AXERERA

ADDIS ABABA UNIVERSITY

DEPARTMENT OF MATHEMATICS

Thursday, January **15, 2008**
Time allowed **3** hours

APPLIED MATHEMATICS I (MATH 231B)

MID EXAM

Name: ______________________________

ID: ______________________________

Instructor's Name: ______________________

MAKE SURE THAT THERE ARE **10** PROBLEMS IN PART I, **6** PROBLEMS IN PART II A AND **7** PAGES INCLUDING THIS COVER PAGE.

DO **NOT** USE A CALCULATOR AND **NOT** USE YOUR OWN ROUGH PAPERS, WE WILL PROVIDE ENOUGH PAPERS.

FOR INSTRUCTORS' USE

PART I	PART II						TOTAL
	1	2	3	4	5	6	

PART I. WRITE YOUR SHORT AND SIMPLIFIED ANSWER ON THE SPACE PROVIDED. **(1.5 points each)**

1. If **A** = (1, -2, 3) and **B** = (3, 1, 2) are vectors in $\mathbf{R}^3$, then find scalars x and y such that $\mathbf{C} = x\mathbf{A} + y\mathbf{B}$ is a non-zero vector with $\mathbf{C} \cdot \mathbf{B} = 0$.

$x =$ ________ $y =$ __________

2. Suppose the angle between two vectors **A** and **B** in $\mathbf{R}^3$ be $\frac{\pi}{3}$ and $\|\mathbf{A}\| = 2$, $\|\mathbf{B}\| = 1$. If $\mathbf{C} = \mathbf{A} \times \mathbf{B}$, then $\|\mathbf{A} + \mathbf{B} - \mathbf{C}\| =$ _________________

3. Let **A** be an n x n matrix such that $\mathbf{A}^2 = \mathbf{A}$ and $\mathbf{A} \neq \mathbf{I}_n$. Then $\boldsymbol{det}\,(\mathbf{A}) =$ _________________

4. Find the value(s) of λ for which $\mathbf{A} = \begin{pmatrix} \lambda-3 & 0 & 3 \\ 0 & \lambda+2 & 0 \\ -5 & 0 & \lambda+5 \end{pmatrix}$ is invertible. _________________

5. If $f(x) = \begin{cases} 2 & , \quad x \le -1 \\ ax+b & , \quad -1 < x < 3 \\ -2 & , \quad x \ge 3 \end{cases}$ is continuous on **R**, then $a =$ ________ and $b =$ ________

6. $\lim\limits_{x\to\infty} \sqrt{x^2 - 3x + 2} - \sqrt{x^2 + 1} =$ _________________

7. $\lim\limits_{x\to 9^-} \left(\sqrt{9-x} - [\![x+1]\!]\right) =$ _________________

8. Find the distance from the point (3,-2,-1) to the line $x = 2 - 3t$, $y = 4 + 2t$, $z = 3 - 5t$ $t \in \mathbf{R}$.

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9. Find the area of the triangle with vertices $P_1(2,-2,1)$, P_2 (-1,0,3) and $P_3(5,-3,4)$

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10. Let $f(x) = \dfrac{\sqrt{x + c^2} - c}{x}$ for $c > 0$. Find $f(0)$ so that f is continuous on **R**.

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PART II. SHOW ALL THE NECESSARY STEPS IN THE FOLLOWING WORKOUT PROBLEMS.

1. Discuss the consistency or inconsistency and if the system is consistent find the solution set.

$$\begin{aligned} x + 2y - 3z &= 4 \\ 2x + 4y - 6z &= 8 \\ 3x + 6y - 9z &= 12 \end{aligned}$$

(5 points)

2. Find all the eigenvalues and corresponding eigenvectors of the matrix **(5 points)**

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 6 & -1 & 0 \\ -1 & 0 & -1 \end{pmatrix}$$

3. Prove Using the $(\varepsilon - \delta)$ definition of limit that $\lim_{x \to 2} 7x + 3 = 17$. **(4 points)**

4. Use the Intermediate Value Theorem to show that there exist at least one real root of the polynomial $x^3 + 2x = x^2 + 1$ between (0, 1). **(3 points)**

5. Find the equation of the plane containing the point P(0,4,-7) and the line with parametric equation x = 1 + t, y= -3 +2t, z= -2 -t . **(4 points)**

6. Find the inverse of the matrix $\mathbf{A} = \begin{pmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{pmatrix}$. **(4 points)**