5118014 Programming Language Theory

Ch 7. Identifiers

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AE, the Working Example

- we will use AE as a working example of our study
  - gradually extends AE by adding different programming language aspects

- proglang: `src/AE`

Identifiers: Binding, Bound and Free

```
f(0)

def f(x: Int): Int = {
  val y = 2
  x + y
}

f(1)
x - z
```

- binding occurrence
  - the identifier occurs to be defined
  - every binding occurrence has a scope
- bound occurrence
  - the identifier occurs to use the entity related to itself
- free identifiers
  - neither binding nor bound

Scope

- shadowing: innermost binding of an identifier shadows the outer binding occurrences of the same identifier.
- example

```
def f(x: Int): Int = {
  def g(x: Int): Int = x
  g(x)
}
```

VAE: Arithmetic Expr. with Immutable Variable

- add variables to AE
  - ex. `1 + 2`

```
1 + (val x=1 in (val y=x+1 in (x + y)))
```

- update syntax
  - `<expr> ::= ⋯ | “val” <id> “=“ <expr> “in” <expr>`

```
| <id>
```

VAE: Semantics (1/3)

- an environment is a map (partial function) from identifiers to values
  - $Env = Id \rightarrowtail \mathbb{Z}$
  - $\sigma \in Env$

- add environment as a factor of semantics function
  - $\Rightarrow \in Env \times E \rightarrow \mathbb{Z}$
  - $\Rightarrow \subseteq Env \times E \times \mathbb{Z}$
  - $(\sigma, e, n) \in \Rightarrow$ if and only if e evaluates to n under σ (i.e., $\sigma \vdash e \Rightarrow n$)

VAE: Semantics (2/3)

AE

$$n \Rightarrow n \quad [\textsc{Num}]$$

$$\frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow n_2}{e_1 + e_2 \Rightarrow n_1 +_z n_2} \quad [\textsc{Add}]$$

$$\frac{e_1 \Rightarrow n_1 \qquad e_2 \Rightarrow n_2}{e_1 - e_2 \Rightarrow n_1 -_z n_2} \quad [\textsc{Sub}]$$

VAE

$$\sigma \vdash n \Rightarrow n \quad [\textsc{Num}]$$

$$\frac{\sigma \vdash e_1 \Rightarrow n_1 \qquad \sigma \vdash e_2 \Rightarrow n_2}{\sigma \vdash e_1 + e_2 \Rightarrow n_1 + n_2} \quad [\textsc{Add}]$$

$$\frac{\sigma \vdash e_1 \Rightarrow n_1 \qquad \sigma \vdash e_2 \Rightarrow n_2}{\sigma \vdash e_1 - e_2 \Rightarrow n_1 - n_2} \quad [\textsc{Sub}]$$

VAE: Semantics (3/3)

<expr> ::= ”val” <id> “=“
<expr> “in” <expr>

$$\frac{\sigma \vdash e_1 \Rightarrow n_1 \qquad \sigma[x \mapsto n_1] \vdash e_2 \Rightarrow n_2}{\sigma \vdash \text{val } x{=}e_1 \text{ in } e_2 \Rightarrow n_2}$$

$$\sigma[x \mapsto n](x') = \begin{cases} n & \text{if } x = x' \\ \sigma(x') & \text{if } x \neq x' \end{cases}$$

<expr> ::= <id>

$$\frac{x \in Domain(\sigma)}{\sigma \vdash x \Rightarrow \sigma(x)}$$

Example

$$\dfrac{\emptyset \vdash 1 \Rightarrow 1 \qquad \dfrac{\dfrac{x \in Domain([x \mapsto 1])}{[x \mapsto 1] \vdash x \Rightarrow 1} \qquad \dfrac{x \in Domain([x \mapsto 1])}{[x \mapsto 1] \vdash x \Rightarrow 1}}{[x \mapsto 1] \vdash x + x \Rightarrow 2}}{\emptyset \vdash \text{val } x=1 \text{ in } x + x \Rightarrow 2}$$