AXERERA

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Math. 325 Mid-exam.(Reg)

<table>
<tr><td rowspan="8">Name ____

ID.No. ________________

Section : ______________

Instructor's Name: ____________________________

Date: may18, 2011

Time allowed **2** hrs</td><td colspan="3">For instructor's use only</td></tr>
<tr><td colspan="2">Part I</td><td></td></tr>
<tr><td rowspan="4">Part II</td><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td colspan="2">Total</td><td></td></tr>
</table>

► This exam booklet contains 6 short answer questions and 4 workout problems.

Part I. *Give the precise and simplified short answer on the space provided.* [1.5 points each]

1. If A and B are orthogonal vectors such that $\|A\| = \sqrt{3}$ and $\|B\| = 1$, then the angle between vectors $A+B$ and $A-B$ is ____________

2. If a nonzero vector A makes $30^0, 60^0$ & 45^0 from the positive $x-axis$, $y-axis$ and $z-axis$ respectively and $\|A\| = \sqrt{3}$, then A = ____________

3. If $A = \begin{pmatrix} 1 & \cos\beta & \cos\theta \\ \cos\beta & 1 & \cos\alpha \\ \cos\theta & \cos\alpha & 1 \end{pmatrix}$ where $\cos\alpha, \cos\beta$ & $\cos\theta$ are directional cosines of a vector v in $\mathbb{R}^3$. Then $\det A =$ ____________

4. Given matrices A and B of size 3x3. If detA = 4 and detB = -3 ,then $\det\left(A^{-1}({}^tB)^2 adjA\right) =$__________________

5. Given vectors A=(1,1,1), B=(2,1,0) and C=(0,3,1), then $\left\|proj_A(2B+C)\right\| =$ ________

6. The eigenvalues of the matrix A = $\begin{pmatrix} 2 & c \\ 3 & -1 \end{pmatrix}$ are -4 and c-1.

   Then c = _______________

7. If $A = \begin{pmatrix} k-3 & 2 & 1 \\ -1 & k & 1 \\ 0 & 0 & 1 \end{pmatrix}$,then the linear system AX=0 have a non-trivial solution for

   k =________________

8. If the matrix $\begin{pmatrix} 0 & -1 & 1 & 2 \\ 1 & x & 3 & -1 \\ -1 & y & 0 & z \\ -2 & 1 & -2 & 0 \end{pmatrix}$ is skew-symmetric, then the values of x, y and z

   respectively are ________, _______ and __________

9. The volume of the parallelepiped spanned by vectors A=(-2,2,1) ,B =(0,1,0) and C = (-4, 3, 2) is ________________

10. For a positive integer n if

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n - \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 0 & 6 \\ -6 & 0 \end{pmatrix}$$, then $n =$ ______________

Part II. *Attempt all the problems and show the required steps clearly.* *[25 points]*

1. Show that $W = \{(x, y, z) \mid x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$. (3 pts.)

2. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$. Find

a) all eigenvalues and corresponding eigenvectors of matrix A. (3 pts.)

b) inverse of A. (3 pts.)

3. a) Find the equation of the line l of intersection of the planes $\pi_1 : 2x - y + z = 2$ and $\pi_2 : 3x + y + z = 2$. (3 pts.)

b) Find the distance between the line l in part (a) and the point Q = (1, 0, 0). (2 pts.)

4. Show that the vectors $v_1 = (-1,1,1), v_2 = (0,0,1)$ & $v_3 = (2,1,1)$ form basis of $\mathbb{R}^3$.Find coordinate vector of (1, 2,3) with respect to this basis. (4 pts.)

5. (a) Determine all values the constant b for which the system

$$x + y - 2z = 4,$$
$$3x + 5y - 4z = 16,$$
$$2x + 3y - bz = 8,$$

has i) a unique solution, (2 pts.)
ii) an infinite number of solutions, (2 pts.)

(b) Find the solution sets in part (a) (i) and (ii) (3 pts.)