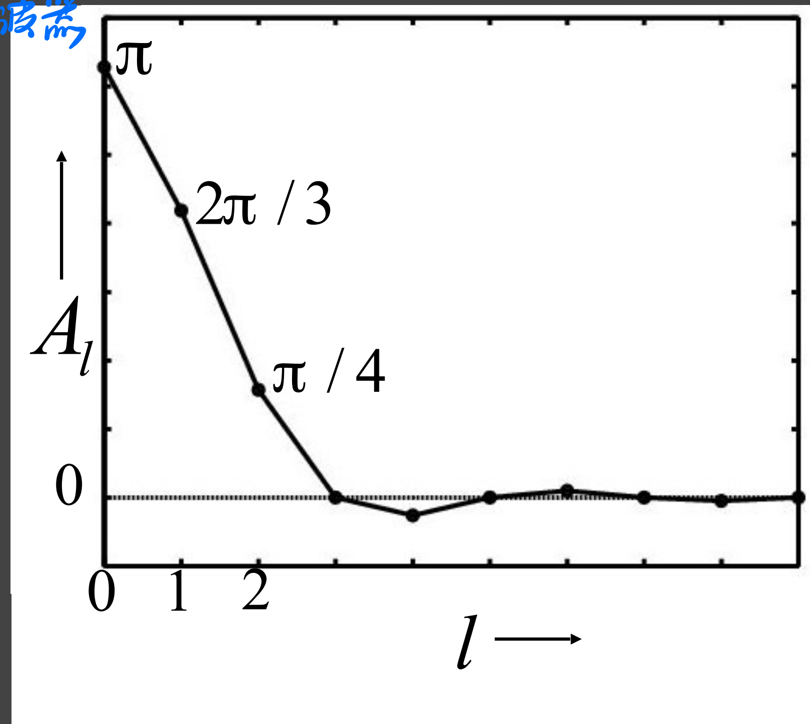
Analytic Irradiance Formula

漫反射BRDF的作用类似于低通滤波器

- Diffuse BRDF acts like a low-pass filter

$$E_{lm} = A_l L_{lm}$$

漫反射的BRDF只需
前3阶的SH就可描述



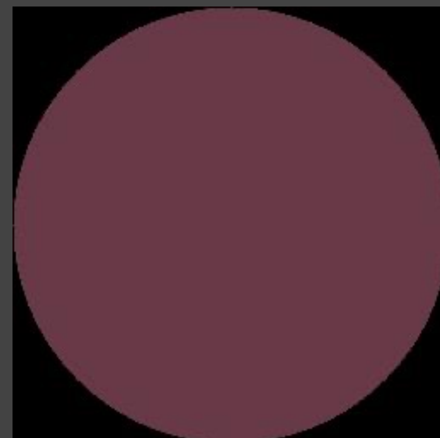
Ramamoorthi and Hanrahan 01
Basri and Jacobs 01

$$A_l = 2\pi \frac{(-1)^{\frac{l}{2}-1}}{(l+2)(l-1)} \left[\frac{l!}{2^l \left(\frac{l}{2}!\right)^2} \right] \quad l \text{ even}$$

9 Parameter Approximation

精确图像

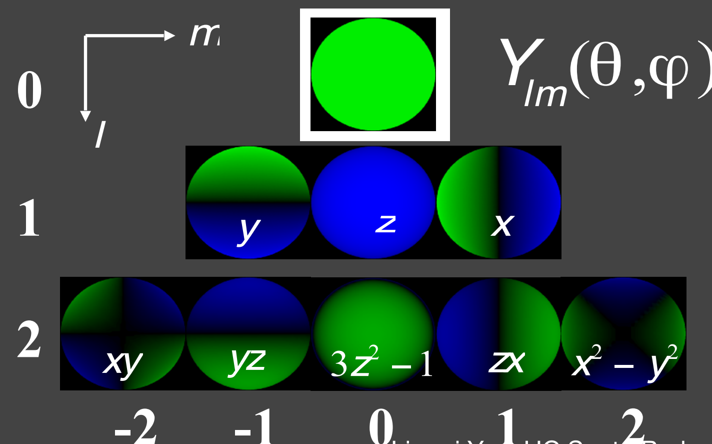
Exact image



Order 0
1 term

用前1阶的SH描述
<描述光照>

RMS error = 25 %



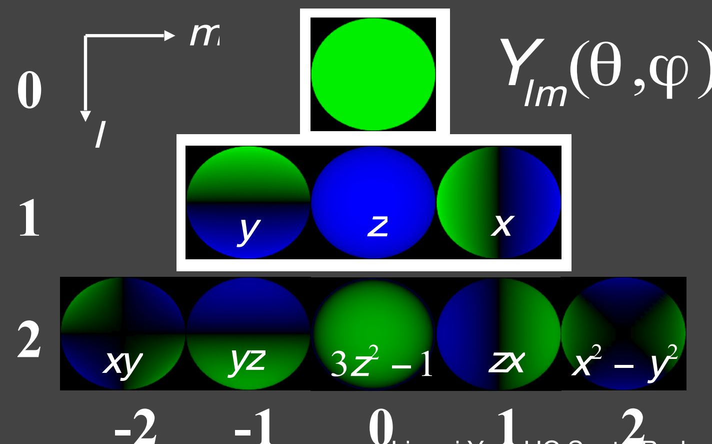
9 Parameter Approximation

Exact image

Order 1
4 terms

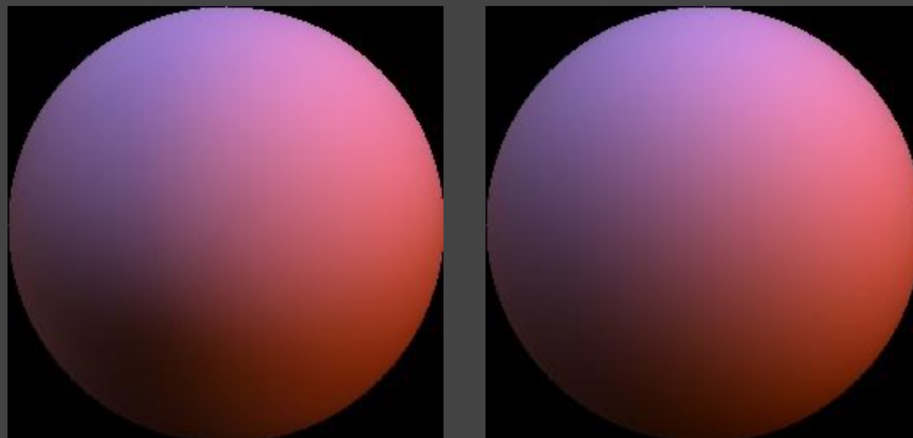
前2阶

RMS Error = 8%



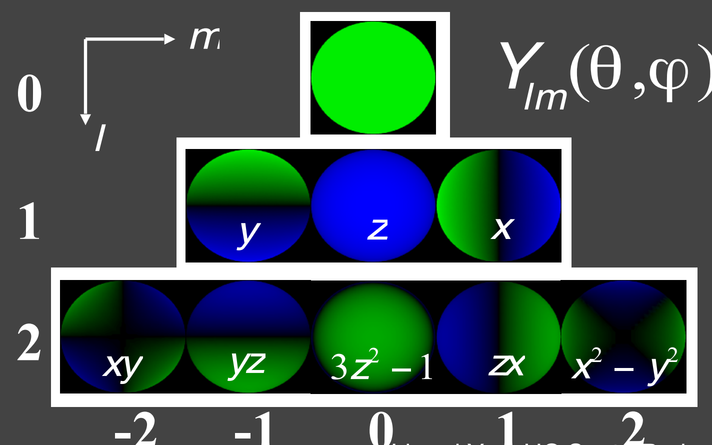
9 Parameter Approximation

Exact image



Order 2
9 terms

前3阶



RMS Error = 1%

For any illumination, average error < 3% [Basri Jacobs 01]

对于任何照明，只要物体是 diffuse 就可以用前3阶的SH来描述光照

In Real-Time Rendering (FYI)

$$E(n) = n^t M n$$

Simple procedural rendering method (no textures)

- Requires only matrix-vector multiply and dot-product
- In software or NVIDIA vertex programming hardware

Widely used in Games (AMPED for Microsoft Xbox), Movies (Pixar, Framestore CFC, …)

```
surface float1 irradmat (matrix4 M, float3 v) {
        float4 n = {v , 1} ;
        return dot(n , M*n) ;
}
```

A Brief Summarization

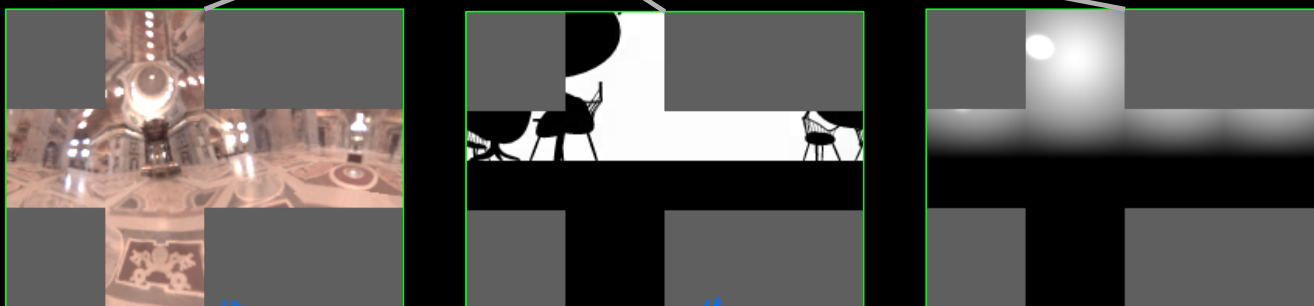
- We have seen the usage of basis functions
  - Representing any function (with enough #basis)
  - Keeping a certain frequency contents (with a low #basis)
  - Reducing integrals to dot products (?)
- But here it's still shading from environment lighting
  - No shadows yet
- Next: Precomputed Radiance Transfer (PRT)
  - Handles shadows and global illumination!
  - What's the cost?

Next Slides Courtesy of Prof. Kun Xu from Tsinghua University

Rendering under environment lighting

$$L(\mathbf{o}) = \int_{\Omega} \underbrace{L(\mathbf{i})}_{\text{lighting}} \underbrace{V(\mathbf{i})}_{\text{visibility}} \underbrace{\rho(\mathbf{i},\mathbf{o}) \max(0, \boldsymbol{n} \cdot \mathbf{i})}_{\text{BRDF}} \mathrm{d}\mathbf{i}$$

lighting visibility BRDF



球面函数 球面函数

- **i**/**o**: incoming/view directions
- Brute-force computation
  - Resolution: 6*64*64
  - Needs 6*64*64 times for each point!

预计算 Radiance 传递

Precomputed Radiance Transfer (PRT)

- Introduced by Sloan in SIGGRAPH 2002
  - *Precomputed Radiance Transfer for Real-Time Rendering in Dynamic, Low-Frequency Lighting Environments* [Sloan 02]

Basic idea of PRT [Sloan 02]

PRT
假设场景中只有光照发生变化，其他皆不变.

shading point 本身性质

$$L(\mathbf{o}) = \int_{\Omega} \underbrace{L(\mathbf{i})}_{\text{lighting}} \underbrace{V(\mathbf{i})\,\rho(\mathbf{i},\mathbf{o})\max(0, \boldsymbol{n} \cdot \mathbf{i})}_{\text{light transport}} \mathrm{d}\mathbf{i}$$

由 L(o) 确定

lighting　　light transport

都是球面函数构成.

用基函数近似 lighting

- Approximate lighting using basis functions
  - $L(\mathbf{i}) \approx \sum l_i B_i(\mathbf{i})$
- Precomputation stage 预计算 light transport
  - compute light transport, and project to basis function space
- Runtime stage
  - dot product (diffuse) or matrix-vector multiplication (glossy)

Diffuse Case

$$L(\mathbf{o}) = \rho \int_{\Omega} L(\mathbf{i}) V(\mathbf{i}) \max(0, \boldsymbol{n} \cdot \mathbf{i}) \mathrm{d}\mathbf{i}$$

BRDF
常数

$$L(\mathbf{i}) \approx \sum \underbrace{l_i}_{\text{lighting coefficient}} \underbrace{B_i(\mathbf{i})}_{\text{basis function}}$$

场景不变

$$L(\mathbf{o}) \approx \rho \sum l_i \int_{\Omega} \mathrm{B}_i(\mathbf{i}) V(\mathbf{i}) \max(0, \boldsymbol{n} \cdot \mathbf{i}) \mathrm{d}\mathbf{i}$$

环境光贴图

Precompute

$$L(\mathbf{o}) \approx \rho \sum l_i T_i$$

- Reduce rendering computation to dot product

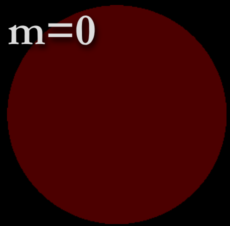
将渲染计算减少为卷积

Basis functions $B(\mathrm{i})$

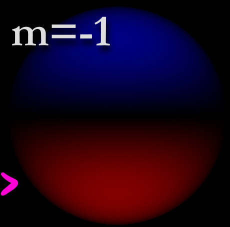
球谐函数

- Spherical Harmonics (SH)
- SH have nice properties:
  - orthonormal 正交性.
  - simple projection/reconstruction
  - simple rotation 支持旋转 <旋转原图 <=> 旋转任意一个 basis>
    旋转后可以被同阶的SH基函数线性组合得到.
  - simple convolution
  - few basis functions: low freqs
    低频.

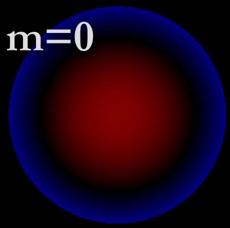
l=0 m=0



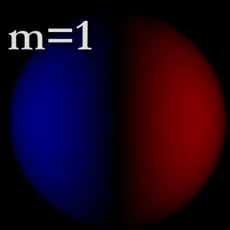
l=1 m=-1

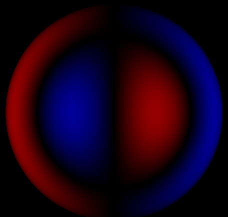


l=1 m=0



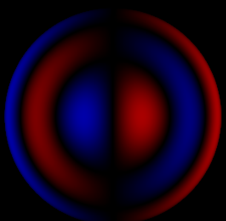
l=1 m=1





l=2 m=1

l=3 m=-1



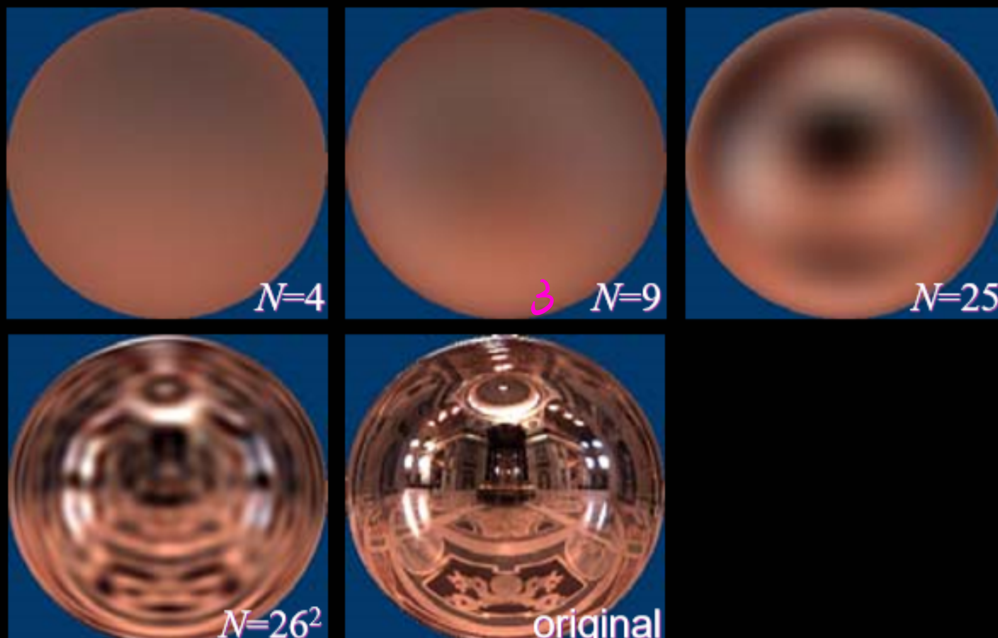
l=3 m=2

l=4 m=-2

Basis functions $B(\mathbf{i})$

- Spherical Harmonics (SH)
- Light Approximation Examples

前2阶



5阶

26阶

Low frequency

Basis functions $B(\mathbf{i})$

正交基 SH.

- SH is orthonormal, we have:

$$\int_{\Omega} B_i(\mathbf{i}) \cdot B_j(\mathbf{i}) \mathrm{d}\mathbf{i} = \mathbf{1} \quad (\mathbf{i} = j)$$

$$\int_{\Omega} B_i(\mathbf{i}) \cdot B_j(\mathbf{i}) \mathrm{d}\mathbf{i} = \mathbf{0} \quad (\mathbf{i} \neq j)$$

正交性.

Basis functions $B(\mathbf{i})$

Original space

lighting

$$L(\mathbf{i}) \approx \sum l_i B_i(\mathbf{i})$$

SH space

lighting coefficients

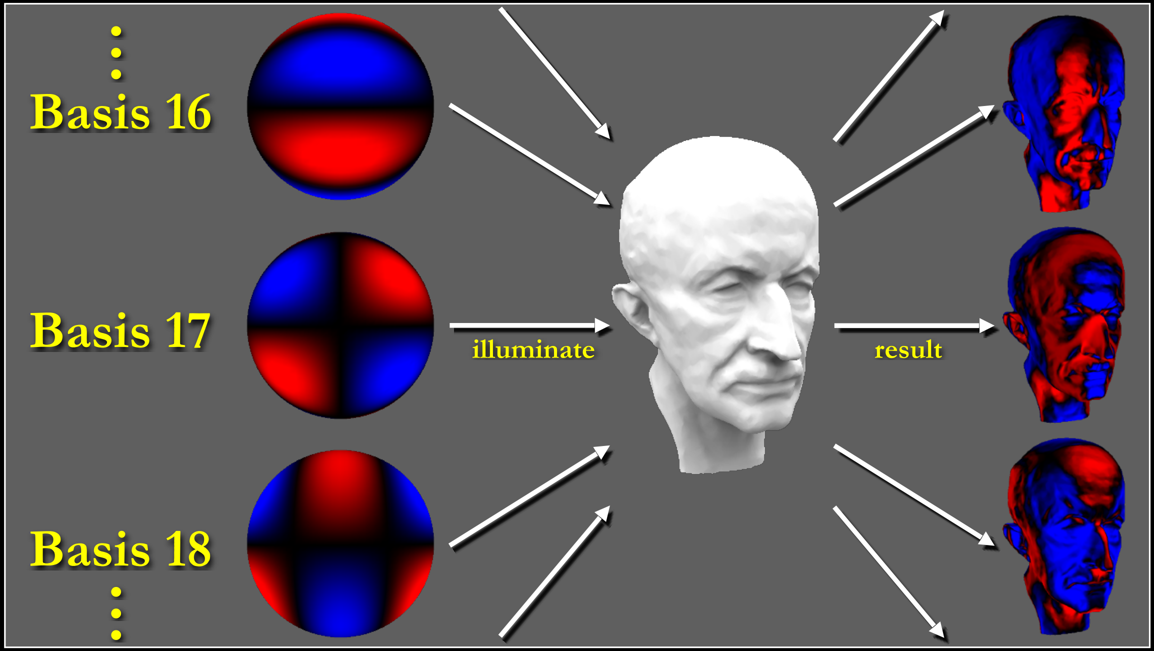
- Projection to SH space $$l_i = \int_{\Omega} L(\mathbf{i}) \cdot B_i(\mathbf{i})\,\mathrm{d}\mathbf{i}$$
- Reconstruction $$L(\mathbf{i}) \approx \sum l_i B_i(\mathbf{i})$$

Precomputation

light transport $$T_i \approx \int_{\Omega} \underline{B_i(\mathbf{i})}\, \underline{V(\mathbf{i}) \max(0, \boldsymbol{n} \cdot \mathbf{i})}\, d\mathbf{i}$$

对每个环境光基函数计算物体上每个点的光照结果.

- No shadow/ shadow/ inter-reflection



Run-time Rendering

$$L(\mathbf{o}) \approx \rho \sum l_i T_i$$

- Rendering at each point is reduced to a dot product
  - First, project the lighting to the basis to obtain l_i
  - Or, rotate the lighting instead of re-projection
  - Then, compute the dot product
- Real-time: easily implemented in shader

Diffuse Rendering Results

No Shadows

Shadows

Shadows+Inter

light 多次 Bounce

Questions?

Next Lecture

- Real-time global illumination cont.
  - ~~By precomputation~~
  - In 3D (LPV, VXGI, RTXGI, etc.)
  - In the image space (SSR, etc.)

[VXGI by NVIDIA]

Thank you!