AXERERA

Addis Ababa University
Department of Mathematics
Math 1041: Applied Mathematics I common Test

Date: December 18, 2021. Time Allotted: $1\&\frac{1}{2}$ hour.

Name: ANSWER KEY ID.NO. ______ section ______

Part I: Short Answer

Write the most simplified answer for each of the following questions on the space provided. Each correct answer is worth 1 points.

1. Find a real number α such that the vectors $A = (\alpha, 4, 2)$ and $B = (\alpha, 2\alpha, 8)$ are perpendicular. $\alpha =$ -4

2. If a nonzero vector A makes an angle of $30^o, 45^o$ and 60^o from the positive $x-$axis, $y-$axis and $z-$axis respectively and $\|A\| = \sqrt{2}$.
Then the vector $A =$ BOnus

3. The volume of a parallelpiped whose three edges are the vectors $u = i - j - 2k$, $v = 3i - 4j + k$ and $w = 3i + j + 2k$ is 36 sq. units

4. The point of intersection of the lines $2 - x = \dfrac{y-1}{2} = \dfrac{1-z}{3}$ and $x - 1 = \dfrac{y+1}{2} = \dfrac{z+2}{3}$ is (2,1,1).

5. The distance from the point $p(-1, 1, 6)$ to the line $x = 1, y - 2 = z + 1$ is
6 units

6. For what values of t is the set $\{(t, -1), (t+3, -2t)\}$ linearly dependent.
$t =$ -1 and $\frac{3}{2}$

7. The values of x and y so that the matrix $A = \begin{pmatrix} 0 & 3y-4 & 2 \\ 2x-3 & 0 & 2y-3 \\ -2 & 4x-7 & 0 \end{pmatrix}$ is skew symmetric $x =$ 2 and $y =$ 1

8. If A and B are square matrices of order 3 with $\det A = 4$ and $\det B = -3$ then
$\det(-2A^{-1}B^{2t}) =$ -18

9. The eigenvalue(s) of the matrix $\begin{pmatrix} 3 & 4 \\ 0 & 3 \end{pmatrix}$ is(are) 3

Part II: Workout

Answer each of the following questions. Show all the necessary steps.

1. Find the equation of the plane passing through $(-2, 0, 3)$ and is perpendidular to the vector $3i - j - k$. (2pts)

Solution: $N = (3, 1, -1)$ is a normal vector to the plane and $P(-2, 0, 3)$ is a point on the plane.

If $Q(x, y, z)$ is a point on the plane, then $\vec{PQ} \cdot N = 0$

This implies, $3(x+2) + 1(y-0) + (-1)(z-3) = 0$

$\Rightarrow 3x + y - z + 6 + 3 = 0$

$\Rightarrow \boxed{3x + y - z = -9}$ (Equation of the plane)

2. Show that $W = \{(x, y, -x) | x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$ under the usual addition and scalar multiplication. (3pts)

Proof: (i) $0 = -0 \Rightarrow (0, 0, 0) \in W$.

This implies, $W \neq \phi$.

(ii) Let $w_1 = (x, y, z)$, $w_2 = (a, b, c) \in W$.

Then $z = -x$ and $c = -a$.

$w_1 + w_2 = (x+a, y+b, z+c)$ and $z + c = (-x) + (-a) = -(x+a)$

This implies, $w_1 + w_2 = (x+a, y+b, -(x+a)) \in W$.

(iii) Let $w \in W$ and $\alpha \in \mathbb{R}$.

Then $w = (x, y, z)$ and $z = -x$.

$\alpha w = (\alpha x, \alpha y, \alpha z)$ and $\alpha z = \alpha(-x) = -(\alpha x)$.

$\Rightarrow \alpha w = (\alpha x, \alpha y, -(\alpha x)) \in W$.

Therefore, W is a subspace ~~over~~ of $\mathbb{R}^3$.

3. Given a system of equations

$$x + y + z = 0$$
$$2x + 3y + z = 1$$
$$x - z = 2$$

(a) Find the rank of the **augmented** matrix and the **coefficient** matrix(3pts)

(b) Solve the system by using **ONLY** Gaussian elimination method (2pts)

Solution: The system in matrix form is:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

Then the augmented matrix of the system is:

$$\left(\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 2 & 3 & 1 & 1 \\ 1 & 0 & -1 & 2 \end{array}\right) \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & -2 & 2 \end{array}\right) \xrightarrow{R_3 \to R_3 + R_2} \left(\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & 3 \end{array}\right)$$

The last matrix is in REF and the ~~xon~~ coefficient matrix is $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -3 \end{pmatrix}$ whose rank is 3

and the augmented matrix is $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -3 & 3 \end{pmatrix}$

whose rank is 3.

Therefore, the system has a unique ~~matrix~~ solution given by

$$\left.\begin{array}{r} x + y + z = 0 \\ y - z = 1 \\ -3z = 3 \end{array}\right\} \Rightarrow \begin{array}{l} z = -1 \\ y = 1 + z = -1 + 1 = 0 \\ x = 0 - (y + z) = 0 - (-1) = 1. \end{array}$$

$\therefore$ S.S. $= \{(1, 0, -1)\}$