

# Real-Time High Quality Rendering

GAMES202, Lingqi Yan, UC Santa Barbara

## Lecture 4: Real-Time Shadows 2

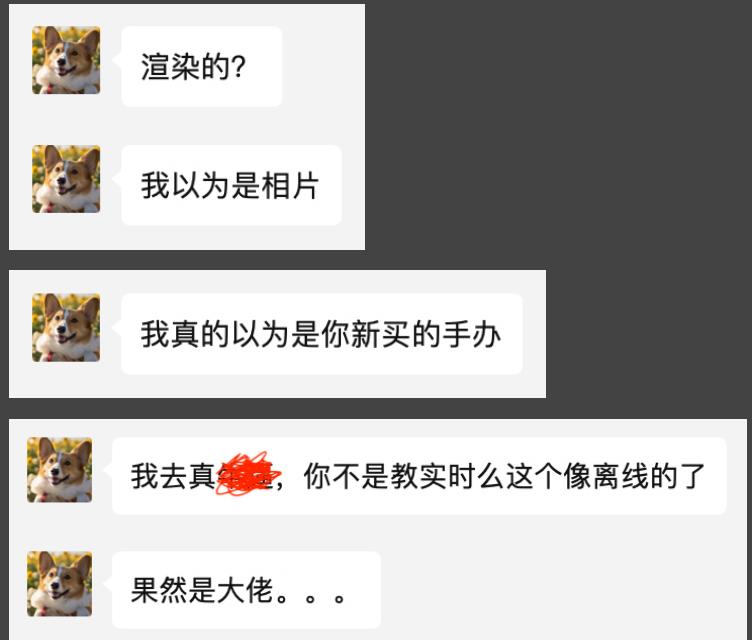


# Announcement

- Assignment 1 will be released today or tomorrow
  - Due in 1.5 weeks

# Before Assignment 1 is Released

- @Maxwell has already implemented a stunning version



— Edward (Shiqiu) Liu, NVIDIA

# Announcement

- Assignment 1 will be released today or tomorrow
  - Due in 1.5 weeks
- Quite a lot to discuss today
  - WHY?

# ★★ Mysterious Power ★★



不可能中的呀

[Maru vs. Solar, IEM 2021]

# ★★ Great News ★★

- **4 papers accepted to SIGGRAPH 2021!**
  - Thanks to the all-star students and collaborators
  - Might be the winner of most #papers in this SIGGRAPH in the **rendering** domain
  - Career #SIGGRAPH (including SIGGRAPH Asia and ToG) papers has reached 20
- Some deeper thoughts

# ★★ Greater News ★★

- I got my PlayStation 4 BACK!!!!!!
  - My story
- Next Goals
  - 30 to unlock PlayStation 5!
  - La Campanella to unlock Steinway & Sons!



[Sony PlayStation 4]

# ★★ Food for Thought ★★

- My industrial friends
  - I need your help **immediately**
  - \$50,000 can support a Ph.D student for 1 year
- Various ways
  - **Fellowships** and **gift funding** are certainly welcomed
  - I am also happy to be your **consultant** except you-know-who
  - ...



<https://modkat.com/blogs/modkat-purrr-how-to-prevent-or-stop-your-cat-from-begging-for-food>

## 11 Acknowledgements

The authors are indebted to Alexander Alvarado and Jared Reisweber, who created the cat and chipmunk models respectively. We also thank Matt Chiang and Brent Burley for helpful discussions. The fur samples are donated by Lyons and O'Haver Taxidermy. This work was supported in part by ~~NSF grant 1451828, Intel, and gifts from Pixar, Sony, Adobe and QualComm to the UC San Diego Center for Visual Computing.~~

YOU!

# Last Lecture

- Shadow mapping
  - Issues from shadow mapping and solutions
- The math behind shadow mapping
- Percentage closer soft shadows

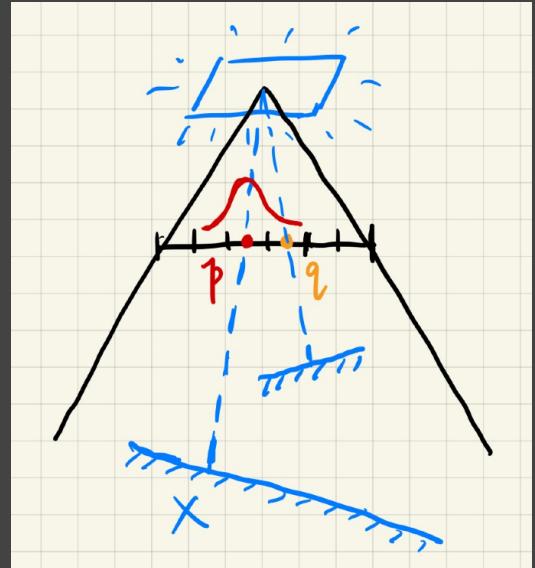
# Today

- More on PCF and PCSS
- Variance soft shadow mapping VSSM
- MIPMAP and Summed-Area Variance Shadow Maps
- Moment shadow mapping

# A Deeper Look at PCF

- The math behind PCF
- Filter / convolution:

$$[w * f](p) = \sum_{q \in \mathcal{N}(p)} w(p, q)f(q)$$



- In PCSS

$$V(x) = \sum_{q \in \mathcal{N}(p)} w(p, q) \cdot \chi^+ [D_{\text{SM}}(q) - D_{\text{scene}}(x)]$$

*chi* *阶梯函数* {  $\begin{cases} > 0 & \Rightarrow 1 \\ < 0 & \Rightarrow 0 \end{cases}$  }

# A Deeper Look at PCF

$$V(x) = \sum_{q \in \mathcal{N}(p)} \underbrace{w(p, q)}_{\text{weights}} \cdot \chi^+ [D_{\text{SM}}(q) - D_{\text{scene}}(x)]$$

- Therefore, PCF is not filtering the shadow map then compare

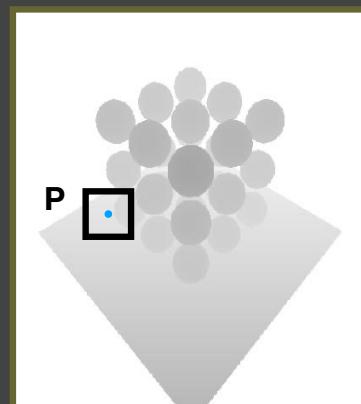
$$V(x) \neq \chi^+ \{ \underbrace{[w * D_{\text{SM}}](q)}_{\text{PCF result}} - D_{\text{scene}}(x) \}$$

- And PCF is not filtering the resulting image with binary visibilities

$$V(x) \neq \sum_{q \in \mathcal{N}(p)} \underbrace{w(p, q)}_{\text{weights}} \underbrace{\chi^+}_{\text{visibilities}} \underbrace{V(q)}_{\text{PCF result}}$$

# Revisiting PCSS

- The complete algorithm of PCSS
  - Step 1: Blocker search  
(getting the average blocker depth in a certain region)
  - Step 2: Penumbra estimation  
(use the average blocker depth to determine filter size)
  - Step 3: Percentage Closer Filtering PcF.
- Which step(s) can be slow?
  - Looking at every texel inside a region (steps 1 and 3)
  - Softer -> larger filtering region -> slower



方差软阴影贴图 VSSM

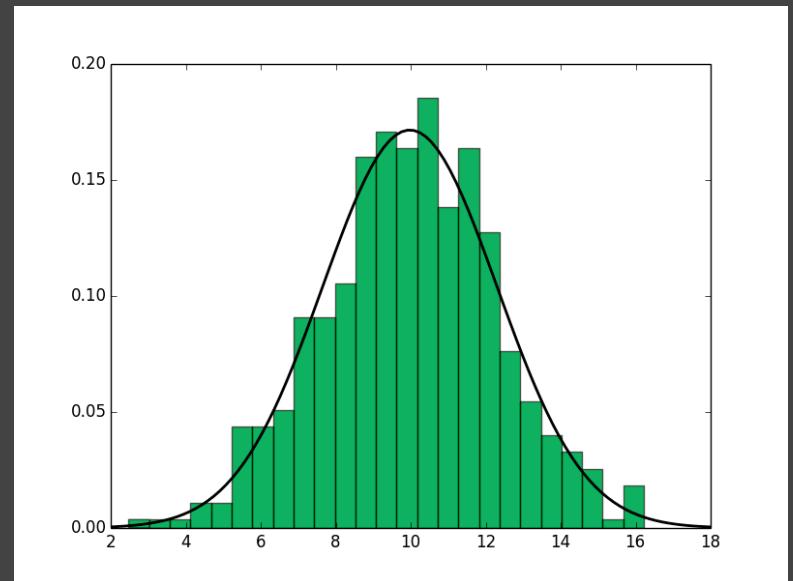
# Variance Soft Shadow Mapping

- Fast blocker search (step 1) and filtering (step 3)  
[Yang et al.]
- Let's think from “percentage closer” filtering
  - The percentage of texels that are in front of the shading point, i.e.,
  - how many texels are closer than  $t$  in the search area, i.e.,
  - how many students did better than you in an exam

# Variance Soft Shadow Mapping

- How many students did better than you in an exam?
  - Using a histogram -> accurate answer!
  - Using a **Normal distribution** -> approximate answer!
  - What do you need to define a normal distribution?

均值 方差  
(期望)



# Variance Soft Shadow Mapping

- Key idea
    - Quickly compute the mean and variance of depths in an area

- Mean (average)

- # Hardware MIPMAPing

- ## 2 Summed Area Tables (SAT)

- Variance 方差.

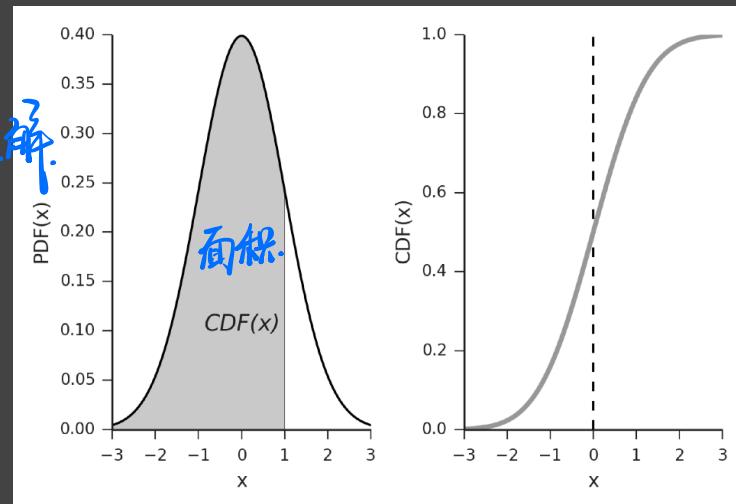
$$\text{Var}(X) = E(X^2) - E^2(X)$$

- So you just need the mean of ( $\text{depth}^2$ )

- Just generate a “square-depth map” along with the shadow map!

# Variance Soft Shadow Mapping

- Back to the question
  - Percentage of texels that are closer than the shading point
  - You want to calculate the shade's area
  - Accurate answer exists (hint: What's the CDF of a Gaussian PDF?)



[<http://work.thaslwanter.at/Stats/html/statsDistributions.html>]

# Variance Soft Shadow Mapping

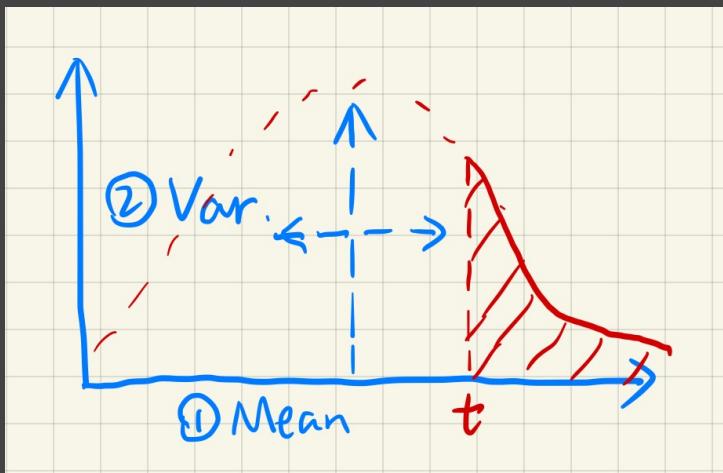
- It doesn't have to be too accurate!

- Chebychev's inequality (one-tailed version, for  $t > \mu$ )  
切比雪夫不等式

$$P(x > t) \leq \frac{\sigma^2}{\sigma^2 + (t - \mu)^2}$$

$\mu$  : mean

$\sigma^2$  : variance



Doesn't even assume  
Gaussian distribution!

不是由高斯分布

\*  $t$  越远大于 mean  
否则准确性很低

# Variance Soft Shadow Mapping

三个步骤 PCF < visibility>

- Performance
  - Shadow map generation:  
    - “square depth map”: parallel, along with shadow map, #pixels
  - Anything else?
- Run time
  - Mean of depth in a range:  $O(1)$   $\bar{d}$   $\text{得平均值}$
  - Mean of depth square in a range:  $O(1)$   $\bar{d}^2$   $\text{得方差}$
  - Chebychev:  $O(1)$   $\text{得到在该区域内有多少像素不可见从而有缺口。}$
  - No samples / loops needed!  $\text{得 visibility}$
- Step 3 (filtering) solved perfectly (?)

# Variance Soft Shadow Mapping

Step 1: 得到该区域内的所有遮挡物的平均深度

- Back to Step 1: blocker search (within an area)
  - Also require sampling (loop) earlier, also inefficient
  - The average depth of blockers ✓
  - Not the average depth  $z_{avg}$  ✗
  - The average depth of those texels whose depth  $z < t$
- Key idea
  - Blocker ( $z < t$ ), avg.  $z_{occ}$
  - Non-blocker ( $z > t$ ), avg.  $z_{unocc}$

4	4	8	8	8
4	4	8	8	8
4	4	6	8	8
6	6	6	8	9
8	8	8	9	9

# VSSM Variance Soft Shadow Mapping

- Key idea

- Blocker ( $z < t$ ), avg.  $z_{occ}$  (we want to compute)
- Non-blocker ( $z > t$ ), avg.  $z_{unocc}$

- $$\cancel{\star} \quad \frac{N_1}{N} z_{unocc} + \frac{N_2}{N} z_{occ} = z_{Avg}$$

4	4	8	8	8
4	4	8	8	8
4	4	6	8	8
6	6	6	8	9
8	8	8	9	9

- Approximation:  $N_1 / N = \underbrace{P(x > t)}$ , Chebychev!
- Approximation:  $N_2 / N = \underbrace{1 - P(x > t)}$
- $z_{unocc}$ , we really don't know
- Approximation:  $z_{unocc} = t$  (i.e. shadow receiver is a plane)  
*假设本遍挡物深度都和当前 shader point 相等*

- Step 1 solved with negligible additional cost

# Variance Soft Shadow Mapping



[[https://developer.nvidia.com/gpugems/GPUGems3/gpugems3\\_ch08.html](https://developer.nvidia.com/gpugems/GPUGems3/gpugems3_ch08.html)]

# Questions?

# Today

- More on PCF and PCSS
- Variance soft shadow mapping
- MIPMAP and Summed-Area Variance Shadow Maps
- Moment shadow mapping

# Variance Soft Shadow Mapping

得到  $E[X]$ ,  $E[X^2]$

- Key observation: in order to accelerate

为了加速  
需要快速从任意范围  $\langle \mu, \sigma \rangle$  抓取.

- Need to quickly grab  $\mu$  and  $\sigma$  from an arbitrary range (rectangular)

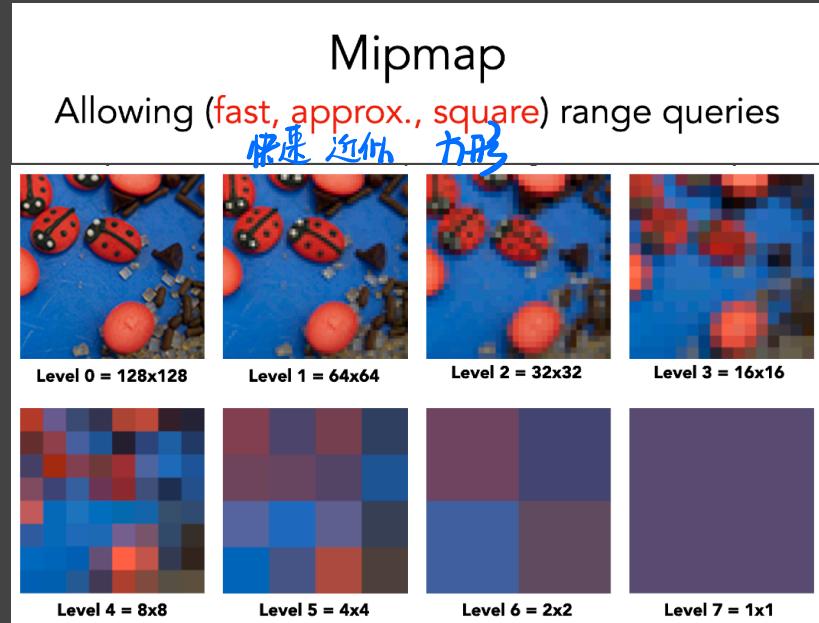
$$P(x \geq t) \leq p_{max}(t) \equiv \frac{\sigma^2}{\sigma^2 + (t - \mu)^2}$$

- For the average  $\mu$ , this is rectangular range query

- Can be handled by both MIPMAP and Summed Area Table (SAT)

# MIPMAP for Range Query

- Recall: MIPMAP



- Note: still approximate even with trilinear interpolation  
*即使使用三线性插值仍为近似.*

## Summed Area Table

# SAT for Range Query

- Classic data structure and algorithm (prefix sum)
  - In 1D:

Input:

1	3	5	3	7	1	3	8	6	4
---	---	---	---	---	---	---	---	---	---



sum = ?

“预处理”

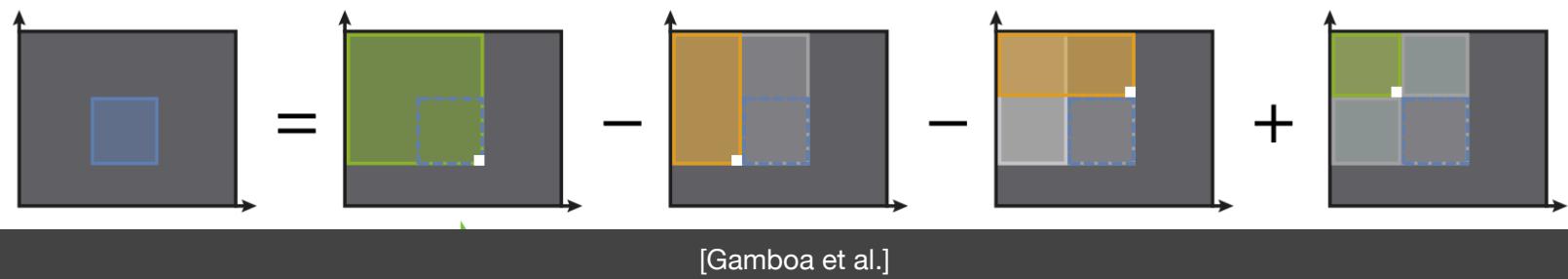
SAT:

1	4	9	12	19	20	23	31	37	41
---	---	---	----	----	----	----	----	----	----

# SAT for Range Query

- Classic data structure and algorithm

- In 2D:



- Note: accurate, but need  $O(n)$  time and storage to build
  - Storage might not be an issue
  - Can we speed up building SAT?

# Questions?

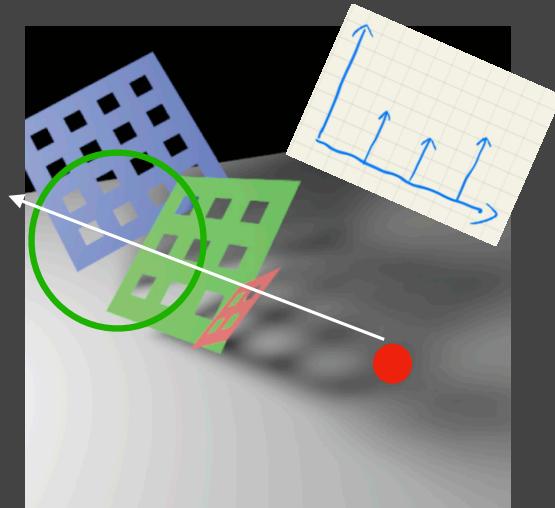
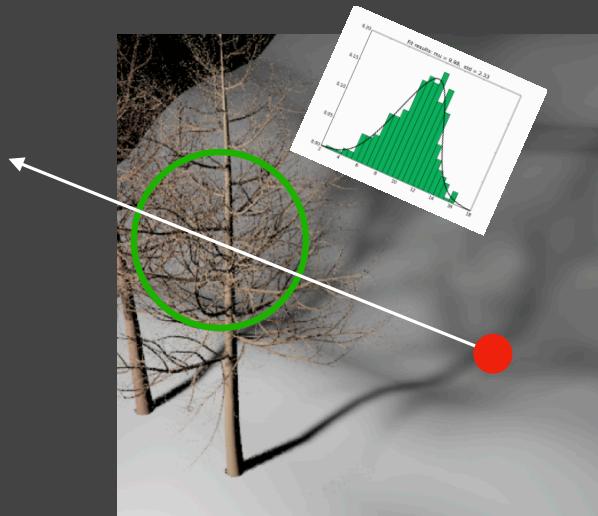
# Today

- More on PCF and PCSS
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# Revisit: VSSM

- Is a normal distribution always good enough to approximate the distribution of fragments' distances?

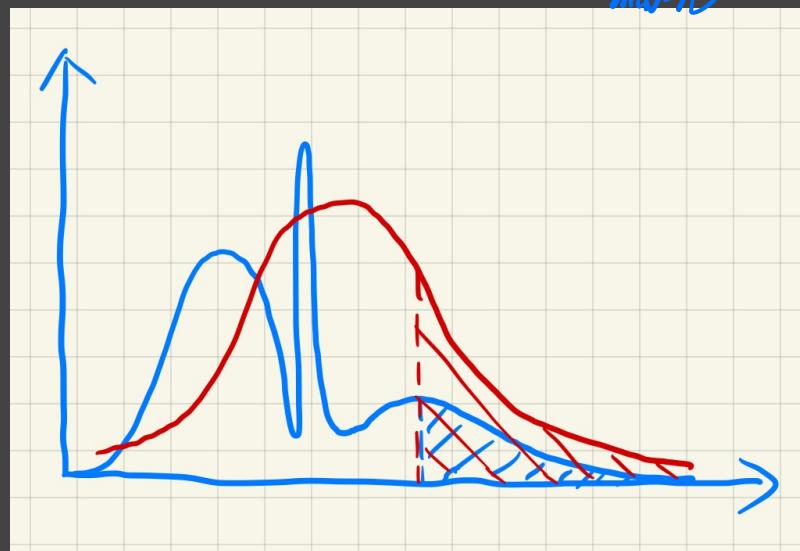
碎片的距离分布 用正态分布模拟不太好.



# Revisit: VSSM

深度分布不准确

- Issues if the depth distribution is inaccurate
  - Overly dark: may be acceptable 深暗 ✓
  - Overly bright: LIGHT LEAKING! 太亮 XXX  
漏光

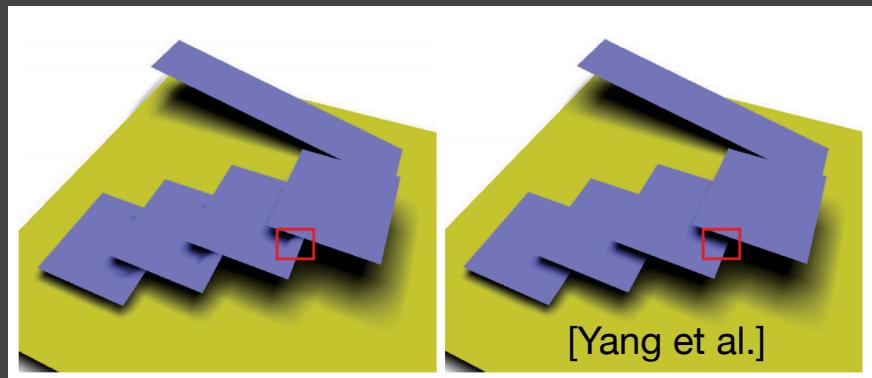


# Revisit: VSSM

- Limitations?
  - Light leaking
  - non-planarity artifact 非平面性  
伪影
- Chebychev is to blame?
  - Only valid when  $t > z_{avg}$
- Can we do better?



[[https://developer.nvidia.com/gpugems/GPUGems3/gpugems3\\_ch08.html](https://developer.nvidia.com/gpugems/GPUGems3/gpugems3_ch08.html)]



# Moment Shadow Mapping

- Goal

更准确地表示分布

- Represent a distribution more accurately  
(but still not too costly to store)

- Idea

用更多阶的矩 描述分布

- Use higher order moments to represent a distribution

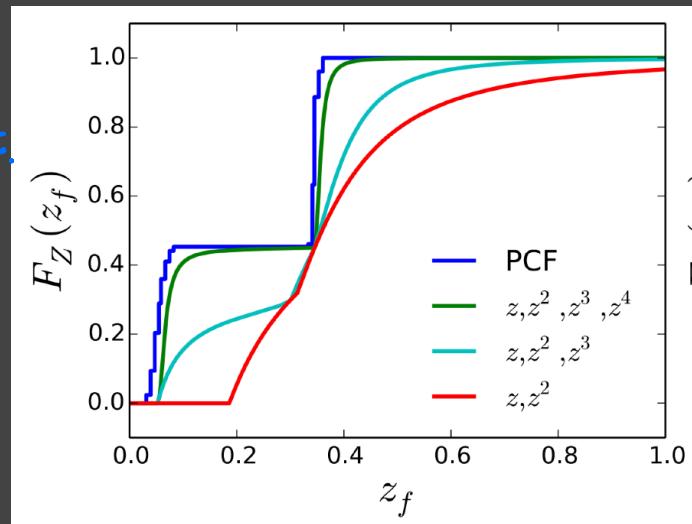
# Moment Shadow Mapping

- Moments
  - Quite a few variations on the definition
  - We use the simplest:  
 $x, x^2, x^3, x^4, \dots$
  - So, VSSM is essentially using the **first two** orders of moments

VSSM 本质上 使用前两个的矩

# Moment Shadow Mapping

- What can moments do?
  - Conclusion: 前  $m$  阶矩可以表示  $m/2$  个台阶的阶跃函数。  
first  $m$  orders of moments can represent a function with  $m/2$  steps
  - Usually, 4 is good enough to approximate the actual CDF of depth dist.  
前4阶矩是拟合深度与实际CDF.
  - How to restore a CDF whose moments match the given moments



[Peters et al., Moment Shadow Mapping]

# Moment Shadow Mapping

- Moment Shadow Mapping
  - Extremely similar to VSSM 与VSSM相似
  - When generating the shadow map, record  $z, z^2, z^3, z^4$
  - Restore the CDF during blocker search & PCF  
存储 CDF (> 当进行 blocker search 和 PCF 时)  
step 01

# Moment Shadow Mapping

- Pro: very nice results
- Cons
  - Costly storage (might be fine)
  - Costly performance (in the reconstruction)  
花费性能。



Variance shadow mapping, 1.15ms



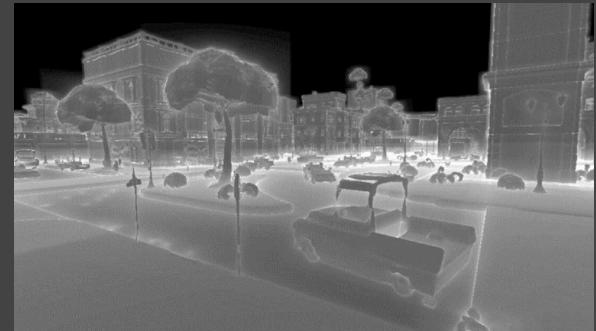
Moment shadow mapping, 1.46ms

[Peters et al., Moment Shadow Mapping]

# Questions?

# Next Lecture

- Distance Field Soft Shadows
- Real-Time Environment Lighting
  - Prefiltering
  - The split sum method



[https://docs.unrealengine.com/en-US/BuildingWorlds/  
LightingAndShadows/  
RayTracedDistanceFieldShadowing/index.html](https://docs.unrealengine.com/en-US/BuildingWorlds/LightingAndShadows/RayTracedDistanceFieldShadowing/index.html)



<https://cdn2.unrealengine.com/Resources/files/2013SiggraphPresentationsNotes-26915738.pdf>

Thank you!