Addis Ababa Universtity
Department of Mathematics
Applied Mathematics I (Math. 231B)

Mid-Semester Examination

Date: January 10, 2004
Time allowed: $2\frac{1}{2}$ hrs.

Name : ______________________ ID. No.: ____________________

Instructor's Name (OR Section):______________________

INSTRUCTION: THIS EXAMINATION HAS TWO PARTS. ATTEMPT ALL THE QUESTIONS AND PROBLEMS IN BOTH PARTS.

PART I : In this part there are 14 short answer questions and each correct answer to a corresponding blank space worths 1.5 points. Give a short and precise answer on the space provided.

PART II: In this part there are 5 workout problems. Write down your solution with all the necessary steps for each of the questions on the space provided as neatly as possible. Please note that unreadable answers may not be corrected.

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FOR INSTRUCTOR'S USE ONLY

Part I	Part II					Total
	1	2	3	4	5	

I: Short Answer Questions.

1. Let $\mathbf{v}$ and $\mathbf{w}$ be orthogonal vectors in $\mathbb{R}^n$ such that $\|\mathbf{v}\| = 3$ and $\|\mathbf{w}\| = 5$. If the vectors $(x\mathbf{v} + \mathbf{w})$ and $(x\mathbf{v} - 3\mathbf{w})$ are also orthogonal, then the value(s) of x is (are) ________________.

2. The parametric equation of the line through the point $(1, -1, 1)$ and orthogonal to the plane $\pi : x + 2y - 3z = 6$ is ________________.

3. The value of
$$\lim_{x \to -4^+} \frac{1 + |x - 4|}{|x|} \quad \text{is} \ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

4. If $\begin{pmatrix} x & 1 \\ -1 & 3y \end{pmatrix} - 2\begin{pmatrix} y-1 & 0 \\ 3 & 2-x \end{pmatrix} = \begin{pmatrix} y & 1 \\ -7 & -x \end{pmatrix}$, then $x =$ __________ and $y =$ __________.

5. The possible value(s) of α, where the vectors $\mathbf{u}_1 = (\alpha, 0, 1)$, $\mathbf{u}_2 = (1, 1, \alpha)$, and $\mathbf{u}_3 = (1, 2, \alpha)$ are linearly dependent in $\mathbb{R}^3$ is (are) ______________.

6. The area of the triangle whose vertices are (1,-1,2), (3,1,-1) and (0,2,1) is ____________ square units.

7. Let A and B be 4×4 matrices such that $\det(A) = 3$. If $\det(2BA^{-1}) = 8$, then $\det(B) =$ ______________.

8. Give two **nonzero** vectors A and B in $\mathbb{R}^3$ such that $A \times B = \mathbf{0}$. $A =$ ____________ and $B =$ ____________.

9. The rank of matrix $A = \begin{pmatrix} 4 & 1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 5 \end{pmatrix}$ is ____________.

10. Let $f(x)$ be the greatest integer function defined by $f(x) = [x] = n$ for $n \leq x \leq n + 1$, $n \in \mathbb{Z}$. then the set at which f is continuous is ____________.

11. Let $A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$, then the inverse of A, $A^{-1} =$ ____________.

12. The equation of the plane containing the point $P = (5, 3, 1)$ and the line $\ell : (3, 0, 3) + t(2, 0, -1)$ is ________________.

13. The value of
$$\lim_{x \to -\infty} \frac{\sqrt{x^2 + x + 1}}{\sqrt[3]{x^3 + 2x^2 + x + 1}} = \_\_\_\_\_\_\_\_\_\_\_\_.$$

14. If $A = 3i - j + 5k$ and $B = i + j - k$, then the three direction cosines of $A - B$ are ____________, ____________, and ____________.

II: Work Out Problems

1. Let $\ell_1 : x = 4 - 2t, y = -5 + 3t, 2z = -4 - 3t$ and $\ell_2 : 3x = 18 + 8t, y = -7 - 4t, z = 1 + 2t$ represent equations of lines in $\mathbb{R}^3$. Determine whether ℓ_1 and ℓ_2 are intersecting or parallel or none. Moreover find the intersecting point if intersecting, and the distance between them if parallel. **[5 Pts.]**

2. Show that $W = \{(x, y, z) | x = y \text{ and } 2y = z\}$ is a subspace of $\mathbb{R}^3$. **[3 Pts.]**

3. Determine the value(s) of k such that the system of equations

$$\begin{aligned} 2x + ky - z &= -2 \\ x + 2y + kz &= 1 \\ x \quad - 3z &= -3 \end{aligned}$$

has **[4 Pts.]**

(i) a unique solution
(ii) no solution
(iii) more than one solution.

4. Solve the following system of linear equations by using Crammer's rule. **[3 Pts.]**

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 + 2x_2 + x_3 &= 2 \\ x_1 + 3x_2 - x_3 &= -1 \end{aligned}$$

5. Find all the eigenvalues and the corresponding eigenvectors to the matrix

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}.$$

[4 Pts.]